

09 - RELACIÓN ENTRE CONSTANTES ELÁSTICAS:

viernes, 9 de julio de 2021 10:26

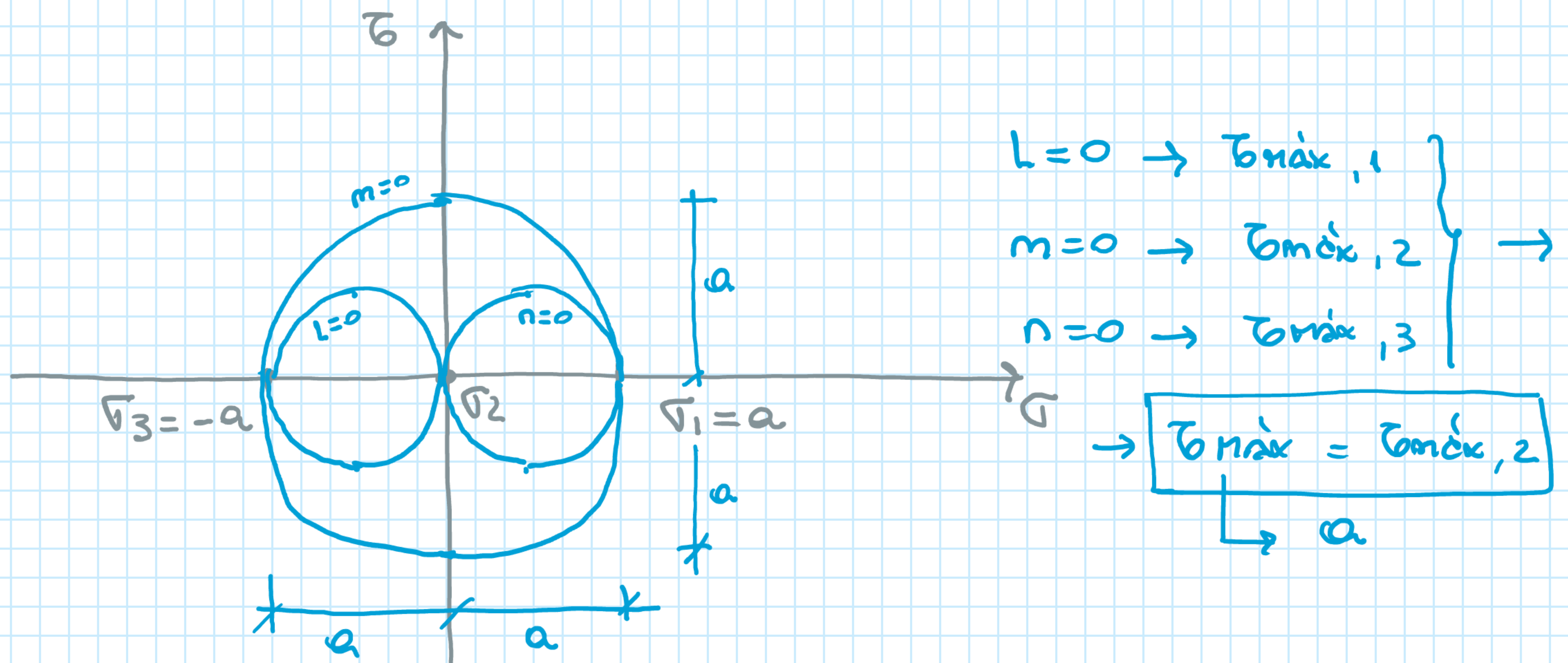
• MATERIAL ELÁSTICO E ISÓTROPICO \rightarrow 3 CRES PROPIAS DEL MATERIAL.

$$E ; G ; \mu.$$

• ESTUDIO \rightarrow TERNA PRINCIPAL. \rightarrow ESTADO PARTICULAR DE TENSIONES.

$$\begin{cases} \sigma_1 = a \\ \sigma_2 = 0 \\ \sigma_3 = -a \end{cases}$$

• ESTE ESTADO DE TENSION \rightarrow CIRCUNF. DE MOHR.



$$\epsilon_1 = \frac{1}{E} [\sigma_1 - \mu(\sigma_2 + \sigma_3)] = \frac{1}{E} [a - \mu(0 - a)] = \frac{1 + \mu}{E} \cdot a \quad \checkmark$$

$$\epsilon_2 = \frac{1}{E} [\sigma_2 - \mu(\sigma_1 + \sigma_3)] = \frac{1}{E} [0 - \mu(a - a)] = 0 \quad \checkmark$$

$$\epsilon_3 = \frac{1}{E} [\sigma_3 - \mu(\sigma_1 + \sigma_2)] = \frac{1}{E} [-a - \mu(a + 0)] = -\frac{(1 + \mu)}{E} a \quad \checkmark$$

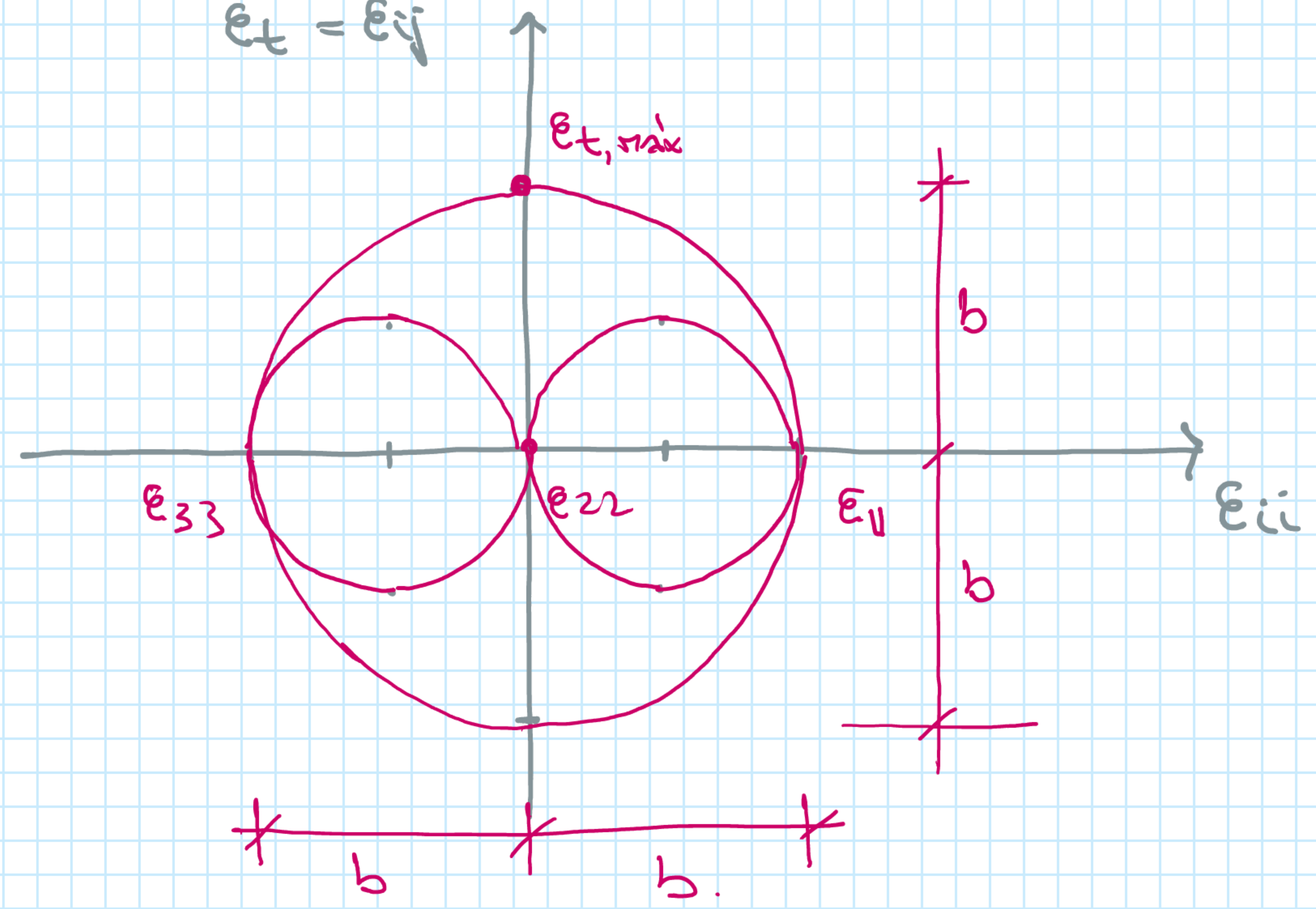
$$\tau_{\max} = a \rightarrow \epsilon_{t,\max} = \frac{\tau_{\max}}{2G} = \frac{a}{2G} = b$$

