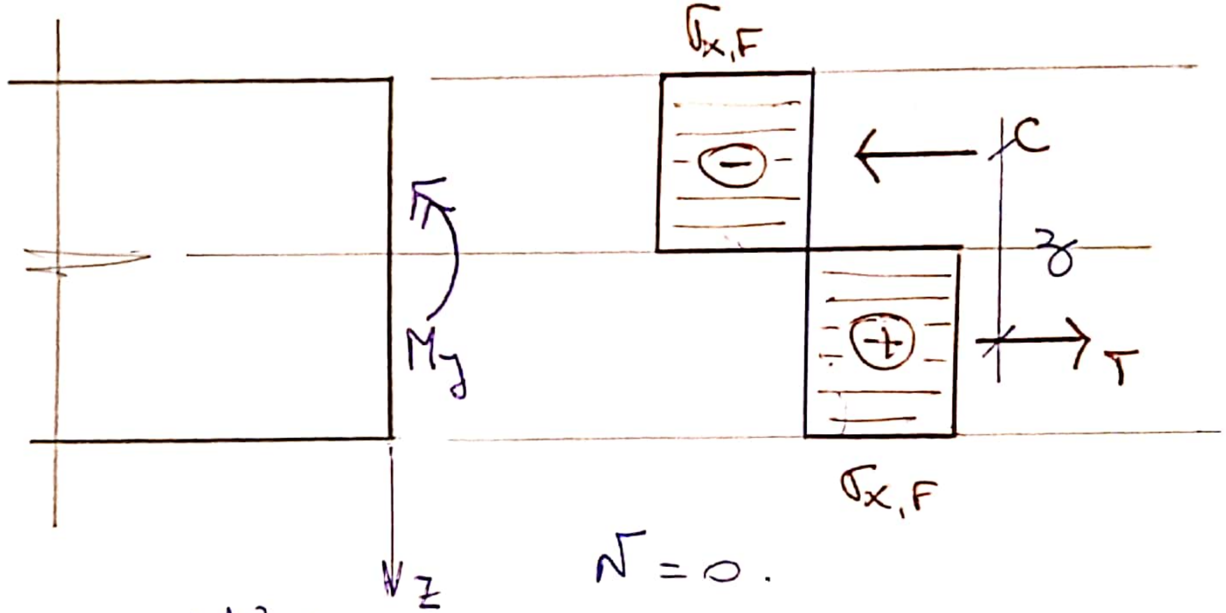
SECCIÓN TOTALMENTE PLASTIFICADA:

$$M_y = bh^2 \sigma_{x,F} - \frac{1}{3} b z_E^2 \sigma_{x,F}$$

$$M_y = bh^2 \sigma_{x,F} \left[1 - \frac{1}{3} \frac{z_E^2}{h^2} \right].$$

$$M_y = \frac{3}{2} M_E \left[1 - \frac{1}{3} \frac{z_E^2}{h^2} \right]$$



$N = 0.$

$T + C = 0.$

$$M_P = T \cdot z = C \cdot z = \underbrace{\sigma_{x,F} \cdot b \cdot h}_{T=C} \cdot \underbrace{h}_{z} = bh^2 \sigma_{x,F}.$$

$$T = \sigma_{x,F} \cdot b\,h = C$$

$$z = 2 \cdot \frac{h}{2} = h.$$

$$M_y = M_P = bh^2 \sigma_{x,F} = \frac{3}{2} M_E \left[1 - \frac{1}{3} \frac{0}{h^2} \right] = \frac{3}{2} M_E.$$

$$\rightarrow \boxed{M_P = \frac{3}{2} M_E}$$

- $\dfrac{M_P}{M_E} = \dfrac{3}{2} = k =$ FACTOR DE FORMA.

$$\boxed{M_P = k\, M_E.}$$

- $\dfrac{M_P}{\sigma_{x,F}} = \dfrac{b h^2 \cancel{\sigma_{x,F}}}{\cancel{\sigma_{x,F}}} = b h^2 = Z$

Z: MÓDULO PLÁSTICO DE LA SECCIÓN.

- M_P: → MOMENTO DE PLASTIFICACIÓN TOTAL.
  → MOMENTO ÚLTIMO O DE COLAPSO DE LA SECCIÓN.

- $$\boxed{M_P = Z \cdot \sigma_{x,F}}$$

RECTANGULAR $k = 1{,}50$

CUADRADA $k = 2{,}00$

I $k \to 1{,}08$ y $1{,}14$.

- $M_E = \dfrac{2}{3} b h^2 \sigma_{x,F}$.

$$\frac{M_E}{\sigma_{x,F}} = \frac{\frac{2}{3} b h^2 \cancel{\sigma_{x,F}}}{\cancel{\sigma_{x,F}}} = \frac{2}{3} b h^2 = S$$

S: MÓDULO ELÁSTICO DE LA SECCIÓN.

