# ESTADO DE DEFORMACIÓN EN PUNTO DE UN MEDIO CONTINUO

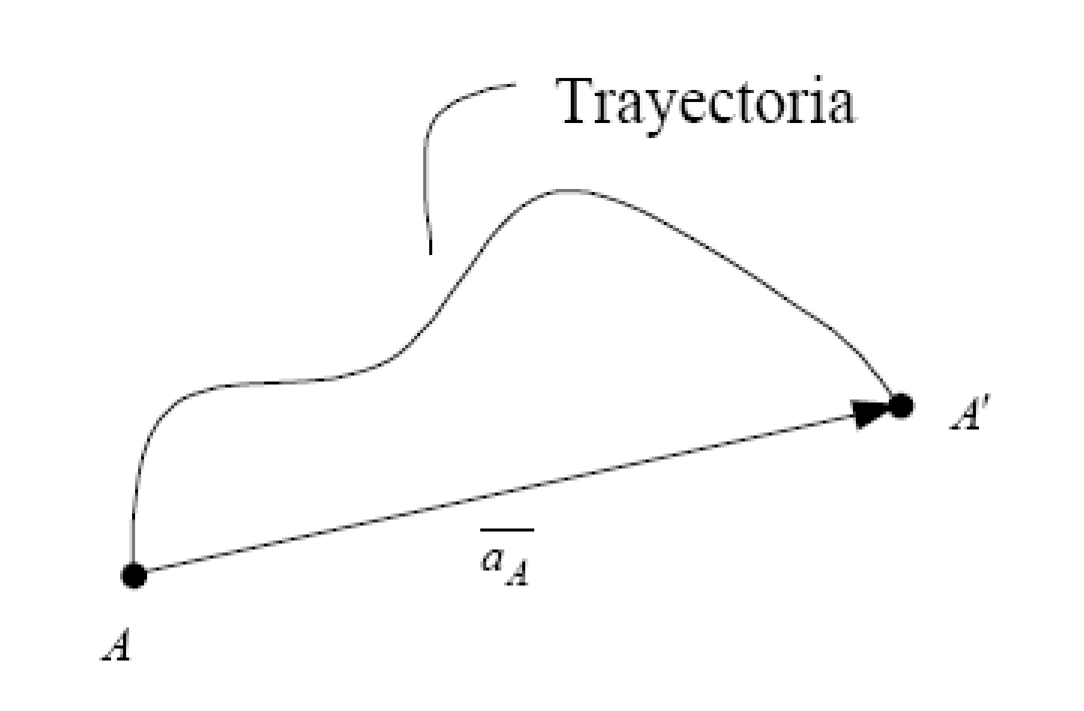
# HIPÓTESIS

- Cuerpo continuo
- Material isótropo y homogéneo
- Deformaciones infinitamente pequeñas
- Linealidad cinemática

# CINEMATICA DEL CUERPO RIGIDO

## HIPOTESIS

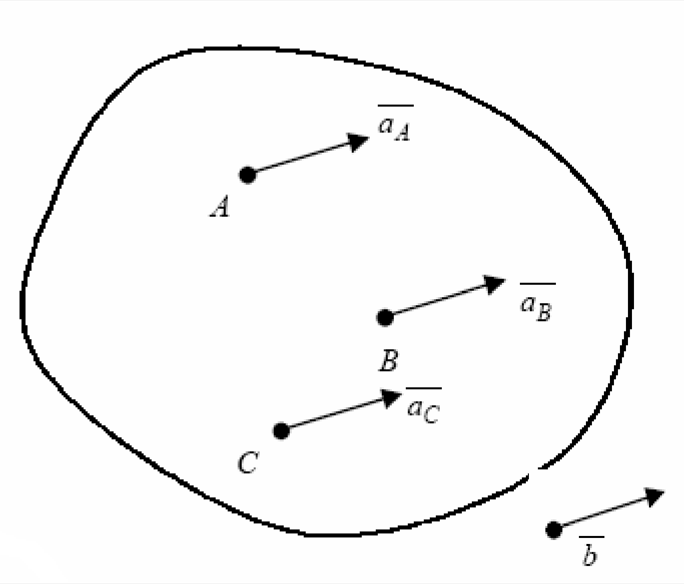
- Cuerpos rígidos
- Estudio atemporal (se prescinde de la variable tiempo)
- Desplazamientos infinitamente pequeños respecto de las dimensiones del cuerpo




