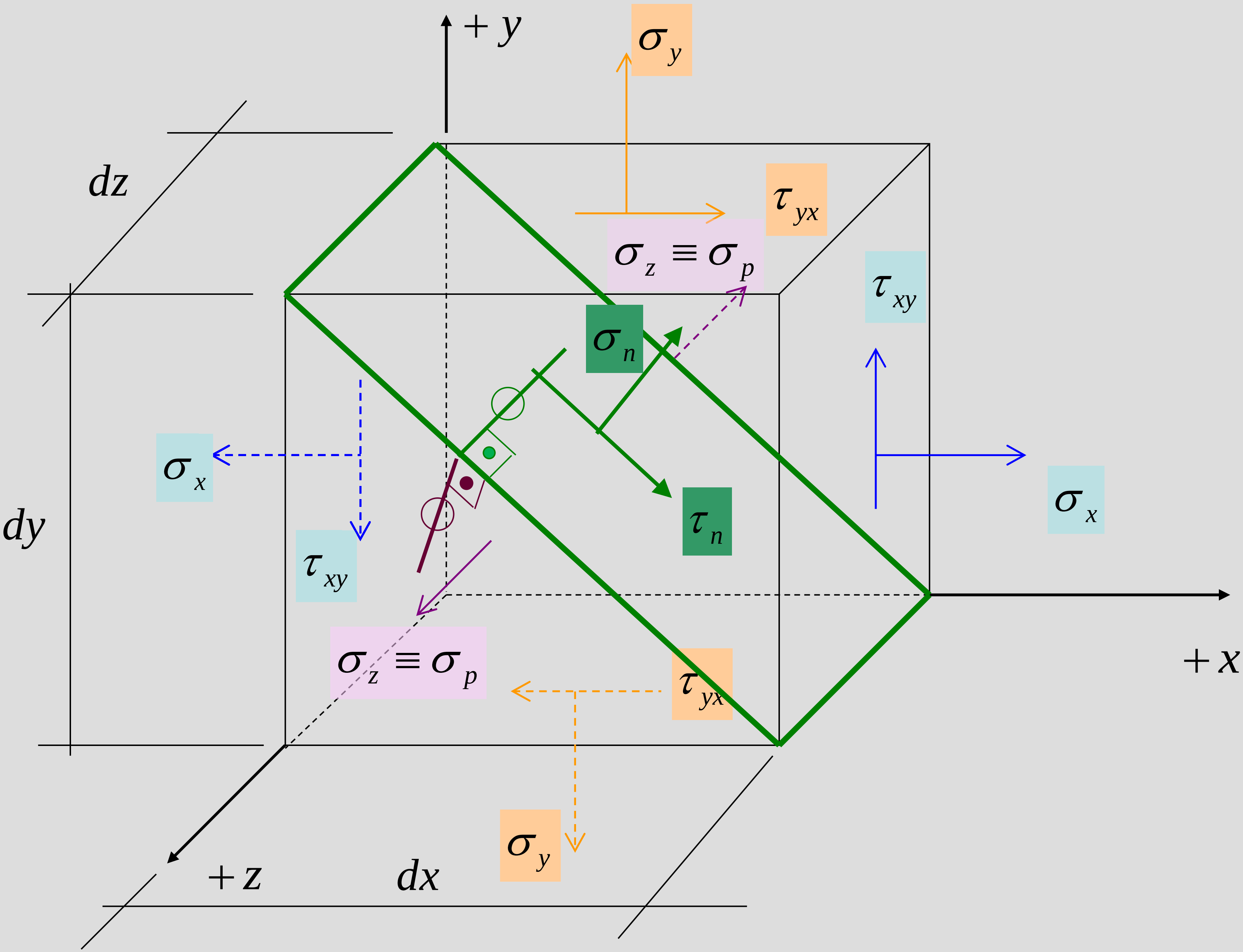
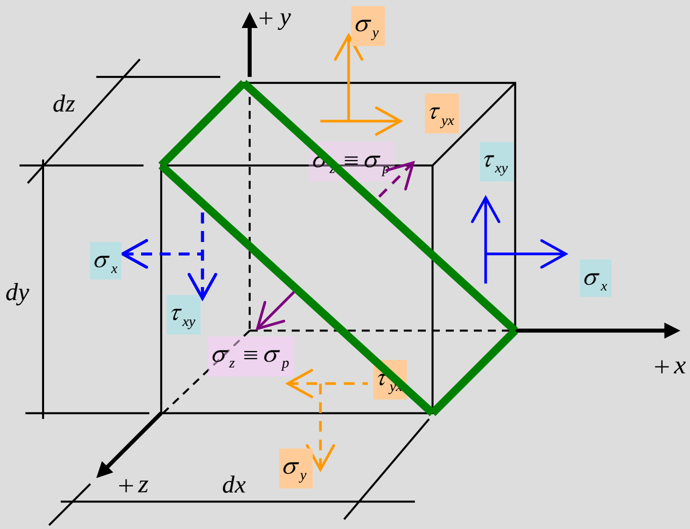
Tensiones en el haz de planos de eje sostén una dirección principal