→ M_{ADM}
→ M_E
→ M_P

$$I_y = \frac{b(2h)^3}{12} = \frac{8}{12} b h^3 = \frac{2}{3} b h^3.$$

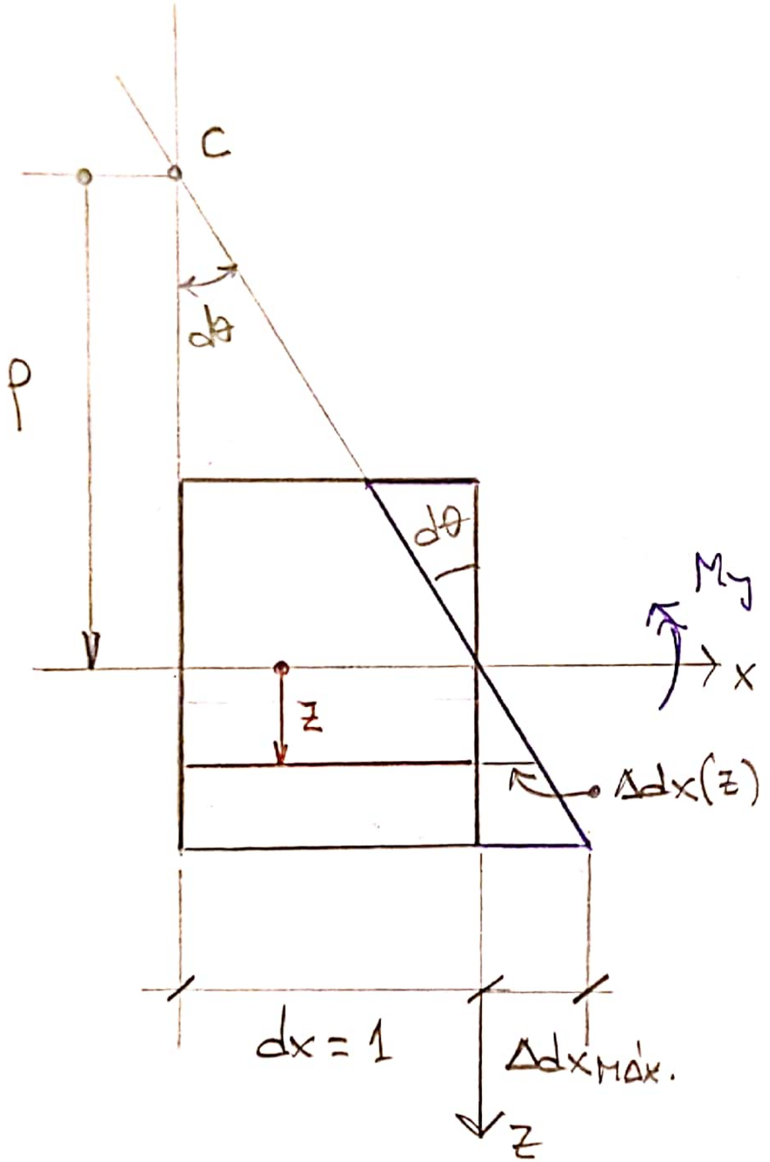
$$z_{máx} = h$$

$$W_y = S_y = \frac{I_y}{z_{máx}} = \frac{2}{3} b h^2$$

$$\frac{Z}{S} = \frac{bh^2}{\frac{2}{3} b\cancel{h^2}} = \frac{3}{2} = k.$$

$$k = \frac{M_P}{M_E} = \frac{Z}{S}$$

- DEFORMACIONES:



- si $dx = 1 \rightarrow \frac{d\theta}{\underbrace{dx}_{=1}} = \chi$ = CURVATURA DE FLEXIÓN.

$$\boxed{d\theta = \chi}$$

- $\varepsilon_x = \frac{\Delta dx}{\underbrace{dx}_{=1}} \rightarrow \boxed{\Delta dx = \varepsilon_x.}$

- $\Delta dx_{,z} = z \cdot d\theta \rightarrow \boxed{\varepsilon_x = z \cdot \chi}$

- $\rho \cdot d\theta = dx \rightarrow \rho \cdot \chi = 1 \rightarrow$

$$\boxed{\chi = \frac{1}{\rho} \quad ; \quad \rho = \frac{1}{\chi}}$$

- $\boxed{\varepsilon_x = z \cdot \frac{1}{\rho}}$

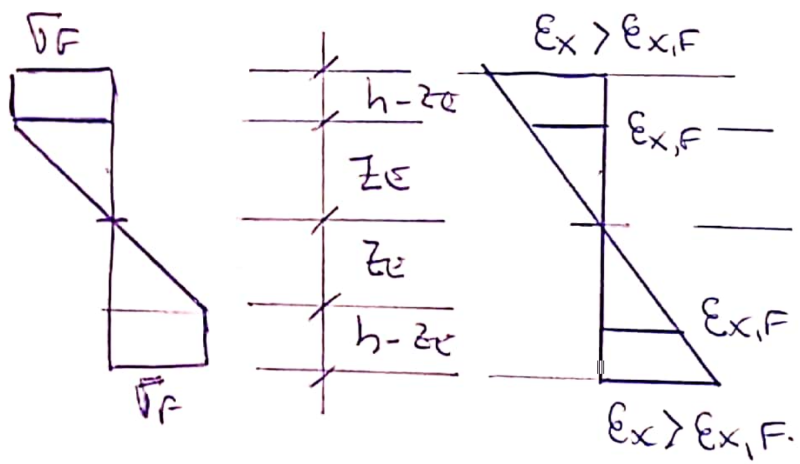
- si $z_E = h \rightarrow \varepsilon_{x,F} = h \cdot \chi_F \rightarrow \boxed{\chi_F = \frac{\varepsilon_{xF}}{h}}$

$\hookrightarrow \varepsilon_{x,F} = h \frac{1}{\rho_F} \rightarrow \boxed{\rho_F = \frac{h}{\varepsilon_{x,F}}}$

$$\rightarrow \text{A} \begin{cases} h = \dfrac{\varepsilon_{x,F}}{x_F} \\[2ex] h = \rho_F \, \varepsilon_{x,F} \end{cases}$$

- Si $z_E < h$:

$$z = z_E \rightarrow \varepsilon_x = \varepsilon_{x,F}$$



$$z_E \cdot x = \varepsilon_{x,F}. \longrightarrow x = \frac{\varepsilon_{x,F}}{z_E} \rightarrow \boxed{z_E = \frac{\varepsilon_{x,F}.}{x}} \ \text{B}$$

$$z_E \cdot \frac{1}{\rho} = \varepsilon_{x,F} \longrightarrow \rho = \frac{z_E}{\varepsilon_{x,F}} \rightarrow \boxed{z_E = \rho \cdot \varepsilon_{x,F}}$$

$$\frac{\text{B}}{\text{A}} \rightarrow \frac{z_E}{h} = \frac{\cancel{\varepsilon_{x,F}}}{x} \cdot \frac{x_F}{\cancel{\varepsilon_{x,F}}} = \boxed{\frac{x_F}{x}}$$

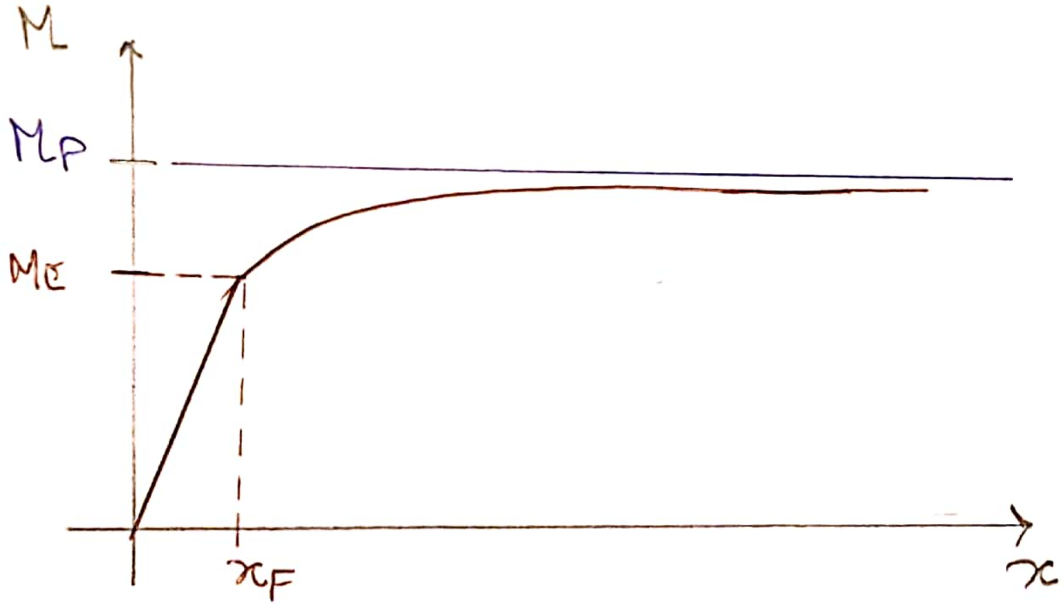
$$\rightarrow \frac{z_E}{h} = \frac{\rho \cdot \cancel{\varepsilon_{x,F}}}{\rho_F \cdot \cancel{\varepsilon_{x,F}}} = \boxed{\frac{\rho}{\rho_F}}$$

