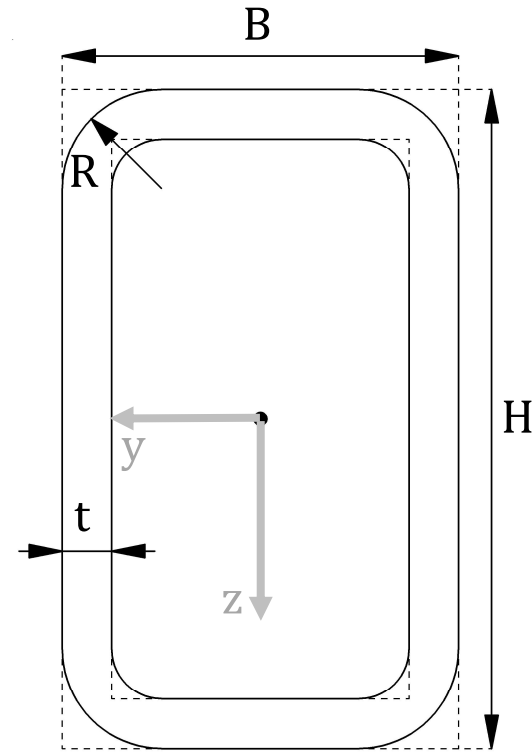
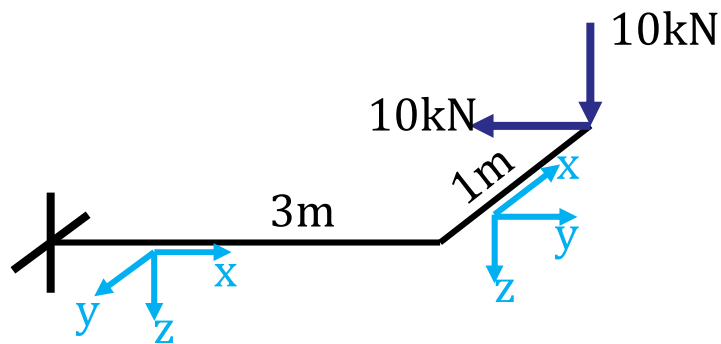




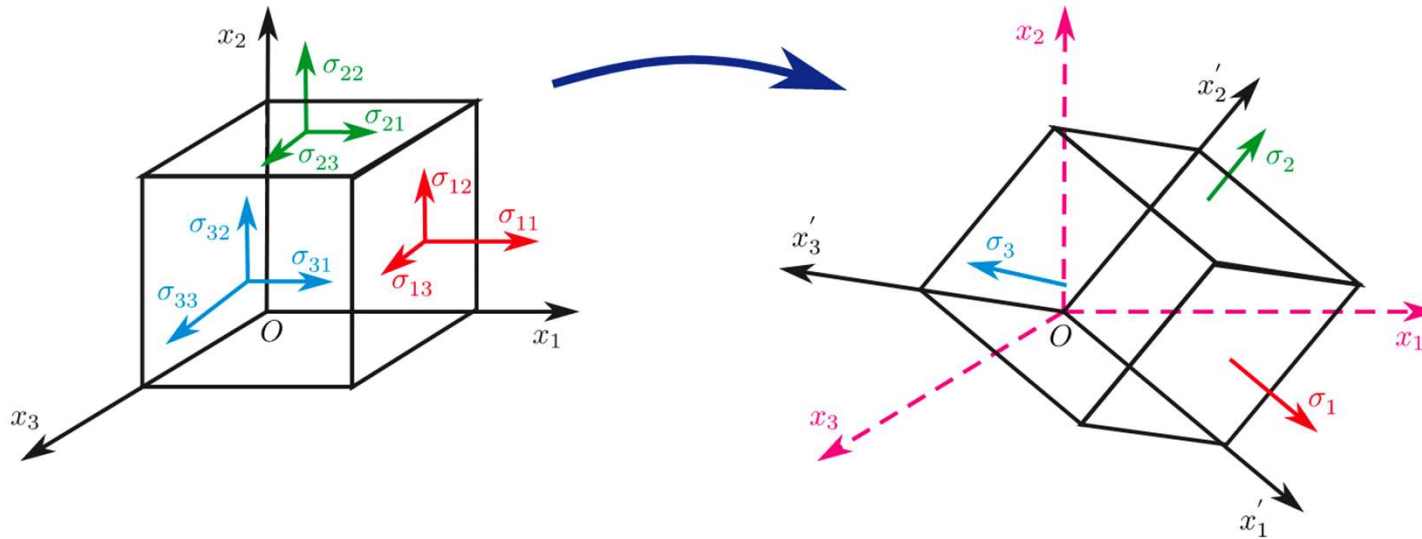
# Teoría de Estados Límites



Augusto La Colla – Manuela Medina – Constanza Ruffinelli – Bautista Chesta



# Repaso Estado de Tensión



$$T_T = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$$\bar{\rho} = [T_T] \cdot \check{n}$$

$$|\sigma_n| = \bar{\rho} \cdot \check{n}$$

$$\overline{\sigma_n} = |\sigma_n| \check{n}$$

$$\overline{\tau_n} = \bar{\rho} - \overline{\sigma_n}$$

Dirección principal es cuando

$$\bar{\rho} = \overline{\sigma_n} \quad [T_T] \cdot \check{n} = |\sigma_n| \check{n} \quad ([T_T] - \sigma_i [I]) \cdot \check{n}_i = \bar{0}$$

Estado triple: Ninguna tensión principal es nula

Estado doble: Una tensión principal es nula

Estado simple: Dos tensiones principales son nulas



# Repaso Estado de Deformación

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \frac{1}{2} \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{n_x} \\ \varepsilon_{n_y} \\ \varepsilon_{n_z} \end{pmatrix} = \begin{pmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix} \quad \overline{\varepsilon}_n = [T_D] \cdot \check{n}$$

$$\overline{\varepsilon}_n = [T_D] \cdot \check{n} \quad \varepsilon_l = \overline{\varepsilon}_n \cdot \check{n}$$

$$\overline{\varepsilon}_l = \varepsilon_l \check{n} \quad \overline{\varepsilon}_t = \overline{\varepsilon}_n - \overline{\varepsilon}_l$$

Dirección principal es cuando  $\overline{\varepsilon}_n = \overline{\varepsilon}_l$

$$[T_D] \cdot \check{n} = \varepsilon_l \check{n}$$

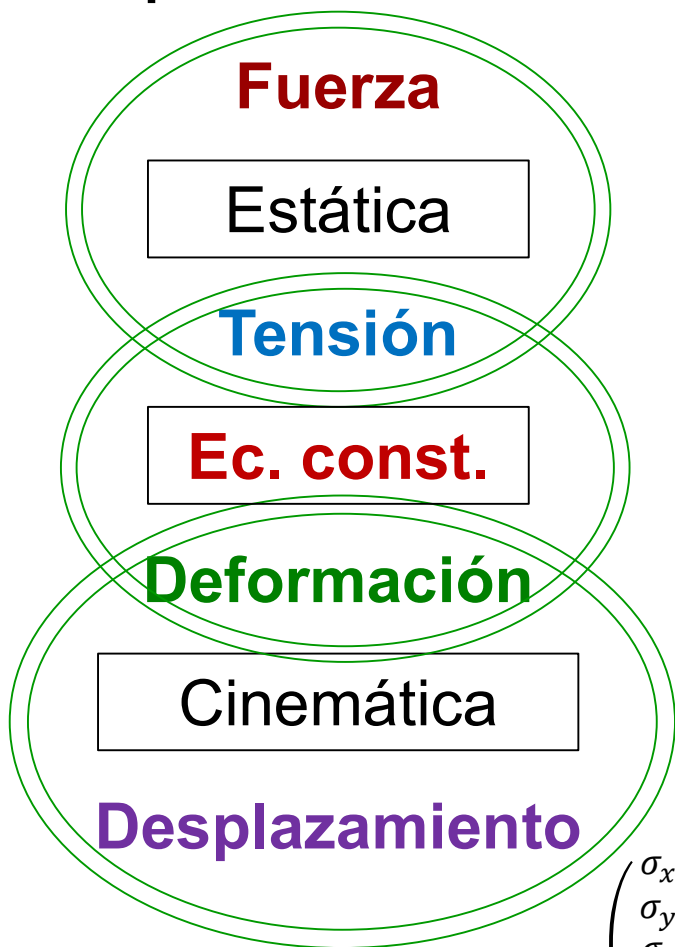
Estado triple: Ninguna deformación principal es nula

Estado doble: Una deformación principal es nula

Estado simple: Dos deformaciones principales son nulas



# Repaso Relaciones Constitutivas



$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\varepsilon_{xy} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{yz} \end{pmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\mu}{E} & -\frac{\mu}{E} & 0 & 0 & 0 \\ -\frac{\mu}{E} & \frac{1}{E} & -\frac{\mu}{E} & 0 & 0 & 0 \\ -\frac{\mu}{E} & -\frac{\mu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \cdot \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{pmatrix}$$

$\sigma$  desacopladas de  $\varepsilon_{xy} / \varepsilon_{xz} / \varepsilon_{yz}$

$\tau$  desacopladas de  $\varepsilon_x / \varepsilon_y / \varepsilon_z$

$$G = \frac{E}{2(1 + \mu)}$$

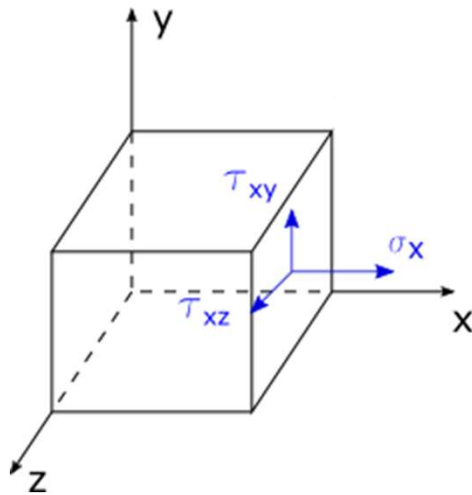
$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{pmatrix} = \frac{E(1 - \mu)}{(1 + \mu)(1 - 2\mu)} \begin{bmatrix} 1 & \frac{\mu}{1 - \mu} & \frac{\mu}{1 - \mu} & 0 & 0 & 0 \\ \frac{\mu}{1 - \mu} & 1 & \frac{\mu}{1 - \mu} & 0 & 0 & 0 \\ \frac{\mu}{1 - \mu} & \frac{\mu}{1 - \mu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1 - 2\mu}{(1 - \mu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1 - 2\mu}{(1 - \mu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1 - 2\mu}{(1 - \mu)} \end{bmatrix} \cdot \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{pmatrix}$$



# Introducción teórica

Dependiendo del material, existen ciertos límites en las tensiones.  
Por ejemplo, el estado de tensiones que establece el fin del periodo elástico se denomina fluencia.

En nuestra materia trabajaremos con barras, que son estados dobles de tensiones



$$[T_T] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix}$$



$$[T_T] = \begin{bmatrix} \sigma_x & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tau = \sqrt{\tau_{xy}^2 + \tau_{xz}^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \frac{\sqrt{4 \cdot \tau^2 + \sigma^2}}{2}$$

$$\sigma_3 = \frac{\sigma}{2} - \frac{\sqrt{4 \cdot \tau^2 + \sigma^2}}{2}$$



- Saint Venant  
(máxima deformación principal)

$$\max(|\varepsilon_1|, |\varepsilon_2|, |\varepsilon_3|) \leq |\varepsilon_{limite}|$$

$$EPT \quad \boxed{|\sigma_1 - \mu \cdot \sigma_3| \leq \sigma_{adm}}$$

$$\boxed{|\sigma_3 - \mu \cdot \sigma_1| \leq \sigma_{adm}}$$

- Rankine  
(máxima tensión principal)

$$\max(|\sigma_1|, |\sigma_2|, |\sigma_3|) \leq |\sigma_{adm}|$$

$$EPT \quad \boxed{|\sigma_1| \leq \sigma_{adm}}$$

$$\boxed{|\sigma_3| \leq \sigma_{adm}}$$

- Guest - Tresca  
(máxima tensión tangencial)

$$|\sigma_1 - \sigma_3| \leq |\sigma_{adm}|$$

$$EPT \quad \boxed{\sqrt{4 \cdot \tau^2 + \sigma^2} \leq \sigma_{adm}}$$

- Beltrami  
(máxima energía total)

$$EPT \quad \boxed{\sqrt{2\mu \cdot \tau^2 + 2 \cdot \tau^2 + \sigma^2} \leq \sigma_{adm}}$$



- Von Mises (Se usa para acero)  
(máxima energía de distorsión)

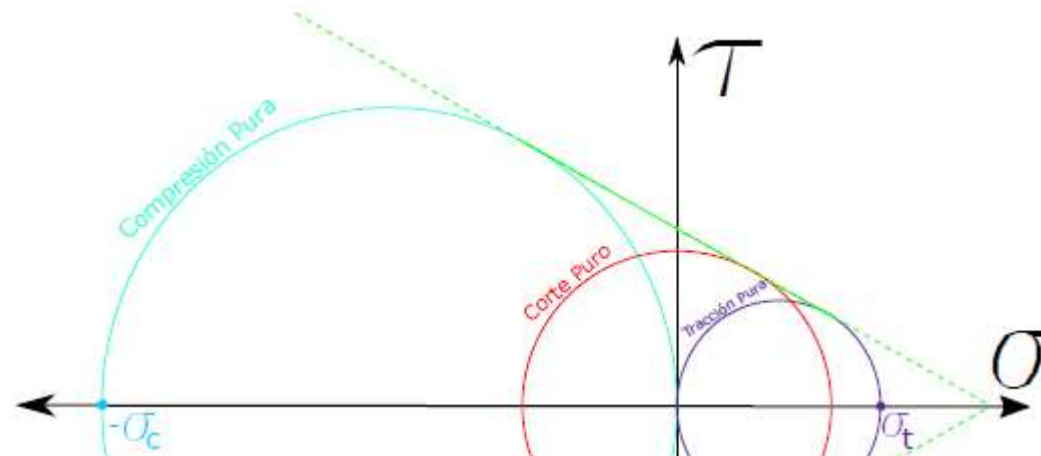
$$\sqrt{\frac{\sigma_1^2 + \sigma_3^2 + (\sigma_1 - \sigma_3)^2}{2}} \leq |\sigma_{adm}|$$

EPT  $\boxed{\sqrt{3 \cdot \tau^2 + \sigma^2} \leq \sigma_{adm}}$

- Coulomb (Se usa para suelos)

$$\boxed{\sigma_1 - K\sigma_3 \leq \sigma_{adm}}$$

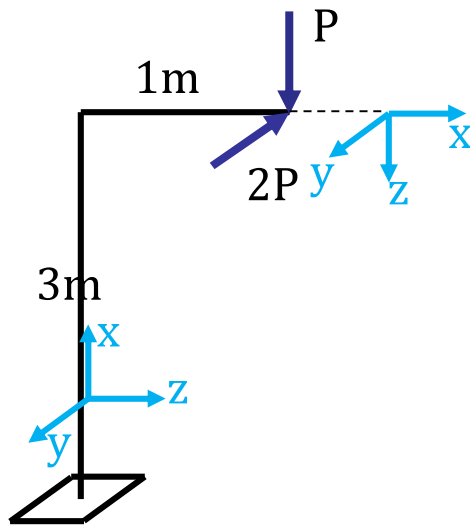
$$K = \frac{\sigma_{fl-tracción}}{\sigma_{fl-compresión}} = \frac{\sigma_{rotura-tracción}}{\sigma_{rotura-compresión}} \quad K > 0$$





## Ejercicio 1:

- a) Calcular el máximo valor de  $P$  que admite la estructura, considerando la Teoría de Tresca (Máxima Tensión Tangencial).