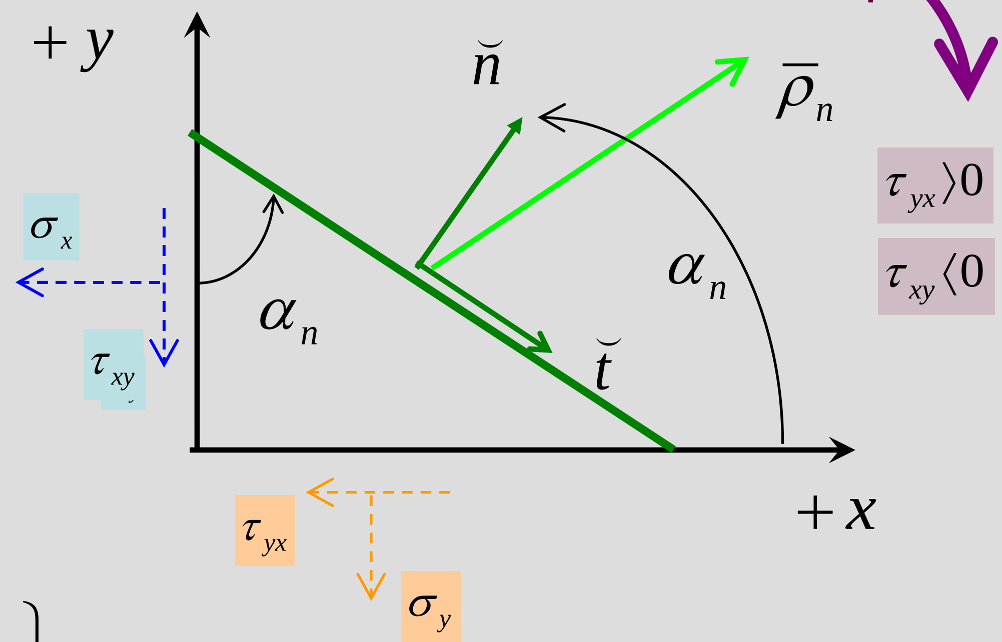


+ y
σ y
τ yx
σ z ≡ σ p
τ xy
dz
σ n
σ x
τ n
σ x
dy
τ xy
σ z ≡ σ p
τ yx
+ x
σ y
+ z
dx

HAZ DE PLANOS $n_z = 0$



Nueva convención de τ



$$\begin{Bmatrix} \rho_{nx} \\ \rho_{ny} \\ \rho_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \begin{Bmatrix} \cos\alpha_n \\ sen\alpha_n \\ 0 \end{Bmatrix}$$

$$\breve{n} = \cos\alpha_n \breve{i} + sen\,\alpha_n \breve{j} + 0\breve{k}$$

$$\sigma_n = \bar{\rho}_n \cdot \breve{n}$$

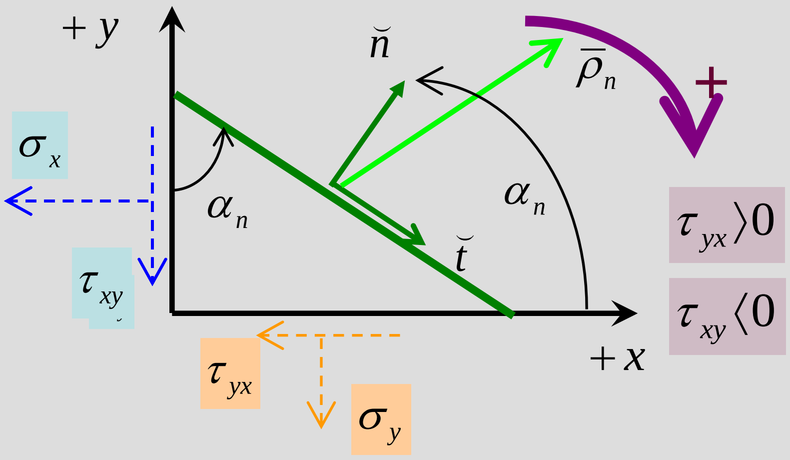
$$\tau_n = \bar{\rho}_n \cdot \breve{t}$$

$$\breve{t} = sen\,\alpha_n \breve{i} - \cos\alpha_n \breve{j} + 0\breve{k}$$

$$\rho_{nx} = \sigma_x \cos\alpha_n + \tau_{yx} sen\,\alpha_n$$

$$\rho_{ny} = \tau_{yx} \cos\alpha_n + \sigma_y sen\,\alpha_n$$

$$\rho_{nz} = 0$$



$$\breve{n} = \cos\alpha_n \breve{i} + sen\,\alpha_n \breve{j} + 0\breve{k}$$

$$\breve{t} = sen\,\alpha_n \breve{i} - \cos\alpha_n \breve{j} + 0\breve{k}$$

$$\rho_{nx} = \sigma_x \cos\alpha_n + \tau_{yx} sen\,\alpha_n$$

$$\rho_{ny} = \tau_{yx} \cos\alpha_n + \sigma_y sen\,\alpha_n$$

$$\rho_{nz} = 0$$

$$\sigma_n = \bar{\rho}_n \cdot \breve{n}$$

$$\sigma_n = \sigma_x \cos^2\alpha_n + \tau_{yx} sen\,\alpha_n \cos\alpha_n + \tau_{yx} \cos\alpha_n sen\,\alpha_n + \sigma_y sen^2\alpha_n$$

$$\boxed{\sigma_n = \sigma_x \cos^2\alpha_n + \sigma_y sen^2\alpha_n + 2\tau_{yx} sen\,\alpha_n \cos\alpha_n}$$

$$\tau_n = \bar{\rho}_n \cdot \breve{t}$$

$$\tau_n = \sigma_x \cos\alpha_n sen\,\alpha_n + \tau_{yx} sen^2\alpha_n - \tau_{yx} \cos^2\alpha_n - \sigma_y sen\,\alpha_n \cos\alpha_n$$

$$\boxed{\tau_n = (\sigma_x - \sigma_y)\cos\alpha_n sen\,\alpha_n - \tau_{yx}(\cos^2\alpha_n - sen^2\alpha_n)}$$

Expresiones en función del ángulo doble

$$\sigma_n = \sigma_x \cos^2 \alpha_n + \sigma_y sen^2 \alpha_n + 2\tau_{yx} sen\,\alpha_n \cos\alpha_n$$

$$\tau_n = \left(\sigma_x - \sigma_y\right)\cos\alpha_n sen\,\alpha_n - \tau_{yx}\left(\cos^2\alpha_n - sen^2\alpha_n\right)$$

$$\cos^2\alpha + sen^2\alpha = 1 \qquad sum.m.a.m \qquad \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\cos^2\alpha - sen^2\alpha = \cos 2\alpha \qquad rest.m.a.m \qquad sen^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$2 sen\,\alpha \cos\alpha = sen 2\alpha$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha_n + \tau_{yx} sen 2\alpha_n$$

$$\tau_n = \left(\frac{\sigma_x - \sigma_y}{2}\right) sen 2\alpha_n - \tau_{yx}\cos 2\alpha_n$$