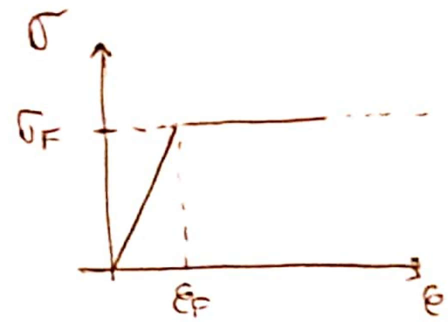
REEMPLAZO EN LAS PRIMERAS 3 EXPRESIONES DE LA HOJA 1 :

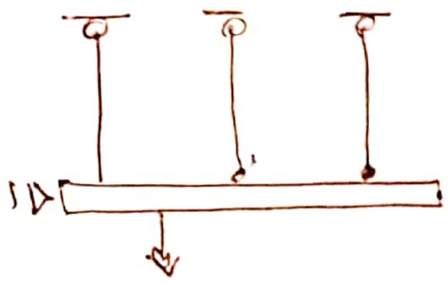
$$\boxed{\begin{aligned} M_y &= \frac{3}{2} M_e \left[1 - \frac{1}{3} \frac{x_F^2}{x^2} \right] \\ M_y &= \frac{3}{2} M_e \left[1 - \frac{1}{3} \frac{\rho^2}{\rho_F^2} \right] \end{aligned}}$$

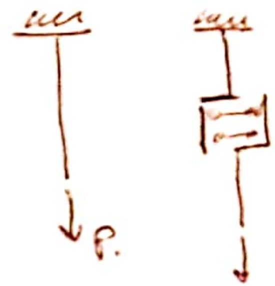
I) Si $z_E = h \rightarrow \chi = \chi_F \wedge \rho = \rho_F$
(MOMENTO ELÁSTICO).

II) Si $z_E = 0 \rightarrow \chi \rightarrow \infty \wedge \rho \rightarrow 0$.
(MOMENTO PLÁSTICO).









ESQUEMA SERIE.

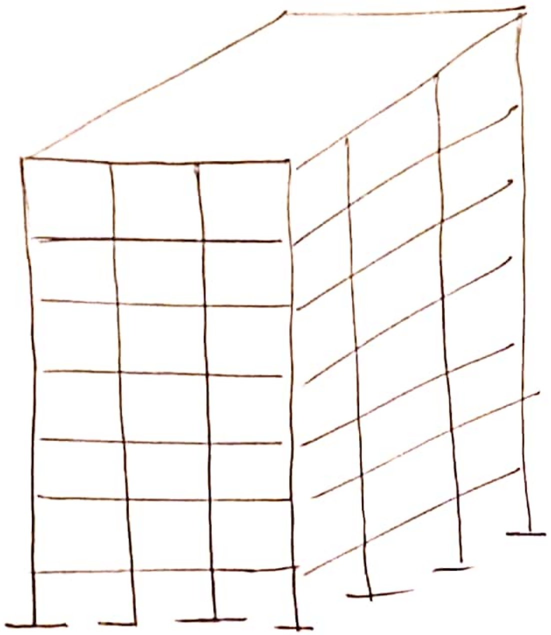
• Si $M \leq M_E \rightarrow \chi = \dfrac{M_y}{EI}$. $\dfrac{1}{\rho} = \dfrac{M_y}{EI}$.

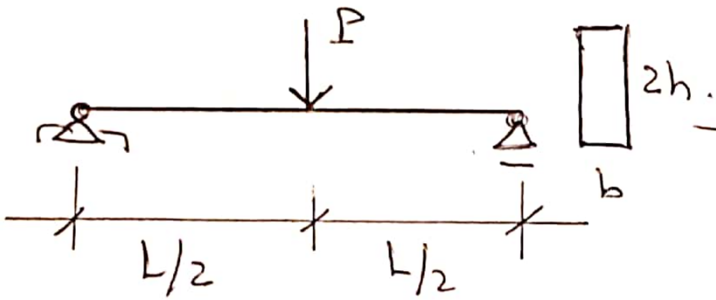
$M_y = EI\chi$.

RÓTULA O ARTICULACIÓN PLÁSTICA.

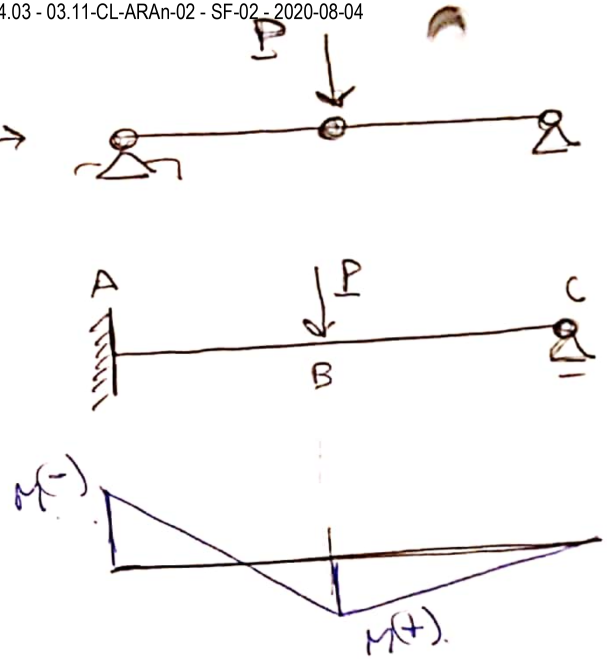
$\rightarrow$ SE FORMA CUANDO

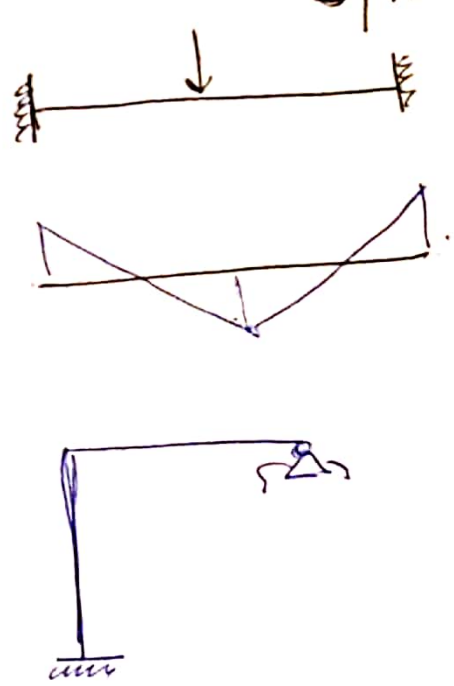
$M = M_P$.



