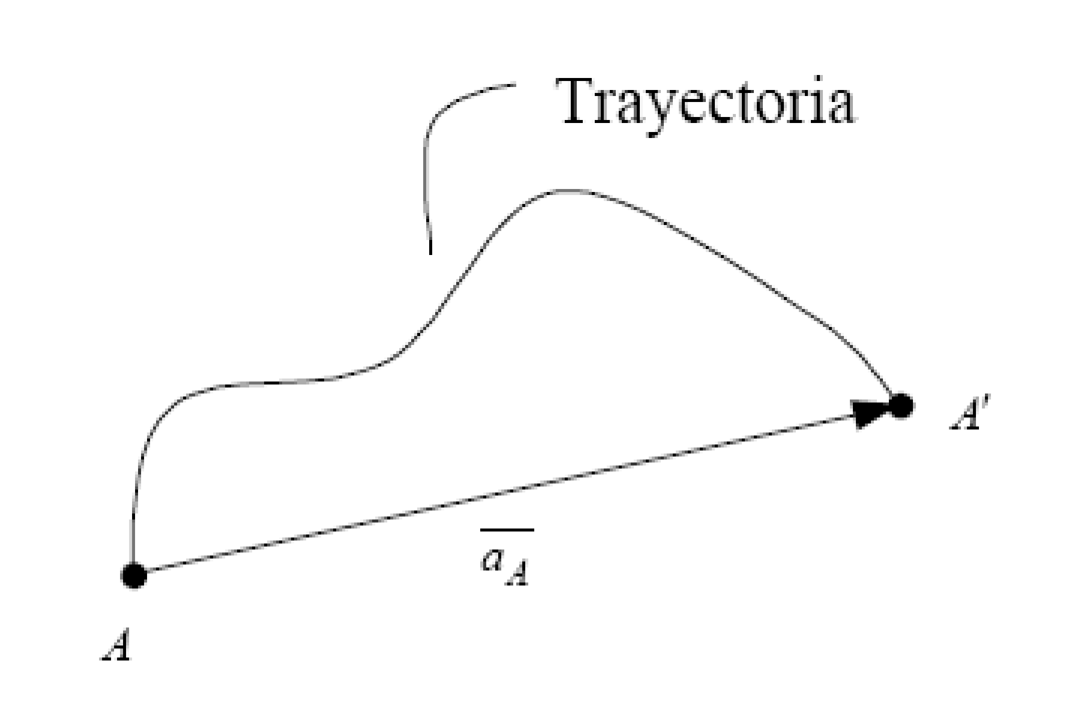
ESTADO DE DEFORMACIÓN EN PUNTO DE UN MEDIO CONTINUO

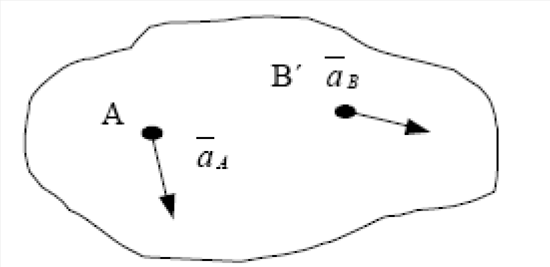
HIPÓTESIS

- Cuerpo continuo
- Material isótropo y homogéneo
- Deformaciones infinitamente pequeñas
- Linealidad cinemática



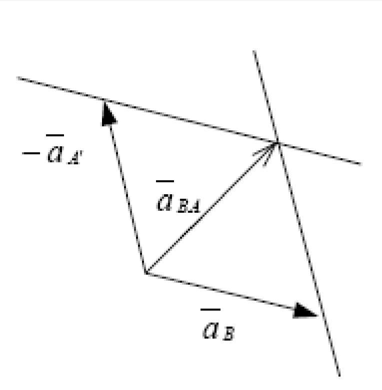
$\overline{a}_A$: *Vector desplazamiento del punto A*

