ESTADO DE DEFORMACIÓN:

01 - INTRODUCCIÓN

→ CAUSAS DEFORMANTES:

- I) CAUSA FUERZA 'P' o 'F'
- II) " ΔT.
- III) " CV (CEDIMIENTO DE VÍNCULO)

MAGNITUDES ESTÁTICAS → ET

" CINEMÁTICAS → ED

QUÉ EFECTOS PRODUCEN ESTAS CAUSAS SOBRE EL CUERPO ???

(PTO DE VISTA → EFECTOS CINEMÁTICOS).

$\boxed{i}$ → VARIACIÓN DE LA DISTANCIA ENTRE PLANOS.

$\boxed{ii}$ → VARIACIÓN DE LOS ÁNGULOS ENTRE DIRECCIONES.

$\boxed{iii}$ → VARIACIÓN DE VOLUMEN.

02 - HIPÓTESIS BÁSICAS P/ EL ANÁLISIS:

ENTORNO DE UN PUNTO → DEF.

OBJETIVO: VAMOS A ESTUDIAR LA DEFORMACIÓN EN EL ENTORNO DE UN PUNTO.

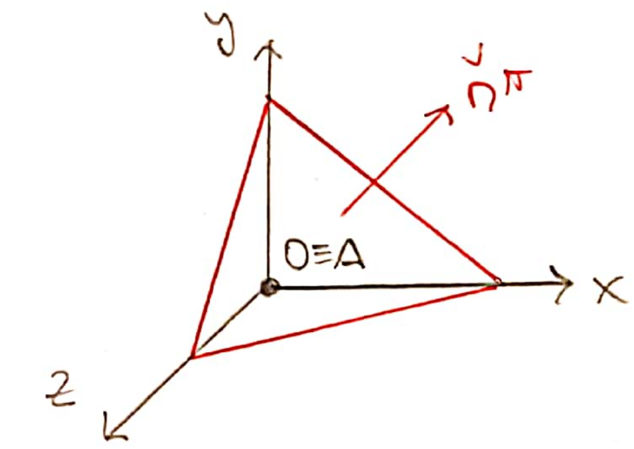
(H1) → LA DEFORMACIÓN ES CONTINUA.

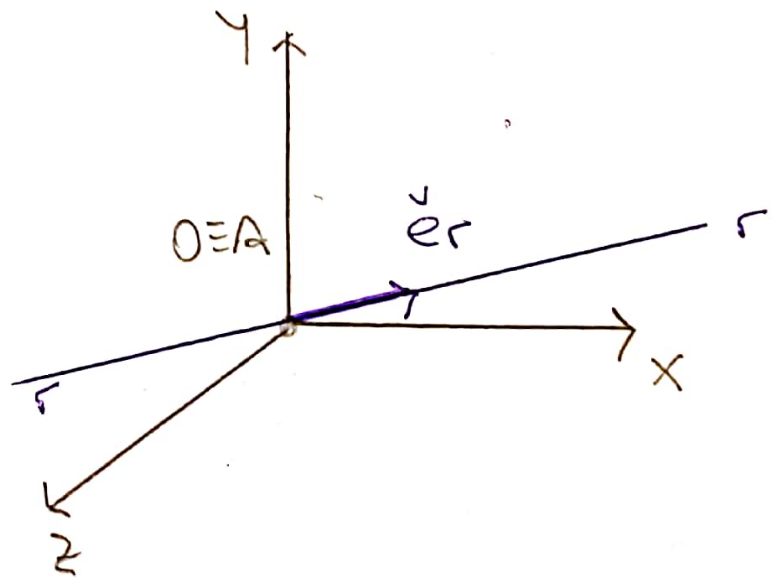
(H2) → LAS CURVAS EN EL ENTORNO DE UN PUNTO MANTIENEN SU GRADO DURANTE LA DEFORMACIÓN.

(H3) → $\boxed{\text{HLG}}$ → LOS DESPLAZAMIENTOS SON INFINITAMENTE PEQUEÑOS CON RESPECTO A LAS DIMENSIONES DEL CUERPO.

