

→ SOLICITACIÓN POR FLEXIÓN EN RÉGIMEN ELÁSTICO. ✓

↳ I) → FLEXIÓN SIMPLE. ARAn - FS

↳ II) → " COMPUESTA ARAn - FCm

1º) → se asumirá siempre válida la "HIPÓTESIS DE BERNOLLI - NAVIER"  
→ HBN.

$$\begin{cases} \epsilon_{x,z} = \frac{z}{z_{máx}} \cdot \epsilon_{x,máx} \\ \epsilon_{x,z} = \frac{z}{r} \cdot \bar{\epsilon}_x \end{cases}$$

2) ECS. DE EQUILIBRIO:

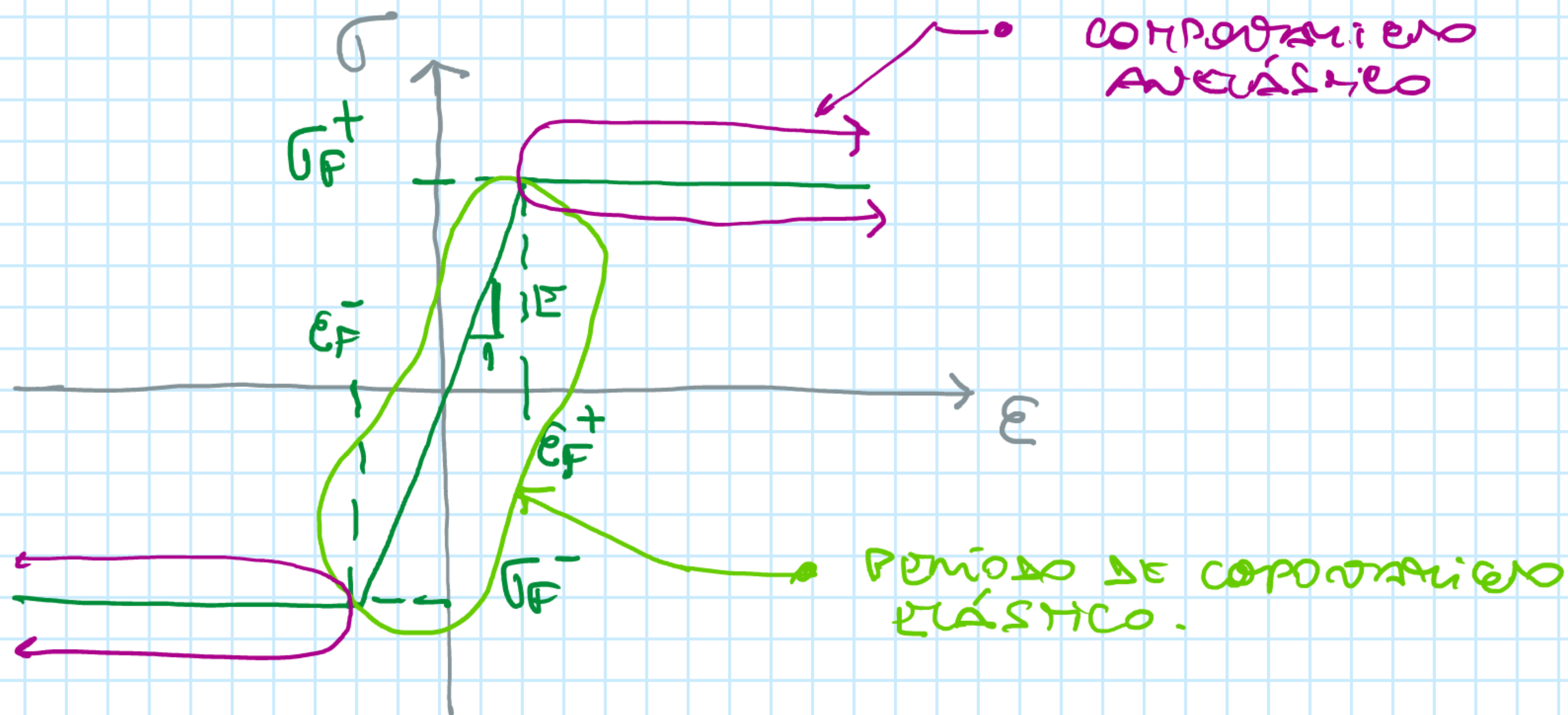
**FS**

$$\begin{cases} N = 0 = \int_A \sigma_x \cdot dA \\ M_y = \int_A \underbrace{\sigma_x \cdot z \cdot dA}_{dM} \end{cases}$$

**FCm**

$$\begin{cases} N = \int_A \sigma_x \cdot dA \\ M_y = \int_A \sigma_x \cdot z \cdot dA \end{cases}$$

3) - DIAGRAMA  $\sigma - \epsilon$ :



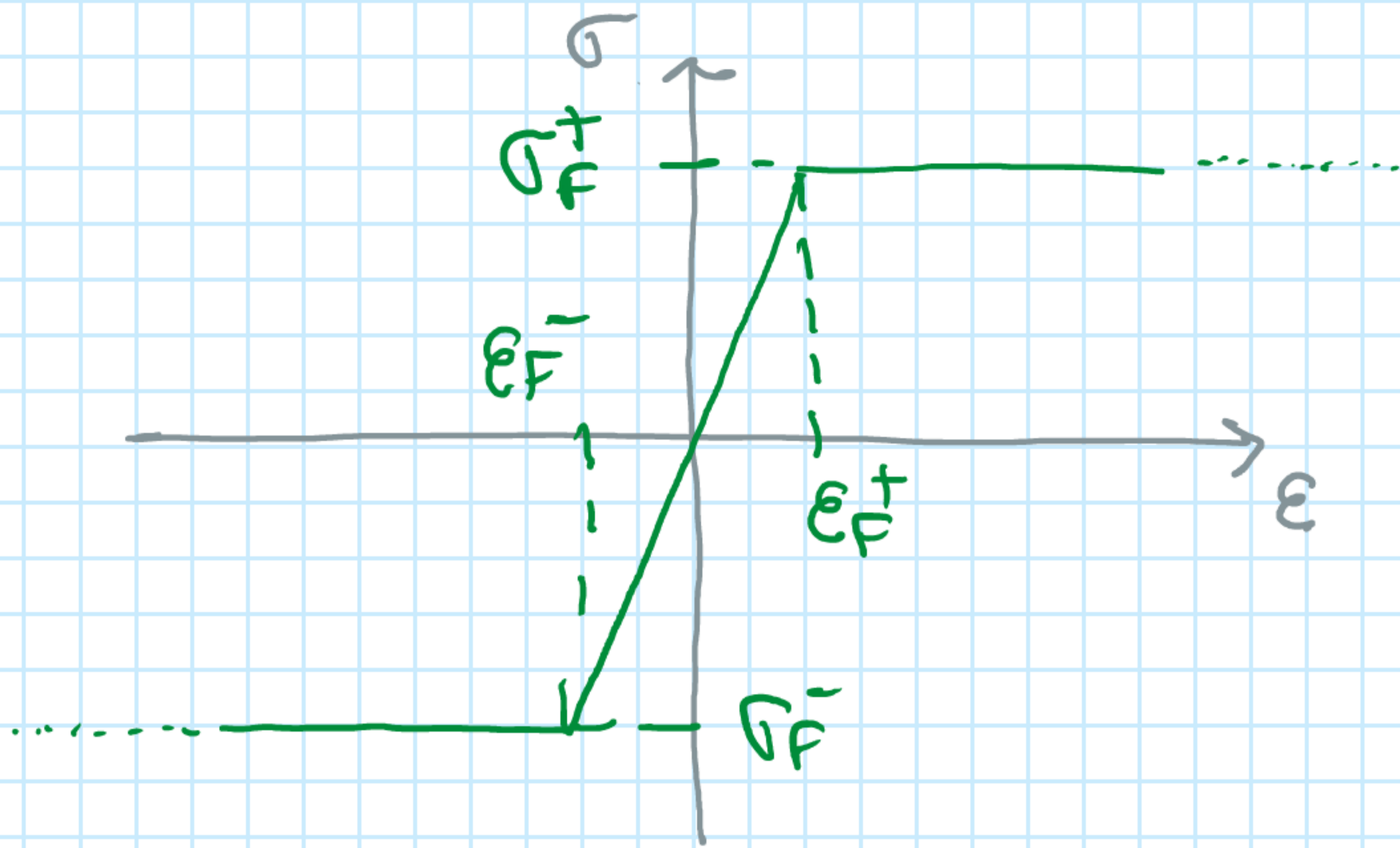
4) Expresión de flexión P) EL PERIODO ELÁSTICO:

$$\sigma_x = \frac{M_y(x)}{I_y} \cdot z \quad \text{FLEXIÓN SIMPLE USCM}$$

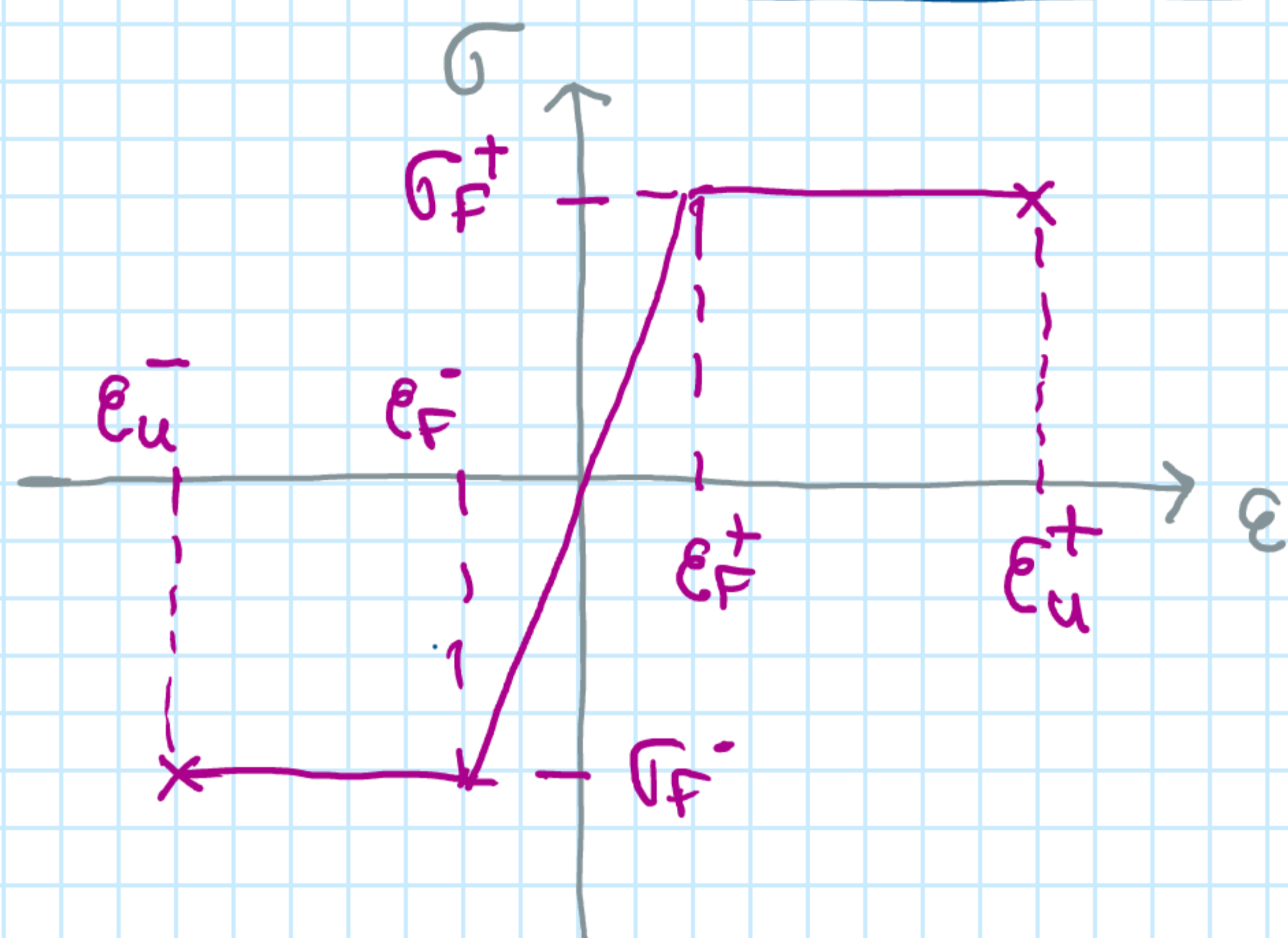


I) TIPOS DE COMPORTAMIENTO DEL MATERIAL

I.1 - EPI: ELASTO-PLÁSTICO IDEAL

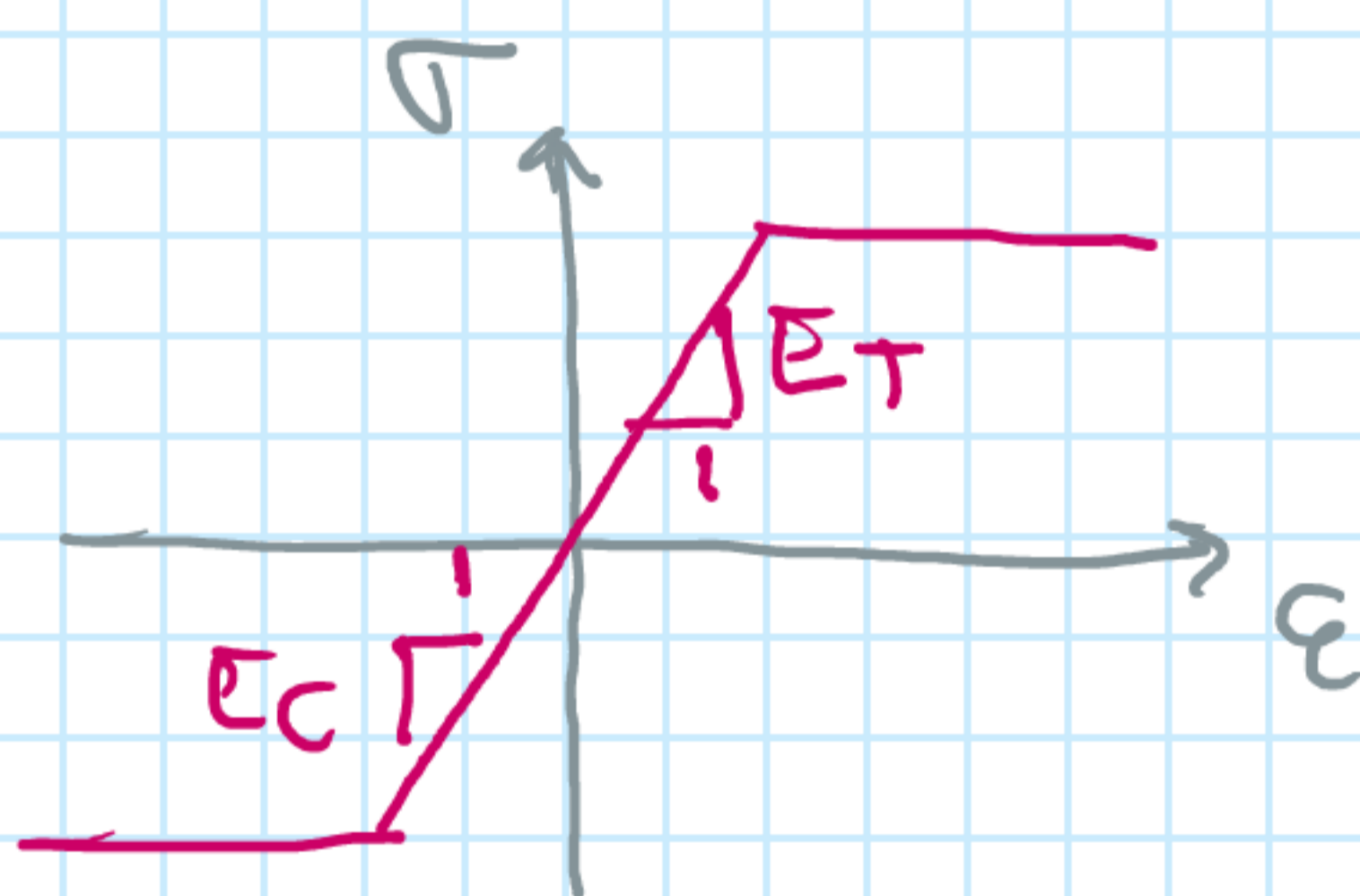


I. II - EPR: ELASTO-PLÁSTICO REAL



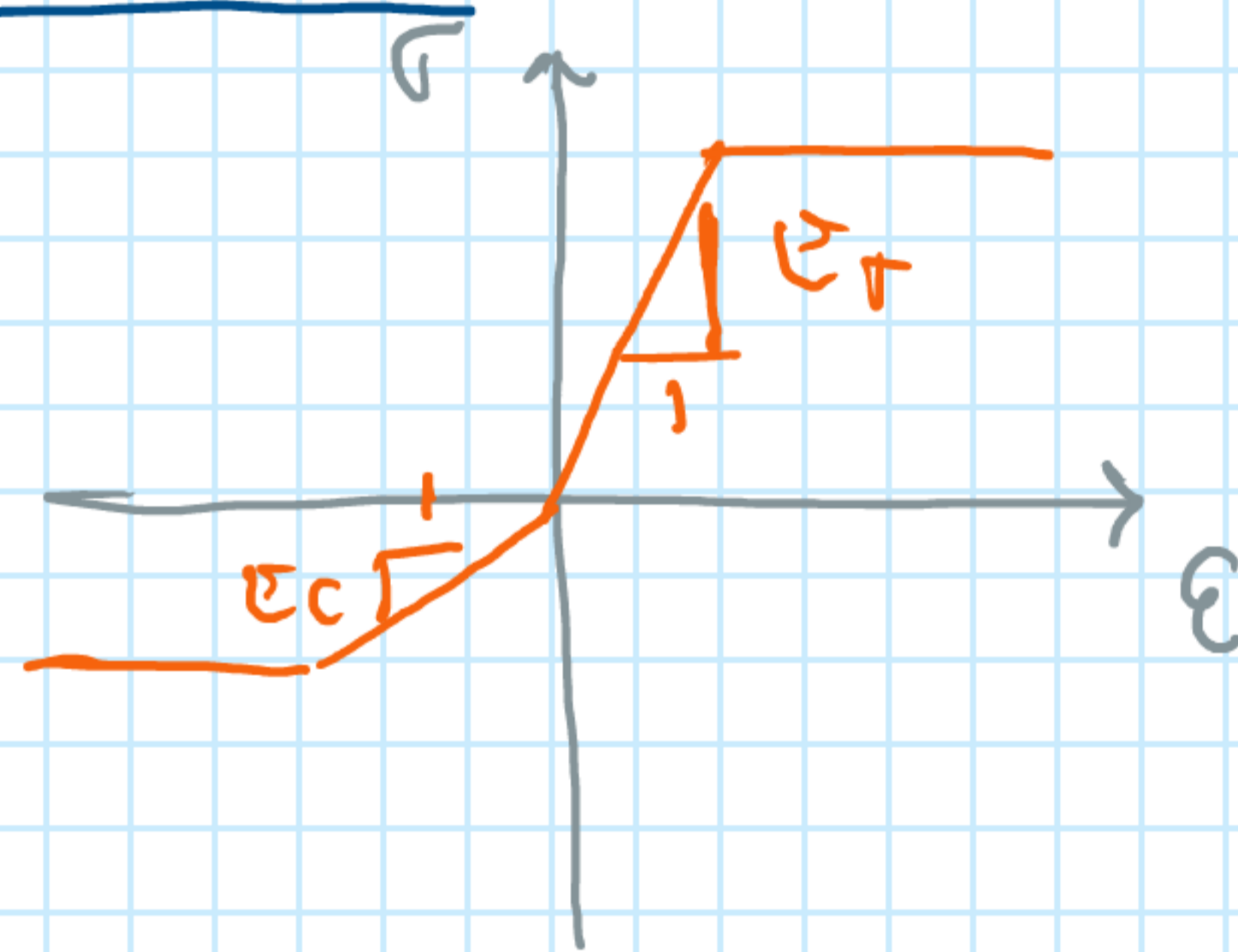
II) - VARIOS:

caso 'a':



$E_T = E_C$

caso 'b':



$E_T \neq E_C$

caso 'c':

$$\begin{cases} \sigma_F^+ = \sigma_F^- \\ \sigma_F^+ \neq \sigma_F^- \end{cases}$$

caso 'd':

$$\begin{cases} \epsilon_F^+ = \epsilon_F^- \\ \epsilon_F^+ \neq \epsilon_F^- \end{cases}$$

III) → OTROS DO SUBCORTES:

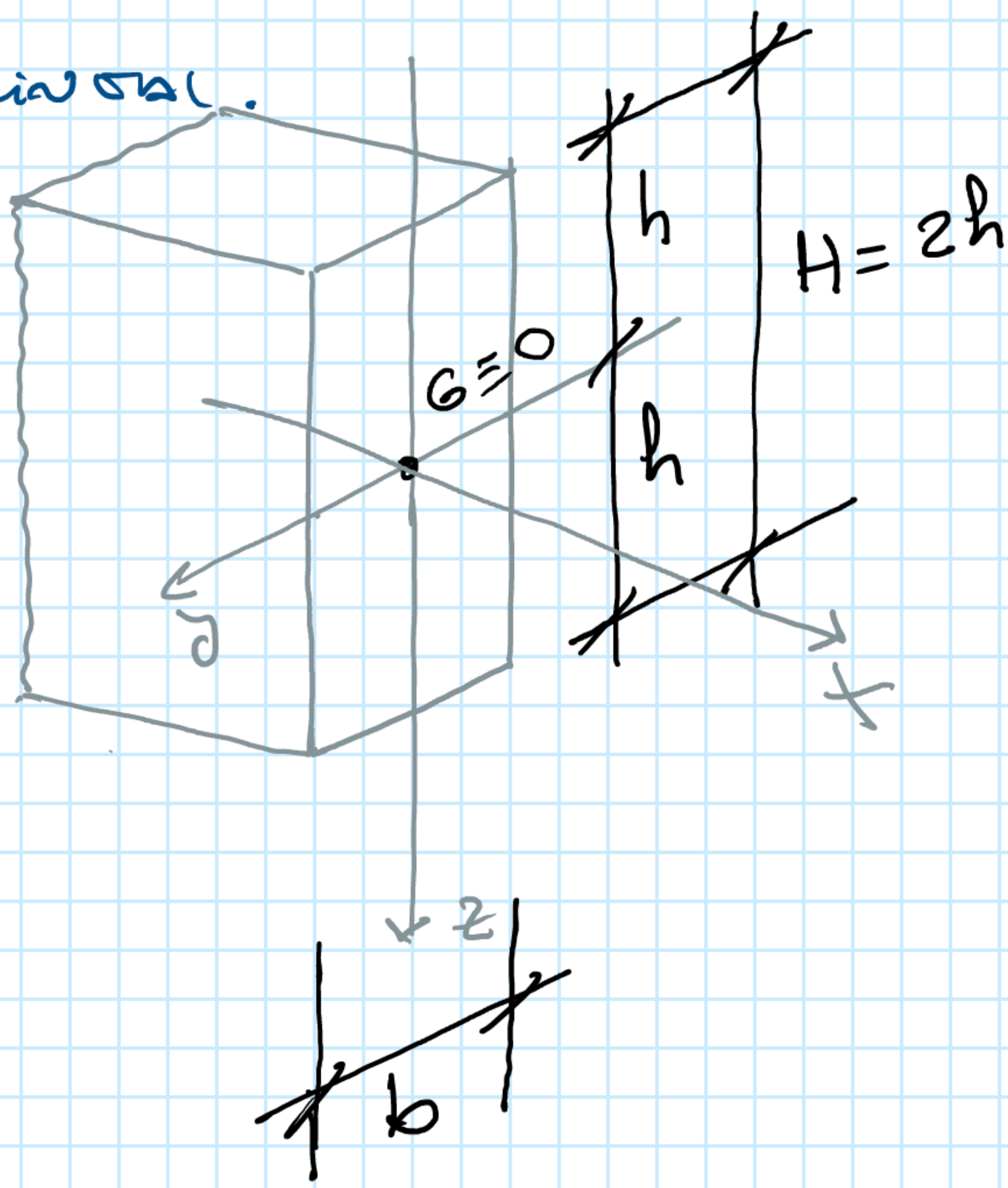
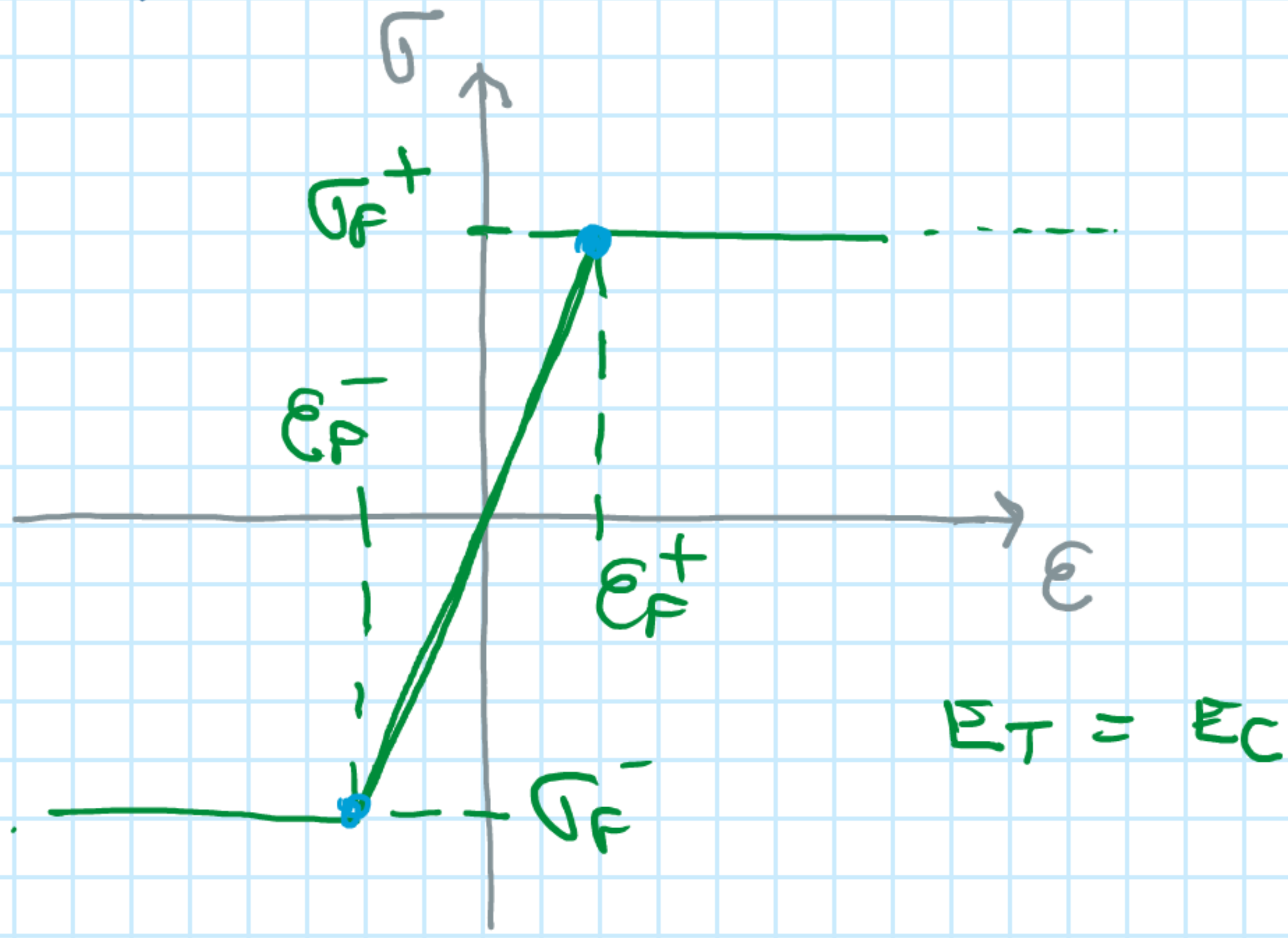
i) → DOS DO SUBCORTES ←

ii) → SIMPLER "



→ EPI

→ DIAGRAMA  $\sigma - \epsilon \rightarrow$  BILINEAL.



**Período elástico**

↳  $\sigma_{x, \max} \leq \sigma_F \rightarrow \sigma_{x, \max} = \frac{M_y}{I_y} \cdot h$

•  $\sigma_{adm} = \frac{\sigma_F}{CS} \rightarrow \sigma_{x, \max} = \sigma_{adm} = \frac{M_{y, adm}}{I_y} \cdot h$

$M_{y, adm} = \sigma_{adm} \cdot \frac{I_y}{h} = \sigma_{adm} \cdot \frac{b(2h)^3}{12 \cdot h}$

$M_{y, adm} = \frac{2}{3} b h^2 \sigma_{adm} \quad (1)$

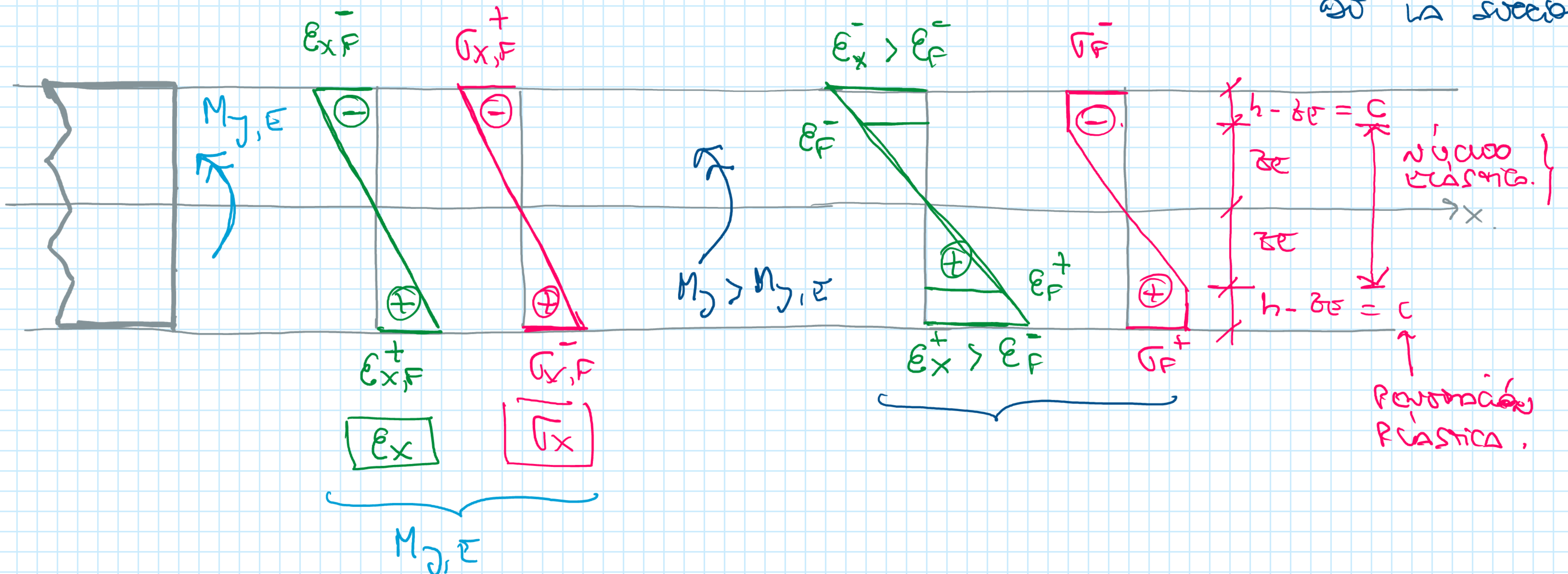
• si  $\sigma_{x, \max} = \sigma_F \rightarrow \sigma_{x, \max} = \sigma_F = \frac{M_{y, \epsilon}}{I_y} \cdot h = \frac{M_{y, \epsilon}}{I_y/h}$

$M_{y, \epsilon} = M_{y, F}$  : momento fuerza elástico.

$M_{y, \epsilon} = \sigma_F \cdot \frac{I_y}{h} \rightarrow M_{y, \epsilon} = \frac{2}{3} b h^2 \sigma_F \quad (2)$

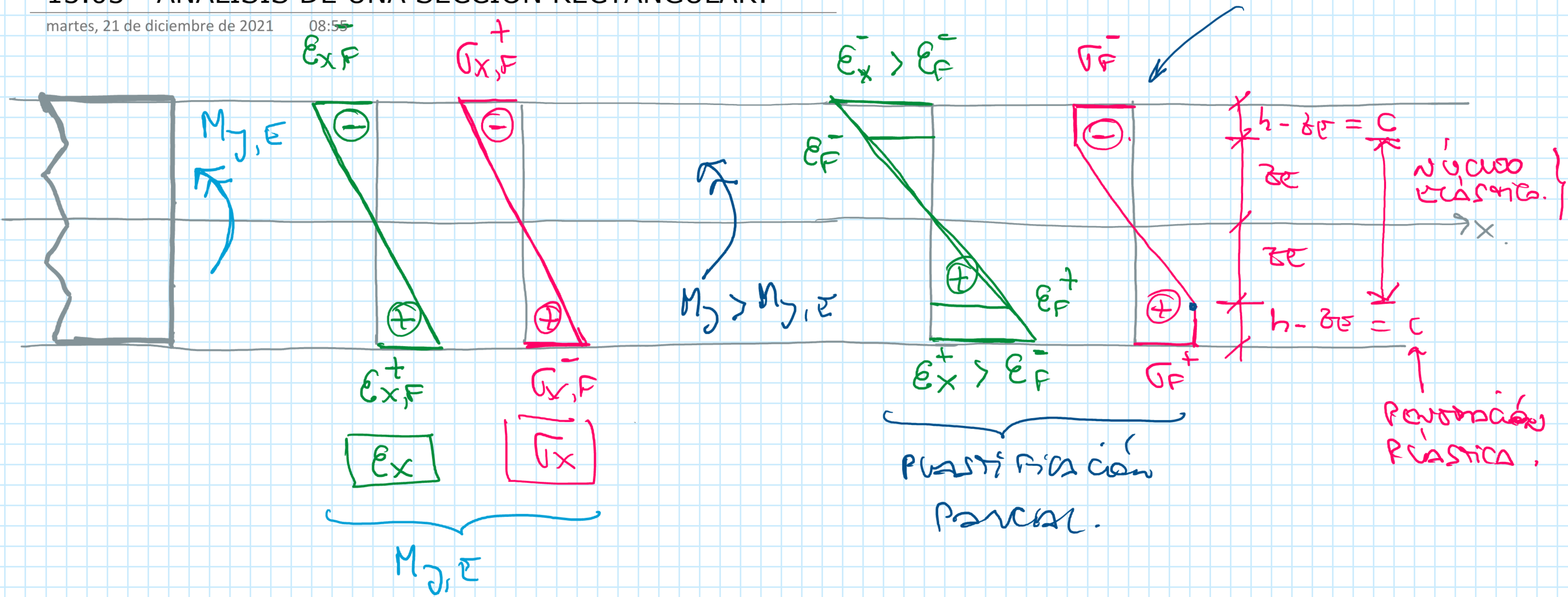
$I_y/h$

↳  $S_y$   
momento o momento  
resistencia elástico  
de la sección.



$M_{y, \epsilon} = \sigma_F \cdot S_y \quad (3)$





NÚCLEO ELÁSTICO:

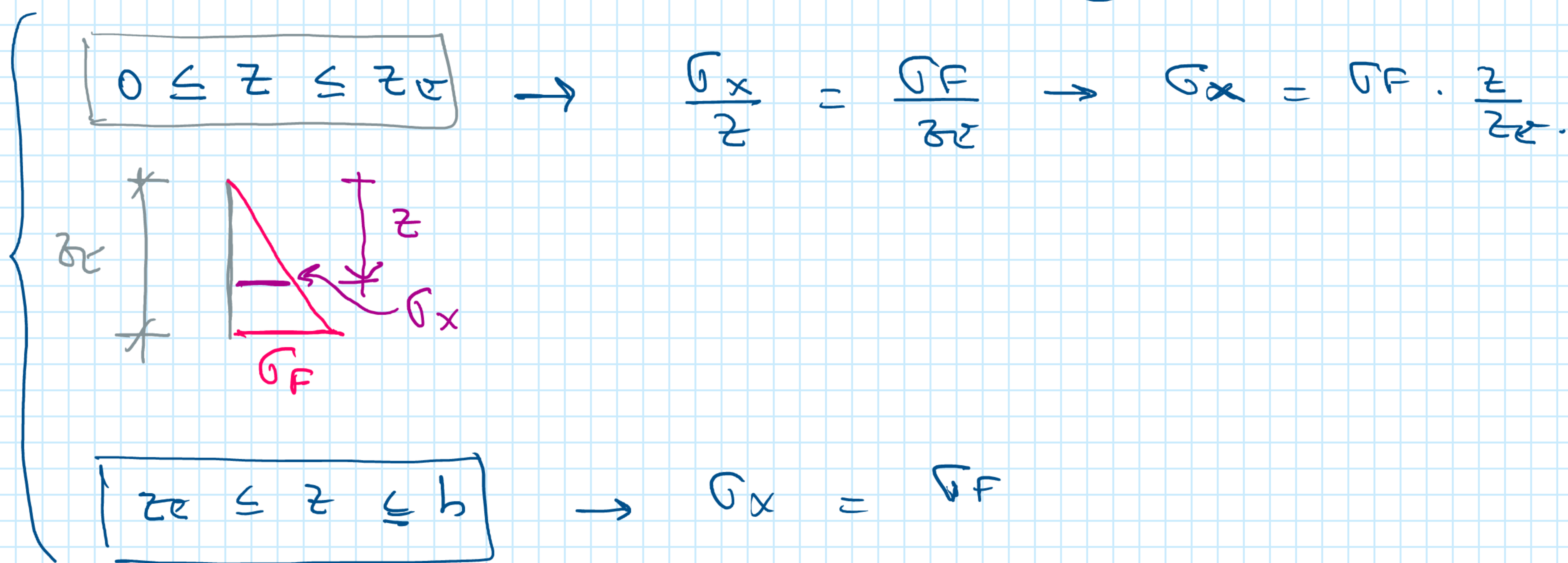
$$-z_{\epsilon} \leq z \leq +z_{\epsilon}$$

ZONA PLÁSTICA:

$$|(h - z_{\epsilon})|$$

Ⓘ  $z_{\epsilon} \leq z \leq h$

Ⓡ  $-h \leq z \leq -z_{\epsilon}$



$$z_{\epsilon} \leq z \leq h \rightarrow \sigma_x = \sigma_F$$

EC. DE EQUIVALENCIAS:

$$M_j = \int_A \sigma_x \cdot z \cdot dA = \int_{-h}^h \sigma_x \cdot z \cdot \underbrace{b \cdot dz}_{dA} = 2 \int_0^h \sigma_x \cdot z \cdot b \cdot dz$$

$$M_j = 2 \cdot \left[ \int_0^{z_{\epsilon}} \left( \frac{\sigma_F b}{z_{\epsilon}} \right) \cdot z^2 \cdot dz + \int_{z_{\epsilon}}^h (\sigma_F \cdot b) \cdot z \cdot dz \right]$$

$$M_j = 2 \cdot \left[ \frac{\sigma_F b}{z_{\epsilon}} \int_0^{z_{\epsilon}} z^2 \cdot dz + \sigma_F b \int_{z_{\epsilon}}^h z \cdot dz \right]$$

$$M_j = 2 \cdot \left[ \frac{\sigma_F b}{z_{\epsilon}} \left. \frac{z^3}{3} \right|_0^{z_{\epsilon}} + \sigma_F \cdot b \left. \frac{z^2}{2} \right|_{z_{\epsilon}}^h \right]$$

$$M_j = \frac{2}{3} \frac{\sigma_F \cdot b}{z_{\epsilon}} \cdot z_{\epsilon}^3 + \sigma_F b \cdot (h^2 - z_{\epsilon}^2)$$

$$M_j = \frac{2}{3} b z_{\epsilon}^2 \sigma_F + b h^2 \sigma_F - b z_{\epsilon}^2 \sigma_F$$

$$M_j = b h^2 \sigma_F - \frac{1}{3} b z_{\epsilon}^2 \sigma_F \quad (4)$$

$$M_j = b h^2 \sigma_F \left[ 1 - \frac{1}{3} \frac{z_{\epsilon}^2}{h^2} \right] \quad (5)$$

