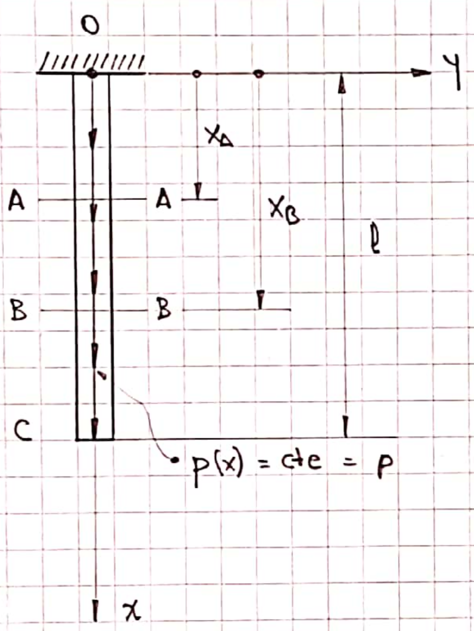
EJERCICIO N° 2:



c) Reacciones de Vínculo Externo:

c.1). Ecuaciones de Equilibrio de Proyección:

$\Sigma F_x = 0 \qquad p.\ell + R^O_x = 0 \qquad (1)$

$\Sigma F_y = 0 \qquad R^O_y = 0 \qquad (2)$

$\Sigma F_z = 0 \qquad R^O_z = 0 \qquad (3)$

c.2). Ecuaciones de Equilibrio de Momentos:

$\Sigma M_x = 0 \qquad M^O_{Rx} = 0 \qquad (4)$

$\Sigma M_y = 0 \qquad M^O_{Ry} = 0 \qquad (5)$

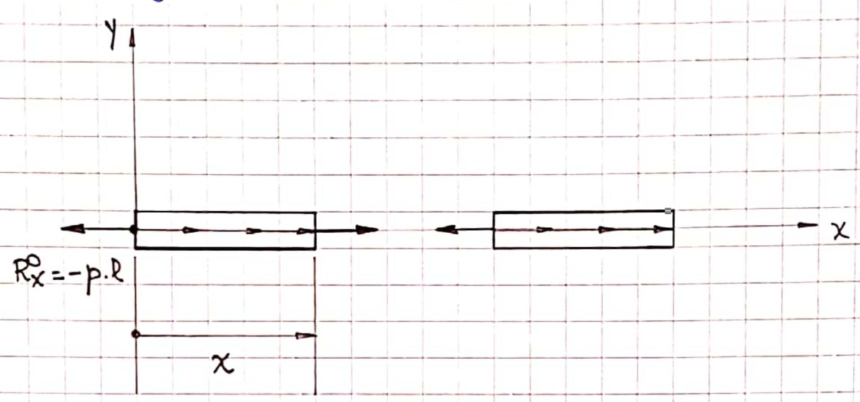
$\Sigma M_z = 0 \qquad M^O_{Rz} = 0. \qquad (6)$

$$R^O_x = - p.\ell$$

d) Trazado del Diagrama de Cuerpo Libre "DCL":

(ver esquemas).

e) Trazado de Diagramas de Características: Diagrama de Esfuerzos Normales a lo largo de la barra "$N(x)$":



$$N(x) = N(0 \le x \le \ell) = -\left(R_x^o + p.x\right) = -\left(-p.\ell + p.x\right) = p\ell - px$$

$$N(x) = p(\ell - x) \longrightarrow \text{lineal}$$

$$N(x=0) = p.(\ell - 0) = p.\ell$$

$$N(x=\ell) = p.(\ell - \ell) = 0$$

f) Trazado del Diagrama de Tensiones Normales Máximas a lo largo de la Barra:

$$N(x) = p.(\ell - x).$$

$$F(x) = \frac{\pi D^2}{4} = cte \quad \text{sección circular.}$$

$$\sigma(x) = \frac{N(x)}{F(x)} = \underbrace{\frac{p.(\ell - x)}{F}}_{\text{válida p/ cualquier sección}} = \underbrace{\frac{4p}{\pi.D^2}(\ell - x)}_{\text{válido p/ sección circular.}} \longrightarrow \text{lineal.}$$

g) Diagrama de Tensiones Normales en la sección A-A (Volumen de Tensiones):

idem Ejercicio N° 1.