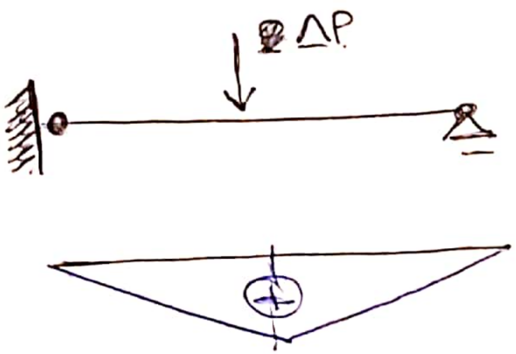


P crece $\rightarrow M_{y,adm} \rightarrow M_{y,e} \rightarrow M_{y,p}$



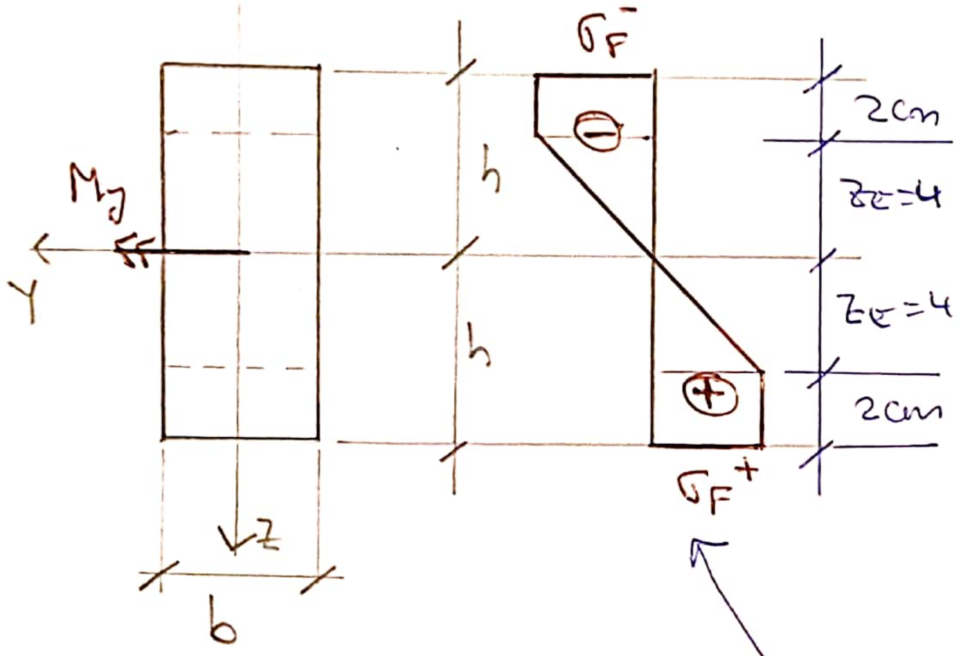


$M_A = M_p$.



$P_c = P_u = P_j + \Delta P$

EJEMPLO:



DATOS:

$b = 50\,mm.$

$h = 60\,mm$

$H = 120\,mm$

$\sigma_F^+ = \sigma_F^- = 24\,kN/cm^2$

$E = 20000\,kN/cm^2.$

$M_y = 36{,}8\,kNm.$

- $I_y = \dfrac{b(2h)^3}{12} = \dfrac{2}{3} bh^3 \qquad z_{máx} = h.$

$\dfrac{I_y}{z_{máx}} = \dfrac{2}{3} bh^2 = \dfrac{2}{3}\, 5 \cdot 6^2 = 120\,cm^3.$

- $M_E = \dfrac{I_y}{z_{máx}} \cdot \sigma_F = 120\,cm^3 \cdot 24\,\dfrac{kN}{cm^2}$

$M_E = 2880\,kNcm = 28{,}80\,kNm$

- $M_P = \dfrac{3}{2} M_E = 1{,}50 \cdot 28{,}80 = 43{,}20\,kNm.$

$$\boxed{M_E < M_y < M_P.}$$

$$M_y = \frac{3}{2} M_E \left[1 - \frac{1}{3}\frac{z_e^2}{h^2}\right]$$

$$\frac{2}{3}\frac{M_y}{M_E} = 1 - \frac{1}{3}\frac{z_e^2}{h^2}$$

$$\frac{1}{3}\frac{z_e^2}{h^2} = 1 - \frac{2}{3}\frac{M_y}{M_E}$$

$$Z_E = \sqrt{\left(1 - \frac{2}{3}\frac{M_y}{M_E}\right) 3h^2}$$

$$Z_E = \sqrt{\left(1 - \frac{2}{3}\frac{36{,}8}{28{,}8}\right) \cdot 3 \cdot 6^2}$$

$$\boxed{Z_E = 4 \text{ cm.}}$$

CURVATURA Y RADIO DE CURVATURA:

$$\frac{Z_E}{h} = \frac{\chi_F}{\chi} \longrightarrow \boxed{\chi = \chi_F \frac{h}{Z_E}}$$

$h = 6 \text{ cm}$

$Z_E = 4 \text{ cm.}$

$E = 20000 \text{ kN/cm}^2$

$I_y = \frac{2}{3} b h^3 = \frac{2}{3} \cdot 5 \cdot 6^3$

$I_y = 720 \text{ cm}^4.$

$$\chi_F = \frac{M_E}{E I_y}$$

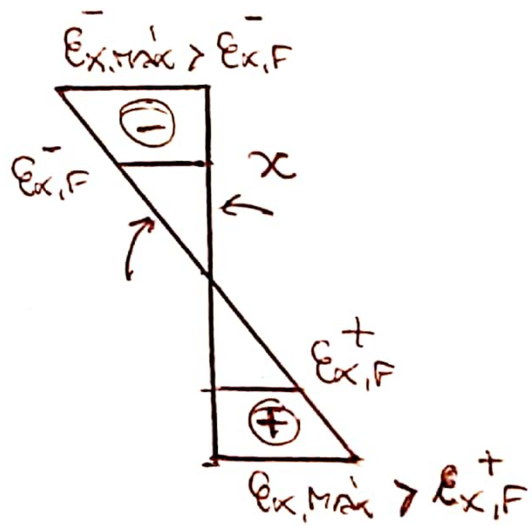
$$\chi_F = \frac{M_E}{E I_y} = \frac{2880 \text{ kN cm}}{20000 \frac{\text{kN}}{\text{cm}^2} \cdot 720 \text{ cm}^4}$$

$$\boxed{\chi_F = 2 \cdot 10^{-4} \frac{1}{\text{cm.}}}$$

$$\chi = 2 \cdot 10^{-4} \cdot \frac{6}{4} = 3 \cdot 10^{-4} \frac{1}{\text{cm.}}$$