03) - ENFOQUE:

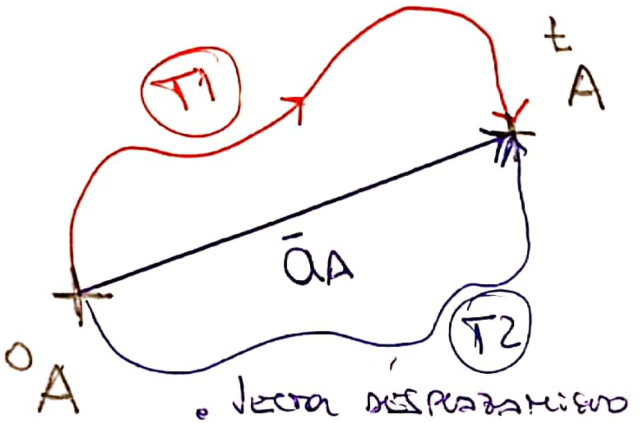
$\boxed{\text{ME}}$





04 - Repaso cinemática básica:

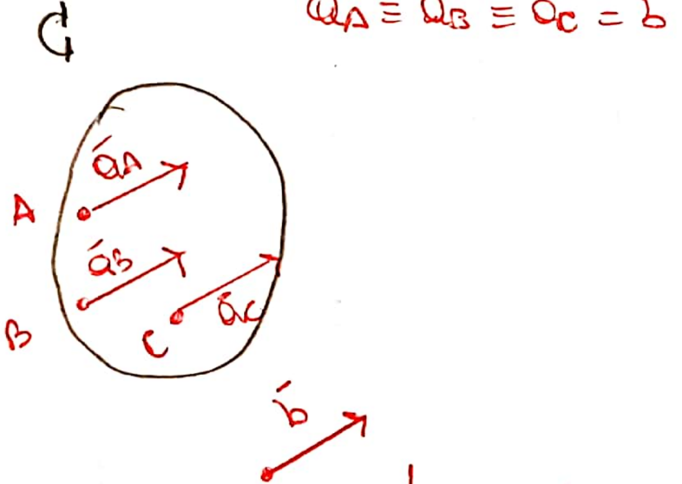
I



- Vector desplazamiento $\bar{\delta}_o$
- Trayectorias T_i

II Traslación:

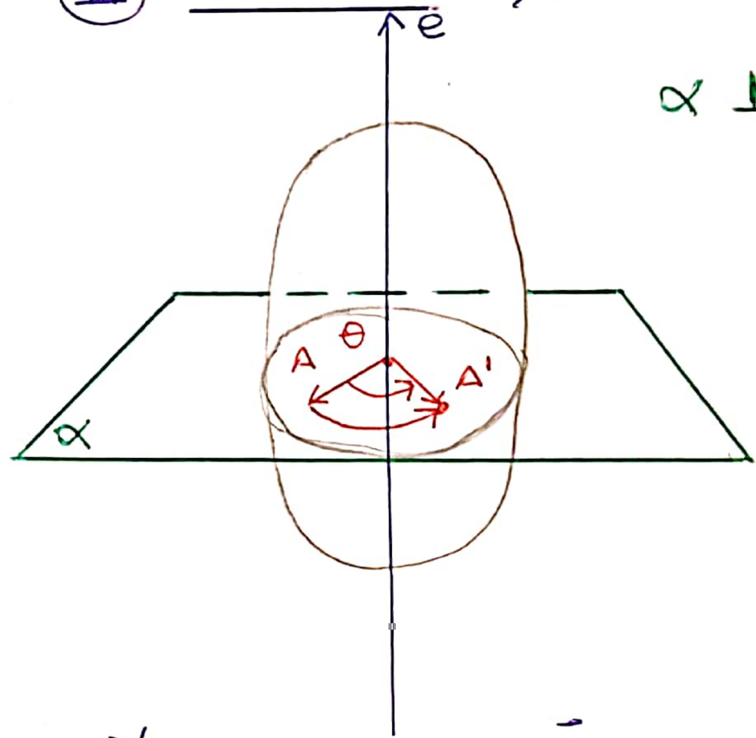
$\bar{\delta}_A \equiv \bar{\delta}_B \equiv \bar{\delta}_C = \bar{b}$



Vector libro

III Rotación: → Alrededor de 1 eje.