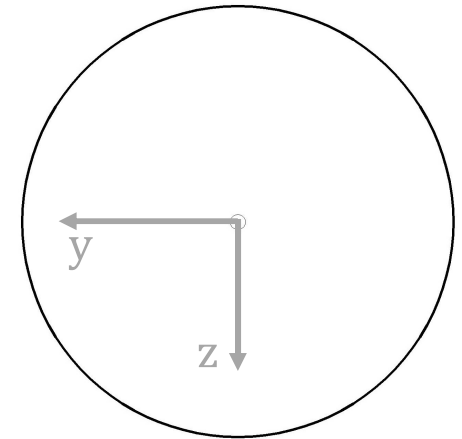
Sección circular maciza

$$D = 10\text{cm}$$

Acero F24

$$\sigma_{fl} = 24 \frac{\text{kN}}{\text{cm}^2} \quad CS = 1,6$$

$$\sigma_{adm} = \frac{\sigma_{fl}}{CS} = 15 \frac{\text{kN}}{\text{cm}^2}$$



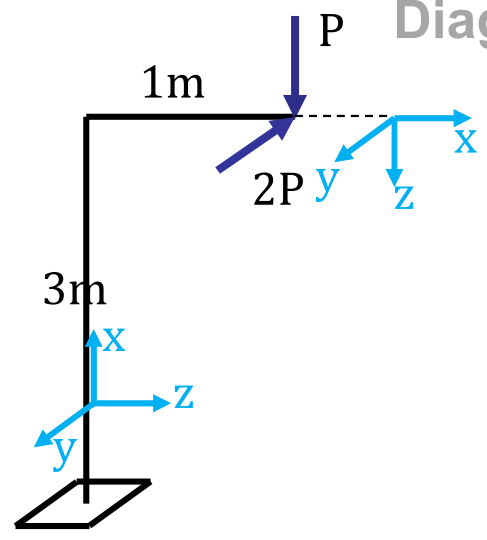
$$A = \frac{\pi \cdot D^2}{4} = 78,54 \text{ cm}^2$$

$$I = \frac{\pi \cdot D^4}{64} = 490,87 \text{ cm}^4$$

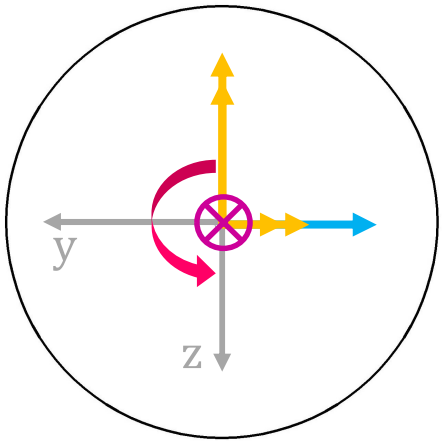
$$J_P = \frac{\pi \cdot D^4}{32} = 981,75 \text{ cm}^4$$



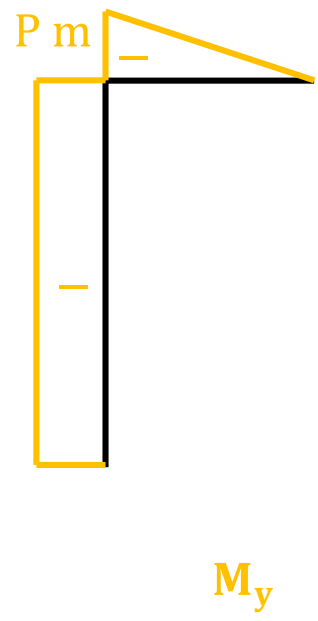
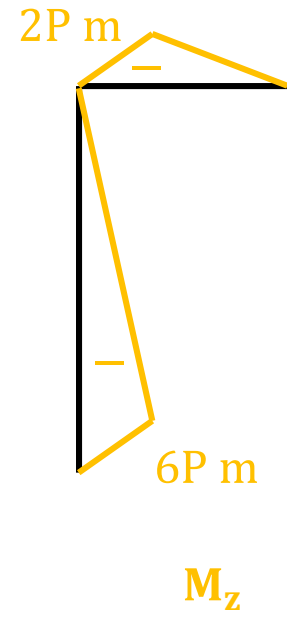
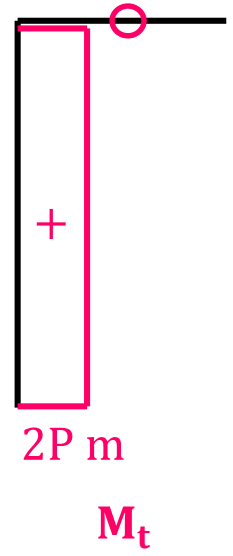
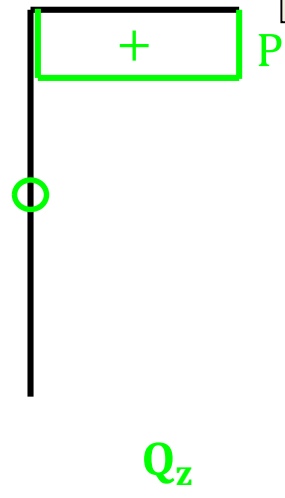
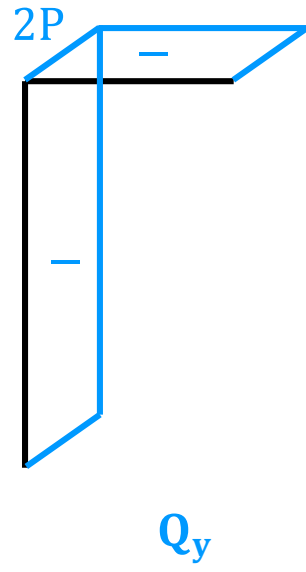
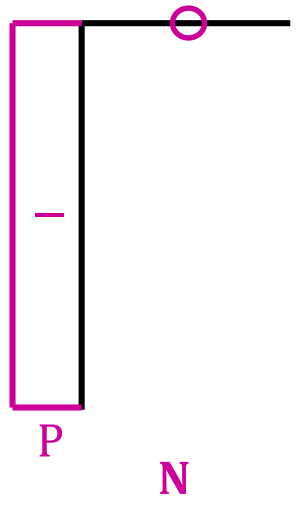
## Diagramas de características



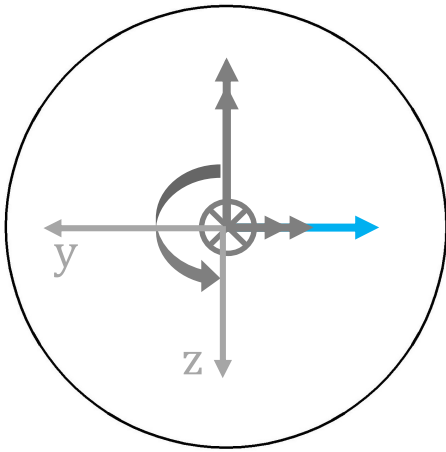
Sección más solicitada



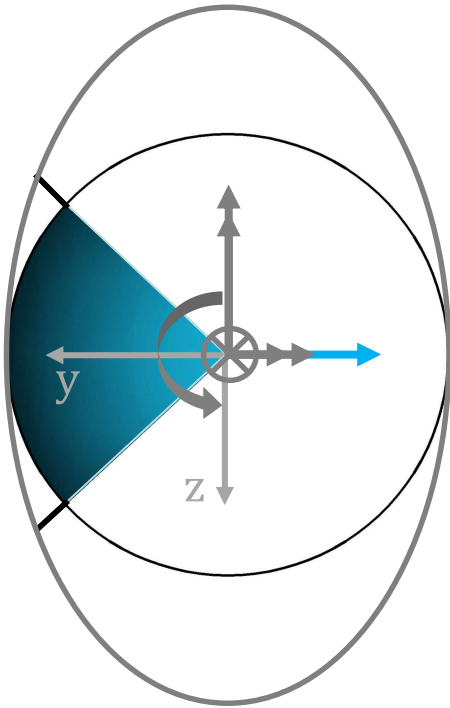
$$\begin{aligned}
 N &= -P \\
 Q_y &= -2P & M_z &= -6P \text{ m} \\
 M_t &= 2P \text{ m} & M_y &= -P \text{ m}
 \end{aligned}$$



Corte  $Q_y = -2P$

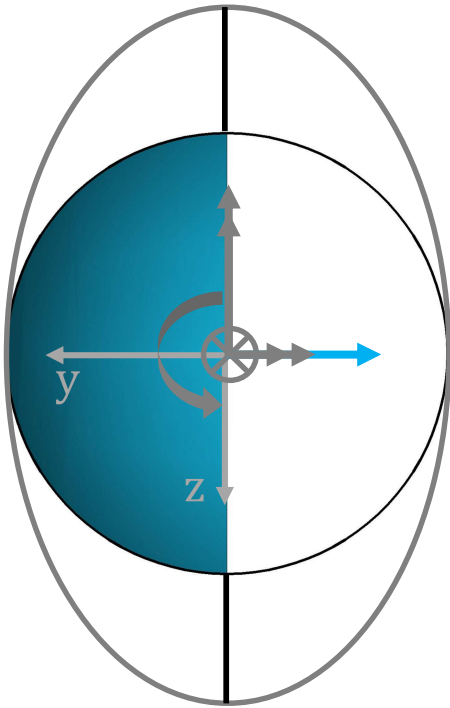


Corte  $Q_y = -2P$





Corte  $Q_y = -2P$



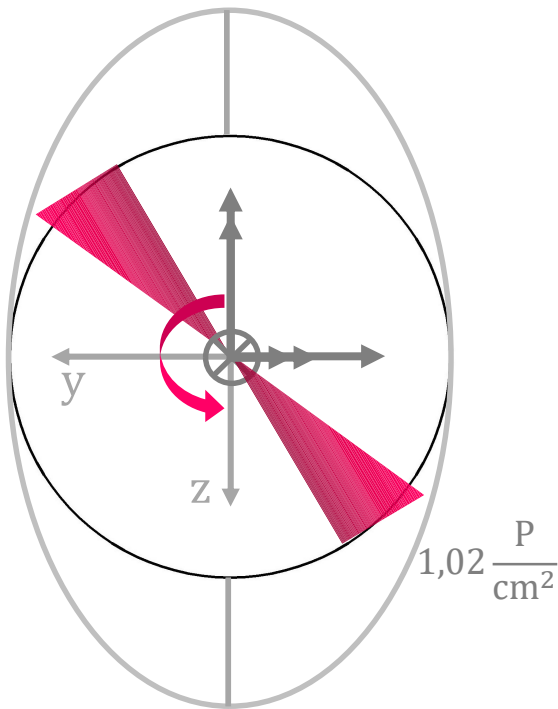
$$\tau_{\text{máx}}^{Q_y} = \frac{Q_y \cdot S_{\text{EN}}^*}{I_{\text{EN}} \cdot b} = \frac{2P \cdot S_{\text{EN}}^*}{490,87 \text{ cm}^4 \cdot D}$$

$$S_{\text{EN}}^* = \frac{A}{2} \cdot \frac{4}{3\pi} \frac{D}{2} = 83,33 \text{ cm}^3$$

$$\tau_{\text{máx}}^{Q_y} = 0,034 \frac{P}{\text{cm}^2}$$

Torsión

$$M_t = 2P \text{ m}$$



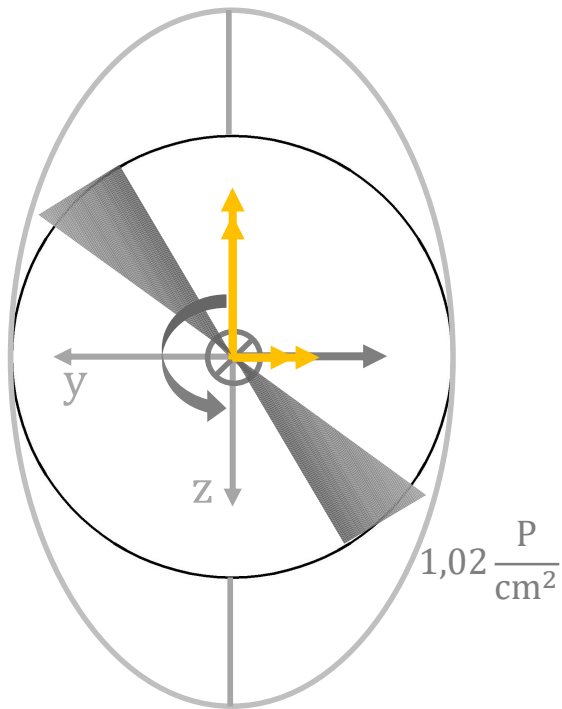
$$\tau_{\text{máx}}^{M_t} = \frac{M_t}{J_P} \cdot \frac{D}{2} = \frac{2P \text{ m}}{981,75 \text{ cm}^4} \cdot \frac{10 \text{ cm}}{2} = 1,02 \frac{P}{\text{cm}^2}$$



Flexión

$$M_y = -P \text{ m}$$

$$M_z = -6P \text{ m}$$

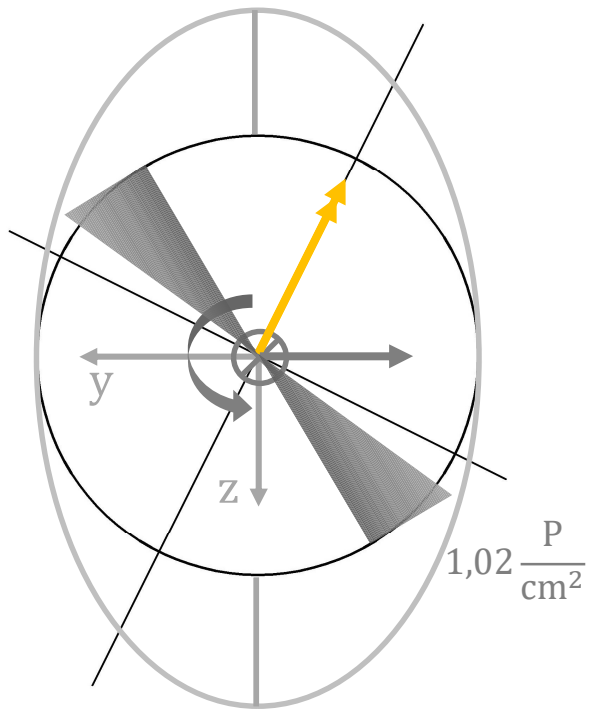


Flexión compuesta **recta**

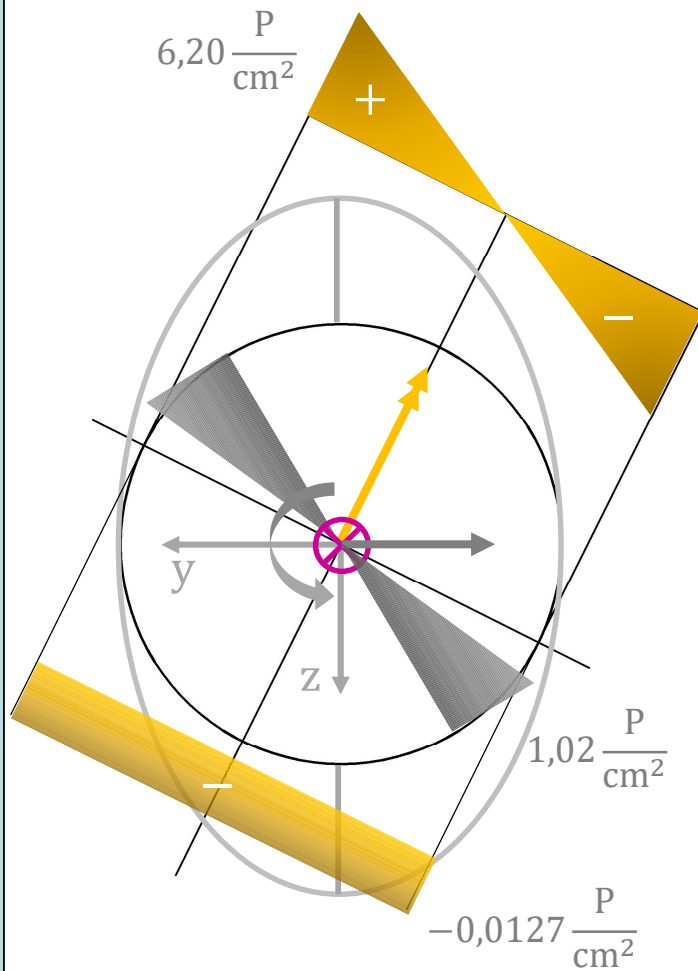
$$M_f = \sqrt{(M_y)^2 + (M_z)^2} = 6,083P \text{ m}$$