$\overline{a}_A$ : *Vector desplazamiento del punto A*

# Movimientos simples del cuerpo rígido

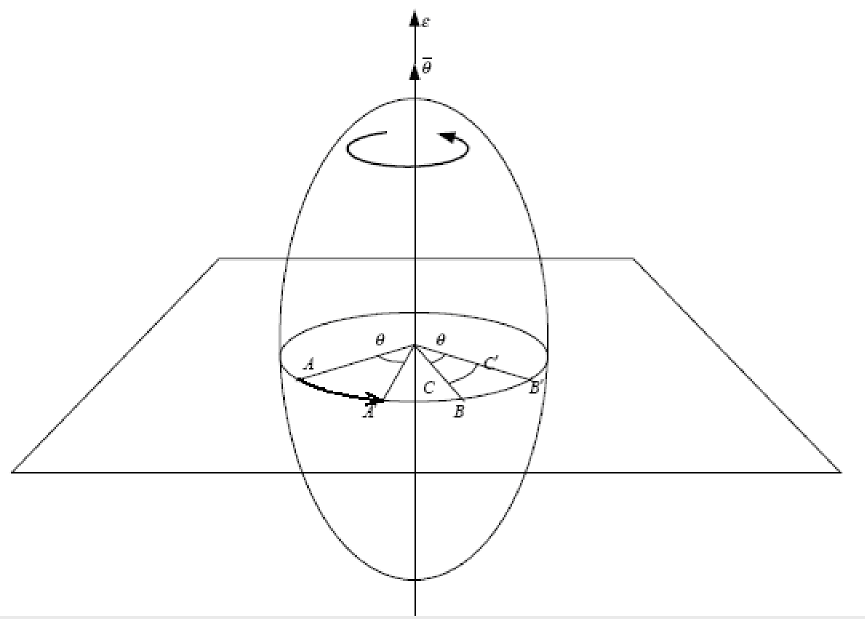
## Traslación




$$\left|\overline{a}_A\right| = \left|\overline{a}_B\right| = \left|\overline{a}_C\right| = \left|\overline{b}\right|$$

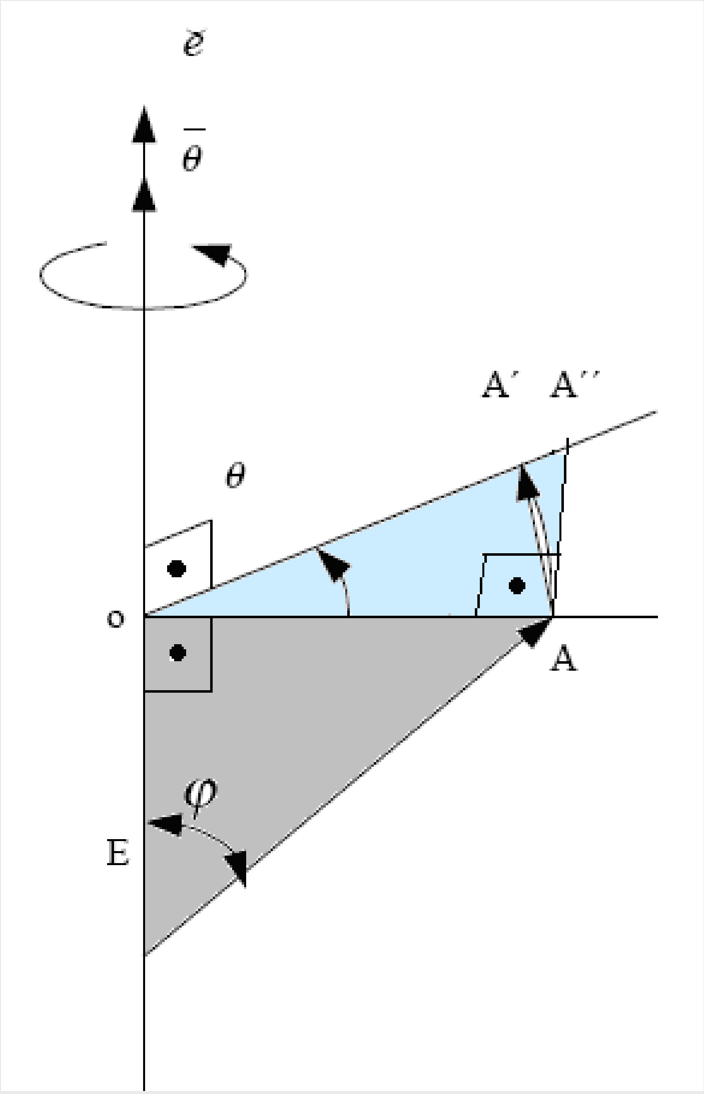
$\overline{b}$ : Vector Traslación

## Rotación en torno a un eje




$\overline{\theta}$ : Vector Rotación

# Desplazamiento de un punto en una rotación




Rotaciones pequeñas

$$\theta \langle\langle\langle 1 \Rightarrow \quad \theta \cong sen\,\theta \cong tg\,\theta$$

