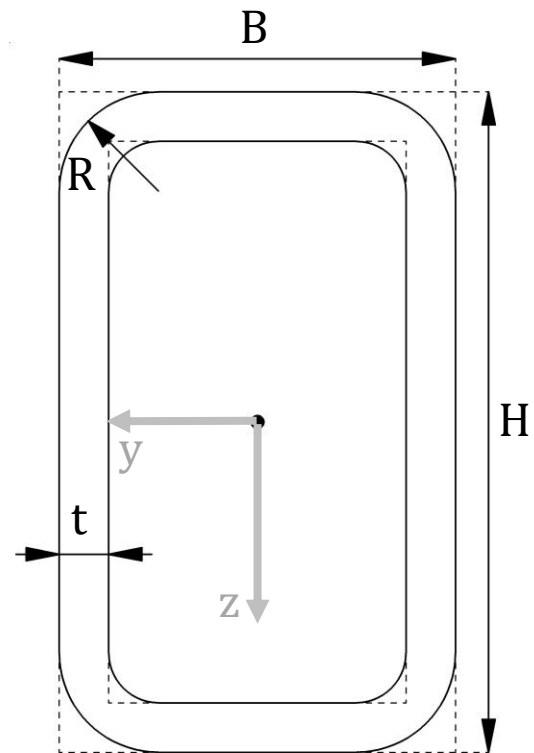
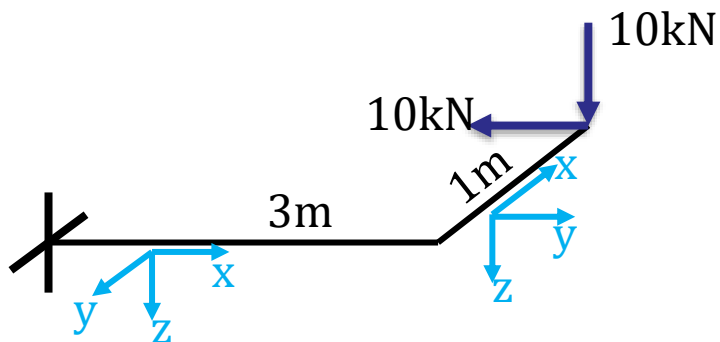




Teoría de Estados Límites



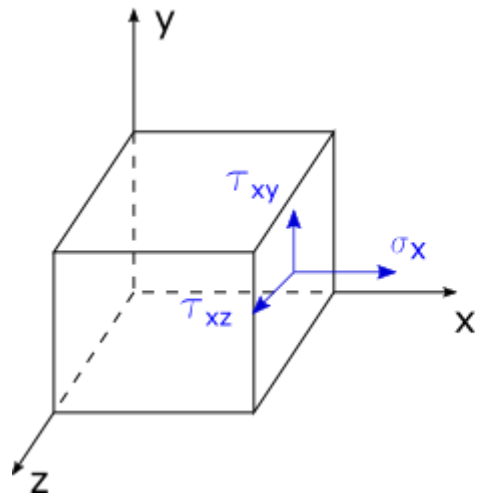
Augusto La Colla – Manuela Medina – Constanza Ruffinelli – Bautista Chesta

Introducción teórica



Dependiendo del material, existen ciertos límites en las tensiones. Por ejemplo, el estado de tensiones que establece el fin del periodo elástico se denomina fluencia.

En nuestra materia trabajaremos con barras, que son estados dobles de tensiones



$$[T_T] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix}$$



$$[T_T] = \begin{bmatrix} \sigma_x & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tau = \sqrt{\tau_{xy}^2 + \tau_{xz}^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \frac{\sqrt{4 \cdot \tau^2 + \sigma^2}}{2}$$

$$\sigma_3 = \frac{\sigma}{2} - \frac{\sqrt{4 \cdot \tau^2 + \sigma^2}}{2}$$



- Saint Venant
(máxima deformación principal)

$$\max(|\varepsilon_1|, |\varepsilon_2|, |\varepsilon_3|) \leq |\varepsilon_{limite}|$$

$$EPT \quad \boxed{|\sigma_1 - \mu \cdot \sigma_3| \leq \sigma_{adm}}$$
$$\boxed{|\sigma_3 - \mu \cdot \sigma_1| \leq \sigma_{adm}}$$

- Rankine
(máxima tensión principal)

$$\max(|\sigma_1|, |\sigma_2|, |\sigma_3|) \leq |\sigma_{adm}|$$

$$EPT \quad \boxed{|\sigma_1| \leq \sigma_{adm}}$$
$$\boxed{|\sigma_3| \leq \sigma_{adm}}$$

- Guest - Tresca
(máxima tensión tangencial)

$$|\sigma_1 - \sigma_3| \leq |\sigma_{adm}|$$

$$EPT \quad \boxed{\sqrt{4 \cdot \tau^2 + \sigma^2} \leq \sigma_{adm}}$$

- Beltrami
(máxima energía total)

$$EPT \quad \boxed{\sqrt{2\mu \cdot \tau^2 + 2 \cdot \tau^2 + \sigma^2} \leq \sigma_{adm}}$$



- Von Mises (Se usa para acero)
(máxima energía de distorsión)

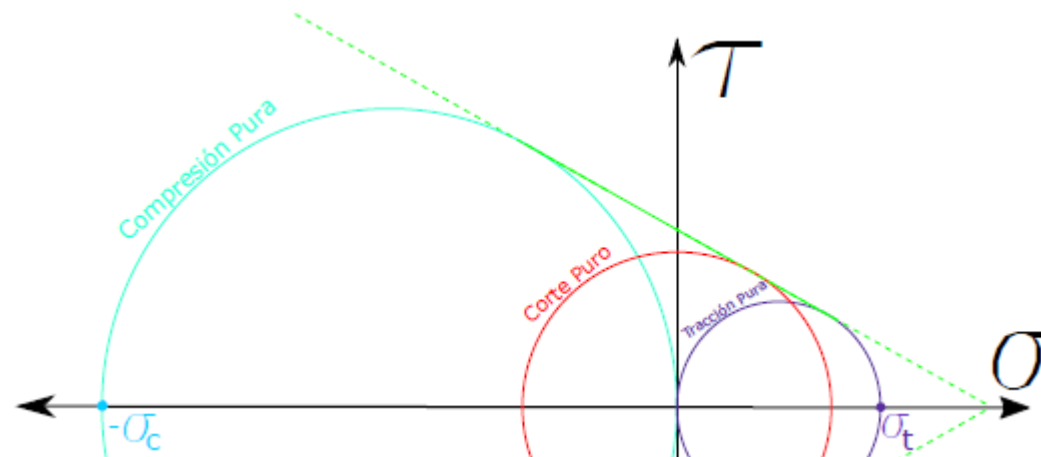
$$\sqrt{\frac{\sigma_1^2 + \sigma_3^2 + (\sigma_1 - \sigma_3)^2}{2}} \leq |\sigma_{adm}|$$

EPT $\boxed{\sqrt{3 \cdot \tau^2 + \sigma^2} \leq \sigma_{adm}}$

- Coulomb (Se usa para suelos)

$$\boxed{\sigma_1 - K\sigma_3 \leq \sigma_{adm}}$$

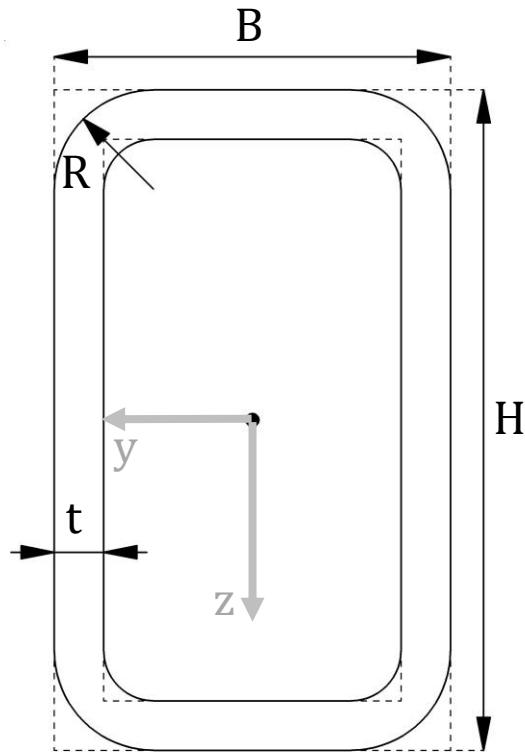
$$K = \frac{\sigma_{fl-tracción}}{\sigma_{fl-compresión}} = \frac{\sigma_{rotura-tracción}}{\sigma_{rotura-compresión}} \quad K > 0$$





Ejercicio 1:

- a) Determinar diagramas de tensiones parciales en la sección más solicitada.
- b) Identificar el/los punto/s más peligroso/s de la estructura
- c) Calcular el coeficiente de seguridad con la Teoría de Von Mises



Perfil Tubo Rectangular 100*250*10

Acero F24 $\sigma_{fl} = 24 \frac{\text{kN}}{\text{cm}^2}$

Datos de tabla

$B = 100 \text{ mm}$

$t = 10 \text{ mm}$

$R = 2 t$

$H = 250 \text{ mm}$

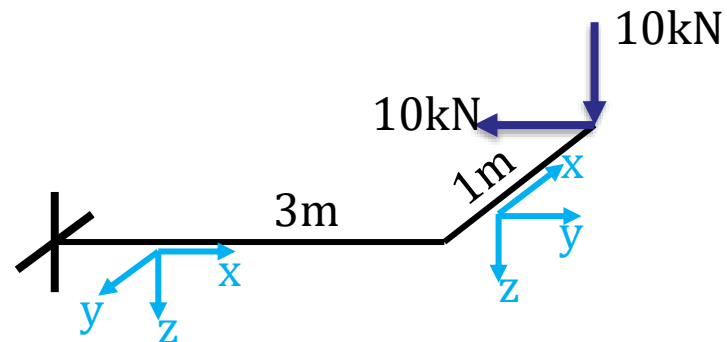
$C = 428,14 \text{ cm}^3$

$I_{\text{máx}} = 4515,84 \text{ cm}^4$

$W_{\text{máx}} = S_{\text{máx}} = 361,27 \text{ cm}^3$

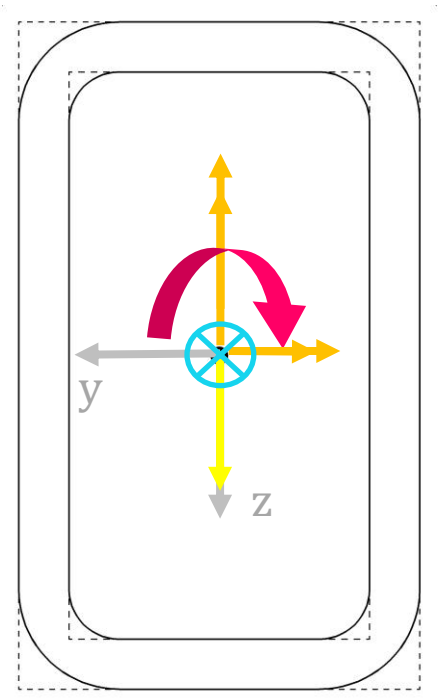
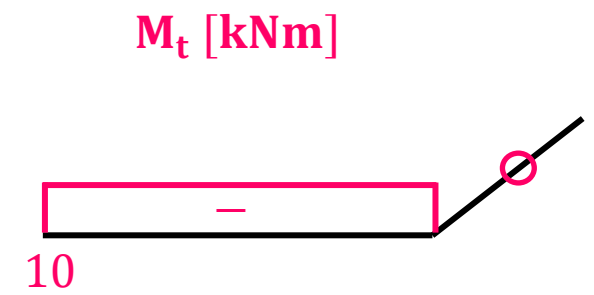
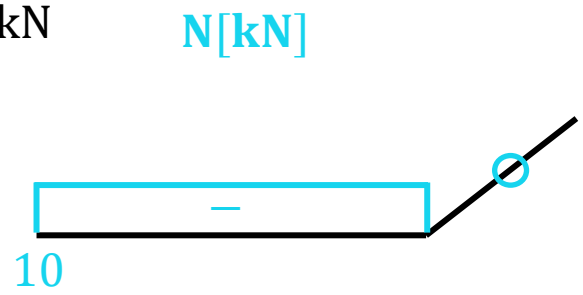
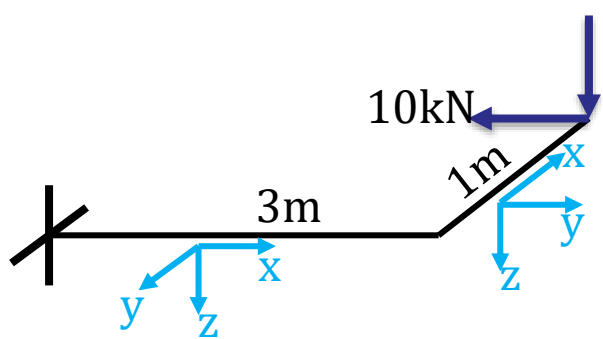
$I_{\text{mín}} = 1040,04 \text{ cm}^4$

$W_{\text{mín}} = S_{\text{mín}} = 208,01 \text{ cm}^3$

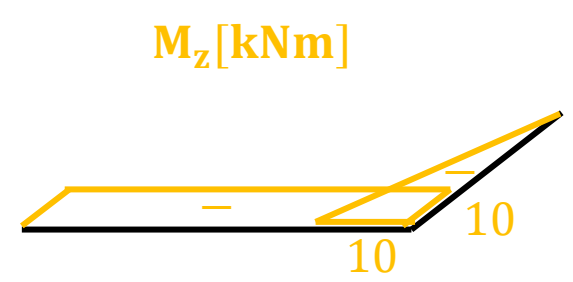
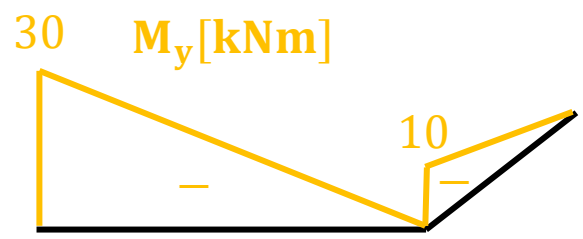
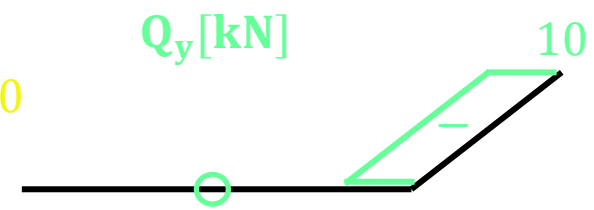
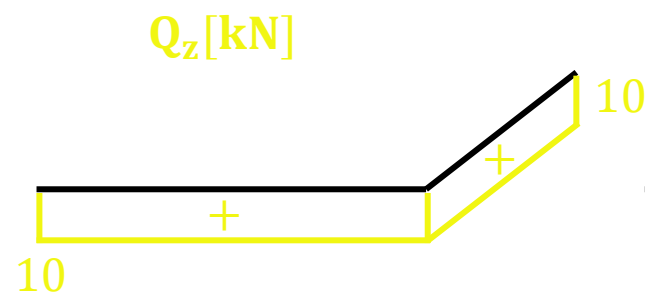




