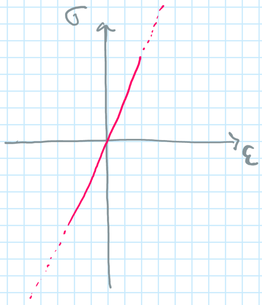
11 - ARAn - 11.01 - SA - INTRODUCCIÓN:

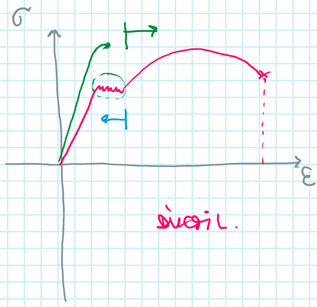
martes, 7 de diciembre de 2021 11:27



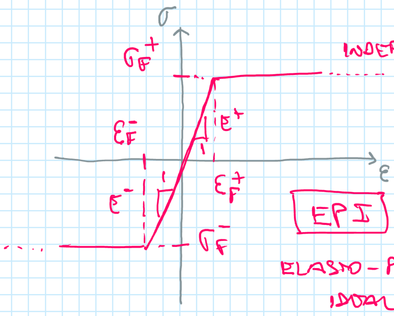
LINEAL INDEFINIDA

$\sigma = k\,\varepsilon$ → ε

$\tau = k'\cdot\gamma$ → τ



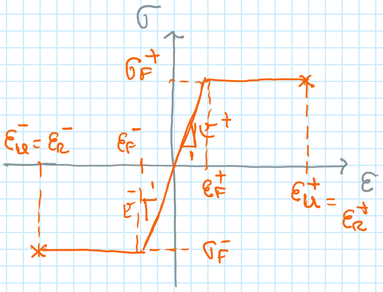
DÚCTIL.



INDEFINIDO

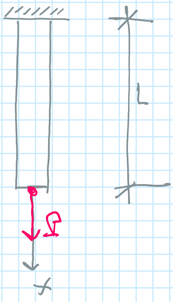
EPI

ELÁSTICO - PLÁSTICO IDEAL



EPR ELÁSTICO - PLÁSTICO REAL.

ARAn → SA → SOLICITACIÓN AXIL.

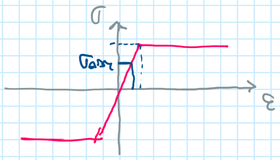


E, G, μ

A

MATERIAL

MIS SECCIÓN



EPI $\varepsilon_T = \varepsilon_C$

$|\sigma_F^+| = |\sigma_F^-|$

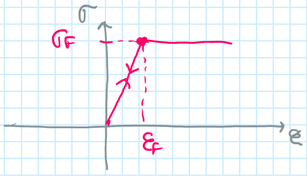
$P \rightarrow N = N(x)$; $A = A(x) = cte \rightarrow \sigma_x(x) = \frac{N(x)}{A} = cte$

Si $P \uparrow \rightarrow \sigma_x \uparrow \rightarrow \sigma_x = \sigma_{x,adm} = \frac{P_{adm}}{A}$

Si $P \uparrow > P_{adm} \rightarrow \sigma_x \uparrow > \sigma_{x,adm} \rightarrow$ SIGUE AUMENTANDO

HASTA QUE $\sigma_x = \sigma_F \rightarrow$ SE PRODUCE LA FALLA ESTRUCTURAL DE LA BARRA.

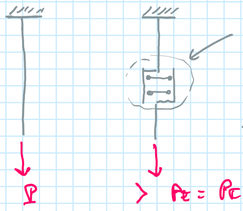
$P \rightarrow \sigma_x = \sigma_F$



$\sigma_x = \sigma_F = \frac{P_F}{A} = \frac{P_E}{A}$

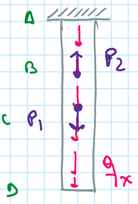
$P_F = P_E$ ← CARGA ELÁSTICA

↑ CARGA DE FLUENCIA.