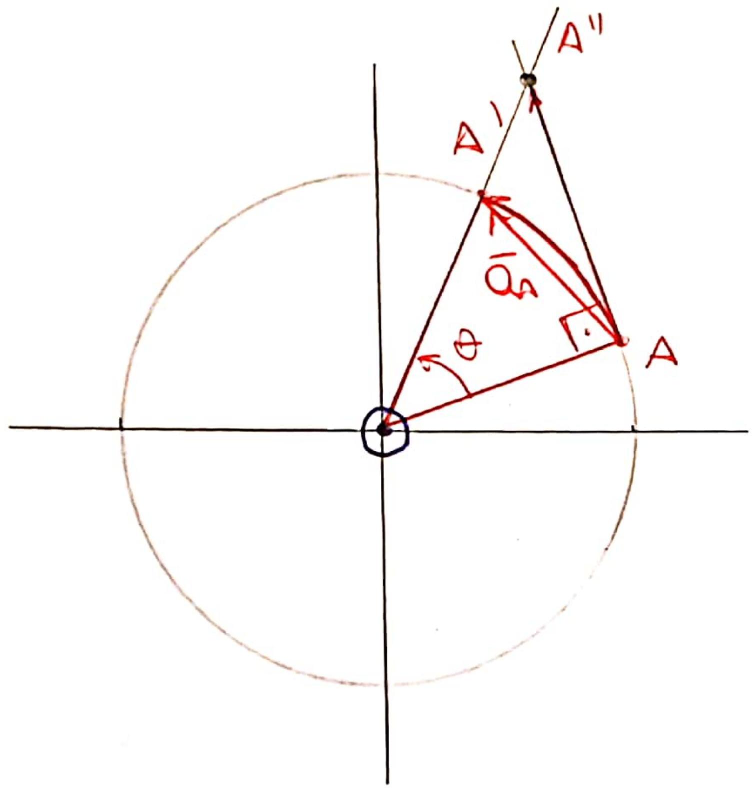
$\alpha \perp \check{e}$



Rotación → Vector $\bar{\theta}$ → Axilmente libre

- Dirección → eje.
- Sentido
- Magnitud → θ.

IV) ROTACIONES MUY PEQUEÑAS

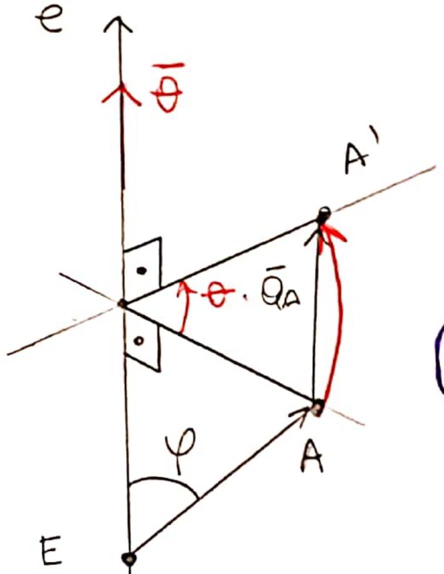


TRAYECTORIA $\overset{\frown}{AA'}$

$\bar{a}_A \equiv \underbrace{\overline{AA'}}_{\text{CUERDA SECANTE}}$

• Si θ es muy pequeño:

$$\bar{a}_A \equiv \underbrace{\overline{AA'}}_{\text{CUERDA}} \equiv \underbrace{\overset{\frown}{AA'}}_{\text{ARCO}} \equiv \underbrace{\overline{AA''}}_{\text{TANGENTE}}$$



$$\bar{a}_A \equiv \bar{\theta} \times (\overline{A - E})$$

E: PTO CUALQUIERA DEL EJE

$$\bar{a}_A \equiv \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ \theta_x & \theta_y & \theta_z \\ (x_A - x_E) & (y_A - y_E) & (z_A - z_E) \end{vmatrix}$$

$$\begin{cases} a_{Ax} = \theta_y (z_A - z_E) - \theta_z (y_A - y_E) \\ a_{Ay} = \theta_z (x_A - x_E) - \theta_x (z_A - z_E) \\ a_{Az} = \theta_x (y_A - y_E) - \theta_y (x_A - x_E) \end{cases}$$