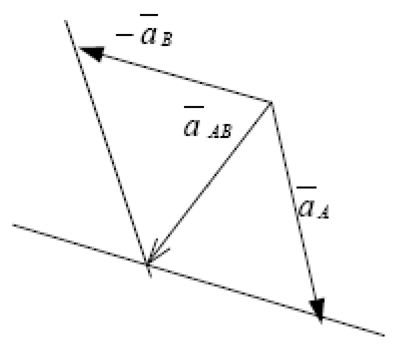
Desplazamiento relativo de un punto respecto a otro considerado fijo



$$\overline{a}_{B,A} = \overline{a}_B - \overline{a}_A = \overline{a}_B + \left(-\overline{a}_A\right)$$

$$\overline{a}_{A,B} = \overline{a}_A - \overline{a}_B = \overline{a}_A + \left(-\overline{a}_B\right)$$





$$Si\ \overline{a}_A = \overline{a}_B \Rightarrow \overline{a}_{A,B} = \overline{a}_{B,A} = 0$$

Desplazamiento relativo en el entorno e un punto debido a un movimiento rígido del entorno de un punto

Si se fija el origen de coordenadas en el punto considerado fijo (A)

El vector posición de un punto B cualquiera será $\overline{B} = x_B \breve{i} + y_B \breve{j} + z_B \breve{k}$

Los movimientos de un cuerpo rígido son: Traslación y Rotación

El desplazamiento del punto B respecto del A debido a una rotación rígida

$$\begin{Bmatrix} a_{Bx} \\ a_{By} \\ a_{Bz} \end{Bmatrix} = \begin{vmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{vmatrix} \begin{Bmatrix} x_B \\ y_B \\ z_B \end{Bmatrix} \qquad \{\overline{a}_B\} = [D_\theta]\{B\}$$

$[D_\theta]$ Matriz antisimétrica

Desplazamiento relativo en el entorno de un punto en un continuo

Sean u, v, w las componentes del vector desplazamiento de un punto del continuo

$u = u(x; y; z) \quad v = v(x; y; z) \quad w = w(x; y; z)$ Funciones continuas y derivables

$$a_{Ax} = u_A = u(x_A; y_A; z_A)$$

$$a_{Ay} = v_A = v(x_A; y_A; z_A) \qquad \bar{a}_A = a_{Ax}\breve{i} + a_{Ay}\breve{j} + a_{Az}\breve{k}$$