$$\overline{AA'} \cong arc\ AA' \cong \overline{AA''}$$

Adoptamos

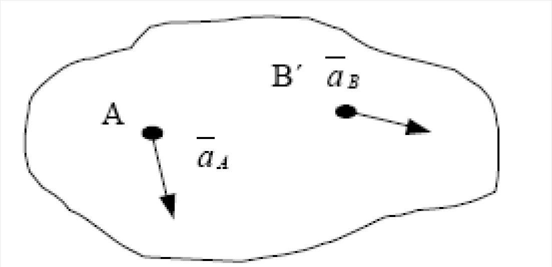
$$dir\ \bar{a}_A \equiv \overline{AA''} \qquad |\bar{a}_a| \equiv \overline{AA''}$$

$$\bar{a}_A = \bar{\theta} \times \overline{A - E}$$

$$dir\ \bar{\theta} \times \overline{A-E} \perp \bar{\theta} \wedge \perp \overline{A-E} \Rightarrow dir\ AA''$$

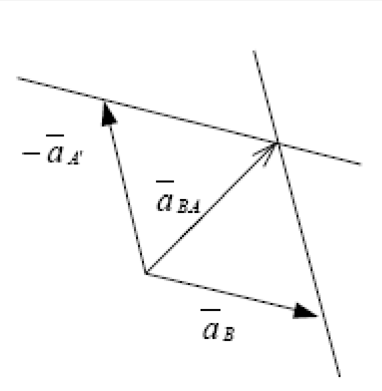
$$|\bar{\theta} \times \overline{A-E}| = |\theta| \times |\overline{A-E}|\ sen\varphi = \theta \times \overline{OA} = arc\ AA'$$

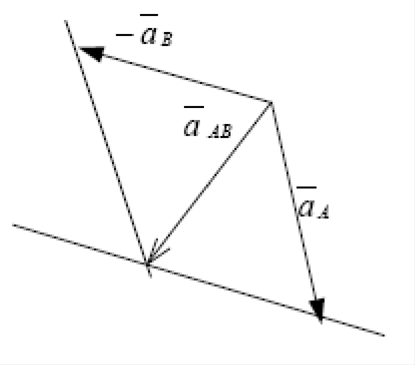
# Desplazamiento relativo




$$\bar{a}_{B,A} = \bar{a}_B - \bar{a}_A = \bar{a}_B + \left(-\bar{a}_A\right)$$

$$\bar{a}_{A,B} = \bar{a}_A - \bar{a}_B = \bar{a}_A + \left(-\bar{a}_B\right)$$




$$Si\ \bar{a}_A = \bar{a}_B \Rightarrow \bar{a}_{A,B} = \bar{a}_{B,A} = 0$$

# Desplazamiento relativo en un movimiento rígido

Desplazamiento relativo debido a una traslación

$$\bar{a}_A = \bar{a}_B = \bar{b} \Rightarrow \bar{a}_{A,B} = \bar{a}_{B,A} = 0$$

Desplazamiento relativo debido a una rotación rígida

$$\bar{a}_A = \bar{\theta} \times \overline{A - E}$$

$$\bar{a}_B = \bar{\theta} \times \overline{B - E}$$

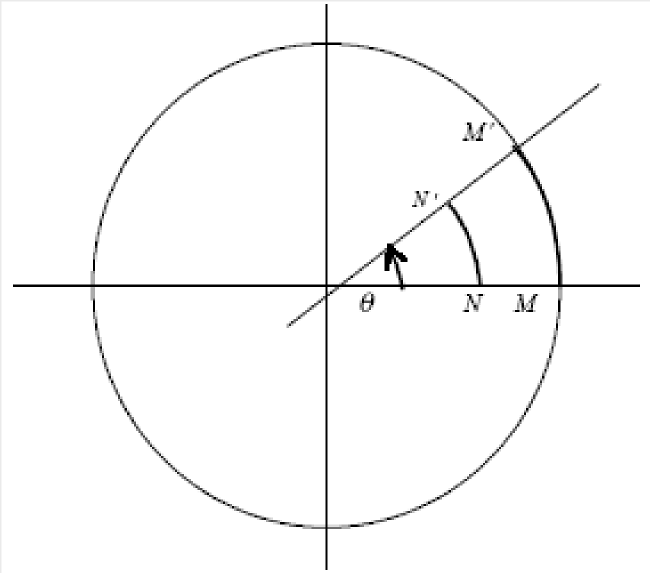
$$\bar{a}_{B,A} = \bar{\theta} \times \overline{B - E} - \bar{\theta} \times \overline{A - E} = \bar{\theta} \times \left( \overline{B - E} - \overline{A - E} \right)$$