Diagramas de características



Sección más solicitada



$N = -10 \text{ kN}$

$M_t = -10 \text{ kNm}$

$Q_z = 10 \text{ kN}$

$M_y = -30 \text{ kNm}$

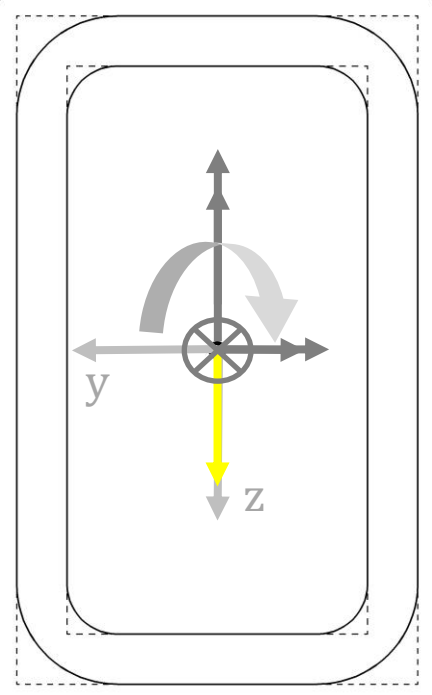
$M_z = -10 \text{ kNm}$

Corte

$$Q_z = 10\text{kN}$$



Se aproxima el diagrama considerando sección con esquinas en ángulo recto

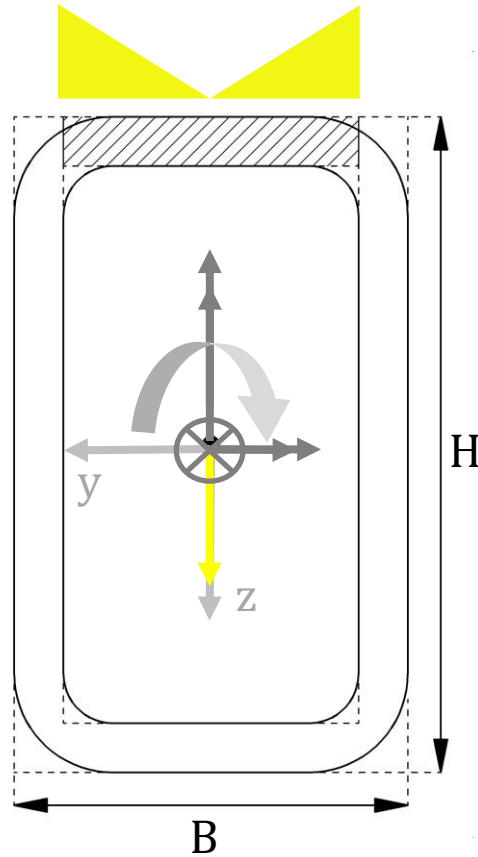




Corte

$$Q_z = 10 \text{ kN}$$

0,1063



Se aproxima el diagrama considerando sección con esquinas en ángulo recto

$$\tau_{xy}^{Q_z}_{\text{máx}} = \frac{Q_z \cdot S_{EN}^{*1}}{I_{EN} \cdot b}$$

$$S_{EN}^{*1} = ((B - 2t)t) \left(\frac{H}{2} - \frac{t}{2} \right)$$

$$S_{EN}^{*1} = 96 \text{ cm}^3$$

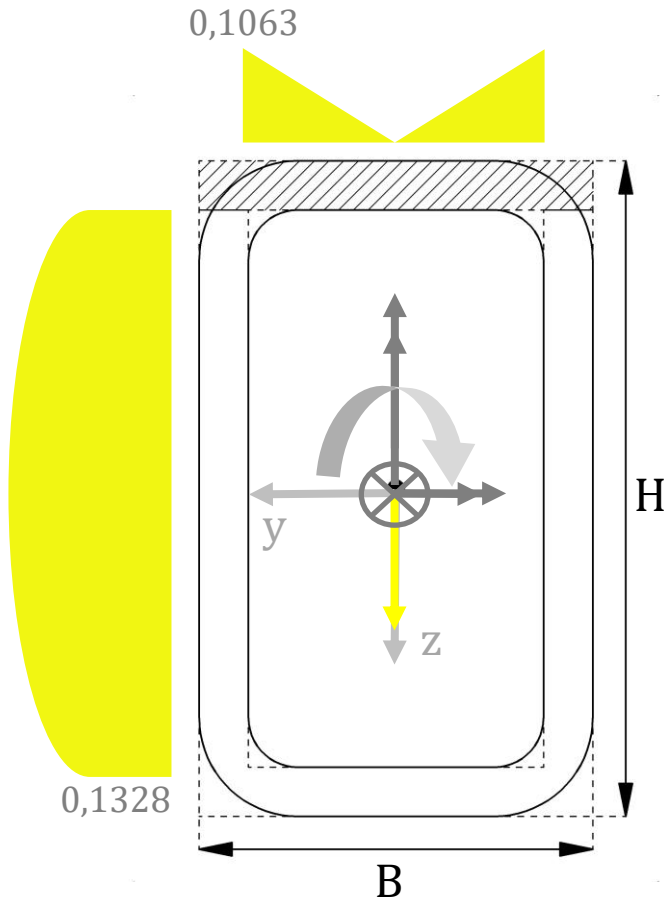
$$\tau_{xy}^{Q_z}_{\text{máx}} = 0,1063 \frac{\text{kN}}{\text{cm}^2}$$

Se expresarán las tensiones en los diagramas en $\frac{\text{kN}}{\text{cm}^2}$



Corte

$$Q_z = 10 \text{ kN}$$



Se aproxima el diagrama considerando sección con esquinas en ángulo recto

$$\tau_{xz}^{Q_z} \text{ mín} = \frac{Q_z \cdot S_{EN}^{*2}}{I_{EN} \cdot b}$$

$$S_{EN}^{*2} = Bt \left(\frac{H}{2} - \frac{t}{2} \right)$$

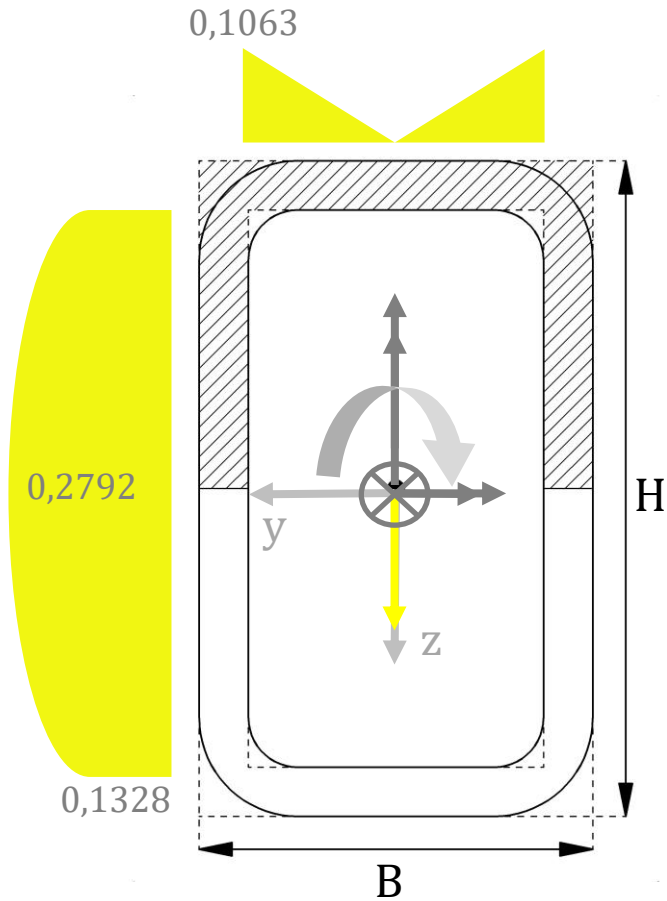
$$S_{EN}^{*2} = 120 \text{ cm}^3$$

$$\tau_{xz}^{Q_z} \text{ mín} = 0,1328 \frac{\text{kN}}{\text{cm}^2}$$



Corte

$$Q_z = 10 \text{ kN}$$



Se aproxima el diagrama considerando sección con esquinas en ángulo recto

$$\tau_{xz}^{Q_z} \text{ máx} = \frac{Q_z \cdot S_{EN}^{*3}}{I_{EN} \cdot b}$$

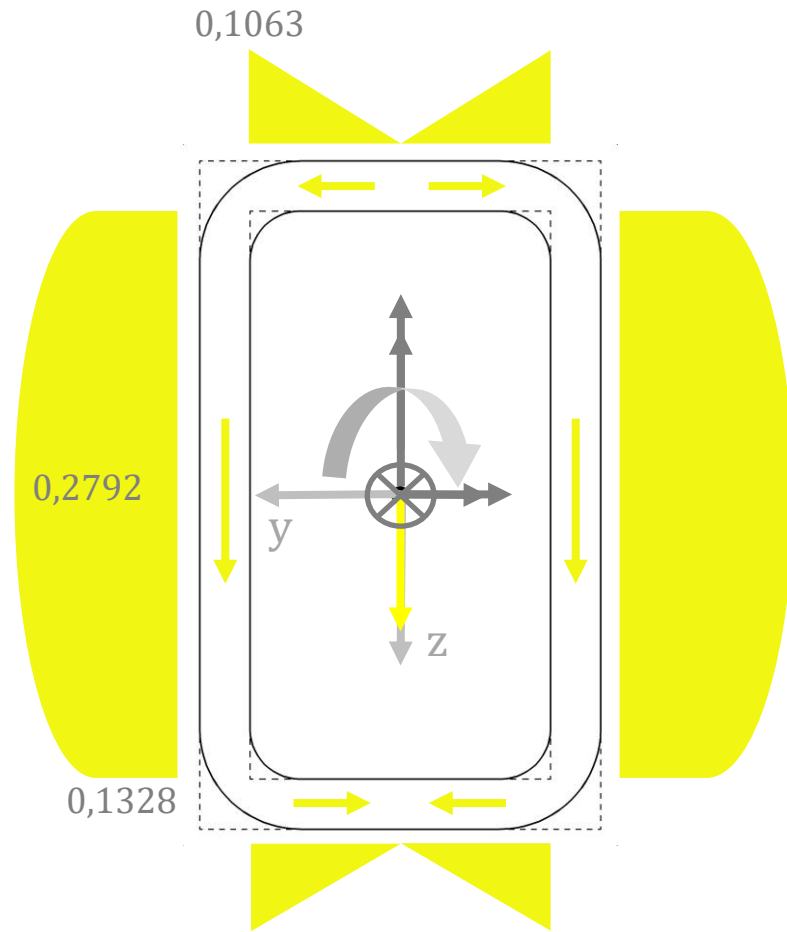
$$S_{EN}^{*3} = S_{EN}^{*2} + 2 \left[t \left(\frac{H}{2} - t \right) \frac{1}{2} \left(\frac{H}{2} - t \right) \right]$$