$$\rho_F = \frac{1}{\chi_F} = 5000 \text{ cm.}$$

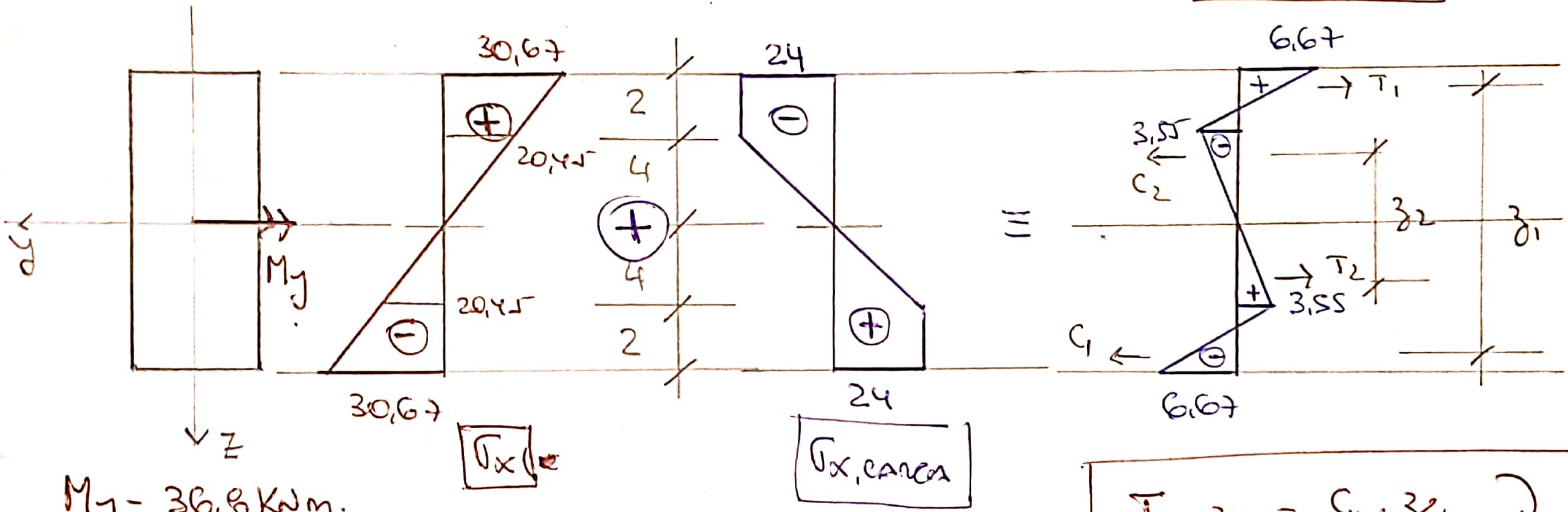
$$\rho = \frac{1}{\chi} = 3333^{\overline{33}} \text{ cm.}$$



DESCARGA:

TENSIONES CARGA

TENSIONES RESIDUALES



$M_y = 36{,}8\ \text{KNm}.$

$$\sigma_{x,DESC} = \frac{M_y}{I_y} \cdot z_{máx} = -\frac{3680\ \text{KNcm} \cdot 6\text{cm}}{720\ \text{cm}^4}.$$

$$\sigma_{x,DESC} = 30{,}67\ \text{KN/cm}^2.$$

$$\frac{30{,}67}{6} = \frac{\sigma_{x,z=4}}{4}$$

$$\sigma_{x,z=4} = 20{,}45\ \text{KN/cm}^2$$

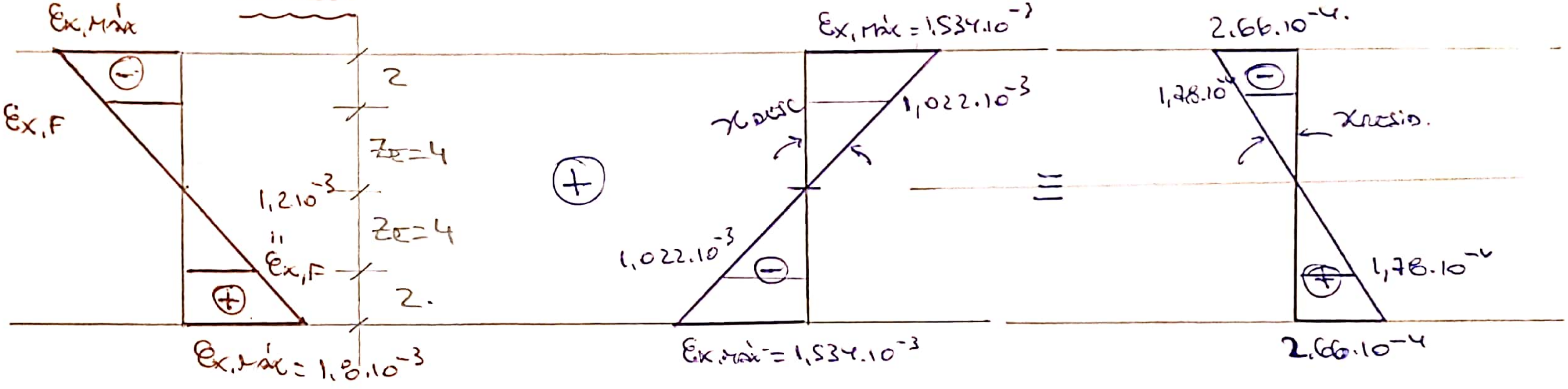
$$\boxed{\begin{aligned} T_1 \cdot z_1 &= C_1 \cdot z_1 \\ T_2 \cdot z_2 &= C_2 \cdot z_2 \end{aligned}}$$

$$|T_1 \cdot z_1| = |T_2 \cdot z_2|$$

$$|C_1 \cdot z_1| = |T_1 \cdot z_1|$$

DEFORMACIONES

CARGA: DESCARGA: RESIDUAL



$$\varepsilon_{x,F} = \frac{\sigma_{x,F}}{E} = \frac{24\,kN/cm^2}{20000\,kN/cm^2}$$

$$\varepsilon_{x,F} = 1{,}2.10^{-3} = 0{,}12\,\%.$$

$$\frac{\varepsilon_{x,máx}}{6} = \frac{\varepsilon_{x,F}}{4} \rightarrow \boxed{\varepsilon_{x,máx} = 1{,}8.10^{-3}}$$

$$\varepsilon_{x,máx} = \frac{\sigma_{x,máx}}{E}$$

$$\varepsilon_{x,máx} = \frac{30{,}67}{20000} = 1{,}534.10^{-3}$$

$$\frac{\varepsilon_{x,máx}}{6} = \frac{\varepsilon_{x,z_E}}{4}$$

$$\varepsilon_{x,z_E} = 1{,}022.10^{-3}$$

$$\underbrace{\frac{\sigma_{x,res,z_E}}{E} = \varepsilon_{x,res,z_E}}$$

ACA $-z_E \leq z \leq z_E$ ES VÁLIDA LA LEY DE HOOKE.

$$\frac{3{,}57\,kN/cm^2}{20000\,kN/cm^2} = \boxed{1{,}78.10^{-4}}$$

CURVATURA + RADIOS DE CURVATURA : $\chi_F = 2.10^{-4}\ 1/cm$. $\rho_F = 5000\,cm$.