Planos y Tensiones principales en este haz de planos

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha_n + \tau_{yx} sen 2\alpha_n$$

$$\tau_n = \left(\frac{\sigma_x - \sigma_y}{2}\right) sen 2\alpha_n - \tau_{yx}\cos 2\alpha_n$$

$$\tau_p = 0 = \left(\frac{\sigma_x - \sigma_y}{2}\right) sen 2\alpha_p - \tau_{yx}\cos 2\alpha_p \quad \Rightarrow tg 2\alpha_p = \frac{2\tau_{yx}}{\sigma_x - \sigma_y}$$

$$2\alpha_1$$

$$2\alpha_2 = 2\alpha_1 + \pi$$

$$\alpha_2 = \alpha_1 + \frac{\pi}{2}$$

Tensiones normales máxima y mínima

$$\frac{\partial \sigma_n}{\partial \alpha} = 0 = -(\sigma_x - \sigma_y) sen 2\alpha_m + 2\tau_{yx}\cos 2\alpha_m \Rightarrow tg 2\alpha_m = \frac{2\tau_{yx}}{\sigma_x - \sigma_y}$$

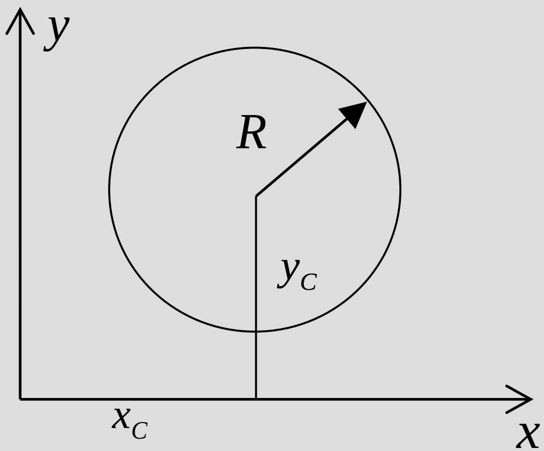
Los planos con valores máximo y mínimo de la tensión normal coinciden con los planos principales

Circunferencia de Mohr

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha_n + \tau_{yx} sen 2\alpha_n$$

$$\tau_n = \left(\frac{\sigma_x - \sigma_y}{2}\right) sen 2\alpha_n - \tau_{yx}\cos 2\alpha_n$$

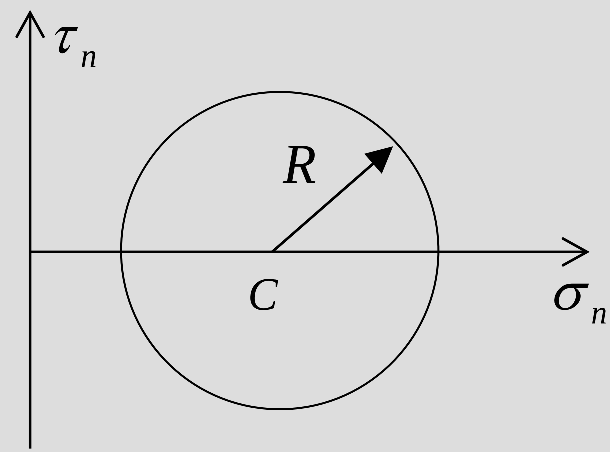
$$\sigma_n - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha_n + \tau_{yx} sen 2\alpha_n$$



$$(y - y_C)^2 + (x - x_C)^2 = R^2$$

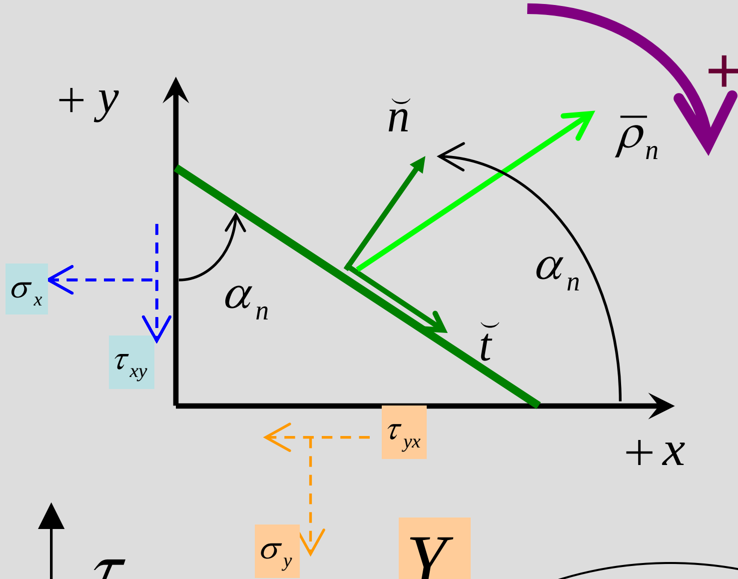
Elevando ambas al cuadrado y sumando m.a.m

$$\tau_n^2 + \left[\sigma_n - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{yx}^{\ 2}$$



$$C \equiv \left(\frac{\sigma_x + \sigma_y}{2};0\right) \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{yx}^{\ 2}}$$

