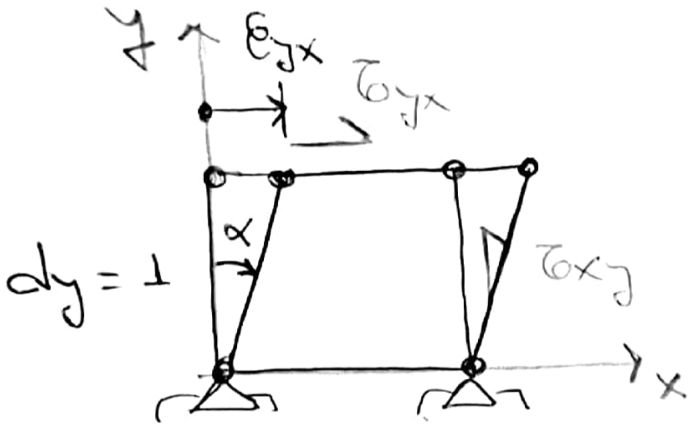
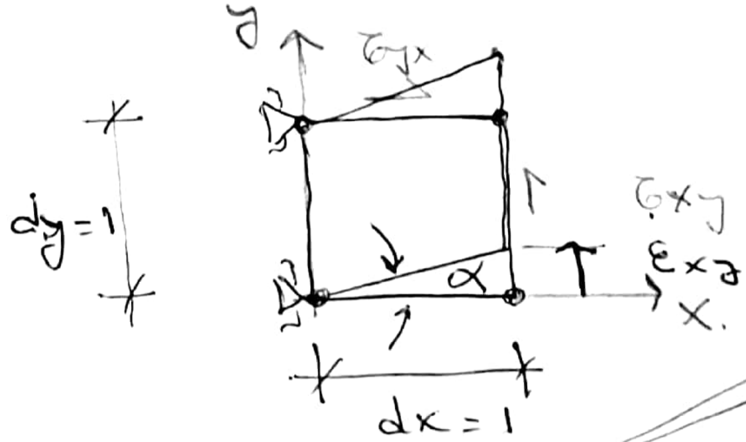
TENSIONES TANGENCIALES Y DEFORMACIONES TRANSVERSALES:

→ HIPÓTESIS DE PEQUEÑOS DESPLAZAMIENTOS.





$$\alpha \cdot \underbrace{dx}_{=1} = \varepsilon_{xy}$$

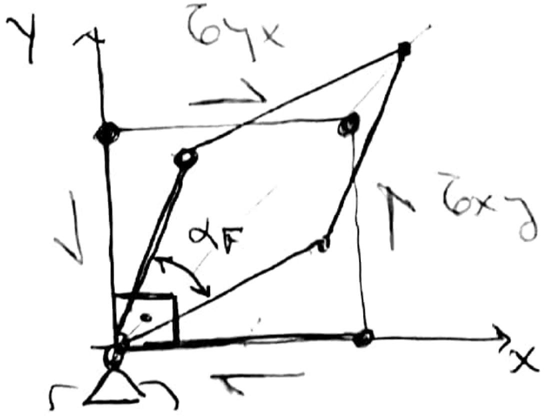
$$\boxed{\alpha = \varepsilon_{xy}}$$

ARCO ≡ CUERDA ≡ TANGENTE.

$$\alpha \cdot \underbrace{dy}_{=1} = \varepsilon_{yx} \rightarrow \boxed{\alpha = \varepsilon_{yx}}$$

$$\boxed{\varepsilon_{xy} = \varepsilon_{yx}.}$$

$$\boxed{\varepsilon_{ij} = \varepsilon_{ji}}$$



$$\left.\begin{array}{l}\alpha_{ORIGINAL,\,XY} = \frac{\pi}{2} = 90^\circ. \\ \hline \alpha_{FINAL}\end{array}\right\}$$