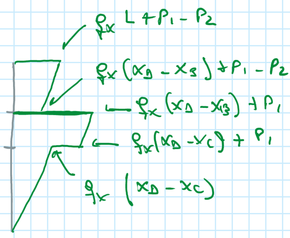


NO DEBIDO SOLO UNA SECCIÓN GENÉRICA PUNTO DE ROTURA LA FALLA SE PRODUCE EN TODAS LAS SECCIONES AL MISMO → → SIMULTÁNEAMENTE

$> P_E = P_E \rightarrow$ LA ESTRUCTURA SE CONVIERTE EN UN MECANISMO



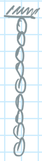
$N_{máx} \rightarrow \sigma_{x\,máx}$



$A = A(x)$

$N = cte = P$

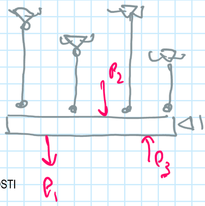
$\rightarrow A_{MIN}$

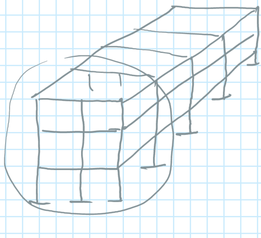


ESTRUCTURA DE SERIE

→ LA FALLA DE UNA ELEM. TIPICA LA FALLA DEL SISTEMA.

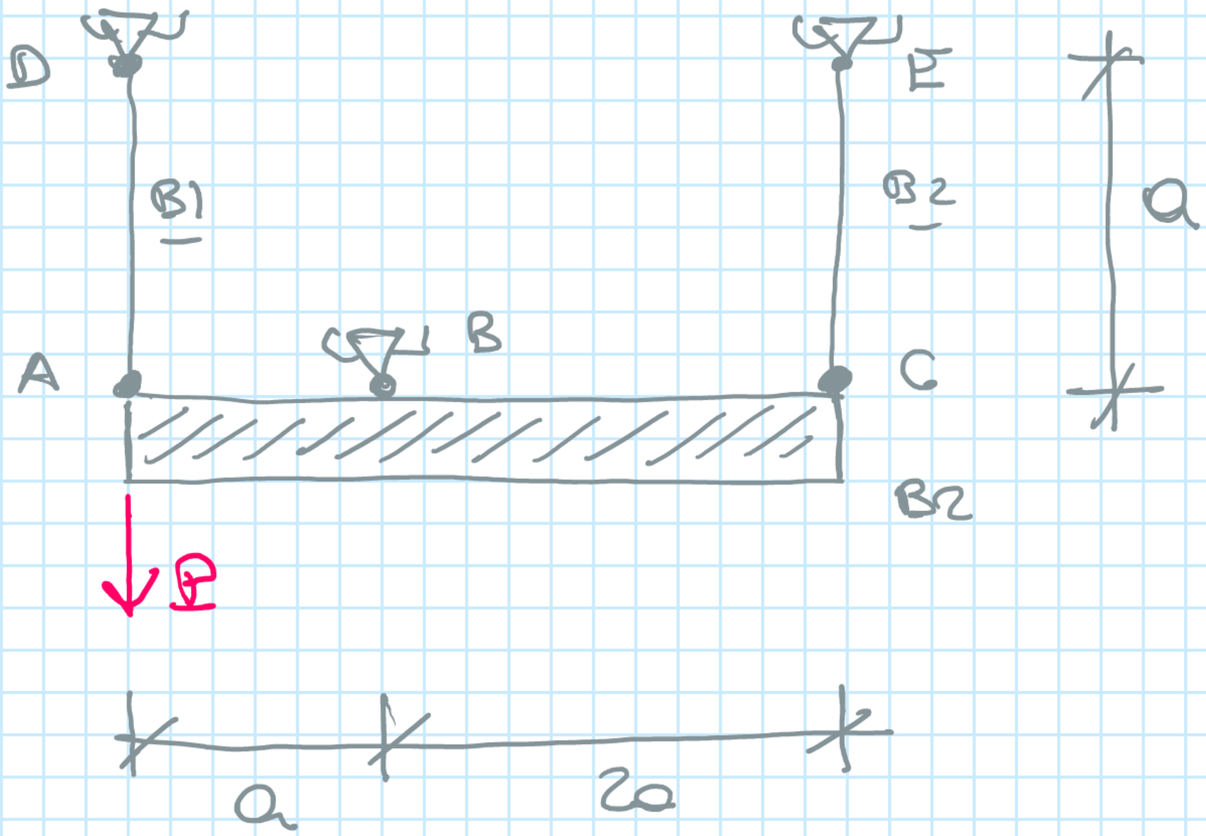
SISTEMAS DE PARALELO:





11.02 - EJERCICIO ARAn - SA:

martes, 7 de diciembre de 2021 12:07



$$\begin{cases} A_1 = 4\ cm^2 \\ A_2 = 2\ cm^2 \end{cases}$$

ACERO F-24

$$\begin{cases} \sigma_F = 24\ kN/cm^2 \\ CS = 1{,}60 \end{cases}$$

$$\sigma_{adm} = \frac{\sigma_F}{CS} = 15\ kN/cm^2$$

$$E = 20000\ kN/cm^2.$$

1) TIPO DE ESTRUCTURA:

- CADENA CINEMÁTICA ABIERTA DE 3 CHAPAS: B1, B2 y BR
- $GL = N + 2 = 3 + 2 = 5.$

1) CV = 2 EN B
2 EN D
2 EN E

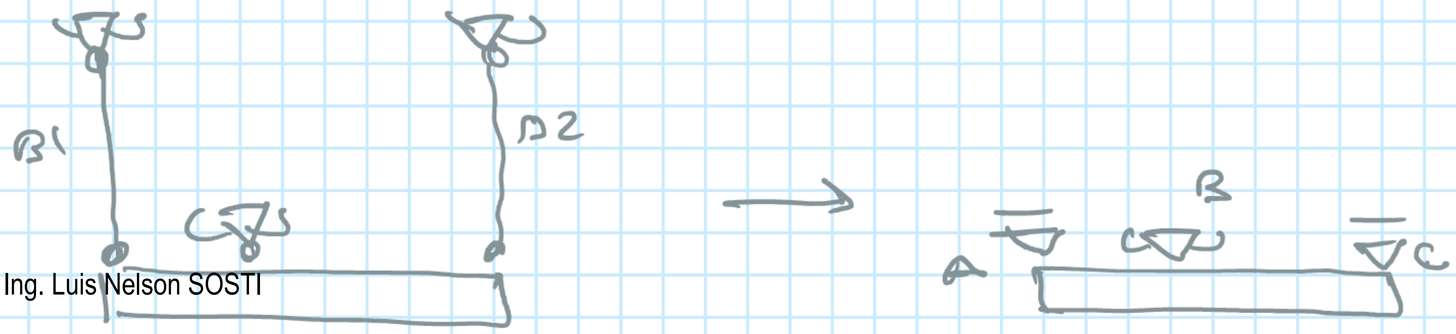
CV = 6

CV > GL ⟶ HIPERESTÁTICA
GH = 1

2) $CV_i \leq 3$

B1 → 2CV EN D.
B2 → 2CV EN E
BR → 2CV EN B

3) ANÁLISIS CINEMÁTICO:



$$\boxed{GH = 1}$$

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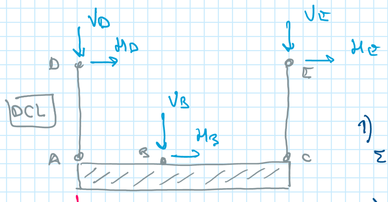
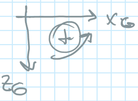
02) - RESOLUCIÓN GRÁFICA — ECS. DE EQUILIBRIO:





1) $\boxed{EEM}$

$\Sigma M^{A}_{F,DCL} = 0 \quad -H_D \cdot a = 0 \rightarrow \boxed{H_D = 0}$

2) $\boxed{EEM}$

$\Sigma M^{C}_{F,DCL} = 0 \quad -H_E \cdot a = 0 \rightarrow \boxed{H_E = 0}$

3) $\boxed{EEA}$

$\Sigma F_x = 0 \qquad H_B + \underbrace{H_D}_{=0} + \underbrace{H_E}_{=0} = 0 \rightarrow \boxed{H_B = 0}$

4) $\boxed{EEA}$

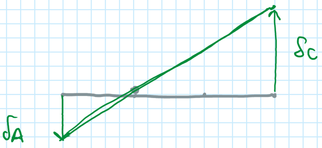
$\Sigma F_z = 0 \qquad \boxed{P + V_B + V_D + V_E = 0} \qquad (4)$ ✓