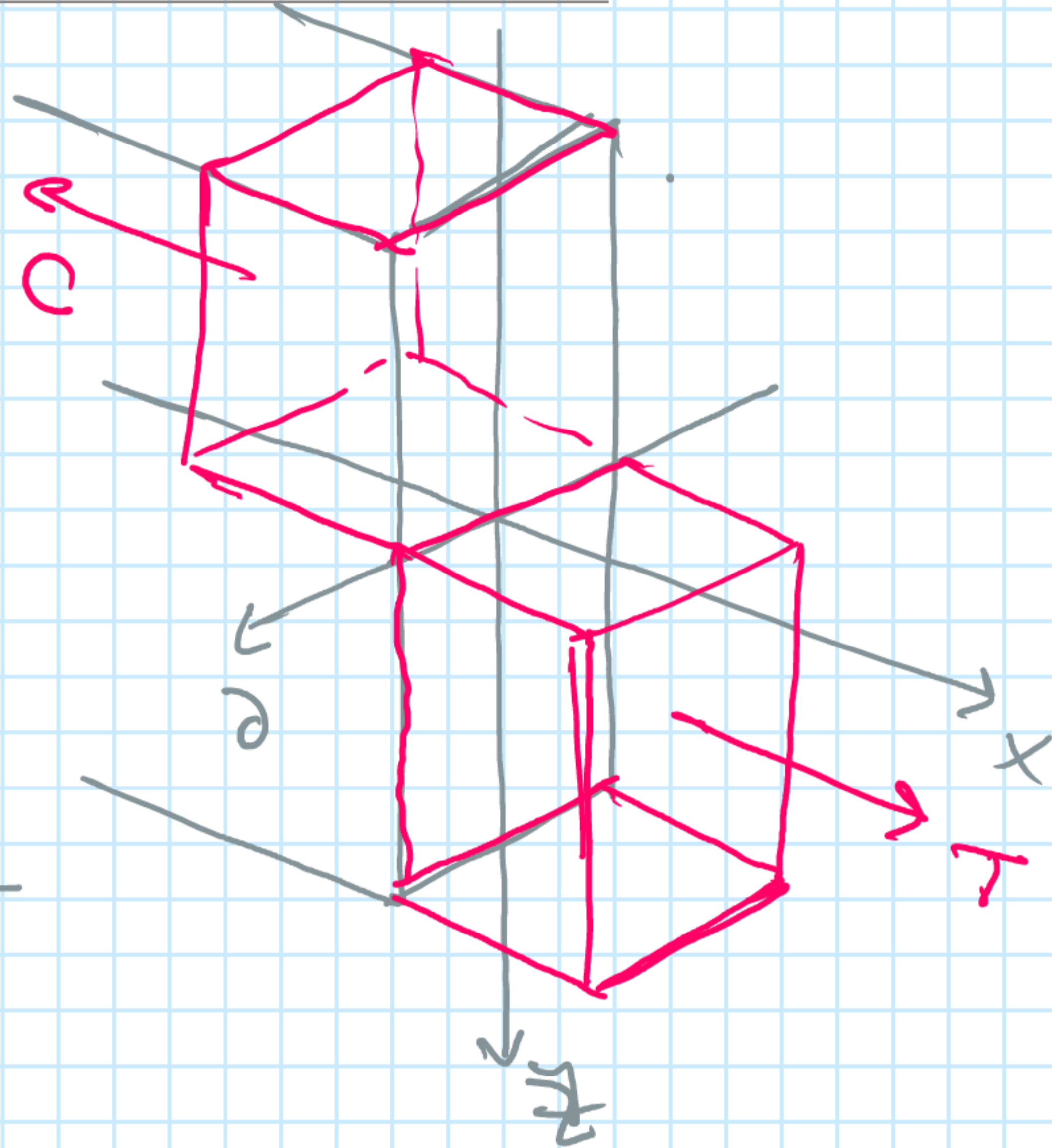
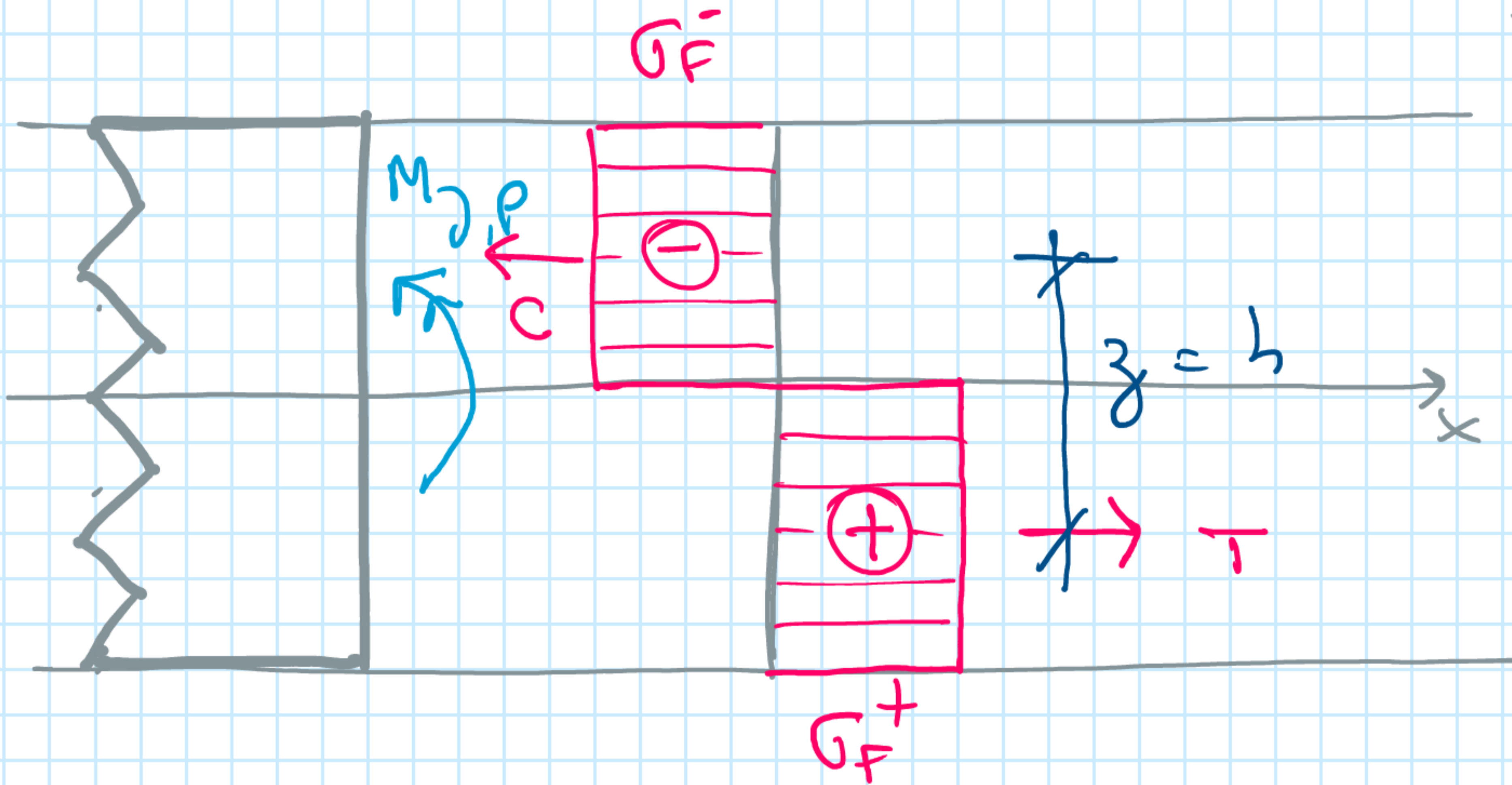
$$M_{j,E} = \frac{2}{3} b h^2 \sigma_F \rightarrow b h^2 \sigma_F = \frac{3}{2} M_{j,E}$$

$$M_j = \frac{3}{2} M_{j,E} \left[ 1 - \frac{1}{3} \frac{z_{\epsilon}^2}{h^2} \right] \quad (6)$$



# 13.03 - SECCIÓN RECTANGULAR - PLASTIFICACIÓN TOTAL:

martes, 21 de diciembre de 2021 10:08



$$M_{y,P} = m_{y,u} = m_{y,c}$$

$$C = T = \sigma_{x,F} \cdot b \cdot h ; \quad z = h ; \quad M_{y,P} = C \cdot z = T \cdot z$$

$$M_{y,P} = b h h \cdot \sigma_{x,F} \rightarrow \boxed{m_{y,P} = b b^2 \sigma_{x,F}} \quad (7)$$

EN LA PLASTIFICACIÓN TOTAL  $\rightarrow z_e = 0$   $\leftarrow$  NO HAY NINGUNO CLASICO

$$\left. \begin{array}{l} (4) \quad M_y = b b^2 \sigma_{x,F} \\ (5) \quad M_y = b h^2 \sigma_{x,F} \\ (6) \quad M_y = \frac{2}{2} m_{y,e} \end{array} \right\} M_y = m_{y,P}$$

$$\boxed{M_{y,P} = \sum m_{y,e}}$$



$$\left. \begin{aligned} M_{D,E} &= \frac{2}{3} b h^2 \sigma_{x,F} \\ M_{D,P} &= b h^2 \sigma_{x,F} \end{aligned} \right\} \rightarrow M_{D,P} = \frac{3}{2} M_{D,E}$$

$$\frac{M_{D,P}}{M_{D,E}} = \frac{\cancel{b h^2} \sigma_{x,F}}{\frac{2}{3} \cancel{b h^2} \sigma_{x,F}} = \frac{3}{2} = k = \text{FACTOR DE FORMA.}$$

f (sección).

Es una característica de la sección con la que se trabaja.

$$\frac{M_{D,E}}{\sigma_{x,F}} = \frac{\frac{2}{3} b h^2 \sigma_{x,F}}{\sigma_{x,F}} = \frac{2}{3} b h^2 = S_y$$

$$I_y = \frac{b(2h)^3}{12} = \frac{8}{12} b h^3 = \frac{2}{3} b h^3$$

$$S_y = \frac{I_y}{z_{\max}} = \frac{\frac{2}{3} b h^3}{h} = \frac{2}{3} b h^2$$

$$\frac{M_{D,P}}{\sigma_{x,F}} = \frac{b h^2 \sigma_{x,F}}{\sigma_{x,F}} = b h^2 = Z_y$$

Módulo plástico de la sección.  
Es una característica de la sección.

$$\left\{ \begin{aligned} M_{D,P} &= Z_y \cdot \sigma_{x,F} & (9a) \\ M_{D,E} &= S_y \cdot \sigma_{x,F} & (9b) \end{aligned} \right.$$

$$\frac{(9a)}{(9b)} = \frac{M_{D,P}}{M_{D,E}} = \frac{Z_y}{S_y} = k = \frac{b h^2}{\frac{2}{3} b h^2} = \frac{3}{2}$$

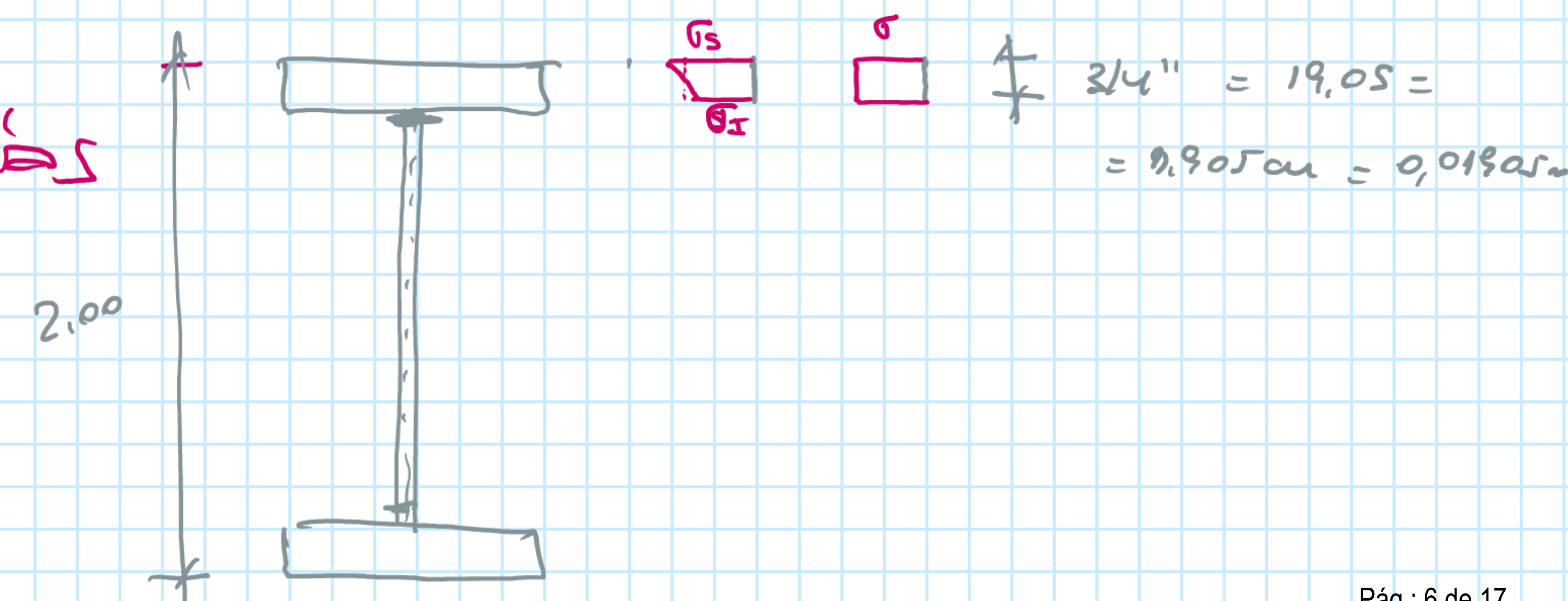
P/ sección rectangular  $\rightarrow k = \frac{3}{2} = 1,50.$

P/ sección cuadrada  $\rightarrow k = 2,00.$

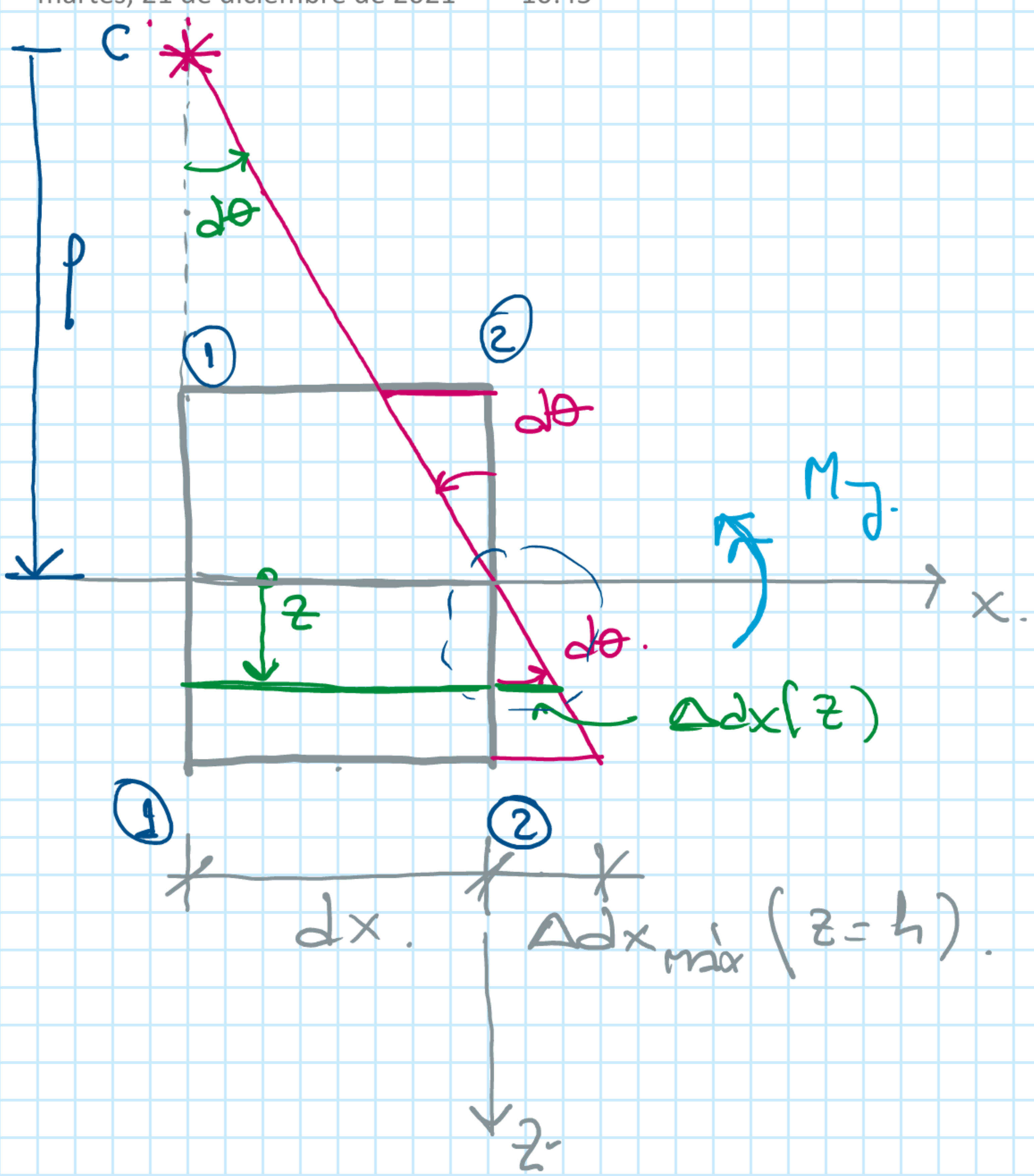
P/ perfil  $T'$   $\rightarrow$  IPN 300  $\left\{ \begin{aligned} S_y &= 653 \text{ cm}^3 \\ Z_y &= 762 \text{ cm}^3 \end{aligned} \right.$

$$k = \frac{Z_y}{S_y} = \frac{762}{653} = 1,167$$

Cuanto más curvo es 'k'  $\rightarrow$  más se aproxima la sección.







• si  $dx = 1 \rightarrow \frac{d\theta}{dx} = \kappa =$  curvatura por flexión.

$$\boxed{d\theta = \kappa} \quad (10)$$

•  $\epsilon_x = \frac{\Delta dx}{dx} \rightarrow \boxed{\epsilon_x = \Delta dx}$  (11a)

•  $\frac{\Delta dx}{\epsilon_x} = z \cdot \frac{d\theta}{\kappa} \rightarrow \boxed{\epsilon_x = z \cdot \kappa}$  (11b)

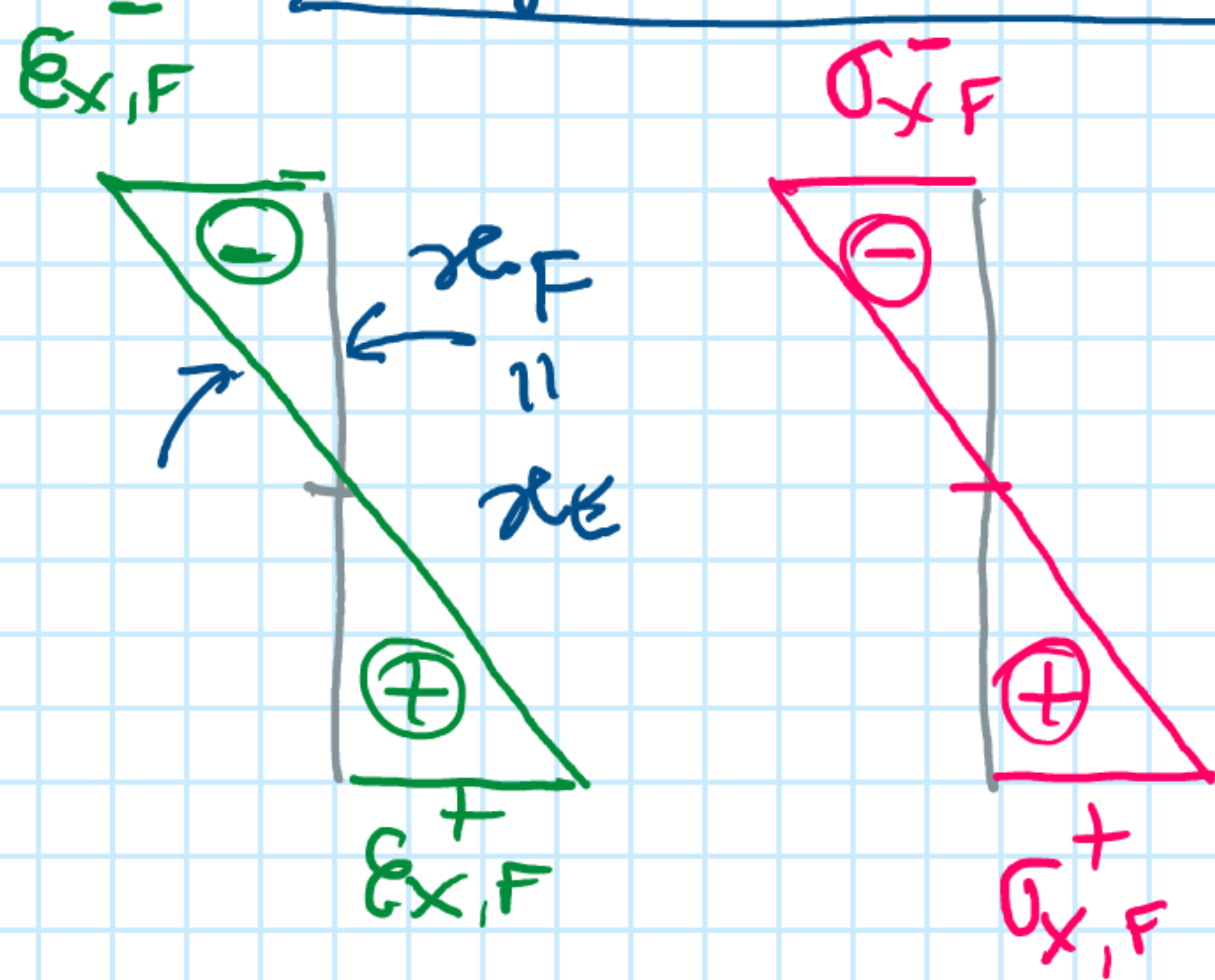
•  $dx = \rho \cdot \frac{d\theta}{\kappa} \rightarrow \rho \cdot \kappa = 1$

$$\boxed{\rho = \frac{1}{\kappa} ; \kappa = \frac{1}{\rho}} \quad (12)$$

la (11b)  $\rightarrow \boxed{\epsilon_x = z \cdot \kappa = z \cdot \frac{1}{\rho}}$  (13)