$$S_{EN}^{*3} = 252,25 \text{ cm}^3$$

$$\tau_{xz}^{Q_z} \text{ máx} = 0,2792 \frac{\text{kN}}{\text{cm}^2}$$

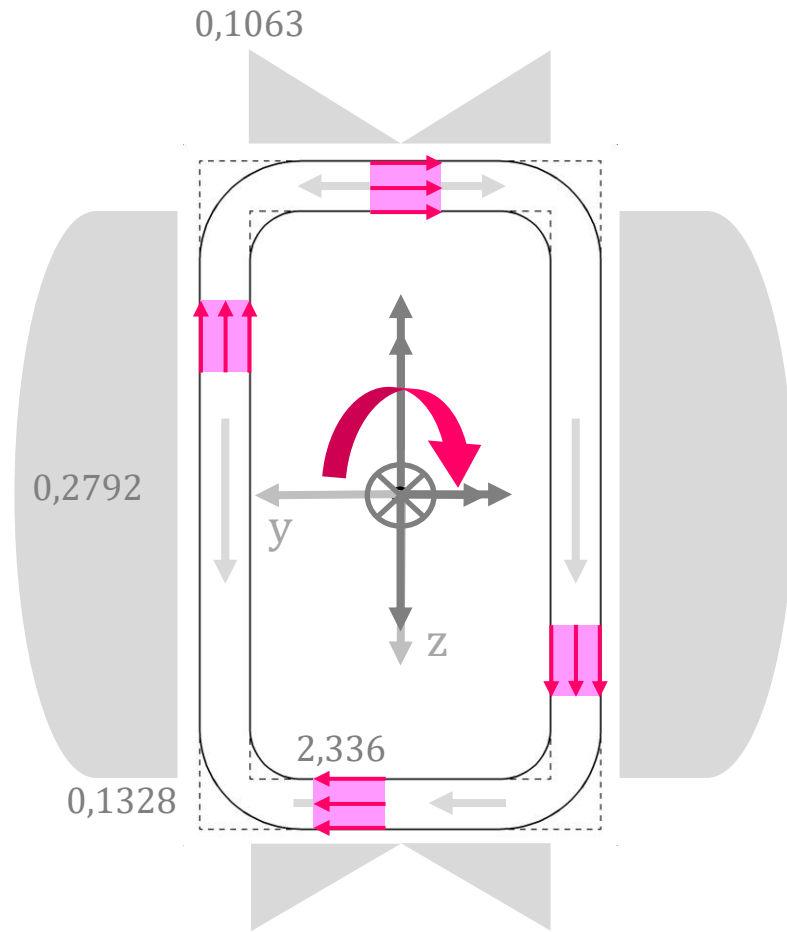
Corte

$$Q_z = 10 \text{ kN}$$



Torsión

$$M_t = -10 \text{ kNm}$$

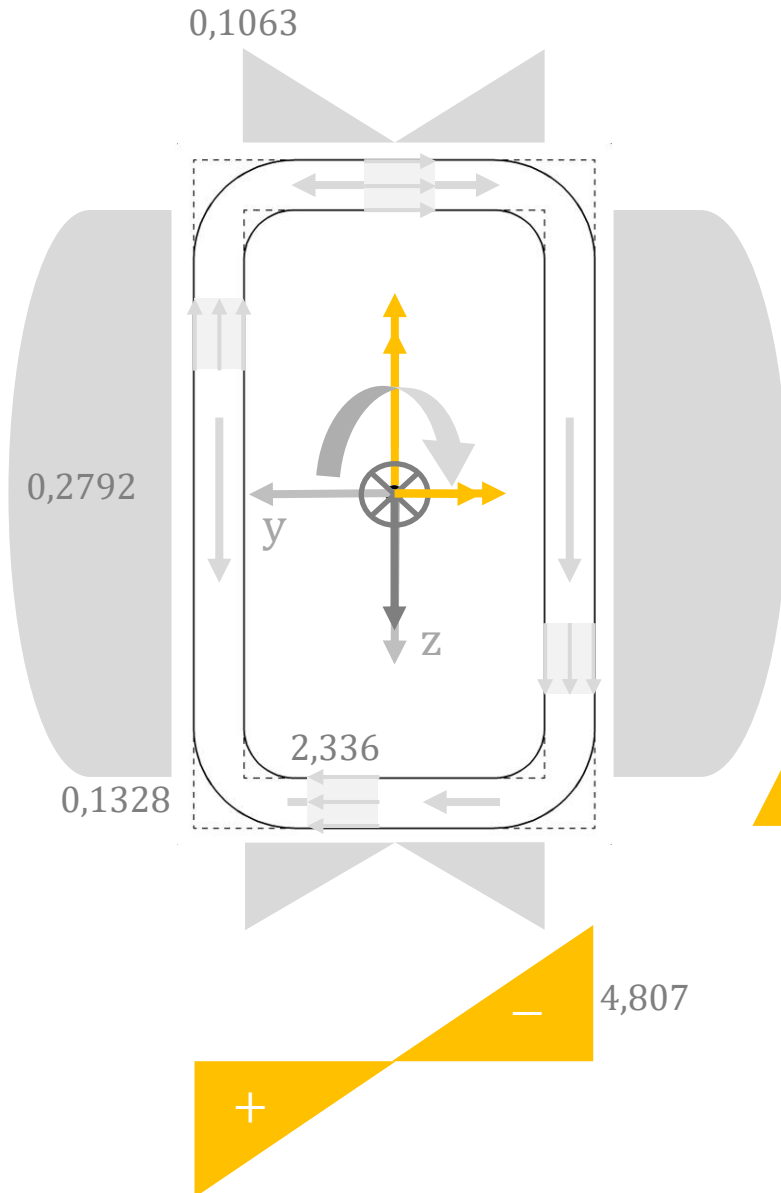


$$\tau_{Mt} = \frac{M_t}{C} = \frac{1000 \text{ kNcm}}{428,14 \text{ cm}^3} = 2,336 \frac{\text{kN}}{\text{cm}^2}$$

Flexión

$$M_y = -30 \text{ kNm}$$

$$M_z = -10 \text{ kNm}$$



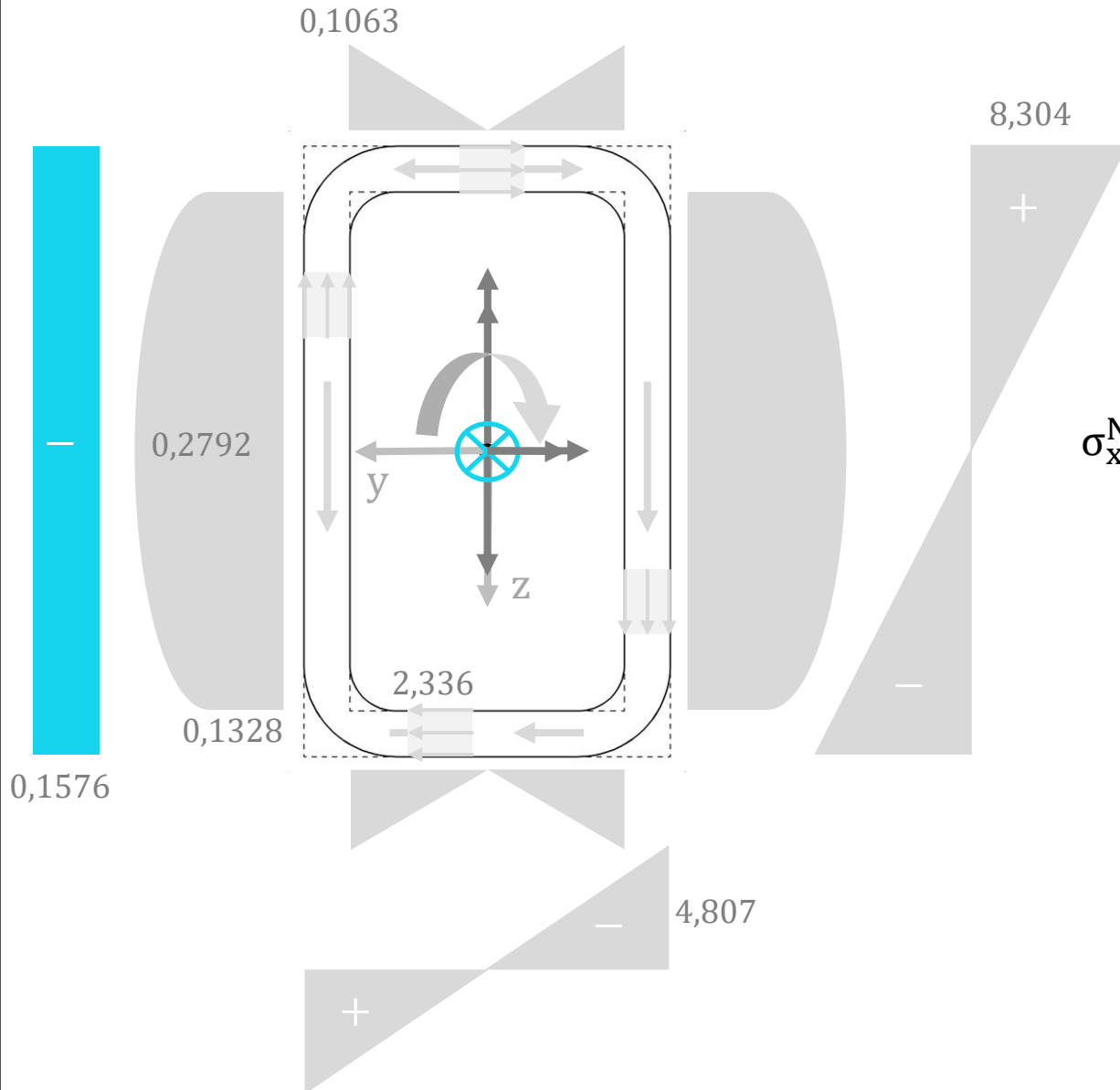
$$\sigma_x^{M_y} = \frac{M_y}{W_y} = \frac{3000 \text{ kNcm}}{361,27 \text{ cm}^3} = 8,304 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_x^{M_z} = \frac{M_z}{W_z} = \frac{1000 \text{ kNcm}}{208,01 \text{ cm}^3} = 4,807 \frac{\text{kN}}{\text{cm}^2}$$



Axial

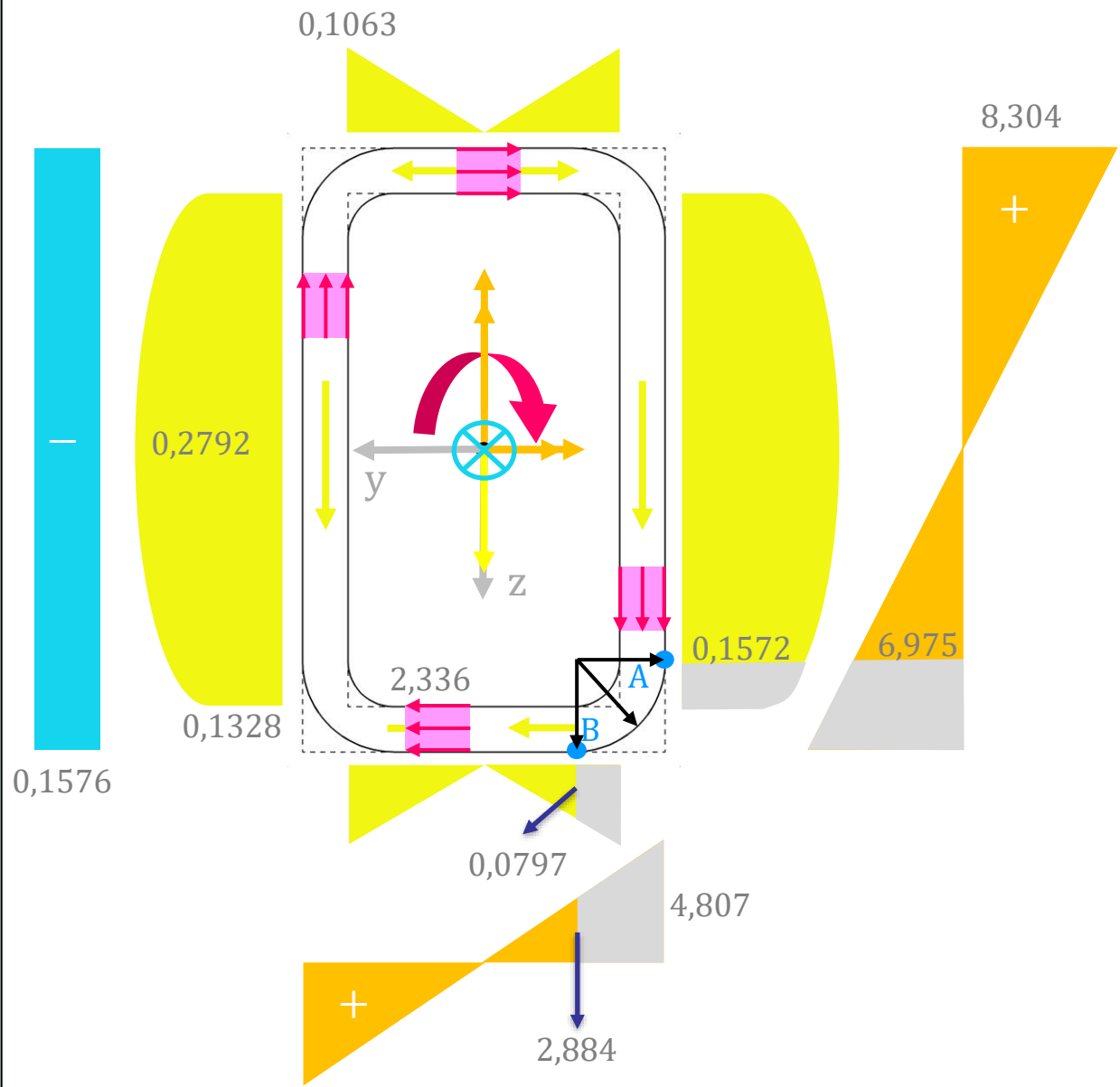
$$N = -10 \text{ kN}$$



$$\sigma_x^N = \frac{N}{A} = \frac{-10 \text{ kN}}{63,425 \text{ cm}^2} = -0,1576 \frac{\text{kN}}{\text{cm}^2}$$



¿Cuál es el punto más peligroso?



Punto i Terna XYZ

$$[T_T]_{XYZ}^i = \begin{pmatrix} \sigma_X & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & \sigma_Y & \tau_{YZ} \\ \tau_{ZX} & \tau_{ZY} & \sigma_Z \end{pmatrix}$$

$$[T_T]_{XYZ}^i = \begin{pmatrix} \sigma_X & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & 0 & 0 \\ \tau_{ZX} & 0 & 0 \end{pmatrix}$$

$$\sigma_X^A = (-0,1576 - 6,975 - 4,807) \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^A = 0$$

$$\tau_{XZ}^A = (2,336 + 0,1572) \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_X^B = (-0,1576 - 2,884 - 8,304) \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^B = (2,336 + 0,0797) \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XZ}^B = 0$$



$$\sigma_X^A = -11,94 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^A = 0$$

$$\tau_{XZ}^A = 2,49 \frac{\text{kN}}{\text{cm}^2}$$

$$[T_T]_{XYZ}^A = \begin{pmatrix} -11,94 & 0 & 2,49 \\ 0 & 0 & 0 \\ 2,49 & 0 & 0 \end{pmatrix} \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_X^B = -11,35 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^B = 2,41 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XZ}^B = 0$$

$$[T_T]_{XYZ}^B = \begin{pmatrix} -11,35 & 2,41 & 0 \\ 2,41 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{\text{kN}}{\text{cm}^2}$$

Von Mises

$$\sqrt{\sigma_X^2 + 3\tau^2} \leq \frac{\sigma_{fl}}{CS}$$

$$\sigma_{fl} = 24 \frac{\text{kN}}{\text{cm}^2}$$

Punto A

$$CS \leq 1,89$$

Punto B

$$CS \leq 1,98$$

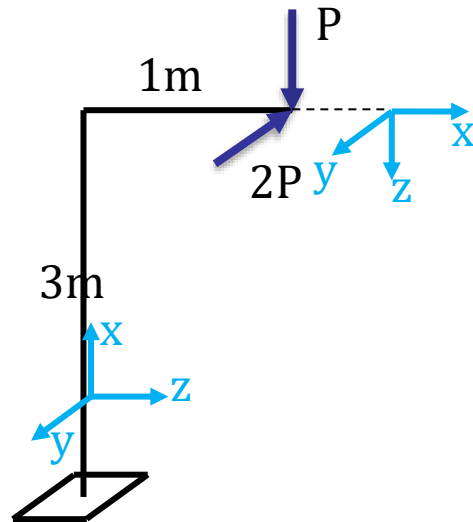
¿Cuál es el punto más peligroso?

¿Cuál es el coeficiente de seguridad?



Ejercicio 2:

- a) Calcular el máximo valor de P que admite la estructura, considerando la Teoría de Tresca (Máxima Tensión Tangencial).