Flexión

$$M_f = 6,083P \text{ m}$$



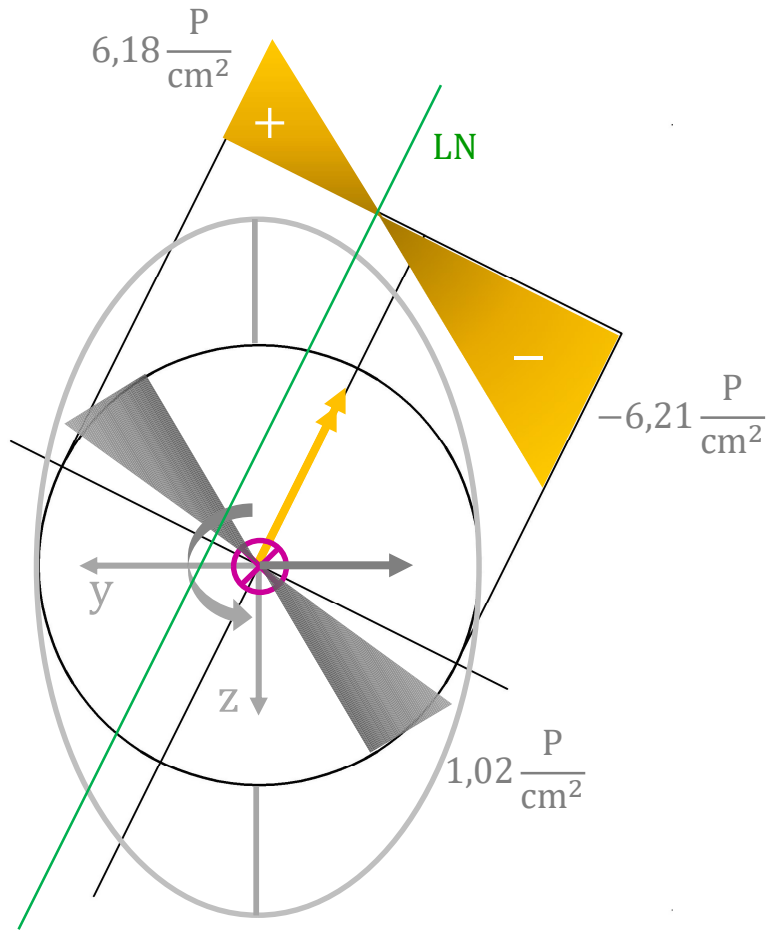
## Flexión



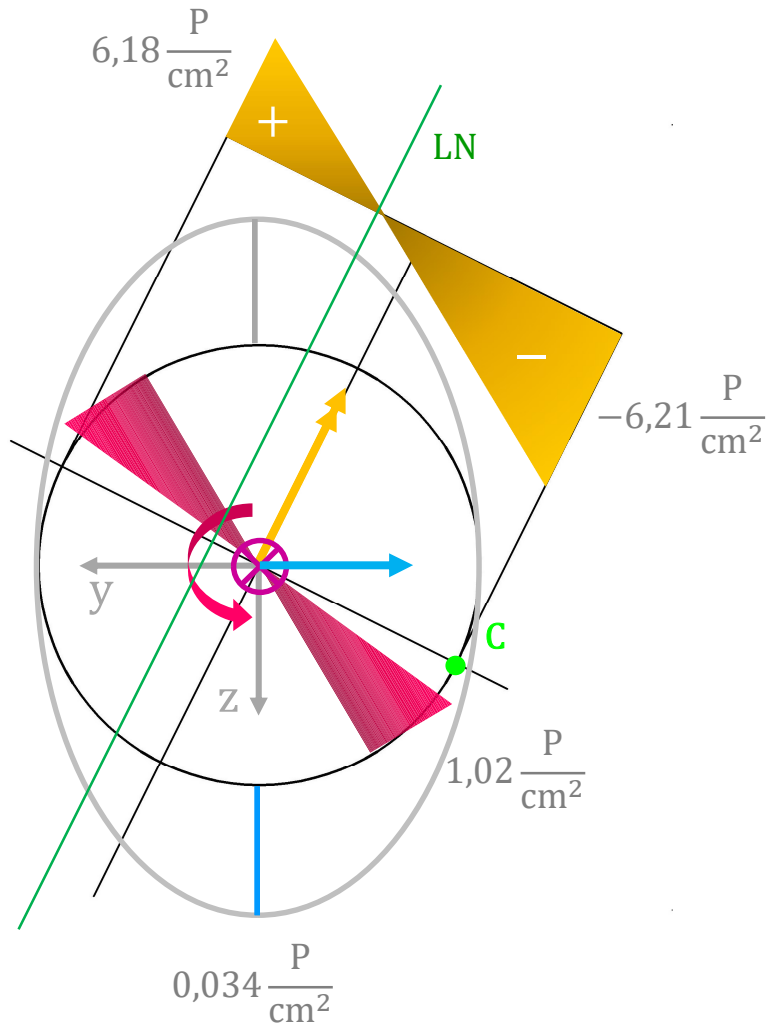
$$\sigma_x^{M_f} = \frac{M_f}{I} \cdot \frac{D}{2} = \frac{6,083P \text{ m}}{I} \cdot \frac{D}{2} = 6,20 \frac{P}{\text{cm}^2}$$

$$\sigma_x^N = \frac{N}{A} = \frac{-P}{78,54 \text{ cm}^2} = -0,0127 \frac{P}{\text{cm}^2}$$





¿Cuál es el punto más peligroso?



Tresca

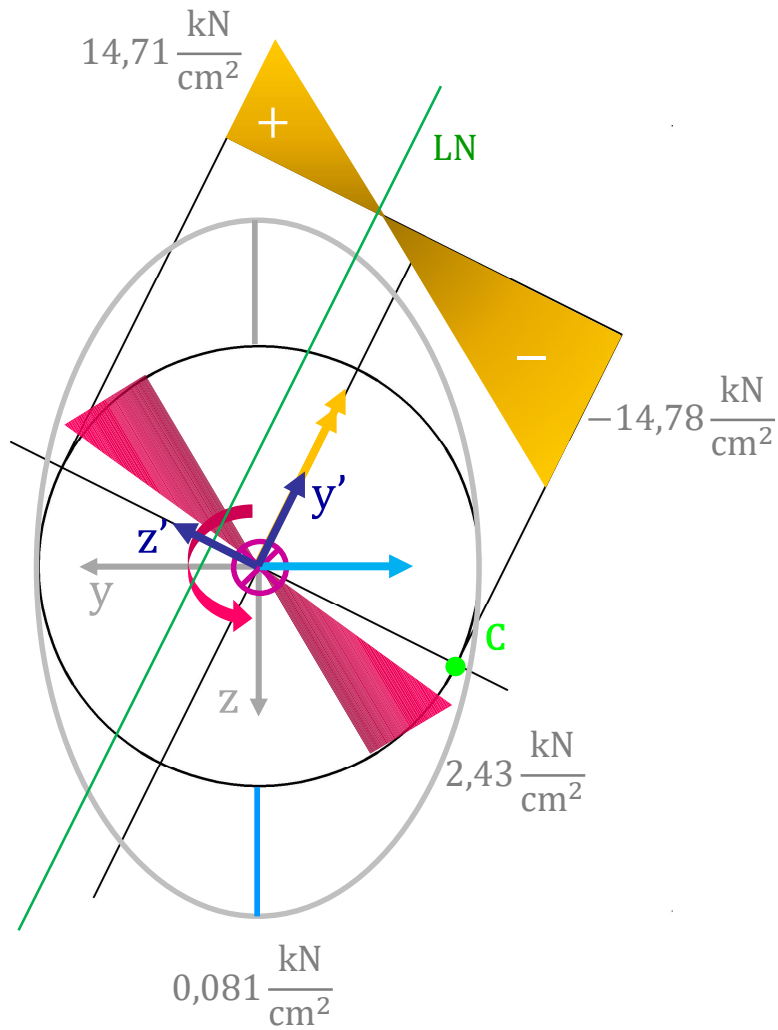
$$\sqrt{\sigma_X^2 + 4\tau_X^2} \leq \frac{\sigma_{fl}}{CS}$$

$$\sigma_x^c = -6,21 \frac{P}{\text{cm}^2}$$

$$\tau_x^c = 1,02 \frac{P}{\text{cm}^2}$$

$$6,29 \frac{P}{\text{cm}^2} \leq 15 \frac{\text{kN}}{\text{cm}^2}$$

$$P \leq 2,38 \text{ kN}$$



Considerando  $P = 2,38 \text{ kN}$

$$\sigma_{x'}^C = -14,78 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{x'y'}^C = 2,43 \frac{\text{kN}}{\text{cm}^2}$$

$$[T_T]_{x'y'z'}^C = \begin{pmatrix} -14,78 & 2,43 & 0 \\ 2,43 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{\text{kN}}{\text{cm}^2}$$



## Ejercicio 2:

- Verificar la estructura para las teorías de Tresca y Von Mises
- Calcular los tensores de tensión y de deformación para el punto más solicitado, y calcular las direcciones principales
- Realizar la circunferencia de Mohr para el caso del punto anterior

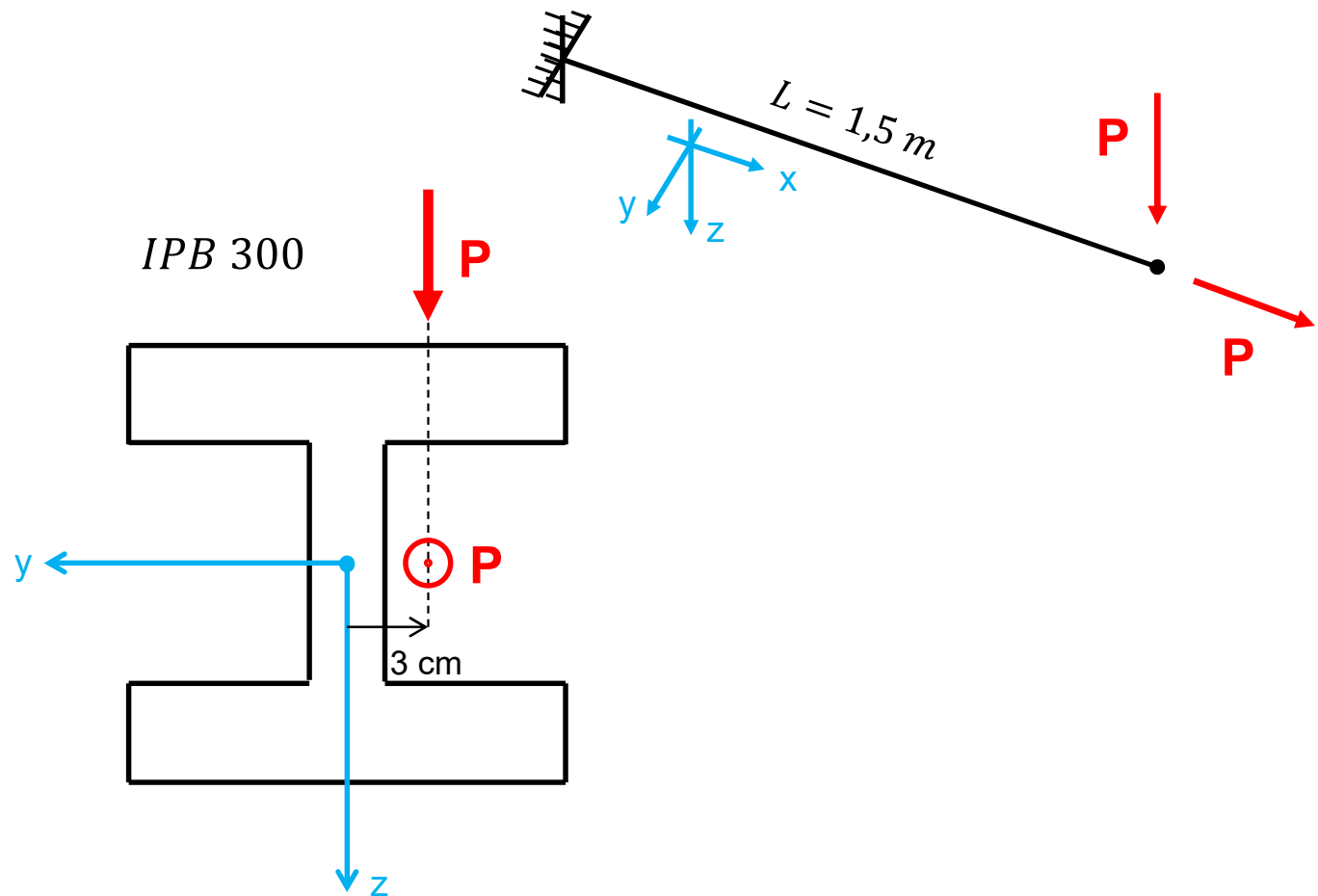
Datos:

$$P = 60 \text{ kN}$$

$$E = 21000 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_F = 24 \frac{\text{kN}}{\text{cm}^2}$$