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• REPRESENTAR EN ϵ_{ij} CORRESPONDIENTE AL ϵ_{ij} PARTICULAR ANTERIOR.

$\epsilon_t = \epsilon_{ij}$



$\epsilon_{11} = \frac{1+\mu}{E} \cdot a = b$; $\epsilon_{t,max} = \frac{a}{2G} = b$
 $\rightarrow \frac{(1+\mu) \cdot a}{E} = \frac{a}{2G}$

$E = (1+\mu) 2G = 2(1+\mu) G$
 $G = \frac{E}{2(1+\mu)}$
 $\mu = \frac{E}{2G} - 1$

$\epsilon_{ij} = \frac{\tau_{ij}}{2G} = \frac{\tau_{ij}}{2 \cdot \frac{E}{2(1+\mu)}}$

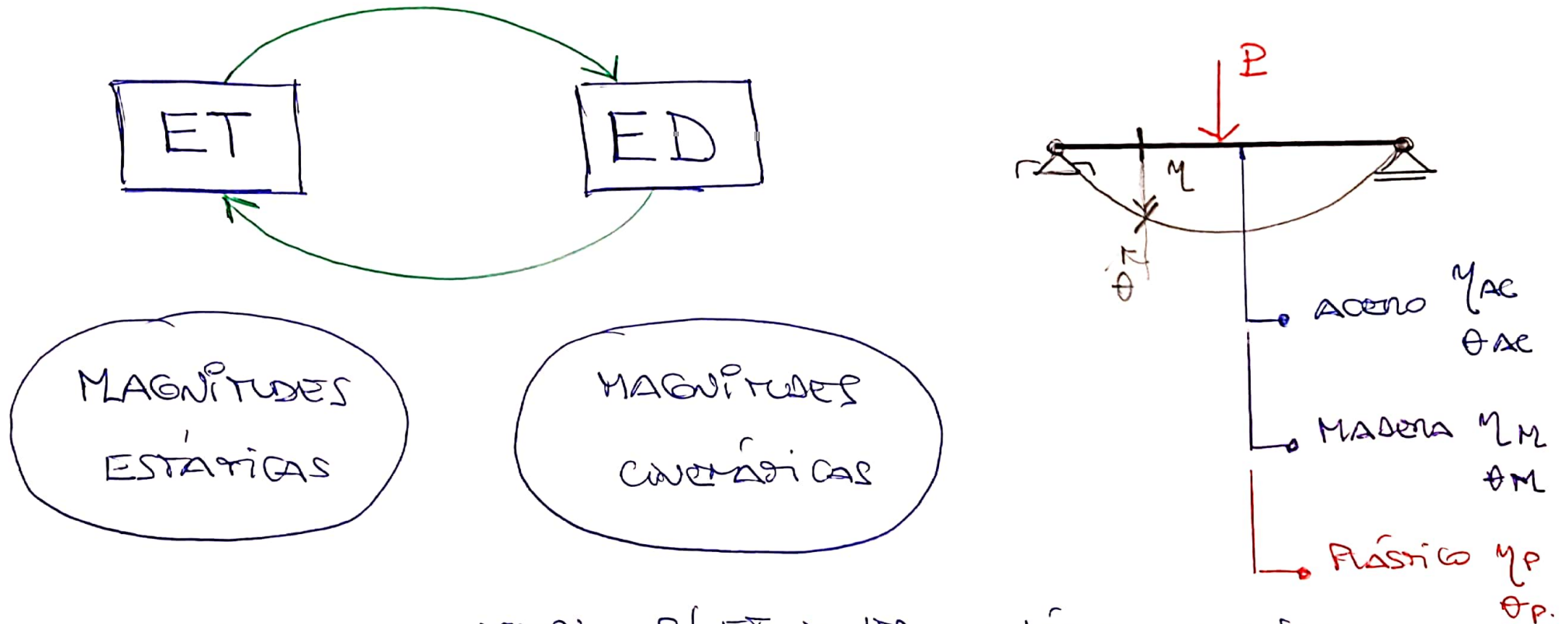
$\epsilon_{ij} = \frac{1+\mu}{E} \cdot \tau_{ij}$

10 - LEY GENERALIZADA DE HOOKE:

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$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\mu & -\mu & 0 & 0 & 0 \\ -\mu & 1 & -\mu & 0 & 0 & 0 \\ -\mu & -\mu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & (1+\mu) & 0 \\ 0 & 0 & 0 & 0 & 0 & (1+\mu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

RELACIONES ENTRE TENSIONES Y DEFORMACIONES:



• EL CAMINO DE ESTUDIO P/ ET y ED → LÓGICO - ANALÍTICO -
 - DEDUCTIVO → **ACÁ NO VA A SER APLICABLE**

• EL CAMINO A SEGUIR P/ RTyD →

- EXPERIMENTAL
- LÓGICO
- ANALÍTICO
- DEDUCTIVO

$[TT] \rightarrow 3 \times 3 \rightarrow 6$ simétricas.