5) $\boxed{EEM}$

$\Sigma M^{B}_{F} = 0 \qquad \boxed{P \cdot a + V_D \cdot a - V_E \cdot 2a = 0}$

$\boxed{V_E = \frac{P}{2} + \frac{V_D}{2}} \qquad (5)$ ✓

6º) EC. DE COMPATIBILIDAD DE LOS DESPLAZAMIENTOS:



$\frac{\delta_A}{a} = \frac{\delta_C}{2a}$ ✓

$\boxed{\delta_A = \frac{\delta_C}{2}} \quad \boxed{\delta_C = 2\delta_A} \leftarrow (6)$

$\delta_A = \frac{N_1 \cdot a}{E A_1}$

$\delta_C = \frac{N_2 \cdot a}{E \cdot A_2}$

$\delta_A > 0 \rightarrow N_1 > 0 \rightarrow V_D < 0$

$N_1 = -V_D \rightarrow \boxed{V_D = -N_1}$

$\boxed{N_2 = +V_E}$

$\delta_C < 0 \rightarrow N_2 < 0 \rightarrow V_E > 0$

✓ $\delta_A = \frac{N_1 a}{E A_1} = -\frac{V_D a}{E A_1}$

✓ $\delta_C = \frac{V_E a}{E A_2}$

$\delta_C = 2\delta_A \rightarrow \frac{V_E \cancel{a}}{\cancel{E} A_2} = -2\frac{V_D \cancel{a}}{\cancel{E} A_1}$

$\boxed{V_E = -2V_D \frac{A_2}{A_1}} = -2V_D \cdot \frac{2cm^2}{4cm^2}$

$\boxed{V_E = -V_D} \quad \boxed{V_D = -V_E}$

DE (5):

$V_E = \frac{P}{2} + \frac{V_D}{2} \rightarrow -V_D = \frac{P}{2} + \frac{V_D}{2}$

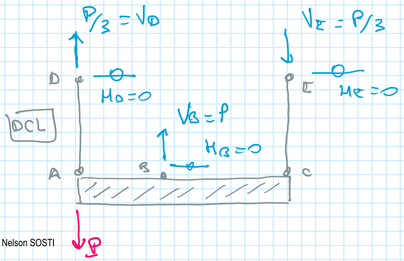
$-\frac{P}{2} = \frac{V_D}{2} + V_D = \frac{3}{2} V_D.$

$\boxed{V_D = -\frac{P}{3}} \uparrow \rightarrow V_E = -V_D = -\left(-\frac{P}{3}\right)$

$\boxed{V_E = +\frac{P}{3}} \downarrow$

DE (4): $P + V_D + V_B + V_E = 0.$

$P + \cancel{\left(-\frac{P}{3}\right)} + V_B + \cancel{\frac{P}{3}} = 0 \rightarrow \boxed{V_B = -P} \uparrow$



$\boxed{DCLE}$ ✓

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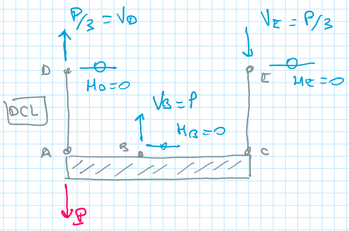


DCLE

03 - DETERMINACIÓN PADM:

(B1) → $\sigma_{ADM} = \frac{N_{1,ADM}}{A_1}$ ⇒ $N_{1,ADM} = A_1 \sigma_{ADM} = 4 \cdot 15 = 60\,kN$

(B2) → $\sigma_{ADM} = \frac{N_{2,ADM}}{A_2}$ → $N_{2,ADM} = A_2 \sigma_{ADM} = 2 \cdot 15 = 30\,kN$

(B1) $N_1 = \frac{P}{3}$ → $P_{ADM,B1} = 3\, N_{1,ADM} = 3 \cdot 60 = 180\,kN.$

(B2) $N_2 = \frac{P}{3}$ → $P_{ADM,B2} = 3 \cdot N_{2,ADM} = 3 \cdot 30 = 90\,kN$

$P_{ADM,\,SISTEMA} = \min(P_{ADM,B1};\ P_{ADM,B2}) = 90\,kN.$ ←

04 - DETERMINACIÓN $P_E = P_F$:

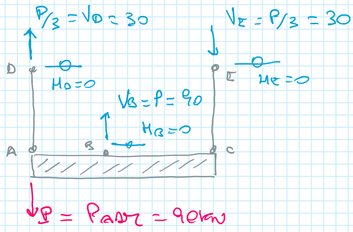
(B1) $\sigma_F = \frac{N_{1E}}{A_1}$ → $N_{1E} = A_1 \sigma_F = 4 \cdot 24 = 96\,kN.$ ←

(B2) $\sigma_F = \frac{N_{2E}}{A_2}$ → $N_{2E} = A_2 \cdot \sigma_F = 2 \cdot 24 = 48\,kN.$ ←

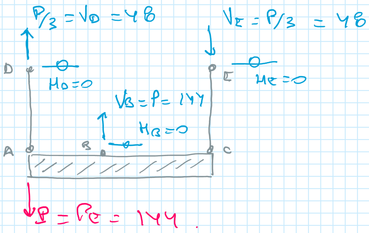
(B1) $N_{1E} = \frac{P_{E1}}{3}$ → $P_{E1} = 3 N_{1E} = 3 \cdot 96 = 288\,kN$

(B2) $N_{2E} = \frac{P_{E2}}{3}$ → $P_{E2} = 3 N_{2E} = 3 \cdot 48 = 144\,kN.$

$P_{E,\,SISTEMA} = \min(P_{E1};\ P_{E2}) = 144\,kN$ ←

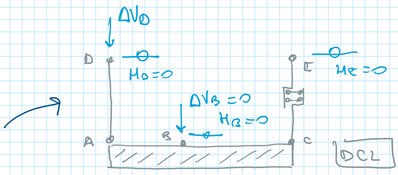


DCLE P_{ADM}



DCLE P_E ←

05 - DETERMINACIÓN DE $P_{COLAPSO} = P_C = P_U$:



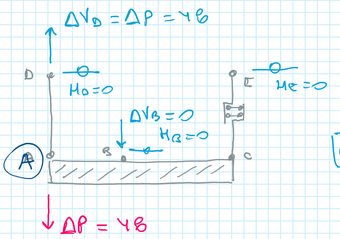
$\Sigma M^B_{Fc} = 0$

$\Delta P \cdot a + \Delta V_D \cdot a = 0$

$\boxed{\Delta V_D = -\Delta P}$

$\Sigma F_z = 0.$ $\cancel{\Delta V_B} + \Delta V_D + \cancel{\Delta P} = 0$

$\boxed{\Delta V_B = 0}$



DCLE ←

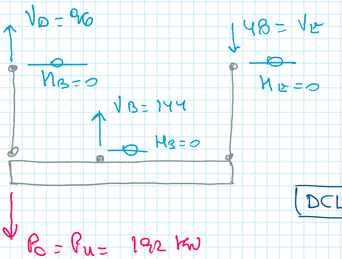
ΔV_D → B1 → COLAPSO $P_{UB1} = P_{CB1} = N_{1,PE} + \Delta V_D =$

$96 = 48 + \Delta V_D$

$\boxed{\Delta V_D = 48\,kN}$

$\boxed{\Delta P = \Delta V_D = 48\,kN}$

$\boxed{P_C = P_U = P_E + \Delta P = 144 + 48 = 192\,kN.}$



$\Sigma F_z = 0$ $192 + 48 - 96 - 144 = 0$ ✓

$\Sigma M^A_{F_D} = 0$ $144 \cdot a - 48 \cdot 3a = 0$

$144 \cdot \cancel{a} = 48 \cdot 3\cancel{a}$ ✓

DCLE $P_C = P_{UB}$

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06 - DESPLAZAMIENTOS DEL PUNTO (A):