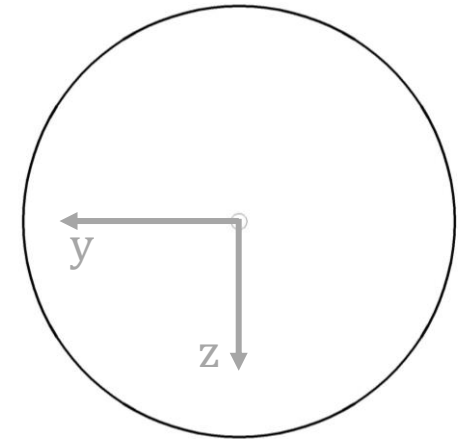
Sección circular maciza

$$D = 10\text{cm}$$

Acero F24

$$\sigma_{fl} = 24 \frac{\text{kN}}{\text{cm}^2} \quad CS = 1,6$$

$$\sigma_{adm} = \frac{\sigma_{fl}}{CS} = 15 \frac{\text{kN}}{\text{cm}^2}$$

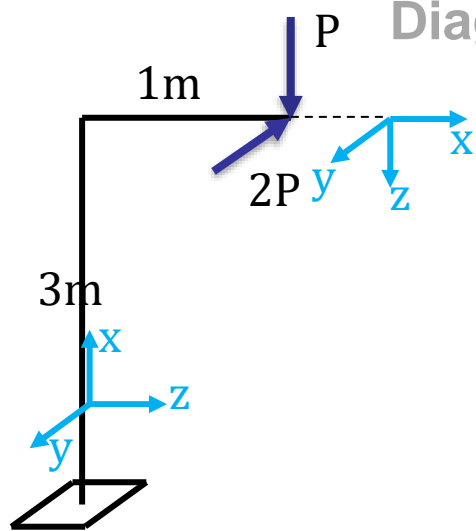


$$A = \frac{\pi \cdot D^2}{4} = 78,54 \text{ cm}^2$$

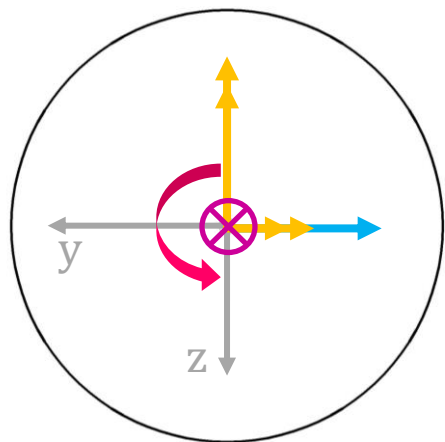
$$I = \frac{\pi \cdot D^4}{64} = 490,87 \text{ cm}^4$$

$$J_P = \frac{\pi \cdot D^4}{32} = 981,75 \text{ cm}^4$$

Diagramas de características



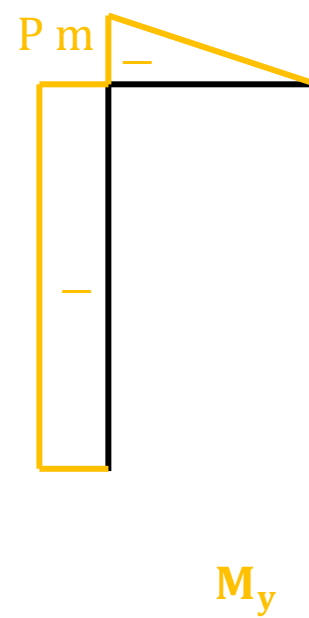
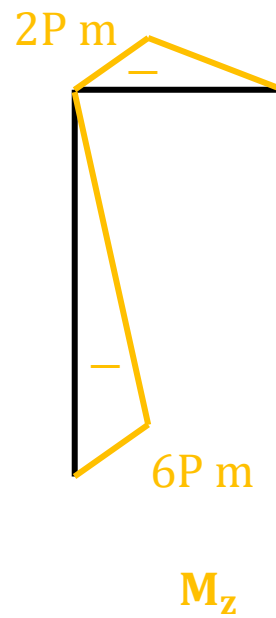
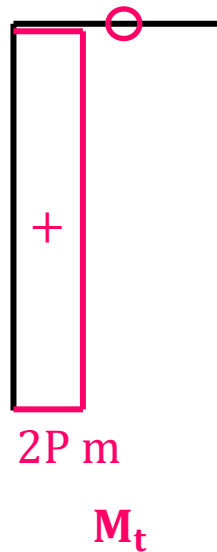
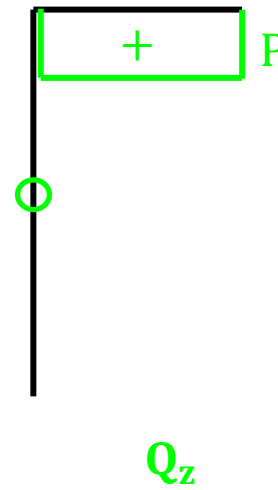
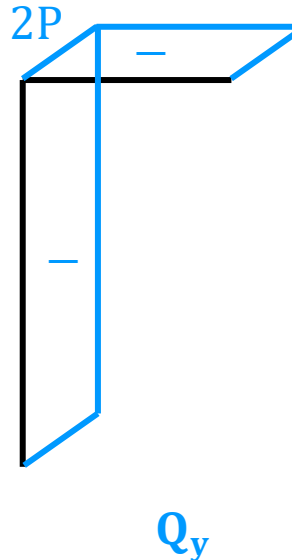
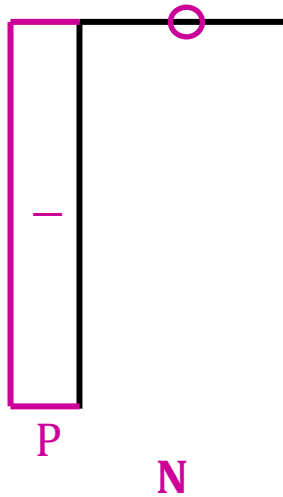
Sección más solicitada



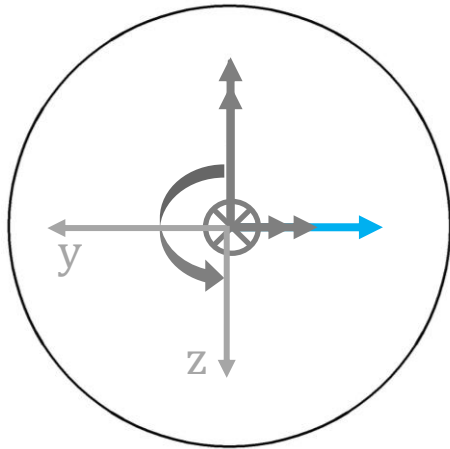
$$N = -P$$

$$Q_y = -2P \quad M_z = -6P \text{ m}$$

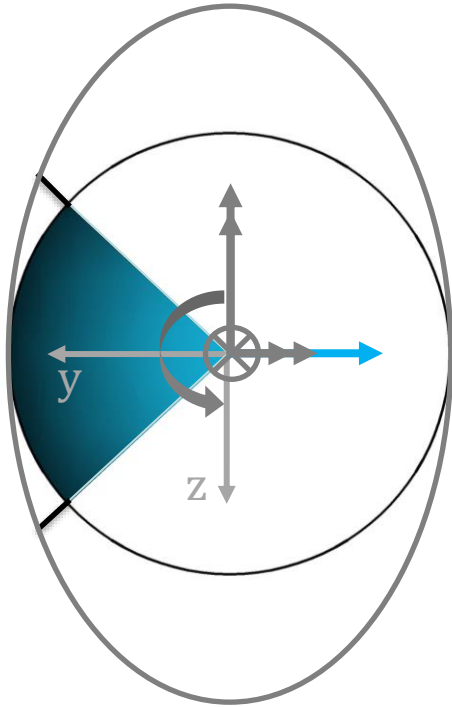
$$M_t = 2P \text{ m} \quad M_y = -P \text{ m}$$



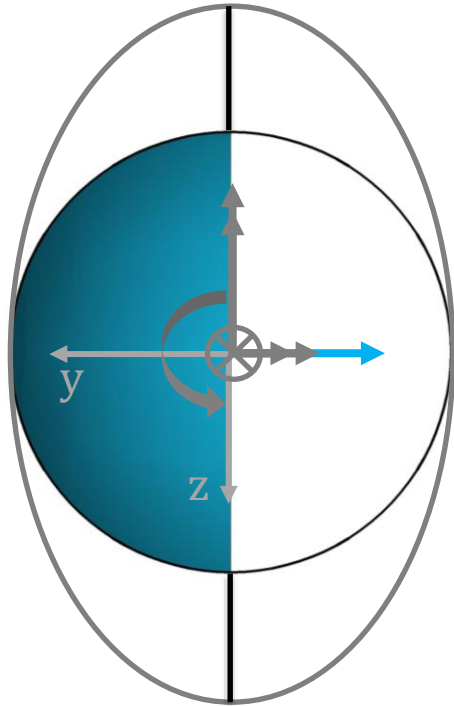
Corte $Q_y = -2P$



Corte $Q_y = -2P$



Corte $Q_y = -2P$



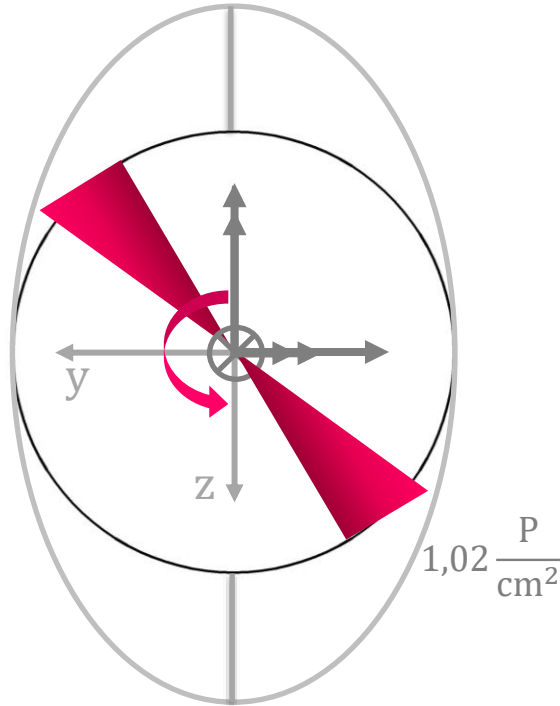
$$\tau_{\text{máx}}^{Q_y} = \frac{Q_y \cdot S_{\text{EN}}^*}{I_{\text{EN}} \cdot b} = \frac{2P \cdot S_{\text{EN}}^*}{490,87 \text{ cm}^4 \cdot D}$$

$$S_{\text{EN}}^* = \frac{A}{2} \cdot \frac{4D}{3\pi} = 83,33 \text{ cm}^3$$

$$\tau_{\text{máx}}^{Q_y} = 0,034 \frac{P}{\text{cm}^2}$$

Torsión

$$M_t = 2P m$$

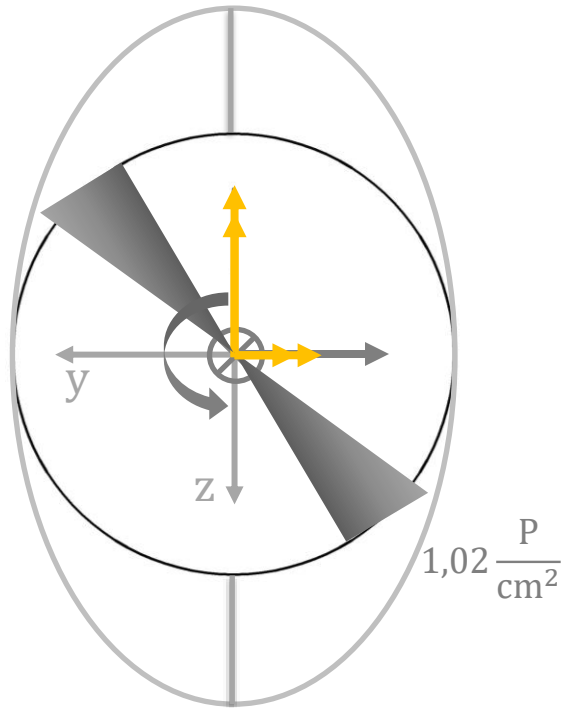


$$\tau_{\text{máx}}^{M_t} = \frac{M_t}{J_P} \cdot \frac{D}{2} = \frac{2P m}{981,75 \text{ cm}^4} \cdot \frac{10 \text{ cm}}{2} = 1,02 \frac{P}{\text{cm}^2}$$