$[TD] \rightarrow 3 \times 3 \rightarrow 6$ "

$$\boxed{\{\bar{\epsilon}_d\} = [B] \{\bar{\sigma}\}}$$

\uparrow \uparrow \uparrow
 $[TD]$ 6×6 $[TT]$
 6×1 6×1

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{bmatrix} = \begin{bmatrix} Q_{11} & \dots & Q_{16} \\ \vdots & \ddots & \vdots \\ Q_{61} & \dots & Q_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

6×1 6×6 6×1

VAMOS A PASSAR

- MATERIAL \rightarrow ISOTRÓPICO
 \rightarrow HOMOGENEO
- PERÍODO \rightarrow ELÁSTICO

• DE LA REALIZACIÓN DE LOS ENSAYOS Y EXPERIMENTOS OBSERVADOS:

I) → POR ESTAR DENTRO DEL PERIODO ELÁSTICO → SE CUMPLE * LEY DE BETTI " → LEY DE RECÍPROCIDAD:

$$\boxed{a_{ij} = a_{ji}} \rightarrow [B] \rightarrow \text{SIMÉTRICA.}$$

II) → EXISTE INDEPENDENCIA ENTRE:

$\underbrace{\epsilon_{ii} \text{ y } \sigma_{ij}}_{\text{DISOCIADAS}} \leftarrow \rightarrow \text{LAS DEFORM. ESPECÍFICAS LONGITUDINALES y LAS TENSIONES '}\sigma\text{'}$

$\underbrace{\epsilon_{ij} \text{ y } \sigma_{ii}}_{\text{DISOCIADAS.}} \leftarrow \rightarrow \text{LAS DEFORM. ESPECÍFICAS TRANSV. y LAS TENSIONES '}\sigma\text{'}$

• LA EXPRESIÓN MATEMÁTICA PUEDE SIMPLIFICARSE:

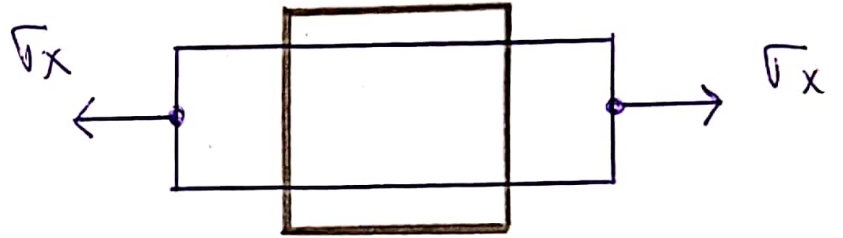
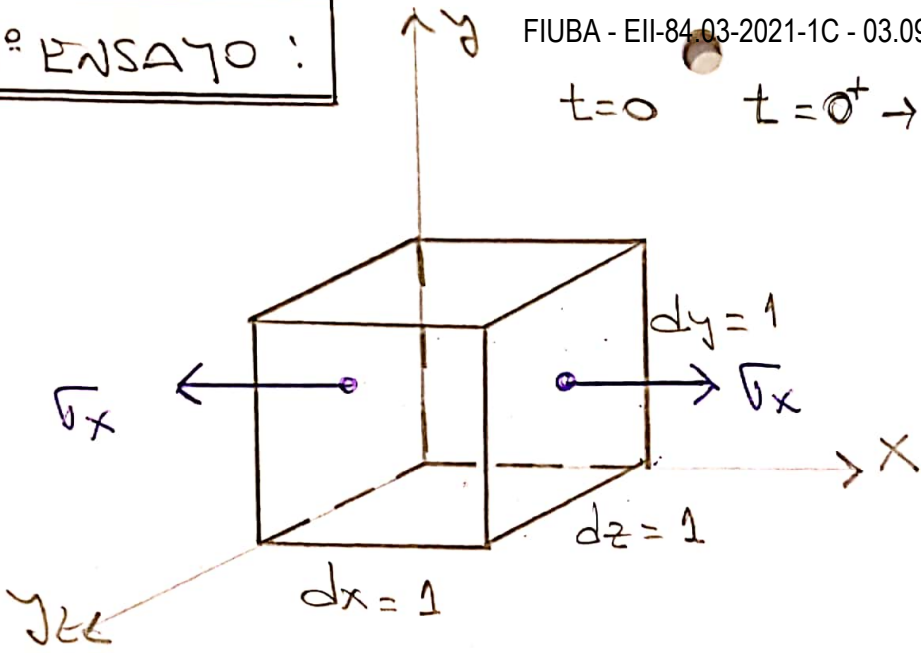
$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \dots \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & & & & \\ a_{21} & a_{22} & a_{23} & & & & \\ a_{31} & a_{32} & a_{33} & & & & \\ \hline & & & a_{44} & a_{45} & a_{46} \\ & & & a_{54} & a_{55} & a_{56} \\ & & & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \dots \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

$\{\bar{\epsilon}_d\} = [B] \{\bar{\sigma}\}$
 MATRICES CONSTITUTIVAS
 $\{\bar{\sigma}\} = [E] \{\bar{\epsilon}_d\}$

→ [B] y [E] → MATRICES DE ELASTICIDAD.

1º ENSAYO :

$t=0$ $t=0^+ \rightarrow$ APUNTO FIBAS



$$\epsilon_{ii} = \frac{\Delta L_{ii}}{L_{0,i=1}} \rightarrow \epsilon_{ii} = \Delta L_{ii}$$

1º CASO: $\sigma_x \neq 0$ $\sigma_y = \sigma_z = 0$

$$\epsilon_{xx} = \frac{\sigma_x}{E}$$

$$\epsilon_{yy} = -\mu_{yx} \epsilon_{xx} = -\mu_{yx} \frac{\sigma_x}{E}$$

$$\epsilon_{zz} = -\mu_{zx} \epsilon_{xx} = -\mu_{zx} \frac{\sigma_x}{E}$$

2º CASO: $\sigma_y \neq 0$ $\sigma_x = \sigma_z = 0$

$$\epsilon_{yy} = \frac{\sigma_y}{E}$$

$$\epsilon_{xx} = -\mu_{xy} \epsilon_{yy} = -\mu_{xy} \frac{\sigma_y}{E}$$

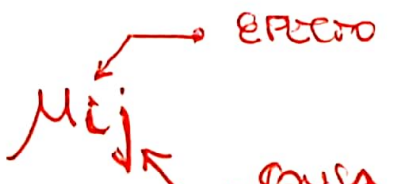
$$\epsilon_{zz} = -\mu_{zy} \epsilon_{yy} = -\mu_{zy} \frac{\sigma_y}{E}$$