$$a_{Az} = w_A = w(x_A; y_A; z_A)$$

El desplazamiento relativo de un punto B del entorno respecto de A

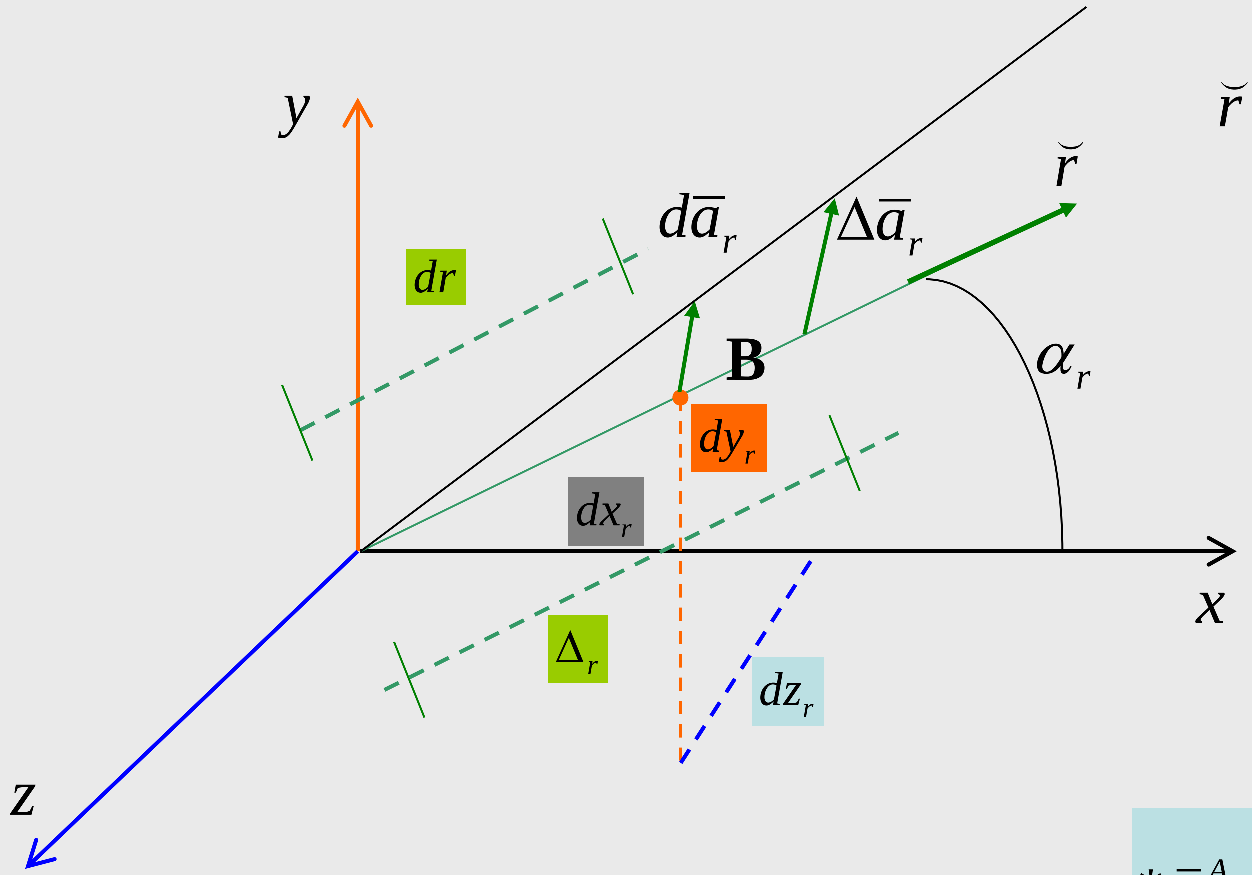
de coordenadas $x_B - x_A = dx \,;\, y_B - y_A = dy \,;\, z_B - z_A = dz$

$$a_{B,Ax} = a_{Bx} = u_B - u_A = du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$a_{B,Ay} = a_{By} = v_B - v_A = dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

$$a_{B,Az} = a_{Bz} = w_B - w_A = dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

Desplazamiento Relativo Específico



$$\breve{r} = n_{rx}\breve{i} + n_{ry}\breve{j} + n_{rz}\breve{k}$$

$$n_{rx} = \frac{dx_r}{dr} = \cos\alpha_r$$

$$n_{ry} = \frac{dy_r}{dr} = \cos\beta_r$$

$$n_{rz} = \frac{dz_r}{dr} = \cos\gamma_r$$

$$*\,\bar{\varepsilon}_r^{A} = \underset{\Delta_r \to 0}{\text{lím}} \frac{\Delta\bar{a}_r}{\Delta_r} = \frac{d\bar{a}_r}{d_r}$$

$$\breve{r} = n_{rx}\breve{i} + n_{ry}\breve{j} + n_{rz}\breve{k} \qquad n_{rx} = \frac{dx_r}{dr} = \cos\alpha_r \qquad n_{ry} = \frac{dy_r}{dr} = \cos\beta_r \qquad n_{rz} = \frac{dz_r}{dr} = \cos\gamma_r$$

$$d\bar{a}_r = du_r\breve{i} + dv_r\breve{j} + dw_r\breve{k} \qquad *\bar{\varepsilon}_r^A = \frac{d\bar{a}_r}{dr} = \frac{du_r}{dr}\breve{i} + \frac{dv_r}{dr}\breve{j} + \frac{dw_r}{dr}\breve{k}$$

$$u_B - u_A = du_r = \frac{\partial u}{\partial x}dx_r + \frac{\partial u}{\partial y}dy_r + \frac{\partial u}{\partial z}dz_r \qquad *\varepsilon_{rx}^A = \frac{du_r}{dr} = \frac{\partial u}{\partial x}\frac{dx_r}{dr} + \frac{\partial u}{\partial y}\frac{dy_r}{dr} + \frac{\partial u}{\partial z}\frac{dz_r}{dr}$$

