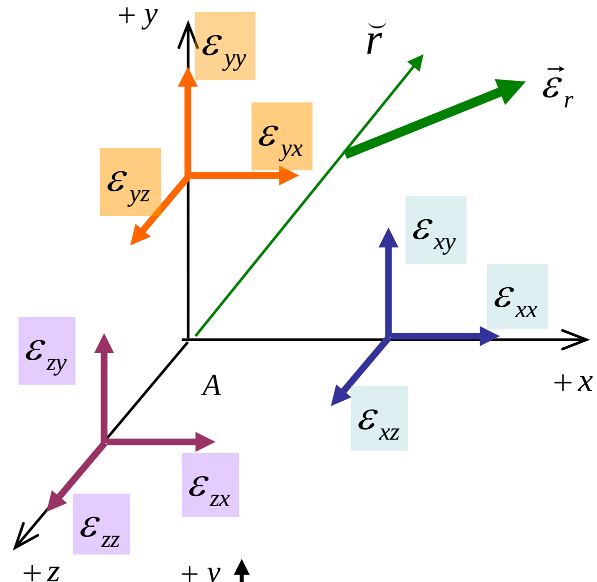
RELACION ENTRE TENSIONES Y DEFORMACIONES

Hipótesis

- Cuerpo continuo
- Material homogéneo
- Material isótropo
- Linealidad cinemática
- Linealidad mecánica (Ley de Hooke)
- Linealidad estática
- Principio desuperposición de efectos (P.S.E)

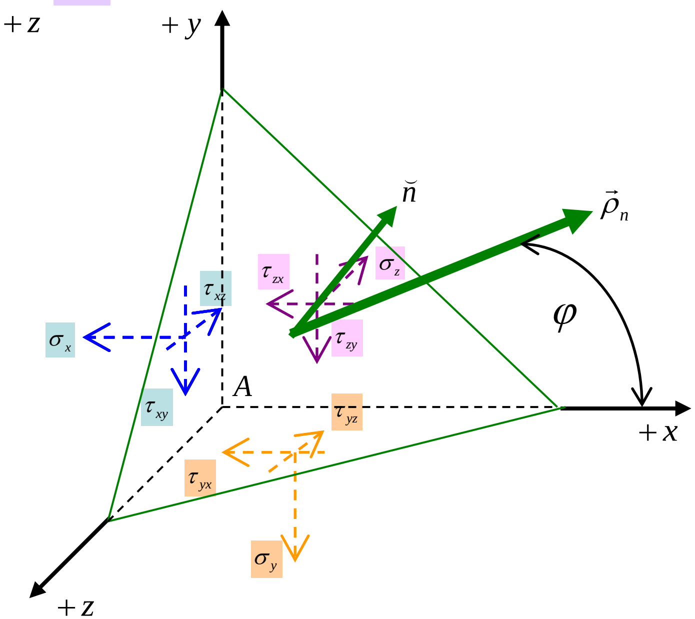
Estado de deformación

$$\begin{Bmatrix} \varepsilon_{rx} \\ \varepsilon_{ry} \\ \varepsilon_{rz} \end{Bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} \begin{Bmatrix} n_{rx} \\ n_{ry} \\ n_{rz} \end{Bmatrix}$$



Estado de tensión

$$\begin{Bmatrix} \rho_{nx} \\ \rho_{ny} \\ \rho_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_{x} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{y} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{z} \end{bmatrix} \begin{Bmatrix} n_{nx} \\ n_{ny} \\ n_{nz} \end{Bmatrix}$$



$$
\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{x} \\ \sigma_{y} \\ \sigma_{z} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}
$$

