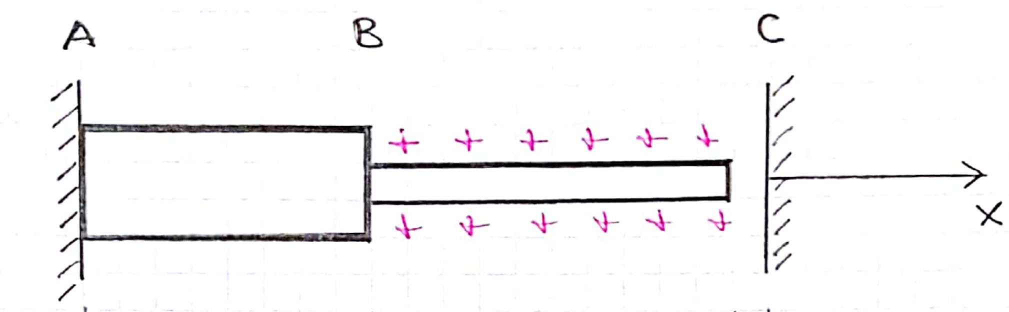
DATOS :



$L_1 = 0{,}80 m$	$L_2 = 1{,}00 m$ — 0,5 mm
$E_1 = 20000 kN/cm^2$	$E_2 = 20000 kN/cm^2$
$A_1 = 10 cm^2$	$A_2 = 5 cm^2$
$\alpha_1 = 1.10^{-5} 1/°C$	$\alpha_2 = 1.10^{-5} 1/°C$
$\sigma_{F1} = 30 kN/cm^2$	$\sigma_{F2} = 30 kN/cm^2$
$CS_1 = 1{,}60$	$CS_2 = 1{,}60.$
$\Delta T_1 = 0{,}0°C$	$\Delta T_2 = 180°C$
ACERO	ACERO

02.01 -

- CONSIDERANDO LA BARRA (2) COMO LIBREMENTE DILATABLE, SE TENDRÁ QUE:

$$\Delta L_2 = \alpha_2 . \Delta T_2 . L_{o2} = 1.10^{-5} \frac{1}{°C} . 180°C . 1000 mm$$

$\Delta L_2 = 1{,}80 mm$ SE PASA EL ERROR DE MONTADO.

SE CALCULA AHORA LA ΔT PARA $\Delta L_2 = 0{,}5 mm$

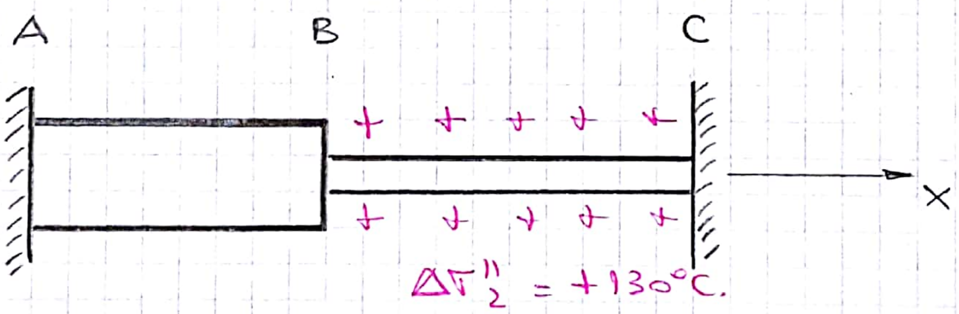
$$\Delta T_2' = \frac{\Delta L_2}{\alpha \cdot L_{02}} = \frac{0{,}5\,mm}{1 \cdot 10^{-5}\,\frac{1}{°C} \cdot 1000\,mm} = 50°C.$$

Es decir, se necesitan $\Delta T_2' = 50°C$ para que se alcance $\Delta L_2 = 0{,}5\,mm$.

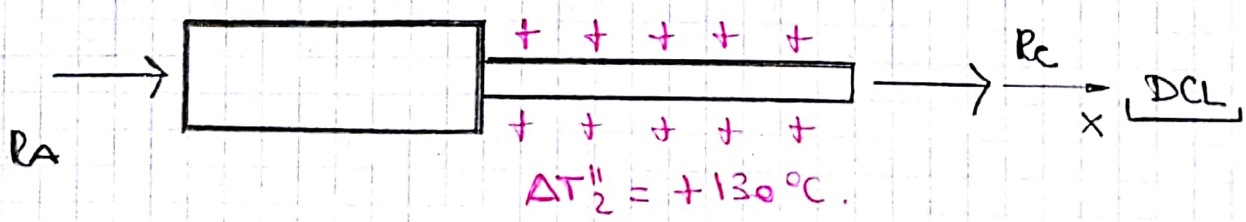
Por lo tanto, resta un $\Delta T_2'' = 180°C - 50°C = 130°C$

Durante los $\Delta T_2' = 50°C$ la barra (2) se dilata libremente y no se generan esfuerzos ni tensiones.