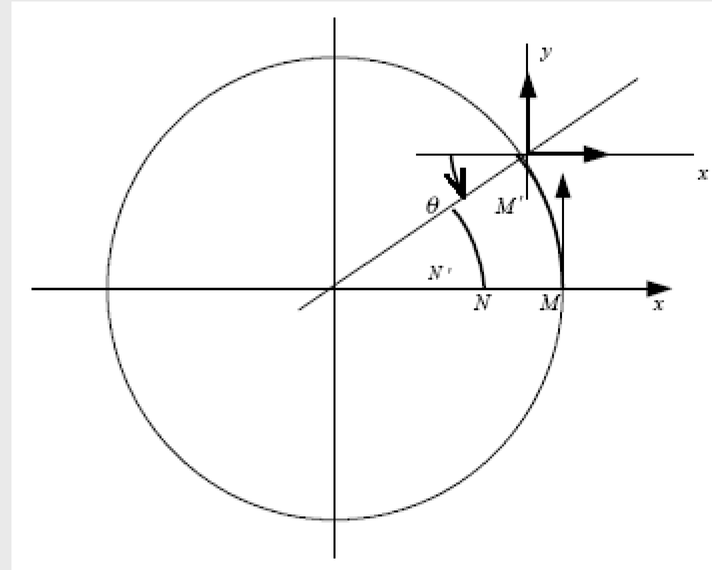
$$\bar{a}_{B,A} = \bar{\theta} \times \overline{B - A}$$

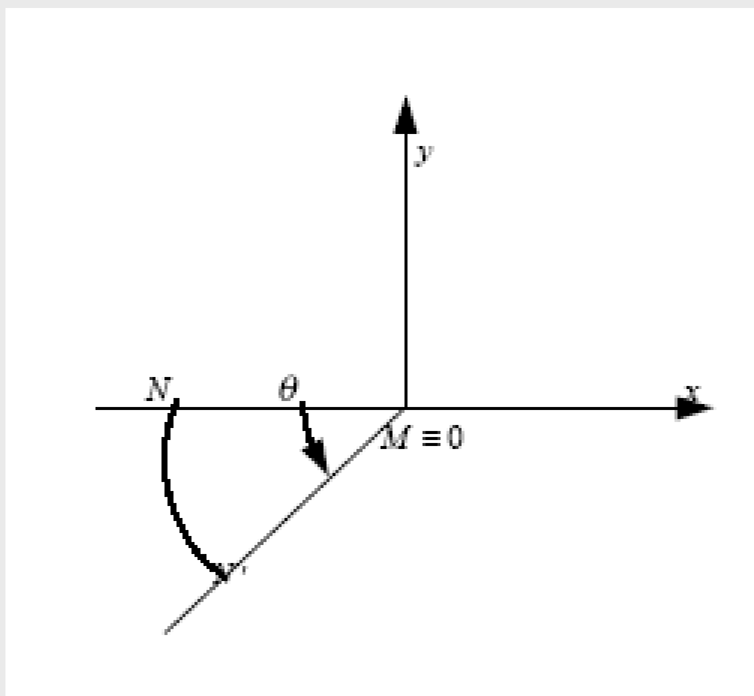
$$\bar{a}_{A,B} = \bar{\theta} \times \overline{A - E} - \bar{\theta} \times \overline{B - E} = \bar{\theta} \times \left( \overline{A - E} - \overline{B - E} \right)$$

$$\bar{a}_{A,B} = \bar{\theta} \times \overline{A - B}$$

El desplazamiento relativo debido a una rotación rígida es igual al desplazamiento del punto si se aplica una rotación cuyo eje pasa por el punto considerado fijo


M'
N'
θ
N
M


y
x
θ
M'
N'
N
M
x


y
N
θ
x
M ≡ 0

Si se fija el origen de coordenadas en el punto considerado fijo (A)

El vector posición de un punto B será $\bar{B} = x_B \breve{i} + y_B \breve{j} + z_B \breve{k}$

Y el desplazamiento del punto B respecto del A debido a una rotación rígida

$$\begin{Bmatrix} a_{Bx} \\ a_{By} \\ a_{Bz} \end{Bmatrix} = \begin{vmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{vmatrix} \begin{Bmatrix} x_B \\ y_B \\ z_B \end{Bmatrix} \qquad \{\bar{a}_B\} = [D_\theta]\{B\}$$

$[D_\theta]$ Matriz antisimétrica

## Desplazamiento relativo en el entorno de un punto en un continuo

$$u = u(x; y; z) \qquad w = w(x; y; z) \qquad v = v(x; y; z)$$

$u, v, w$ Funciones continuas y derivables

$$a_{Ax} = u_A = u(x_A; y_A; z_A)$$

$$a_{Ay} = v_A = v(x_A; y_A; z_A) \qquad \bar{a}_A = a_{Ax}\breve{i} + a_{Ay}\breve{j} + a_{Az}\breve{k}$$