Haz de planos $n_z = 0$



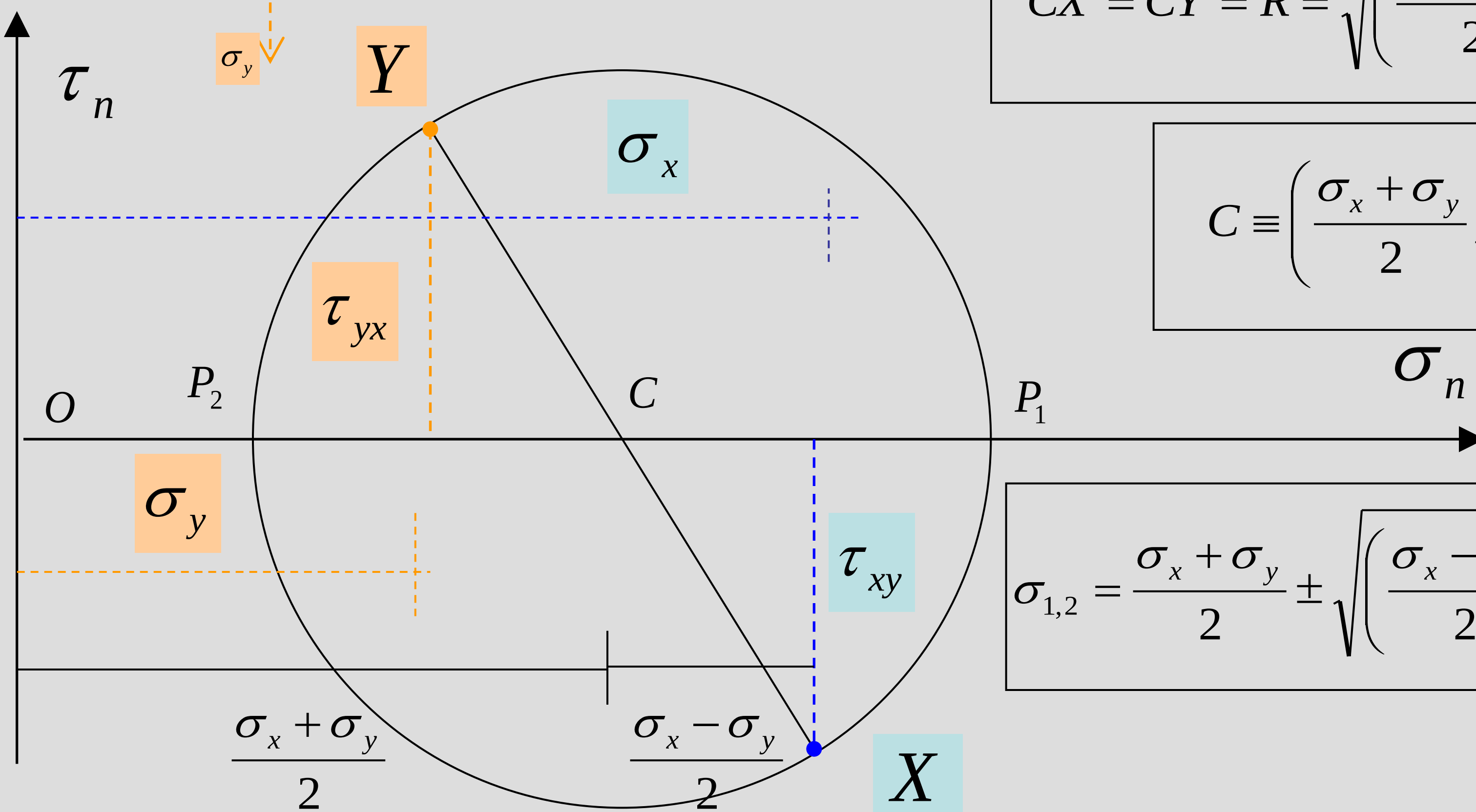
$$\tau_{xy} \langle 0$$

$$\sigma_x \rangle \sigma_y$$

$$\tau_{yx} \rangle 0$$

$$CX = CY = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{yx}^{\ 2}}$$

$$C \equiv \left(\frac{\sigma_x + \sigma_y}{2};0\right)$$



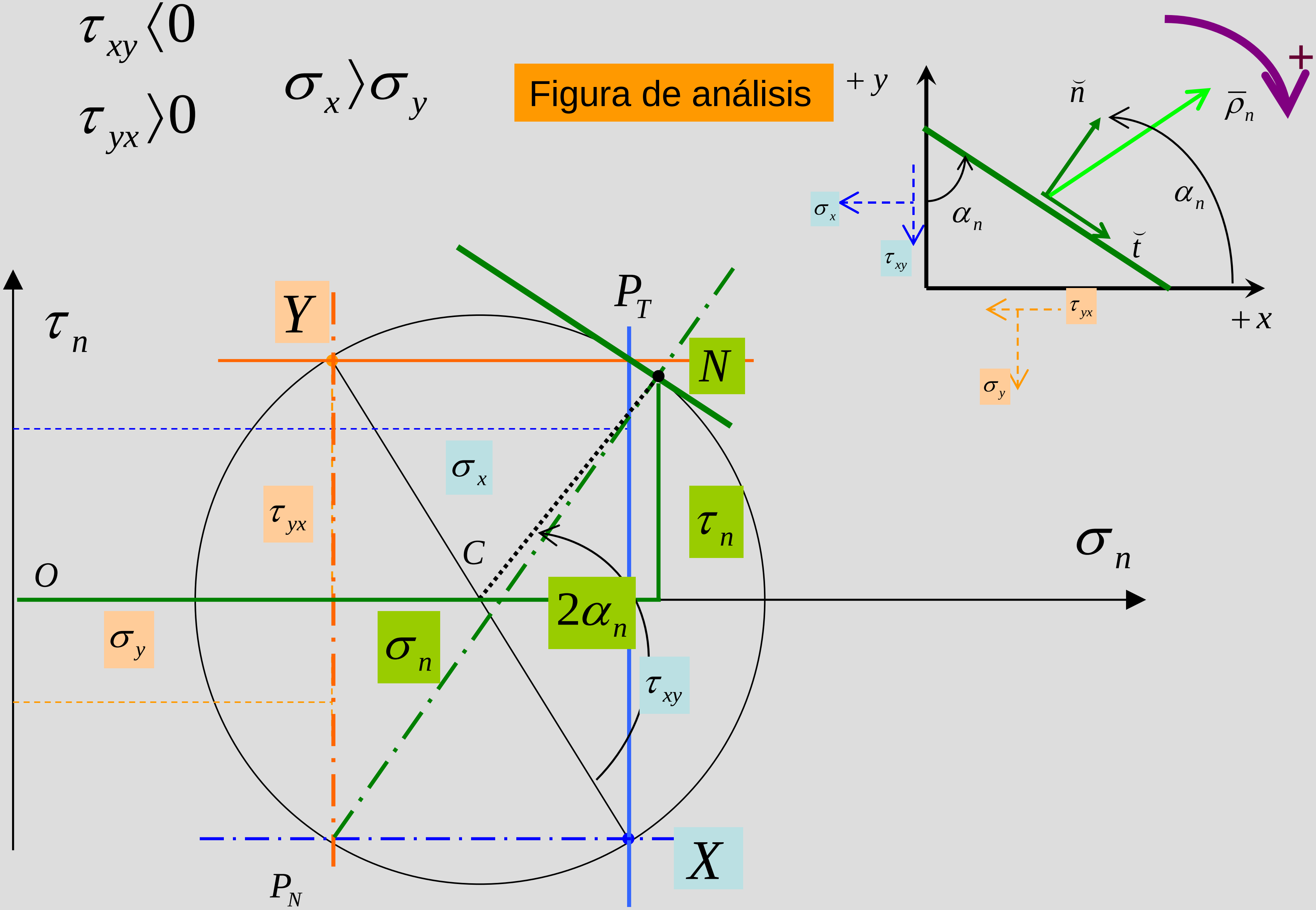
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{yx}^{\ 2}}$$

