TEORIAS DE LOS ESTADOS LIMITES

- Estado Límite de un punto
- Estado Límite de una sección
- Estado Límite de la estructura

HIPÓTESIS

- Cuerpo continuo
- Material isótropo y homogéneo
- Deformaciones infinitamente pequeñas
- Linealidad mecánica
- Linealidad cinemática
- Linealidad estática
- Principio de superposición de efectos
- Principio de Saint Venant

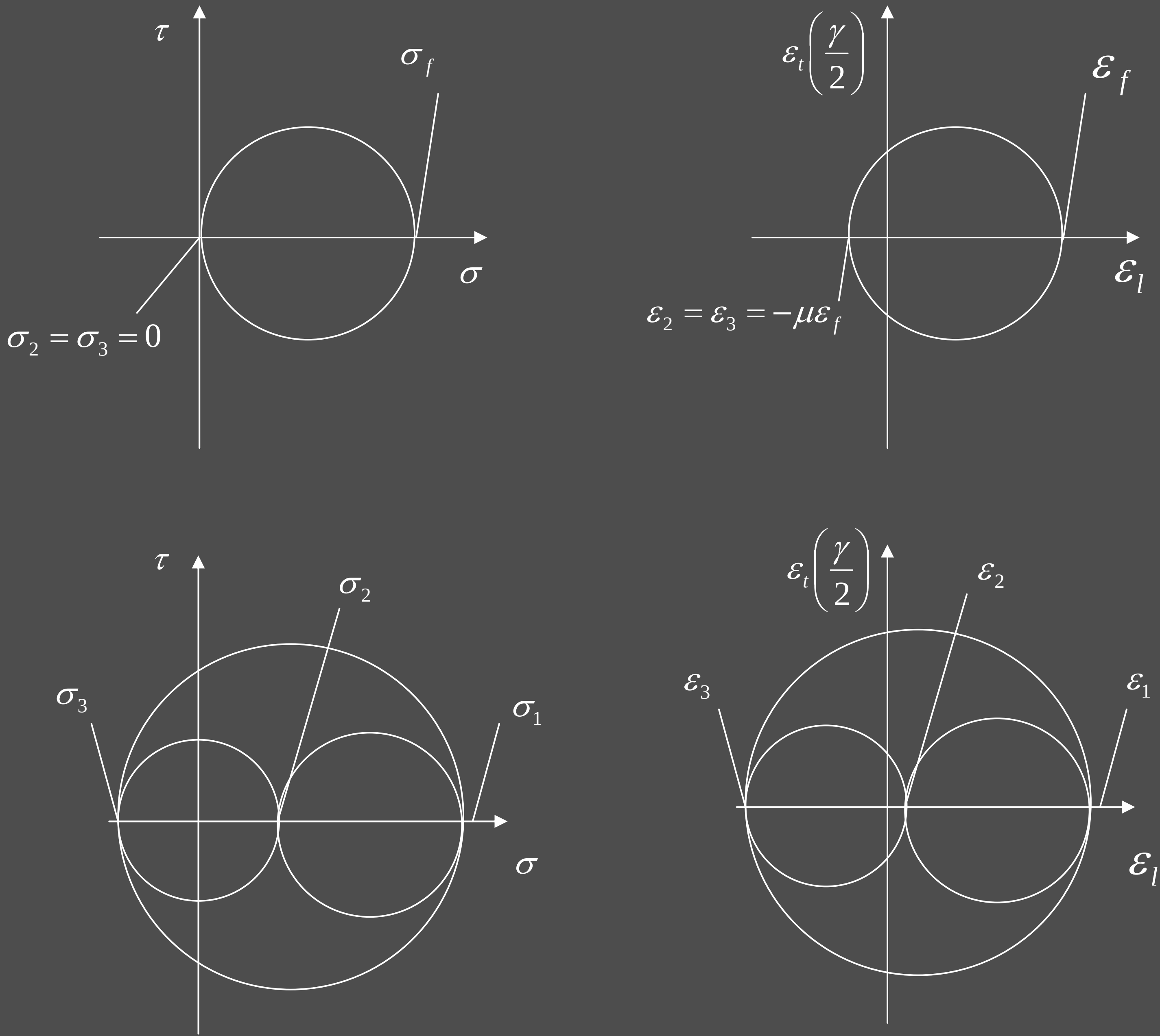
Relación Tensión - Deformación

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_x - \mu\sigma_y - \mu\sigma_z\right) \qquad \varepsilon_{xy} = \frac{\gamma_{xy}}{2} = \frac{\tau_{xy}}{2G}$$

$$\varepsilon_{yy} = \frac{1}{E}\left(\sigma_y - \mu\sigma_x - \mu\sigma_z\right) \qquad \varepsilon_{yz} = \frac{\gamma_{yz}}{2} = \frac{\tau_{yz}}{2G}$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_z - \mu\sigma_x - \mu\sigma_y\right) \qquad \varepsilon_{zx} = \frac{\gamma_{zx}}{2} = \frac{\tau_{zx}}{2G}$$

$$\varepsilon_V = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{1-2\mu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right) = \frac{1-2\mu}{E} I_\sigma^1$$



τ
σ_f
σ
$\sigma_2 = \sigma_3 = 0$
$\varepsilon_t\left(\frac{\gamma}{2}\right)$
ε_f
ε_l
$\varepsilon_2 = \varepsilon_3 = -\mu\varepsilon_f$
τ
σ_2
σ_3
σ_1
σ
$\varepsilon_t\left(\frac{\gamma}{2}\right)$
ε_2
ε_3
ε_1
ε_l

Teorías de estados límites

Indicadores

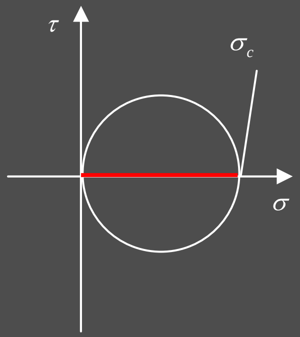
1. Máxima tensión normal (Rankine)
2. Máxima tensión tangencial (Tresca)
3. Máxima deformación longitudinal (Saint-Venant)
4. Energía total de deformación (Beltrami)
5. Energía de distorsión (Von Mises)
6. Tensión tangencial octaédrica

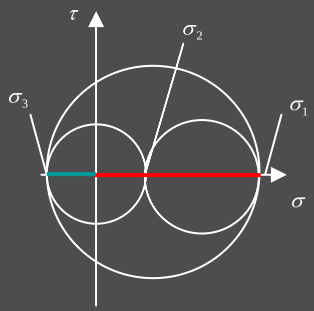
Teoría empírica

Teoría de Mohr

Máxima tensión normal (Rankine)

$$\boxed{\sigma_c = \sigma_1}$$

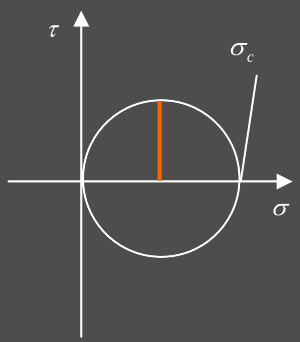


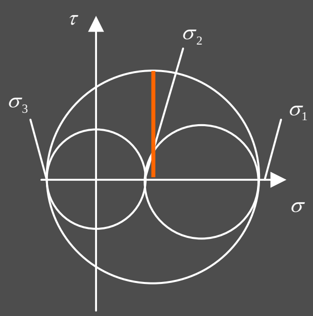


Máxima tensión tangencial (Tresca)

$$\frac{\sigma_c}{2} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\boxed{\sigma_c = \sigma_1 - \sigma_3}$$