$$\mu = 0,25$$





## Datos del perfil

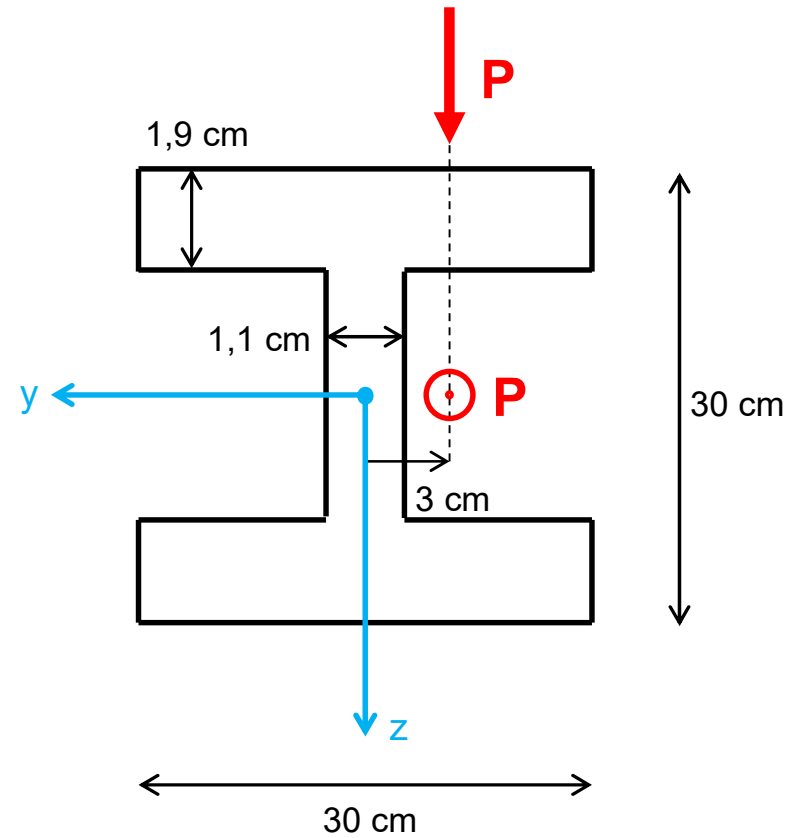
*De tabla:*

$$A = 149 \text{ cm}^2$$

$$J_y = 25170 \text{ cm}^4$$

$$J_z = 8560 \text{ cm}^4$$

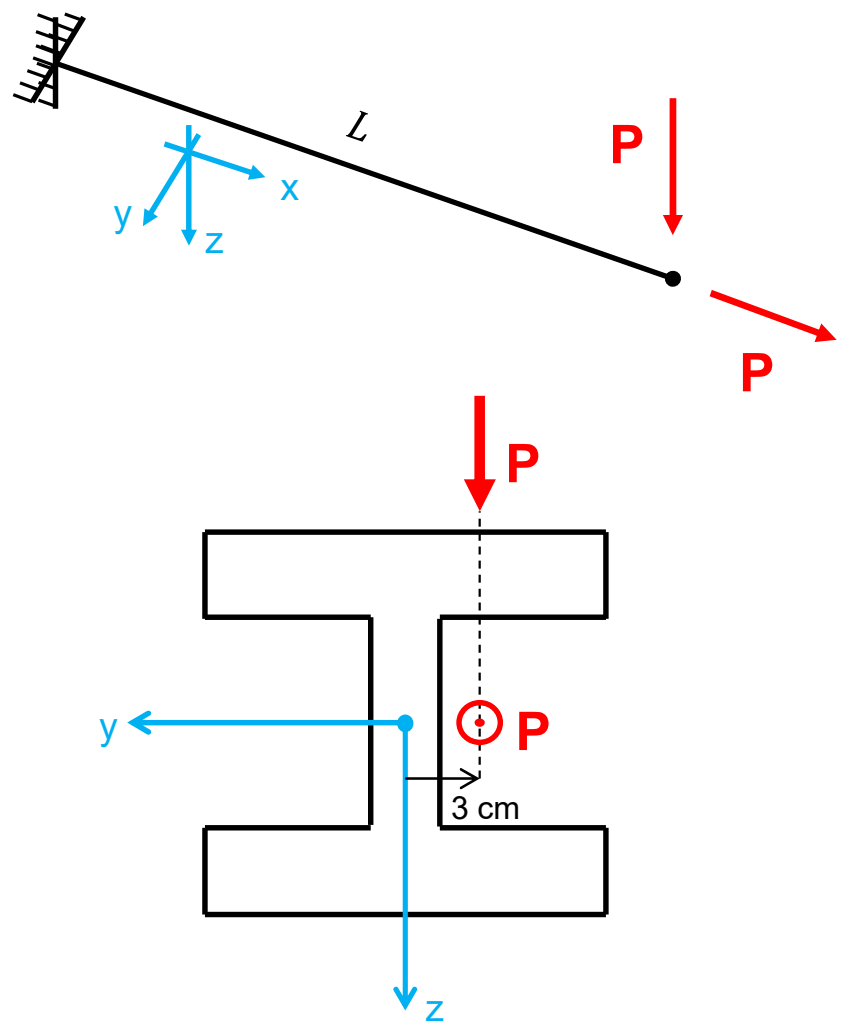
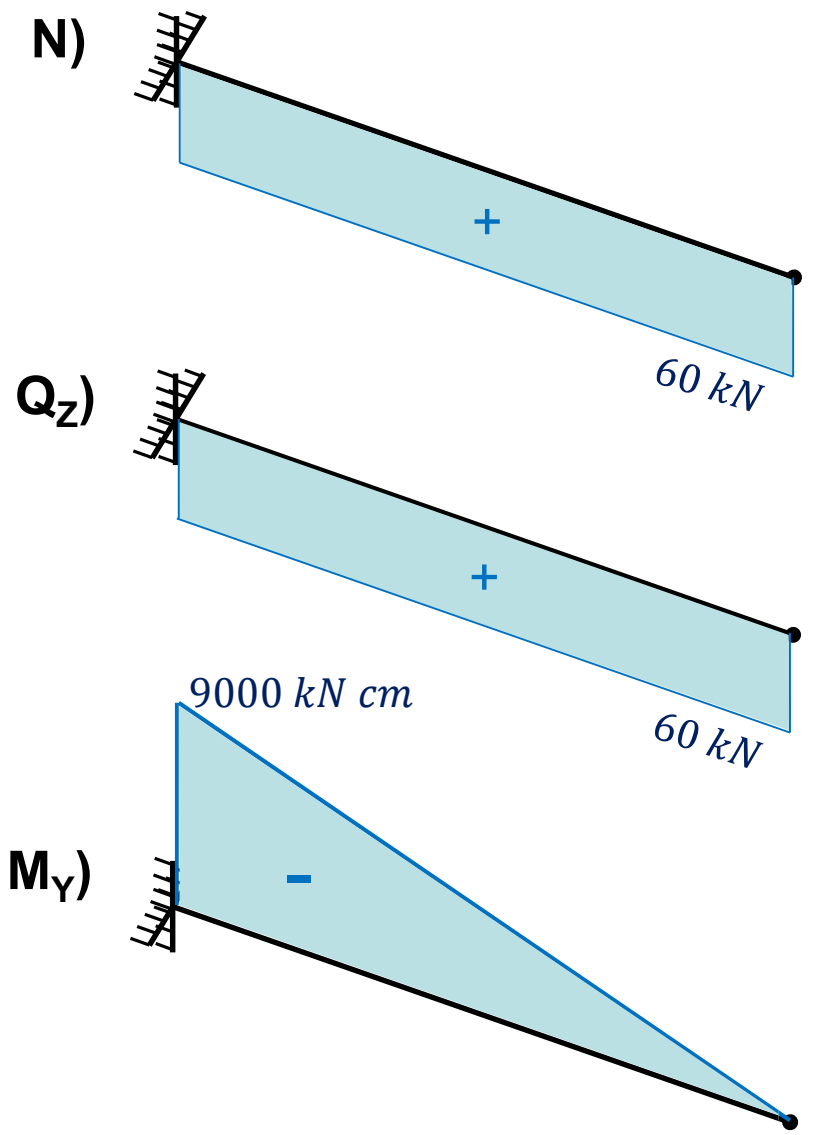
$$J_T = 149 \text{ cm}^4$$



# Diagramas de características



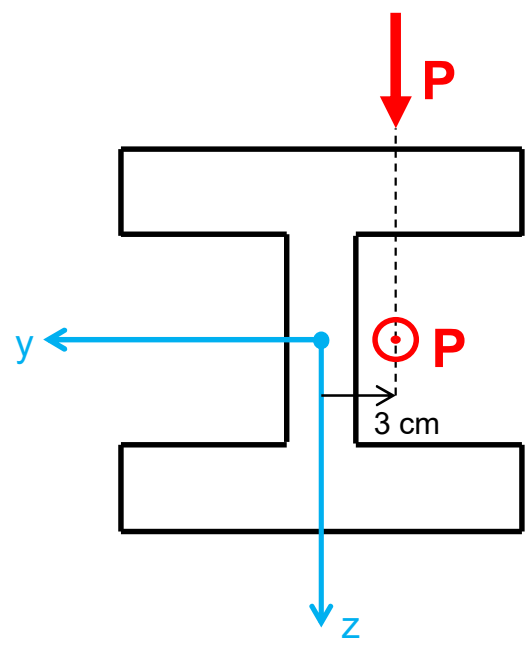
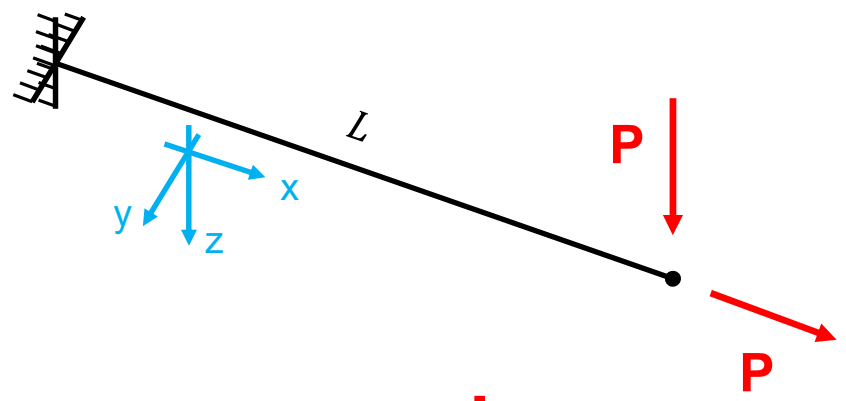
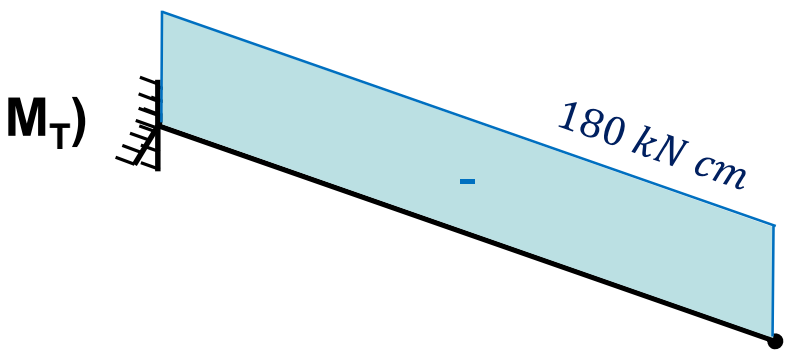
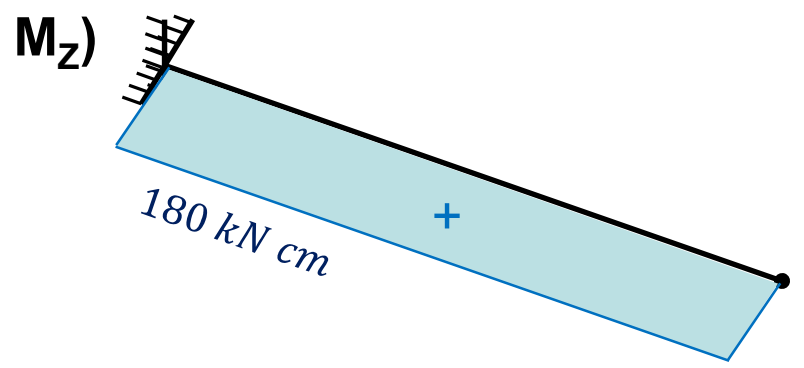
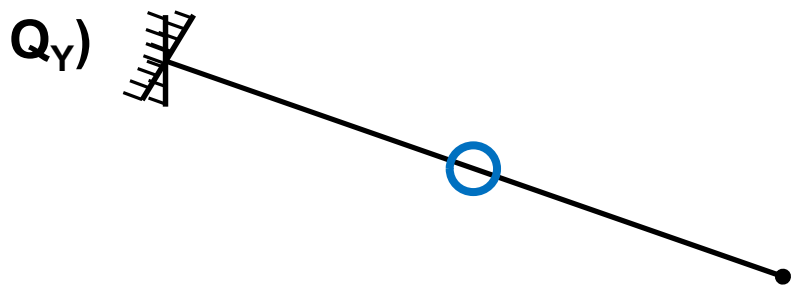
Datos:  
 $P = 60 \text{ kN}$   
 $L = 150 \text{ cm}$

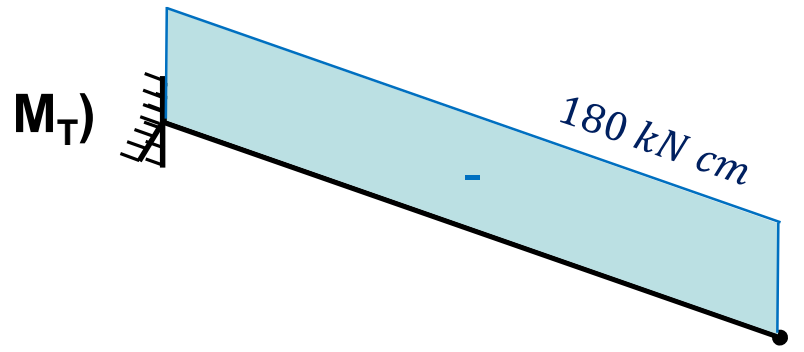
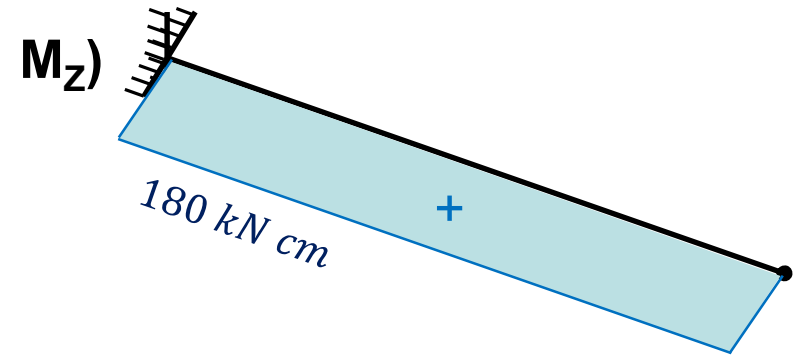
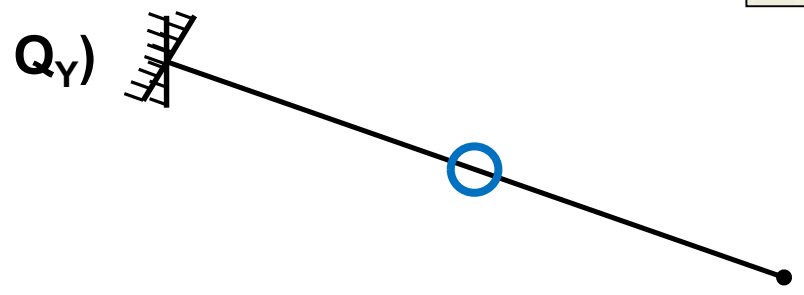
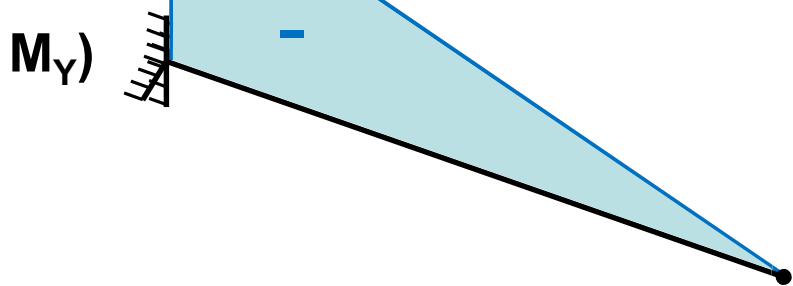
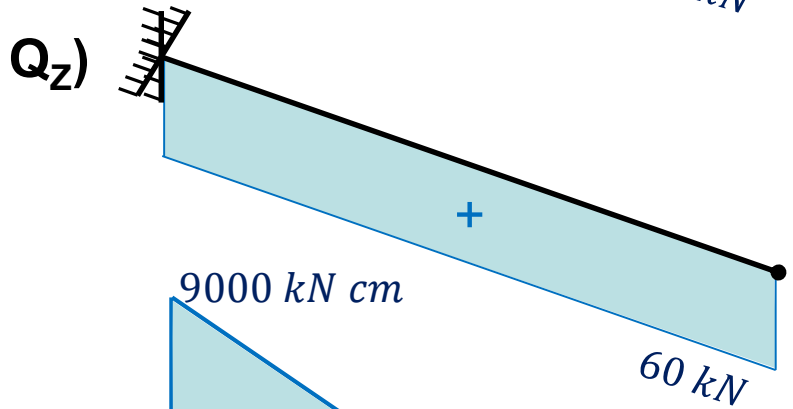
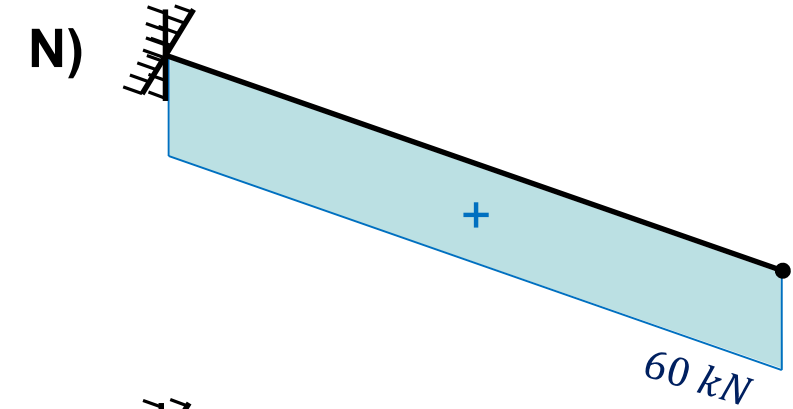


# Diagramas de características



Datos:  
 $P = 60 \text{ kN}$   
 $L = 150 \text{ cm}$





¿Qué sección verificamos?