POLOS DE LA CIRCUNFERENCIA

$\tau_{xy} \langle 0$

$\tau_{yx} \rangle 0$

$\sigma_x \rangle \sigma_y$

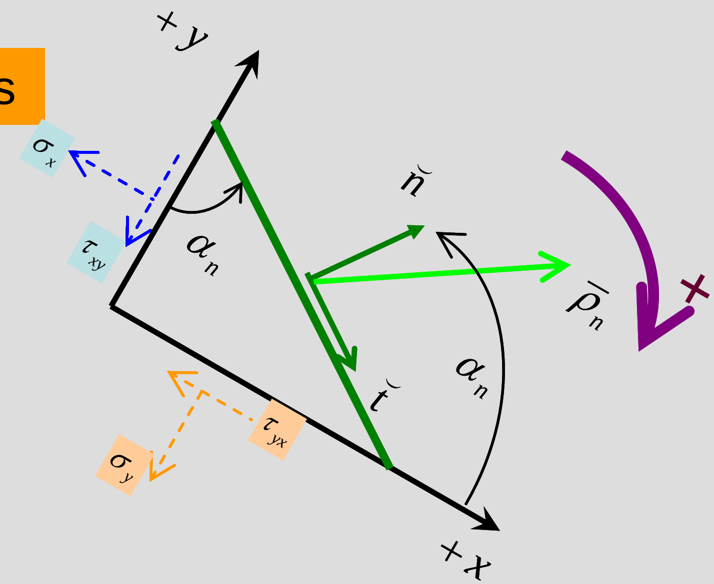