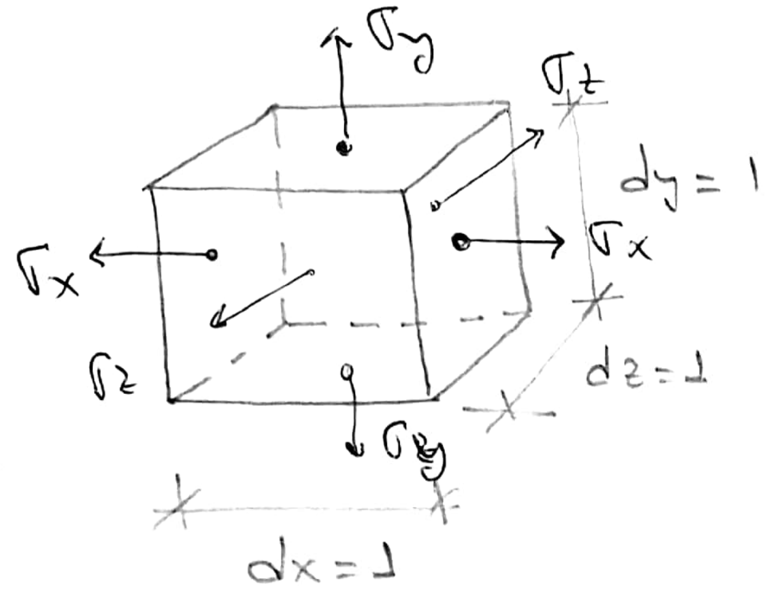
$$\alpha_{ORIG} - \alpha_{FINAL} = \frac{\pi}{2} - \alpha_{FINAL} = \gamma_{xy}$$

$$\gamma_{xy} = 2\,\varepsilon_{xy} = 2\,\varepsilon_{yx} = \gamma_{yx}.$$

$$\boxed{\begin{aligned} \gamma_{xy} &= \frac{\tau_{xy}}{G} = 2\,\varepsilon_{xy}. \\ \gamma_{yz} &= \frac{\tau_{yz}}{G} = 2\,\varepsilon_{yz} \\ \gamma_{zx} &= \frac{\tau_{zx}}{G} = 2\,\varepsilon_{zx} \end{aligned}} \begin{aligned} &\rightarrow \\ &\rightarrow \\ &\rightarrow \end{aligned} \boxed{\begin{aligned} \varepsilon_{xy} &= \frac{\tau_{xy}}{2G} \\ \varepsilon_{yz} &= \frac{\tau_{yz}}{2G}. \\ \varepsilon_{zx} &= \frac{\tau_{zx}}{2G} \end{aligned}}$$

$$\left\{\begin{array}{c} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \hdashline \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{array}\right\} = \left[\begin{array}{ccc|ccc} 1/E & -\mu/E & -\mu/E & & & \\ -\mu/E & 1/E & -\mu/E & & 0 & \\ -\mu/E & -\mu/E & 1/E & & & \\ \hline & & & 1/2G & 0 & 0 \\ & 0 & & 0 & 1/2G & 0 \\ & & & 0 & 0 & 1/2G \end{array}\right] \left\{\begin{array}{c} \sigma_x \\ \sigma_y \\ \sigma_z \\ \hdashline \tau_{xy}. \\ \tau_{yz} \\ \tau_{zx} \end{array}\right\}$$

VARIACIÓN ESPECÍFICA DE VOLUMEN:



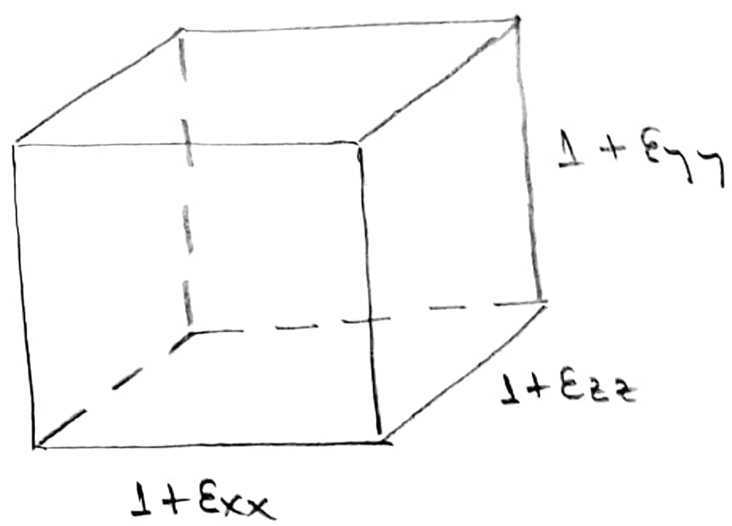
$$V_0 = dx \cdot dy \cdot dz = 1 \cdot 1 \cdot 1 = 1.$$

$$L_{fx} = \underbrace{dx}_{L_{0x} = 1} + \Delta dx = 1 + \Delta dx$$

$$\frac{L_{fx} - L_{0x}}{L_{0x}} = \frac{\Delta dx}{1} = \varepsilon_{xx} \rightarrow$$

