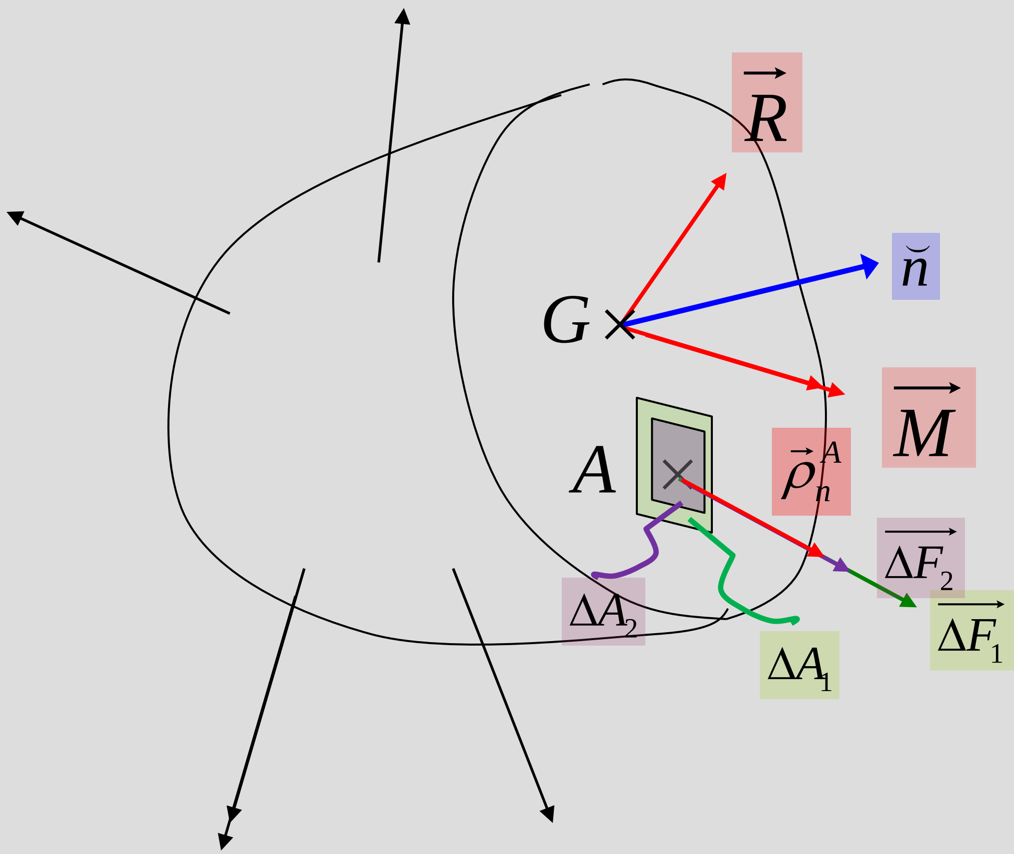
Estado de Tensión en puntos de un medio continuo

Tensión en un punto de un plano de un medio continuo



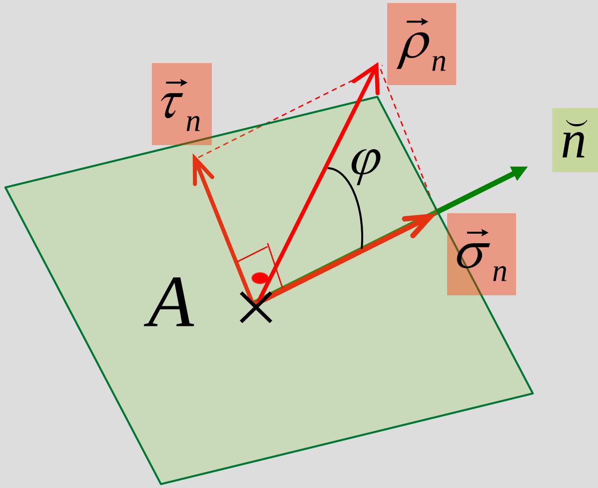
$$\frac{\Delta \vec{F}_1}{\Delta A_1} \rightarrow \frac{\Delta \vec{F}_2}{\Delta A_2}$$

$$\vec{\rho}_n^{\,A} = \lim_{\Delta_A \rightarrow 0} \frac{\Delta \vec{F}}{\Delta A} = \frac{d\vec{F}}{dA}$$