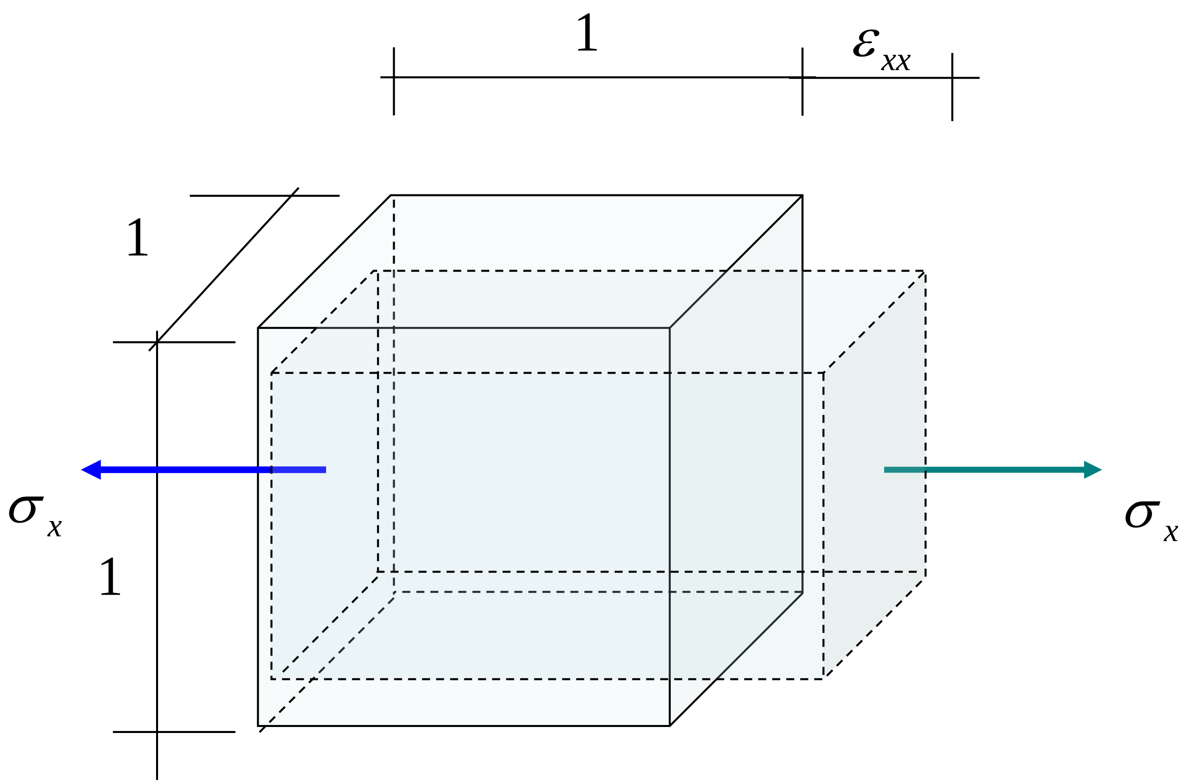
$$
\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{14} & a_{15} & a_{16} \\ a_{24} & a_{25} & a_{26} \\ a_{34} & a_{35} & a_{36} \end{bmatrix} \\ \begin{bmatrix} a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \\ a_{61} & a_{62} & a_{63} \end{bmatrix} & \begin{bmatrix} a_{44} & a_{45} & a_{46} \\ a_{54} & a_{55} & a_{56} \\ a_{64} & a_{65} & a_{66} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \sigma_{x} \\ \sigma_{y} \\ \sigma_{z} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}
$$

Independencia entre deformaciones longitudinales y tensiones tangenciales

Independencia entre deformaciones transversales y tensiones normales

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} a_{44} & a_{45} & a_{46} \\ a_{54} & a_{55} & a_{56} \\ a_{64} & a_{65} & a_{66} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

Tensiones normales y deformaciones longitudinales



$$\varepsilon_{xx} = \frac{\sigma_x}{E} \qquad \varepsilon_{yy} = -\mu\varepsilon_{xx} = -\mu\frac{\sigma_x}{E} \qquad \varepsilon_{zz} = -\mu\varepsilon_{xx} = -\mu\frac{\sigma_x}{E}$$

E : Módulo de elasticidad longitudinal

μ : Coeficiente de Poisson

Si actúa $\sigma_x \Rightarrow$ $\varepsilon_{xx} = \frac{\sigma_x}{E}$ $\varepsilon_{yy} = -\mu\varepsilon_{xx} = -\mu\frac{\sigma_x}{E}$ $\varepsilon_{zz} = -\mu\varepsilon_{xx} = -\mu\frac{\sigma_x}{E}$