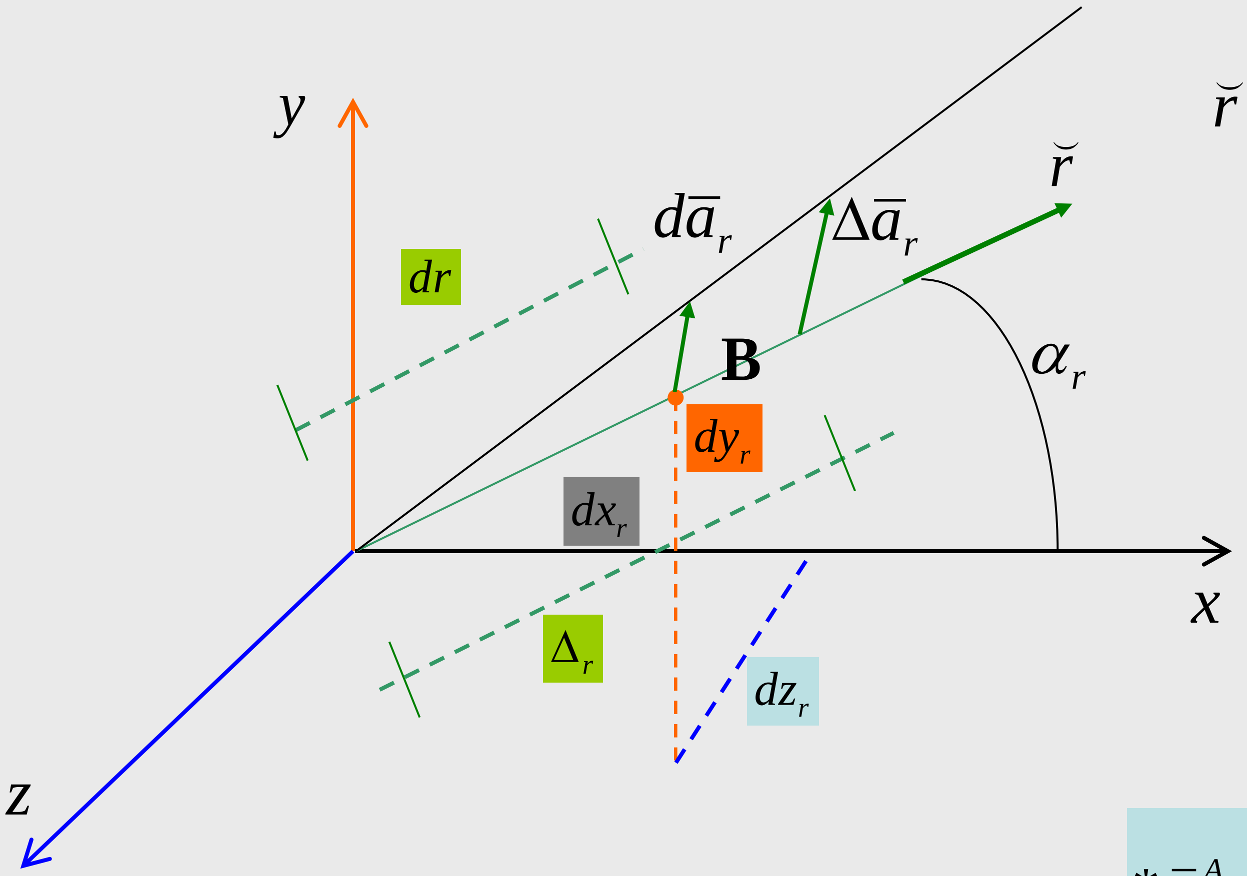
$$a_{Az} = w_A = w(x_A; y_A; z_A)$$

$$a_{B,Ax} = a_{Bx} = u_B - u_A = du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$a_{B,Ay} = a_{By} = v_B - v_A = dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

$$a_{B,Az} = a_{Bz} = w_B - w_A = dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

# Desplazamiento Relativo Específico




$$\breve{r} = n_{rx}\breve{i} + n_{ry}\breve{j} + n_{rz}\breve{k}$$

$$n_{rx} = \frac{dx_r}{dr} = \cos\alpha_r$$

$$n_{ry} = \frac{dy_r}{dr} = \cos\beta_r$$

$$n_{rz} = \frac{dz_r}{dr} = \cos\gamma_r$$

$$*\,\bar{\varepsilon}_r^{A} = \lim_{\Delta_r \to 0} \frac{\Delta\bar{a}_r}{\Delta_r} = \frac{d\bar{a}_r}{d_r}$$

$$\varepsilon_{rx}^{A} = \frac{\partial u}{\partial x} n_{rx} + \frac{\partial u}{\partial y} n_{ry} + \frac{\partial u}{\partial z} n_{rz}$$

