RELACIONES ENTRE TENSIONES Y DEFORMACIONES:

$$ET \rightleftarrows ED$$

MAGNITUDES ESTÁTICAS

MAGNITUDES CINEMÁTICAS

$$\underbrace{\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix}}_{6\times 1} = \underbrace{\begin{bmatrix} a_{11} & \cdots & a_{16} \\ \vdots & & \vdots \\ a_{61} & \cdots & a_{66} \end{bmatrix}}_{6\times 6} \underbrace{\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}}_{6\times 1}$$

- MATERIAL HOMOGÉNEO
- " ISÓTROPO
- PERÍODO ELÁSTICO

SE OBSERVAN LOS SIGUIENTES ASPECTOS:

I) POR SER <u>ELÁSTICO</u> →

→ SE CUMPLE LA LEY DE RECIPROCIDAD DE BETTY

$$a_{ij} = a_{ji}$$

II) EXISTE INDEPENDENCIA ENTRE:

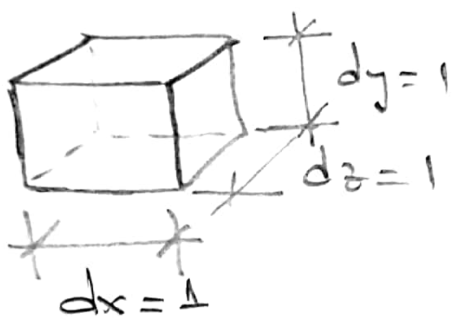
→ DEFORM. LONGITUDINALES Y TENS. TANGENCIALES.

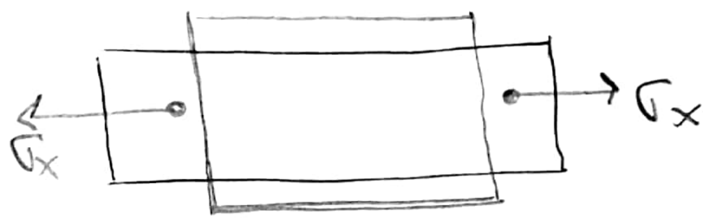
→ DEFORM. TRANSVERSALES Y TENS. NORMALES.

2|4

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & & & \\ a_{21} & a_{22} & a_{23} & & 0 & \\ a_{31} & a_{32} & a_{33} & & & \\ \hline & & & a_{44} & a_{45} & a_{46} \\ & 0 & & a_{54} & a_{55} & a_{56} \\ & & & a_{64} & a_{65} & a_{66} \end{array}\right] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \hline \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

1º ENSAYO:





I) CASO: $\sigma_x \neq 0$ $\quad \sigma_y = 0$ $\quad \sigma_z = 0$

$$\varepsilon_{xx} = \frac{\sigma_x}{E}$$

$$\varepsilon_{yy} = -\mu\, \varepsilon_{xx} = -\frac{\mu}{E}\sigma_x$$

$$\varepsilon_{zz} = -\mu\, \varepsilon_{xx} = -\frac{\mu}{E}\sigma_x$$

II) CASO: $\sigma_y \neq 0$ $\quad \sigma_x = 0$ $\quad \sigma_z = 0$

$$\varepsilon_{yy} = \frac{\sigma_y}{E}$$

$$\varepsilon_{xx} = -\mu\, \varepsilon_{yy} = -\frac{\mu}{E}\sigma_y$$

$$\varepsilon_{zz} = -\mu\, \varepsilon_{yy} = -\frac{\mu}{E}\sigma_y$$

III) CASO $\sigma_z \neq 0$ $\quad \sigma_x = 0$ $\quad \sigma_y = 0$

$$\varepsilon_{zz} = \frac{\sigma_z}{E}$$

$$\varepsilon_{xx} = -\mu\, \varepsilon_{zz} = -\frac{\mu}{E}\sigma_z$$

$$\varepsilon_{yy} = -\mu\, \varepsilon_{zz} = -\frac{\mu}{E}\sigma_z$$

$$\mu = -\frac{\varepsilon_{\perp}}{\varepsilon_{\parallel}}$$

$\varepsilon_{\perp}$ ← DIRECCIÓN ⊥ A LA APLICACIÓN DE LA CARGA

$\varepsilon_{\parallel}$ ← DIRECCIÓN DE APLICACIÓN DE LA CARGA.

$$\mu = -\frac{\varepsilon_{xx}}{\varepsilon_{zz}} = -\frac{\varepsilon_{xx}}{\frac{\sigma_z}{E}}$$

$$\mu = -\frac{\varepsilon_{xx}\, E}{\sigma_z}$$

$$\mu = -\varepsilon_{yy}\frac{E}{\sigma_z}$$

APLICANDO $\sigma_x \neq 0$; $\sigma_y \neq 0$; $\sigma_z \neq 0$ $\rightarrow$ PSE:

$$\varepsilon_{xx} = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} = \frac{1}{E}\left[\sigma_x - \mu\sigma_y - \mu\sigma_z\right].$$

$$\varepsilon_{yy} = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E} = \frac{1}{E}\left[\sigma_y - \mu\sigma_x - \mu\sigma_z\right].$$

$$\varepsilon_{zz} = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} = \frac{1}{E}\left[\sigma_z - \mu\sigma_x - \mu\sigma_y\right].$$

Ley Generalizada de Hooke

$$-\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \mu_{yx}$$

$$-\frac{\varepsilon_{xx}}{\varepsilon_{yy}} = \mu_{xy}$$

$\rightarrow$ Si el material es elástico $\rightarrow$ $\mu_{yx} = \mu_{xy}$

$\mu_{zx} = \mu_{xz} = \mu_{zy} = \mu_{yz} = \mu$