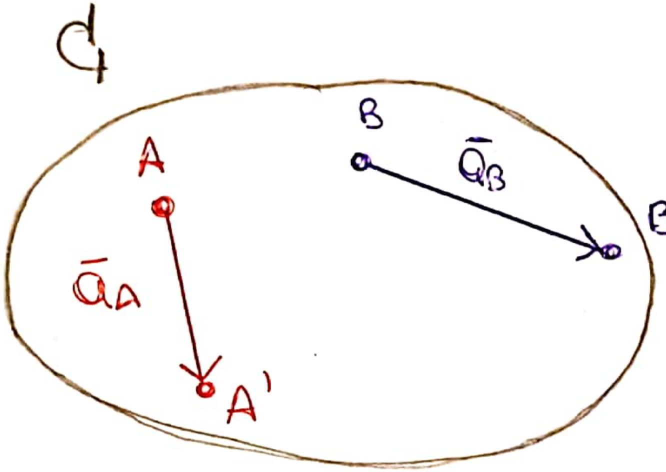
$$\underbrace{\begin{Bmatrix} a_{Ax} \\ a_{Ay} \\ a_{Az} \end{Bmatrix}}_{\bar{a}_A} = \underbrace{\begin{bmatrix} 0 & -\theta_z & +\theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{bmatrix}}_{[\bar{\theta}]} \cdot \underbrace{\begin{Bmatrix} (x_A - x_E) \\ (y_A - y_E) \\ (z_A - z_E) \end{Bmatrix}}_{(\overline{A - E})}$$

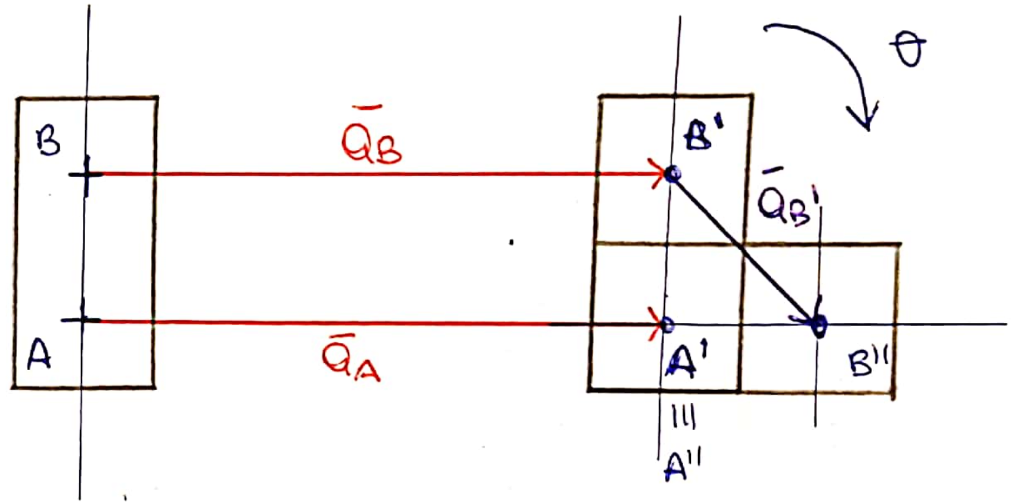
MATRIZ O TENSOR DE ROTACIÓN

$\downarrow$

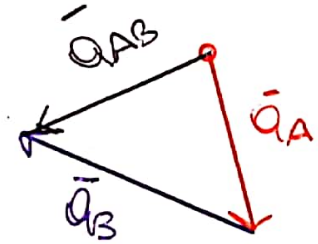
ANTISIMÉTRICA.