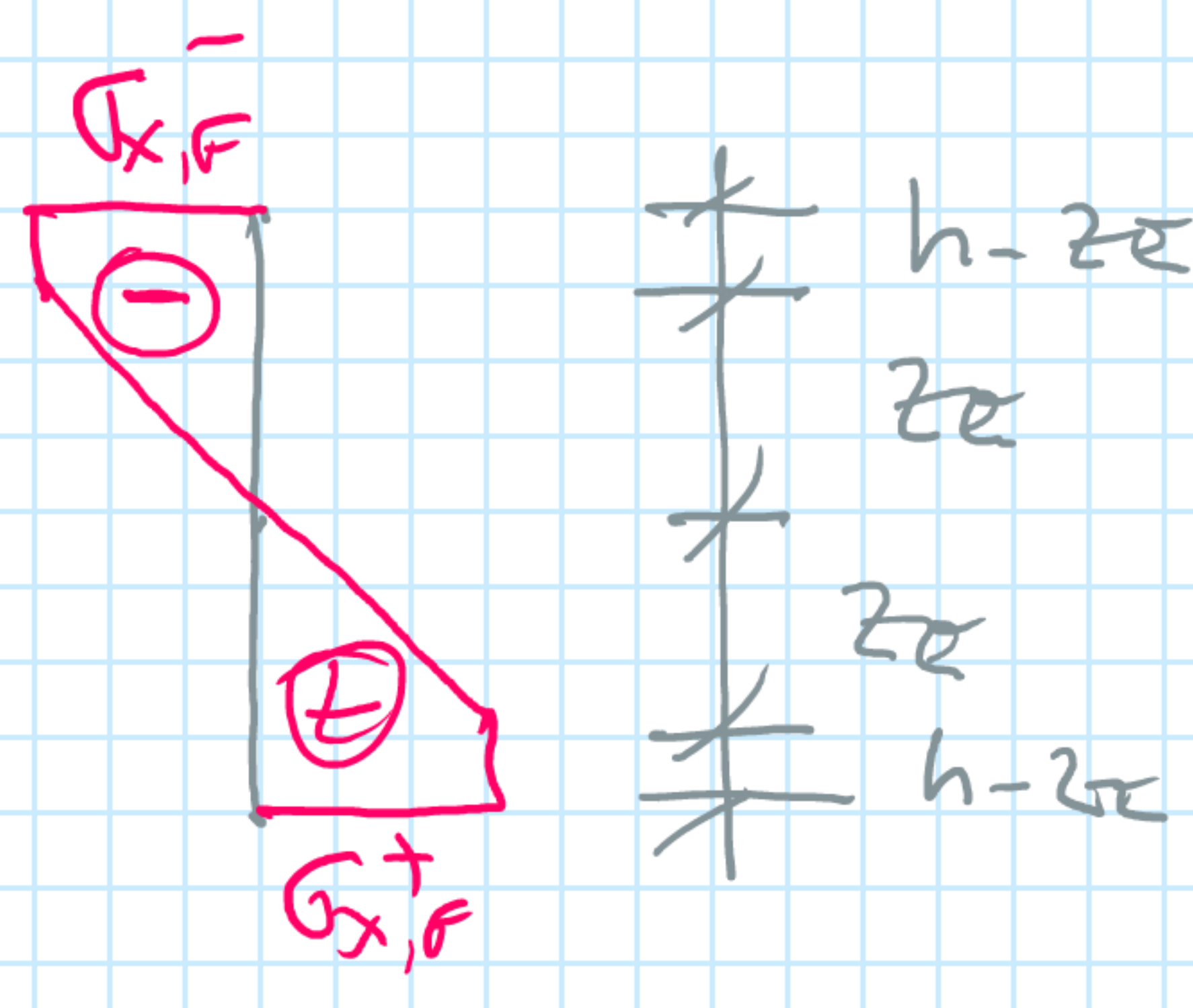
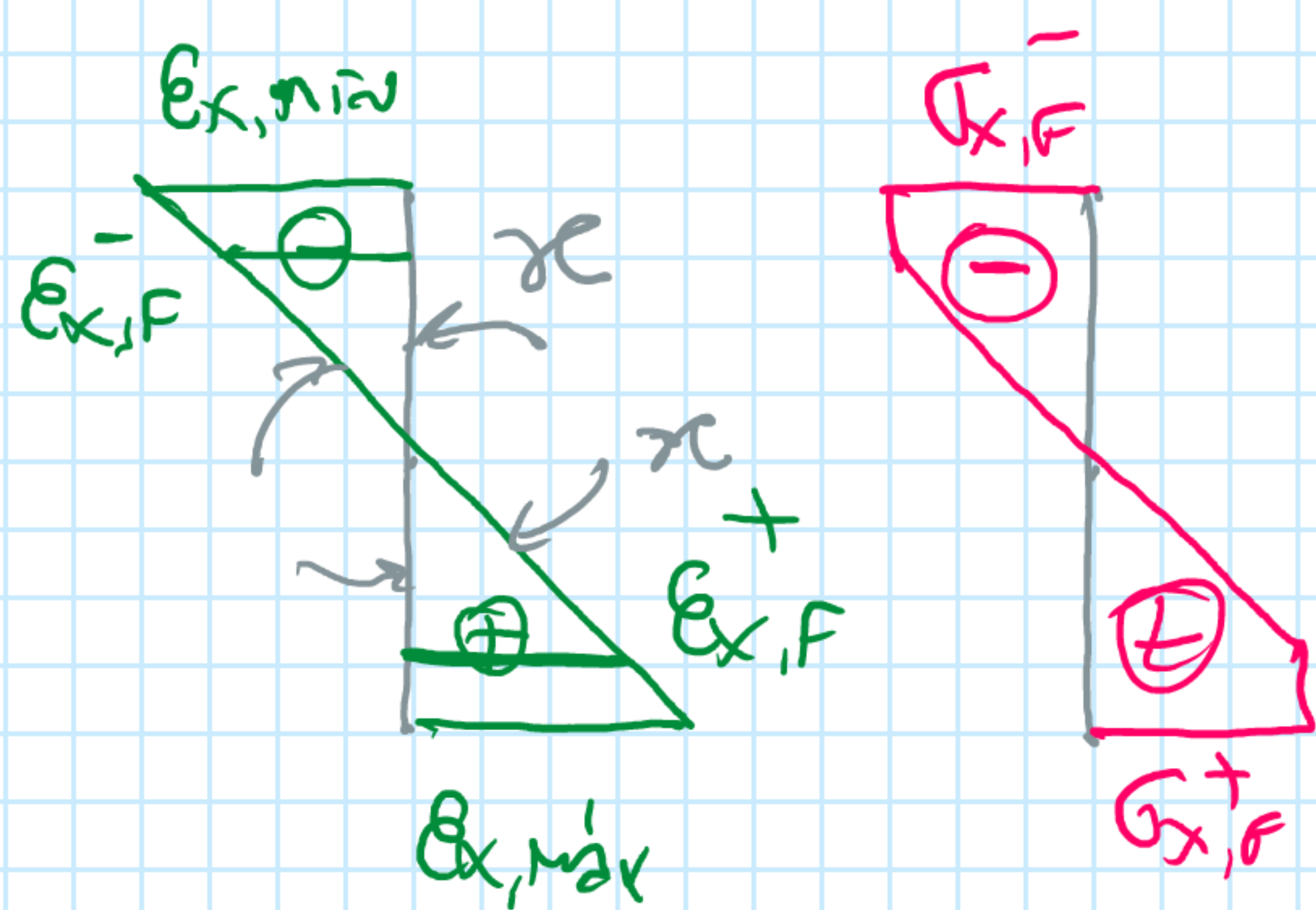
• Aplicación a nuestro problema de la sección metálica:

I) si  $M_y = M_{y,E} \rightarrow z_E = h \rightarrow \epsilon_{x,F} = h \cdot \kappa_E$



$$\boxed{\kappa_E = \frac{\epsilon_{x,F}}{h} \quad \rho_E = \frac{h}{\epsilon_{x,F}}} \quad (14)$$

II)  $M_y > M_{y,E}$ :



$$\boxed{z_E \cdot \kappa = \epsilon_{x,F} \quad \frac{z_E}{\rho} = \epsilon_{x,F}} \quad (15)$$

DE (14)

DE (15)

$$\epsilon_{x,F} = h \cdot \kappa_E$$

$$\epsilon_{x,F} = z_E \cdot \kappa$$

$$h \cdot \kappa_E = z_E \cdot \kappa$$

$$\boxed{\frac{z_E}{h} = \frac{\kappa_E}{\kappa}} \quad (16a)$$

$$\boxed{\frac{z_E}{h} = \frac{\rho}{\rho_E}} \quad (16b)$$

(5)  $\rightarrow M_y = bh^2 \sigma_{x,F} \left[ 1 - \frac{1}{3} \frac{z_E^2}{h^2} \right] = bh^2 \sigma_{x,F} \left[ 1 - \frac{1}{3} \frac{\kappa_E^2}{\kappa^2} \right]$  (17a)

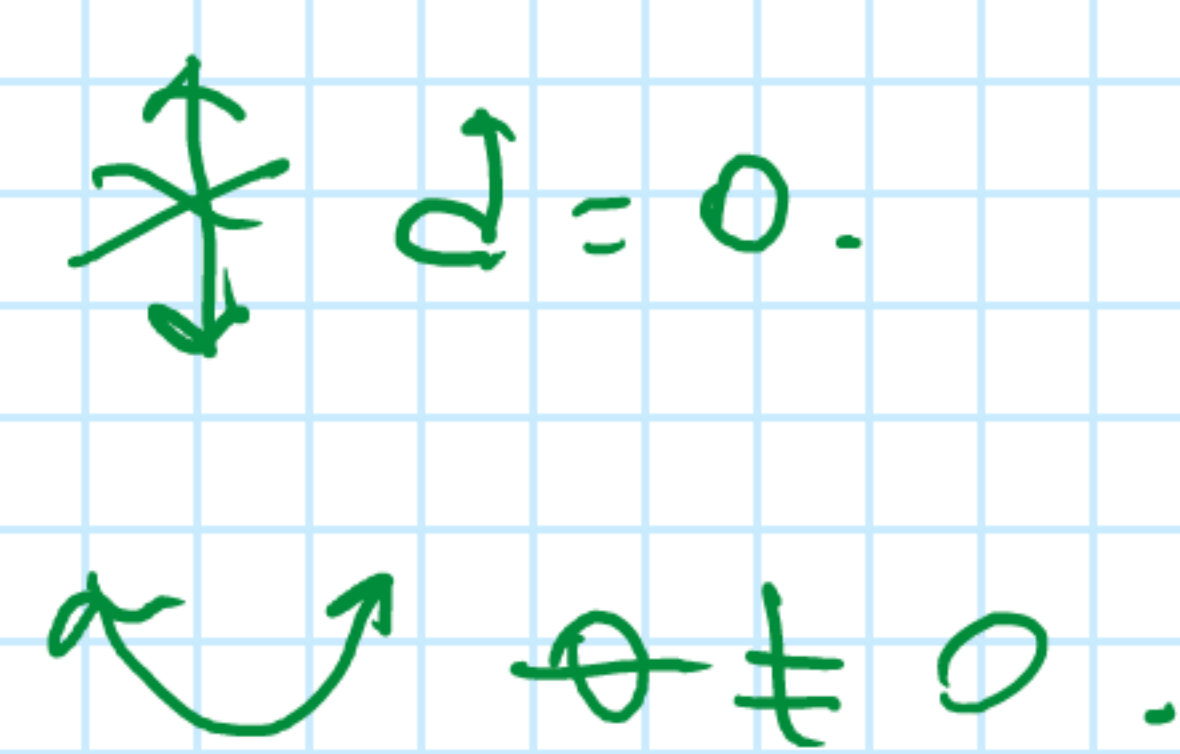
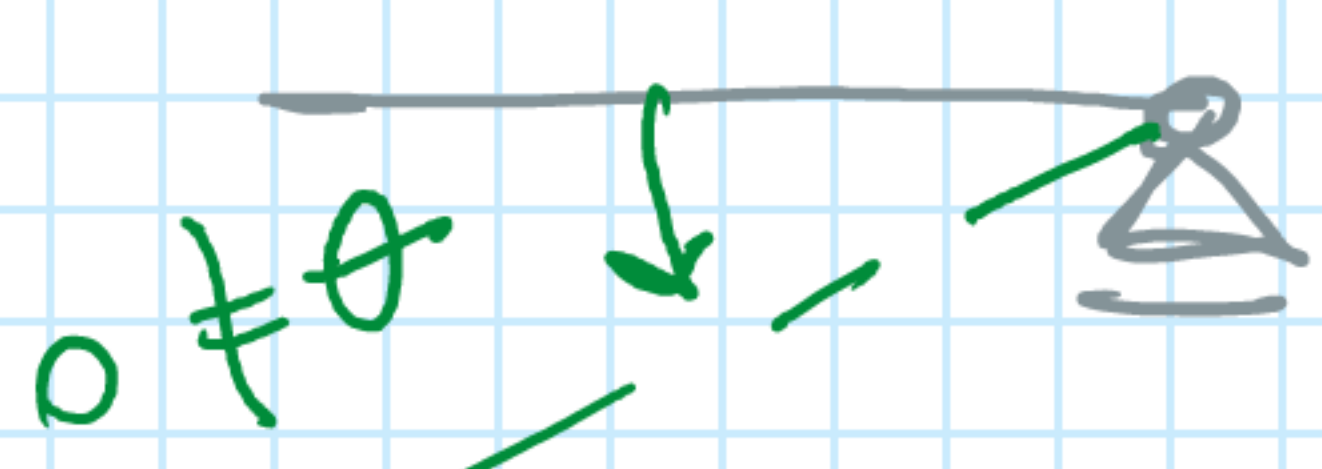
(6)  $\rightarrow M_y = \frac{3}{2} M_{y,E} \left[ 1 - \frac{1}{3} \frac{z_E^2}{h^2} \right] = \frac{3}{2} M_{y,E} \left[ 1 - \frac{1}{3} \frac{\kappa_E^2}{\kappa^2} \right]$  (17b)

$$M_y = bh^2 \sigma_{x,F} \left[ 1 - \frac{1}{3} \frac{\rho^2}{\rho_E^2} \right] \quad (18a)$$

$$M_y = \frac{3}{2} M_{y,E} \left[ 1 - \frac{1}{3} \frac{\rho^2}{\rho_E^2} \right] \quad (18b)$$

I) si  $M_y = M_{y,E} \Rightarrow z_E = h \rightarrow \kappa = \kappa_E \sim \rho = \rho_E$

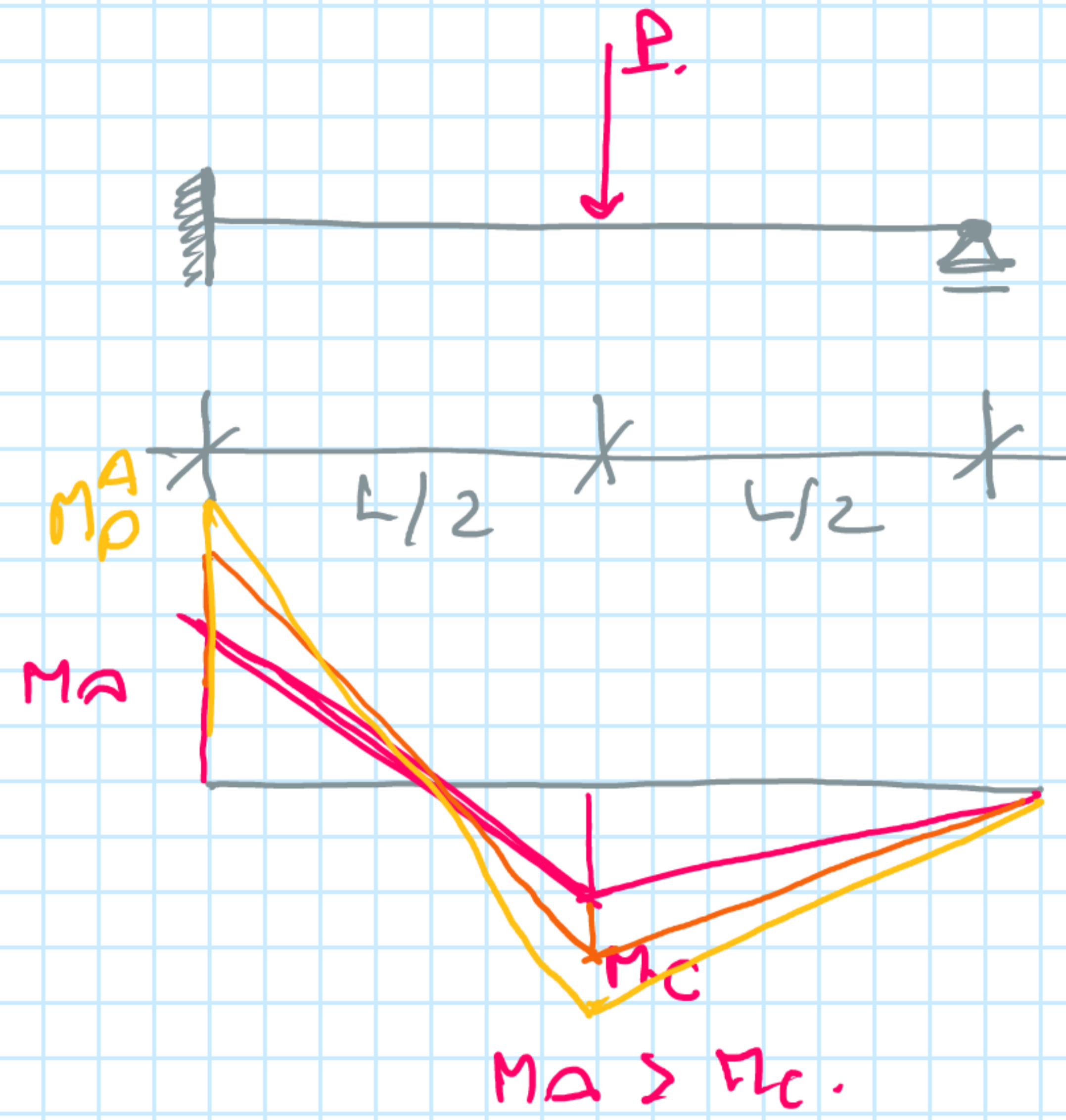
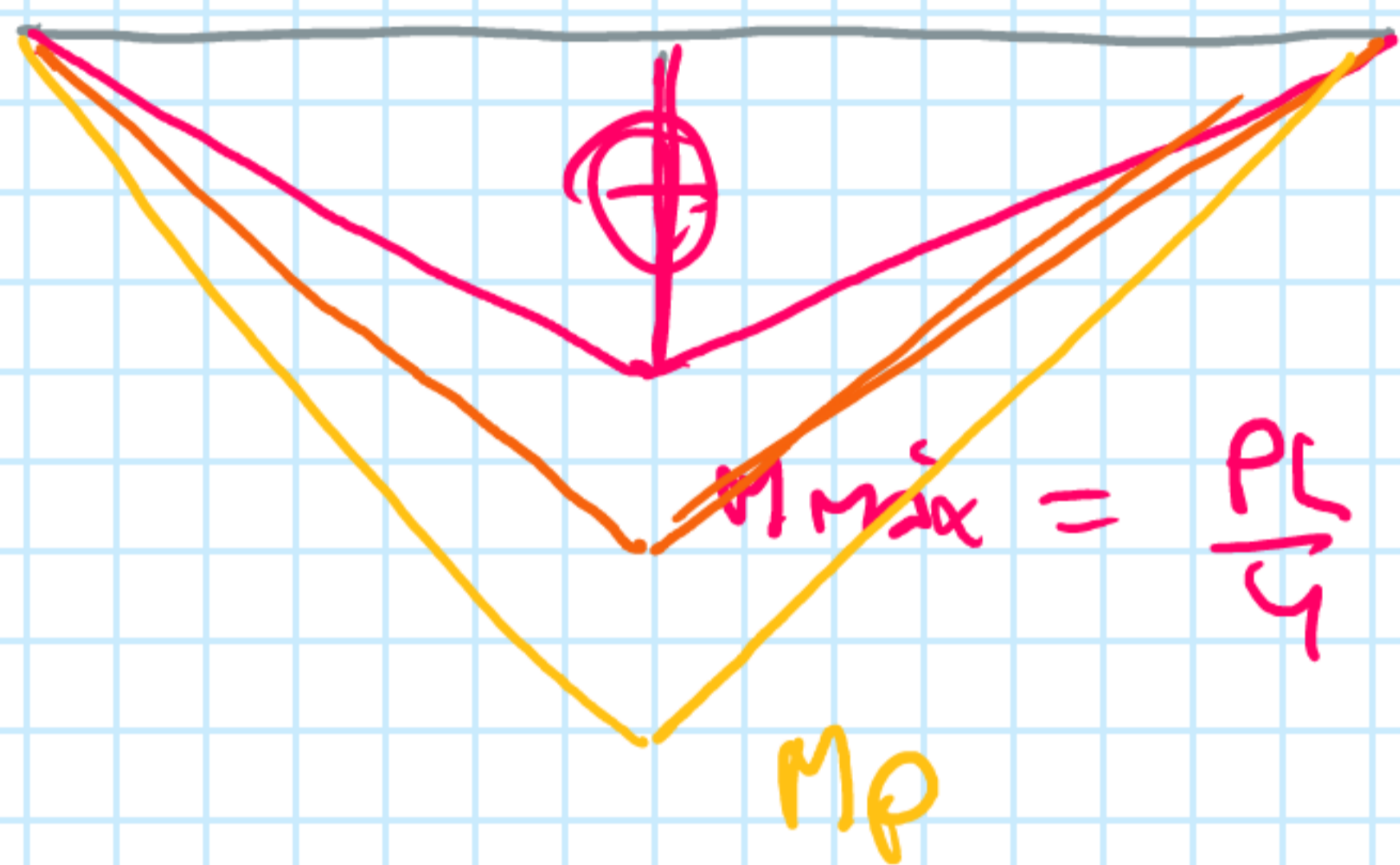
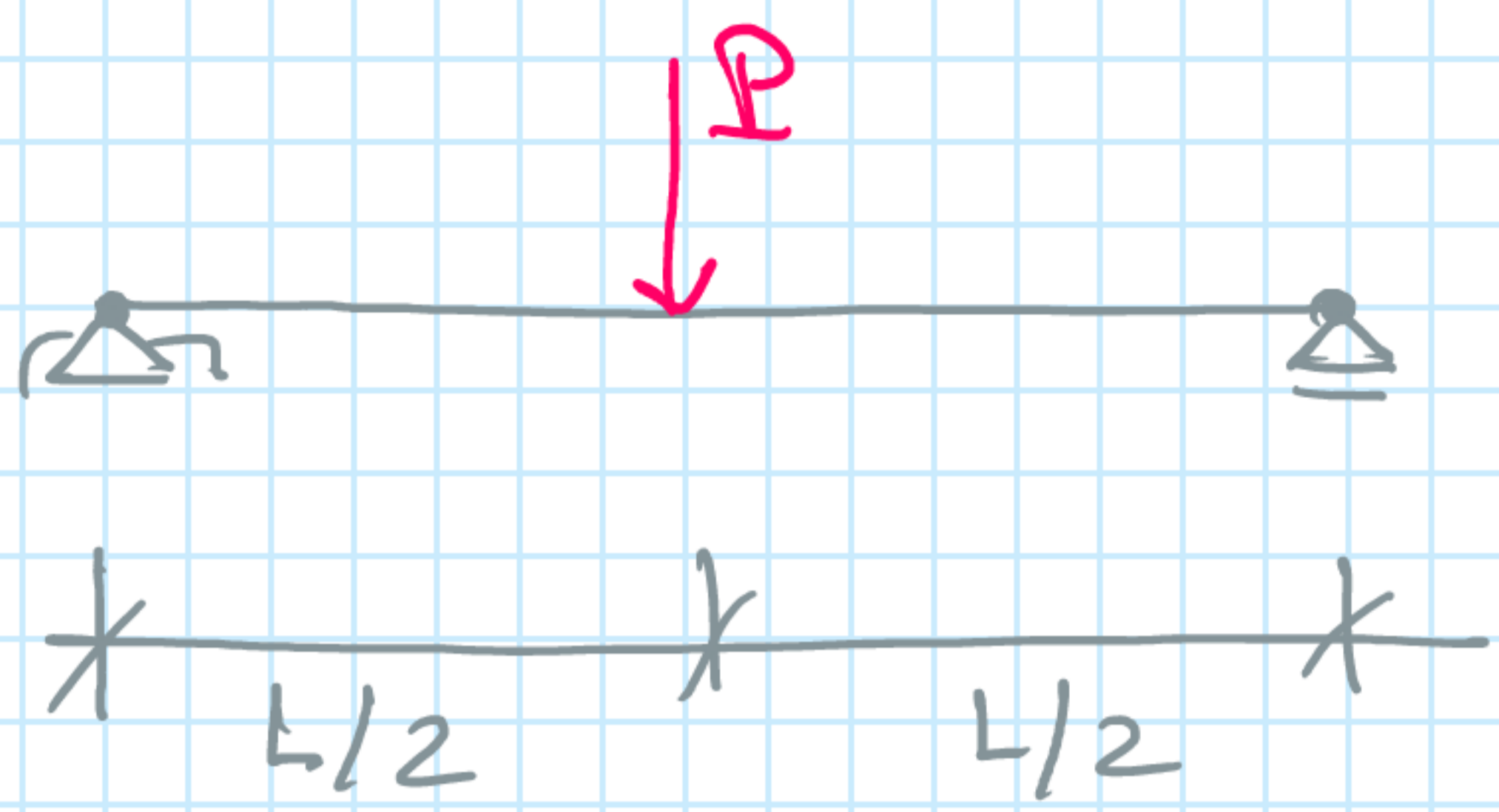
II) si  $M_y = M_{y,p} \rightarrow z_E = 0 \rightarrow \frac{z_E}{h} = 0 = \frac{\kappa_E}{\kappa} \rightarrow \kappa \rightarrow \infty$   
 $\frac{z_E}{h} = 0 = \frac{\rho}{\rho_E} \rightarrow \rho = 0$