Si actúa $\sigma_y \Rightarrow$ $\varepsilon_{xx} = -\mu\varepsilon_{yy} = -\mu\frac{\sigma_y}{E}$ $\varepsilon_{yy} = \frac{\sigma_y}{E}$ $\varepsilon_{zz} = -\mu\varepsilon_{yy} = -\mu\frac{\sigma_y}{E}$

Si actúa $\sigma_z \Rightarrow$ $\varepsilon_{xx} = -\mu\varepsilon_{zz} = -\mu\frac{\sigma_z}{E}$ $\varepsilon_{yy} = -\mu\varepsilon_{zz} = -\mu\frac{\sigma_z}{E}$ $\varepsilon_{zz} = \frac{\sigma_z}{E}$

Aplicando el P.S.E.

$$\varepsilon_{xx} = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} = \frac{1}{E}\left(\sigma_x - \mu\sigma_y - \mu\sigma_z\right)$$

$$\varepsilon_{yy} = -\mu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} = \frac{1}{E}\left(\sigma_y - \mu\sigma_x - \mu\sigma_z\right)$$

$$\varepsilon_{zz} = -\mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} = \frac{1}{E}\left(\sigma_z - \mu\sigma_x - \mu\sigma_y\right)$$

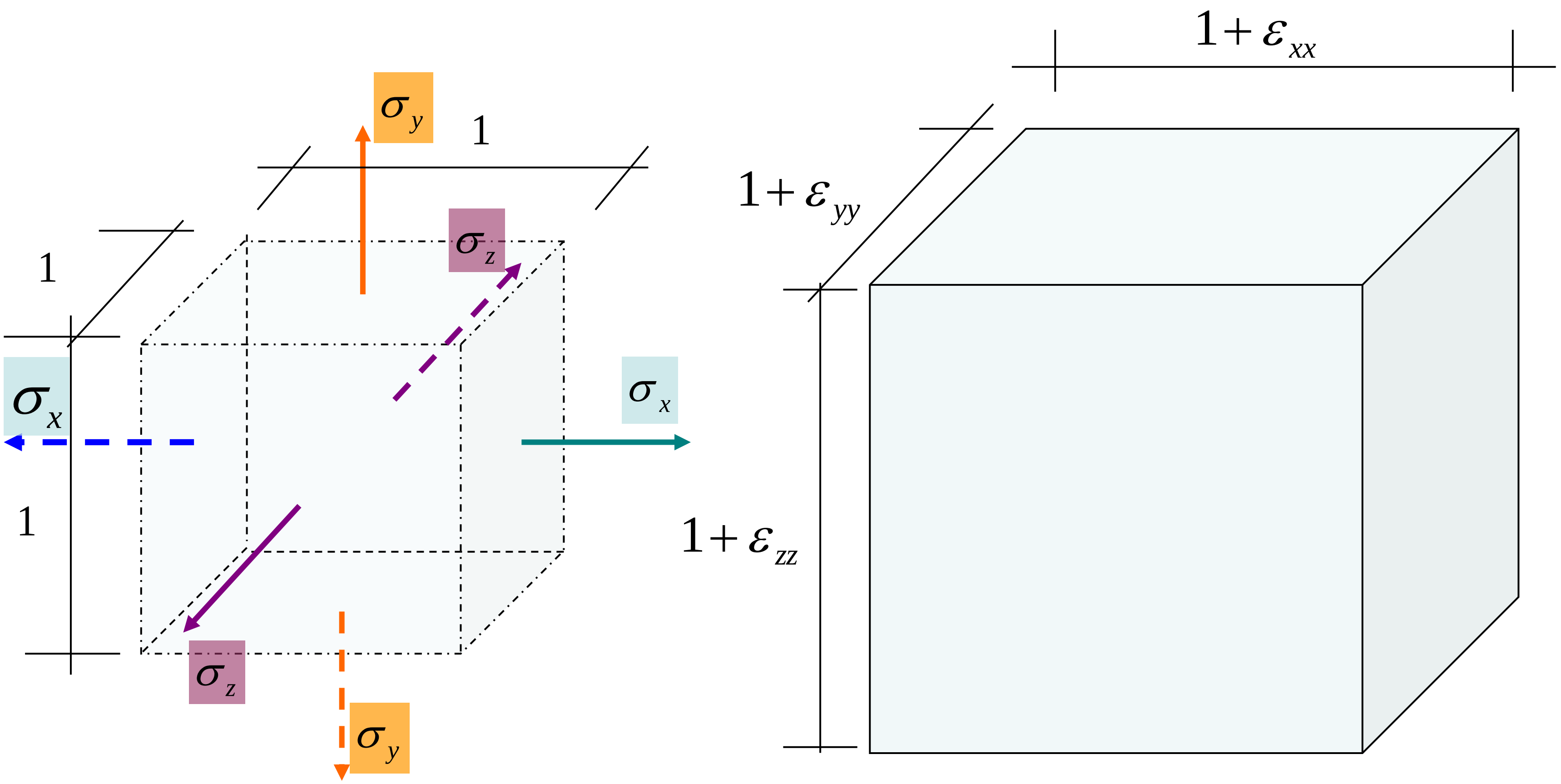
Tensiones tangenciales y deformaciones transversales