08 - DESPLAZAMIENTOS RELATIVOS:





$$\bar{Q}_{AB} = \bar{Q}_A - \bar{Q}_B$$



MOVIM RÍGIDOS

$\nexists$ DESPLAZ. RELATIVOS $\rightarrow$ TRASLACIÓN

$\exists$ " " $\rightarrow$ ROTACIÓN.

06) – DESPLAZAMIENTOS RELATIVOS ESPECÍFICOS:

I) VARIACIONES DE DISTANCIA

II) VARIACIONES ANGULARES

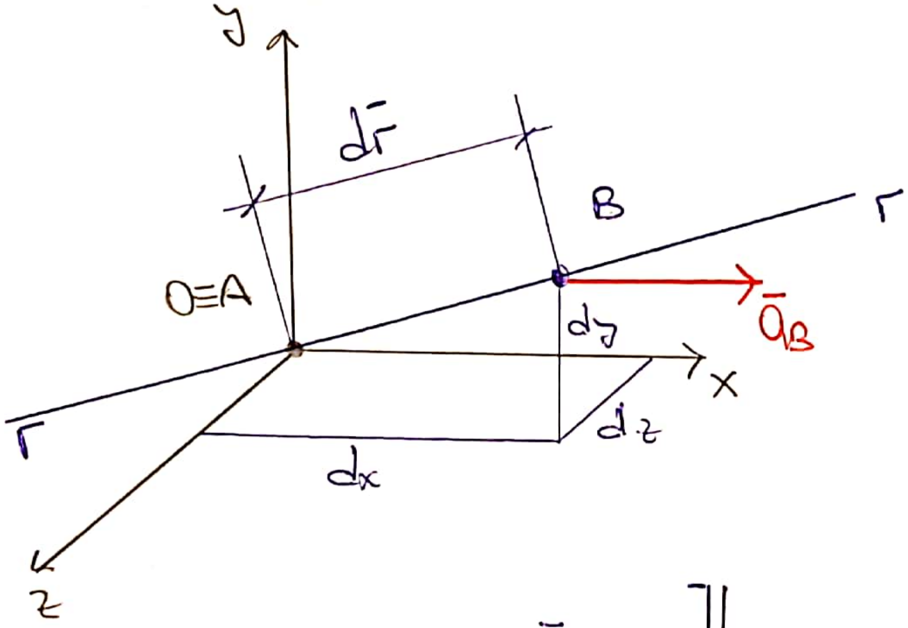
III) CAMBIOS DE VOLUMEN

→ ESTÁN DIRECTAMENTE ASOCIADAS A QUÉ ??)

→ DESPLAZAMIENTOS RELATIVOS

$\boxed{i}$ SI $\exists$ I, II, III → SÍ O SÍ EXISTEN DESPLAZAM. RELATIVOS

$\boxed{ii}$ SI $\exists$ DESPLAZAM. RELATIVOS → PODRÁN O NO EXISTIR I, II ó III



$$\begin{cases} a_{xB} = u_B - u_A = du = \dfrac{\partial u}{\partial x} dx + \dfrac{\partial u}{\partial y} dy + \dfrac{\partial u}{\partial z} dz. \\ a_{yB} = v_B - v_A = dv = \dfrac{\partial v}{\partial x} dx + \dfrac{\partial v}{\partial y} dy + \dfrac{\partial v}{\partial z} dz. \\ a_{zB} = w_B - w_A = dw = \dfrac{\partial w}{\partial x} dx + \dfrac{\partial w}{\partial y} dy + \dfrac{\partial w}{\partial z} dz \end{cases}$$

$$d\bar{r} = dr_x \breve{i} + dr_y \breve{j} + dr_z \breve{k}.$$

Si adopto $|d\bar{r}| = 1 = dr.$

$$d\bar{r} = \underbrace{dx}_{l} \breve{i} + \underbrace{dy}_{m} \breve{j} + \underbrace{dz}_{n.} \breve{k}$$

$$\bar{\varepsilon}_r = \lim_{\Delta r \to 0} \frac{\bar{a}_B}{\Delta r}.$$

$$\bar{\varepsilon}_r = \underbrace{\frac{du}{dx}}_{\varepsilon_{rx}} \breve{i} + \underbrace{\frac{dv}{dy}}_{\varepsilon_{ry}} \breve{j} + \underbrace{\frac{dw}{dz}}_{\varepsilon_{rz}} \breve{k}.$$