Estado de Corte o resbalamiento puro

$$\sigma_1 = -\sigma_3 \quad \sigma_2 = 0$$

$$\boxed{\sigma_c = 2\sigma_1}$$

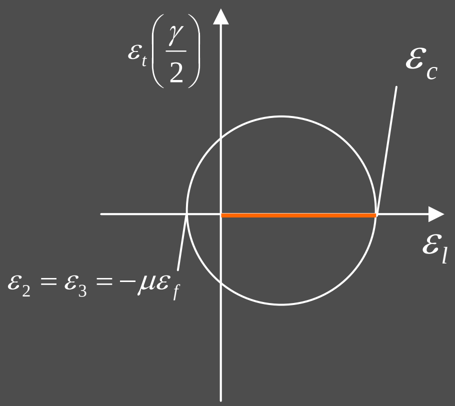
$$\boxed{\sigma_1 = \tau_M = 0.5\,\sigma_c}$$

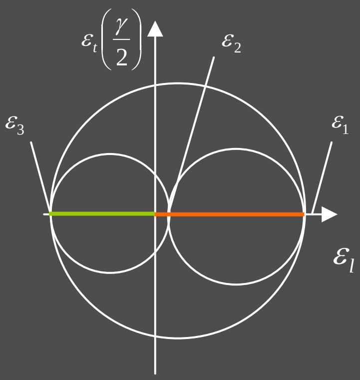
Máxima deformación longitudinal (Saint-Venant)

$$\varepsilon_c = \varepsilon_1$$

$$\frac{1}{E}\sigma_c = \frac{1}{E}\left(\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right)$$

$$\boxed{\sigma_c = \sigma_1 - \mu\sigma_2 - \mu\sigma_3}$$





Estado de Corte o resbalamiento puro

$$\sigma_1 = -\sigma_3 \quad \sigma_2 = 0$$

$$\boxed{\sigma_c = 1.1 a 1.25\,\sigma_1}$$

$$(\mu = 0{,}1 a 0{,}25)$$

$$\boxed{\sigma_1 = \tau_M = 0{,}91\ a\ 0.8\,\sigma_c}$$

Energía total de deformación (Beltrami)

$$u = \frac{1}{2}\left(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3\right)$$

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right]$$

Estado lineal de tensiones $\sigma_1 = \sigma_c$ $\sigma_2 = \sigma_3 = 0$

$$u_c = \frac{1}{2E}\sigma_c^2 = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right]$$

$$\boxed{\sigma_c = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)}}$$

Estado de Corte o resbalamiento puro

$\sigma_1 = -\sigma_3$ $\sigma_2 = 0$

$(\mu = 0{,}1 a 0{,}25)$

$$\boxed{\sigma_c = 1.483\ a\ 1.581\,\sigma_1}$$

$$\boxed{\sigma_1 = \tau_M = 0.674\ a\ 0.633\,\sigma_c}$$

Energía de distorsión (Von Mises)

$$u_d = \frac{1+\mu}{6E}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]$$

Estado lineal de tensiones $\sigma_1 = \sigma_c \qquad \sigma_2 = \sigma_3 = 0$

$$u_{dc} = \frac{1+\mu}{6E}2\sigma_c^2 = \frac{1+\mu}{6E}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]$$

$$\boxed{\sigma_c = \sqrt{\frac{\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2}{2}}}$$

Estado de Corte o resbalamiento puro

$\sigma_1 = -\sigma_3 \quad \sigma_2 = 0$

$$\boxed{\sigma_c = \sqrt{3}\sigma_1 = 1{,}732\,\sigma_1}$$

$$\boxed{\sigma_1 = \tau_M = 0.577\,\sigma_c}$$

Energía de distorsión (Von Mises)

$$[T_T] = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = [T_D] + [T_E]$$

$$[T_T] = \begin{bmatrix} \sigma_1 - p & 0 & 0 \\ 0 & \sigma_2 - p & 0 \\ 0 & 0 & \sigma_3 - p \end{bmatrix} + \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}$$

$$p = \frac{I^1_{\sigma T}}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \Rightarrow I^1_{\sigma D} = 0, I^1_{\sigma E} = I^1_{\sigma T}$$

$$\varepsilon_V = \frac{1 - 2\mu}{E} I^1_\sigma \qquad \varepsilon_{VD} = 0 \qquad \varepsilon_{VE} = \varepsilon_{VT}$$

Tensión tangencial octaédrica

$$n_{1oct}=n_{2oct}=n_{3oct}=\pm\frac{\sqrt{3}}{3}$$

$$\rho_{1oct}=\frac{\sqrt{3}}{3}\sigma_1 \qquad \rho_{2oct}=\frac{\sqrt{3}}{3}\sigma_2 \qquad \rho_{3oct}=\frac{\sqrt{3}}{3}\sigma_3$$

$$\overline{\sigma_{oct}}=\left(\bar{\rho}\cdot\bar{n}\right)\cdot\bar{n}=\left[\frac{1}{3}\left(\sigma_1+\sigma_2+\sigma_3\right)\right]\bar{n} \qquad \bar{\tau}_{oct}=\bar{\rho}_{oct}-\bar{\sigma}_{oct}$$

$$\tau_{1oct}=\left[\sigma_1-\frac{1}{3}\left(\sigma_1+\sigma_2+\sigma_3\right)\right]\frac{\sqrt{3}}{3}$$

$$\tau_{2oct}=\left[\sigma_2-\frac{1}{3}\left(\sigma_1+\sigma_2+\sigma_3\right)\right]\frac{\sqrt{3}}{3}$$

$$\tau_{3oct}=\left[\sigma_3-\frac{1}{3}\left(\sigma_1+\sigma_2+\sigma_3\right)\right]\frac{\sqrt{3}}{3}$$

Tensión tangencial octaédrica

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

Estado lineal de tensiones $\sigma_1 = \sigma_c \qquad \sigma_2 = \sigma_3 = 0$

$$\tau_{coct} = \frac{1}{3}\sqrt{2\sigma_c^2} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\boxed{\sigma_c = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}}$$