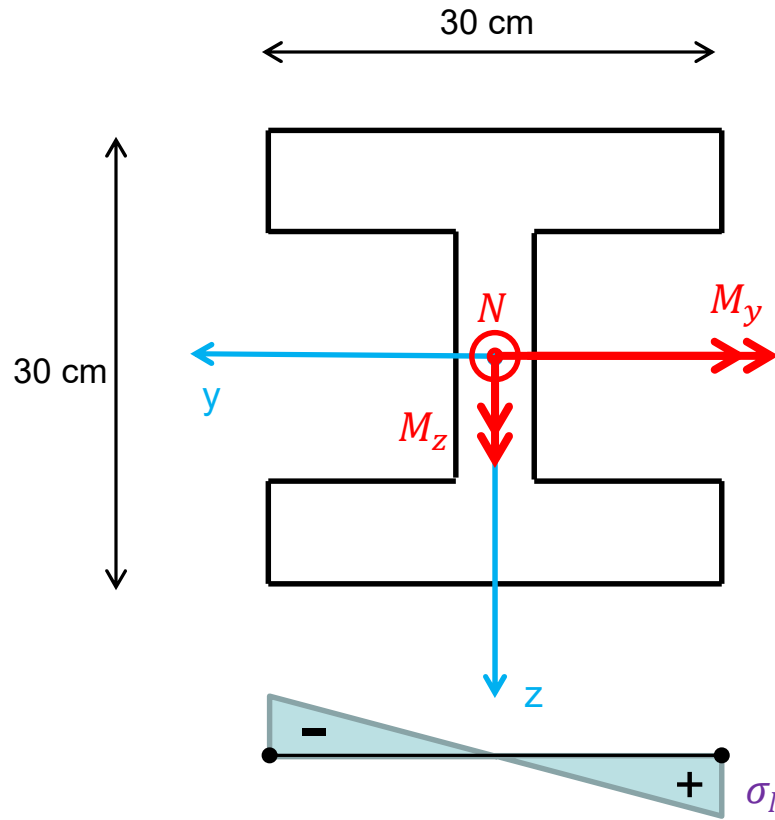




# Diagramas de tensiones normales

Solicitaciones:  $N = 60 \text{ kN}$   
 $M_y = -9000 \text{ kNcm}$   
 $M_z = 180 \text{ kNcm}$

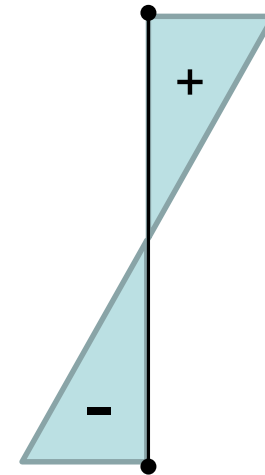
Datos del perfil:  $A = 149 \text{ cm}^2$   
 $J_y = 25170 \text{ cm}^4$   
 $J_z = 8560 \text{ cm}^4$



$$\sigma_N = \frac{N}{A} = 0,4030 \frac{\text{kN}}{\text{cm}^2}$$



$$\sigma_{My} = \frac{M_y}{J_y} \cdot z = 5,364 \frac{\text{kN}}{\text{cm}^2}$$



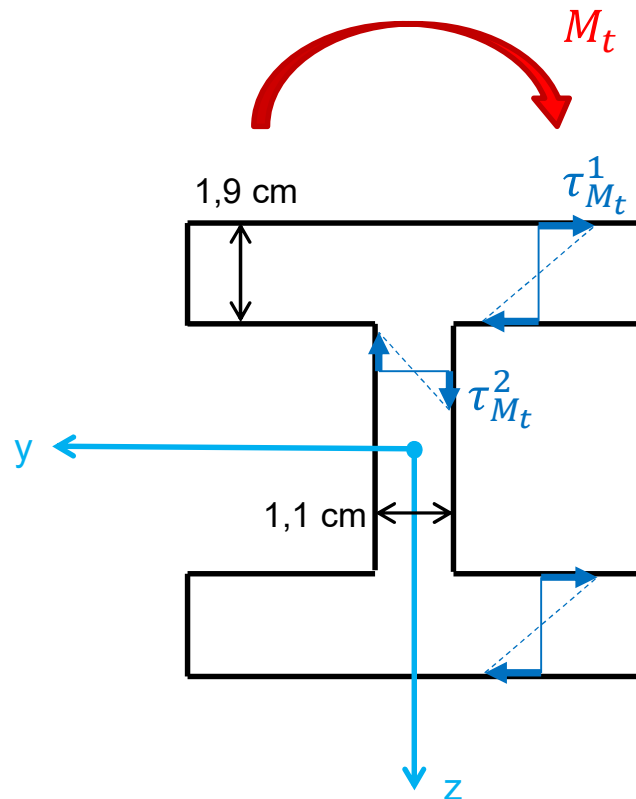
$$\sigma_{Mz} = \frac{M_z}{J_z} \cdot y = 0,315 \frac{\text{kN}}{\text{cm}^2}$$



# Diagramas de tensiones tangenciales por $M_t$

Solicitaciones:  $M_T = -180 \text{ kN cm}$

Datos del perfil:  $J_T = 149 \text{ cm}^4$



Por ser una sección abierta, calculo el  $\tau_{M_t}$  con **Saint Venant**.

$$\tau_{M_t} = \frac{M_T}{J_T} \cdot e$$

$$|\tau_{M_t}^1| = 2,295 \frac{\text{kN}}{\text{cm}^2}$$

$$|\tau_{M_t}^2| = 1,329 \frac{\text{kN}}{\text{cm}^2}$$



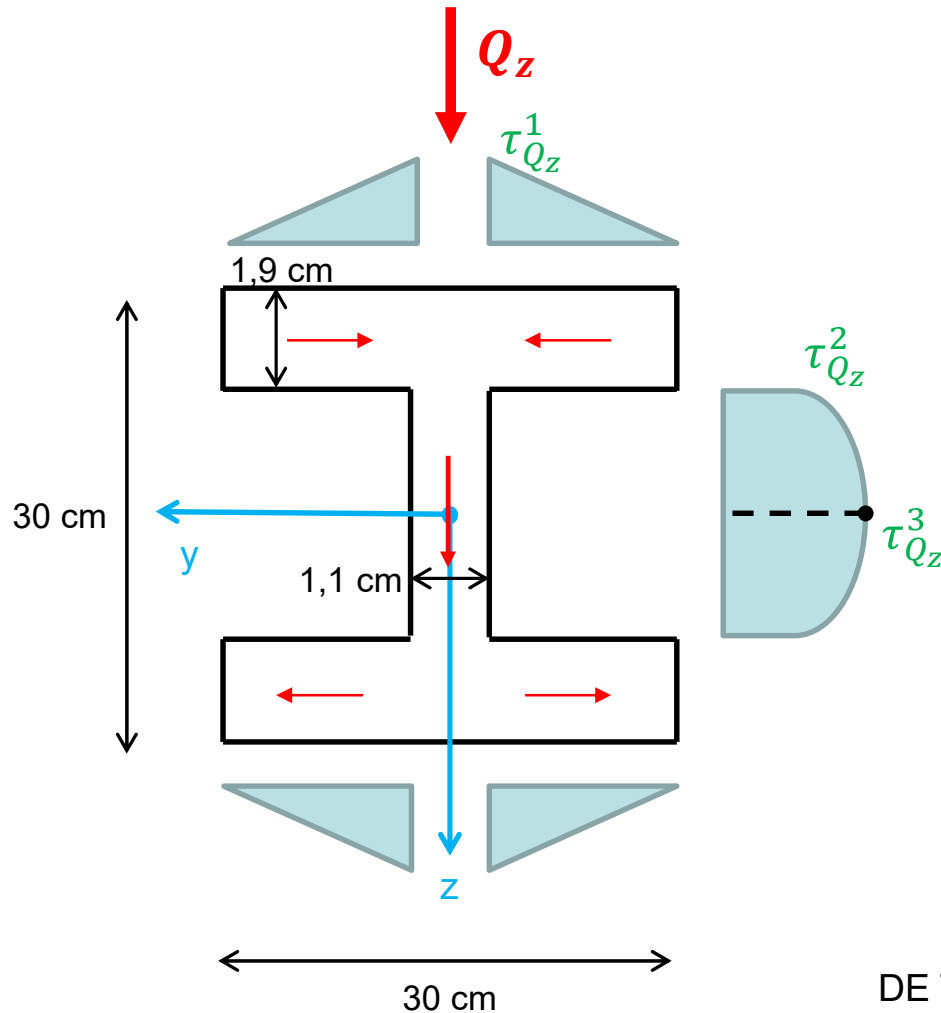
# Diagramas de tensiones tangenciales por $Q_z$

Solicitaciones:  $Q_z = 60 \text{ kN}$

Datos del perfil:  $J_y = 25170 \text{ cm}^4$   
 $J_z = 8560 \text{ cm}^4$

Calculo el  $\tau_{Q_z}$  con Jouravski-Colignon

$$\tau_{Q_z} = \frac{Q \cdot S^*}{J \cdot b}$$

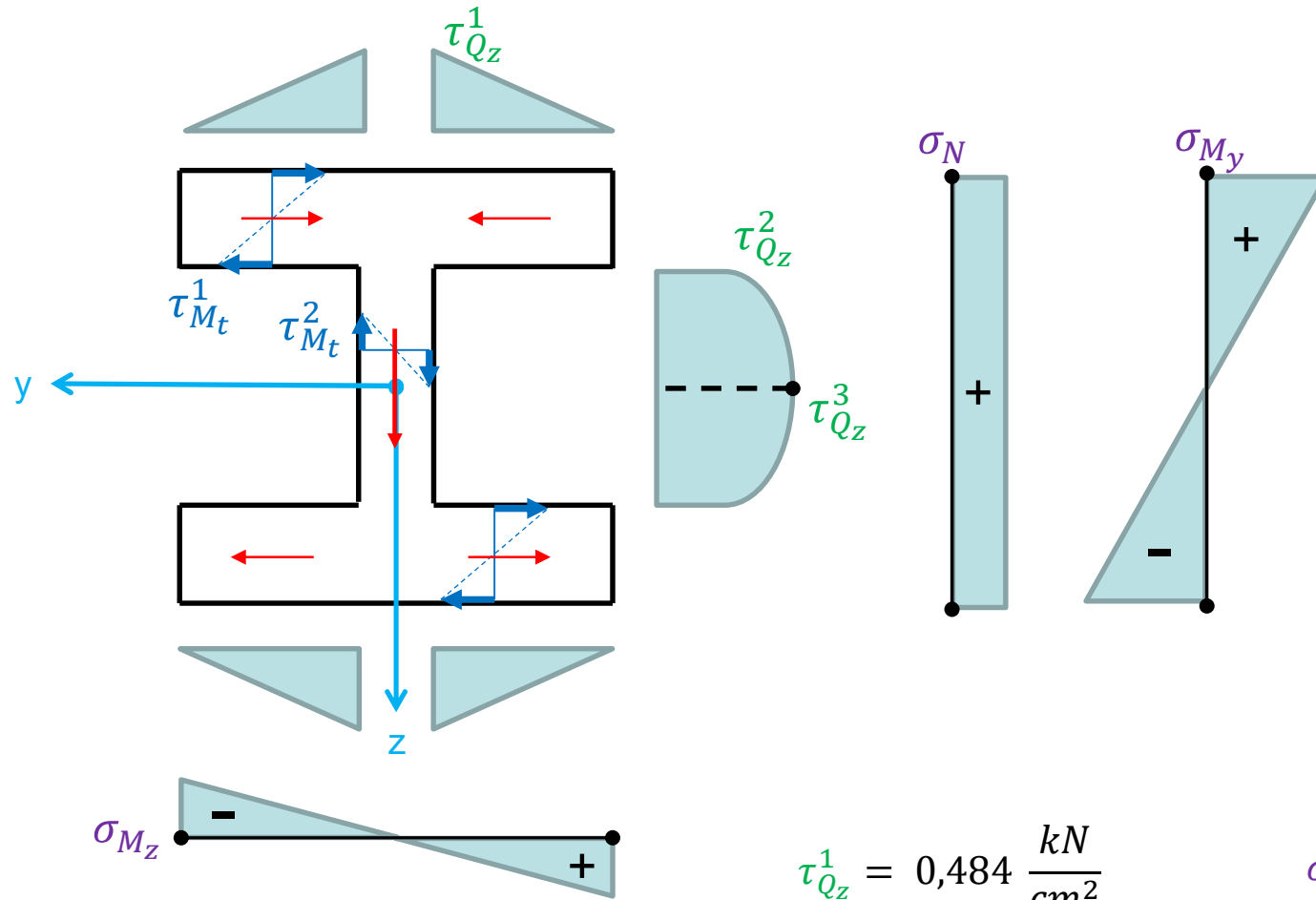


$$\tau_{Q_z}^1 = \frac{Q_z \cdot \left( 1,9 \cdot \left( \frac{30}{2} - \frac{1,1}{2} \right) \cdot \left( \frac{30}{2} - \frac{1,9}{2} \right) \right)}{J_y \cdot 1,9} = 0,484 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{Q_z}^2 = \frac{Q_z \cdot \left( 1,9 \cdot 30 \cdot \left( \frac{30}{2} - \frac{1,9}{2} \right) \right)}{J_y \cdot 1,1} = 1,736 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{Q_z}^3 = \frac{Q_z \cdot (934)}{J_y \cdot 1,1} = 2,024 \frac{\text{kN}}{\text{cm}^2}$$

DE TABLA!