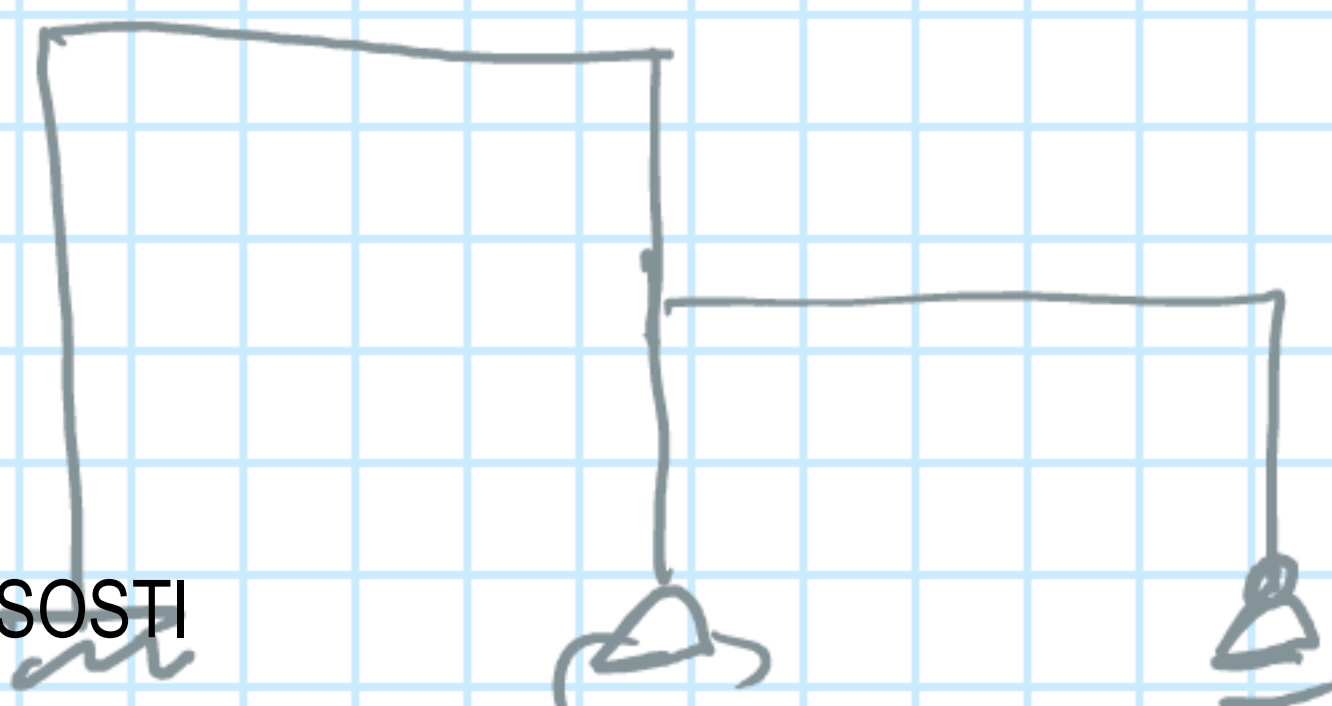
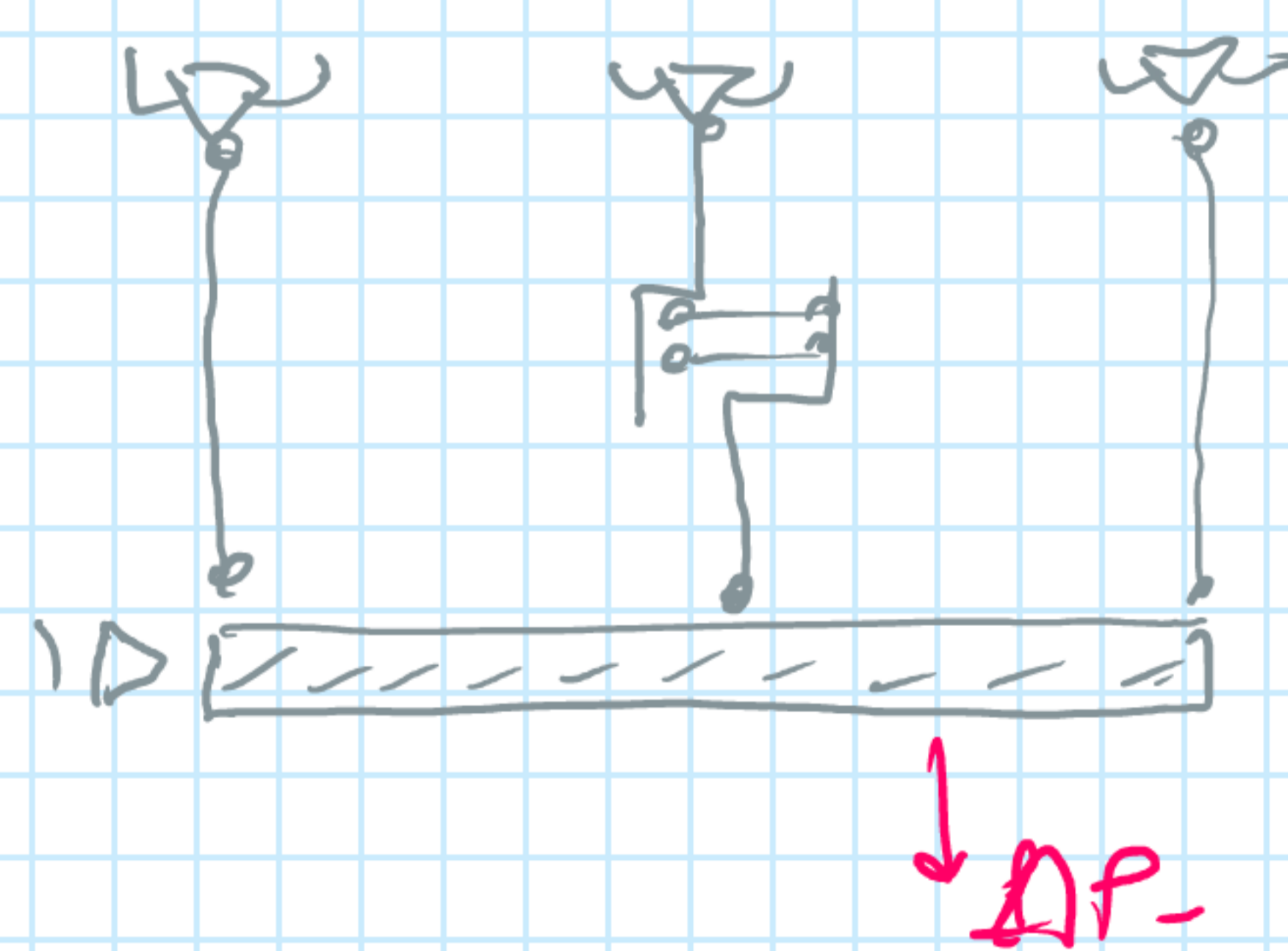
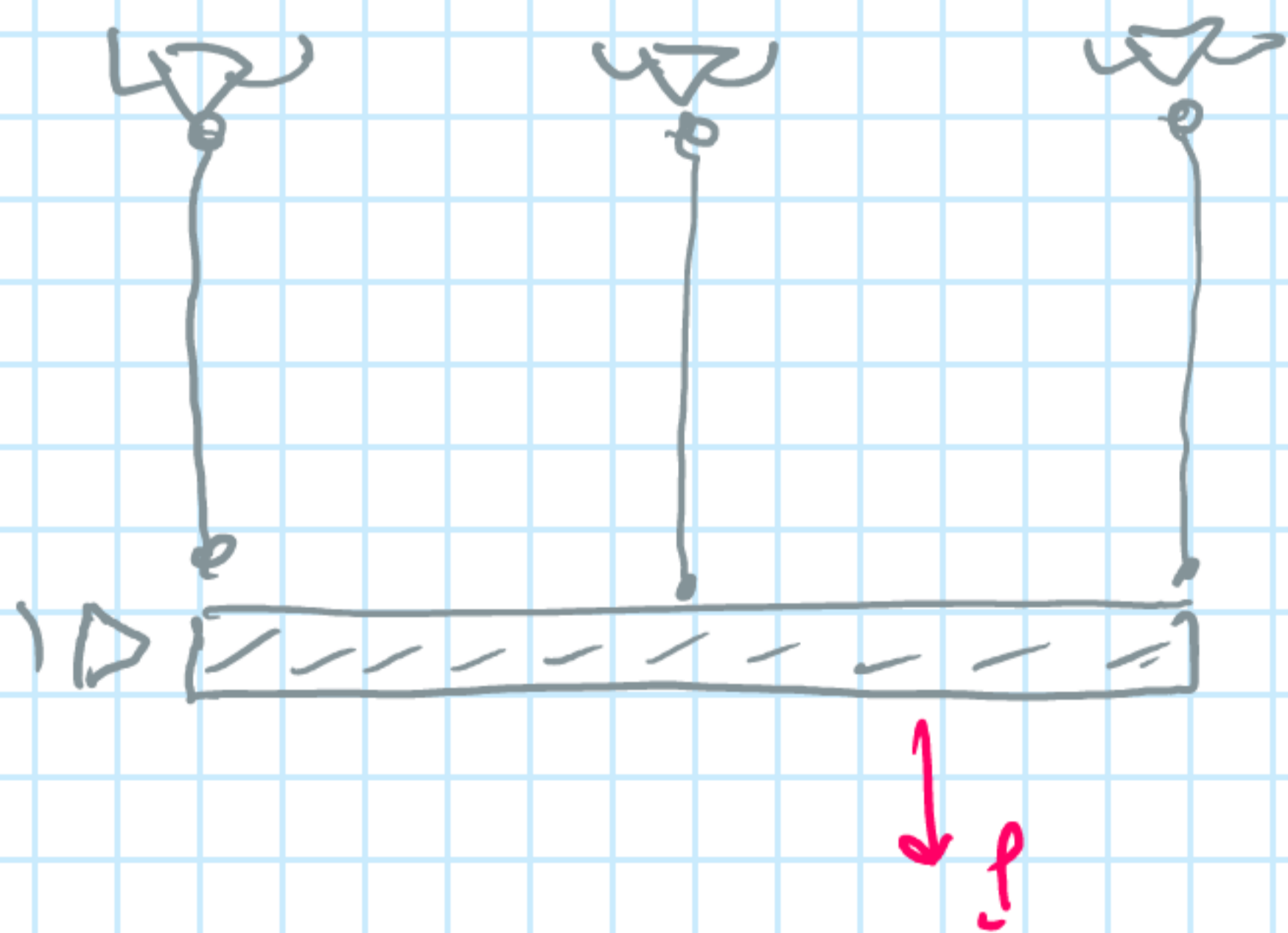
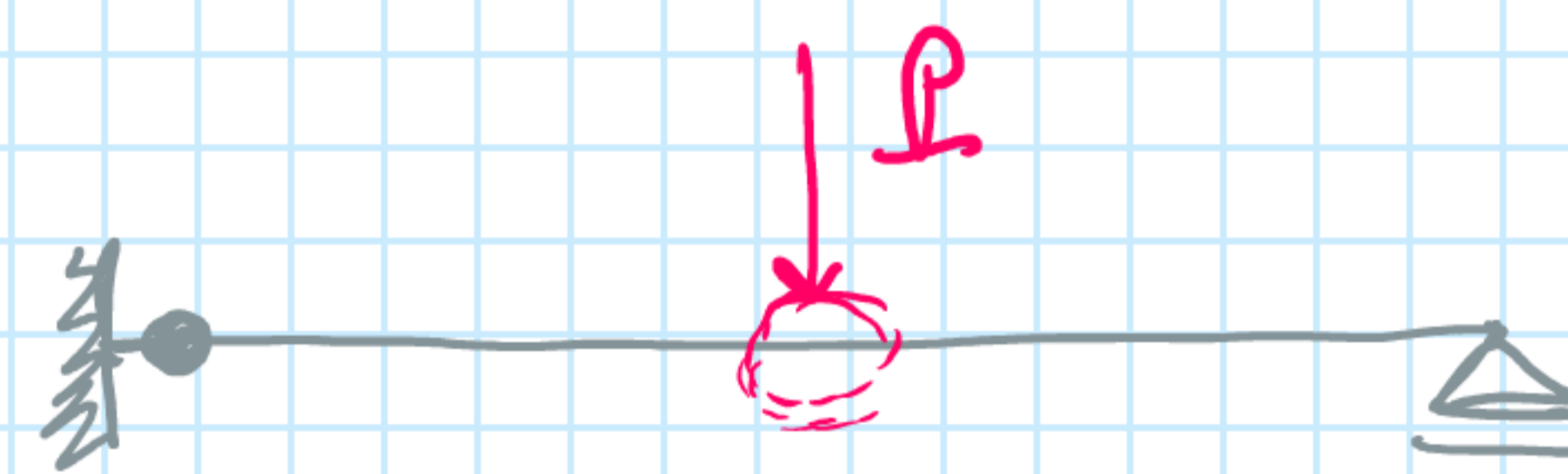
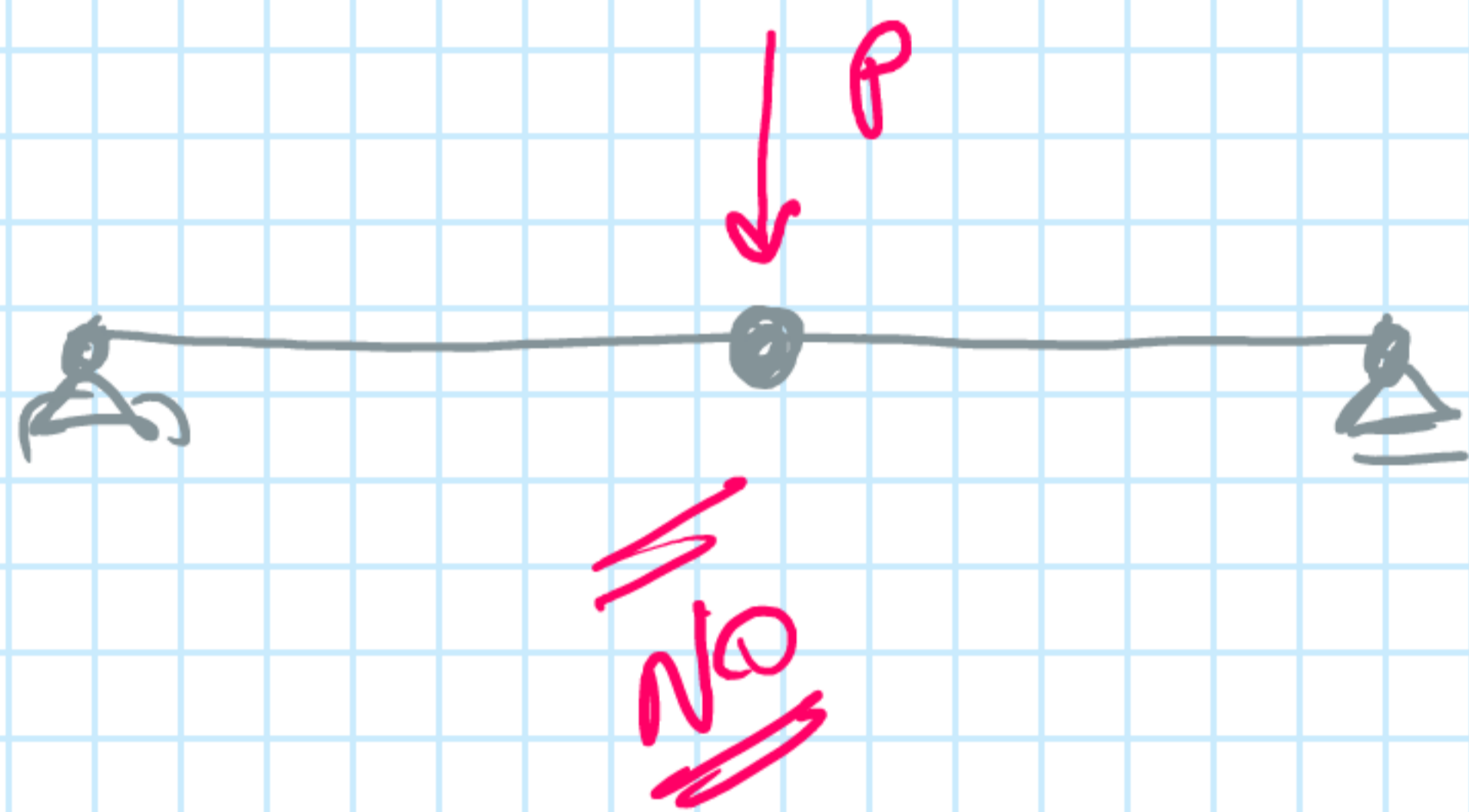


# 13.06 - ARTICULACIONES PLÁSTICAS:

martes, 21 de diciembre de 2021 11:31



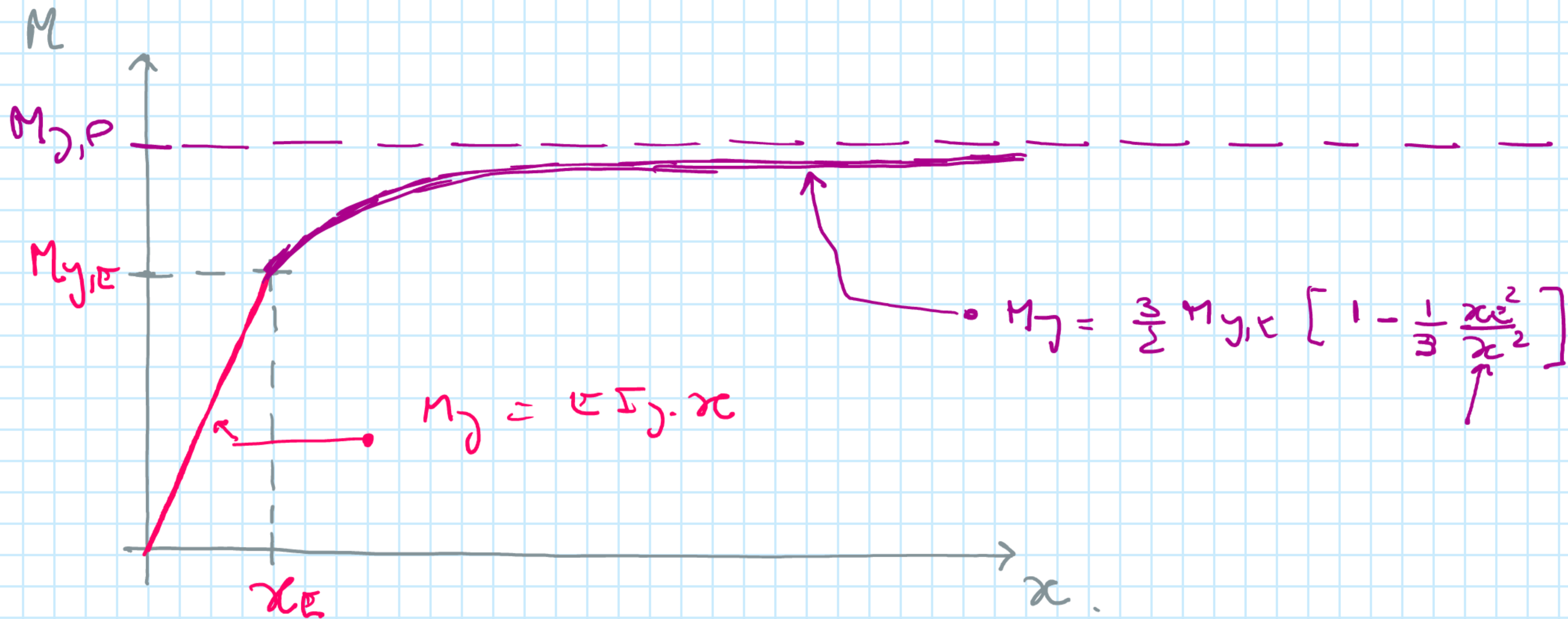
ME → PLASTICAS RENDIMIENTO → MP.