h) Trazado del Diagrama de Desplazamientos:

$$\delta(x) = \int \frac{N(x)}{F(x).E(x)} \, dx$$

como:

$$N(x) = p.(l - x)$$

$$F(x) = F = \pi D^2 / 4 = cte$$

$$E(x) = E = cte$$

$$\delta(x) = \int \frac{p.(l-x)}{F.E} \, dx = \frac{p}{FE} \int (l-x)\, dx = \frac{p}{FE} . \left(lx - \frac{x^2}{2} \right)$$

$$\delta(x) = \underbrace{\frac{p}{F.E} \left(lx - \frac{x^2}{2} \right)}_{\text{válida p/ cualquier sección}} = \underbrace{\frac{4p}{\pi D^2 E} \left(lx - \frac{x^2}{2} \right)}_{\text{válida para sección circular.}} \longrightarrow \text{ec. de 2º grado}$$

$$\delta(x = 0) = 0$$

$$\delta(x = l) = \frac{p l^2}{2 F E} = \frac{2 p l^2}{\pi D^2 E}$$

i) Cálculo de Desplazamientos Absolutos y Relativos:

$$\delta(x = x_A) = \frac{p}{FE} . \left(l . x_A - \frac{x_A^2}{2} \right) = \frac{4p}{\pi D^2 E} \left(l x_A - \frac{x_A^2}{2} \right)$$

$$\delta(x = x_B) = \frac{p}{FE} . \left(l . x_B - \frac{x_B^2}{2} \right) = \frac{4p}{\pi D^2 E} . \left(l . x_B - \frac{x_B^2}{2} \right)$$

$$\delta_{BA} = \frac{p}{FE} . \left[l (x_B - x_A) - \frac{x_B^2}{2} + \frac{x_A^2}{2} \right]$$

j) Trazado del Diagrama de Deformaciones Específicas:

$$\varepsilon(x) = \frac{d\delta(x)}{dx} = \delta'(x) = \frac{p}{FE}(\ell - x) = \frac{4p}{\pi D^2 E}\cdot(\ell - x)$$

verificando de otra manera, nos queda:

$$\sigma(x) = E(x)\cdot\varepsilon(x) \longrightarrow \varepsilon(x) = \frac{\sigma(x)}{E(x)} = \frac{\sigma(x)}{E}$$

$$\varepsilon(x) = \frac{p}{FE}\cdot(\ell - x) = \frac{4p}{\pi D^2 E}(\ell - x)$$

k) Ejemplo con Valores:

Ej.: $NP = 4$

$p = (10000 + 4 \times 1000)$ kg/m $=$ 14.000 kg/m.

$A = 30\ cm^2$.

$L = 2{,}00$ m. ; $x_A = 0.85$ m ; $x_B = 1.35$ m.

Luego:

$R^o_x = -14000$ kg/m $\cdot$ 2,00 m $= -28\,000$ kg.

$N(x) = 14\,000$ kg/m $\cdot$ $(2{,}00\,m - x)$

$N(x=0) = 28000$ kg $\qquad N(x_A) = 14000\ \frac{kg}{m}\,(2 - 0.85)$ kg/m $= 16.100$ kg

$N(x=2) = 0$ kg.

$$\sigma(x) = \frac{14000\ kg/m}{30\ cm^2}\cdot(2{,}00\,m - x)$$

$\sigma(x=0) = 933{,}3$ kg/cm^2. $\qquad \sigma(x_A) = \frac{14000\ kg/m}{30\ cm^2}\cdot(2{,}0 - 0.85)\,m =$

$\sigma(x=2) = 0$ kg/cm^2. $\qquad \sigma_{xA} = 536{,}67$ kg/cm^2.

$$\delta(x) = \frac{14000\ kg/m}{30\ cm^2\cdot 2.1E6\ \frac{kg}{cm^2}}\cdot\left(2.x - \frac{x^2}{2}\right)m^2 =$$

$\delta(x=0) = 0$

$\delta(x=2) = 4.44\cdot 10^{-4}$ m $= 0.444$ mm.