3º CASO: $\sigma_z \neq 0$ $\sigma_x = \sigma_y = 0$

$$\epsilon_{zz} = \frac{\sigma_z}{E}$$

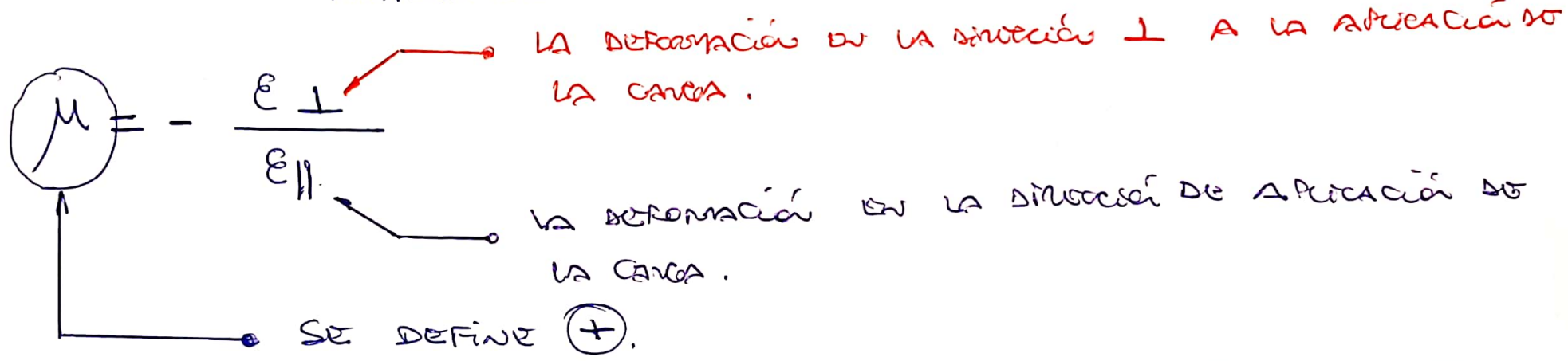
$$\epsilon_{xx} = -\mu_{xz} \epsilon_{zz} = -\mu_{xz} \frac{\sigma_z}{E}$$

$$\epsilon_{yy} = -\mu_{yz} \epsilon_{zz} = -\mu_{yz} \frac{\sigma_z}{E}$$



$\mu_{yx} = \mu_{xy} = \mu_{zx} = \mu_{xz} = \mu_{yz} = \mu_{zy} = \mu$ COEFICIENTE DE POISSON.

MATERIALES ISÓTROPOS



Qué PASARÍA si APLICAMOS $\sigma_x \neq 0$ $\sigma_y \neq 0$ $\sigma_z \neq 0$ EN FORMA SIMULTÁNEA.

→ APLICAMOS PSE POR LA RELACIÓN LINEAL ~~ENTRE~~ LAS VARIABLES:

$$\epsilon_{xx} = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \frac{\mu}{E} \sigma_z = \frac{1}{E} [\sigma_x - \mu(\sigma_y + \sigma_z)]$$

$$\epsilon_{yy} = \frac{\sigma_y}{E} - \frac{\mu}{E} \sigma_x - \frac{\mu}{E} \sigma_z = \frac{1}{E} [\sigma_y - \mu(\sigma_x + \sigma_z)]$$

$$\epsilon_{zz} = \frac{\sigma_z}{E} - \frac{\mu}{E} \sigma_x - \frac{\mu}{E} \sigma_y = \frac{1}{E} [\sigma_z - \mu(\sigma_x + \sigma_y)]$$

LEY DE HOOKE GENERALIZADA

I) μ : ES UNA CARACTERÍSTICA O PROPIEDAD DEL MATERIAL.

II) EN ANISÓTROPOS Y ORTOTRÓPICOS \rightarrow LOS μ_{ij} ^{SON} $\neq \bullet$.

$$\mu_{yx} = - \frac{\epsilon_{17}}{\epsilon_{xx}}$$

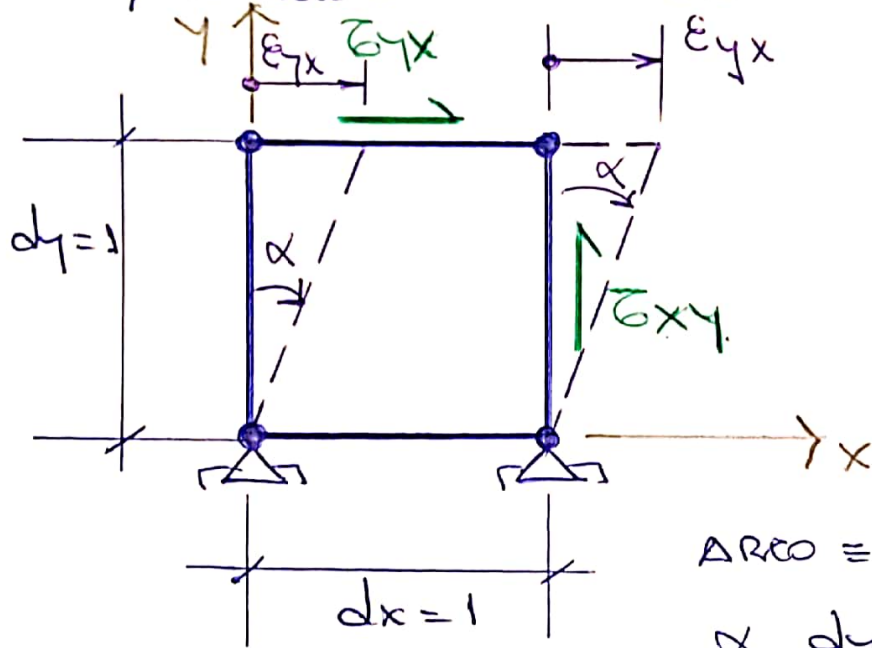
$$\mu_{zy} = - \frac{\epsilon_{22}}{\epsilon_{17}}$$

$$\mu_{ij} = - \frac{\epsilon_{ic}}{\epsilon_{ji}}$$

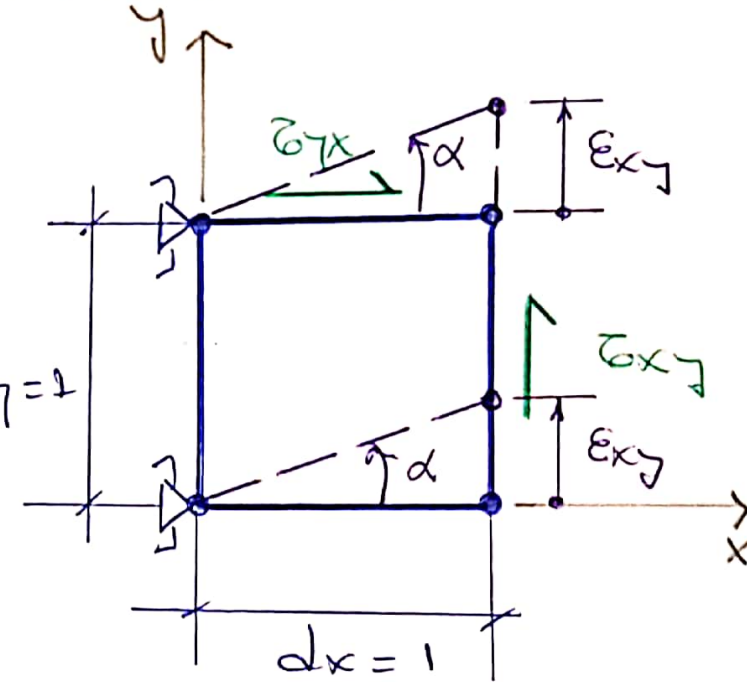
← EFECTO
← CAUSA.