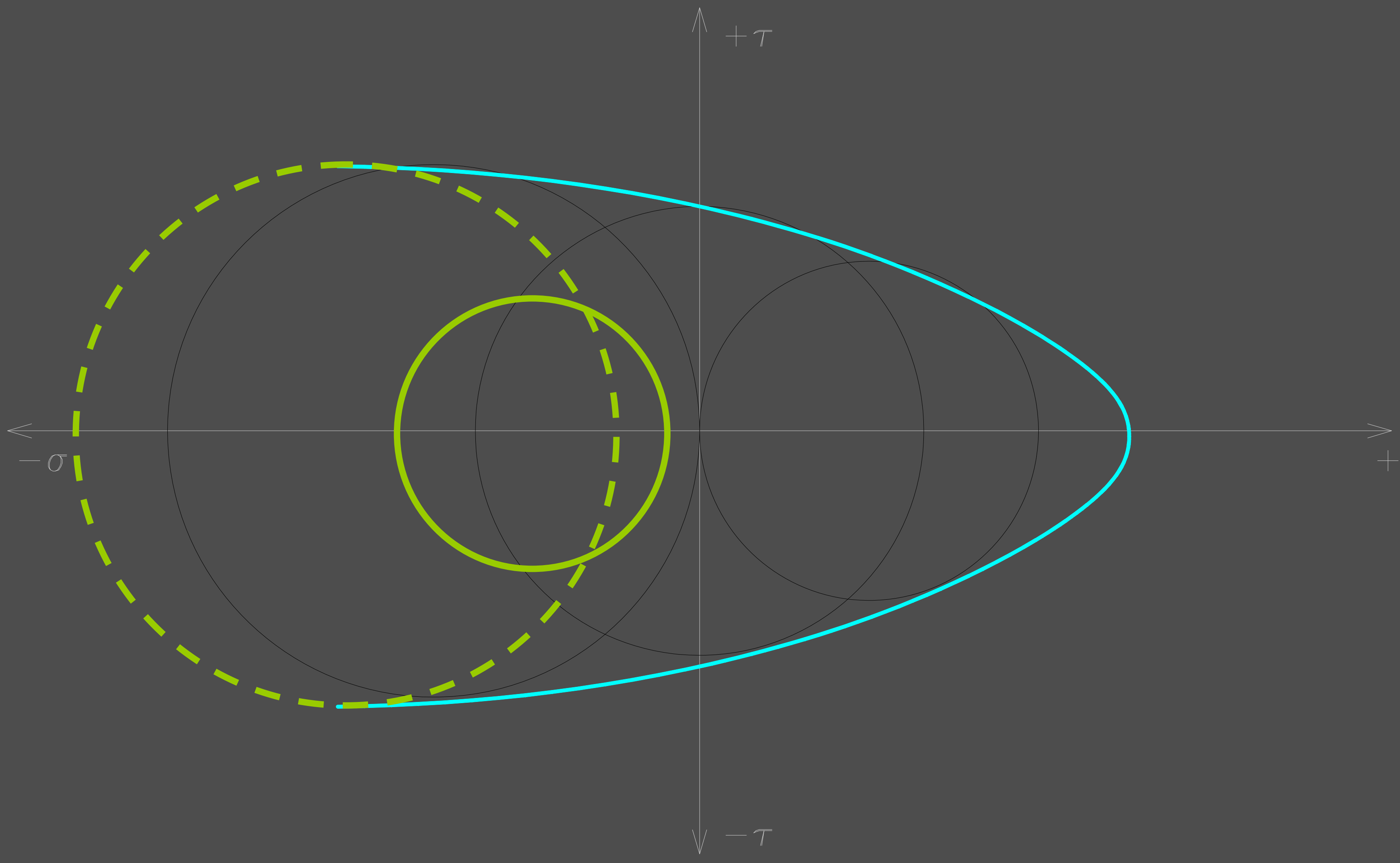
Estado de Corte o resbalamiento puro

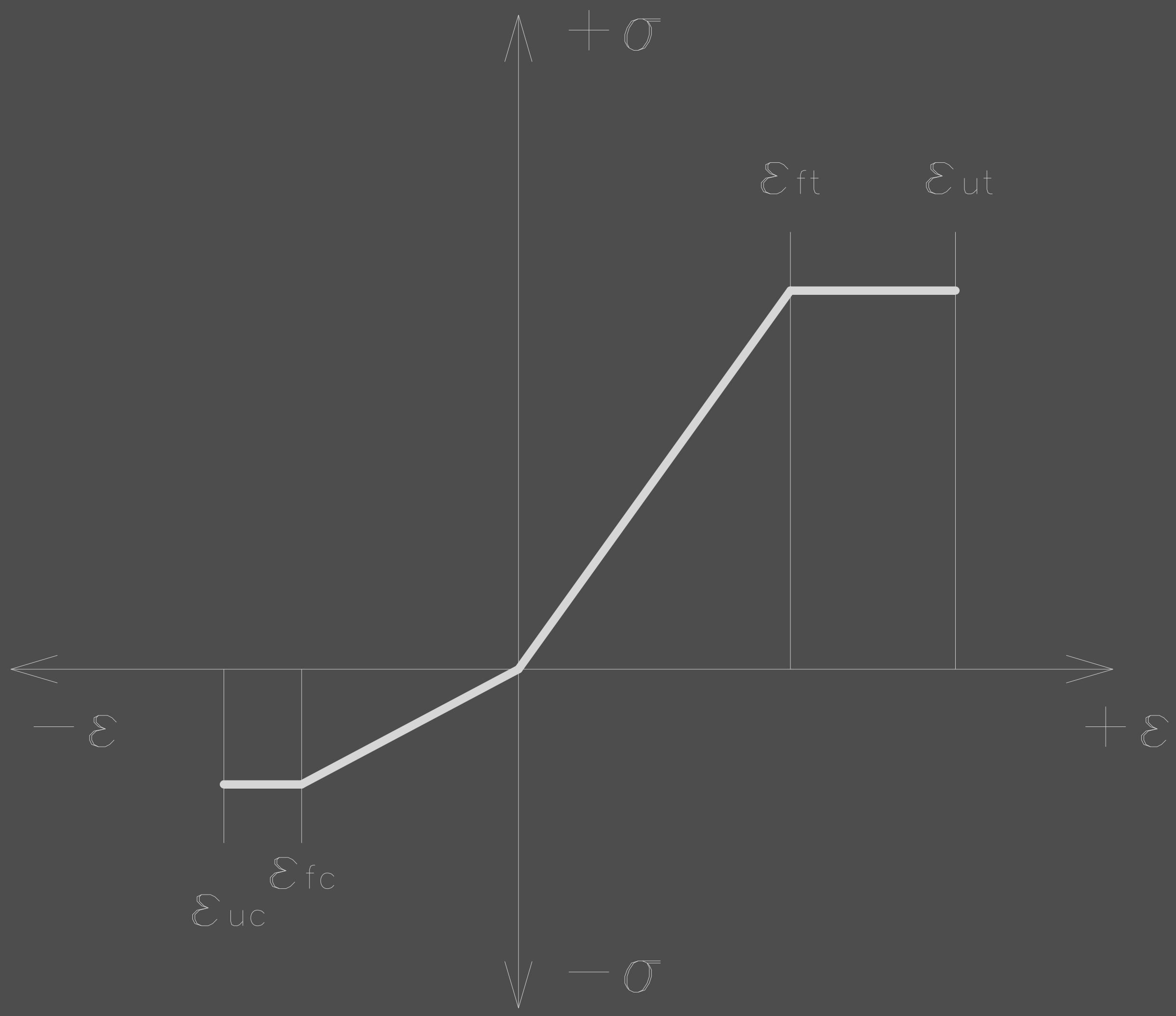
$\sigma_1 = -\sigma_3 \qquad \sigma_2 = 0$