$a = 1{,}00\,m.$

(I) $P_{AM} = 90 \rightarrow V_D = -30 \rightarrow N_1 = +30\,kN.$

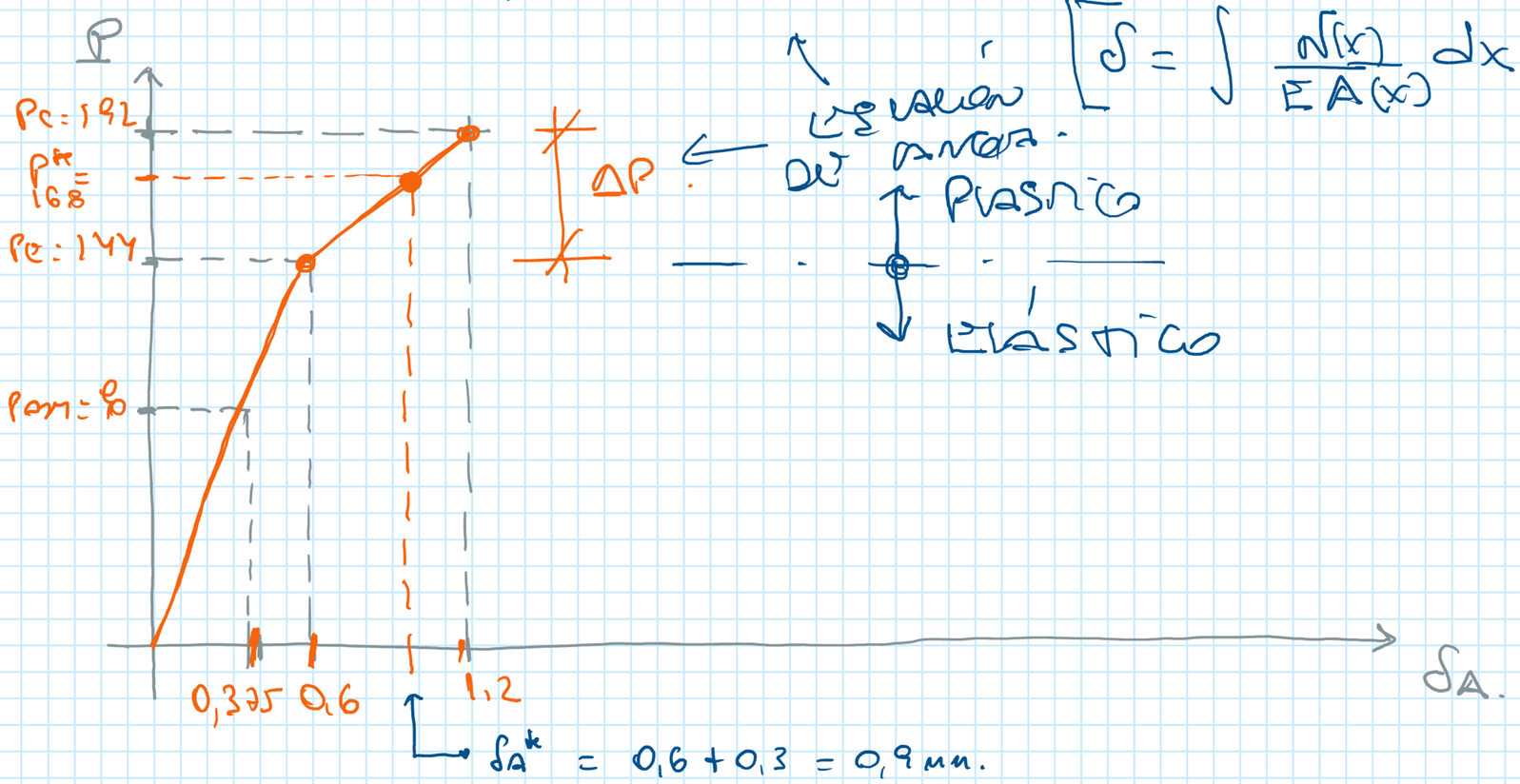
$$\delta_{A,P_{AM}} = \frac{N_1\, a}{E \cdot A_1} = \frac{30\,kN \cdot 100\,cm}{20000\,\frac{kN}{cm^2} \cdot 4\,cm^2} = \boxed{+0{,}0375\,cm.}$$

(II) $P_E = 144 \rightarrow V_D = -144 \rightarrow N_1 = +144\,kN.$

$$\delta_{A,P_E} = \frac{N_1\, a}{E A_1} = \frac{48\,kN \cdot 100\,cm}{20000 \cdot 4} = \boxed{+0{,}06\,cm}$$

(III) $\Delta P = 48\,kN \rightarrow \Delta V_D = -48 \rightarrow \Delta N_1 = +48\,kN$