$$|\tau_{M_t}^1| = 2,295 \frac{kN}{cm^2}$$

$$|\tau_{M_t}^2| = 1,329 \frac{kN}{cm^2}$$

$$\tau_{Q_z}^1 = 0,484 \frac{kN}{cm^2}$$

$$\tau_{Q_z}^2 = 1,736 \frac{kN}{cm^2}$$

$$\tau_{Q_z}^3 = 2,024 \frac{kN}{cm^2}$$

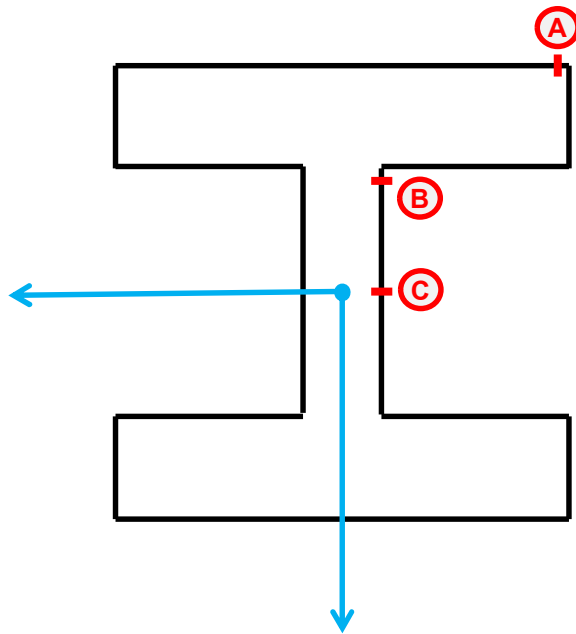
$$\sigma_N = 0,403 \frac{kN}{cm^2}$$

$$\sigma_{M_y} = 5,364 \frac{kN}{cm^2}$$

$$\sigma_{M_z} = 0,315 \frac{kN}{cm^2}$$



## Verificamos puntos críticos



$$\text{Von Mises: } \sigma_{\text{VonMises}} = \sqrt{\sigma^2 + 3 \cdot \tau^2}$$

$$\text{Tresca: } \sigma_{\text{Tresca}} = \sqrt{\sigma^2 + 4 \cdot \tau^2}$$

$$|\sigma_A| = \sigma_N + \sigma_{M_y} + \sigma_{M_z} = 6,082 \frac{\text{kN}}{\text{cm}^2}$$

$$|\tau_A| = \tau_{M_t}^1 = 2,295 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{\text{Tresca}}^A = 7,619 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{\text{VonMises}}^A = 7,266 \frac{\text{kN}}{\text{cm}^2}$$

$$|\sigma_B| = \sigma_N + \sigma_{M_y}' + \sigma_{M_z}' = 5,099 \frac{\text{kN}}{\text{cm}^2}$$

$$|\tau_B| = \tau_{M_t}^1 + \tau_{Q_z}^1 = 3,065 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{\text{Tresca}}^B = 7,973 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{\text{VonMises}}^B = 7,361 \frac{\text{kN}}{\text{cm}^2}$$

$$|\sigma_C| = \sigma_N + \sigma_{M_z}' = 0,415 \frac{\text{kN}}{\text{cm}^2}$$

$$|\tau_C| = \tau_{M_t}^2 + \tau_{Q_z}^3 = 3,353 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{\text{Tresca}}^C = 6,719 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{\text{VonMises}}^C = 5,822 \frac{\text{kN}}{\text{cm}^2}$$



# Tensor de Tensiones

Armo los tensores para el punto B

$$[T_{T_{xyz}}]^B = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} = \begin{pmatrix} 5,099 & 0 & 3,065 \\ 0 & 0 & 0 \\ 3,065 & 0 & 0 \end{pmatrix} \frac{kN}{cm^2}$$

Estado *DOBLE* de tensiones

$$\sigma_{1,3} = \frac{\sigma \pm \sqrt{\sigma^2 + 4 \cdot \tau^2}}{2} \quad \text{Solo vale para estados dobles!}$$

$$\rightarrow \sigma_1 = 6,536 \frac{kN}{cm^2}$$

$$\rightarrow \sigma_3 = -1,437 \frac{kN}{cm^2}$$

$$[T_{T_{123}}]^B = \begin{pmatrix} 6,536 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1,437 \end{pmatrix} \frac{kN}{cm^2}$$



# Tensor de Deformaciones

Datos del material:  $E = 21000 \frac{kN}{cm^2}$   
 $\mu = 0,25$

$$[T_{T_{123}}]^B = \begin{pmatrix} 6,536 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1,437 \end{pmatrix} \frac{kN}{cm^2} \left\{ \begin{array}{l} \varepsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_3) = 3,284 \cdot 10^{-4} \\ \varepsilon_2 = \frac{1}{E}(-\mu\sigma_1 - \mu\sigma_3) = -0,607 \cdot 10^{-4} \\ \varepsilon_3 = \frac{1}{E}(-\mu\sigma_1 + \sigma_3) = -1,463 \cdot 10^{-4} \end{array} \right.$$

$$[T_{D_{123}}]^B = \begin{pmatrix} 3,284 & 0 & 0 \\ 0 & -0,607 & 0 \\ 0 & 0 & -1,463 \end{pmatrix} \cdot 10^{-4}$$

Estado *TRIPLE* de deformaciones

## Direcciones principales

$$[T_{T_{xyz}}]^B = \begin{pmatrix} 5,099 & 0 & 3,065 \\ 0 & 0 & 0 \\ 3,065 & 0 & 0 \end{pmatrix} \frac{kN}{cm^2} \quad V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$V_1 = \begin{pmatrix} (5,099 - 6,536) & 0 & 3,065 \\ 0 & -6,536 & 0 \\ 3,065 & 0 & -6,536 \end{pmatrix} \cdot \begin{pmatrix} V_{x1} \\ V_{y1} \\ V_{z1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-1,437 \cdot V_{x1} + 3,065 \cdot V_{z1} = 0 \quad \rightarrow \quad V_{x1} = \frac{3,065}{1,437} \cdot V_{z1} \quad V_{x1} = 2,133$$

$$-6,536 \cdot V_{y1} = 0 \quad \rightarrow \quad V_{y1} = 0 \quad V_{y1} = 0$$

$$+3,065 \cdot V_{x1} - 6,536 \cdot V_{z1} = 0 \quad \rightarrow \quad V_{z1} = 1 \cdot V_{z1} \quad V_{z1} = 1$$

$$V_1 = \begin{pmatrix} 0,905 \\ 0 \\ 0,424 \end{pmatrix}$$

Normalizo

$$\begin{aligned} \sigma_1 &= 6,536 \frac{kN}{cm^2} \\ \sigma_2 &= 0 \\ \sigma_3 &= -1,437 \frac{kN}{cm^2} \end{aligned}$$







## Direcciones principales

$$V_1 = \begin{pmatrix} 0,905 \\ 0 \\ 0,424 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

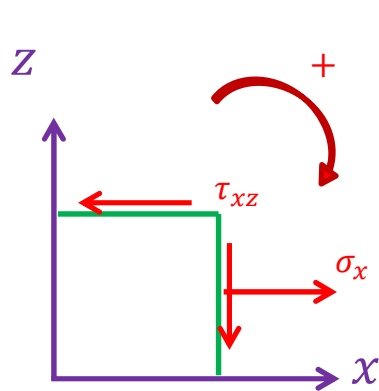
$$V_3 = V_1 \times V_2 = \begin{pmatrix} 0,905 \\ 0 \\ 0,424 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -0,424 \\ 0 \\ 0,905 \end{pmatrix}$$

Las direcciones principales  
son ortogonales

$$V_1 = \begin{pmatrix} 0,905 \\ 0 \\ 0,424 \end{pmatrix} ; V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} ; V_3 = \begin{pmatrix} -0,424 \\ 0 \\ 0,905 \end{pmatrix}$$

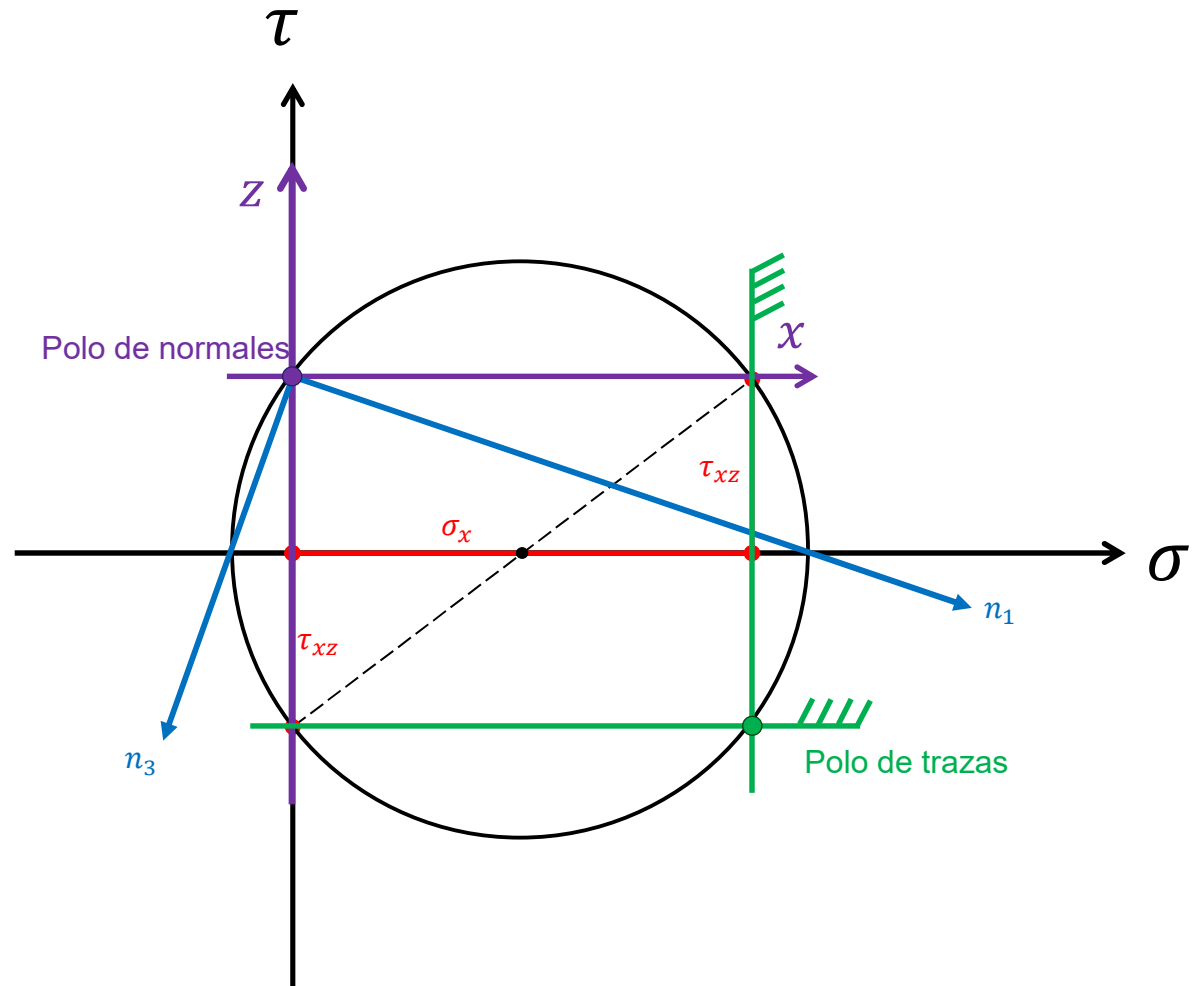
## Circunferencia de Mohr

Tensiones:  $\sigma_x = 5,099 \frac{kN}{cm^2}$   
 $\tau_{xz} = 3,065 \frac{kN}{cm^2}$



$$V_1 = \begin{pmatrix} 0,905 \\ 0 \\ 0,424 \end{pmatrix}$$

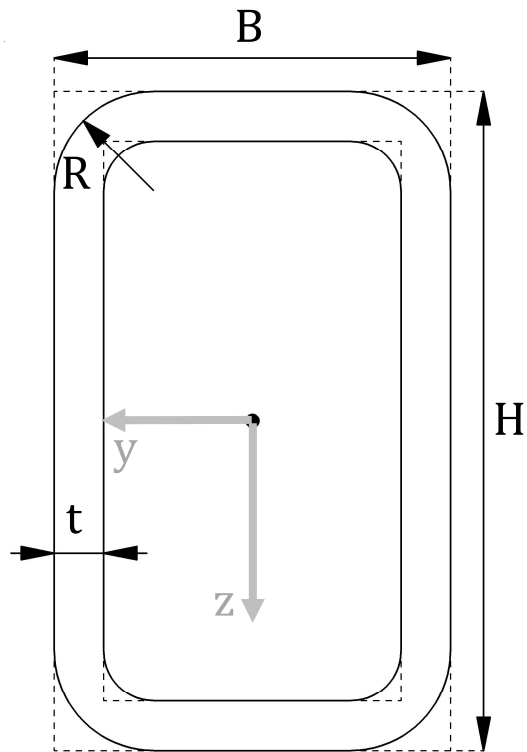
$$V_3 = \begin{pmatrix} -0,424 \\ 0 \\ 0,905 \end{pmatrix}$$





### Ejercicio 3:

- Determinar diagramas de tensiones parciales en la sección más solicitada.
- Identificar el/los punto/s más peligroso/s de la estructura
- Calcular el coeficiente de seguridad con la Teoría de Von Mises



Perfil Tubo Rectangular 100\*250\*10

Acero F24  $\sigma_{fl} = 24 \frac{\text{kN}}{\text{cm}^2}$

#### Datos de tabla

$B = 100 \text{ mm}$

$t = 10 \text{ mm}$

$R = 2 t$

$H = 250 \text{ mm}$

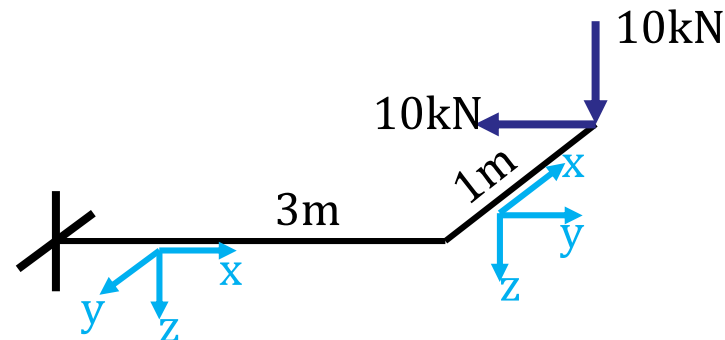
$C = 428,14 \text{ cm}^3$

$I_{\text{máx}} = 4515,84 \text{ cm}^4$

$W_{\text{máx}} = S_{\text{máx}} = 361,27 \text{ cm}^3$

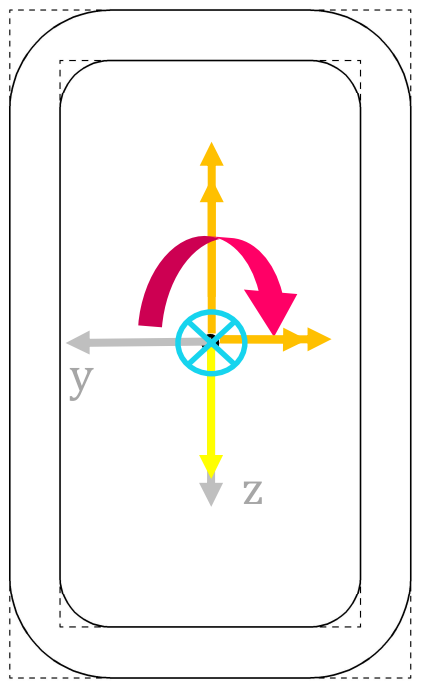
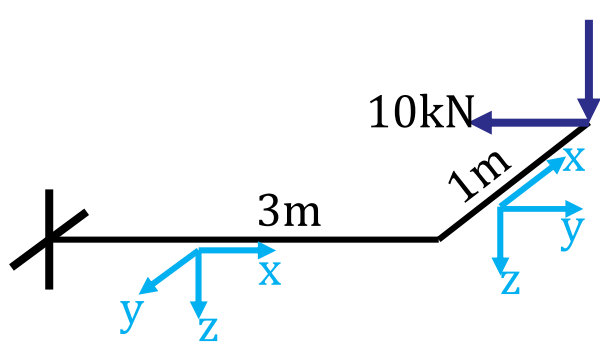
$I_{\text{mín}} = 1040,04 \text{ cm}^4$