$$v_B - v_A = dv_r = \frac{\partial v}{\partial x}dx_r + \frac{\partial v}{\partial y}dy_r + \frac{\partial v}{\partial z}dz_r \qquad *\varepsilon_{ry}^A = \frac{dv_r}{dr} = \frac{\partial v}{\partial x}\frac{dx_r}{dr} + \frac{\partial v}{\partial y}\frac{dy_r}{dr} + \frac{\partial v}{\partial z}\frac{dz_r}{dr}$$

$$w_B - w_A = dw_r = \frac{\partial w}{\partial x}dx_r + \frac{\partial w}{\partial y}dy_r + \frac{\partial w}{\partial z}dz_r \qquad *\varepsilon_{rz}^A = \frac{dw_r}{dr} = \frac{\partial w}{\partial x}\frac{dx_r}{dr} + \frac{\partial w}{\partial y}\frac{dy_r}{dr} + \frac{\partial w}{\partial z}\frac{dz_r}{dr}$$

$$*\varepsilon_{rx}^A = \frac{\partial u}{\partial x}n_{rx} + \frac{\partial u}{\partial y}n_{ry} + \frac{\partial u}{\partial z}n_{rz} \qquad *\varepsilon_{ry}^A = \frac{\partial v}{\partial x}n_{rx} + \frac{\partial v}{\partial y}n_{ry} + \frac{\partial v}{\partial z}n_{rz}$$

$$*\varepsilon_{rz}^A = \frac{\partial w}{\partial x}n_{rx} + \frac{\partial w}{\partial y}n_{ry} + \frac{\partial w}{\partial z}n_{rz}$$

$$\varepsilon_{rx}^{A} = \frac{\partial u}{\partial x} n_{rx} + \frac{\partial u}{\partial y} n_{ry} + \frac{\partial u}{\partial z} n_{rz} \qquad \varepsilon_{ry}^{A} = \frac{\partial v}{\partial x} n_{rx} + \frac{\partial v}{\partial y} n_{ry} + \frac{\partial v}{\partial z} n_{rz} \qquad \varepsilon_{rz}^{A} = \frac{\partial w}{\partial x} n_{rx} + \frac{\partial w}{\partial y} n_{ry} + \frac{\partial w}{\partial z} n_{rz}$$

$$\left\{ * \bar{\varepsilon}_r^{A} \right\} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} \left\{ \breve{r} \right\}$$

$$\left\{ * \bar{\varepsilon}_r^{A} \right\} = \left[\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & 0 & -\frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ -\frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) & 0 \end{bmatrix} \right] \left\{ \breve{r} \right\}$$

Matriz de rotación rígida

$$\theta_{x=} \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \theta_{y=} \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \theta_{z=} \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\begin{Bmatrix} a_{Bx} \\ a_{By} \\ a_{Bz} \end{Bmatrix} = \begin{vmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{vmatrix} \begin{Bmatrix} x_B \\ y_B \\ z_B \end{Bmatrix}$$

Vector deformación específica

$$\left\{ {}^{*}\varepsilon_r^A \right\} = [T_S]\{\breve{r}\} + [T_A]\{\breve{r}\} = \left\{ \varepsilon_r^A \right\} + \left\{ {}^{\theta}\varepsilon_r^A \right\}$$