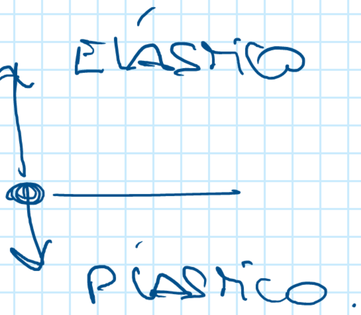
$$\delta_{A,\Delta P} = \frac{\Delta N_1\, a}{E A_1} = \frac{48 \cdot 100}{20000 \cdot 4} = \boxed{+0{,}06\,cm}$$

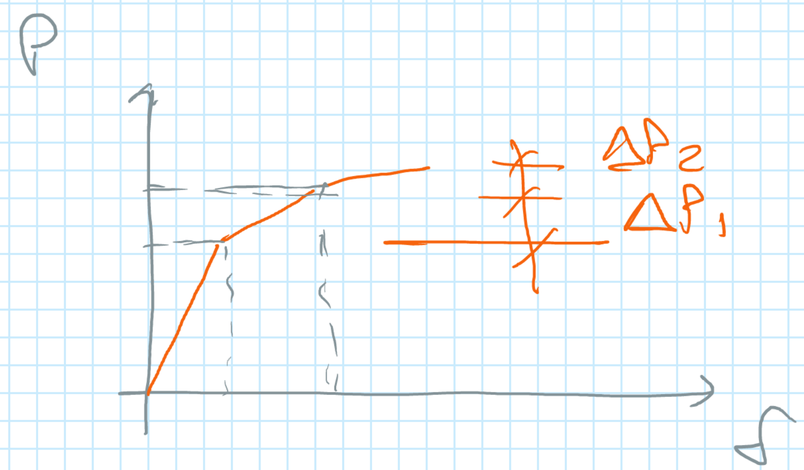


$\delta_A^* = 0{,}6 + 0{,}3 = 0{,}9\,mm.$

$$\begin{cases} P^* = \dfrac{P_U + P_E}{2} = \dfrac{192 + 144}{2} = 168\,kN. \\ \Delta P^* = P^* - P_E = 168 - 144 = 24\,kN. \end{cases}$$

$$\left[\delta = \int \frac{N(x)}{EA(x)}\,dx = \frac{N}{EA}\int_0^L dx = \boxed{\frac{NL}{EA}}\right]$$





BARRA → B1 $\quad \Delta\delta_A = \dfrac{\Delta N_1}{EA_1} \cdot a \qquad \Delta N_1 = \Delta P^*$

$$\Delta\delta_A = \frac{24\,kN \cdot 100\,cm}{20000\,\frac{kN}{cm^2} \cdot 4\,cm^2} = 0{,}03\,cm. = 0{,}3\,mm.$$

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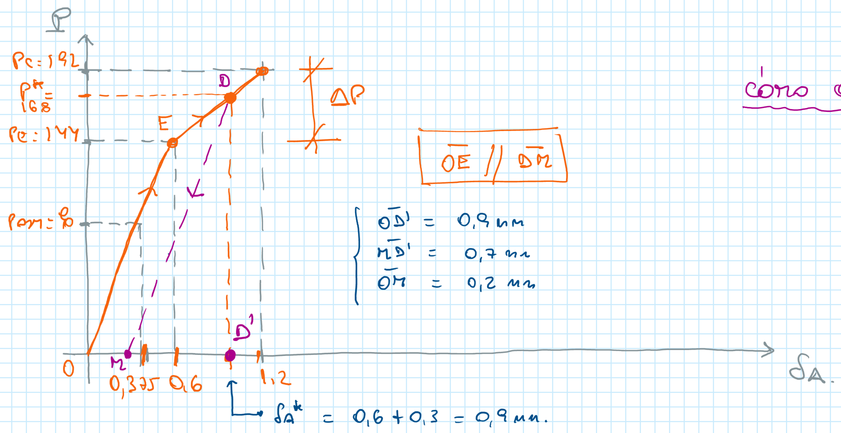
DESCARGA:



¿Cómo ocurre la descarga?