- Se analiza a continuación que sucede cuando se considera $\Delta T_2'' = +130°C$.



- Se plantea el DCL (diagrama de cuerpo libre):



- EC. DE EQUILIBRIO:

$$\Sigma F_x = 0 \qquad R_A + R_C = 0 \rightarrow R_C = -R_A.$$

- EC. DE COMPATIBILIDAD:

$$\Delta L_1 + \Delta L_2 = 0$$

$$\Delta L_1 = \frac{N_1 L_1}{E_1 A_1} = \frac{(-R_A) L_1}{E_1 A_1} \qquad N_1 = -R_A$$

$$\Delta L_2 = \Delta L_{2,N_2} + \Delta L_{2,\Delta T''_2}$$

$$\Delta L_{2,N_2} = \frac{N_2 L_2}{E_2 A_2} = \frac{(-R_A) L_2}{E_2 A_2} \qquad N_2 = -R_A.$$

$$\Delta L_{2,\Delta T''_2} = \alpha_2 . \Delta T''_2 . L_2$$

REUNIENDO TODO EN LA 1° EXPRESIÓN SE TIENE:

$$\underbrace{\frac{(-R_A) L_1}{E_1 A_1}}_{\Delta L_1} + \underbrace{\frac{(-R_A) L_2}{E_2 A_2} + \alpha_2 . \Delta T''_2 . L_2}_{\Delta L_2} = 0.$$

$$\alpha_2 \Delta T''_2 L_2 = R_A \left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right)$$

$$R_A = \frac{\alpha_2 \Delta T''_2 L_2}{\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2}} = \frac{1.10^{-5}\ 1/°C .\ 130°C .\ 1000\ cm}{\frac{80}{20000 . 10} + \frac{100}{20000 . 5}}$$

$$R_A = \frac{0,13}{4.10^{-4} + 1.10^{-3}} \rightarrow \underline{\underline{R_A = +92,86\,kN}} \longrightarrow$$

- DE LA EC. DE EQUILIBRIO:

$$R_C = -R_A \rightarrow \underline{R_C = -92{,}86\ \text{KN}} \leftarrow$$

- CÁLCULO DE TENSIONES:

↳ BARRA (1):

$$N_1 = -R_A = -92{,}86\ \text{KN}.$$

$$\sigma_1 = -\frac{92{,}86\ \text{KN}}{10\ \text{cm}^2} \rightarrow \underline{\sigma_1 = -9{,}286\ \text{KN/cm}^2}$$

↳ BARRA (2):

$$N_2 = -R_A = -92{,}86\ \text{KN}.$$

$$\sigma_2 = -\frac{92{,}86\ \text{KN}}{5\ \text{cm}^2} \rightarrow \underline{\sigma_2 = -18{,}572\ \text{KN/cm}^2}$$

- VERIFICACIÓN:

$$\sigma_{F1} = \sigma_{F2} = 30\ \text{KN/cm}^2$$

$$CS_1 = CS_2 = 1{,}60.$$

$$\sigma_{ADM1} = \sigma_{ADM2} = \frac{\sigma_{F1}}{CS_1} = \frac{\sigma_{F2}}{CS_2} = \frac{30}{1{,}60} = 18{,}75\ \text{KN/cm}^2$$

$$\sigma_{MÁX} = \sigma_{MÁX}\big|_2 = +18{,}572 < 18{,}75 = \sigma_{ADM}.$$

AMBAS BARRAS VERIFICAN.

- **CÁLCULO DE DESPLAZAMIENTOS:**

↳ SON 2 ETAPAS DE DEFORMACIÓN.

↳ EN LA 1º ETAPA, SÓLO EN LA BARRA ② HAY DESPLAZAMIENTOS Y DEFORMACIONES.

↳ EN LA 2º ETAPAS, AMBAS BARRAS SE DESPLAZAN Y DEFORMAN.

1º ETAPA:

$$\delta'_2(x) = \alpha_2 \, \Delta T'_2 \, . \, x = 1.10^{-5} \frac{1}{°C} . (+50°C) . x .$$

$$\delta'_2(x) = 5.10^{-4} x .$$

$$\delta'_2(x = L_2 = 1000\,mm) = \underline{0{,}5\,mm} .$$

2º ETAPA:

$$\delta''_1(x) = \frac{N_1}{E_1 A_1} . x = \frac{-92.86\,kN}{20000 \frac{kN}{cm^2} . 10\,cm^2} . x = -4{,}643.10^{-4} x .$$

$$\delta''_1(x = 800\,mm = L_1) = -0{,}37144\,mm$$

$$\delta''_2 = \delta''_1(L_1) + \frac{N_2 (x - L_1)}{E_2 A_2} + \alpha_2 . \Delta T''_2 . (x - L_1)$$