Desplazamientos debidos a rotación rígida del entorno del punto

$$\begin{Bmatrix} \varepsilon_{rx}^A \\ \varepsilon_{ry}^A \\ \varepsilon_{rz}^A \end{Bmatrix} = [T_S] \begin{Bmatrix} n_{rx} \\ n_{ry} \\ n_{rz} \end{Bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) & \frac{\partial w}{\partial z} \end{bmatrix} \begin{Bmatrix} n_{rx} \\ n_{ry} \\ n_{rz} \end{Bmatrix}$$

$$r \equiv x \Rightarrow n_{xx} = 1; n_{xy} = 0; n_{xz} = 0 \quad \Rightarrow \varepsilon_{rx} = \frac{\partial u}{\partial x} = \varepsilon_{xx}; \varepsilon_{ry} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \varepsilon_{xy}; \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \varepsilon_{xz}$$

$$1^{a}\, columna: \varepsilon_{xx} = \frac{\partial u}{\partial x}; \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right); \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$r \equiv y \Rightarrow n_{yx} = 0; n_{yy} = 1; n_{yz} = 0 \quad \Rightarrow \varepsilon_{rx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \varepsilon_{yx}; \varepsilon_{ry} = \frac{\partial v}{\partial y} = \varepsilon_{yy}; \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \varepsilon_{yz}$$

$$2^{a}\, columna: \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right); \varepsilon_{yy} = \frac{\partial v}{\partial y}; \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$3^{a}\, columna: \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right); \varepsilon_{zy} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right); \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

TENSOR DE DEFORMACIONES

Como el tensor es simétrico

(deformación pura)

$\varepsilon_{xy} = \varepsilon_{yx}$ $\varepsilon_{yz} = \varepsilon_{zy}$ $\varepsilon_{zx} = \varepsilon_{xz}$

•Caracteriza al Estado de Deformación (permite conocer el vector deformación asociado a cada dirección pasante por el punto)

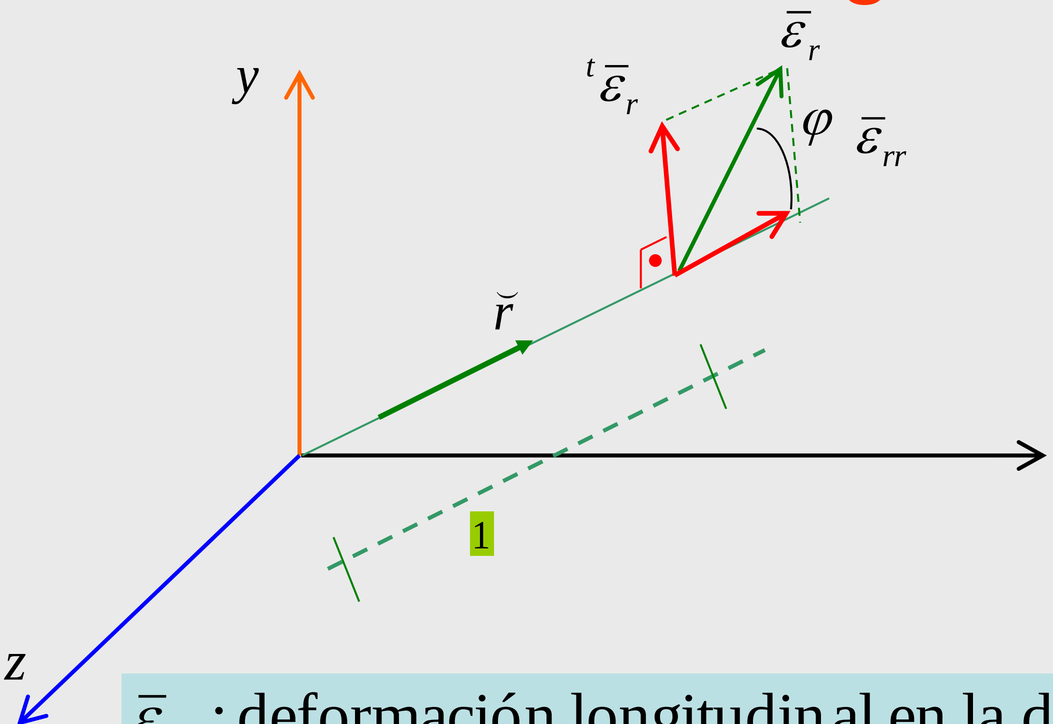
Si se cambian los ejes de referencia

•Al cambiar los ejes coordenados (ejes con que se caracteriza al estado de deformación), varía el tensor de deformaciones