$$\boxed{\sigma_c = \sqrt{3}\sigma_1 = 1{,}732\ \sigma_1}$$

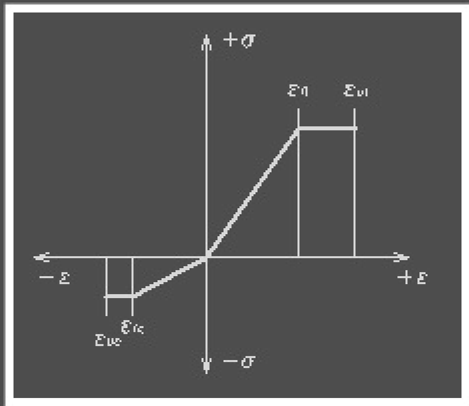
$$\boxed{\sigma_1 = \tau_M = 0.577\ \sigma_c}$$

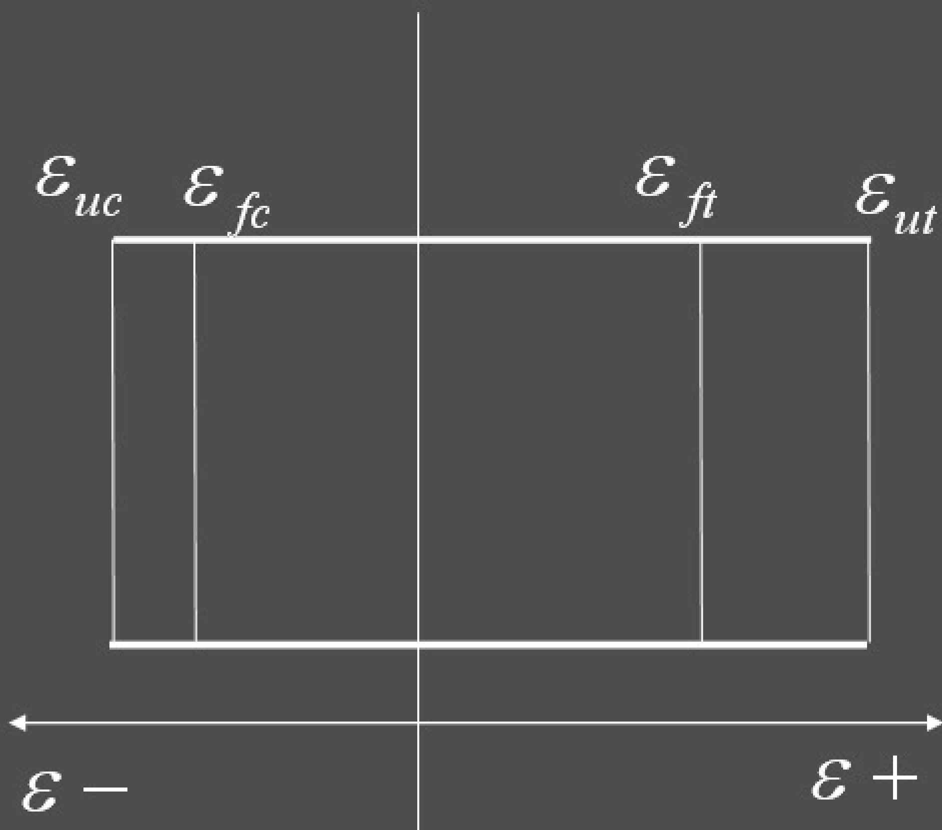
TEORIA DE MOHR

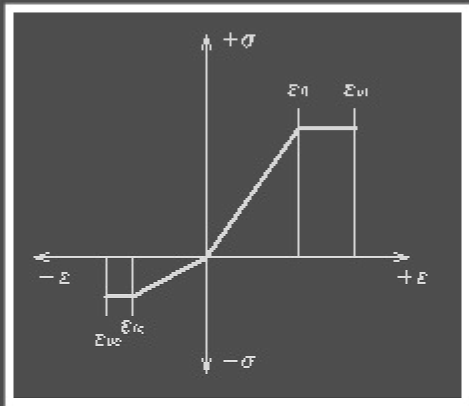


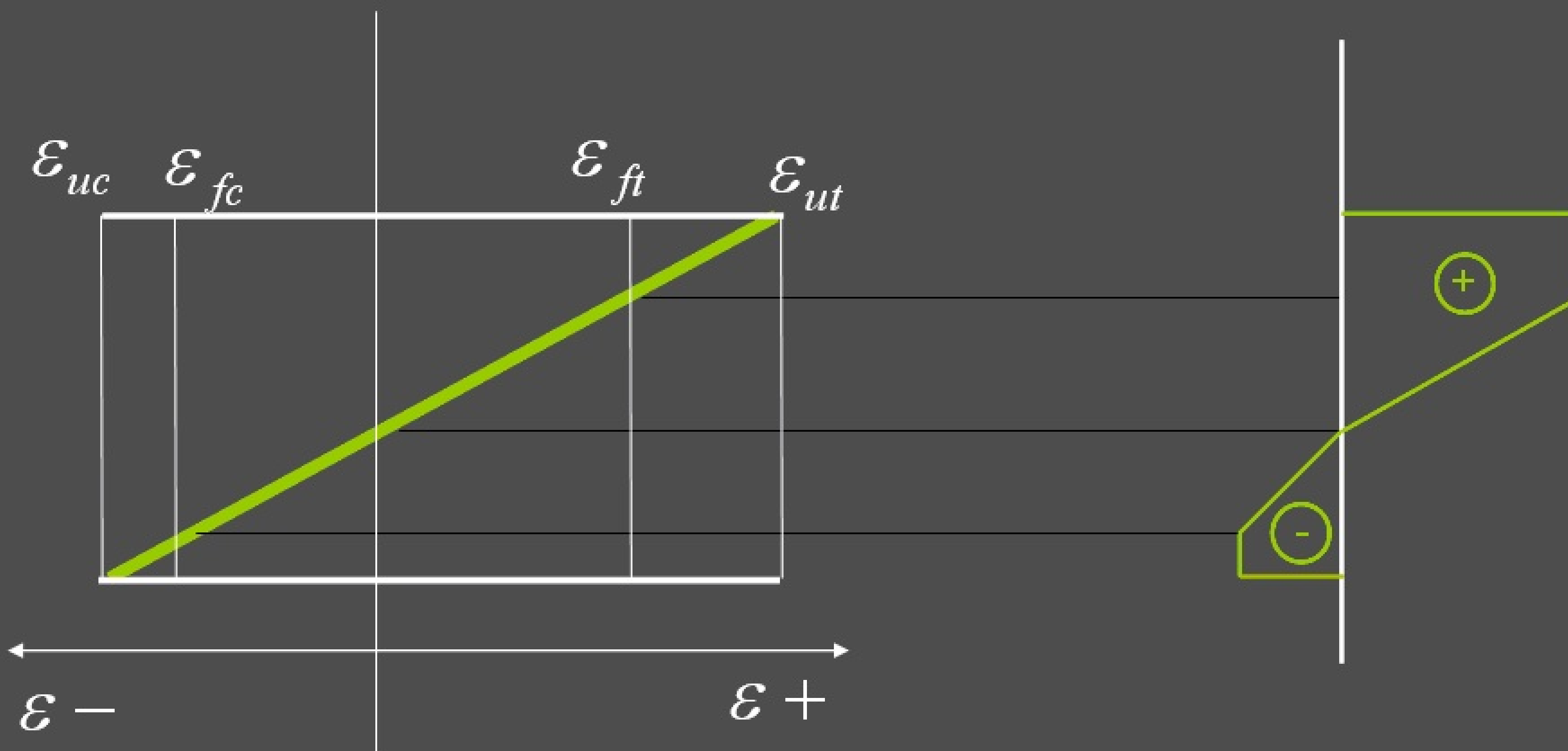


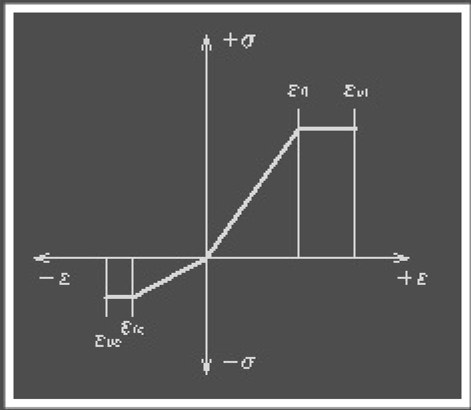
$+\sigma$
ε_{ft}
ε_{ut}
$-\varepsilon$
$+\varepsilon$
ε_{fc}
ε_{uc}
$-\sigma$