$\delta(x = x_A) = 0.2975\ mm.$

$\delta(x = x_B) = 0.3975\ mm.$

$\delta_{BA} = 0.3975\ mm - 0.2975\ mm = 0.100\ mm. > 0$

los puntos se alejan

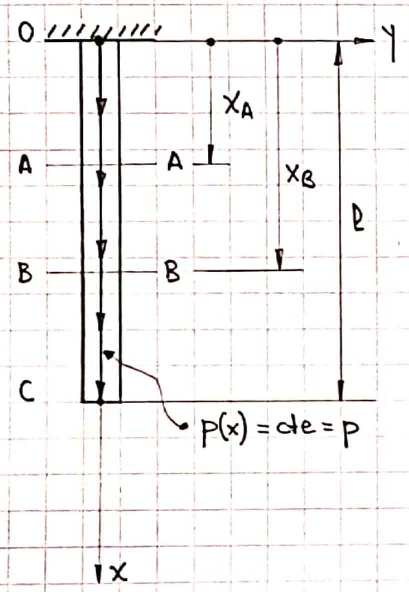
$$\varepsilon(x) = \frac{14000\ kg/m.}{30\ cm^2 \cdot 2.1E6\ \frac{kg}{cm^2}} \cdot (2 - x)\ m$$

$\varepsilon(x=0) = 4.44 \cdot 10^{-4}$

$\varepsilon(x=2) = 0$

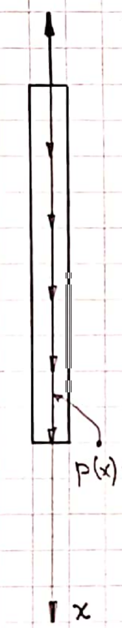
$$\varepsilon(x_A) = \frac{14000\ kg/m.}{30\ cm^2 \cdot 2.1E6\ \frac{kg}{cm^2}} \cdot (2 - 0.85)\ m = 2.56 \cdot 10^{-4} = 0.256\ \%$$

$$\varepsilon(x_B) = \frac{14000\ kg/m.}{30\ cm^2 \cdot 2.1E6\ \frac{kg}{cm^2}} \cdot (2 - 1.15)\ m = 1.89 \cdot 10^{-4} = 0.0189\ \%$$



DCL

$R_x^o = -p \cdot \ell$



$N(x)$

$N(x=0) = p \cdot \ell$

$N(x=\ell) = 0$

$N(x) = p \cdot (\ell - x)$

$\sigma(x)$

$\sigma(x=0) = \dfrac{4p\ell}{\pi D^2}$

$\sigma(x=\ell) = 0$

$\sigma(x) = \dfrac{p}{F} \cdot (\ell - x)$

$\sigma(x) = \dfrac{4p}{\pi D^2} (\ell - x)$

$\delta(x)$

$\delta(x=0) = 0$

$\delta(x=\ell) = \dfrac{2p\ell^2}{\pi D^2 E}$

$\delta(x) = \dfrac{p\ell^2}{2FE}$

$\delta(x) = \dfrac{2p\ell^2}{\pi D^2 E}$

$\varepsilon(x)$

$\varepsilon(x=0) = \dfrac{4p}{\pi D^2 E} (\ell - x)$

$\varepsilon(x=\ell) = 0$

$\varepsilon(x) = \dfrac{p}{FE} (\ell - x)$

$\varepsilon(x) = \dfrac{4p}{\pi D^2 E} (\ell - x)$