$W_{\text{mín}} = S_{\text{mín}} = 208,01 \text{ cm}^3$

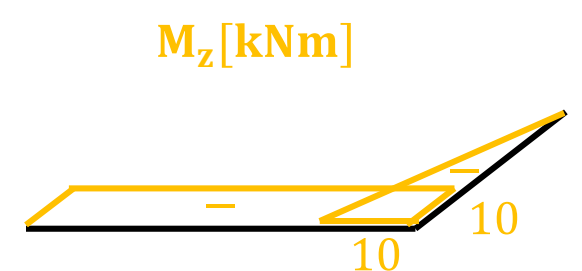
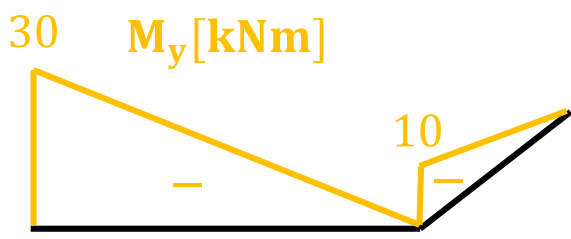
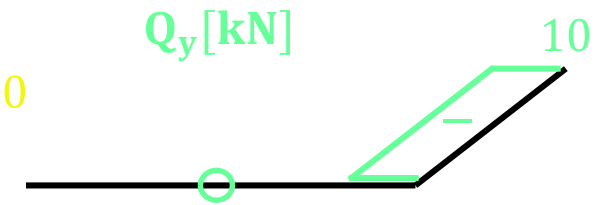
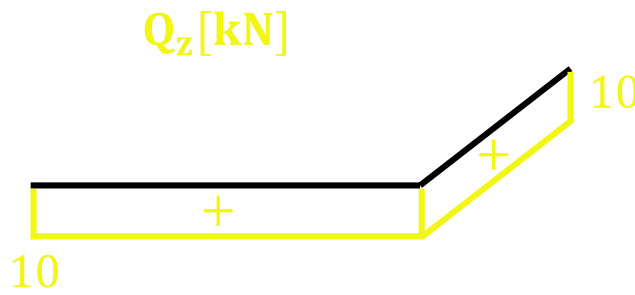
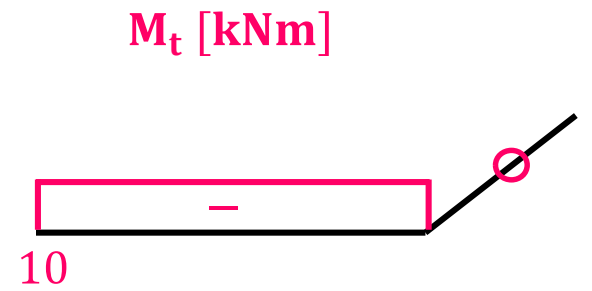
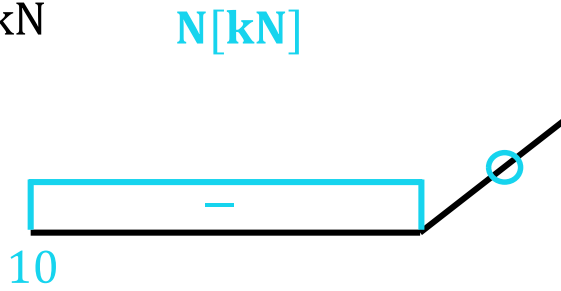




# Diagramas de características



Sección más solicitada



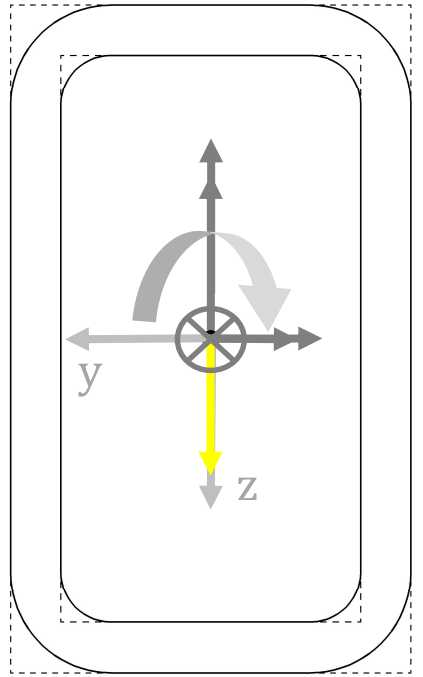
$N = -10 \text{ kN}$	$M_t = -10 \text{ kNm}$	$Q_z = 10 \text{ kN}$
	$M_y = -30 \text{ kNm}$	$M_z = -10 \text{ kNm}$

**Corte**

$$Q_z = 10\text{kN}$$



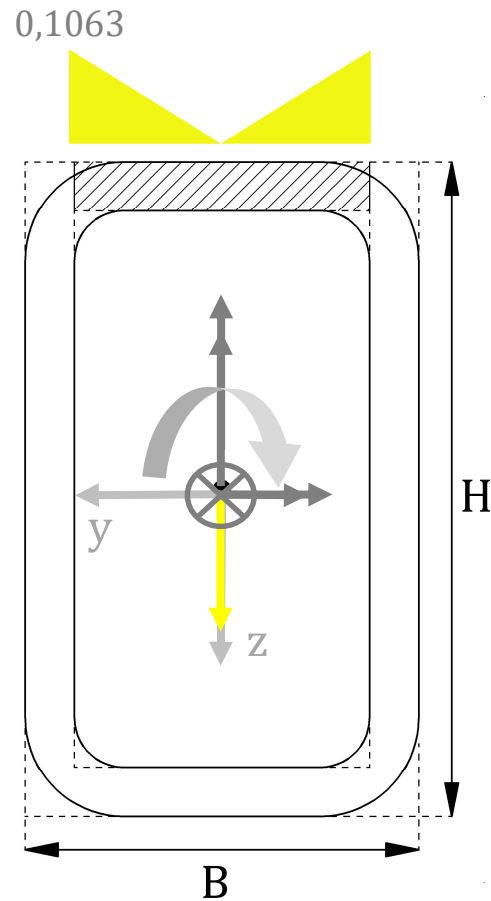
Se aproxima el diagrama considerando sección con esquinas en ángulo recto





Corte

$$Q_z = 10 \text{ kN}$$



Se aproxima el diagrama considerando sección con esquinas en ángulo recto

$$\tau_{xy}^{Q_z} \text{ máx} = \frac{Q_z \cdot S_{EN}^{*1}}{I_{EN} \cdot b}$$

$$S_{EN}^{*1} = ((B - 2t)t) \left( \frac{H}{2} - \frac{t}{2} \right)$$

$$S_{EN}^{*1} = 96 \text{ cm}^3$$

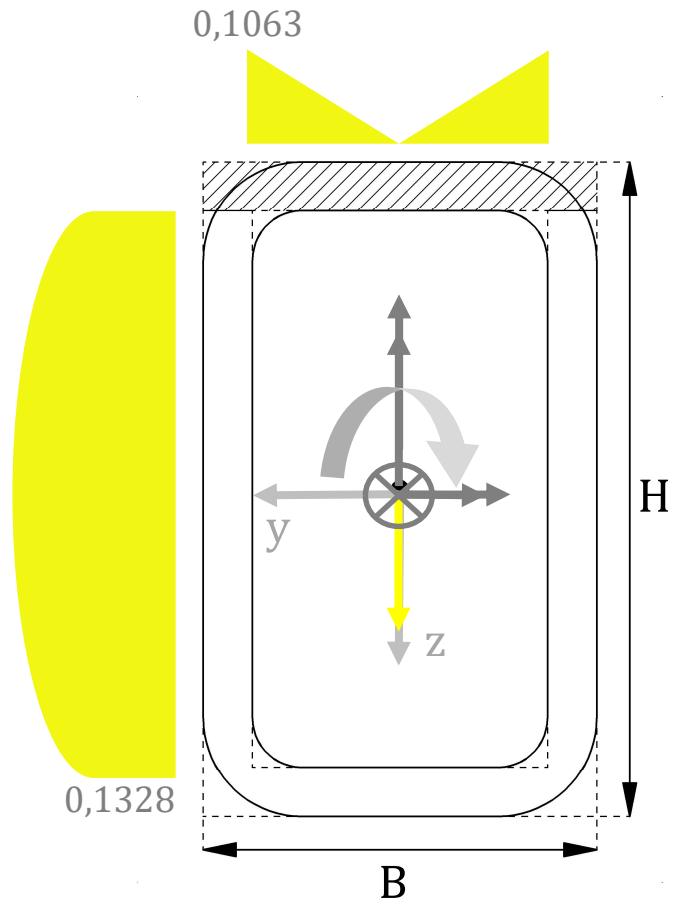
$$\tau_{xy}^{Q_z} \text{ máx} = 0,1063 \frac{\text{kN}}{\text{cm}^2}$$

Se expresarán las tensiones en los diagramas en  $\frac{\text{kN}}{\text{cm}^2}$



Corte

$$Q_z = 10 \text{ kN}$$



Se aproxima el diagrama considerando sección con esquinas en ángulo recto

$$\tau_{xz}^{Q_z} \text{ mín} = \frac{Q_z \cdot S_{EN}^{*2}}{I_{EN} \cdot b}$$

$$S_{EN}^{*2} = Bt \left( \frac{H}{2} - \frac{t}{2} \right)$$

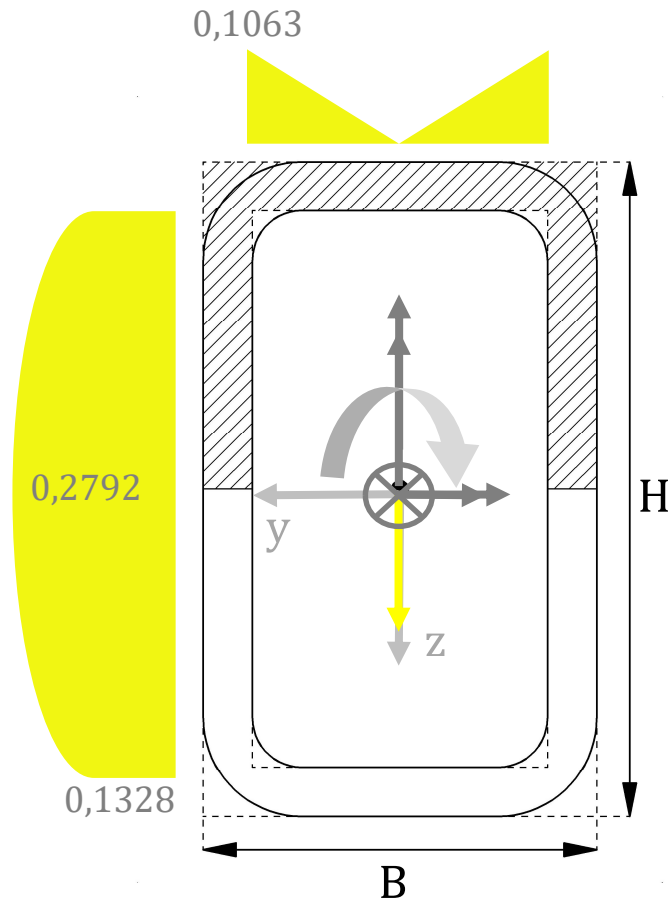
$$S_{EN}^{*2} = 120 \text{ cm}^3$$

$$\tau_{xz}^{Q_z} \text{ mín} = 0,1328 \frac{\text{kN}}{\text{cm}^2}$$



Corte

$$Q_z = 10 \text{ kN}$$



Se aproxima el diagrama considerando sección con esquinas en ángulo recto

$$\tau_{xz \text{ máx}}^{Q_z} = \frac{Q_z \cdot S_{EN}^{*3}}{I_{EN} \cdot b}$$

$$S_{EN}^{*3} = S_{EN}^{*2} + 2 \left[ t \left( \frac{H}{2} - t \right) \frac{1}{2} \left( \frac{H}{2} - t \right) \right]$$

$$S_{EN}^{*3} = 252,25 \text{ cm}^3$$

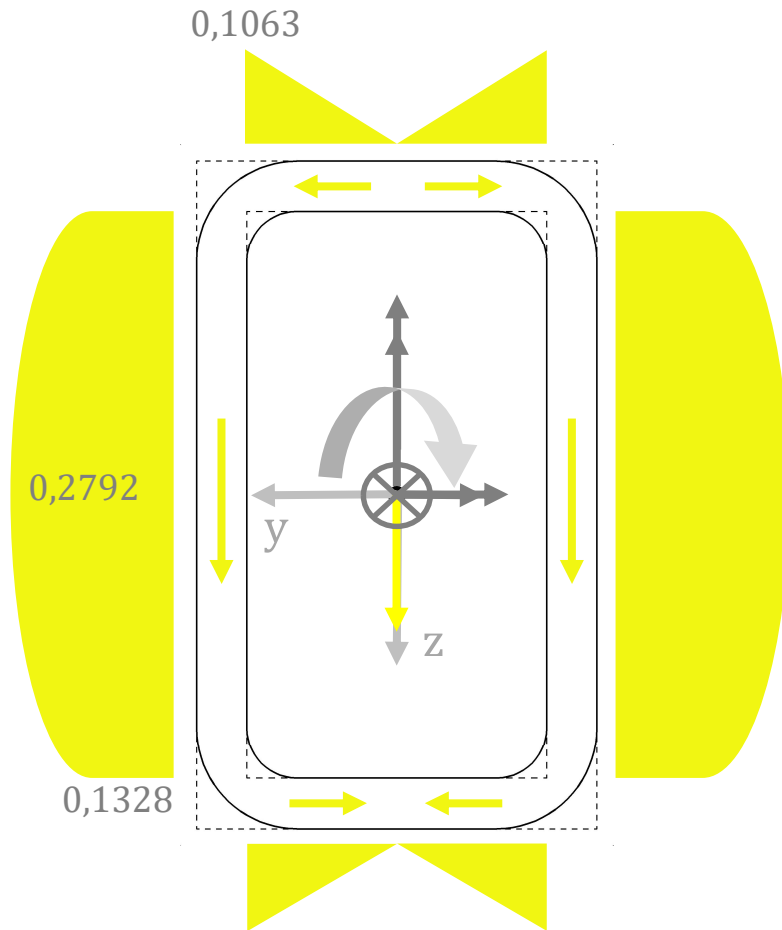
$$\tau_{xz \text{ máx}}^{Q_z} = 0,2792 \frac{\text{kN}}{\text{cm}^2}$$