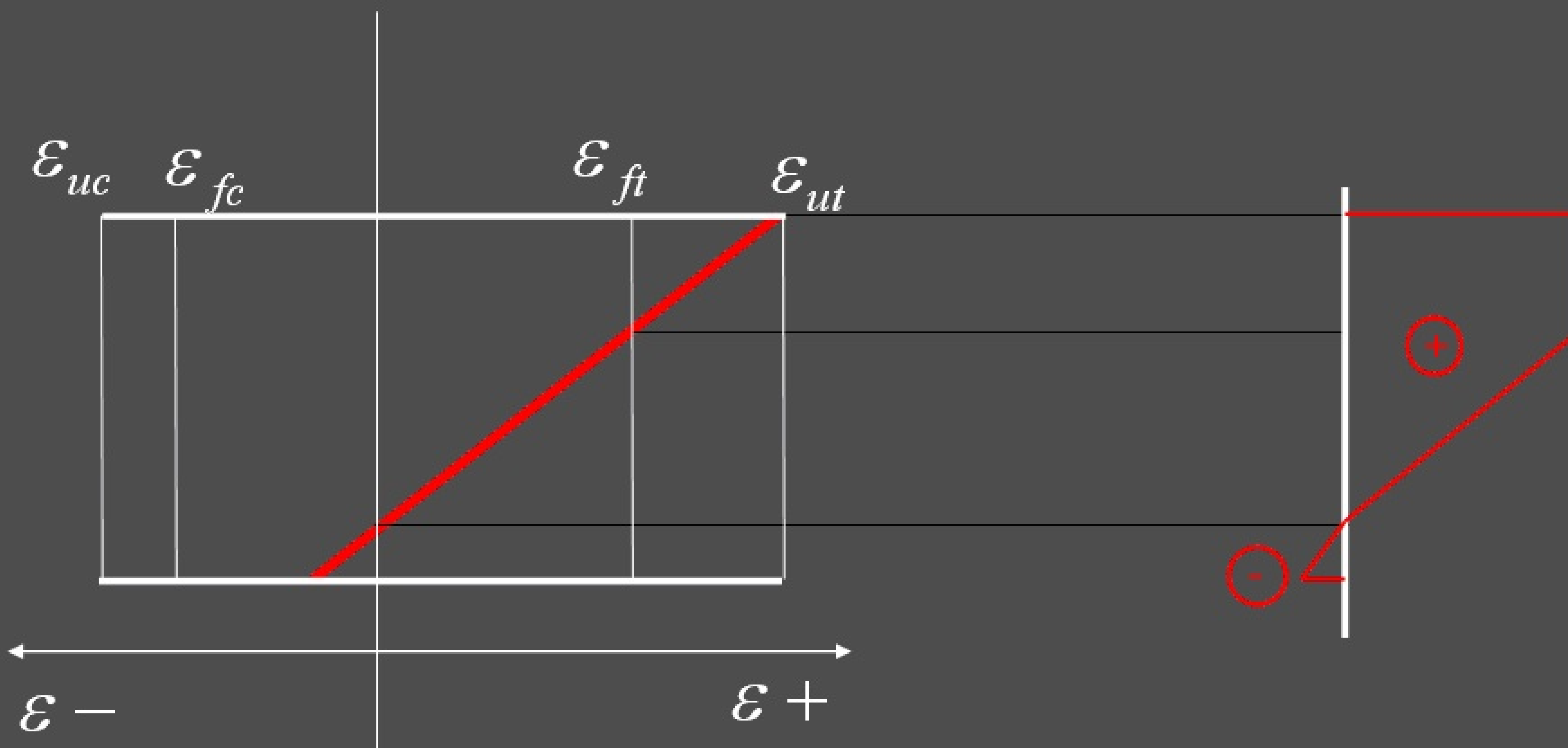

+σ
ε_ft
ε_ut
−ε
+ε
ε_uc
ε_fc
−σ


ε_{uc}
ε_{fc}
ε_{ft}
ε_{ut}
$\varepsilon -$
$\varepsilon +$


$+\sigma$
ε_{ft}
ε_{ut}
$-\varepsilon$
$+\varepsilon$
ε_{uc}
ε_{fc}
$-\sigma$


ε_{uc}
ε_{fc}
ε_{ft}
ε_{ut}
+
-
$\varepsilon -$
$\varepsilon +$


$+\sigma$
ε_{ft}
ε_{ut}
$-\varepsilon$
$+\varepsilon$
ε_{uc}
ε_{fc}