$$\boxed{\varepsilon_{xx} = \Delta dx}$$

$$\boxed{\begin{array}{c} \varepsilon_{yy} = \Delta dy \\ \hline \varepsilon_{zz} = \Delta dz \end{array}}$$



$$V_f = L_{fx} \cdot L_{fy} \cdot L_{fz} = (1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz}).$$

$$V_f = 1 + \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} + \underbrace{\cancel{\varepsilon_{xx}\cdot\varepsilon_{yy}} + \cancel{\varepsilon_{xx}\cdot\varepsilon_{zz}} + \cancel{\varepsilon_{yy}\,\varepsilon_{zz}} + \cancel{\varepsilon_{xx}\,\varepsilon_{yy}\,\varepsilon_{zz}}}_{\text{SON INFINITÉSIMOS DE ORDEN SUPERIOR FRENTE A LOS } \varepsilon_{ii}}.$$

$$V_f = 1 + \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}.$$

$$\varepsilon_V = \frac{\Delta V}{V_0} = \frac{V_f - V_0}{V_0} = \frac{(1+\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz})-1}{1} = \boxed{\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}}$$

$$\varepsilon_V = \frac{1}{E}\left[(\sigma_x - \mu\sigma_y - \mu\sigma_z) + (\sigma_y - \mu\sigma_x - \mu\sigma_z) + (\sigma_z - \mu\sigma_y - \mu\sigma_x)\right] =$$

$$\boxed{\varepsilon_V = \frac{(1-2\mu)}{E}(\sigma_x+\sigma_y+\sigma_z)}$$

$$\boxed{\begin{aligned}\varepsilon_V &= I_{1,\varepsilon} \\ \varepsilon_V &= \frac{(1-2\mu)}{E} I_{1,\sigma}\end{aligned}}$$

Si $\sigma_x = \sigma_y = \sigma_z = p > 0$

$$\varepsilon_V = \frac{(1-2\mu)}{E} \underbrace{(\sigma_x + \sigma_y + \sigma_z)}_{3p} > 0$$

$$\varepsilon_V = \frac{3(1-2\mu)}{E} p > 0.$$

$$1 - 2\mu > 0 \rightarrow 1 > 2\mu \rightarrow$$

$$\rightarrow \boxed{\mu < 0{,}5} \quad \text{I}$$

$$\varepsilon_V = \frac{p}{\frac{E}{3(1-2\mu)}} = \frac{p}{K} > 0$$

$$\boxed{p = K\,\varepsilon_V} \qquad \boxed{K = \frac{E}{3(1-2\mu)}}$$

$\rightarrow$ si $\mu \rightarrow 0{,}5 \rightarrow K \rightarrow \infty \rightarrow$

$\rightarrow \boxed{\varepsilon_V \rightarrow 0}$

MATERIAL INCOMPRESIBLE [INFINITAMENTE RÍGIDO]

$\rightarrow$ si $\mu \rightarrow 0{,}0 \rightarrow K = \frac{E}{3} \rightarrow$

$\rightarrow \boxed{\varepsilon_V = \frac{3p}{E}}$

$\rightarrow$ si $\mu \rightarrow -\infty \rightarrow K \rightarrow 0 \rightarrow$

$\rightarrow \boxed{\varepsilon_V \rightarrow \infty}$

$\rightarrow$ RIGIDEZ NULA.

$\rightarrow$ si $\mu > 0{,}5 \rightarrow K < 0 \rightarrow$ UN MATERIAL TRACCIONADO DISMINUIRÍA SU VOLUMEN.

RELACIÓN ENTRE E, G y μ:

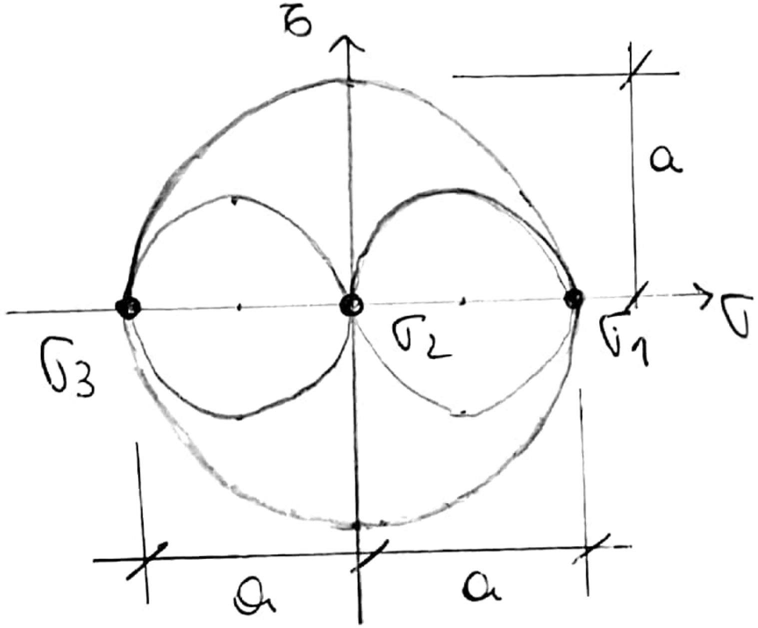
$\boxed{\text{MATERIAL}}$ → 3 CONSTANTES PROPIAS