$u \to$ 'x'

$v \to$ 'y'

$w \to$ 'z'

$$\begin{Bmatrix} \varepsilon_{rx} \\ \varepsilon_{ry} \\ \varepsilon_{rz} \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}} \begin{Bmatrix} l \\ m \\ n \end{Bmatrix}$$

→ MATRIZ O TENSOR DE TRANSFORMACIÓN DE LOS DESPLAZAMIENTOS RELATIVOS.

[TDR]

→ SE DESCOMPONE EN LA SUMA DE 1 MATRIZ SIMÉTRICA Y OTRA ANTISIMÉTRICA.

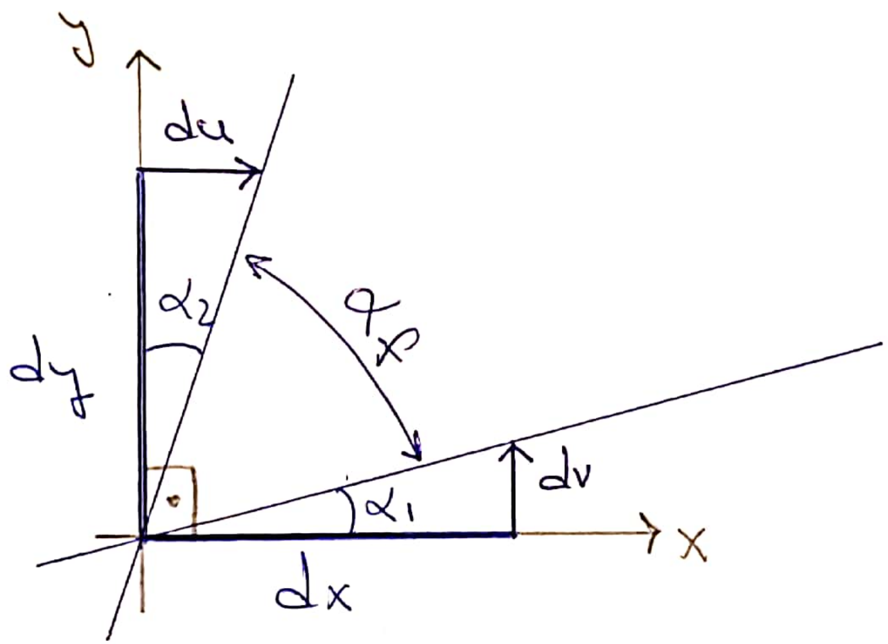
$$\underbrace{\begin{bmatrix} \overset{\varepsilon_{xx}}{\dfrac{\partial u}{\partial x}} & \overset{\varepsilon_{yx}}{\dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right)} & \overset{\varepsilon_{zx}}{\dfrac{1}{2}\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\right)} \\ \underset{\varepsilon_{xy}}{\dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right)} & \underset{\varepsilon_{yy}}{\dfrac{\partial v}{\partial y}} & \underset{\varepsilon_{zy}}{\dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}+\dfrac{\partial v}{\partial z}\right)} \\ \underset{\varepsilon_{xz}}{\dfrac{1}{2}\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\right)} & \underset{\varepsilon_{yz}}{\dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}+\dfrac{\partial v}{\partial z}\right)} & \underset{\varepsilon_{zz}}{\dfrac{\partial w}{\partial z}} \end{bmatrix}}_{\text{SIMÉTRICA.}} + \underbrace{\begin{bmatrix} 0 & -\dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}-\dfrac{\partial u}{\partial y}\right) & -\dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}-\dfrac{\partial u}{\partial z}\right) \\ -\dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}-\dfrac{\partial v}{\partial x}\right) & 0 & -\dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}-\dfrac{\partial v}{\partial z}\right) \\ -\dfrac{1}{2}\left(\dfrac{\partial u}{\partial z}-\dfrac{\partial w}{\partial x}\right) & -\dfrac{1}{2}\left(\dfrac{\partial v}{\partial z}-\dfrac{\partial w}{\partial y}\right) & 0 \end{bmatrix}}_{\text{ANTISIMÉTRICA.}}$$