2º ENSAYO:

↳ vincular a las tens. tangenciales con las deform. espec. transversales.



HIPOTESIS DE PEQUEÑOS DESPLAZAMIENTOS



ARCO \approx CUERVA \approx TANGENTE

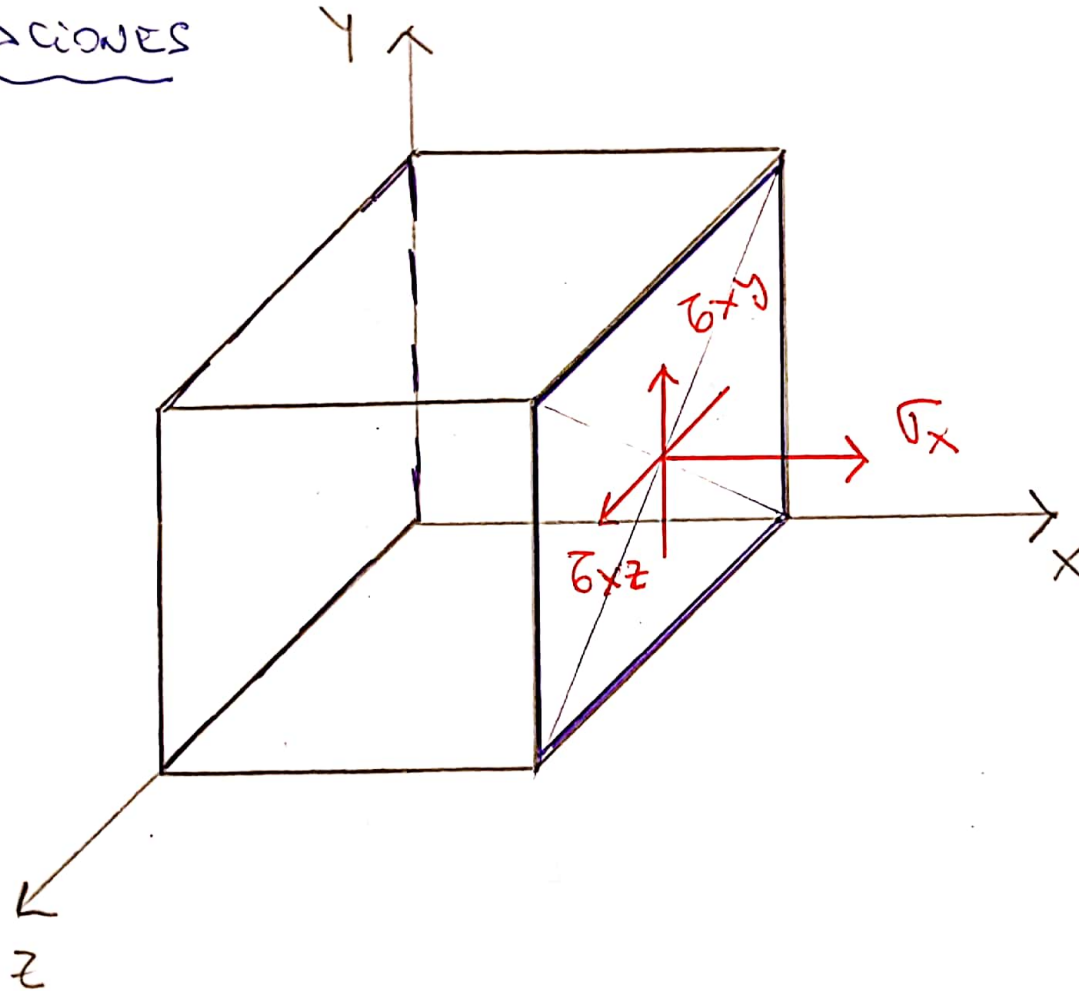
$$\alpha \cdot \underbrace{dy}_{=1} = \epsilon_{yx} \rightarrow \boxed{\alpha = \epsilon_{yx}} \quad (1)$$

ARCO \approx CUERVA \approx TANGENTE

$$\alpha \cdot \underbrace{dx}_{=1} = \epsilon_{xy} \rightarrow \boxed{\alpha = \epsilon_{xy}} \quad (2)$$

EL PLANO EN EL QUE ACTÚA σ_{ij} → LA DIRECCIÓN DE LA TENSIÓN σ .

ACLARACIONES



$\sigma_{ij} \rightarrow i=j \rightarrow$ normal
 $i \neq j \rightarrow$ tangencial

σ_{ij}

DE ① y ②.

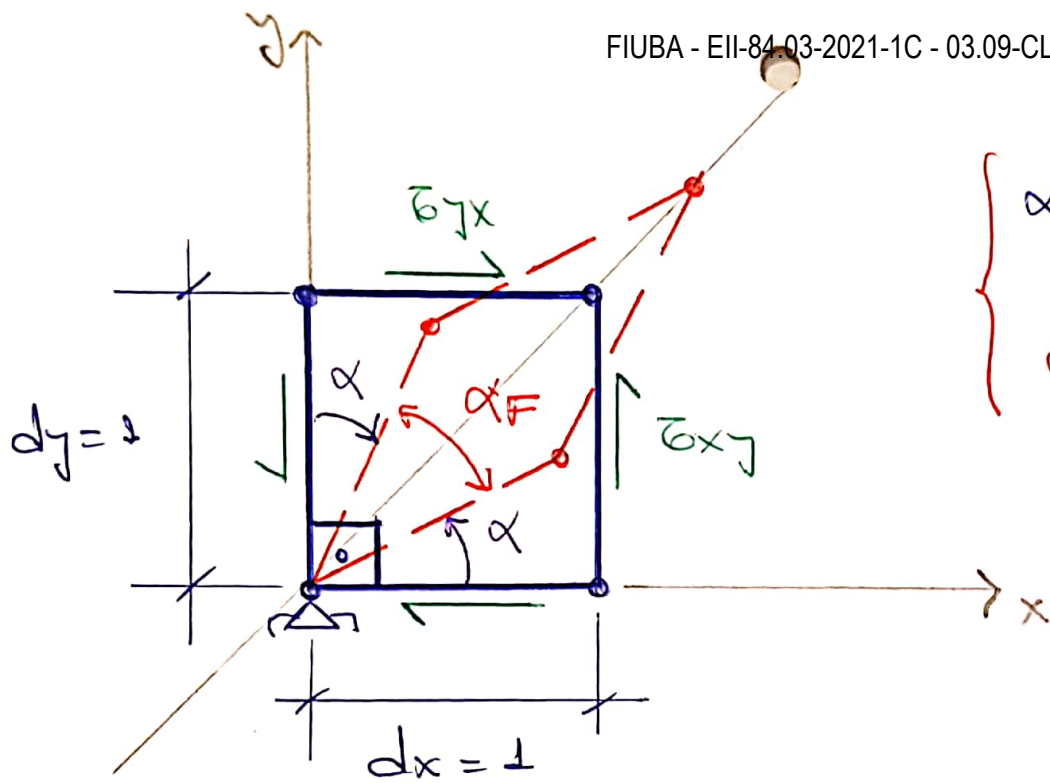
$$\left. \begin{array}{l} \alpha = E_{yx} \quad \textcircled{1} \\ \alpha = E_{xy} \quad \textcircled{2} \end{array} \right\} \rightarrow \boxed{E_{xy} = E_{yx}}$$