$$\varepsilon_{ry}^{A} = \frac{\partial v}{\partial x} n_{rx} + \frac{\partial v}{\partial y} n_{ry} + \frac{\partial v}{\partial z} n_{rz}$$

$$\varepsilon_{rz}^{A} = \frac{\partial w}{\partial x} n_{rx} + \frac{\partial w}{\partial y} n_{ry} + \frac{\partial w}{\partial z} n_{rz}$$

$$\left\{ * \bar{\varepsilon}_{r}^{A} \right\} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} = \left\{ \breve{r} \right\}$$

$$\left\{ * \bar{\varepsilon}_{r}^{A} \right\} = \left[ \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & 0 & -\frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ -\frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) & 0 \end{bmatrix} \right] \left\{ \breve{r} \right\}$$

Matriz de rotación rígida

$$\theta_{x=} \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\theta_{y=} \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\theta_{z=} \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\begin{Bmatrix} a_{Bx} \\ a_{By} \\ a_{Bz} \end{Bmatrix} = \begin{vmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{vmatrix} \begin{Bmatrix} x_B \\ y_B \\ z_B \end{Bmatrix}$$

# Vector deformación específica

$$\left\{\overline{{}^{*}\varepsilon_r^A}\right\}=[T_S]\{\breve{r}\}+[T_A]\{\breve{r}\}=\left\{\overline{\varepsilon_r^A}\right\}+\left\{\overline{{}^{\theta}\varepsilon_r^A}\right\}$$

Desplazamientos debidos a rotación rígida del entorno del punto

$$\begin{Bmatrix}\varepsilon_{rx}^A\\ \varepsilon_{ry}^A\\ \varepsilon_{rz}^A\end{Bmatrix}=[T_S]\begin{Bmatrix}n_{rx}\\ n_{ry}\\ n_{rz}\end{Bmatrix}=\begin{bmatrix}\frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\\ \frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\\ \frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) & \frac{\partial w}{\partial z}\end{bmatrix}\begin{Bmatrix}n_{rx}\\ n_{ry}\\ n_{rz}\end{Bmatrix}$$

$$r\equiv x\Rightarrow n_{xx}=1; n_{xy}=0; n_{xz}=0 \quad \Rightarrow \varepsilon_{rx}=\frac{\partial u}{\partial x}=\varepsilon_{xx};\varepsilon_{ry}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=\varepsilon_{xy};\varepsilon_{rz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=\varepsilon_{xz}$$

$$1^{a}\,columna:\varepsilon_{xx}=\frac{\partial u}{\partial x};\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right);\varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$$

$$r\equiv y\Rightarrow n_{yx}=0; n_{yy}=1; n_{yz}=0 \quad \Rightarrow \varepsilon_{rx}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=\varepsilon_{yx};\varepsilon_{ry}=\frac{\partial v}{\partial y}=\varepsilon_{yy};\varepsilon_{rz}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=\varepsilon_{yz}$$

$$2^{a}\,columna:\varepsilon_{yx}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right);\varepsilon_{yy}=\frac{\partial v}{\partial y};\varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)$$

$$3^{a}\,columna:\varepsilon_{zx}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right);\varepsilon_{zy}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right);\varepsilon_{zz}=\frac{\partial w}{\partial z}$$

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

# TENSOR DE DEFORMACIONES

**Como el tensor es simétrico**

**(deformación pura)**

$\varepsilon_{xy} = \varepsilon_{yx}$ $\varepsilon_{yz} = \varepsilon_{zy}$ $\varepsilon_{zx} = \varepsilon_{xz}$

- Caracteriza al Estado de Deformación (permite conocer el vector deformación asociado a cada dirección pasante por el punto)