$\boxed{\text{CARGA :}}$ $\chi = 3.10^{-4}\ \frac{1}{cm}$

$\rho = 3333^{33}_{-}\ cm.$

$\boxed{\text{RESIDUAL}}$

$\chi_{RESID} = \frac{1{,}78.10^{-4}}{4cm} = 4{,}45.10^{-5}\ \frac{1}{cm}$

$\chi_{RESID} = \frac{2{,}66.10^{-4}}{6cm} = 4{,}44.10^{-5}\ \frac{1}{cm}.$

$\rho_{RESID} = 22471^{9}\ cm.$

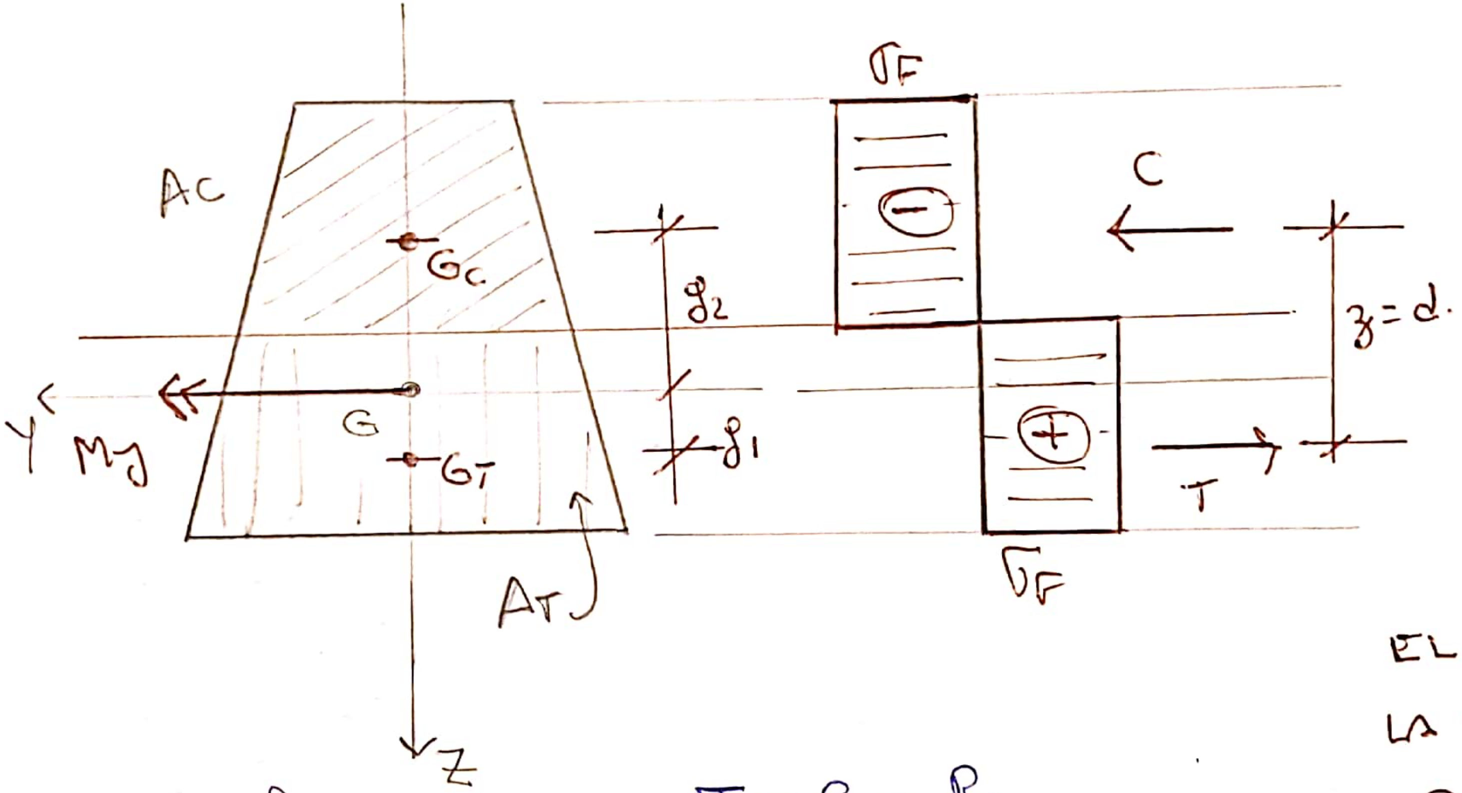
$\boxed{\text{DESCARGA :}}$

$\chi_{DESC} = \frac{\varepsilon_{máx,DESC}}{h} = \frac{1{,}534.10^{-3}}{6cm.}$

$\chi_{DESC} = 2{,}557.10^{-4}\ \frac{1}{cm.}$

$\rho_{DESC} = \frac{1}{\chi_{DESC}} = 3911^{34}_{-}\ cm.$

SECCIONES CON 1 EJE DE SIMETRÍA: → SECCIÓN TOTALMENTE PLASTIFICADA.



$$T = C$$

$$\cancel{\sigma_F} A_T = \cancel{\sigma_F} . A_C.$$

$$\boxed{A_T = A_C}$$

- Si A = ÁREA DE LA SECCIÓN

$$A_T = A_C = \frac{A}{2}$$

EL EJE NEUTRO DIVIDE A LA SECCIÓN TRANSVERSAL EN 2 ÁREAS IGUALES.

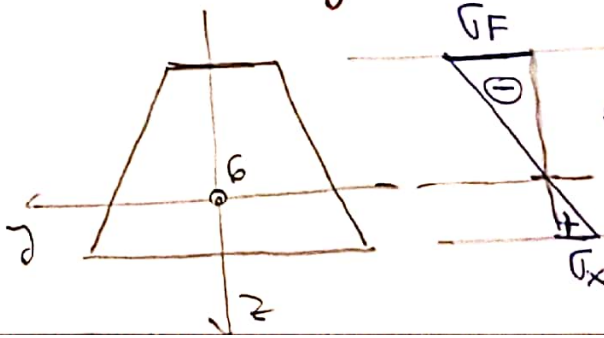
$$C . z = T . z = M_y . = M_P.$$

$$N = 0 = \int_A \sigma_x . dA . \rightarrow T = C = R$$

$$M_y = \int_A \sigma_x . z . dA.$$

RESULTANTE TRACCIÓN $= T = \sigma_F . A_T$

COMPRESIÓN $= C = \sigma_F . A_C.$



HASTA COMIENZO PLASTIFICACIÓN EN → BALCÓNICO.

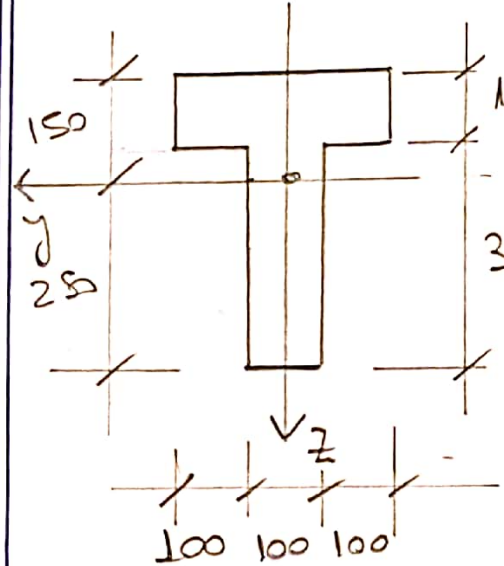
FLEXIÓN COMPUESTA:

- ES SIMILAR AL PROBLEMA DE LA FLEXIÓN SIMPLE → EL INCREMENTO DE LOS ESFUERZOS INTERNOS → HACE QUE $\sigma_x > \sigma_{x,F}$.