SE HACE LO MISMO PARA LOS PARES 'xz' e 'yz'

$$E_{xz} = E_{zx}$$

$$E_{yz} = E_{zy}$$

$$\underline{E_{ij} = E_{ji}}$$



$$\left\{ \begin{aligned} \alpha_{ORIGINAL, xy} &= \alpha_{ORIGINAL, xy} = \frac{\pi}{2} = 90^\circ \\ \alpha_{F, xy} & \end{aligned} \right.$$

$$\alpha_{INT.} - \alpha_F = \frac{\pi}{2} - \alpha_F = \gamma_{xy}$$

$$\gamma_{xy} = \alpha + \alpha = 2\alpha = 2\varepsilon_{xy}$$

$$\gamma_{xy} = 2\varepsilon_{xy} \rightarrow \varepsilon_{xy} = \frac{\gamma_{xy}}{2}$$

$$\boxed{\gamma_{ij} = 2\varepsilon_{ij}} \leftrightarrow \boxed{\varepsilon_{ij} = \frac{\gamma_{ij}}{2}}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 2\varepsilon_{xy}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = 2\varepsilon_{yz}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} = 2\varepsilon_{zx}$$

\leftrightarrow

$$\varepsilon_{xy} = \frac{\tau_{xy}}{2G} = \frac{\gamma_{xy}}{2}$$

\leftrightarrow

$$\varepsilon_{yz} = \frac{\tau_{yz}}{2G} = \frac{\gamma_{yz}}{2}$$

\leftrightarrow

$$\varepsilon_{zx} = \frac{\tau_{zx}}{2G} = \frac{\gamma_{zx}}{2}$$

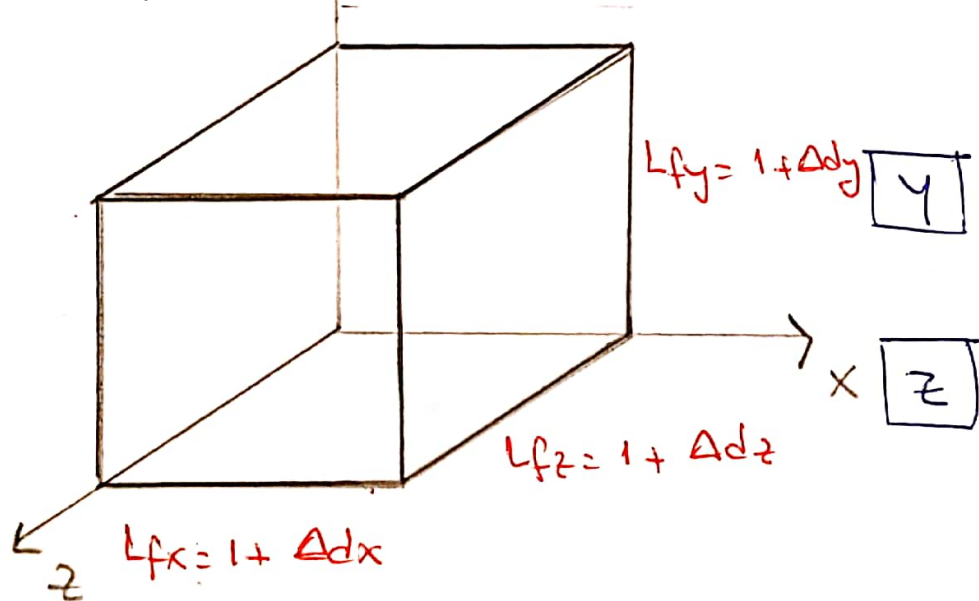
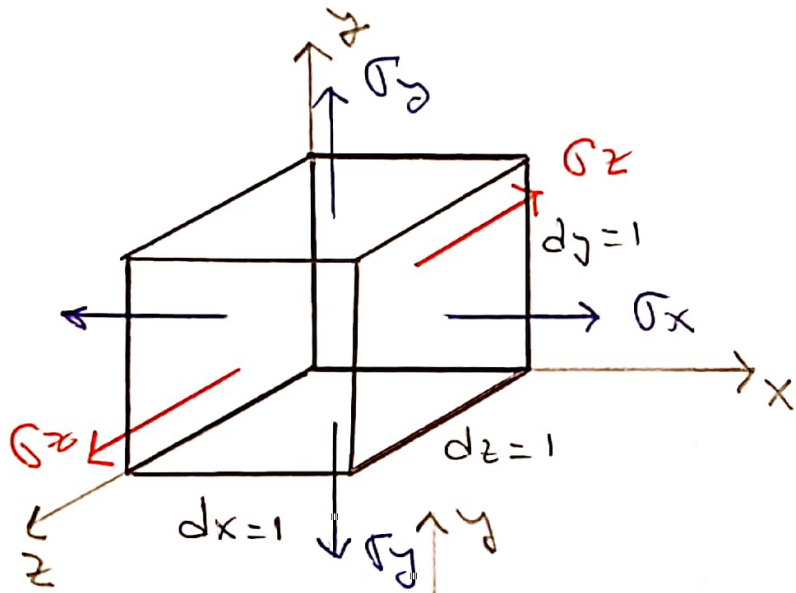
LEY DE HOOKE GENERALIZADA:

$$\begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{pmatrix} = \begin{bmatrix} 1/E & -\mu/E & -\mu/E & & & \\ -\mu/E & 1/E & -\mu/E & & & \\ -\mu/E & -\mu/E & 1/E & & & \\ & & & 1/2G & 0 & 0 \\ & & & 0 & 1/2G & 0 \\ & & & 0 & 0 & 1/2G \end{bmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix}$$

E : MÓDULO DE ELASTICIDAD LONGITUDINAL
 G : " " " " TRANSVERSAL.
 μ : COEF. DE POISSON

CONSTANTES ELÁSTICAS DEL MATERIAL.