$-\sigma$


ε_{uc}
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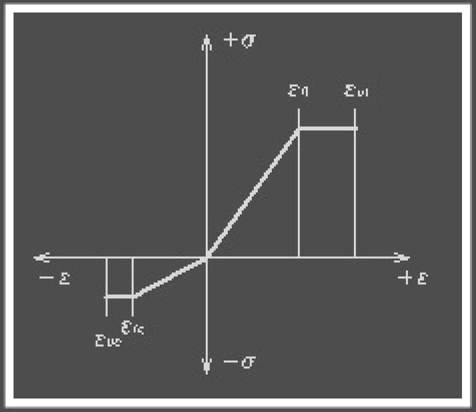
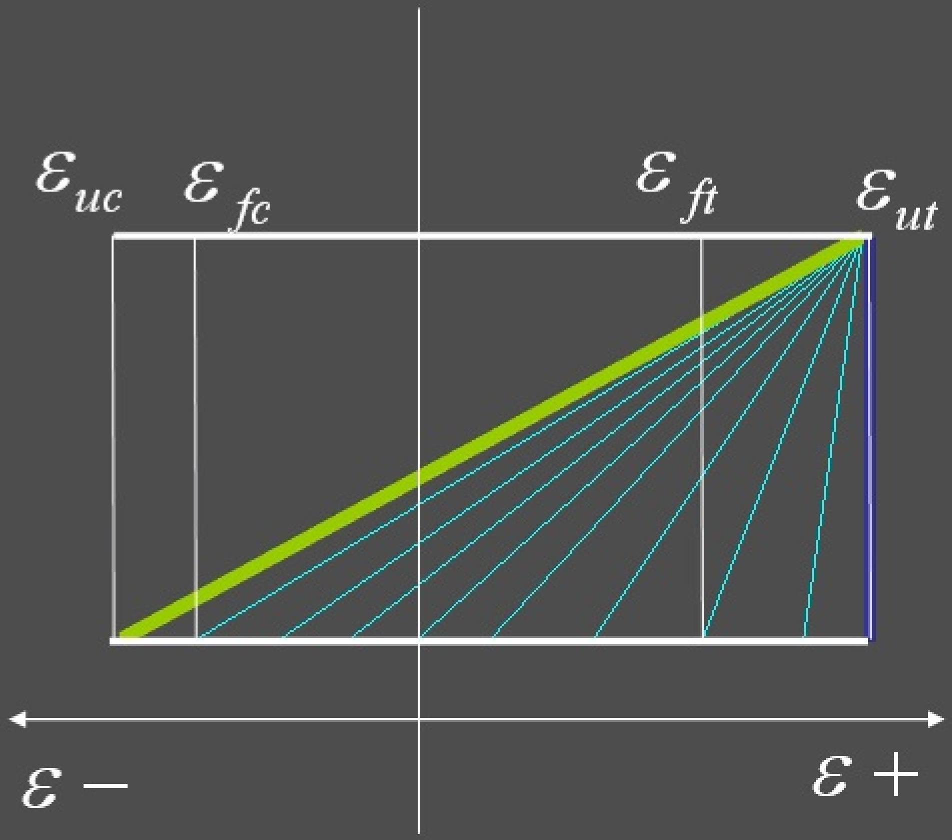
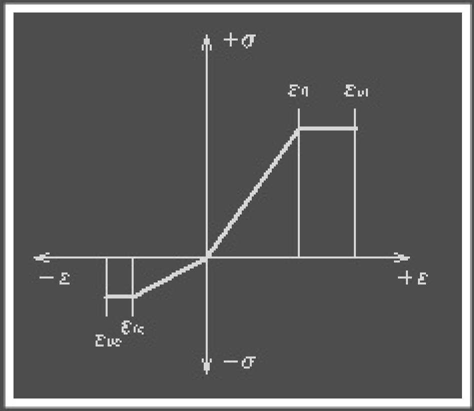
ε_{ut}
+
-
$\varepsilon -$
$\varepsilon +$



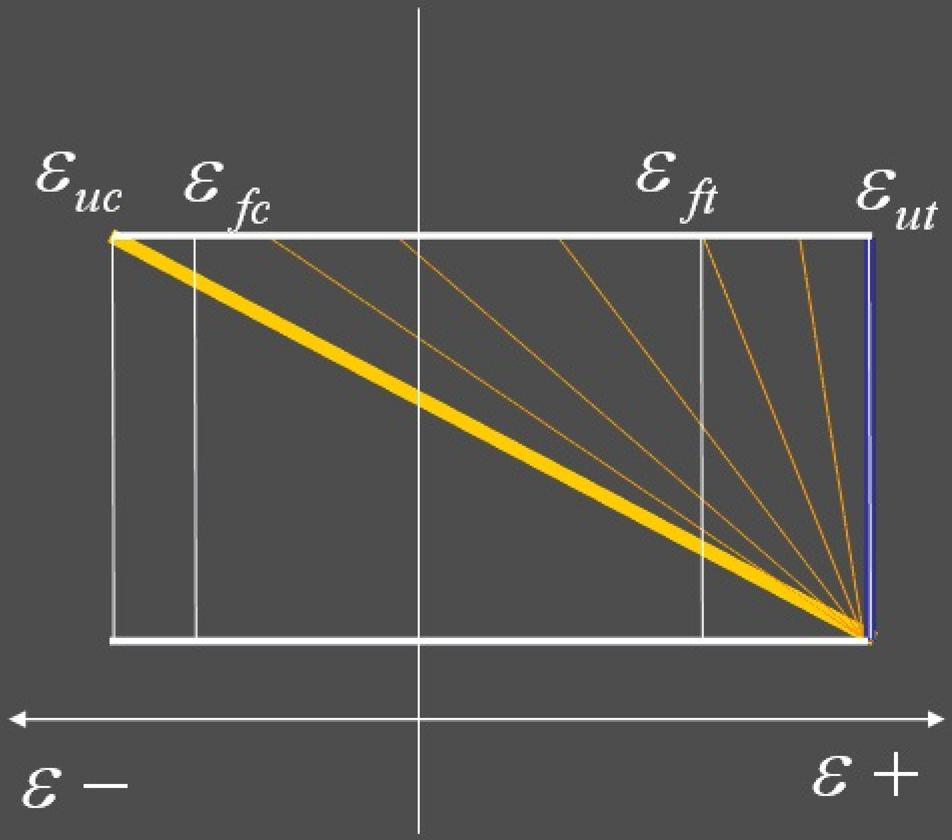
$+\sigma$
ε_{ft}
ε_{ut}
$-\varepsilon$
$+\varepsilon$
ε_{fc}
ε_{uc}
$-\sigma$