- EL PROBLEMA ES MÁS COMPLEJO PORQUE. → SON 2 ~~LOS 2~~ LOS ESF. INTERNOS QUE PRODUCEN TENSIONES.

- $$\begin{cases} N = \int_A \sigma_x \, dA. \\ M_y = \int_A \sigma_x \cdot z \cdot dA \end{cases}$$

EJEMPLO:



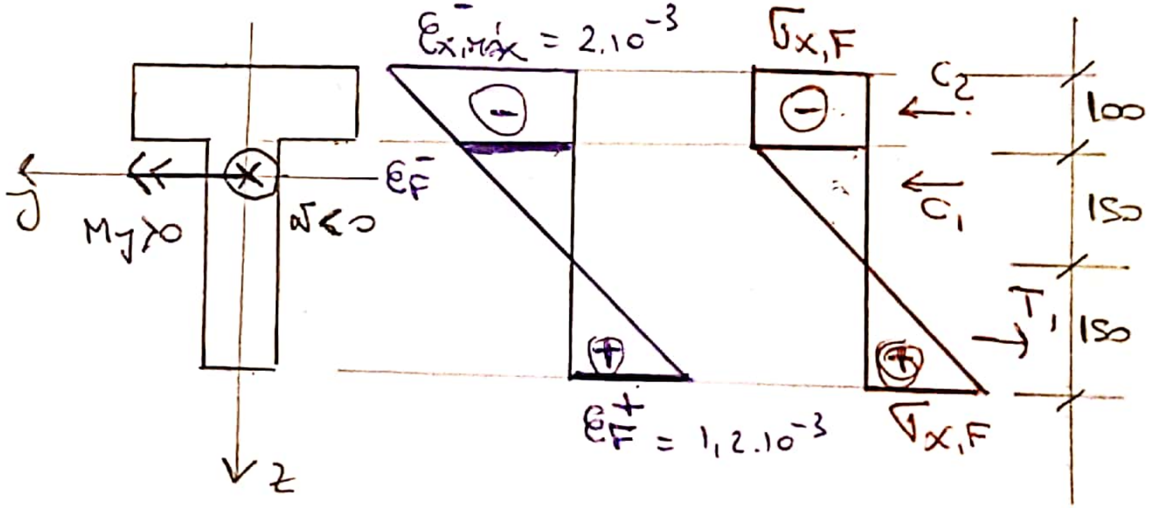
$A = 600\, cm^2.$

$I_y = 85000\, cm^4.$

$W_{yS} = 5666^{\widehat{67}}\, cm^3$

$W_{yI} = 3400\, cm^3.$

$$\begin{cases} \sigma_F = 24\, kN/cm^2 \\ E = 20000\, kN/cm^2 \end{cases}$$



$$\boxed{N} = \underbrace{\cancel{(+24).\frac{10.15}{2}}}_{T_1} - \underbrace{\cancel{(24).\frac{10.15}{2}}}_{C_1} - \underbrace{24.10.30.}_{C_2} = \boxed{-7200\,kN}$$

$$M_y = \underbrace{\frac{24.10.15}{2}.(25-5)}_{M_{y,T_1}} + \underbrace{\frac{24.10.15}{2}.(15-15)}_{M_{y,C_1}} + \underbrace{24.10.30.(15-5)}_{M_{y,C_2}} = 36000+0+72000$$

$$\boxed{M_y} = 108.000\,kN\,cm = \boxed{1080\,kNm}$$

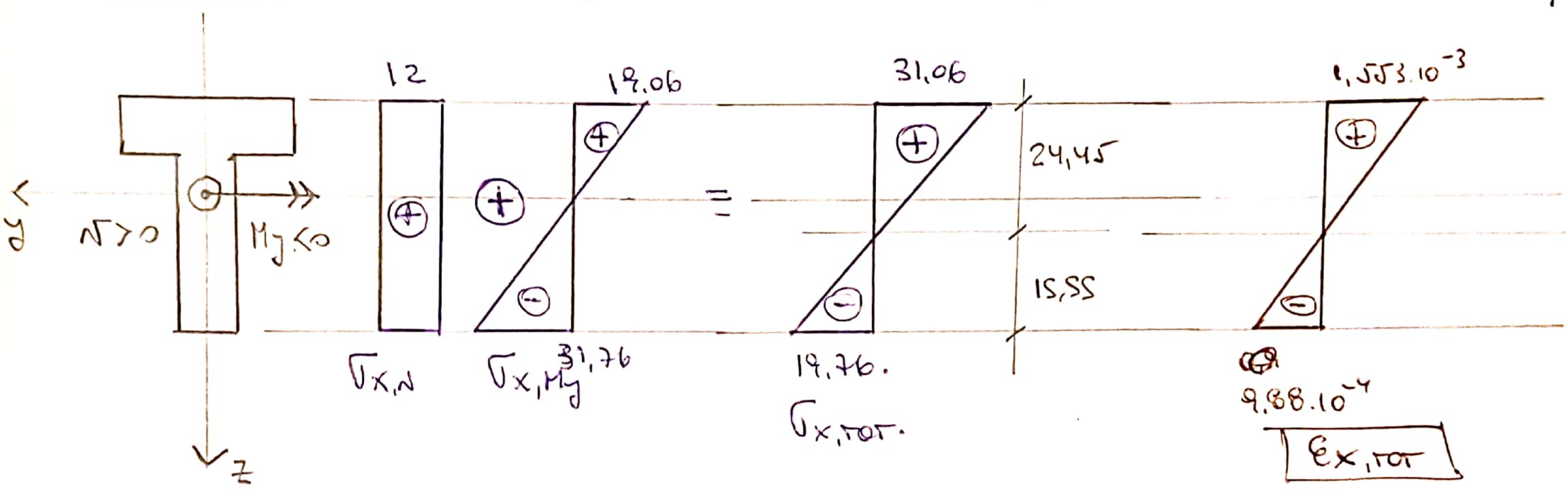
REDUCIDO A G DE LA SECCIÓN.
RESPECTO AL BARICENTRO DE LA SECCIÓN.

FLEXIÓN COMPUESTA RECTA.

$$|\varepsilon_F^+| = |\varepsilon_F^-| = \frac{\sigma_F}{E} = \frac{24}{20000} = 1{,}2.10^{-3}$$

$$\frac{\varepsilon^-_{x,MAX}}{25} = \frac{\varepsilon_F^-}{15} \longrightarrow \varepsilon^-_{x,máx} = 2{,}0.10^{-3}$$

DESCARGA



$$\sigma_{x,N} = \frac{N}{A} = \frac{7200\ kN}{6000\ cm^2} = 12\ kN/cm^2$$

$$\varepsilon_{x,S} = \frac{\sigma_{x,S,TOT}}{E} = \frac{31,06}{20000} = 1,553 \cdot 10^{-3}$$

$$\sigma^{S}_{x,My} = \frac{M_y}{W_{y\,S}} = \frac{108000}{5666,67} = 19,06\ \frac{kN}{cm^2}$$

$$\varepsilon_{x,I} = \frac{\sigma_{x,I,TOT}}{E} = \frac{19,76}{20000} = 9,88 \cdot 10^{-4}$$