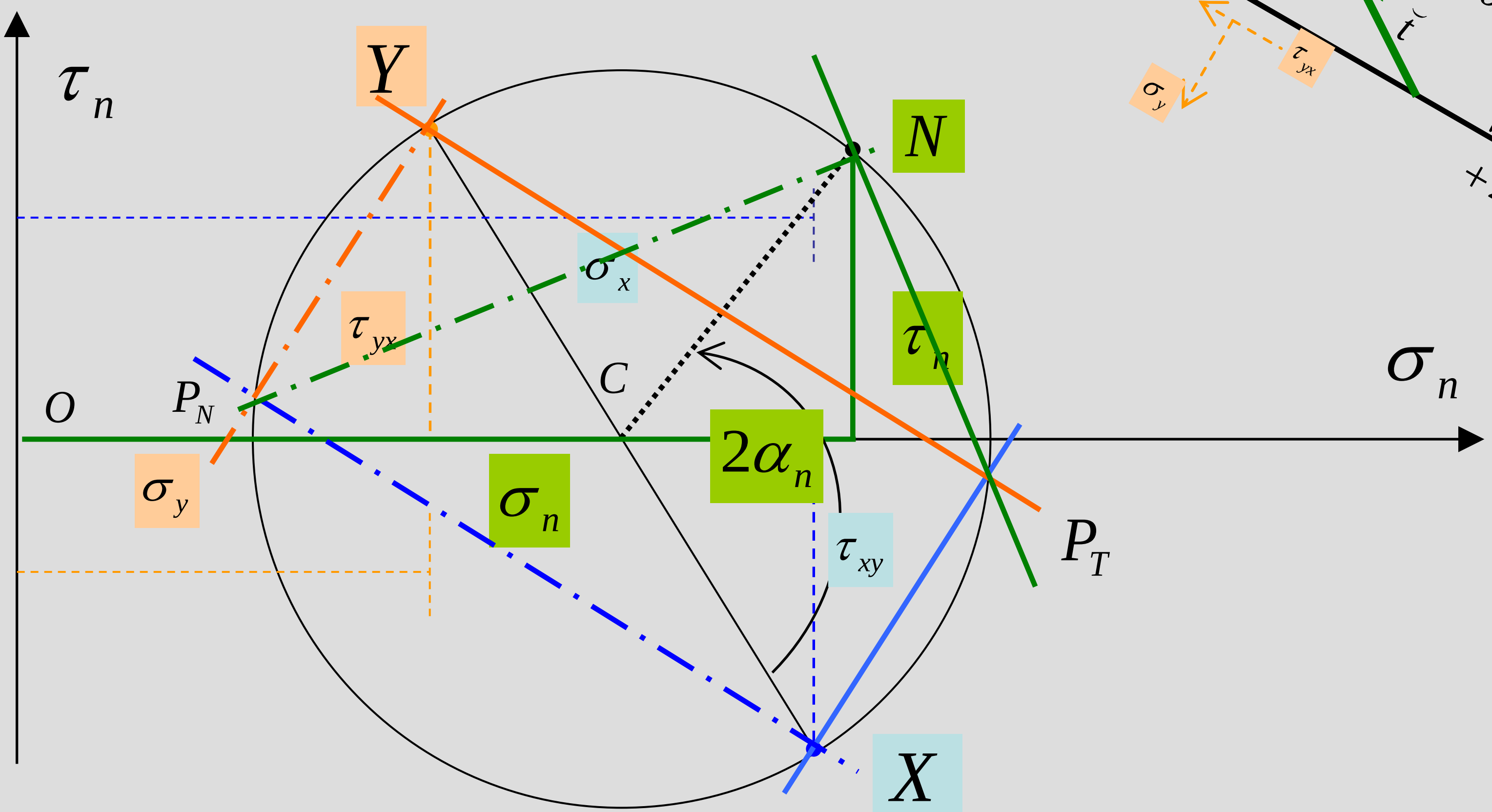
$\tau_{xy} \langle 0$

$\sigma_x \rangle \sigma_y$

$\tau_{yx} \rangle 0$