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 2\varepsilon_{xy} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} = 2\varepsilon_{yz} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G} = 2\varepsilon_{zx}$$

G : Módulo de elasticidad transversal

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} \frac{1}{E} & -\frac{\mu}{E} & -\frac{\mu}{E} \\ -\frac{\mu}{E} & \frac{1}{E} & -\frac{\mu}{E} \\ -\frac{\mu}{E} & -\frac{\mu}{E} & \frac{1}{E} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \frac{1}{2G} & 0 & 0 \\ 0 & \frac{1}{2G} & 0 \\ 0 & 0 & \frac{1}{2G} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

Variación específica de volumen – Valor de μ



V_0 : Volumen inicial

V_f : Volumen final

ε_v : Variación específica de volumen

$$V_0 = 1$$

$$V_f = (1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz})$$

$$\varepsilon_v = \frac{V_f - V_0}{V_0}$$

$$V_f = (1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz})$$

Infinitésimos de orden superior

$$V_f = 1+\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}+\varepsilon_{xx}\varepsilon_{yy}+\varepsilon_{yy}\varepsilon_{zz}+\varepsilon_{zz}\varepsilon_{xx}+\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz}$$

$$\varepsilon_v = \frac{V_f - V_0}{V_0} = \frac{(1+\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz})-1}{1} = \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$$

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_x - \mu\sigma_y - \mu\sigma_z)$$

$$\varepsilon_{yy} = \frac{1}{E}(-\mu\sigma_x + \sigma_y - \mu\sigma_z)$$

$$\varepsilon_{zz} = \frac{1}{E}(-\mu\sigma_x - \mu\sigma_y + \sigma_z)$$

$$\varepsilon_v = \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz} = \frac{1-2\mu}{E}(\sigma_x+\sigma_y+\sigma_z)$$

Si $\sigma_x = \sigma_y = \sigma_z = p \rangle 0 \quad \Rightarrow \varepsilon_V \rangle 0 \quad \Rightarrow 1\text{-}2\mu \rangle 0 \qquad \therefore \qquad \mu \langle 0{,}5$

Relación Tensión - Deformación

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_x - \mu\sigma_y - \mu\sigma_{z_z}\right) \qquad \varepsilon_{xy} = \frac{\gamma_{xy}}{2} = \frac{\tau_{xy}}{2G}$$