•Un versor correpondiente a una dirección determinada tendrá componentes distintos

•El vector deformación asociado a esa dirección, tendrá componentes distintos, pues es el mismo vector físico, representado en otra terna

Deformación longitudinal y transversal



$$\breve{r} = n_{rx}\breve{i} + n_{ry}\breve{j} + n_{rz}\breve{k}$$

$$\left\{\overline{\varepsilon_r}\right\} = [T_D]\{\breve{r}\}$$

$$\bar{\varepsilon}_{rr} =_x (\bar{\varepsilon}_r \bullet \breve{r})\breve{r}$$

$$|\bar{\varepsilon}_{rr}| = |\varepsilon_r| \cos\varphi$$

$$|{}^t\bar{\varepsilon}_r| = |\varepsilon_r| \,\mathrm{sen}\,\varphi$$

$$|{}^t\bar{\varepsilon}_r| = \sqrt{|\bar{\varepsilon}_r|^2 - |\bar{\varepsilon}_{rr}|^2}$$

$${}^t\bar{\varepsilon}_r = \breve{r} \times \bar{\varepsilon}_r \times \breve{r}$$

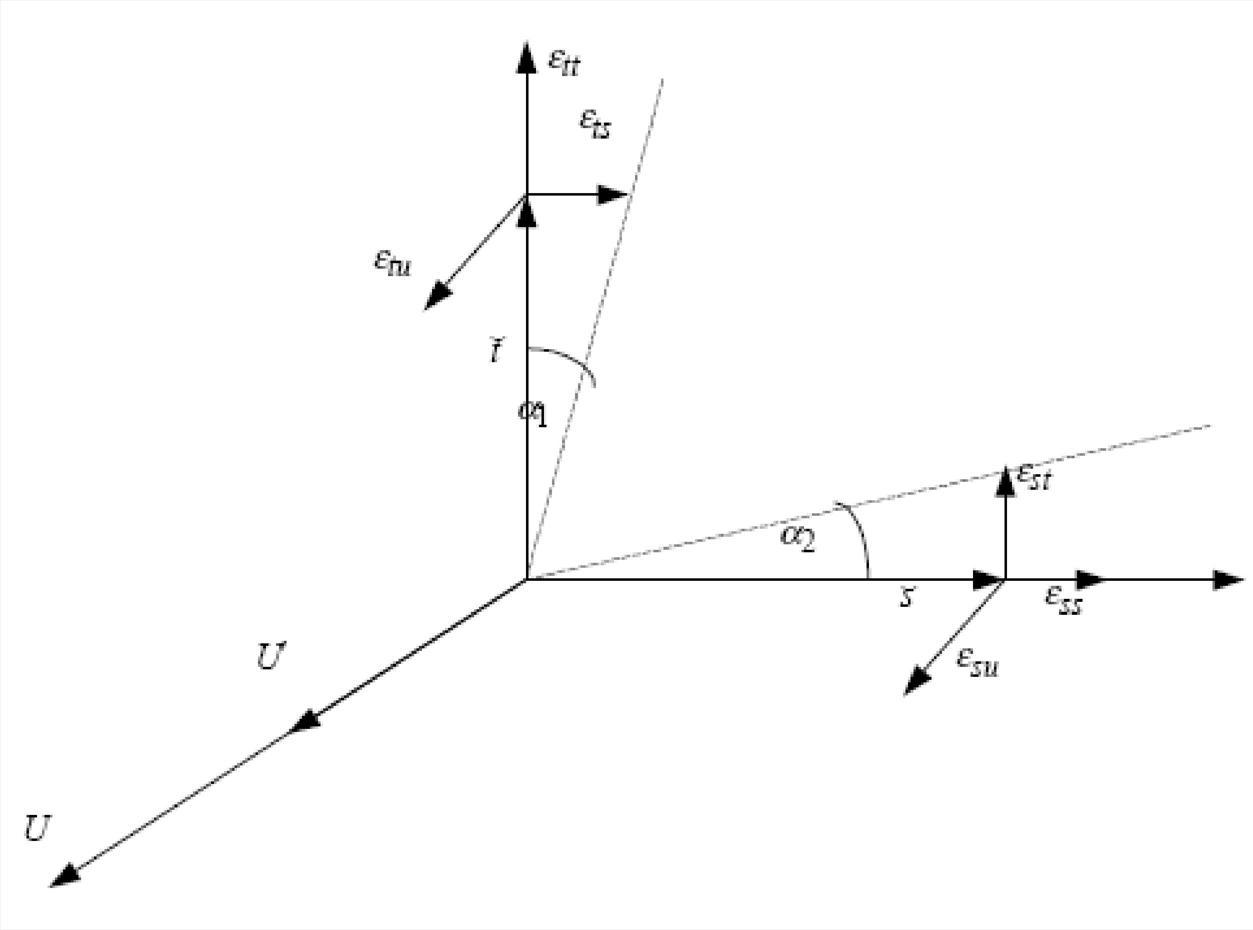
$\bar{\varepsilon}_{rr}$: deformación longitudinal en la dir. r