VARIACION ESPECÍFICA DE VOLUMEN:



$$V_0 = dx dy dz = 1 \cdot 1 \cdot 1 = 1$$

[x]

$$L_{fx} = l_{0x} + \Delta dx = dx + \Delta dx = 1 + \Delta dx$$

$$\Delta L_x = L_{fx} - l_{0x} = L_{fx} - dx = 1 + \Delta dx - 1$$

$$\underline{\underline{\Delta L_x = \Delta dx}}$$

$$e_{xx} = \frac{\Delta L_x}{l_{0x}} = \frac{\Delta dx}{1} = \Delta dx$$

$$\underline{\underline{e_{xx} = \Delta dx}}$$

$$e_{yy} = \Delta dy$$

$$e_{zz} = \Delta dz$$

$$V_f = L_{fx} \cdot L_{fy} \cdot L_{fz} = (1 + \Delta dx)(1 + \Delta dy)(1 + \Delta dz) = (1 + \epsilon_{xx})(1 + \epsilon_{yy})(1 + \epsilon_{zz}) =$$

$$V_f = 1 + \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} + \cancel{\epsilon_{xx} \cdot \epsilon_{yy}} + \cancel{\epsilon_{xx} \cdot \epsilon_{zz}} + \cancel{\epsilon_{yy} \cdot \epsilon_{zz}} + \cancel{\epsilon_{xx} \cdot \epsilon_{yy} \cdot \epsilon_{zz}}$$

• APP. REQ. DESP. + MIP. REQ. DEFORMACIONES. → TÉRMINOS → INFINITESIMOS DE O.S.

$$\boxed{V_f = 1 + \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}}$$

$$\Delta V = V_f - V_0 = 1 + \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} - 1 \rightarrow \Delta V = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}.$$

$$\epsilon_V = \frac{\Delta V}{V_0} = \frac{\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}}{1} \rightarrow \boxed{\epsilon_V = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}}$$

TRAZA DE [TD].

$$\epsilon_V = \frac{1}{E} [\sigma_x - \mu(\sigma_y + \sigma_z)] + \frac{1}{E} [\sigma_y - \mu(\sigma_x + \sigma_z)] + \frac{1}{E} [\sigma_z - \mu(\sigma_x + \sigma_y)]$$

$$\epsilon_V = \frac{1}{E} [(\sigma_x - 2\mu\sigma_x) + (\sigma_y - 2\mu\sigma_y) + (\sigma_z - 2\mu\sigma_z)]$$

$$\epsilon_V = \frac{1}{E} [(1 - 2\mu)\sigma_x + (1 - 2\mu)\sigma_y + (1 - 2\mu)\sigma_z]$$

$$\epsilon_V = \frac{(1-2\mu)}{E} (\sigma_x + \sigma_y + \sigma_z) = \frac{1-2\mu}{E} \cdot I_1 [\text{TT}]$$

$$\epsilon_V = I_1 [\text{TD}] = \frac{1-2\mu}{E} I_1 [\text{TT}]$$

PARTICULARIDADES:

• supondremos que $\sigma_x = \sigma_y = \sigma_z = p > 0$

Ⓘ

$$\epsilon_V = \frac{(1-2\mu)}{E} \cdot 3p > 0$$

$$1-2\mu > 0 \rightarrow$$

$$\rightarrow \mu < \frac{1}{2} \rightarrow$$

$$\mu < 0,5$$

Ⓙ

$$\epsilon_V = \frac{1-2\mu}{E} \cdot 3p.$$

$$\epsilon_V = \frac{p}{E}$$

$$= \frac{p}{3(1-2\mu)}$$

K

$$\epsilon_V = \frac{p}{K} \rightarrow p = K \cdot \epsilon_V$$

$$K = \frac{E}{3(1-2\mu)}$$

$$\rightarrow \text{si } \mu \rightarrow 0,5 \rightarrow K \rightarrow \infty \rightarrow \epsilon_V \rightarrow 0$$

si $\mu \rightarrow 0,5 \rightarrow$ el material es cada vez, más rígido. en EL LÍMITE ~~cuando~~

$\mu = 0,5 \rightarrow$ el material ^{ES} infinitamente rígido.

→ Si $\mu \rightarrow 0 \rightarrow K \rightarrow \frac{E}{3} \rightarrow$

$$E_V = \frac{3p}{E}$$

→ Si $\mu \rightarrow -\infty \rightarrow K \rightarrow 0 \rightarrow$

$$E_V \rightarrow \infty$$

→ Módulo NULA.

$$0 \leq \mu < 0,5$$

→ AUMENTA LA RIGIDEZ

→ DISMINUYE LA AUTOMORFIA.

→ Si $\mu > 0,5 \rightarrow K < 0 \rightarrow$ UN MATERIAL MACIONADO DISMINUYE SU RIGIDEZ.

RELACION ENTRE E, G y μ :

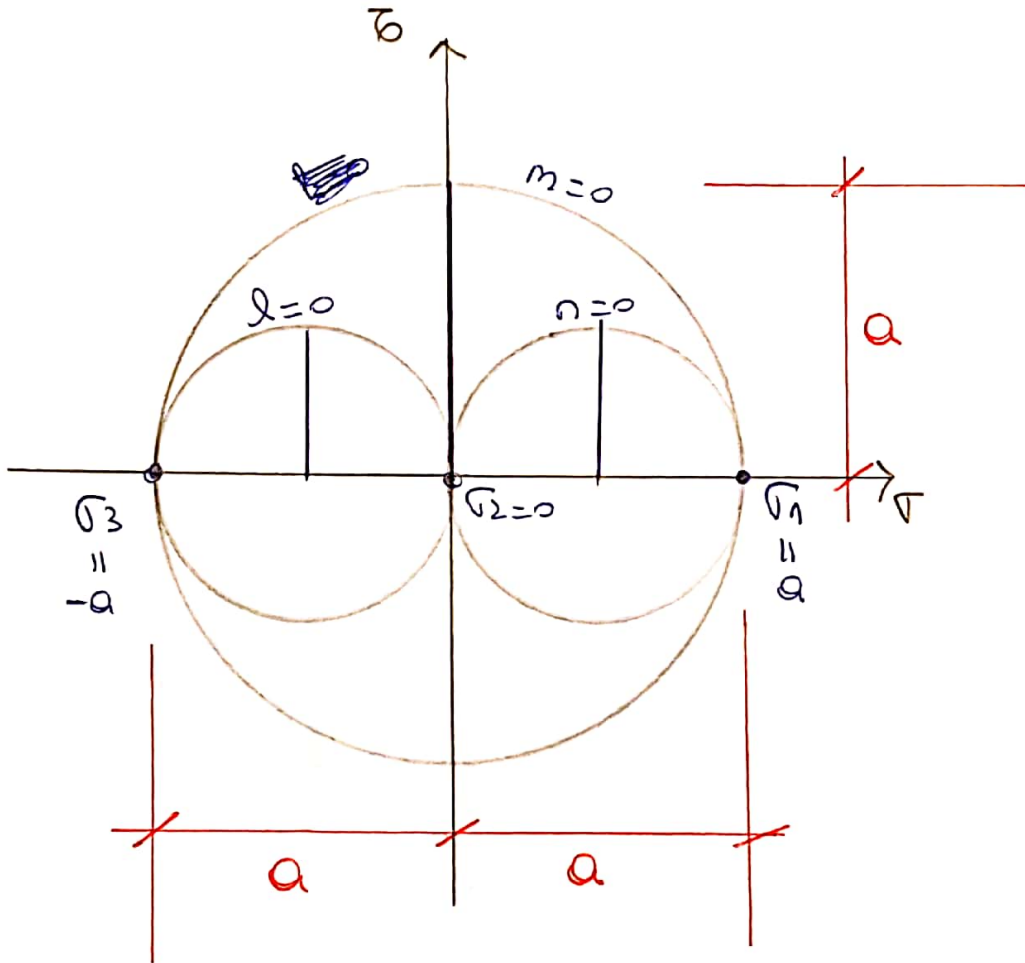
MAT. ISÓTROPAS → 3 ejes propios elásticos