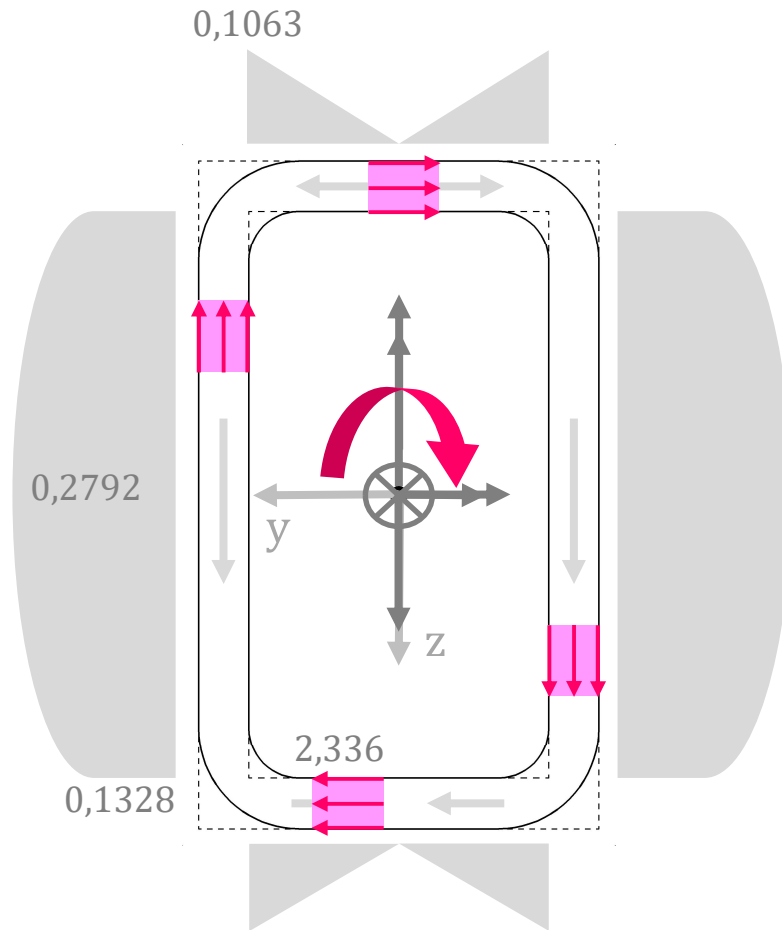
Corte

$$Q_z = 10 \text{ kN}$$



## Torsión

$$M_t = -10 \text{ kNm}$$



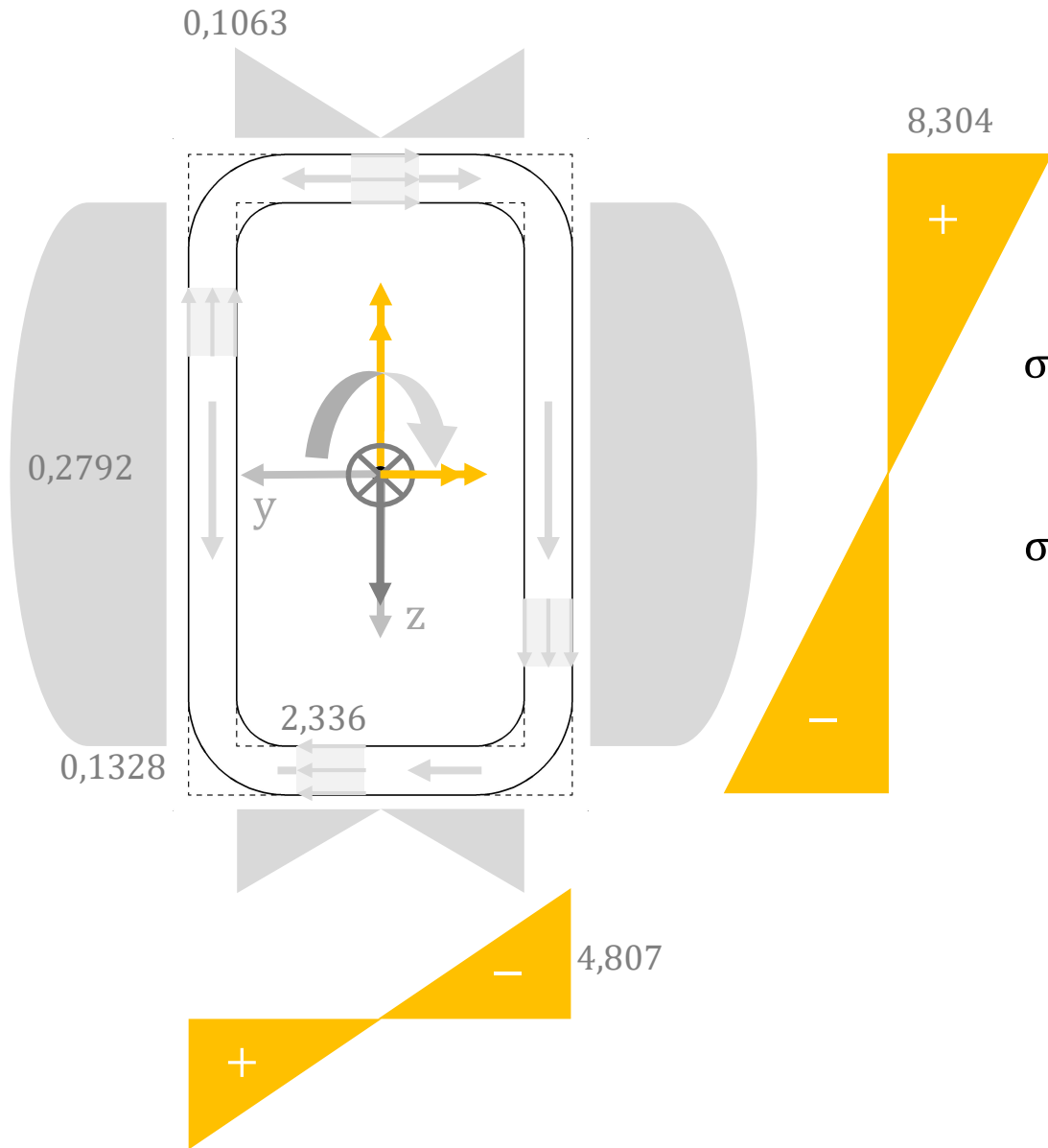
$$\tau_{M_t} = \frac{M_t}{C} = \frac{1000 \text{ kNcm}}{428,14 \text{ cm}^3} = 2,336 \frac{\text{kN}}{\text{cm}^2}$$



# Flexión

$$M_y = -30 \text{ kNm}$$

$$M_z = -10 \text{ kNm}$$



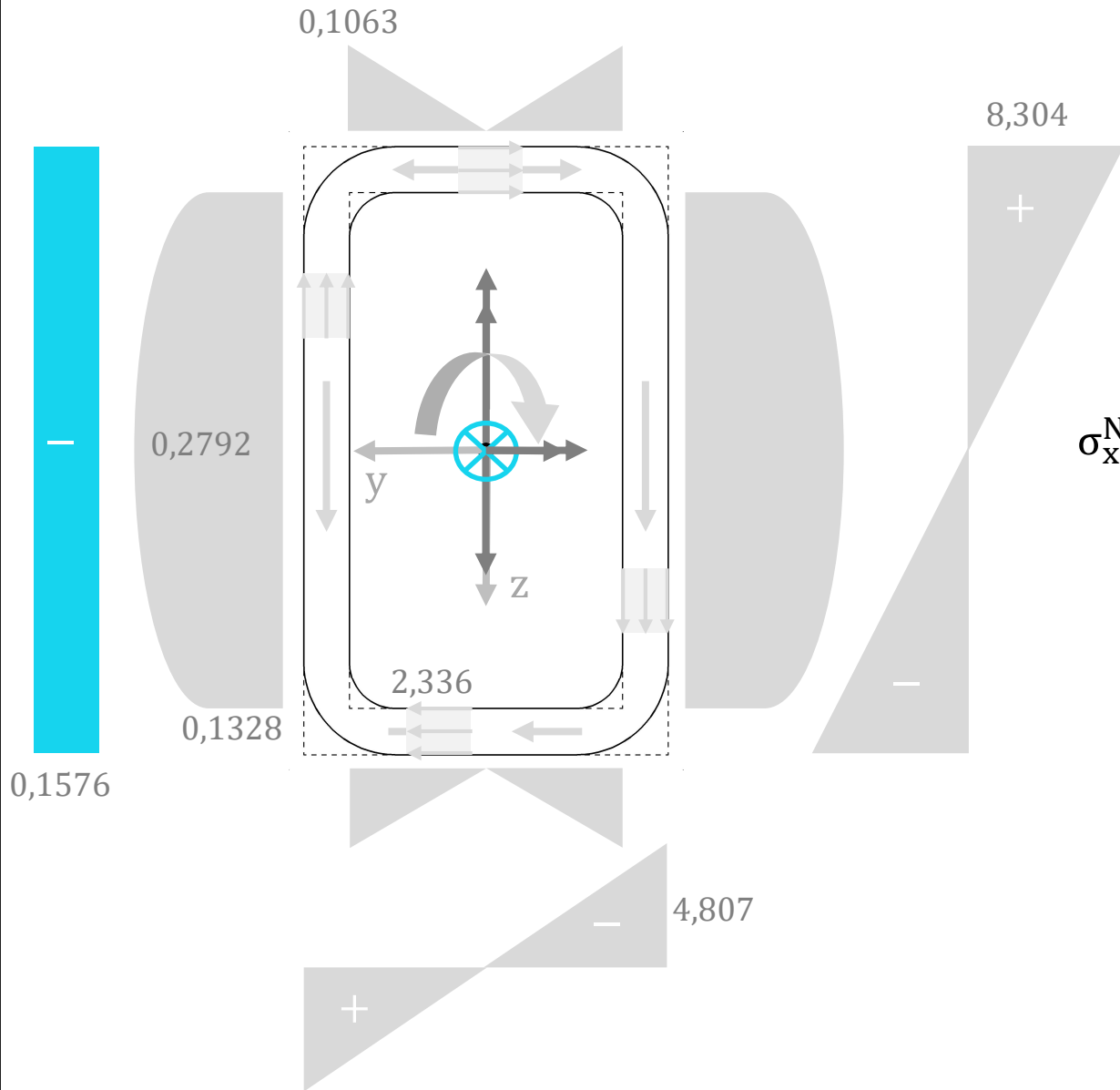
$$\sigma_x^{M_y} = \frac{M_y}{W_y} = \frac{3000 \text{ kNcm}}{361,27 \text{ cm}^3} = 8,304 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_x^{M_z} = \frac{M_z}{W_z} = \frac{1000 \text{ kNcm}}{208,01 \text{ cm}^3} = 4,807 \frac{\text{kN}}{\text{cm}^2}$$



Axil

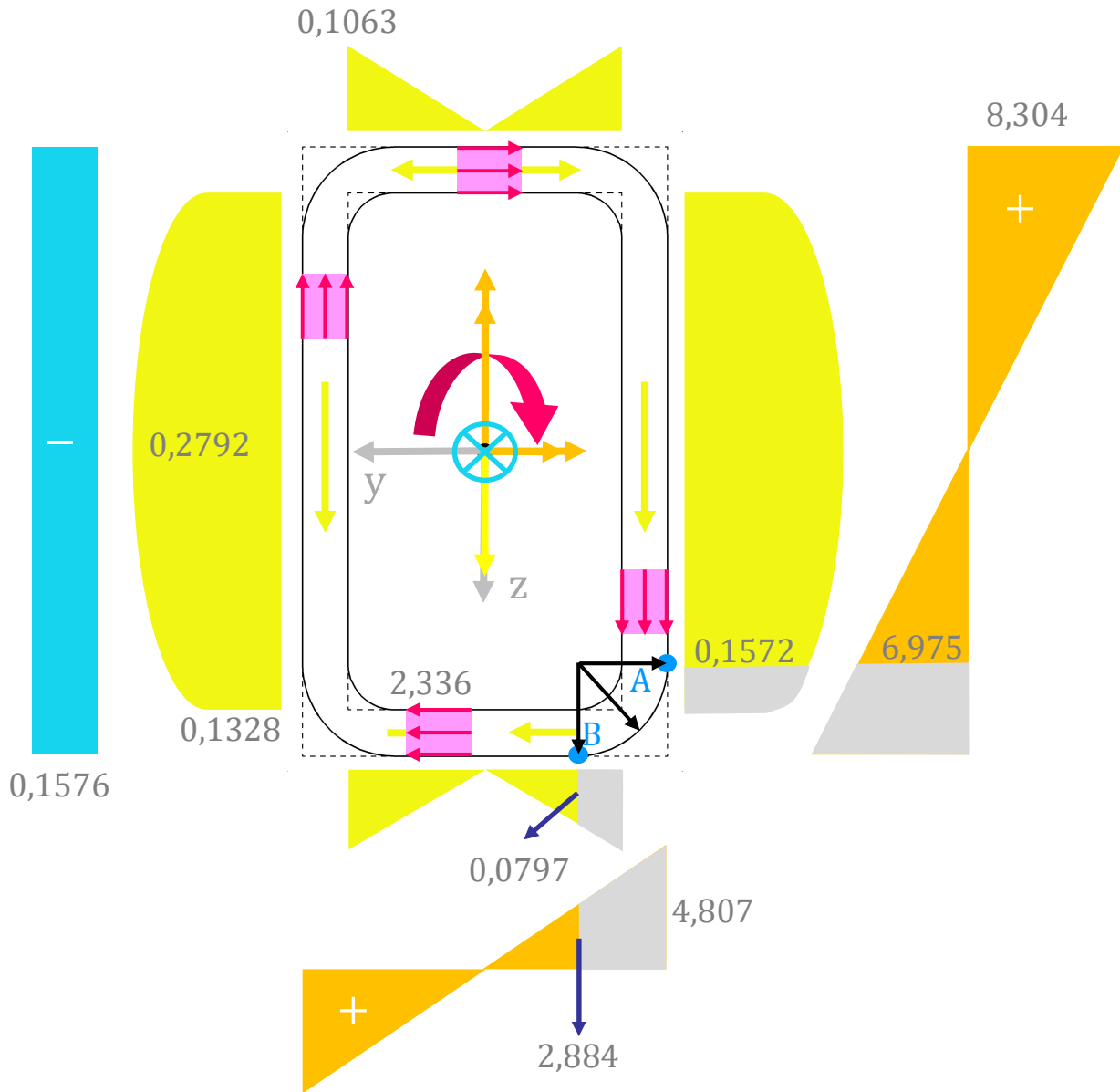
$N = -10 \text{ kN}$



$$\sigma_x^N = \frac{N}{A} = \frac{-10 \text{ kN}}{63,425 \text{ cm}^2} = -0,1576 \frac{\text{kN}}{\text{cm}^2}$$



¿Cuál es el punto más peligroso?



Punto i      Terna XYZ

$$[T_T]_{XYZ}^i = \begin{pmatrix} \sigma_X & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & \sigma_Y & \tau_{YZ} \\ \tau_{ZX} & \tau_{ZY} & \sigma_Z \end{pmatrix}$$

$$[T_T]_{XYZ}^i = \begin{pmatrix} \sigma_X & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & 0 & 0 \\ \tau_{ZX} & 0 & 0 \end{pmatrix}$$

$$\sigma_X^A = (-0,1576 - 6,975 - 4,807) \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^A = 0$$

$$\tau_{XZ}^A = (2,336 + 0,1572) \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_X^B = (-0,1576 - 2,884 - 8,304) \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^B = (2,336 + 0,0797) \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XZ}^B = 0$$



$$\sigma_X^A = -11,94 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^A = 0$$

$$\tau_{XZ}^A = 2,49 \frac{\text{kN}}{\text{cm}^2}$$

$$[T_T]_{XYZ}^A = \begin{pmatrix} -11,94 & 0 & 2,49 \\ 0 & 0 & 0 \\ 2,49 & 0 & 0 \end{pmatrix} \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_X^B = -11,35 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XY}^B = 2,41 \frac{\text{kN}}{\text{cm}^2}$$

$$\tau_{XZ}^B = 0$$

$$[T_T]_{XYZ}^B = \begin{pmatrix} -11,35 & 2,41 & 0 \\ 2,41 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{\text{kN}}{\text{cm}^2}$$

Von Mises

$$\sqrt{\sigma_X^2 + 3\tau^2} \leq \frac{\sigma_{fl}}{CS}$$

$$\sigma_{fl} = 24 \frac{\text{kN}}{\text{cm}^2}$$

Punto A

$$CS_A = 1,89$$

Punto B

$$CS_B = 1,98$$

¿Cuál es el punto más peligroso?

¿Cuál es el coeficiente de seguridad?