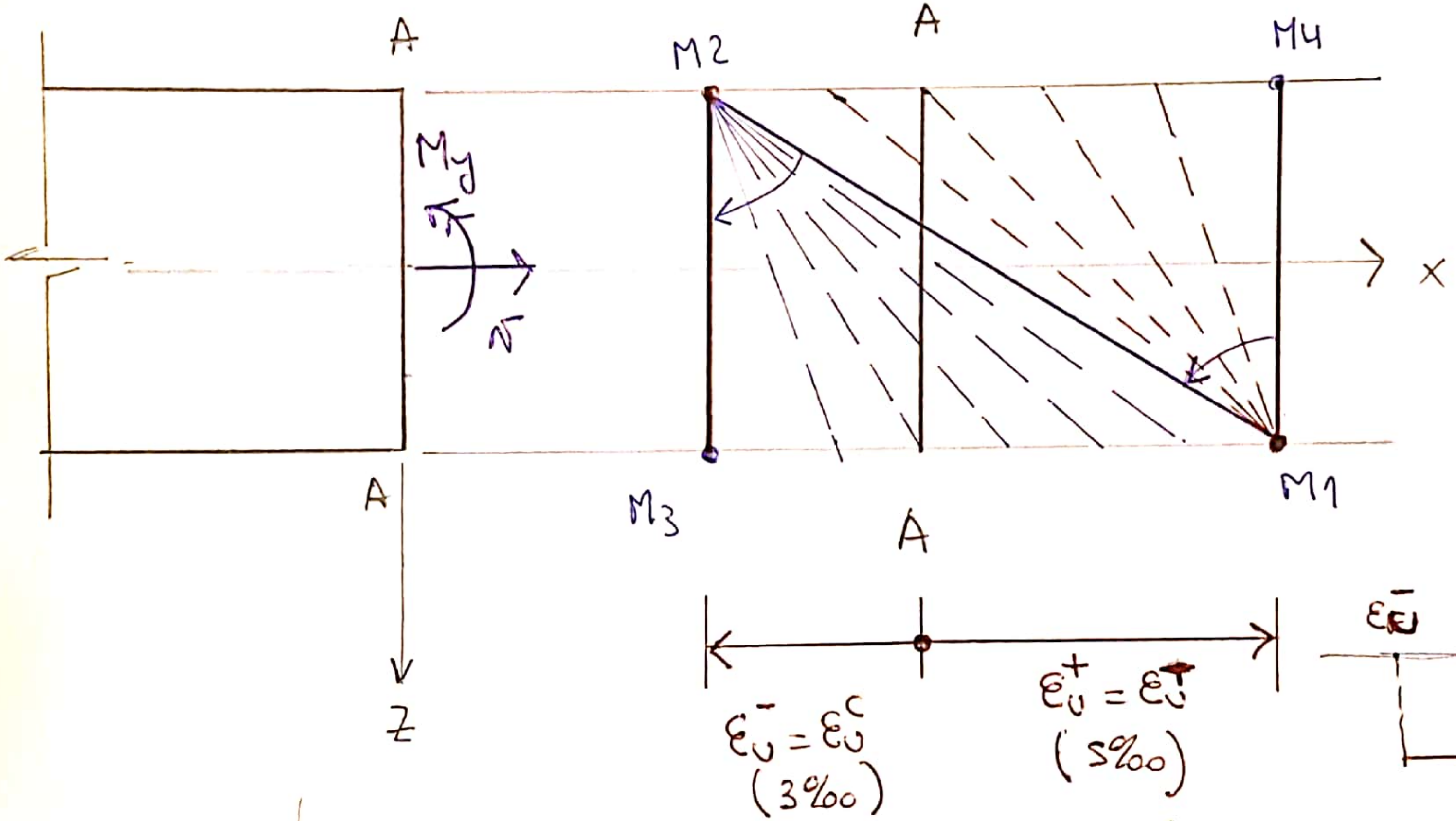
$$\sigma^{I}_{x,My} = \frac{M_y}{W_{y\,I}} = \frac{108000}{34000} = 31,76\ \frac{kN}{cm^2}$$

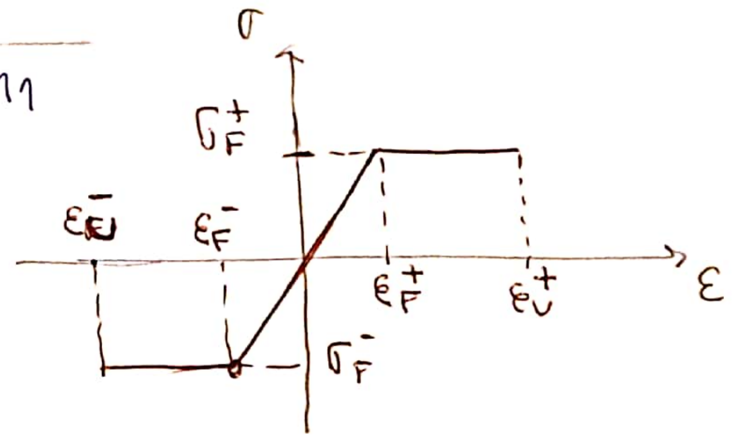
$$\chi_{DESC} = \frac{1,553 \cdot 10^{-3}}{24,45\ cm} = 6,352 \cdot 10^{-5}\ \frac{1}{cm}.$$

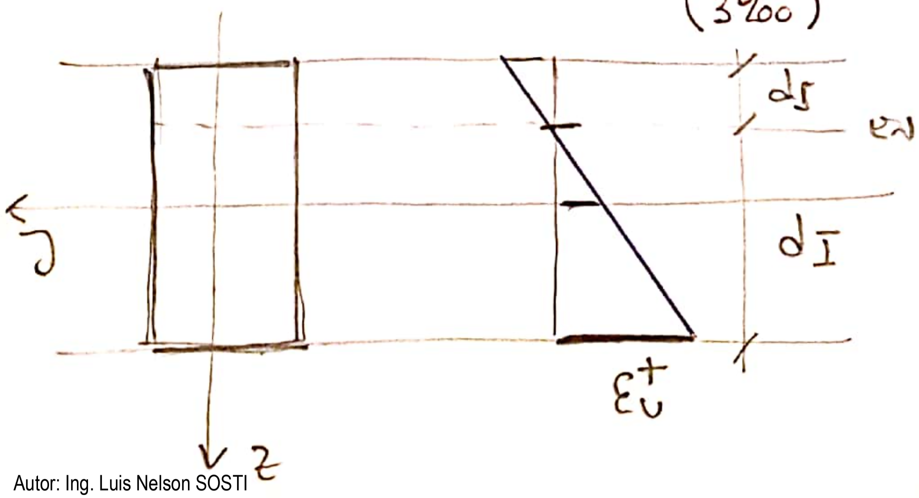
OJO → HACER LA PARTE RESIDUAL

M_i = Ptos de rotación.

Planos límites de deformación.







<u>DATOS</u>

$\varepsilon_U^+ \rightarrow$ FIBRAS INF.

UBICACIÓN EN

- TENS.
- N y M.
- DESCARGA
- TENS y DEF RESIDUALES.