${}^t\bar{\varepsilon}_r$: deformación transversal en la dir. r

$\bar{\varepsilon}_{rr}$ está asociado a la variación de longitud en la dir. r

${}^t\bar{\varepsilon}_r$ está asociado a la variación angular de la dir. r

Deformación transversal y distorsión



$$\gamma_{st} = \alpha_1 + \alpha_2 = \frac{\varepsilon_{ts}}{|\vec{t}|} + \frac{\varepsilon_{st}}{|\vec{s}|} = 2\varepsilon_{st}$$

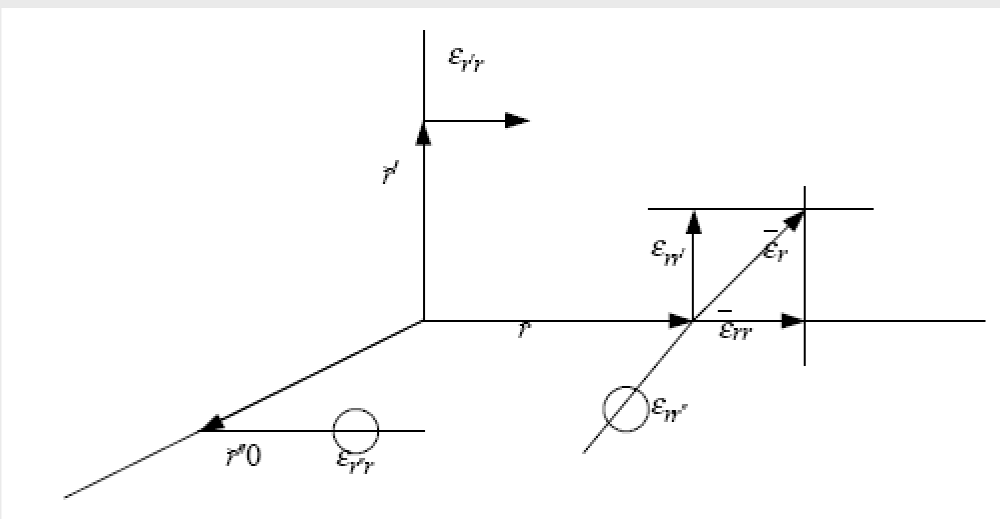
$$\varepsilon_{ts} = \varepsilon_{st}$$

$$\varepsilon_{st} = \frac{\gamma_{st}}{2}$$

$$\gamma_{st} = 2\varepsilon_{st}$$

Distorsión: Variación del ángulo inicialmente recto entre dos direcciones

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} \quad \begin{bmatrix} \varepsilon_{x} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{xy}/2 & \varepsilon_{y} & \gamma_{yz}/2 \\ \gamma_{xz}/2 & \gamma_{yz}/2 & \varepsilon_{z} \end{bmatrix}$$



$$\gamma_{rr'} = 2\varepsilon_{rr'}$$

$$\gamma_{rr''} = 2\varepsilon_{rr''} = 0$$