75.12 ANÁLISIS NUMÉRICO I

FACULTAD DE INGENIERÍA
UNIVERSIDAD DE BUENOS AIRES

ECUACIONES DIFERENCIALES

Métodos de Discretización

Discretización de la ecuación diferencial dy/dt = f(u,t)

1. Método de Euler

$u_{n+1} = u_n + k\, f (u_n, t_n)$

2. Implícito Ponderado

$u_{n+1} = u_n + k\, [ß\, f (u_{n+1}, t_{n+1}) + (1-ß)\, f (u_n, t_n)]$

(Aunque teóricamente $0 < ß \leq 1$, su utilidad es en el rango $0.5 \leq ß \leq 1$)

3. Fuertemente Implícito o Euler Inverso

$u_{n+1} = u_n + k\, f (u_{n+1}, t_{n+1})$

4. Crank-Nicolson o Implícito Ponderado de Orden 2

$u_{n+1} = u_n + k/2\, [f (u_{n+1}, t_{n+1}) + f (u_n, t_n)]$

5. Punto Medio o Predictor-Corrector Explícito (Runge-Kutta de Orden 2)

$u_{n+1/2} = u_n + k/2\, f (u_n, t_n)$

$u_{n+1} = u_n + k\, f (u_{n+1/2}, t_{n+1/2})$

6. Heun (Runge-Kutta de Orden 2)

$u_{n+2/3} = u_n + 2/3\, k\, f (u_n, t_n)$

$u_{n+1} = u_n + k/4\, [f (u_n, t_n) + 3\, f (u_{n+2/3}, t_{n+2/3})]$

7. Euler Modificado (Runge Kutta de Orden 2)

$u_{n+1}^{*} = u_n + k\, f\,(u_n, t_n)$

$u_{n+1} = u_n + k/2\, [\, f\,(u_n, t_n) + f\,(u_{n+1}^{*}, t_{n+1})\,]$

Otra forma :

$q_1 = k\, f\,(u_n, t_n)$

$q_2 = k\, f\,(u_n + q_1, t_{n+1})$

$u_{n+1} = u_n + 1/2\,(q_1 + q_2)$

8. Predictor-Corrector Implícito

$u_{n+1}^{*} = u_n + k\, f\,(u_n, t_n)$

$u_{n+1} = u_n + k\, [\, ß\, f\,(u_{n+1}^{*}, t_{n+1}) + (1 - ß)\, f\,(u_n, t_n)\,]$

(Aunque teóricamente $0 < ß \leq 1$, su utilidad es en el rango $0.5 \leq ß \leq 1$)

9. Runge Kutta de Orden 4

$u_{n+1/2}^{*} = u_n + k/2\, f\,(u_n, t_n)$

$u_{n+1/2}^{**} = u_n + k/2\, f\,(u_{n+1/2}^{*}, t_{n+1/2})$

$u_{n+1}^{*} = u_n + k\, f\,(u_{n+1/2}^{**}, t_{n+1/2})$

$u_{n+1} = u_n + k/6\, [f\,(u_n, t_n) + 2\, f\,(u_{n+1/2}^{*}, t_{n+1/2}) + 2\, f\,(u_{n+1/2}^{**}, t_{n+1/2}) + f\,(u_{n+1}^{*}, t_{n+1})\,]$

Otra forma :

$q_1 = k\, f\,(u_n, t_n)$

$q_2 = k\, f\,(u_n + 1/2\, q_1, t_{n+1/2})$

$q_3 = k\, f\,(u_n + 1/2\, q_2, t_{n+1/2})$

$q_4 = k\, f\,(u_n + q_3, t_{n+1})$

$u_{n+1} = u_n + 1/6\,(q_1 + 2\, q_2 + 2\, q_3 + q_4)$

10. Rayuela (“Leap-frog”)

$u_{n+1} = u_{n-1} + 2 k f (u_n, t_n)$

11. Adams-Bashforth

O(1) $u_{n+1} = u_n + k f_n$

O(2) $u_{n+1} = u_n + k/2 (3 f_n - f_{n-1})$

O(3) $u_{n+1} = u_n + k/12 (23 f_n - 16 f_{n-1} + 5 f_{n-2})$

O(4) $u_{n+1} = u_n + k/24 (55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$

12. Adams-Moulton

O(1) $u_{n+1} = u_n + k f_{n+1}$

O(2) $u_{n+1} = u_n + k/2 (f_{n+1} + f_n)$

O(3) $u_{n+1} = u_n + k/12 (5 f_{n+1} + 8 f_n - f_{n-1})$

O(4) $u_{n+1} = u_n + k/24 (9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2})$

13. Predictor-Corrector de Milne

$u_{n+1} = u_{n-3} + 4/3 k (2 f_n - f_{n-1} + 2 f_{n-2})$

$u_{n+1} = u_{n-1} + 1/3 k (f_{n+1} + 4 f_n + f_{n-1})$