$$\begin{Bmatrix} E \\ G \\ \mu \end{Bmatrix}$$

• VALORES A REALIZAR EL SIGUIENTE ESTUDIO:
→ FORMA PPAAL.

$$\begin{cases} \sigma_1 = a \\ \sigma_2 = 0 \\ \sigma_3 = -a \end{cases}$$

• MOSTRAMOS ESTA SITUACION A TRAVES DE LAS CIRCUNF. MOHR.



$$\epsilon_3 = \frac{1}{E} [\sigma_3 - \mu(\sigma_1 + \sigma_2)] = \frac{1}{E} [-a - \mu(a + 0)]$$

$$\epsilon_3 = -\frac{(1+\mu)a}{E}$$

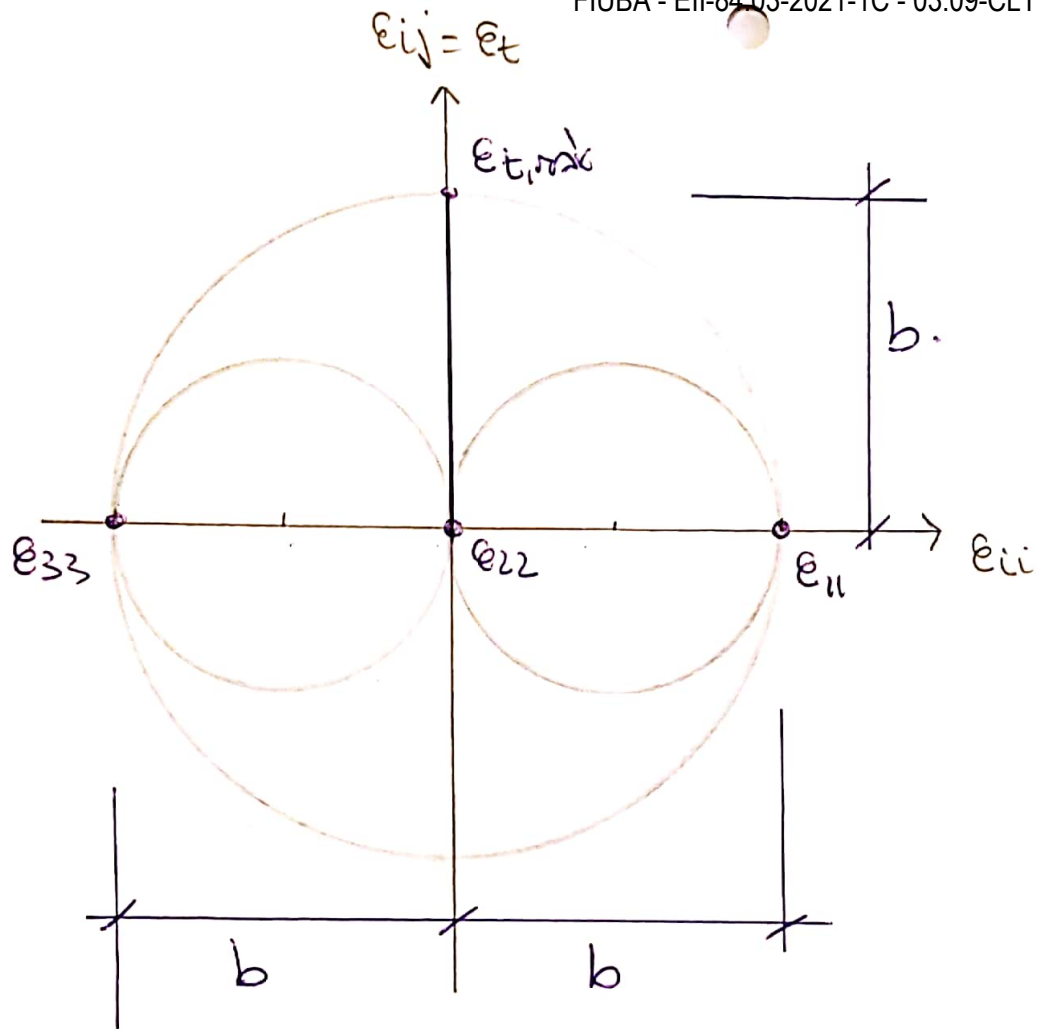
$$\sigma_{\max} = a \rightarrow \epsilon_{t,\max} = \frac{\sigma_{\max}}{2G}$$

$$\boxed{\epsilon_{t,\max} = \frac{a}{2G} = b}$$

$$\epsilon_1 = \frac{1}{E} [\sigma_1 - \mu(\sigma_2 + \sigma_3)] = \frac{1}{E} [a - \mu(0 - a)]$$

$$\epsilon_1 = \frac{1+\mu}{E} a$$

$$\epsilon_2 = \frac{1}{E} [\sigma_2 - \mu(\sigma_1 + \sigma_3)] = \frac{1}{E} [0 - \mu(a - a)] = 0$$



$$\frac{(1+\mu)\sigma}{E} = \frac{\sigma}{2G}$$

$$E = 2 \cdot (1+\mu) G$$

$$G = \frac{E}{2(1+\mu)}$$

$$\mu = \frac{E}{2G} - 1$$

$$\epsilon_{ij} = \frac{\sigma_{ij}}{2G} = \frac{\sigma_{ij}}{2 \cdot \frac{E}{2(1+\mu)}} = \frac{1+\mu}{E} \sigma_{ij}$$

$$\left. \begin{aligned} \epsilon_{11} &= \frac{(1+\mu)\sigma}{E} = b. \\ \epsilon_{t,max} &= \frac{\sigma}{2G} = b. \end{aligned} \right\} \begin{array}{l} \text{LAS} \\ \text{ICUALS.} \end{array}$$

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \dots \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{zx} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\mu & -\mu & & & \\ -\mu & 1 & -\mu & & & \\ -\mu & -\mu & 1 & & & \\ \hline & & & (1+\mu) & 0 & 0 \\ & & & 0 & (1+\mu) & 0 \\ & & & 0 & 0 & (1+\mu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$