E: MÓDULO ELASTICIDAD LONGITUDINAL

G: " " TRANSVERSAL

μ: COEFICIENTE DE POISSON

$$\sigma_1 = a$$
$$\sigma_2 = 0$$
$$\sigma_3 = -a$$



$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right] =$$

$$= \frac{1}{E}\left[a - 0 + \mu a\right] = \boxed{\frac{(1+\mu)}{E}a}$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1 - \mu\sigma_3\right] =$$

$$= \frac{1}{E}\left[0 - \mu \cdot a - \mu(-a)\right] = \boxed{0.}$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \mu\sigma_1 - \mu\sigma_2\right] =$$

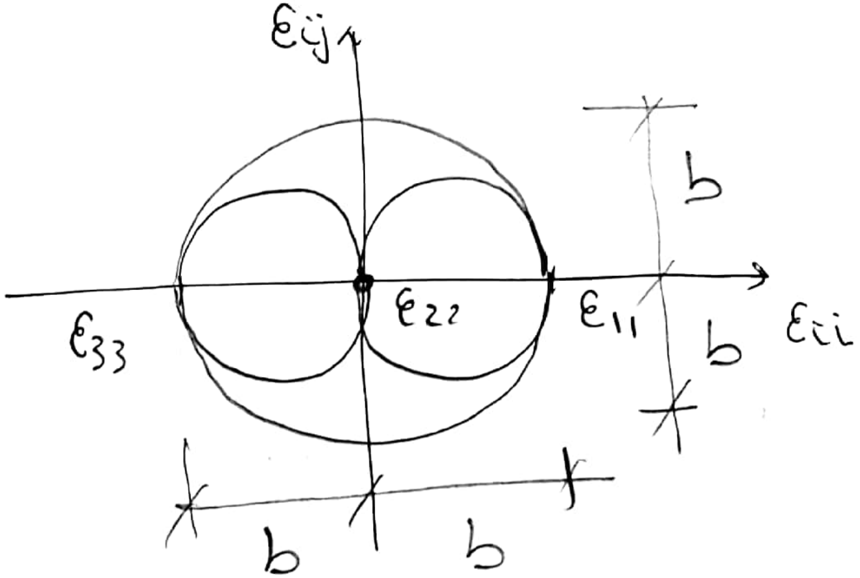
$$= \frac{1}{E}\left[-a - \mu a - 0\right] =$$

$$= \boxed{-\frac{(1+\mu)}{E}a}$$

$$\varepsilon_{t,máx} = \frac{a}{2G} = b$$

$$\left.\begin{array}{l} \varepsilon_1 = \dfrac{(1+\mu)}{E} a \\ \varepsilon_2 = 0. \end{array}\right.$$

$$\left.\begin{array}{l} \varepsilon_3 = -\dfrac{(1+\mu)}{E} a \\ \varepsilon^{t}_{máx} = \dfrac{a}{2G} = b. \end{array}\right.$$



$$\varepsilon_{11} = \varepsilon_{t,máx}$$

$$\frac{(1+\mu)}{E} \cdot \cancel{a} = \frac{\cancel{a}}{2G} \longrightarrow \boxed{G = \frac{E}{2(1+\mu)}}$$

$$\boxed{E = 2(1+\mu)\,G}$$

$$\boxed{\mu = \frac{E}{2G} - 1}$$

$$\tau \longleftrightarrow \varepsilon_{ij} = \varepsilon_t$$

$$\downarrow \qquad\qquad \downarrow$$

$$\tau_{máx} = a \qquad\qquad \varepsilon_{t,máx}.$$

$$\tau = 2G\,\varepsilon_{ij}$$

$$\varepsilon_{ij} = \frac{\tau}{2G}$$