ε_{uc}
ε_{fc}
ε_{ft}
ε_{ut}
$\varepsilon -$
$\varepsilon +$



$+\sigma$
ε_{ft}
ε_{ut}
$-\varepsilon$
$+\varepsilon$
ε_{fc}
ε_{uc}
$-\sigma$



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ε_{fc}
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$\varepsilon -$
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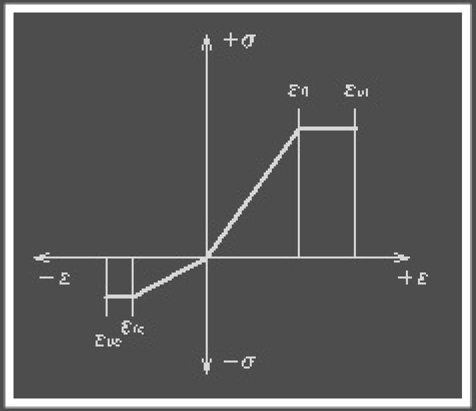
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ε_{ut}
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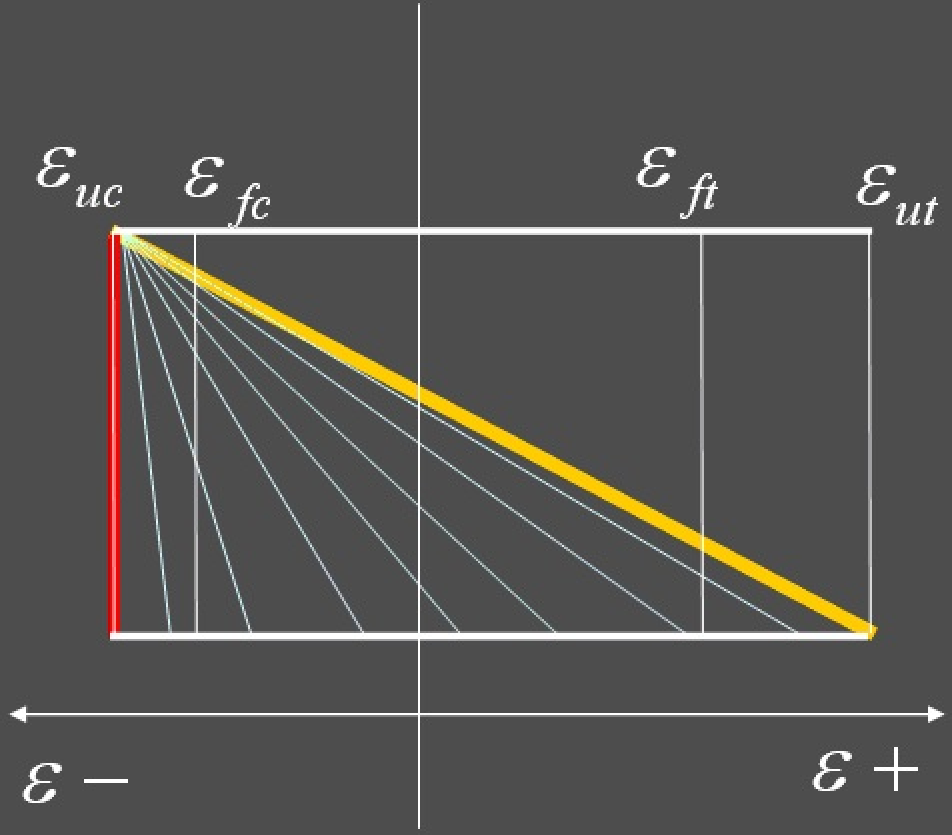
$+\varepsilon$
ε_{uc}
ε_{fc}
$-\sigma$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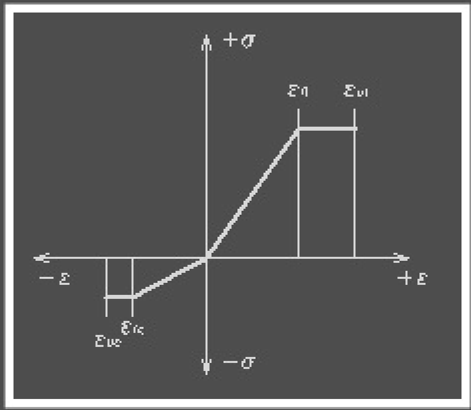
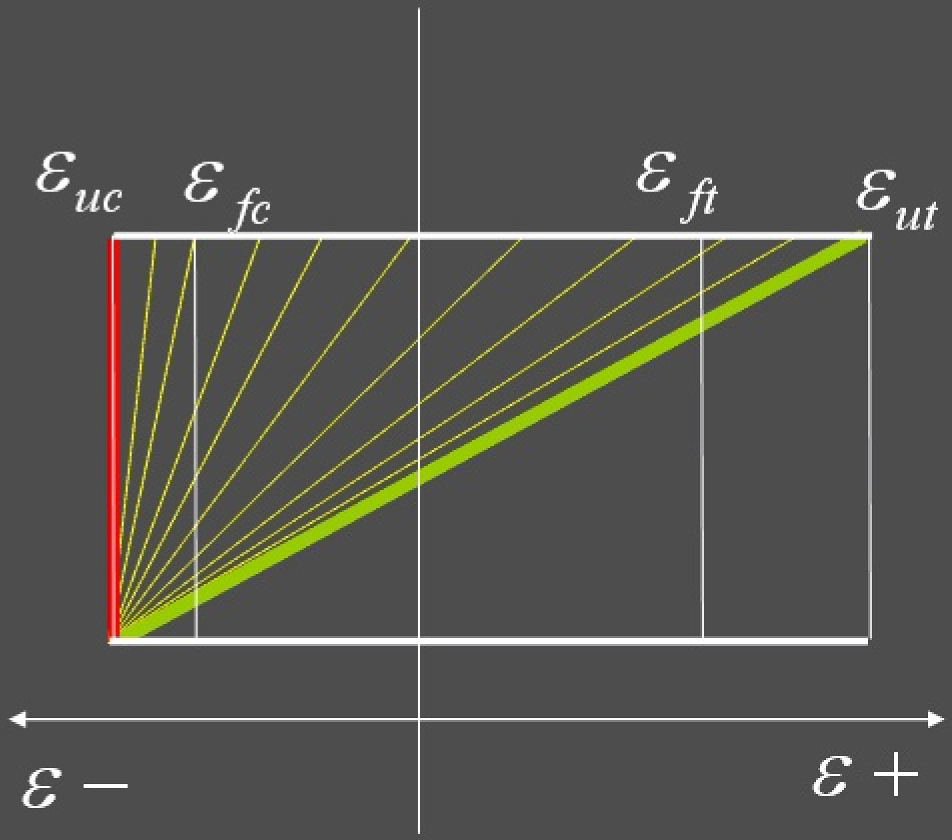
ε_{fc}
ε_{ft}
ε_{ut}
$\varepsilon -$
$\varepsilon +$