$[TD]$

$[TR] = [T\theta]$.

$$\{\bar{\varepsilon}_r\} = [TDR]\{\check{n}_r\} = \Big\{[TD] + [TR]\Big\}\{\check{n}_r\}.$$

$$\{\bar{\varepsilon}_r\} = \underbrace{[TD]\{\check{n}_r\}}_{\{\bar{\varepsilon}_r^{D}\}} + \underbrace{[TR]\{\check{n}_r\}}_{\{\bar{\varepsilon}_r^{R}\}}$$



$$\alpha_1 = \frac{dv}{dx}$$

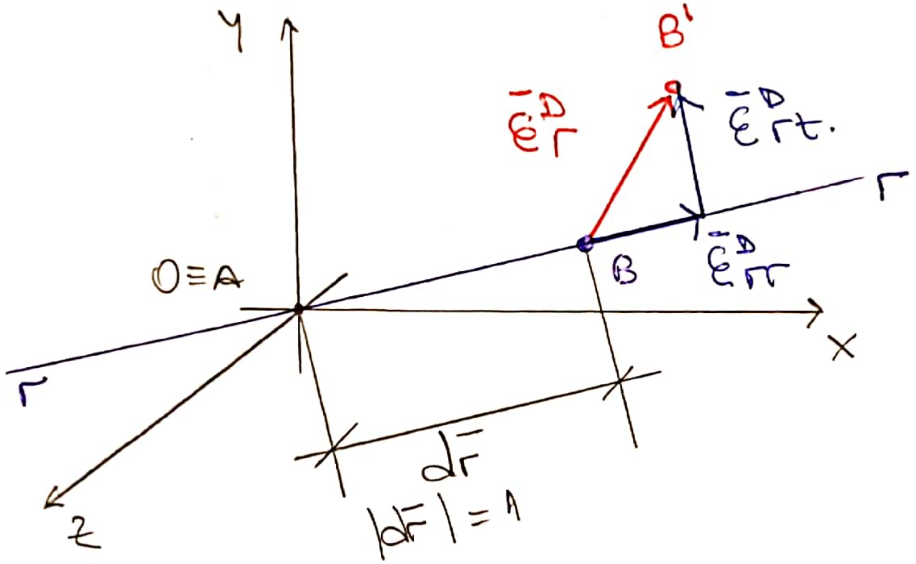
$$\alpha_2 = \frac{du}{dy}$$

$$\underbrace{90^\circ}_{\alpha_0} - \alpha_f = \underbrace{\gamma_{xy}}_{\text{distorsión.}} = \frac{dv}{dx} + \frac{du}{dy} =$$

$$= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}.$$

$$\gamma_{xy} = 2\,\varepsilon_{xy} = 2\,\varepsilon_{yx}.$$

$$\boxed{\gamma_{ij} = 2\,\varepsilon_{ij}}$$



$\bar{\varepsilon}_{rr}^{D}$: DEFORMACIÓN ESPECÍFICA LONGITUDINAL

$\bar{\varepsilon}_{rt}^{D}$: " " TRANSVERSAL. ~~[illegible]~~

$$[TD] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zx} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \frac{\gamma_{yx}}{2} & \frac{\gamma_{zx}}{2} \\ \frac{\gamma_{xy}}{2} & \varepsilon_{yy} & \frac{\gamma_{zy}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \varepsilon_{zz} \end{bmatrix}$$