Figura de análisis





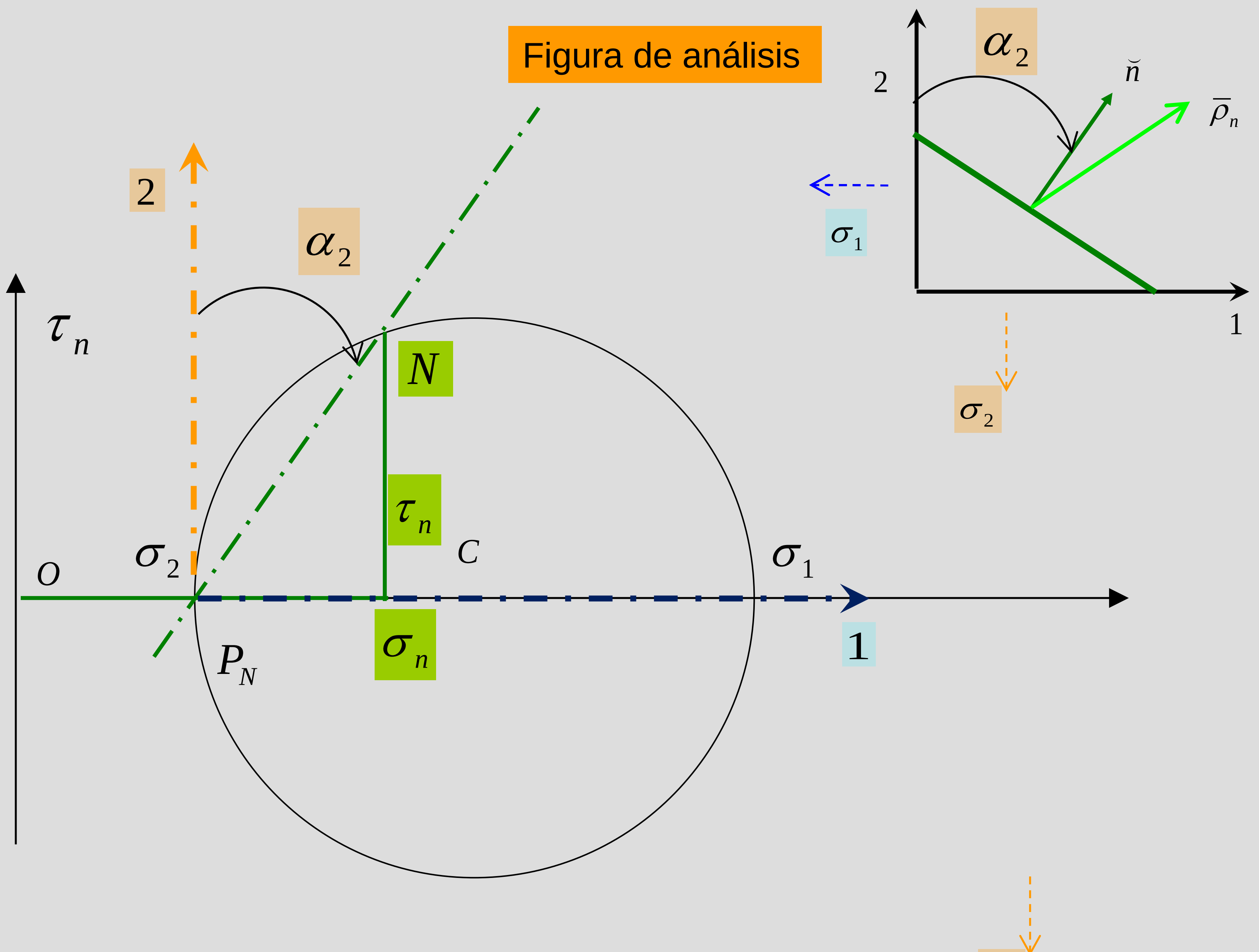
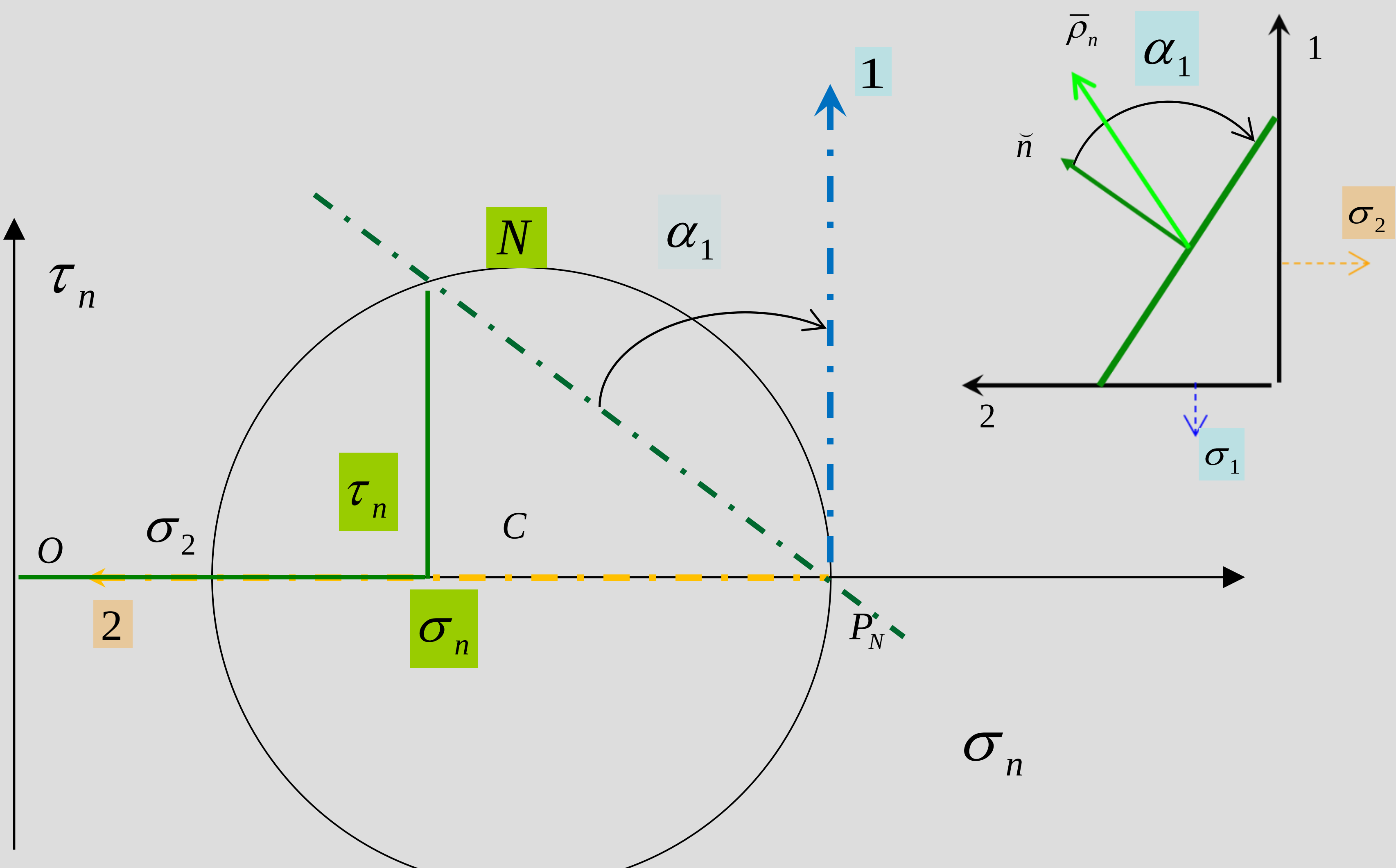
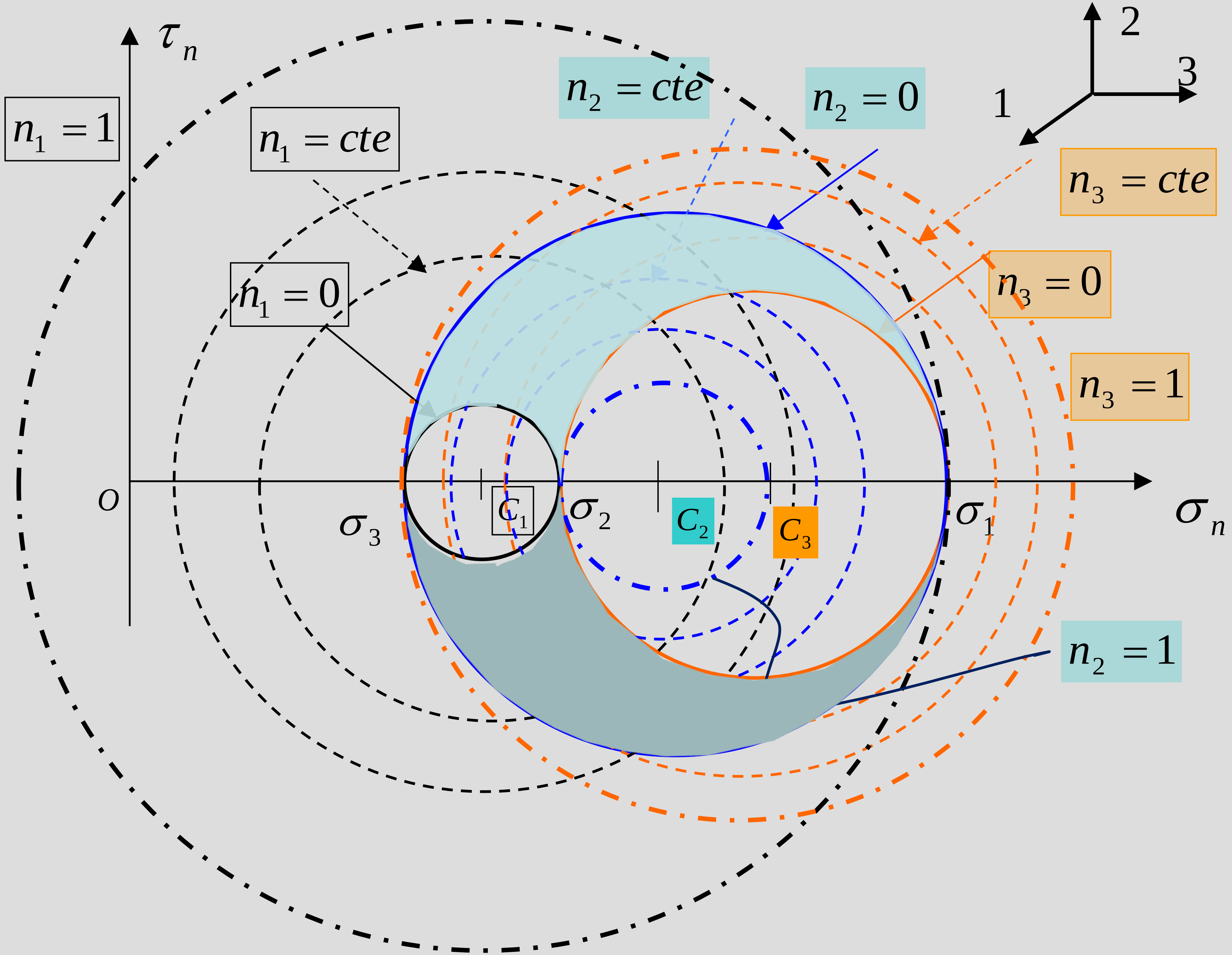