# 13.07 - DIAGRAMA MOMENTO-CURVATURA:

martes, 21 de diciembre de 2021 11:42



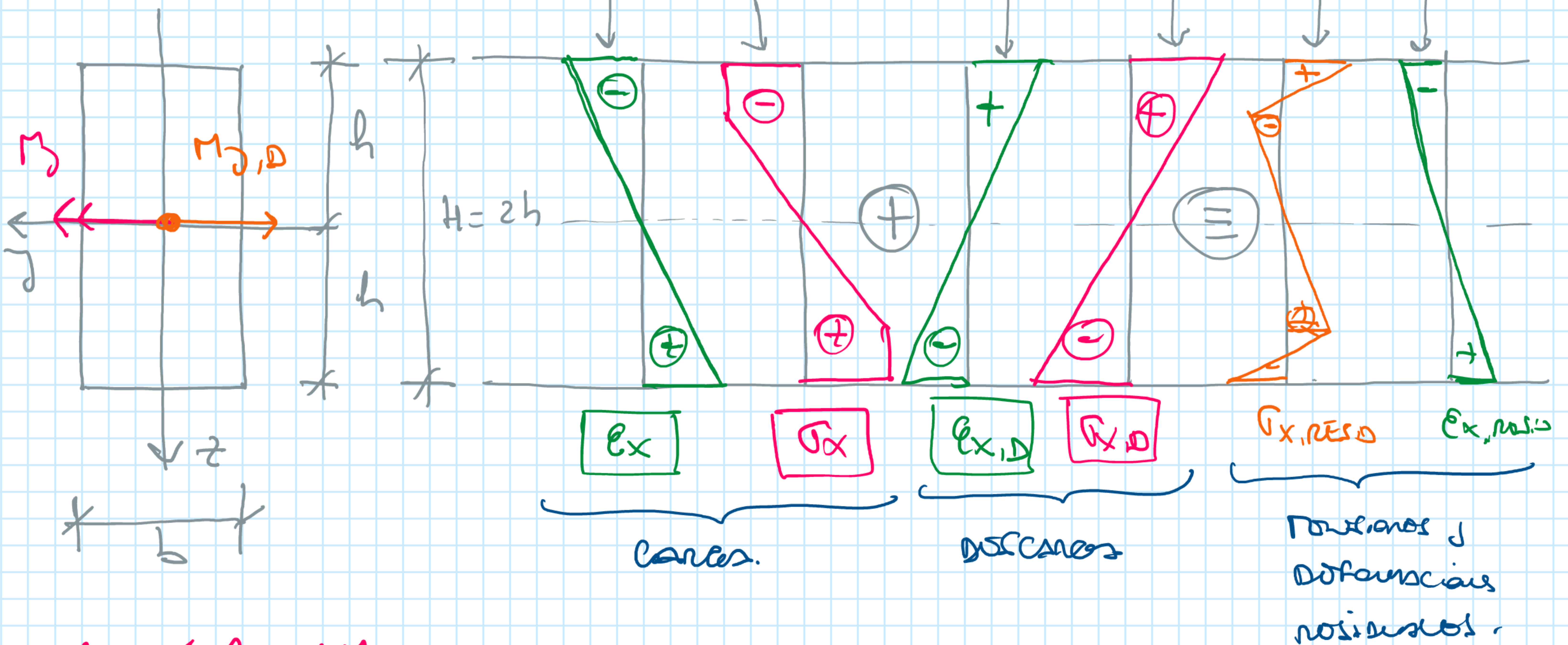
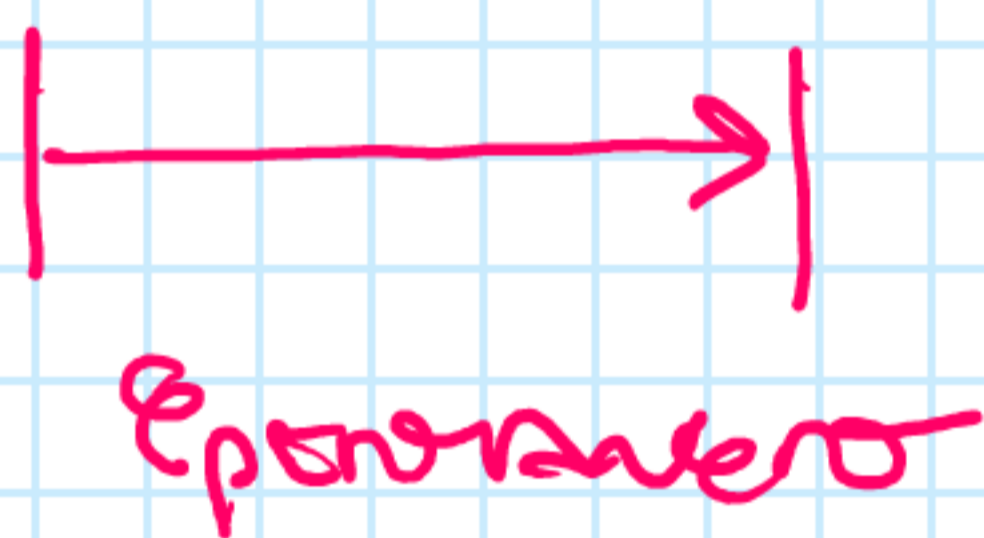
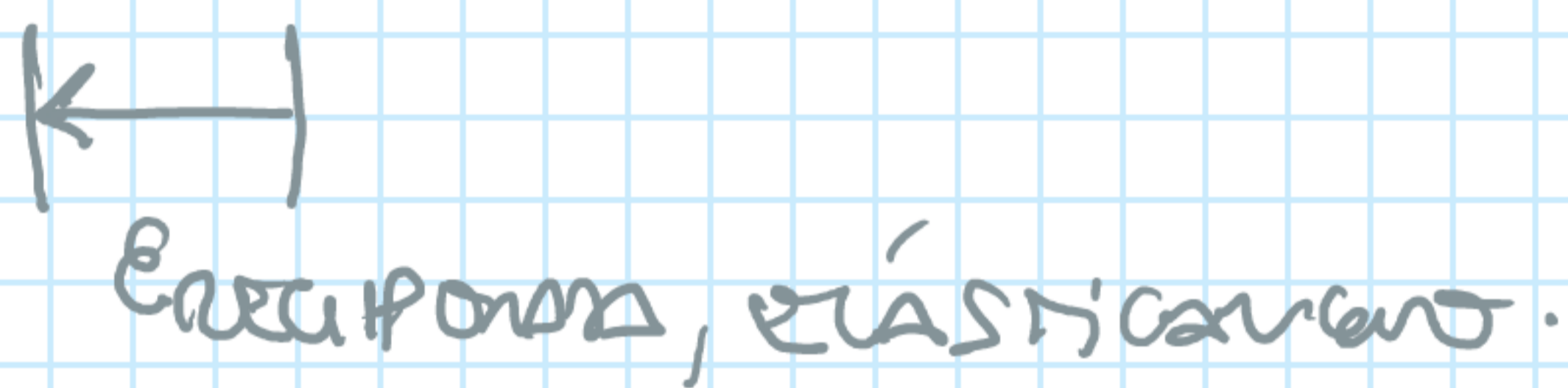
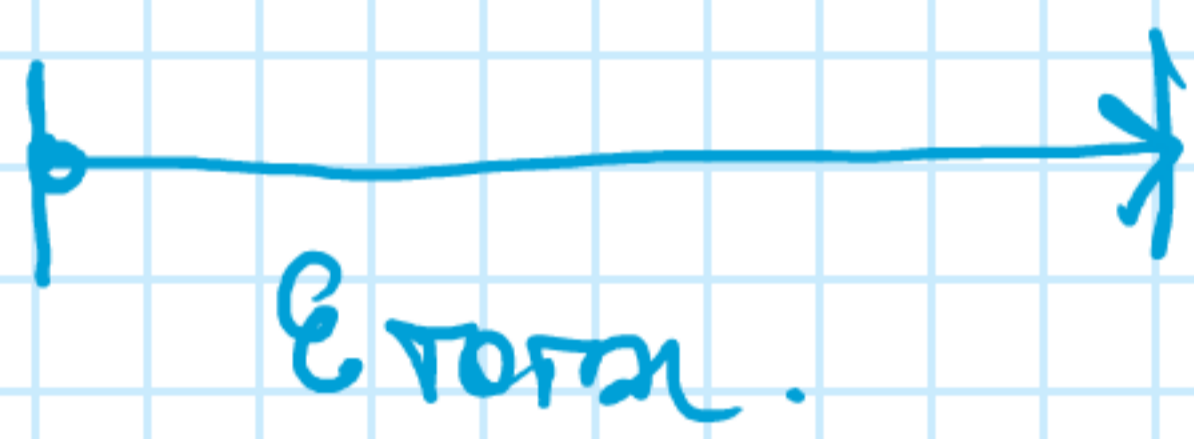
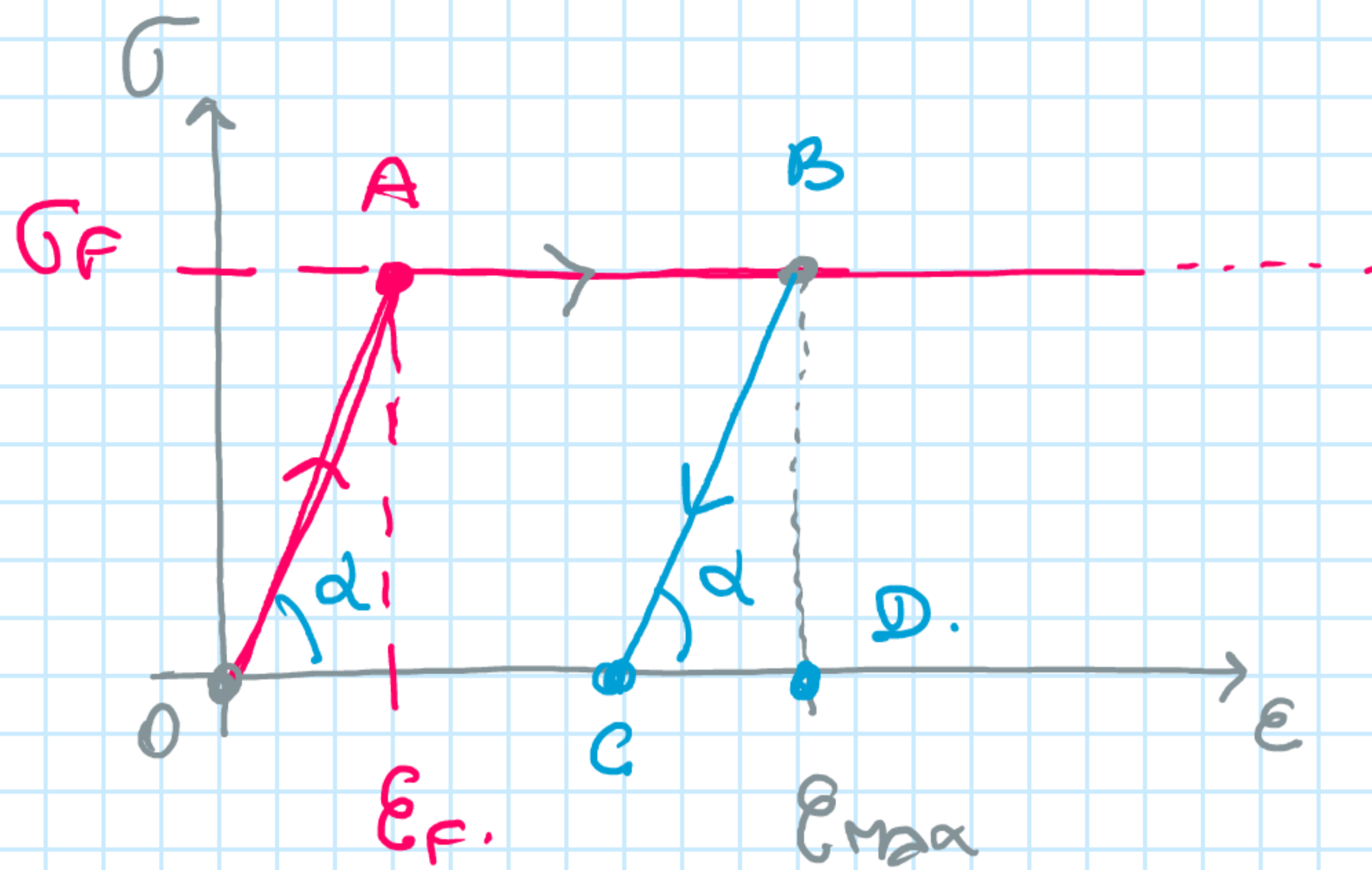
• Si la estructura se comporta elásticamente  $\rightarrow M_y \leq M_{y,E} \rightarrow$

$$\rightarrow x = \theta' = \frac{M}{E I_y} \rightarrow \left\{ \begin{array}{l} M_y = E I_y \cdot x = B_y \cdot x \\ M_y = \frac{E I_y}{\theta} = \frac{E I_y}{\theta} \end{array} \right.$$



# 13.08 - DESCARGA:

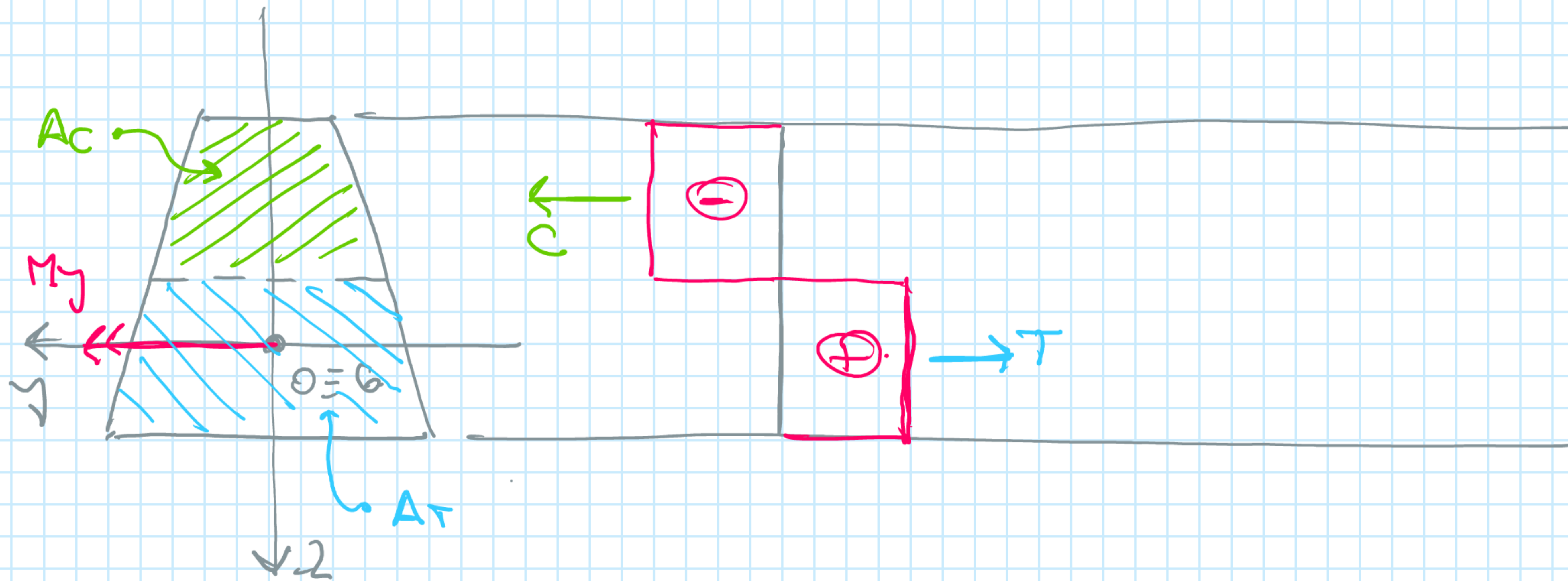
martes, 21 de diciembre de 2021 11:48





# 13.09 - SECCIONES CON 1 EJE DE SIMETRÍA:

martes, 21 de diciembre de 2021 11:59



$$\sum F_x = 0 = \int_A \tau_x \cdot dA = 0 \rightarrow T = C \Rightarrow \tau = C$$