$$\delta''_2(x) = -0{,}37144\,mm + \frac{(-92{,}86)}{20000 . 5} . (x - 800) + 1.10^{-5} \frac{1}{°C} . 130°C (x - 800)$$

$$\delta''_2(x) = -0{,}37144 - 9{,}286.10^{-4} (x - 800) + 1{,}3.10^{-3} (x - 800)$$

$$\delta''_2 (x = L_1 + L_2 = 1800\,mm) =$$

$$= -0{,}37144 - 9{,}286.10^{-4}(1800-800) + 1{,}3.10^{-3}(1800-800)$$

$$= -0{,}37144 - 0{,}9286 + 1{,}30$$

$$\underline{\delta''_2 (L_1 + L_2)} = -1{,}30 + 1{,}30 = \underline{0{,}00}$$

LO CUAL ES CORRECTO PORQUE NO SE PUEDE DEFORMAR MÁS ALLÁ DEL EXTREMO 'C'.

- **CÁLCULO DE DEFORMACIONES:**

$$\varepsilon(x) = \frac{d\delta(x)}{dx} \; ; \quad \varepsilon(x) = \varepsilon(x)\big]_N + \varepsilon(x)\big]_{\Delta T} \; ;$$

$$\varepsilon(x)\big]_N = \frac{d\delta(x)_{,N}}{dx} \; ; \quad \varepsilon(x)\big]_{\Delta T} = \frac{d\delta(x)_{.\Delta T}}{dx}$$

$$\varepsilon(x)\big]_{\Delta T} = \alpha \Delta T$$

1º ETAPA:

$$\varepsilon'_1(x) = 0{,}00.$$

$$\varepsilon'_2(x) = 5.10^{-4}$$

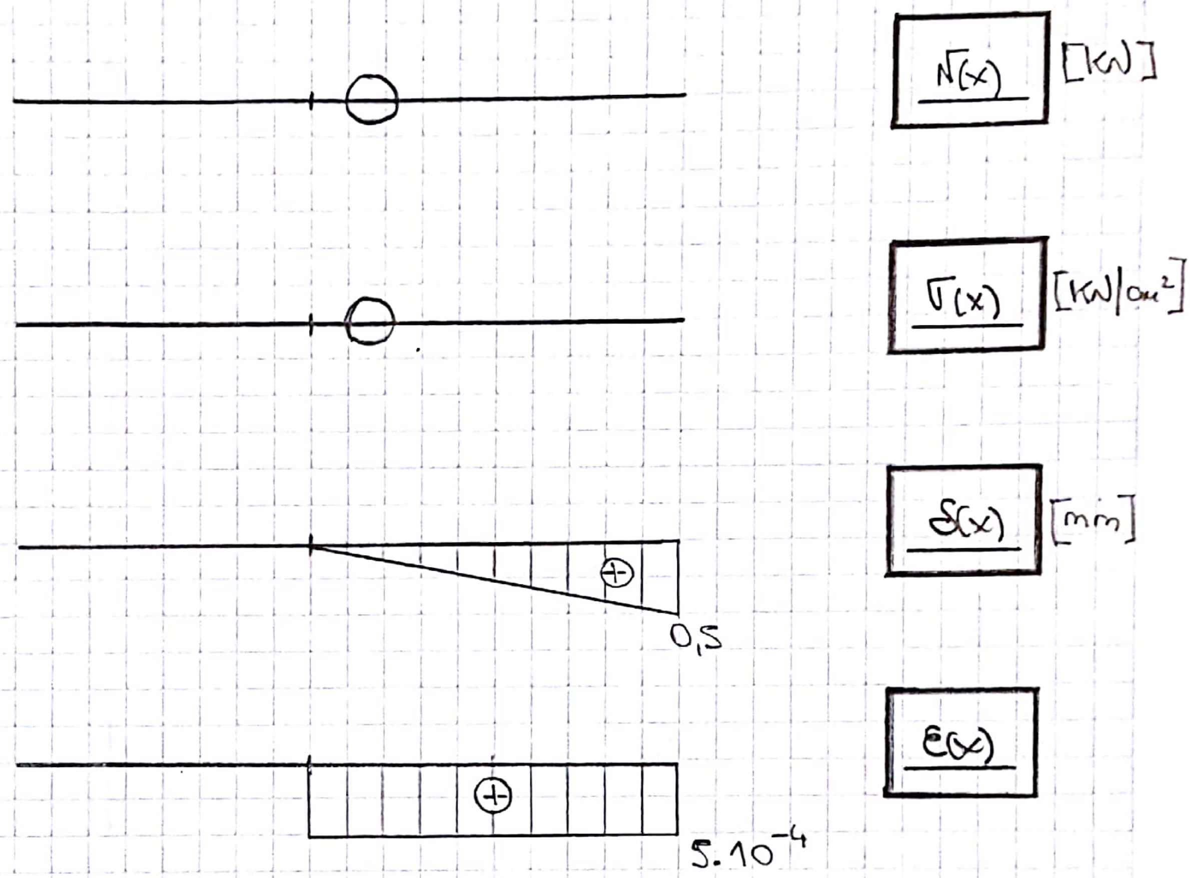
2º ETAPA:

$$\varepsilon''_1(x) = -4{,}643.10^{-4}$$

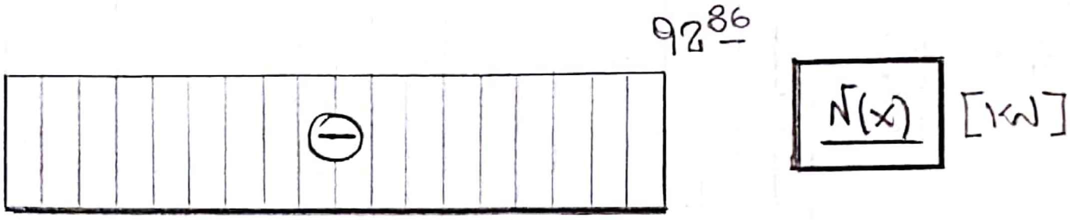
$$\varepsilon''_2(x) = -9{,}286.10^{-4} + 1{,}3.10^{-3} = 3{,}714.10^{-4}$$

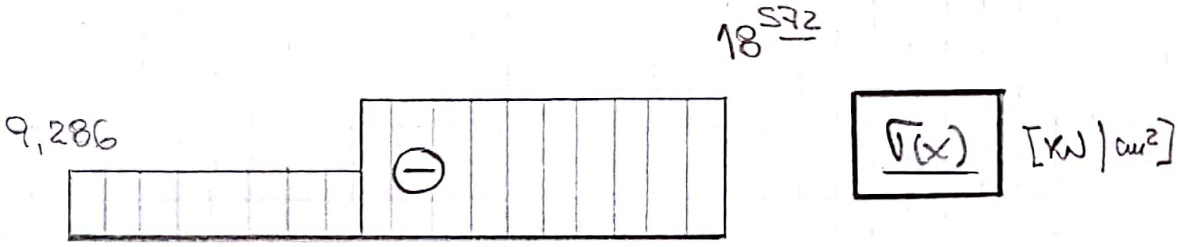
NOTA

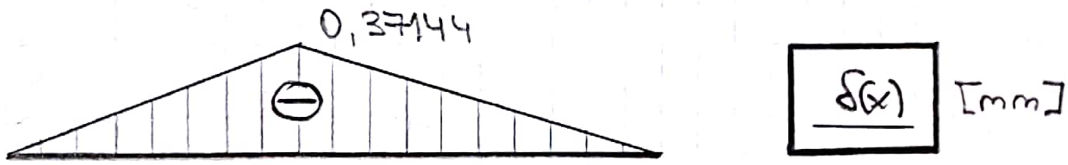
1º ETAPA:

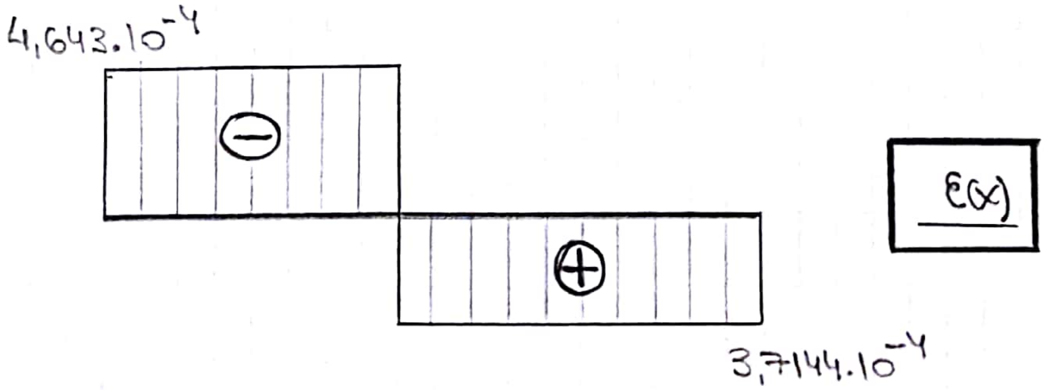


2º ETAPA :


92,86
N(x)
[kN]


18,572
9,286
σ(x)
[kN/cm²]


0,37144
δ(x)
[mm]


4,643.10⁻⁴
ε(x)
3,7144.10⁻⁴

TOTAL : RESUMEN AMBAS ETAPAS :