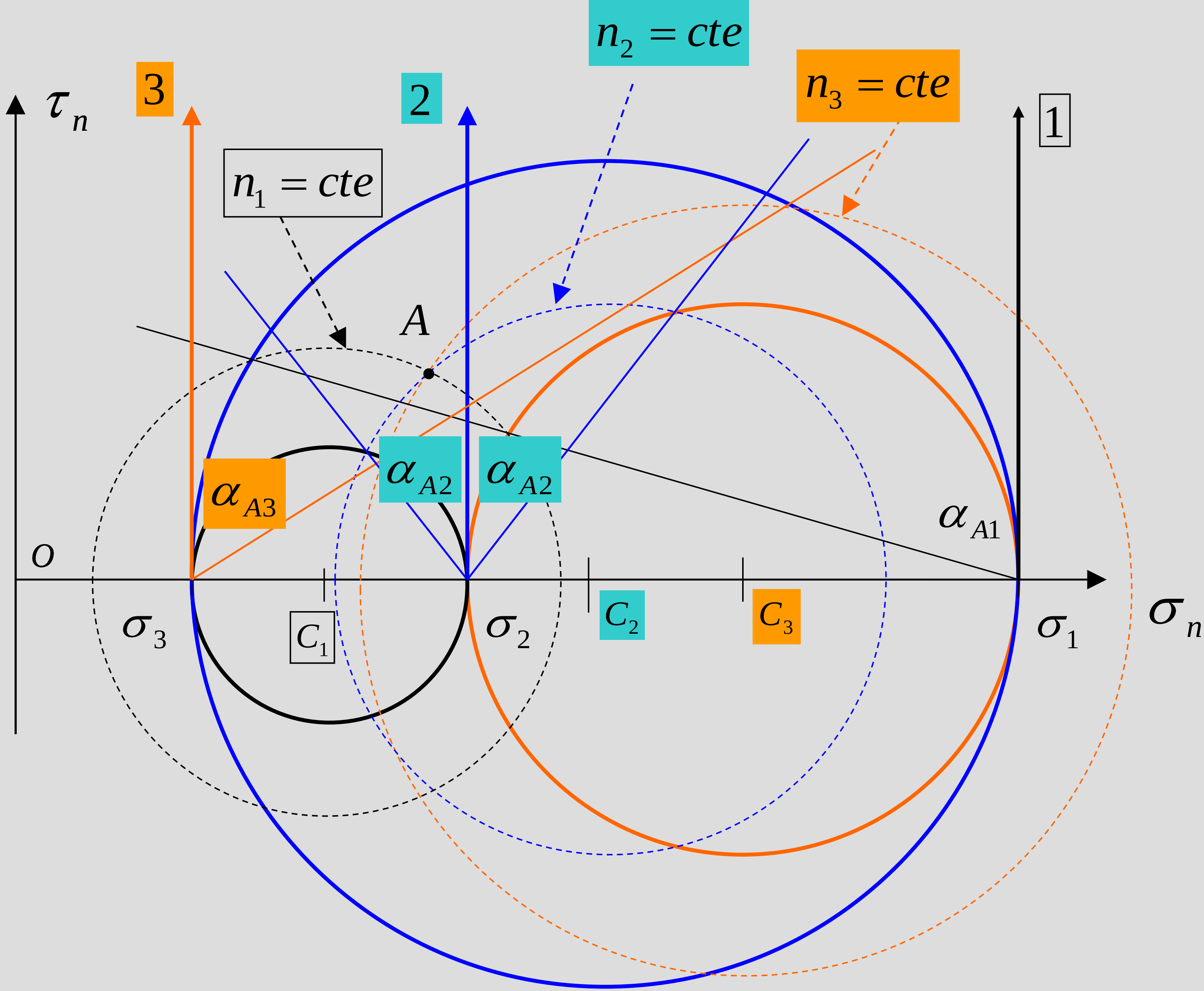
Figura de análisis
2
α_2
τ_n
N
τ_n
C
O
σ_2
σ_1
P_N
σ_n
1
α_2
2
$\breve{n}$
$\bar{\rho}_n$
σ_1
1
σ_2

Figura de análisis





τ_n
$n_1 = 1$
$n_1 = cte$
$n_1 = 0$
$n_2 = cte$
$n_2 = 0$
$n_2 = 1$
$n_3 = cte$
$n_3 = 0$
$n_3 = 1$
2
3
1
O
σ_3
C_1
σ_2
C_2
C_3
σ_1
σ_n



$n_2 = cte$
$n_3 = cte$
3
2
1
τ_n
$n_1 = cte$
A
α_{A3}
α_{A2}
α_{A2}
α_{A1}
O
σ_3
C_1
σ_2
C_2
C_3
σ_1
σ_n