Flexión

$$M_y = -P m$$

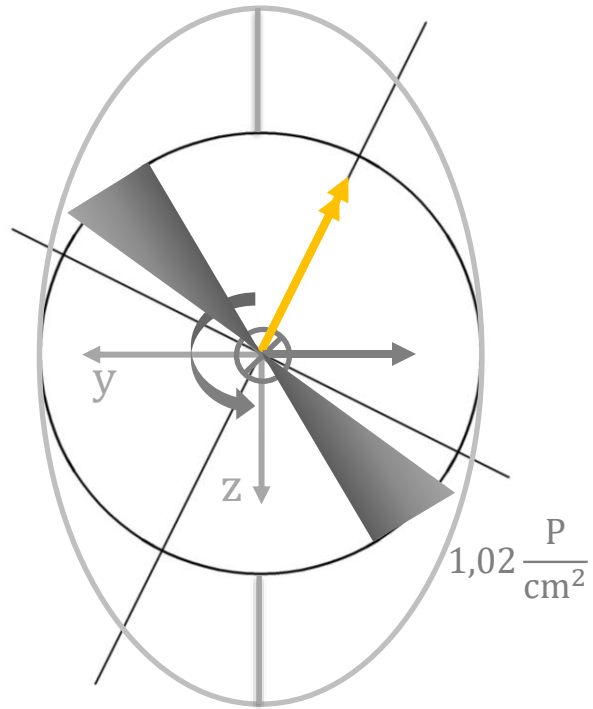
$$M_z = -6P m$$

Flexión compuesta **recta**

$$M_f = \sqrt{(M_y)^2 + (M_z)^2} = 6,083P m$$

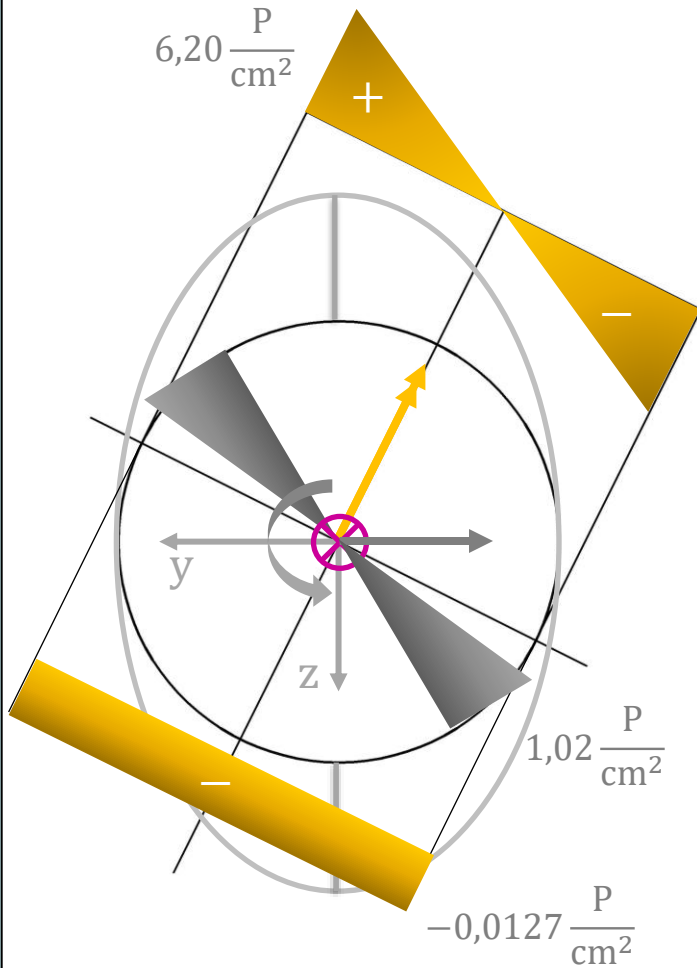
Flexión

$$M_f = 6,083P \text{ m}$$



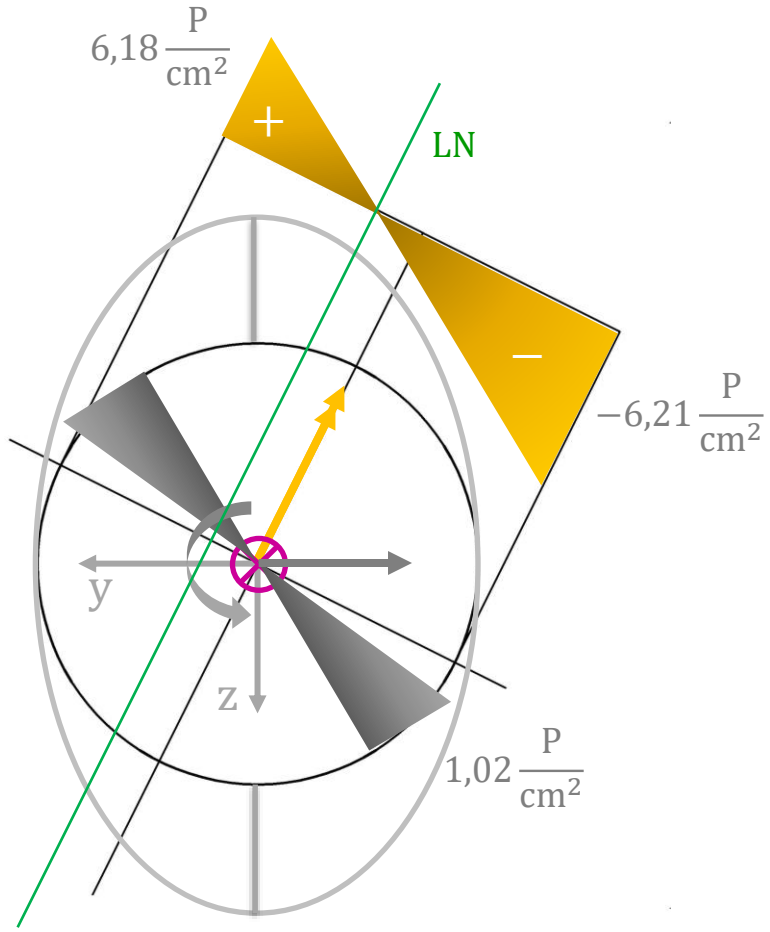


Flexión

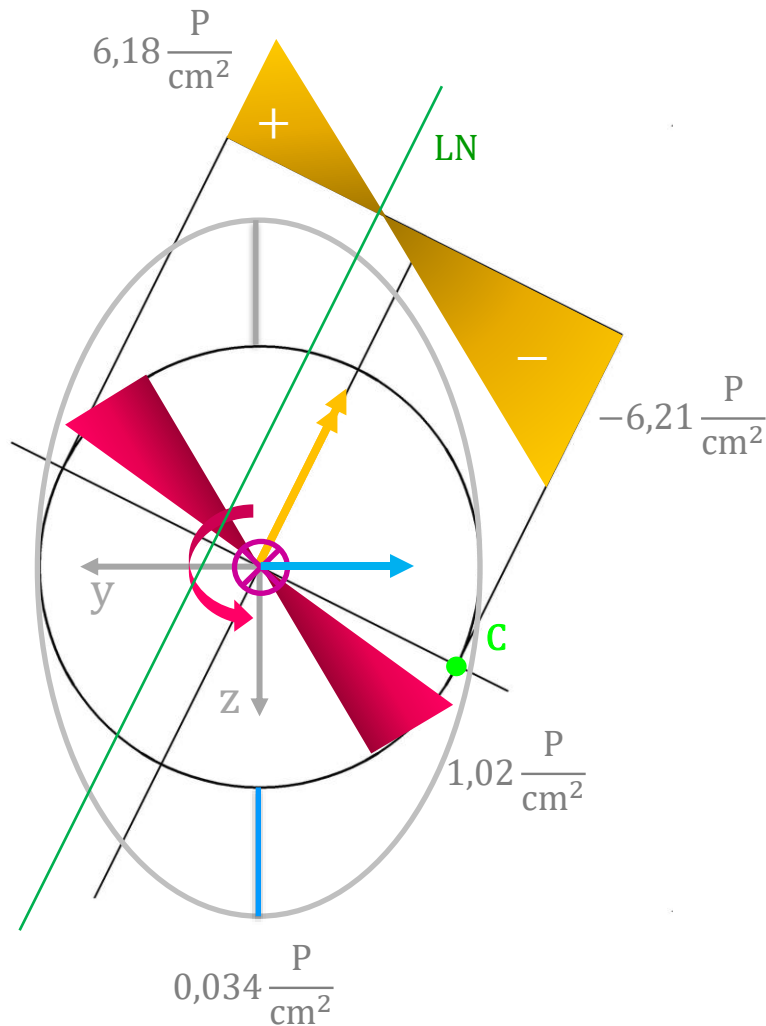


$$\sigma_x^{M_f} = \frac{M_f}{I} \cdot \frac{D}{2} = \frac{6,083P \text{ m}}{I} \cdot \frac{D}{2} = 6,20 \frac{P}{\text{cm}^2}$$

$$\sigma_x^N = \frac{N}{A} = \frac{-P}{78,54 \text{ cm}^2} = -0,0127 \frac{P}{\text{cm}^2}$$



¿Cuál es el punto más peligroso?



Tresca

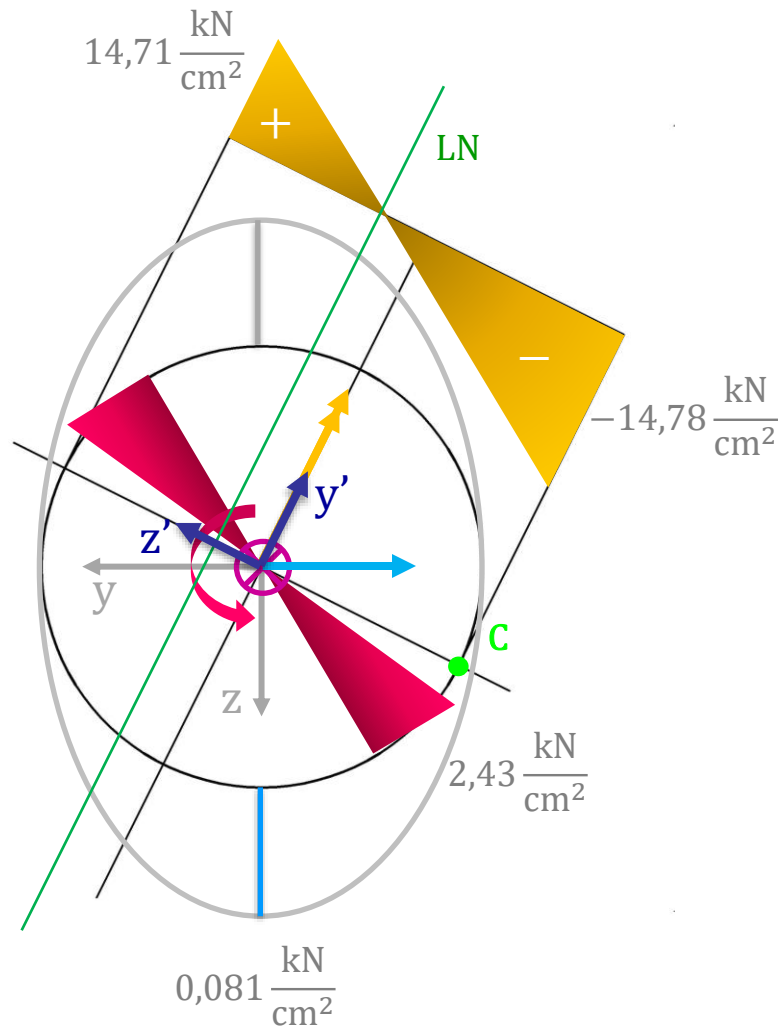
$$\sqrt{\sigma_X^2 + 4\tau_X^2} \leq \frac{\sigma_{fl}}{CS}$$

$$\sigma_x^c = -6,21 \frac{P}{\text{cm}^2}$$

$$\tau_x^c = 1,02 \frac{P}{\text{cm}^2}$$

$$6,29 \frac{P}{\text{cm}^2} \leq 15 \frac{\text{kN}}{\text{cm}^2}$$

$$P \leq 2,38 \text{ kN}$$



Considerando $P = 2,38 \text{ kN}$

$$\sigma_{x'}^C = -14,78 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{x'y'}^C = 2,43 \frac{\text{kN}}{\text{cm}^2}$$

$$[T_T]_{x'y'z'}^C = \begin{pmatrix} -14,78 & 2,43 & 0 \\ 2,43 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{\text{kN}}{\text{cm}^2}$$



Ejercicio 3:

- a) Diagramas característicos
- b) Calcular σ_{ADM} por Tresca y Von Mises
- c) Tensores y direcciones principales del punto más solicitado
- d) Diagrama de Mohr en el punto más solicitado

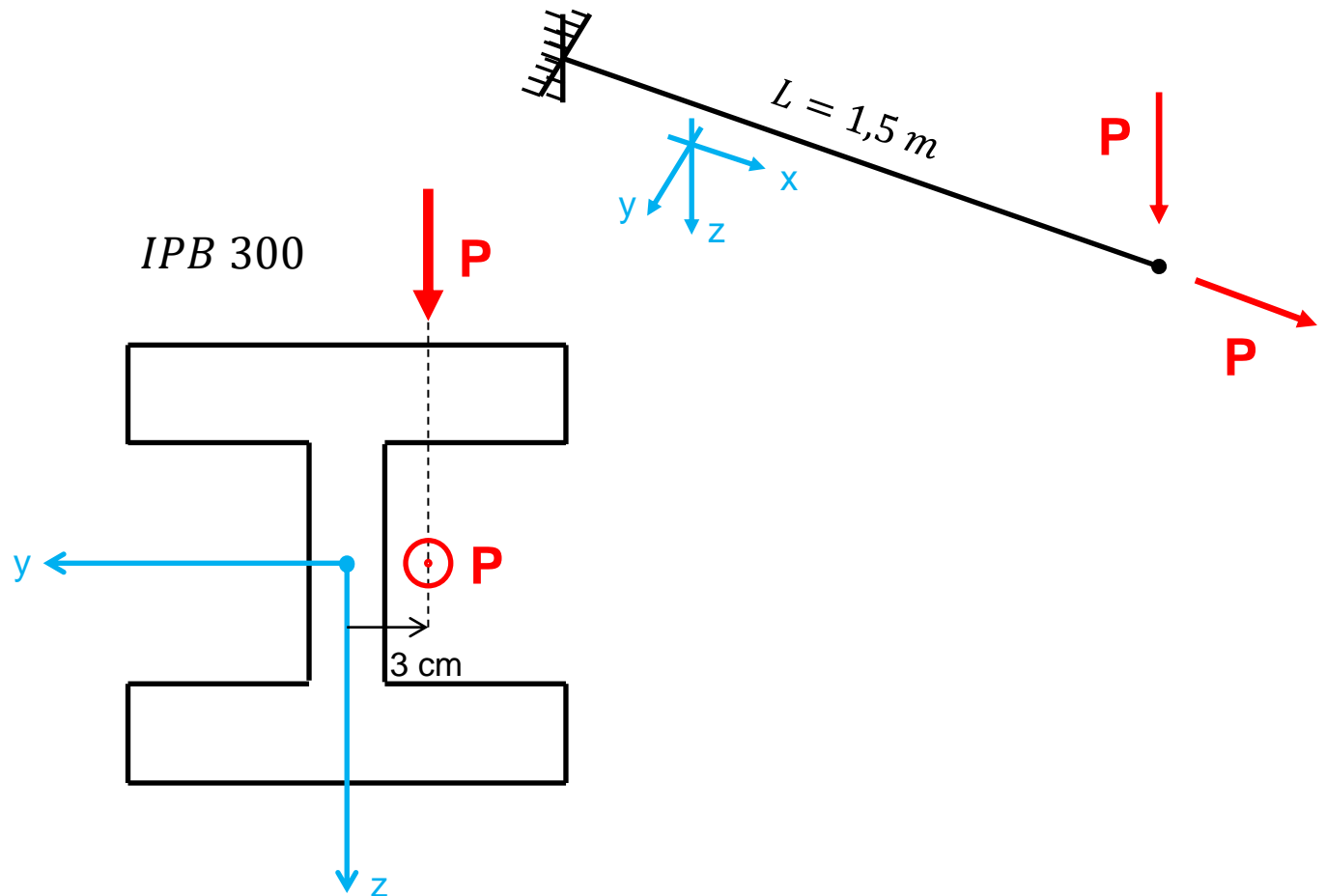
Datos:

$$P = 60 \text{ kN}$$

$$E = 21000 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_F = 24 \frac{\text{kN}}{\text{cm}^2}$$

$$\mu = 0,25$$





Datos del perfil

Revisamos la tabla

Pausa dramática para revisar la Tabla de perfiles CIRSOC...