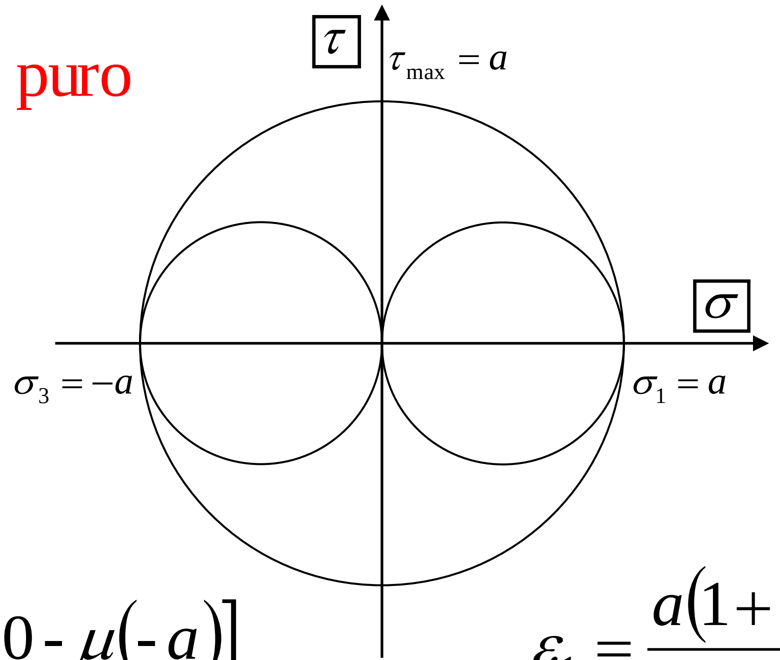
$$\varepsilon_{yy} = \frac{1}{E}\left(\sigma_y - \mu\sigma_x - \mu\sigma_{z_z}\right) \qquad \varepsilon_{yz} = \frac{\gamma_{yz}}{2} = \frac{\tau_{yz}}{2G}$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_z - \mu\sigma_x - \mu\sigma_{y_z}\right) \qquad \varepsilon_{zx} = \frac{\gamma_{zx}}{2} = \frac{\tau_{zx}}{2G}$$

$$\varepsilon_V = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{1-2\mu}{E}\left(\sigma_x + \sigma_y + \sigma_{z_z}\right) = \frac{1-2\mu}{E} I_{\sigma 1}$$