+σ
−σ
−ε
+ε


ε_{uc}
ε_{fc}
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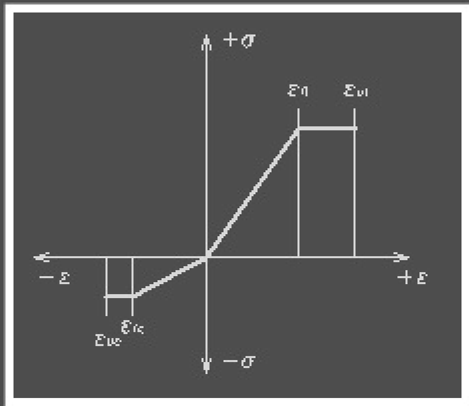
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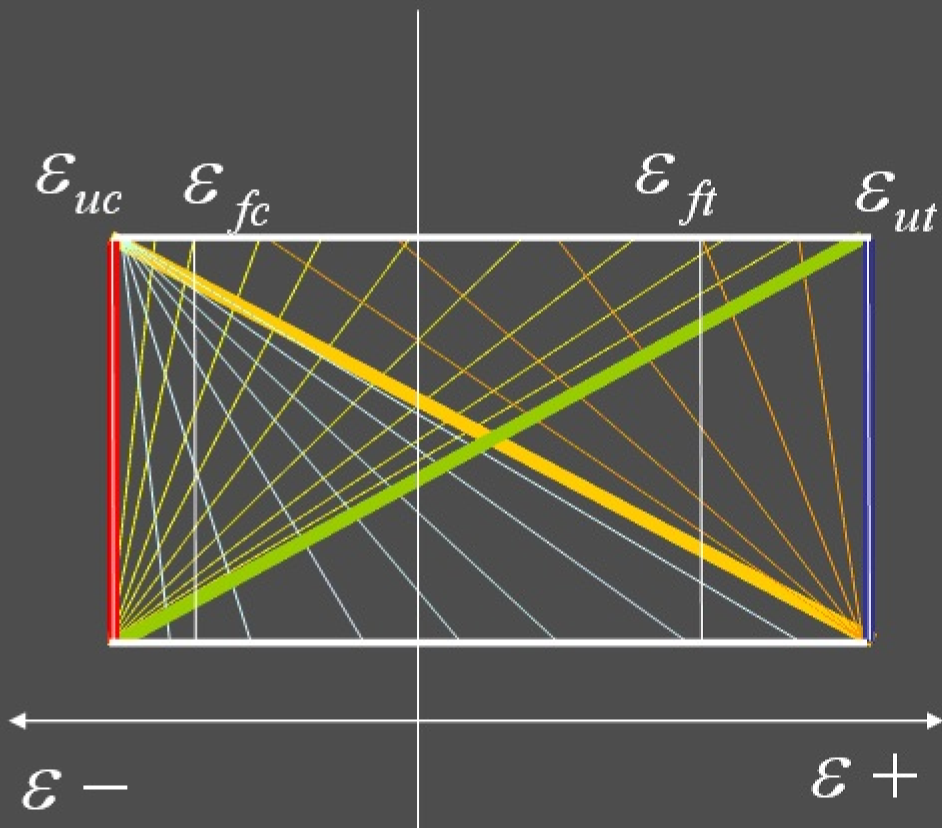
ε_{ft}
ε_{ut}
$-\varepsilon$
$+\varepsilon$
ε_{uc}
ε_{fc}
$-\sigma$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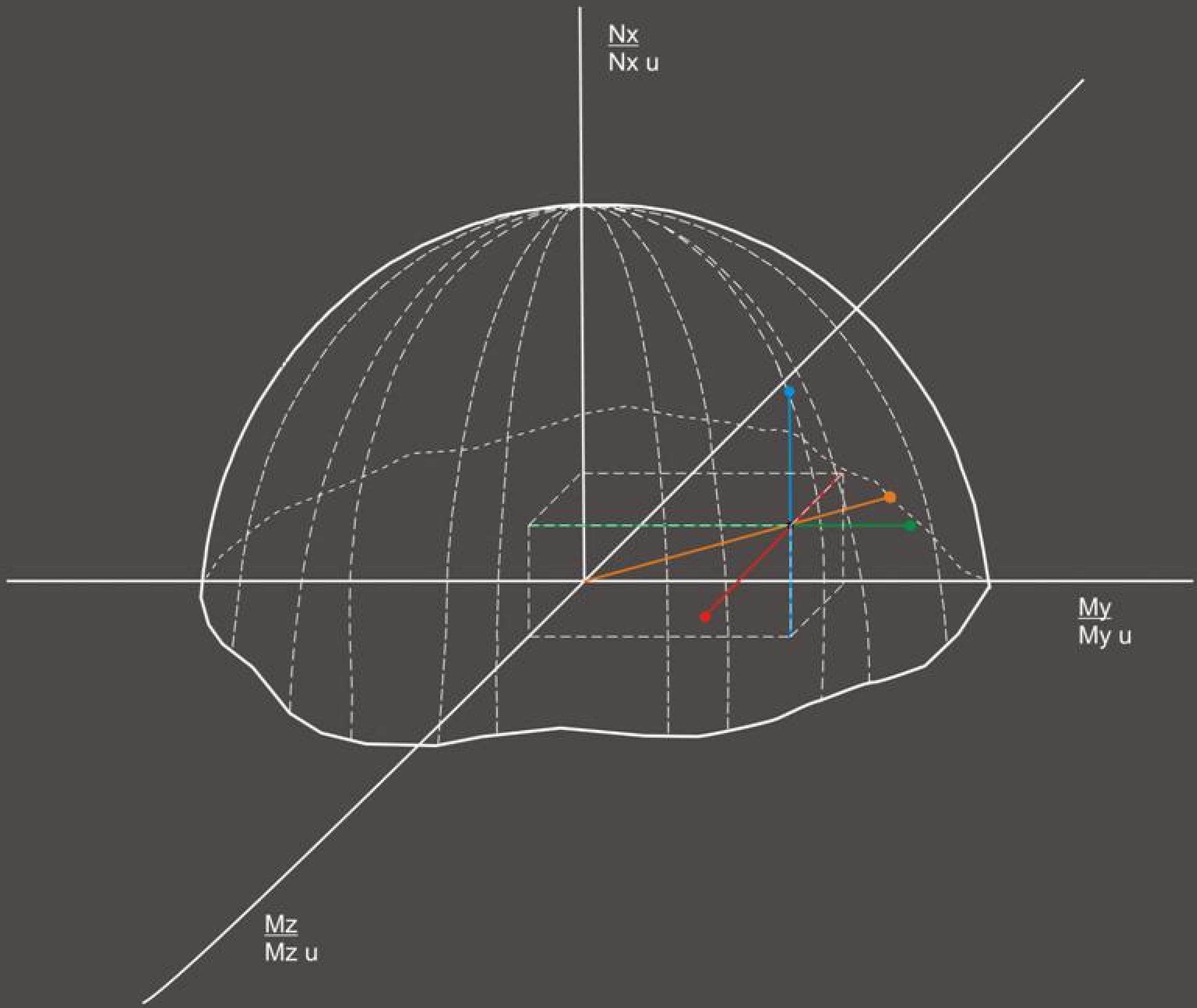

Nx
Nx u
My
My u
Mz
Mz u