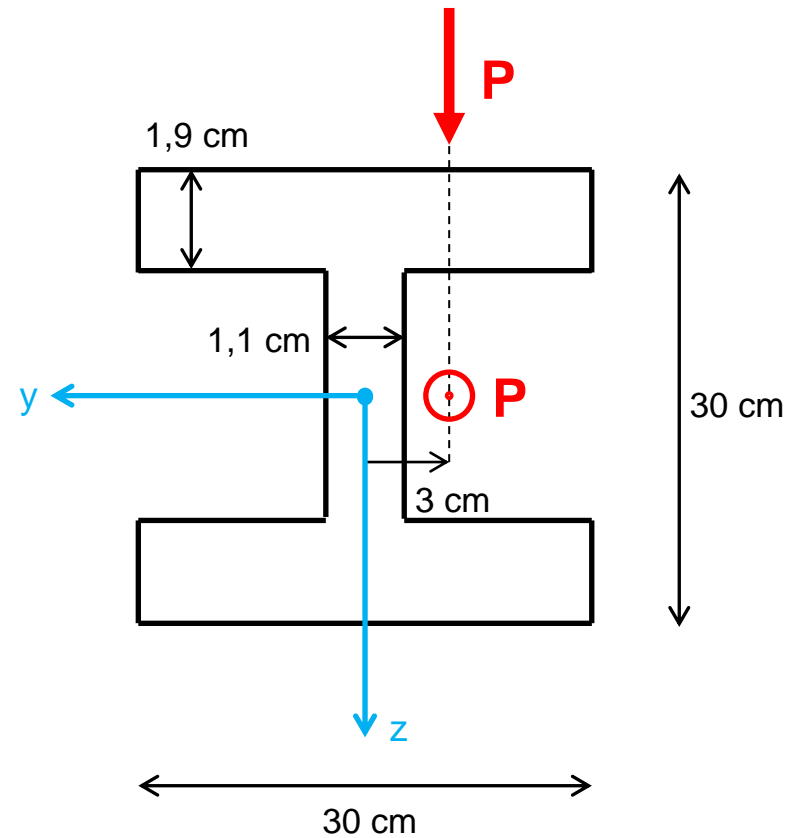
De tabla:

$$A = 149 \text{ cm}^2$$

$$J_y = 25170 \text{ cm}^4$$

$$J_z = 8560 \text{ cm}^4$$

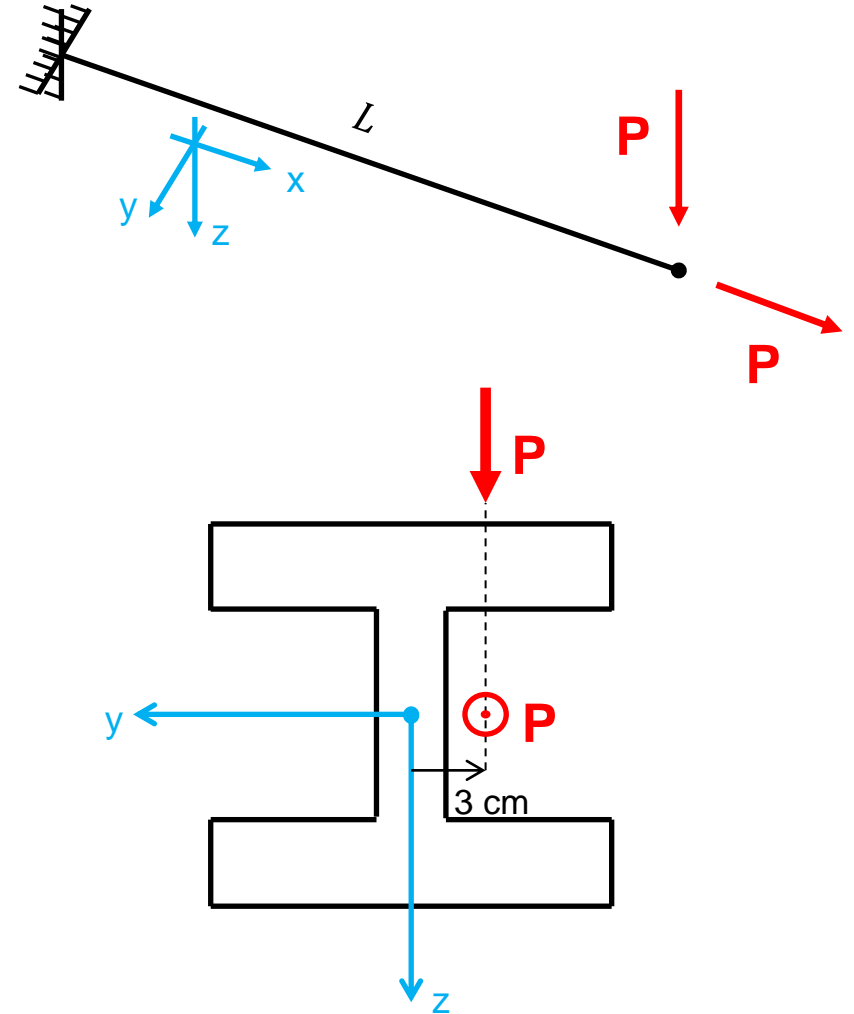
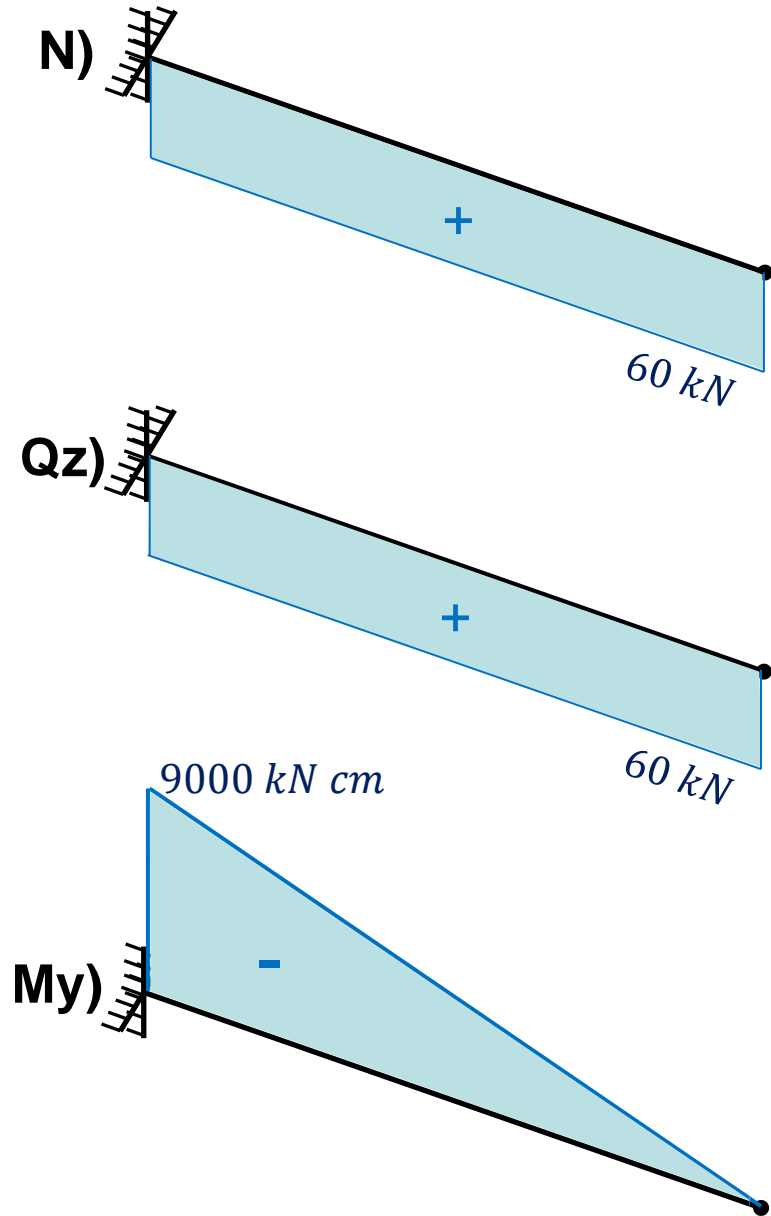
$$J_T = 149 \text{ cm}^4$$



Diagramas de características



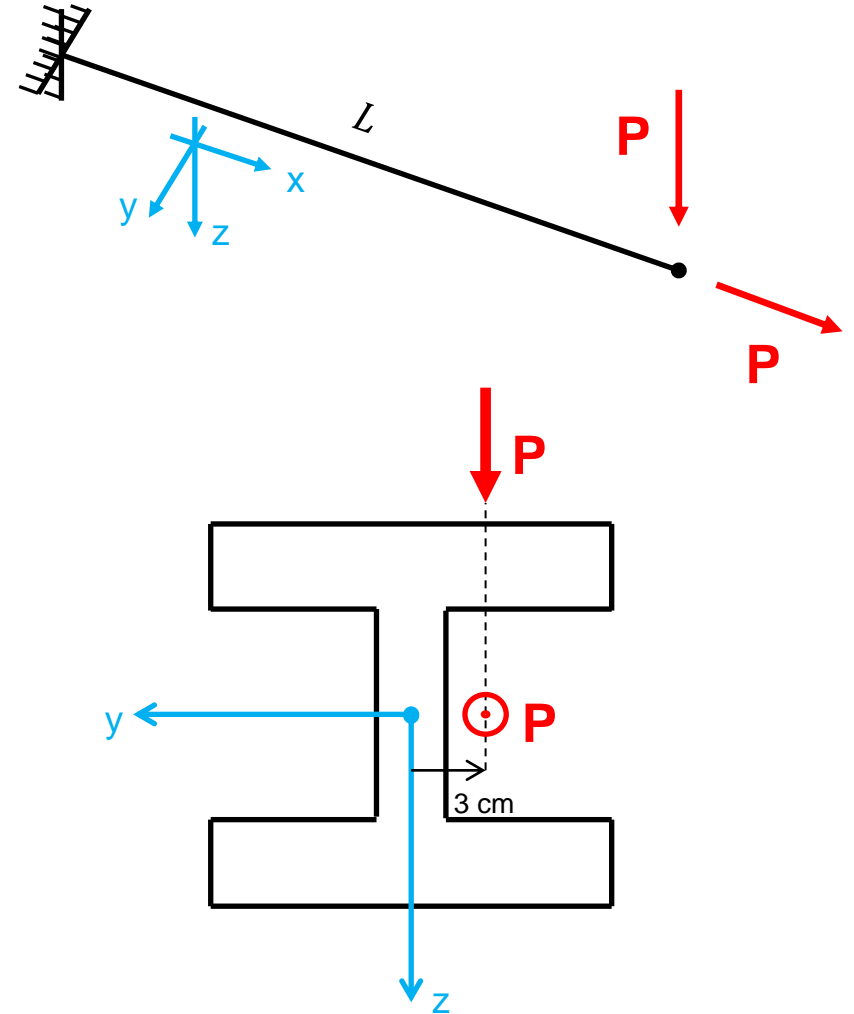
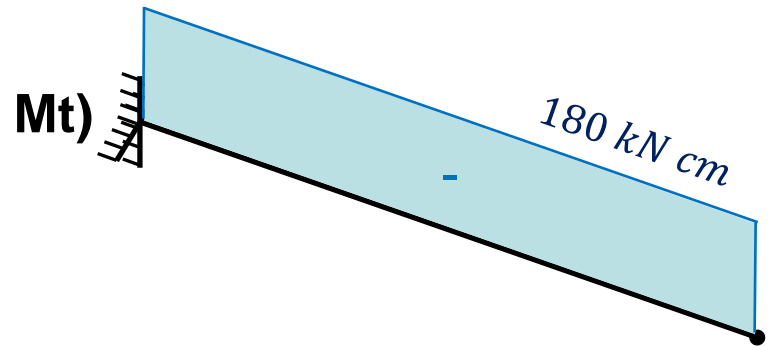
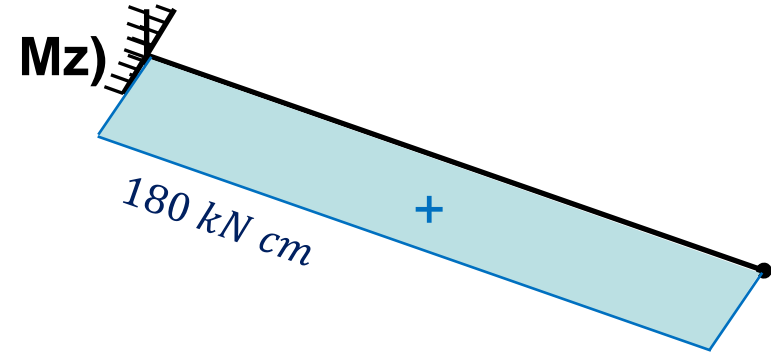
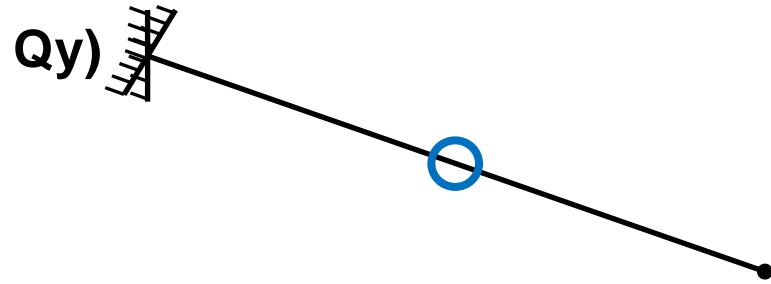
Datos:
 $P = 60 \text{ kN}$
 $L = 150 \text{ cm}$