$$M_y = \int_A \tau_x \cdot z \cdot dA$$

$$\tau = C$$

$$\sigma_F^+ \cdot A_T = \sigma_F^- \cdot A_C$$

$$\text{Si } |\sigma_F^+| = |\sigma_F^-|$$

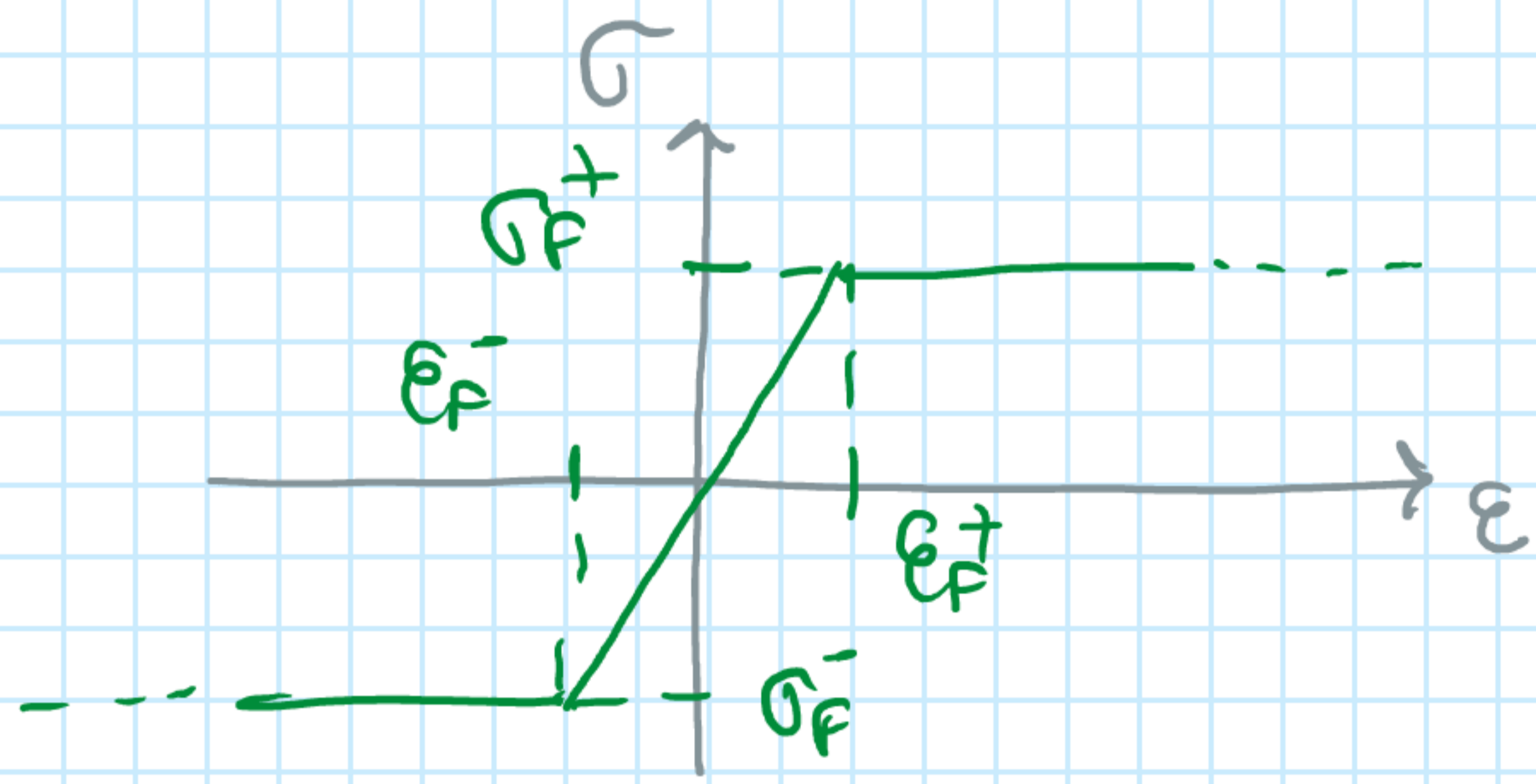
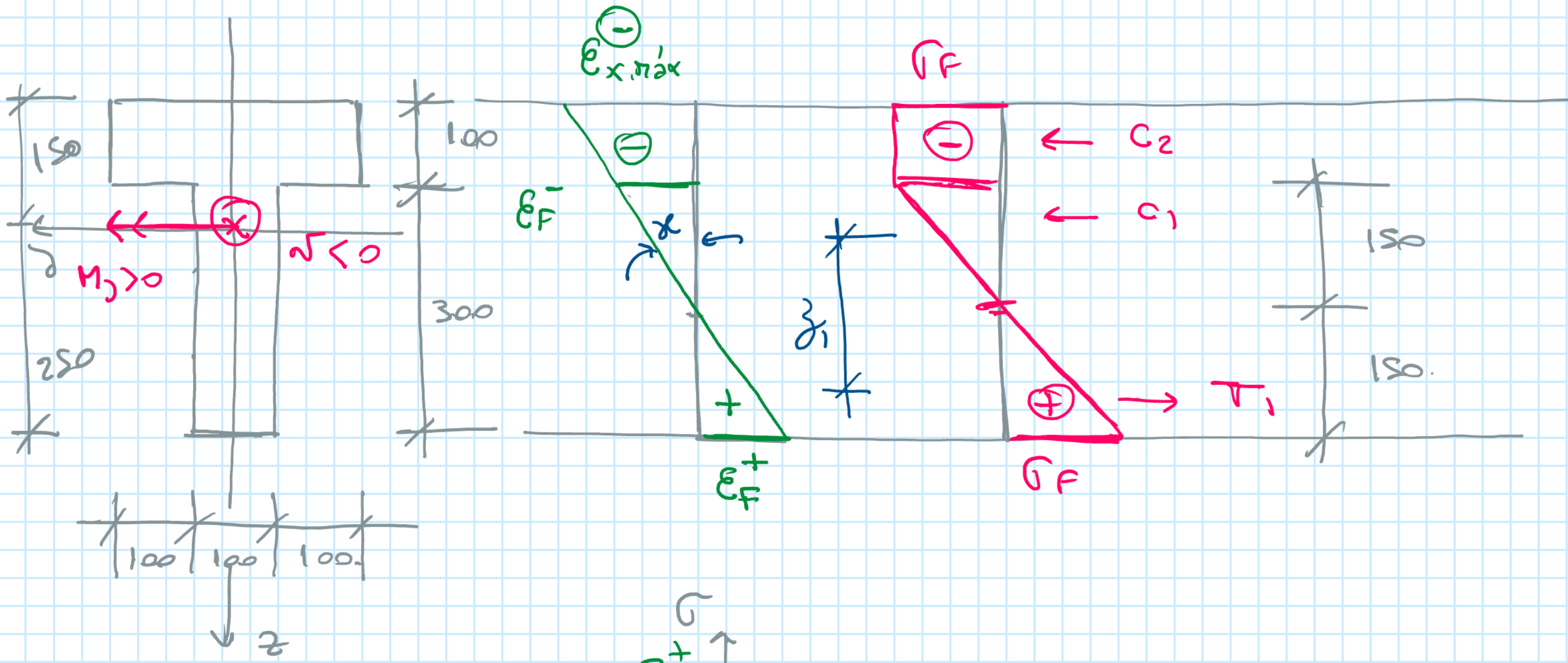
$$\boxed{A_T = A_C}$$



# 13.10 - FLEXIÓN COMPUESTA - EJERCICIO:

martes, 21 de diciembre de 2021 12:07

$$N = \int_A \sigma_x \, dA \quad M_y = \int_A \sigma_x \cdot z \cdot dA$$



$$E = 20000 \text{ kN/cm}^2$$

$$|\sigma_F^+| = |\sigma_F^-| = 24 \text{ kN/cm}^2$$

$$\epsilon_F = \frac{\sigma_F}{E} = 1,2 \cdot 10^{-3}$$

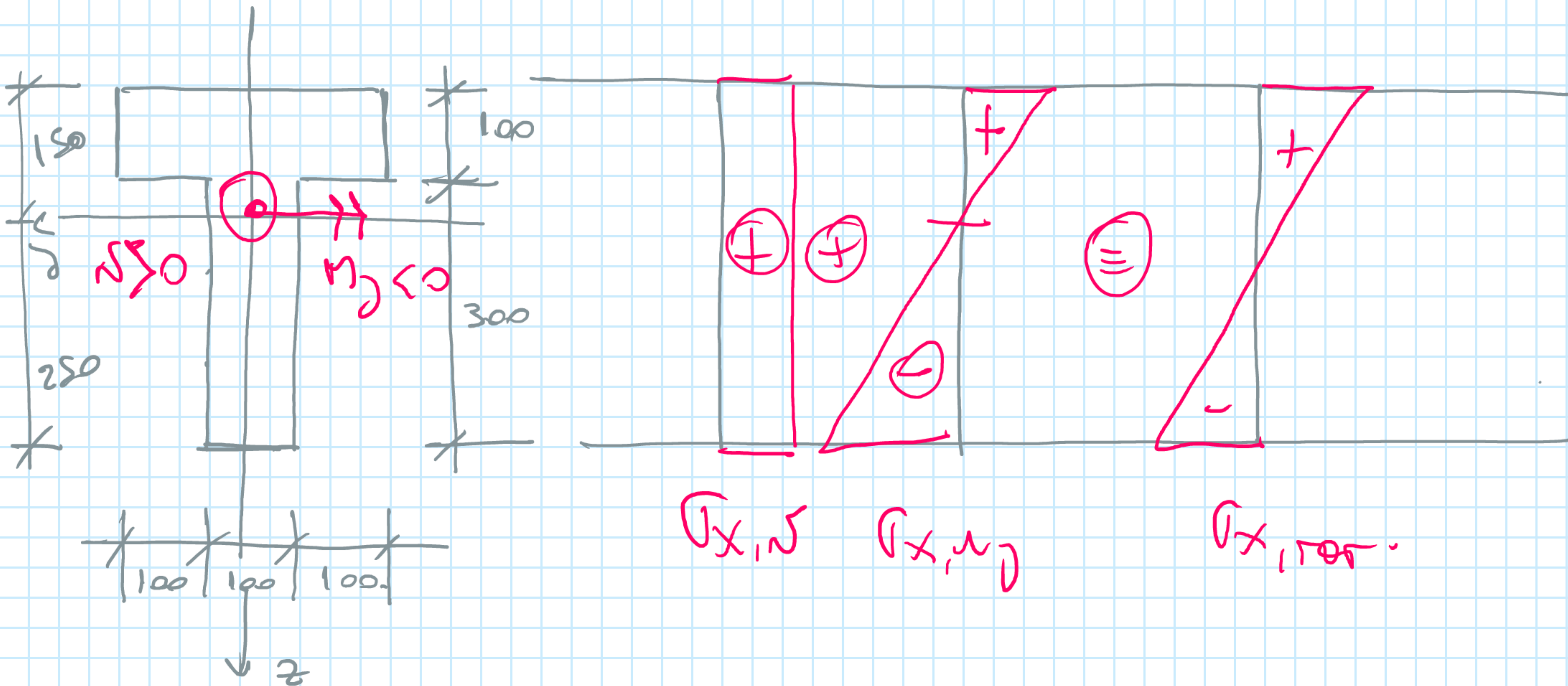
$$= 1,2 \text{‰}$$

$$N = \int_A \sigma_x \cdot dA = 24 \text{ kN/cm}^2 \cdot \frac{10 \text{ cm} \cdot 15 \text{ cm}}{2} - 24 \text{ kN/cm}^2 \cdot \frac{10 \text{ cm} \cdot 15 \text{ cm}}{2} - 24 \frac{\text{kN}}{\text{cm}^2} \cdot 10 \text{ cm} \cdot 30 \text{ cm} =$$

$$N = 1200 \text{ kN} - 1200 \text{ kN} - 7200 \text{ kN} \rightarrow \boxed{N = -7200 \text{ kN}}$$

$$M_y = 24 \cdot \frac{15 \cdot 10}{2} \cdot (25 - 5) + 24 \cdot \frac{15 \cdot 10}{2} \cdot (0) + 24 \cdot 10 \cdot 30 \cdot (15 - 5)$$

$$M_y = 36000 + 0 + 72000 \rightarrow \boxed{M_y = 108.000 \text{ kNcm} = 1080 \text{ kNm}}$$



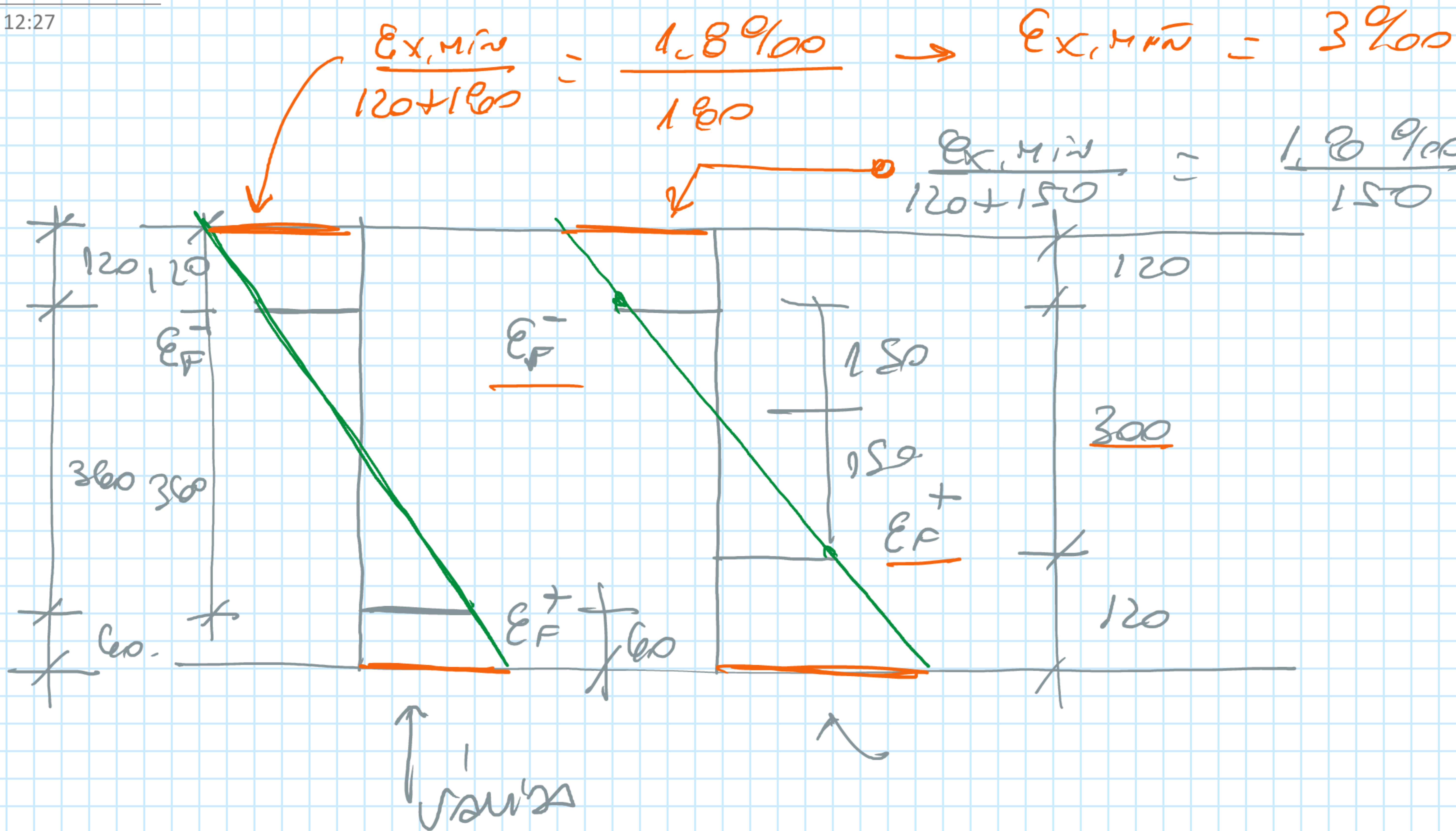
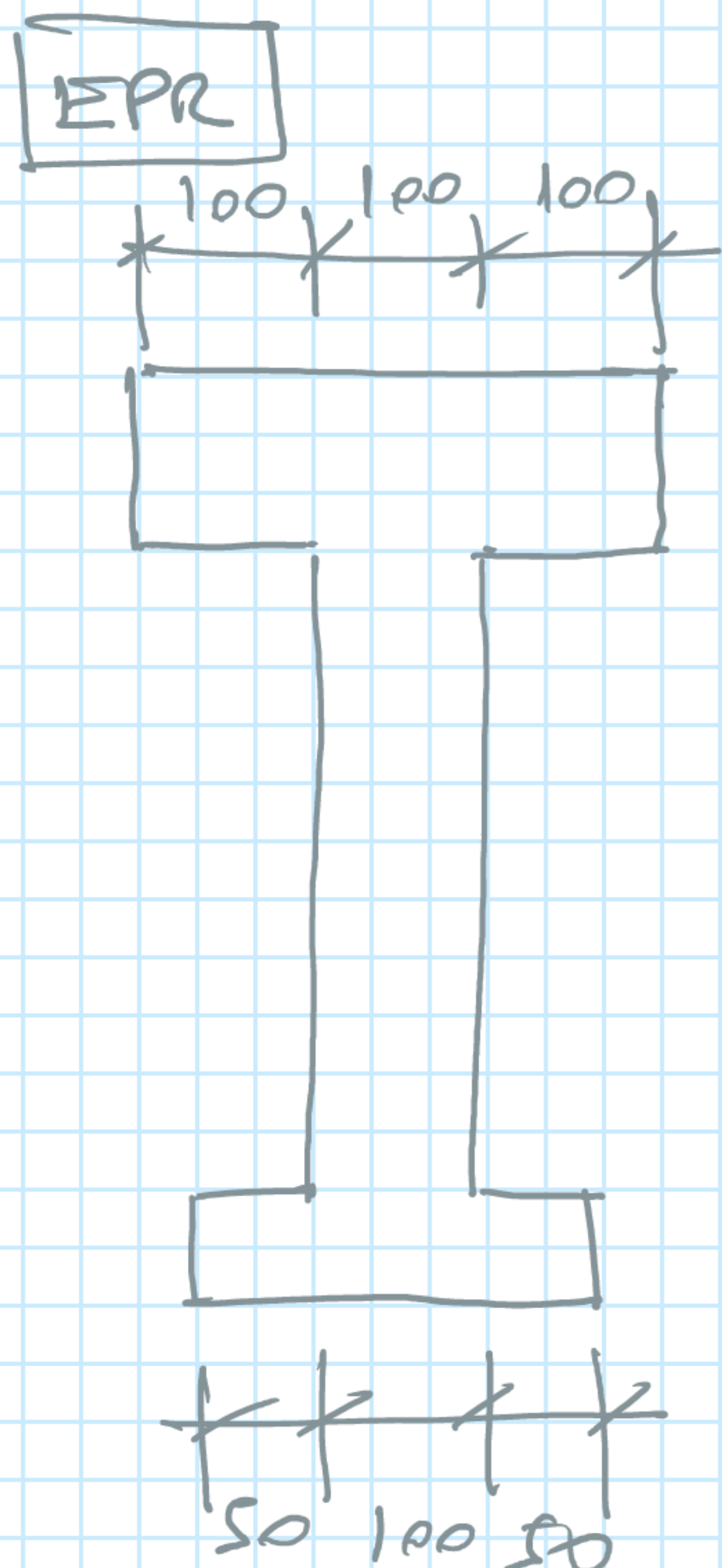
$$M_y = 1080 \text{ kNm}$$

$$N = -7200$$



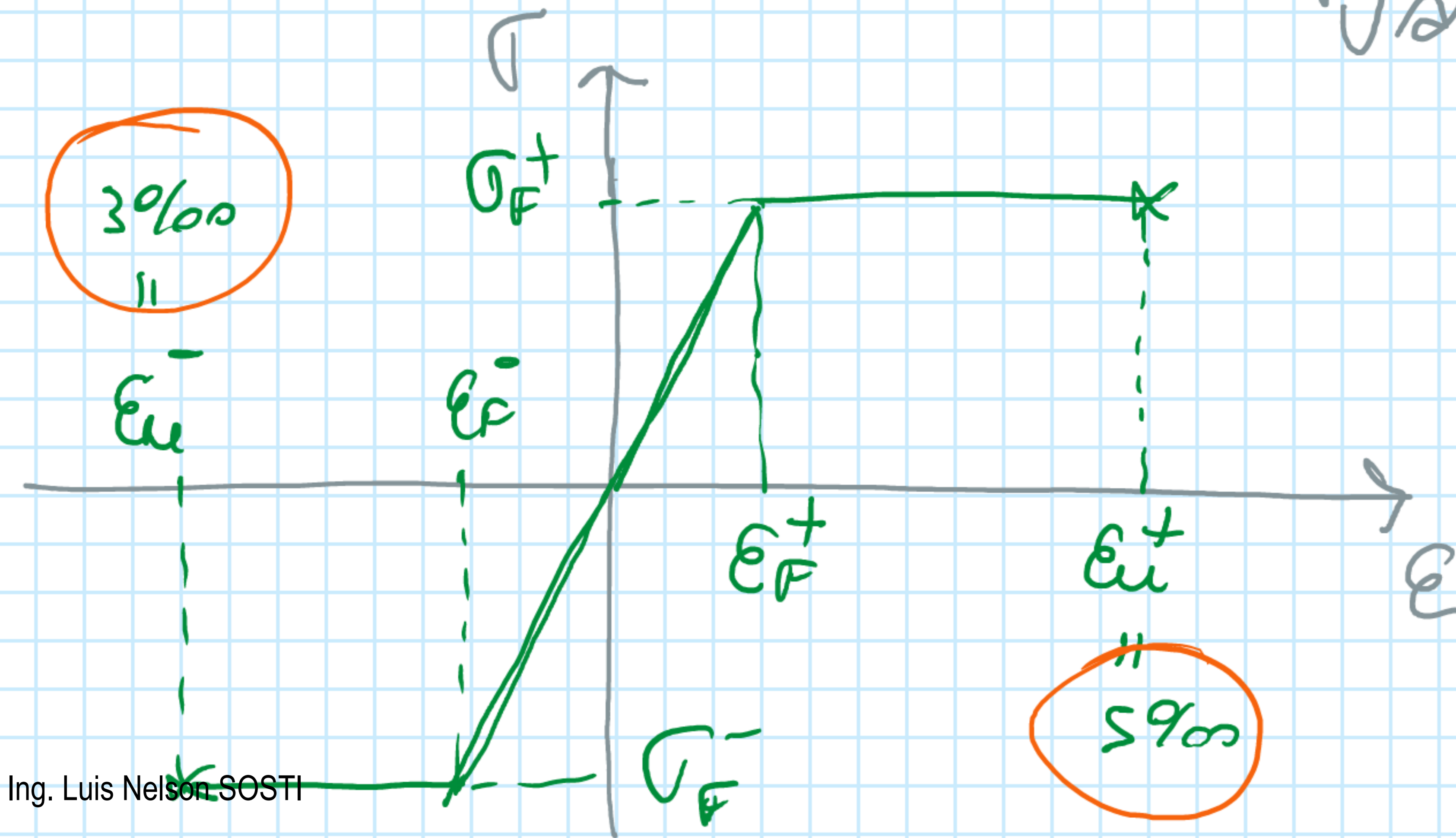
# 13.11 - 2º EJERCICIO:

martes, 21 de diciembre de 2021 12:27



$$E_{x, \min} = \frac{1,8\% \cdot 100}{120+120} \rightarrow E_{x, \min} = 3\% \cdot 100$$

$$E_{x, \min} = \frac{1,8\% \cdot 100}{120+150} \rightarrow E_{x, \min} = 3,27\%$$