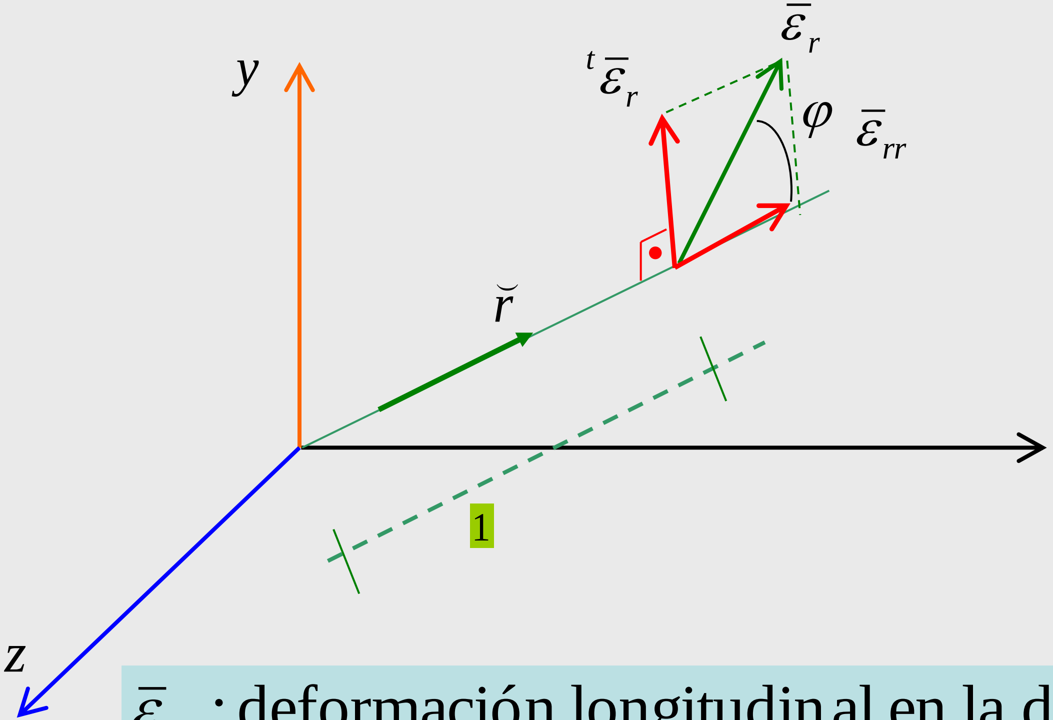
**Si se cambian los ejes de referencia**

- Al cambiar los ejes coordenados (ejes con que se caracteriza al estado de deformación), varía el tensor de deformaciones

- Un versor correpondiente a una dirección determinada tendrá componentes distintos

- El vector deformación asociado a esa dirección, tendrá componentes distintos, pues es el mismo vector físico, representado en otra terna

# Deformación longitudinal y transversal




$$\breve{r} = n_{rx}\breve{i} + n_{ry}\breve{j} + n_{rz}\breve{k}$$

$$\left\{\overline{\varepsilon_r}\right\} = [T_D]\{\breve{r}\}$$

$$\bar{\varepsilon}_{rr} =_x (\bar{\varepsilon}_r \bullet \breve{r})\breve{r}$$

$$|\bar{\varepsilon}_{rr}| = |\varepsilon_r| \cos\varphi$$

$$|{}^t\bar{\varepsilon}_r| = |\varepsilon_r| \operatorname{sen}\varphi$$

$$|{}^t\bar{\varepsilon}_r| = \sqrt{|\bar{\varepsilon}_r|^2 - |\bar{\varepsilon}_{rr}|^2}$$

$$ {}^t\bar{\varepsilon}_r = \breve{r} \times \bar{\varepsilon}_r \times \breve{r}$$

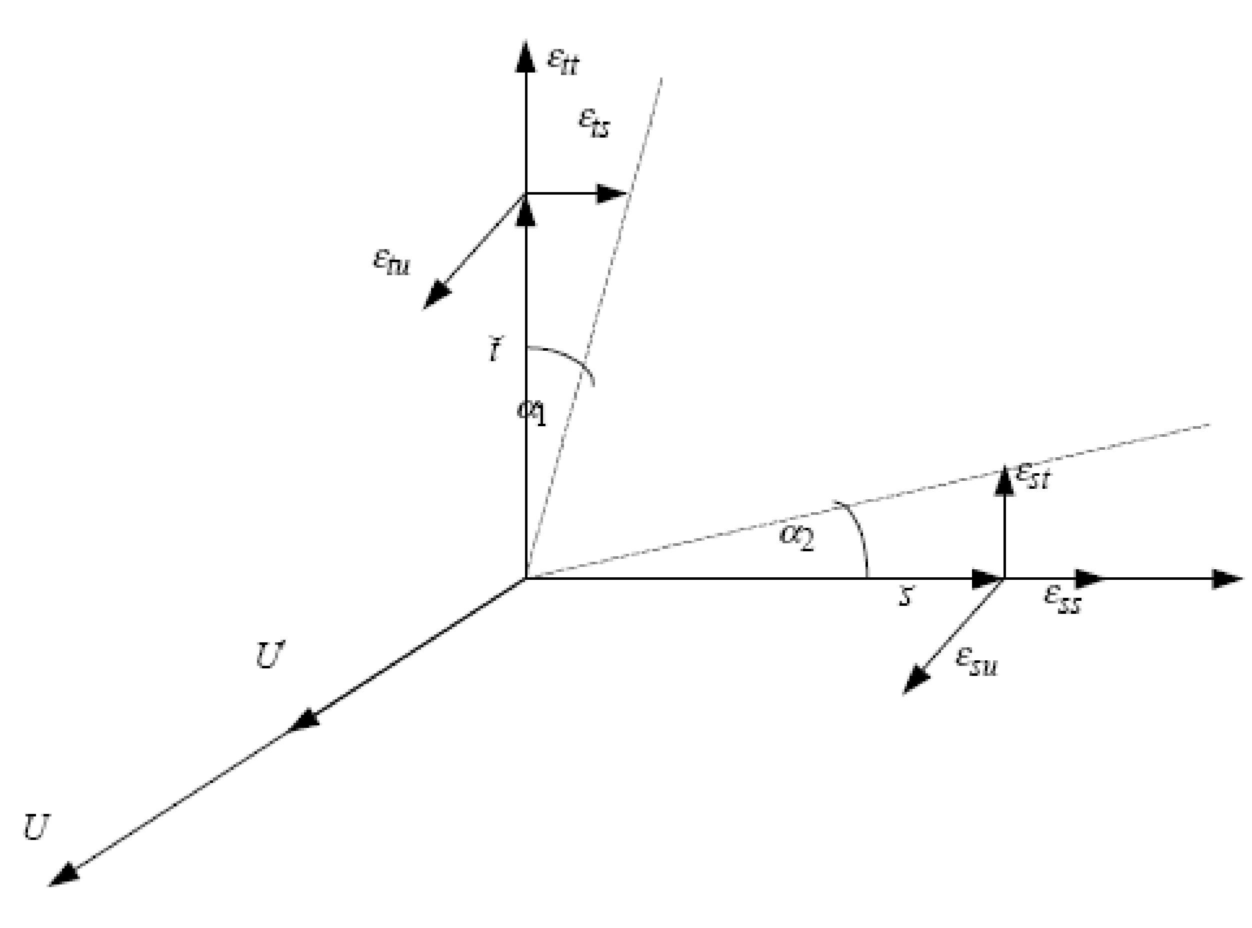
$\bar{\varepsilon}_{rr}$ : deformación longitudinal en la dir. r