Relación entre E G y μ

Estado de corte o resbalamiento puro

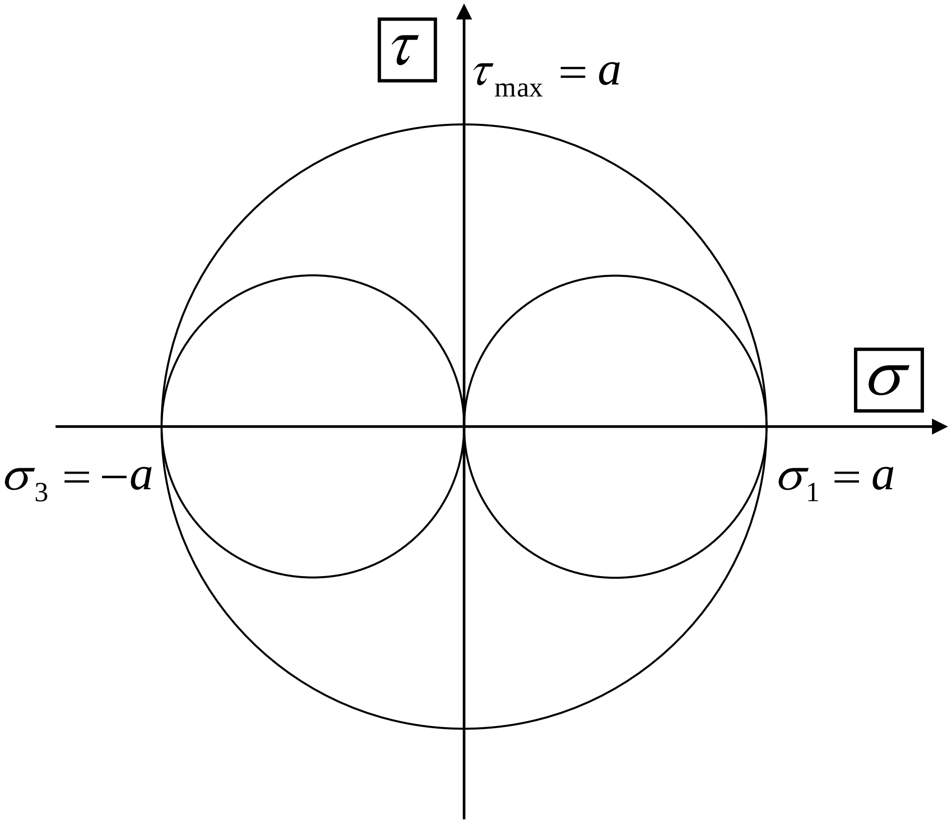


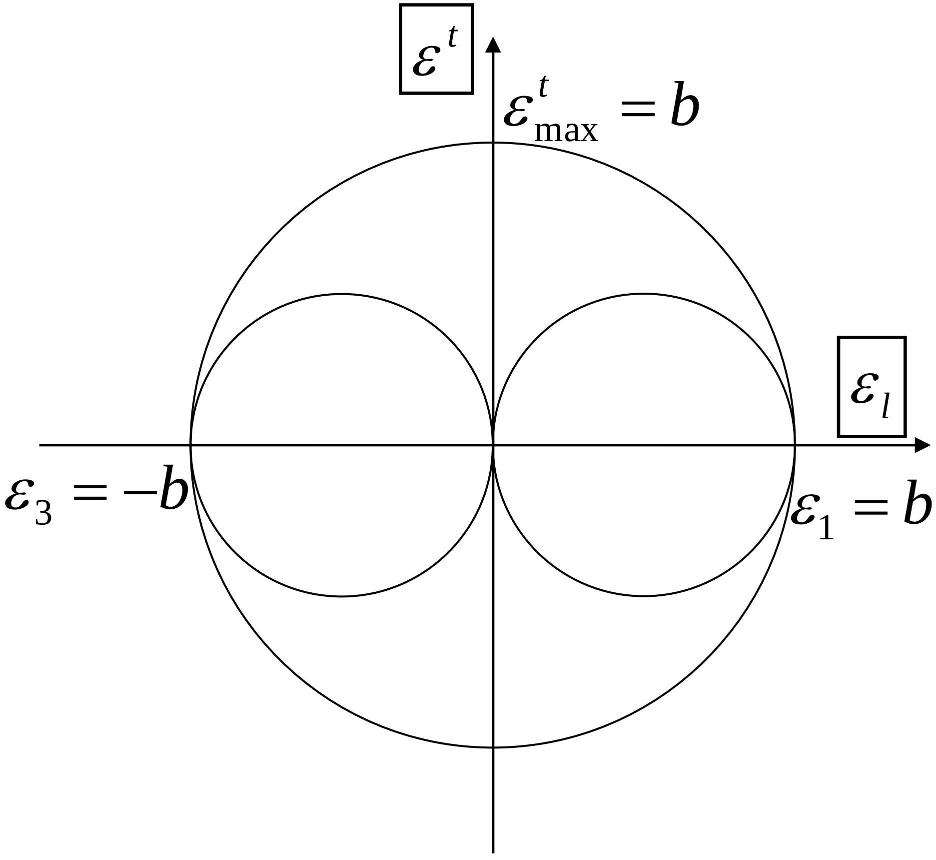
$$\sigma_1 = a \quad \sigma_2 = 0 \quad \sigma_3 = -a$$

$$\varepsilon_1 = \frac{1}{E}\left(\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right) = \frac{1}{E}\left[a - \mu\,0 - \mu(-a)\right] \qquad \varepsilon_1 = \frac{a(1+\mu)}{E}$$

$$\varepsilon_2 = \frac{1}{E}\left(\sigma_2 - \mu\sigma_1 - \mu\sigma_2\right) = \frac{1}{E}\left[0 - \mu\,a - \mu(-a)\right] \qquad \varepsilon_2 = 0$$

$$\varepsilon_3 = \frac{1}{E}\left(\sigma_3 - \mu\sigma_1 - \mu\sigma_2\right) = \frac{1}{E}\left[-a - \mu\,a - \mu\,0\right] \qquad \varepsilon_3 = -\frac{a(1+\mu)}{E}$$





$$\varepsilon^t_{max} = \frac{a}{2G} = b = \varepsilon_1 = \frac{a(1+\mu)}{E}$$

$$\frac{a}{2G} = \frac{a(1+\mu)}{E}$$

$$E = 2G(1+\mu)$$

$$G = \frac{E}{2(1+\mu)}$$