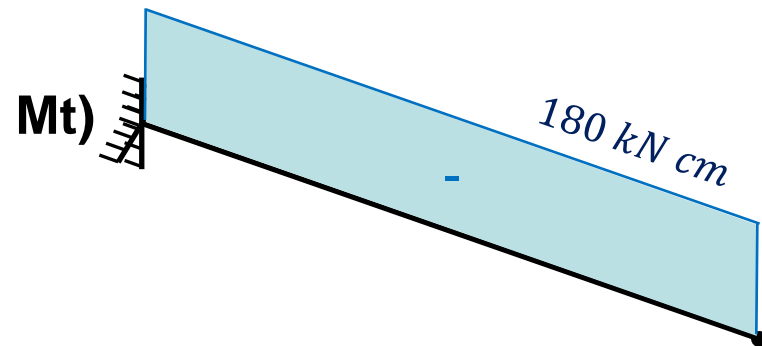
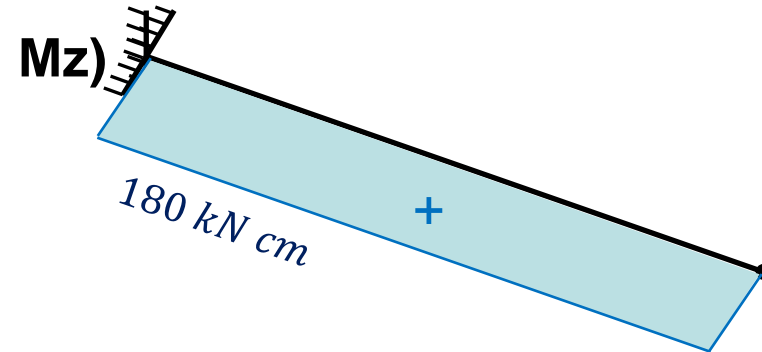
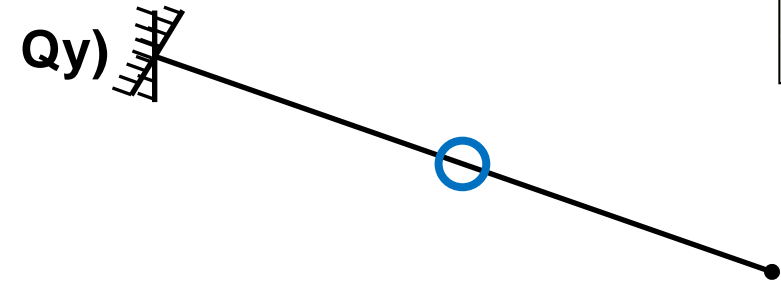
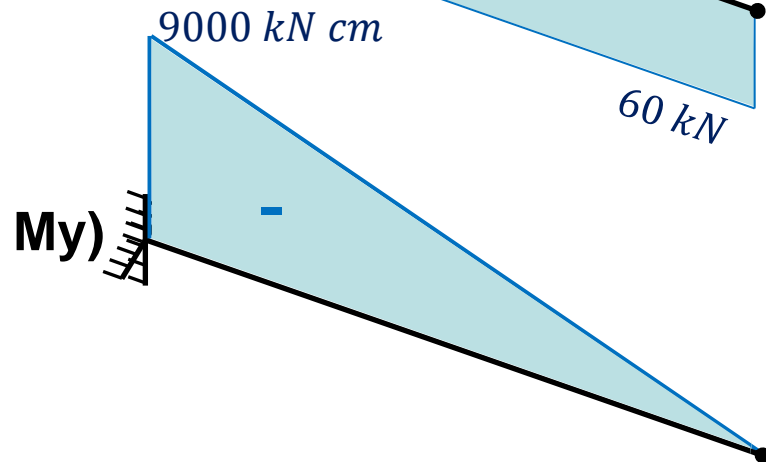
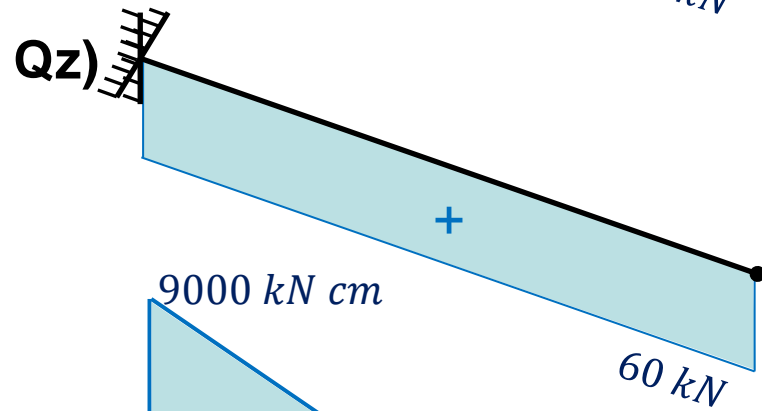
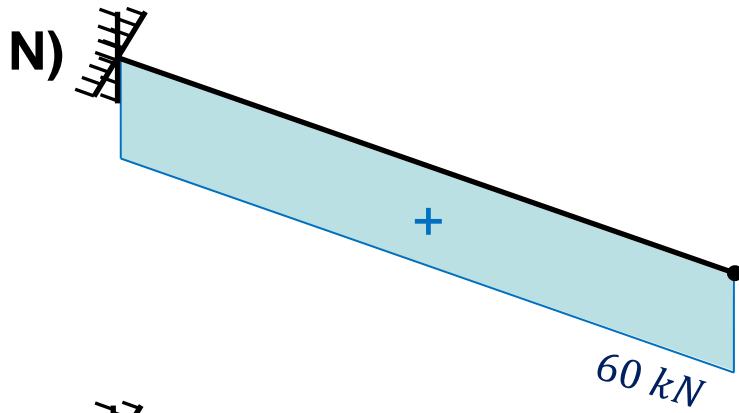


Diagramas de características



Datos:
 $P = 60 \text{ kN}$
 $L = 150 \text{ cm}$





¿Qué sección verificamos?

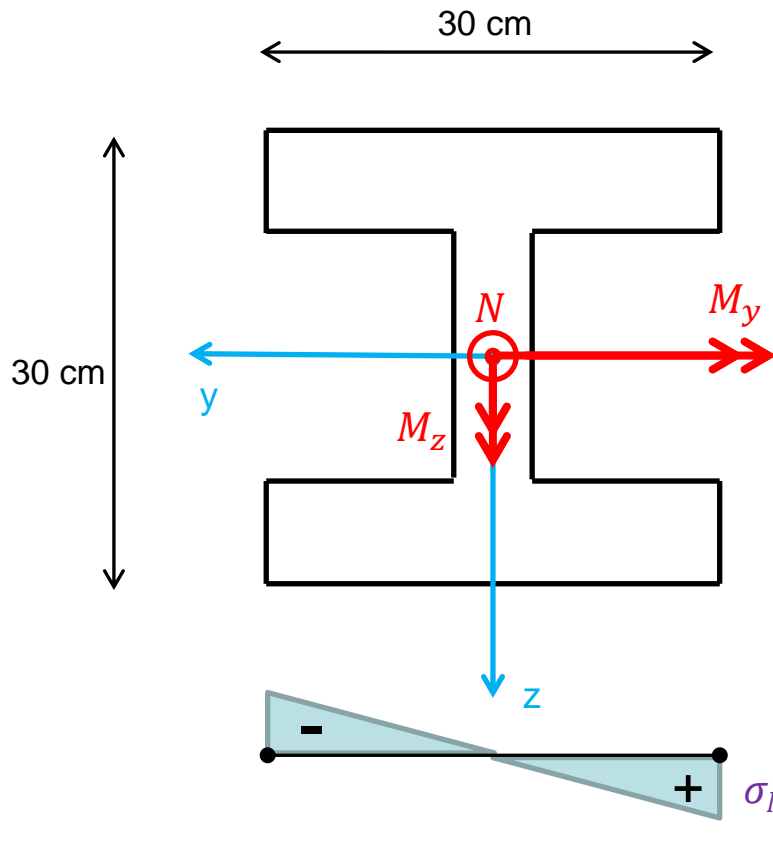
Verificamos el empotramiento



Diagramas de tensiones normales

Solicitaciones: $N = 60 \text{ kN}$
 $M_y = -9000 \text{ kNcm}$
 $M_z = 180 \text{ kNcm}$

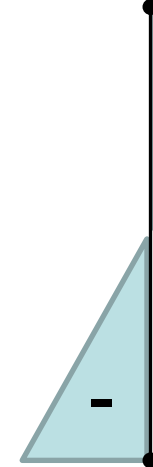
Datos del perfil: $A = 149 \text{ cm}^2$
 $J_y = 25170 \text{ cm}^4$
 $J_z = 8560 \text{ cm}^4$



$$\sigma_N = \frac{N}{A} = 0,403 \frac{\text{kN}}{\text{cm}^2}$$



$$\sigma_{M_y} = \frac{M_y}{J_y} \cdot z = 5,364 \frac{\text{kN}}{\text{cm}^2}$$



$$\sigma_{M_z} = \frac{M_z}{J_z} \cdot y = 0,315 \frac{\text{kN}}{\text{cm}^2}$$

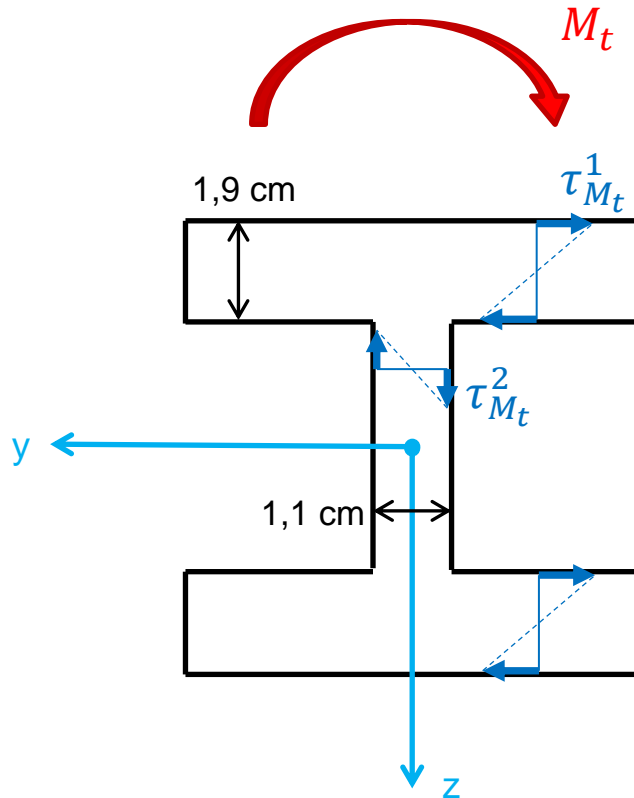




Diagramas de tensiones tangenciales por M_t

Solicitaciones: $M_t = -180 \text{ kN cm}$

Datos del perfil: $J_T = 149 \text{ cm}^4$



Por ser una sección abierta, calculo el τ_{M_t} con Saint Venant.

$$\tau_{M_t} = \frac{M_t}{J_t} \cdot e$$

$$|\tau_{M_t}^1| = 2,295 \frac{\text{kN}}{\text{cm}^2}$$

$$|\tau_{M_t}^2| = 1,329 \frac{\text{kN}}{\text{cm}^2}$$



Diagramas de tensiones tangenciales por Q_z

Solicitaciones: $Q_z = 60 \text{ kN}$

Datos del perfil: $J_y = 25170 \text{ cm}^4$
 $J_z = 8560 \text{ cm}^4$

Calculo el τ_{Q_z} con Jouravski-Colignon

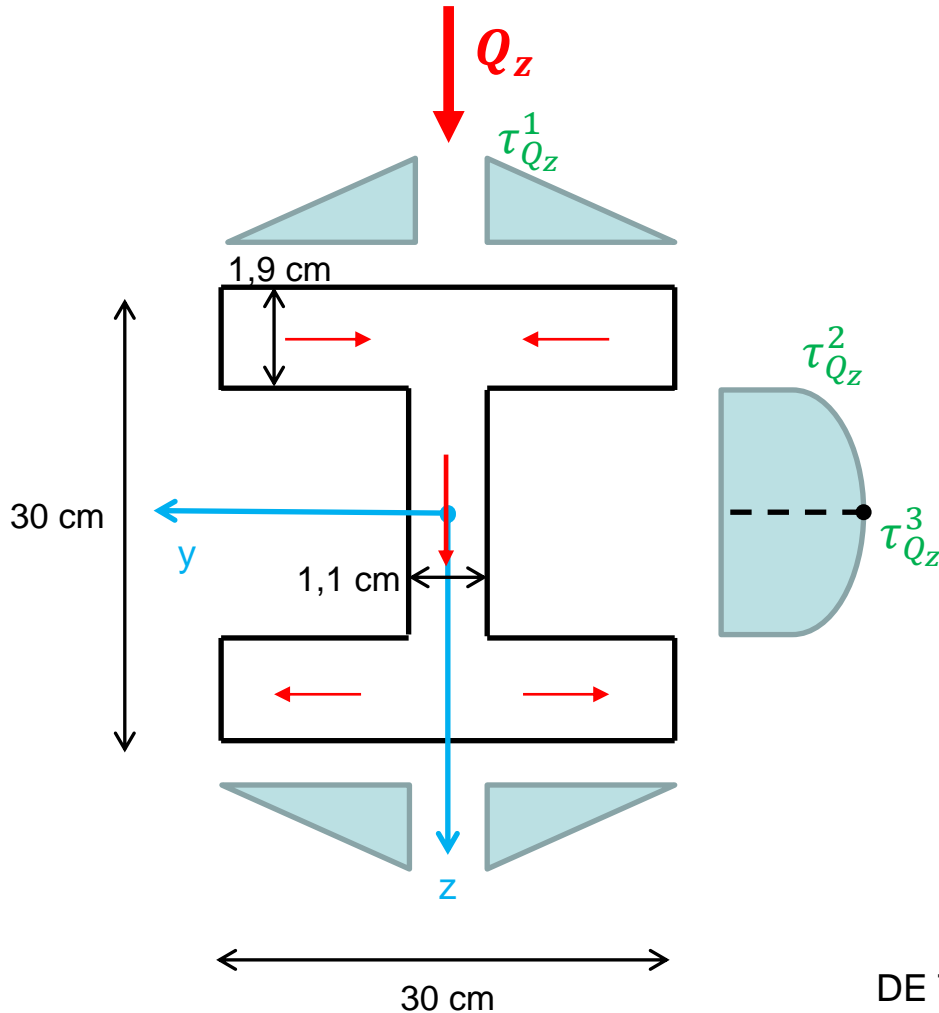
$$\tau_{Q_z} = \frac{Q \cdot S^*}{J \cdot b}$$