${}^t\bar{\varepsilon}_r$ : deformación transversal en la dir. r

$\bar{\varepsilon}_{rr}$ está asociado a la variación de longitud en la dir. r

${}^t\bar{\varepsilon}_r$ está asociado a la variación angular de la dir. r

# Deformación transversal y distorsión




$$\gamma_{st} = \alpha_1 + \alpha_2 = \frac{\varepsilon_{ts}}{|\vec{t}|} + \frac{\varepsilon_{st}}{|\vec{s}|} = 2\varepsilon_{st}$$

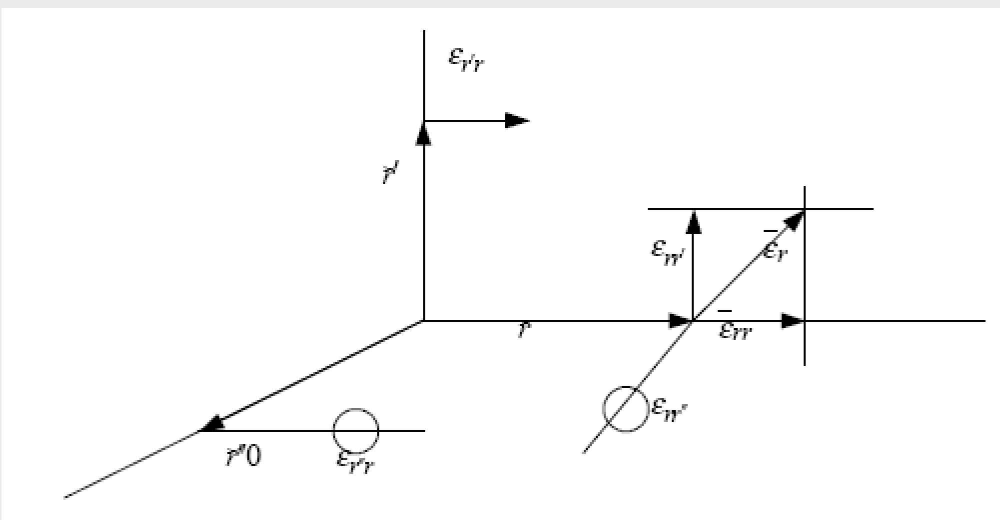
$$\varepsilon_{ts} = \varepsilon_{st}$$

$$\varepsilon_{st} = \frac{\gamma_{st}}{2}$$

$$\gamma_{st} = 2\varepsilon_{st}$$

Distorsión: Variación del ángulo inicialmente recto entre dos direcciones

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} \quad \begin{bmatrix} \varepsilon_{x} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{xy}/2 & \varepsilon_{y} & \gamma_{yz}/2 \\ \gamma_{xz}/2 & \gamma_{yz}/2 & \varepsilon_{z} \end{bmatrix}$$




$$\gamma_{rr'} = 2\varepsilon_{rr'}$$

$$\gamma_{rr''} = 2\varepsilon_{rr''} = 0$$