$$E = 20000 \text{ kN/cm}^2$$

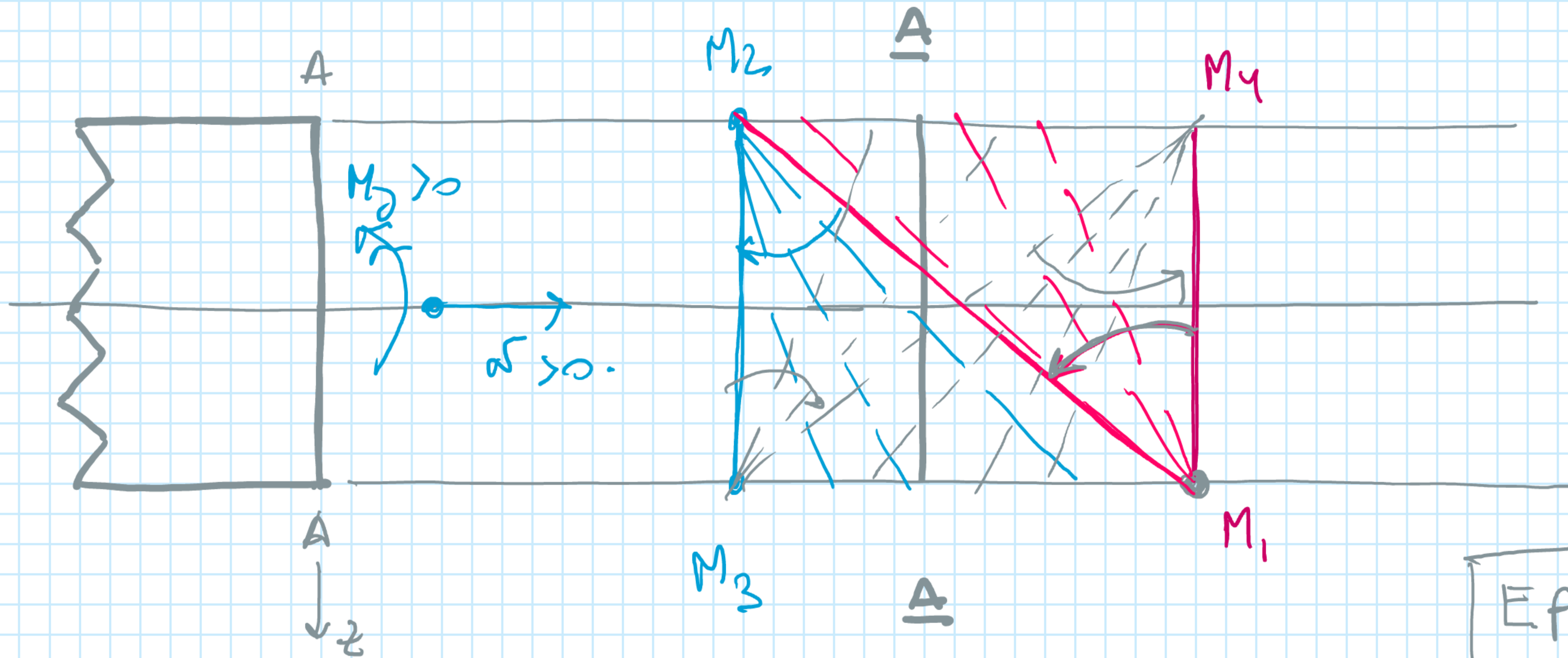
$$|\sigma_F^+| = |\sigma_F^-| = 36 \frac{\text{kN}}{\text{cm}^2}$$

$$|E_F| = \frac{\sigma_F}{E} = \frac{36}{20000} = 1,8 \cdot 10^{-3} = 1,8\%$$



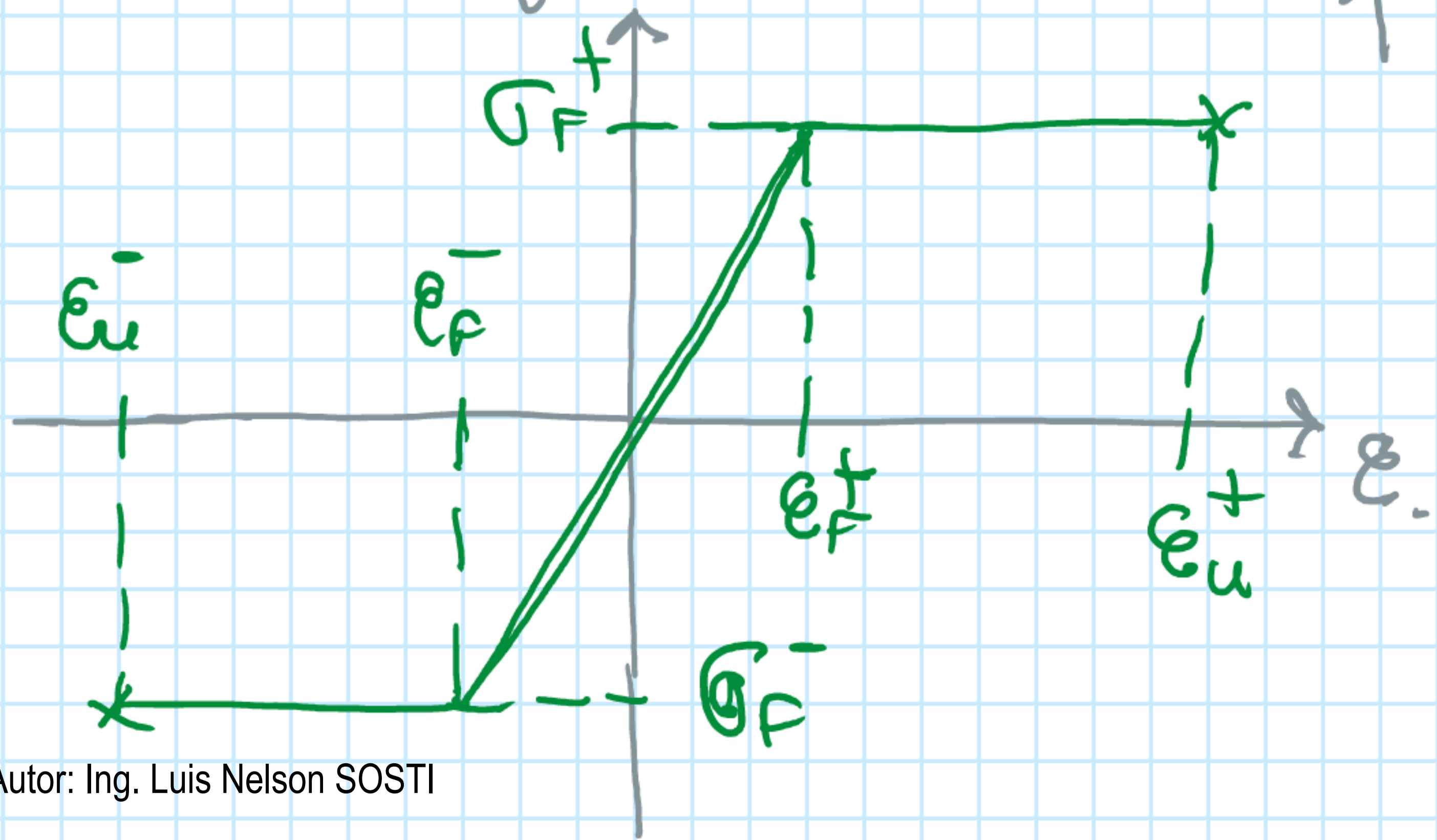
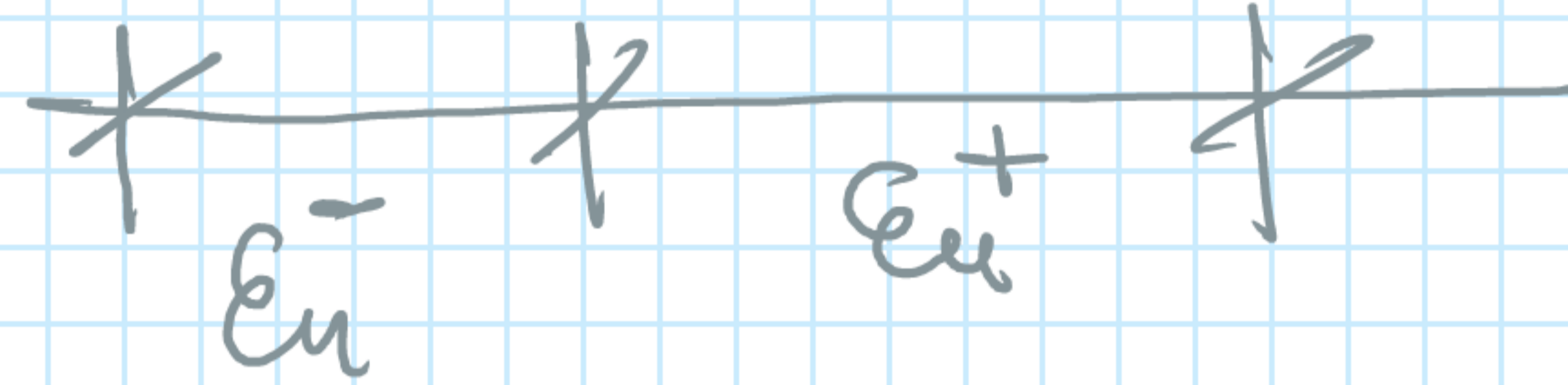
# 13.12 - PLANOS LÍMITES:

martes, 21 de diciembre de 2021 12:39



Planos  
límites

EPR





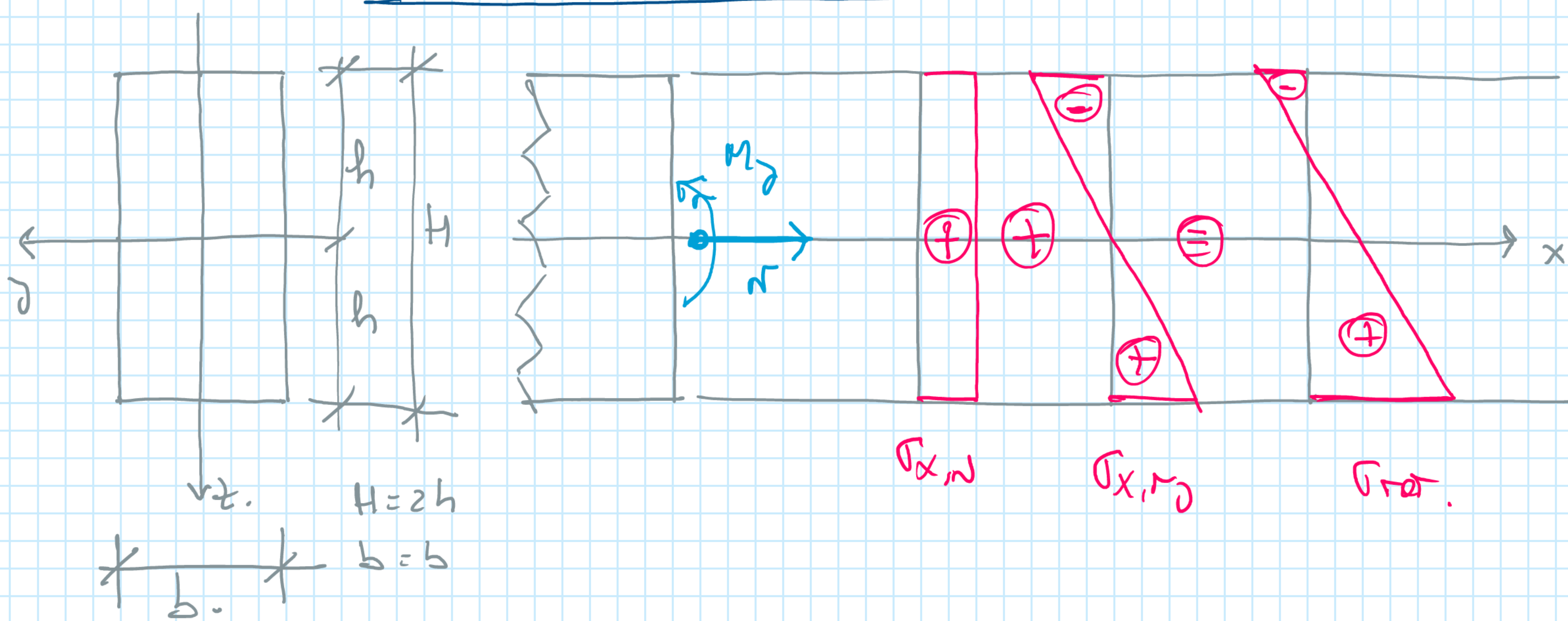
# 13.13 - DIAGRAMA DE INTERACCIÓN:

martes, 21 de diciembre de 2021 12:49

Los diagramas de interacción se crean con el objetivo de poder representar a las combinaciones de los esfuerzos de flexión y normal que conducen a los siguientes estados mecánicos de la sección:

- i.- Comienzo de la plastificación;
- ii.- Plastificación Total;
- iii.- Plastificación parcial con distintos niveles de penetración plástica.

## Comienzo de la Plastificación:



$$\sigma_N = \frac{N}{A}$$

$$\sigma_{x,M}^{max} = \frac{M}{I_y} \cdot b$$

$$\sigma_{tot} = \sigma_N + \sigma_{M_y}$$

$$\sigma_{\sigma, M_y} = \frac{N}{A} - \frac{M}{I_y} h$$

$$\sigma_{\sigma, max} = \frac{N}{A} + \frac{M}{I_y} h$$

Si  $\sigma_{\sigma, max} = \sigma_F = \frac{N}{A} + \frac{M}{I_y} h$ .

$$1 = \frac{N}{A \sigma_F} + \frac{M}{I_y \sigma_F} h \rightarrow M_{y,E} = M_{y,F}$$

$N_E = N_F$

$$\frac{N}{N_F} + \frac{M}{M_{y,F}} = 1$$

si  $M_y = 0 \rightarrow N \neq 0 \rightarrow SA$ .

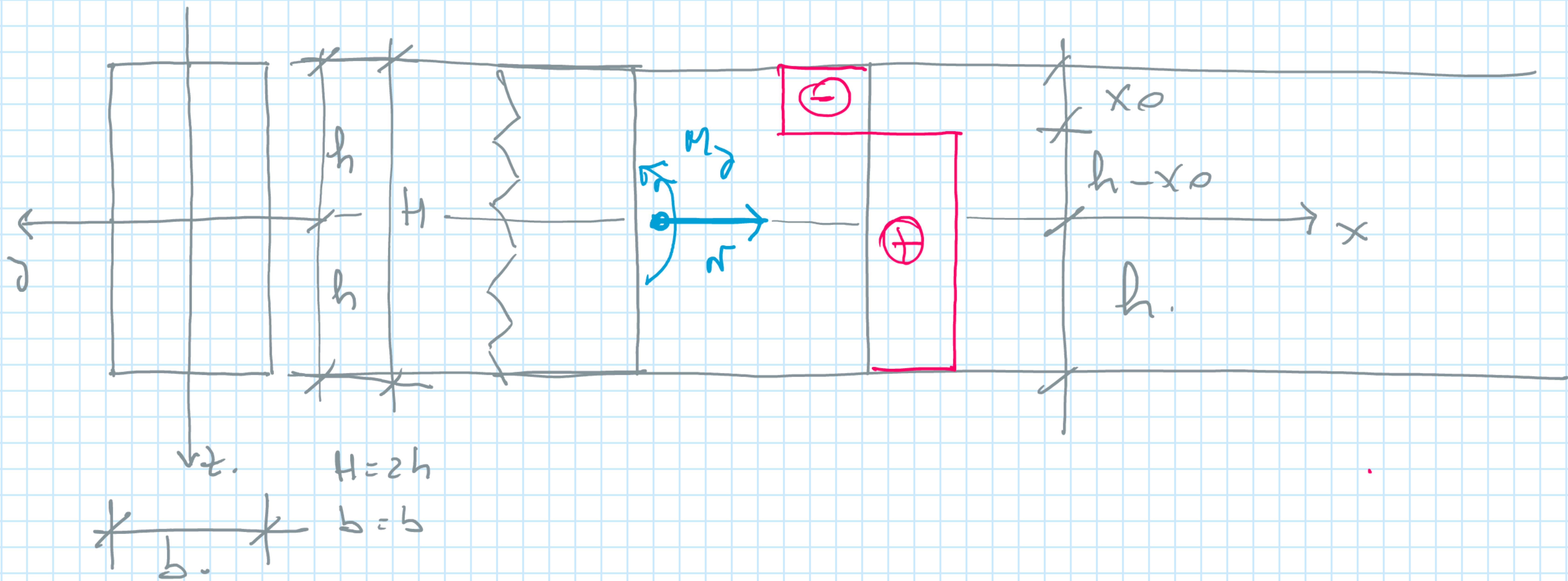
si  $N = 0 \wedge M_y \neq 0 \rightarrow FS$ .



# 13.14 - DIAGRAMA DE INTERACCIÓN:

martes, 21 de diciembre de 2021 12:49

PLASTIFICACIÓN POR LA FLEXIÓN.



$$N_E = N_F = \sigma_F \cdot b \cdot H = \sigma_F \cdot 2bh \quad \text{(SA)}$$

$$\text{(SA)} \quad N = 0 \quad \wedge \quad x_0 = h \quad M_p = \sigma_F b h^2$$

si  $N \neq 0$   $\wedge$   $M_p \neq 0$ .

$$\frac{M_p}{M_{y,p}} + \frac{N^2}{N_F^2} = 1 \quad \rightarrow \quad \frac{M_p}{bh^2\sigma_F} + \frac{N^2}{4b^2h^2\sigma_F^2} = 1$$

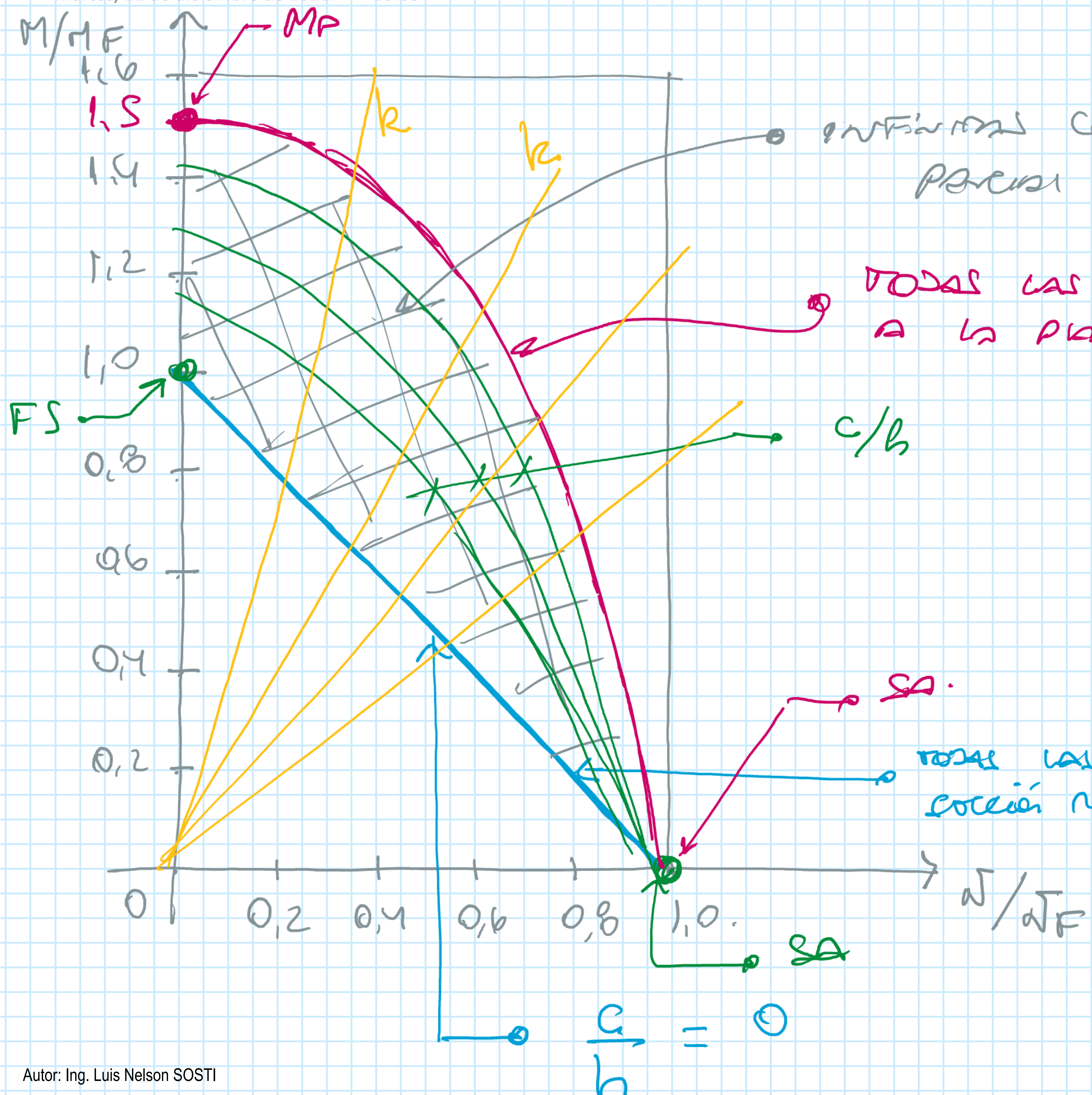
$\frac{3}{2} M_E = \frac{3}{2} M_F$

$$\frac{2}{3} \frac{M_p}{M_F} + \frac{N^2}{N_F^2} = 1$$



# 13.15 - DIAGRAMA PARA SECCIÓN RECTANGULAR:

martes, 21 de diciembre de 2021 13:03



todas las combinaciones de plasticación, para c/distribuidas dentro de la sección

todas las combinaciones de  $N$  y  $M$  que concurren a la plasticación total de la sección.

$$R = \frac{b \cdot e}{H} \Rightarrow e = \frac{M}{N}$$

SA.  
todas las combinaciones de esta sección que concurren al comienzo de la plasticación.