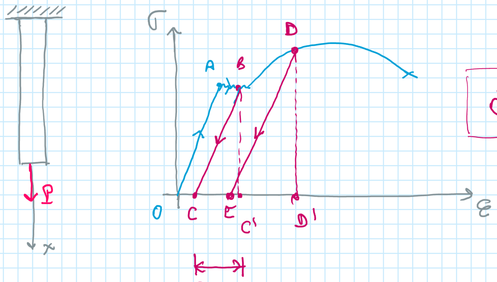
$\boxed{\overline{OE} \parallel \overline{DM}}$

$\overline{OD'} = 0{,}9\ mm$
$\overline{MD'} = 0{,}7\ mm$
$\overline{OM} = 0{,}2\ mm$

$\delta_A^* = 0{,}6 + 0{,}3 = 0{,}9\ mm.$

$P^* = \dfrac{P_u + P_E}{2} = \dfrac{192 + 144}{2} = 168\ kN.$

$\Delta P^* = P^* - P_E = 168 - 144 = 24\ kN.$



$\boxed{\overline{OA} \parallel \overline{BC} \parallel \overline{DE}}$

BAUSCHINGER.
↓
DEMOSTRÓ QUE ESOS SEGMENTOS NO SON EXACTAMENTE PARALELOS.

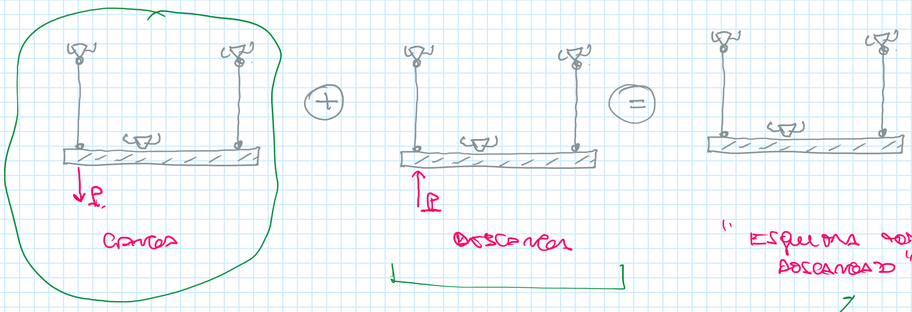
$\delta_E = \delta_{recup.}$

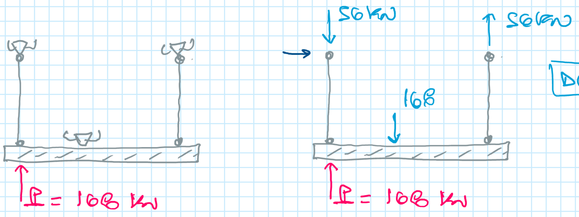
Curvas de descarga:

$\overline{BC}$ y $\overline{DE}$ son $\parallel$ $\overline{OA}$.

- $\overline{OD'}$: desplazamiento total bajando la carga P^*.
- $\overline{D'M}$: " recuperado en forma elástica
- $\overline{OM}$: " permanente o remanente.

$\boxed{\overline{OD'} = \overline{OM} + \overline{D'M}}$

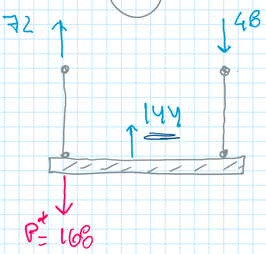




$\boxed{DCLE}$ descarga.

$\delta_A\big|_{descarga} = \dfrac{56 \cdot 100}{20000 \cdot 4} = 0{,}07\ cm = 0{,}7\ mm \uparrow$

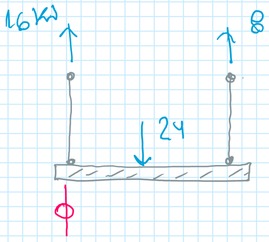
+



$\boxed{DCLE}$ carga

$\delta_A\big|_{carga} = 0{,}09\ cm = 0{,}9\ mm \downarrow$

III



$\boxed{DCLE}$ descargado

De N y σ → esfuerzos y tensiones $\boxed{\text{residuales}}$

$\delta_A\big|_{resid} = 0{,}09 - 0{,}07 = 0{,}02\ cm = 0{,}2\ mm.$

$\delta_A\big|_{resid} = \dfrac{16 \cdot 100}{20000 \cdot 4} = 0{,}02\ cm = 0{,}2\ mm.$