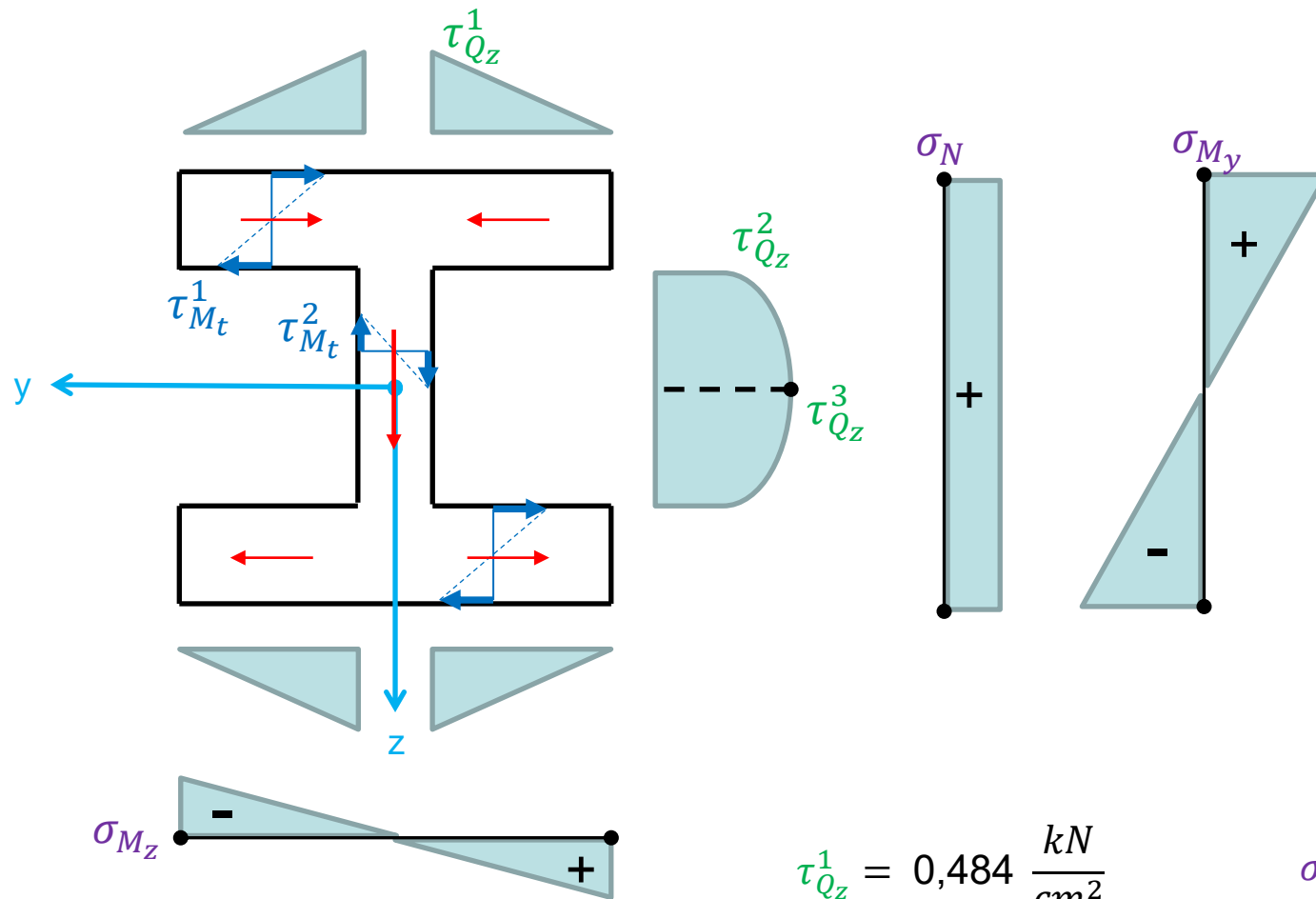
$$\tau_{Q_z}^1 = \frac{Q_z \cdot \left(1,9 \cdot \left(\frac{30}{2} - \frac{1,1}{2}\right) \cdot \left(\frac{30}{2} - \frac{1,9}{2}\right)\right)}{J_y \cdot 1,9} = 0,484 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{Q_z}^2 = \frac{Q_z \cdot \left(1,9 \cdot 30 \cdot \left(\frac{30}{2} - \frac{1,9}{2}\right)\right)}{J_y \cdot 1,1} = 1,736 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{Q_z}^3 = \frac{Q_z \cdot (934)}{J_y \cdot 1,1} = 2,024 \frac{\text{kN}}{\text{cm}^2}$$

DE TABLA!





$$|\tau_{M_t}^1| = 2,295 \frac{kN}{cm^2}$$

$$|\tau_{M_t}^2| = 1,329 \frac{kN}{cm^2}$$

$$\tau_{Q_z}^1 = 0,484 \frac{kN}{cm^2}$$

$$\tau_{Q_z}^2 = 1,736 \frac{kN}{cm^2}$$

$$\tau_{Q_z}^3 = 2,024 \frac{kN}{cm^2}$$

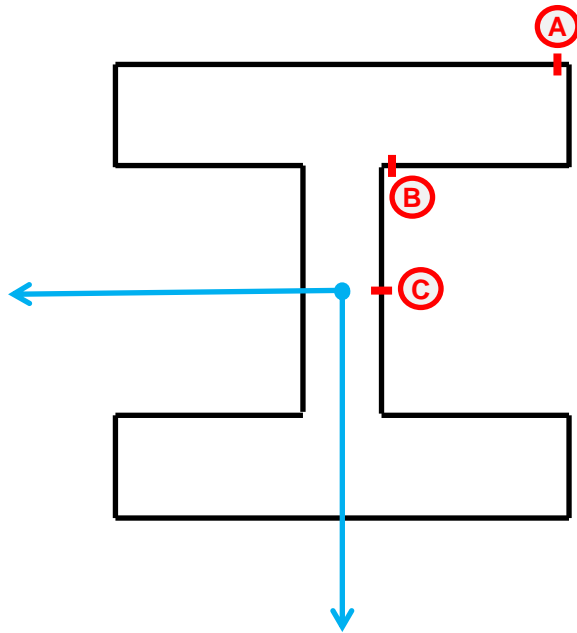
$$\sigma_N = 0,403 \frac{kN}{cm^2}$$

$$\sigma_{M_y} = 5,364 \frac{kN}{cm^2}$$

$$\sigma_{M_z} = 0,315 \frac{kN}{cm^2}$$



Verificamos puntos críticos



$$|\sigma_A| = \sigma_N + \sigma_{M_y} + \sigma_{M_z} = 6,082 \frac{kN}{cm^2}$$

$$|\tau_A| = \tau_{M_t}^1 = 2,295 \frac{kN}{cm^2}$$

$$\sigma_{Tresca}^A = 7,619 \frac{kN}{cm^2}$$

$$\sigma_{VonMises}^A = 7,266 \frac{kN}{cm^2}$$

$$|\sigma_B| = \sigma_N + \sigma_{M_y}' + \sigma_{M_z}' = 5,099 \frac{kN}{cm^2}$$

$$|\tau_B| = \tau_{M_t}^1 + \tau_{Q_z}^1 = 2,779 \frac{kN}{cm^2}$$

$$\sigma_{Tresca}^B = 7,543 \frac{kN}{cm^2}$$

$$\sigma_{VonMises}^B = 7,012 \frac{kN}{cm^2}$$

$$|\sigma_C| = \sigma_N + \sigma_{M_z}' = 0,415 \frac{kN}{cm^2}$$

$$|\tau_C| = \tau_{M_t}^2 + \tau_{Q_z}^3 = 3,353 \frac{kN}{cm^2}$$

$$\sigma_{Tresca}^C = 6,719 \frac{kN}{cm^2}$$

$$\sigma_{VonMises}^C = 5,822 \frac{kN}{cm^2}$$

Von Mises: $\sigma_{VonMises} = \sqrt{\sigma^2 + 3 \cdot \tau^2}$

Tresca: $\sigma_{Tresca} = \sqrt{\sigma^2 + 4 \cdot \tau^2}$



Tensor de Tensiones

Armo los tensores para el punto A

$$[T_{T_{xyz}}]^A = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} = \begin{pmatrix} 6,082 & -2,295 & 0 \\ -2,295 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{kN}{cm^2}$$

Estado *DOBLE* de tensiones

$$\sigma_{1,3} = \frac{\sigma \pm \sqrt{\sigma^2 + 4 \cdot \tau^2}}{2} \quad \text{Solo vale para estados dobles!}$$

$$\rightarrow \sigma_1 = 6,851 \frac{kN}{cm^2}$$

$$\rightarrow \sigma_3 = -0,769 \frac{kN}{cm^2}$$

$$[T_{T_{123}}]^A = \begin{pmatrix} 6,851 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0,769 \end{pmatrix} \frac{kN}{cm^2}$$



Tensor de Deformaciones

Datos del material: $E = 21000 \frac{kN}{cm^2}$
 $\mu = 0,25$

$$[T_{T_{123}}]^A = \begin{pmatrix} 6,851 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0,769 \end{pmatrix} \frac{kN}{cm^2} \left\{ \begin{array}{l} \varepsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_3) = 3,354 \times 10^{-4} \\ \varepsilon_2 = \frac{1}{E}(-\mu\sigma_1 - \mu\sigma_3) = -0,724 \times 10^{-4} \\ \varepsilon_3 = \frac{1}{E}(-\mu\sigma_1 + \sigma_3) = -1,182 \times 10^{-4} \end{array} \right.$$

$$[T_{D_{123}}]^A = \begin{pmatrix} 3,354 & 0 & 0 \\ 0 & -0,724 & 0 \\ 0 & 0 & -1,182 \end{pmatrix} \cdot 10^{-4}$$

Estado *TRIPLE* de deformaciones

Direcciones principales



$$\begin{aligned} \sigma_1 &= 6,851 \frac{kN}{cm^2} \\ \sigma_2 &= 0 \\ \sigma_3 &= -0,769 \frac{kN}{cm^2} \end{aligned}$$

$$[T_{T_{xyz}}]^A = \begin{pmatrix} 6,082 & -2,295 & 0 \\ -2,295 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{kN}{cm^2} \quad V_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$V_1 = \begin{pmatrix} (6,082 - 6,851) & -2,295 & 0 \\ -2,295 & -6,851 & 0 \\ 0 & 0 & -6,851 \end{pmatrix} \cdot \begin{pmatrix} V_{x1} \\ V_{y1} \\ V_{z1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} -0,769 \cdot V_{x1} - 2,295 \cdot V_{y1} &= 0 & \rightarrow & V_{x1} = -\frac{2,295}{0,769} \cdot V_{y1} & V_{x1} &= -2,984 \\ -2,295 \cdot V_{x1} - 6,851 \cdot V_{y1} &= 0 & \rightarrow & V_{y1} = 1 \cdot V_{y1} & V_{y1} &= 1 \\ -6,851 \cdot V_{z1} &= 0 & \rightarrow & V_{z1} = 0 \cdot V_{y1} & V_{z1} &= 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} -0,769 \cdot V_{x1} - 2,295 \cdot V_{y1} &= 0 \\ -2,295 \cdot V_{x1} - 6,851 \cdot V_{y1} &= 0 \\ -6,851 \cdot V_{z1} &= 0 \end{aligned}} \right\} V_1 = \begin{pmatrix} 0,95 \\ -0,32 \\ 0 \end{pmatrix}$$

Normalizo (y cambio el signo para que V_{x1} sea positivo)

$$V_{x1} \text{ (normalizado)} = \frac{V_{x1}}{\sqrt{V_{x1}^2 + V_{y1}^2 + V_{z1}^2}}$$



Direcciones principales

$$V_1 = \begin{pmatrix} 0,95 \\ -0,32 \\ 0 \end{pmatrix} \quad V_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

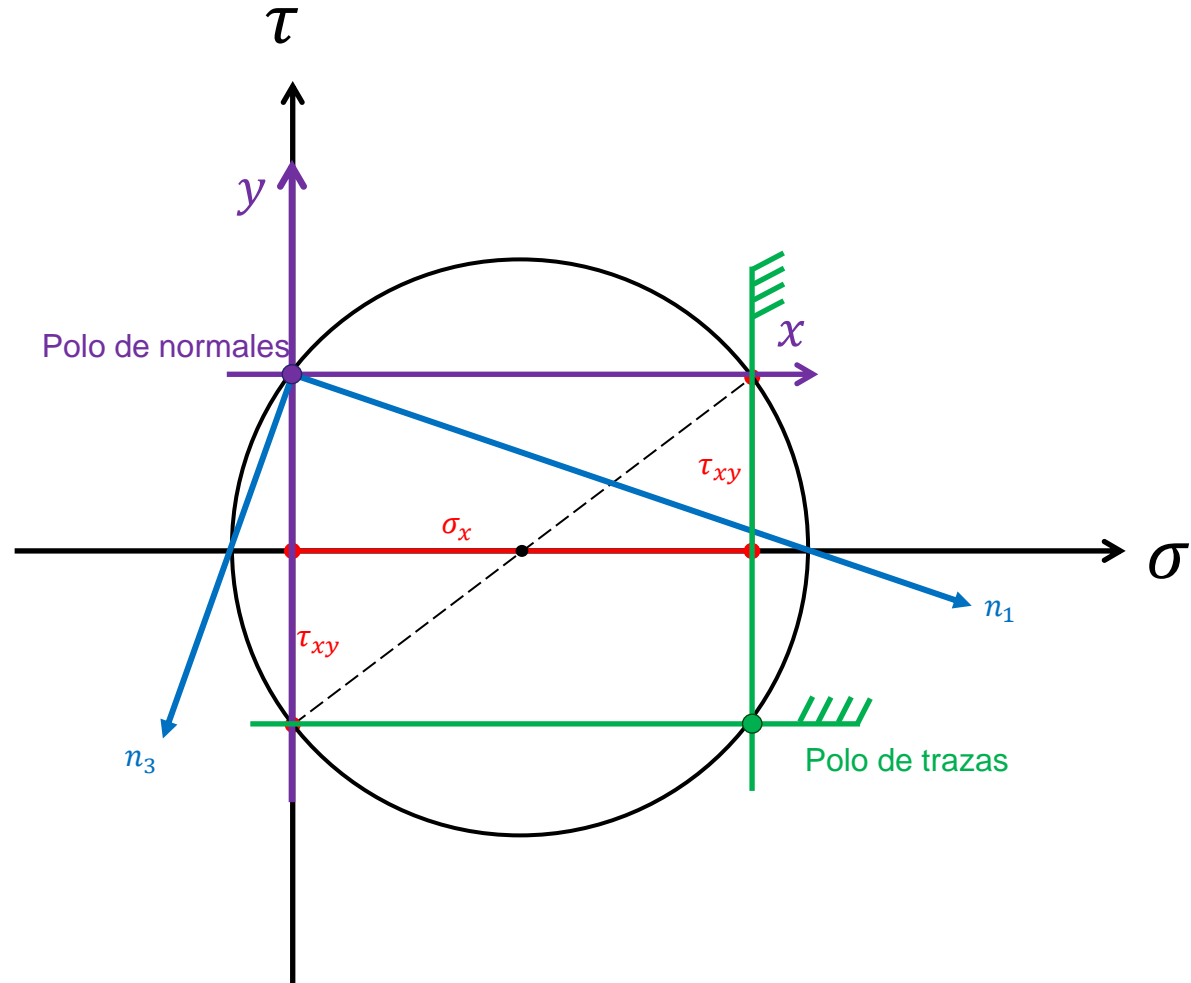
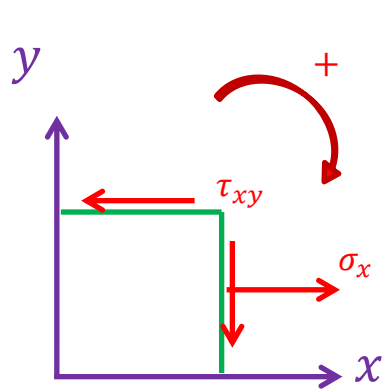
$$V_3 = V_1 \times V_2 = \begin{pmatrix} 0,95 \\ -0,32 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -0,32 \\ -0,95 \\ 0 \end{pmatrix}$$

Las direcciones principales
son ortogonales

$$V_1 = \begin{pmatrix} 0,95 \\ -0,32 \\ 0 \end{pmatrix} ; V_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} ; V_3 = \begin{pmatrix} -0,32 \\ -0,95 \\ 0 \end{pmatrix}$$

Circunferencia de Mohr

Tensiones: $\sigma_x = 6,082 \frac{kN}{cm^2}$
 $\tau_{xy} = -2,295 \frac{kN}{cm^2}$



$$V_1 = \begin{pmatrix} 0,95 \\ -0,32 \\ 0 \end{pmatrix}$$

$$V_3 = \begin{pmatrix} -0,32 \\ -0,95 \\ 0 \end{pmatrix}$$