$$\left[\vec{\rho}_n^{\,A}\right] = \frac{[F]}{[L]^2}$$

$\vec{\rho}_n^{\,A}$: *tensión en el punto A ascociada al plano n*

Tensión Normal y Tangencial



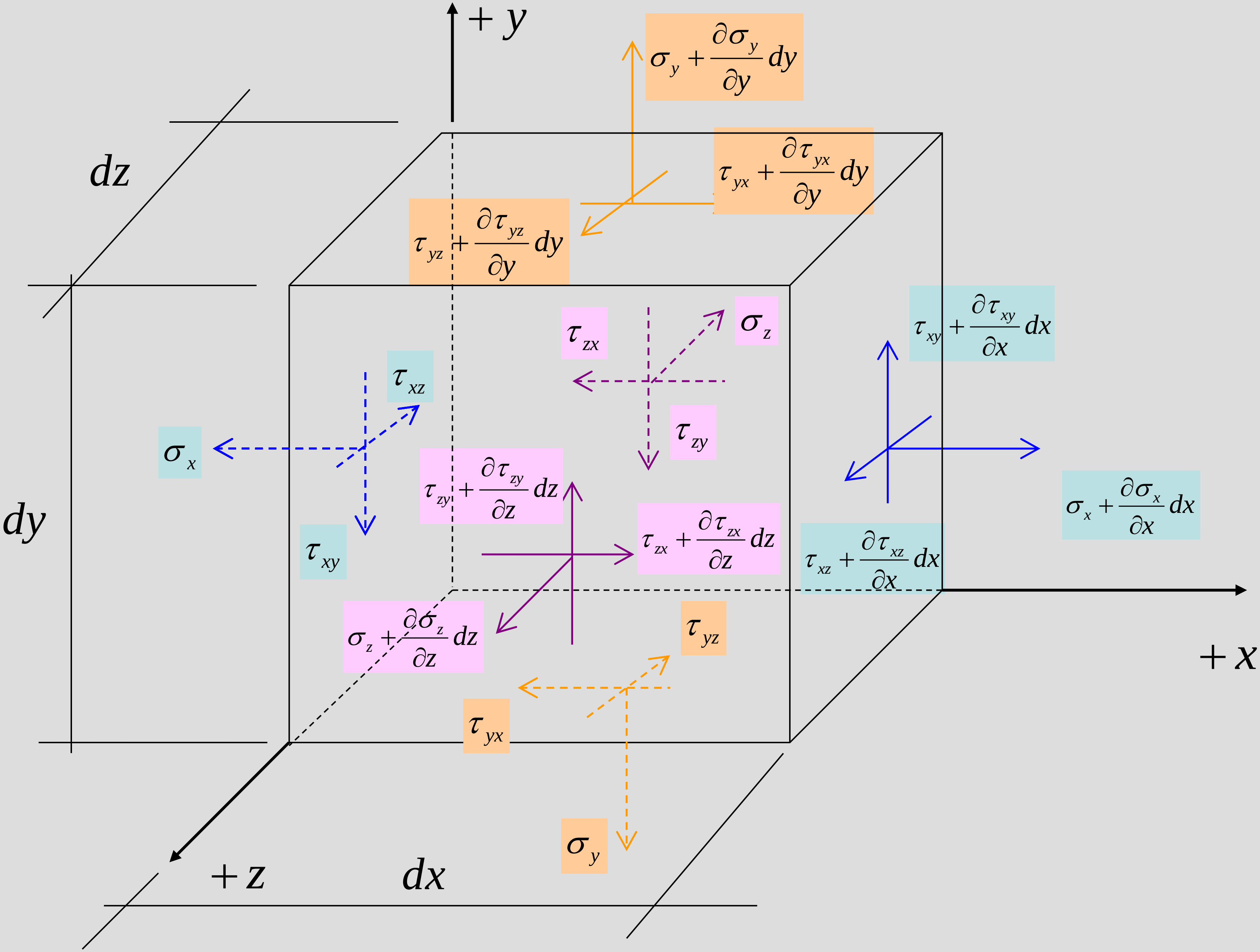
$$\breve{n} = n_x \breve{i} + n_y \breve{j} + n_z \breve{k}$$

$\vec{\sigma}_n$: *tensión normal ascociada al plano n*

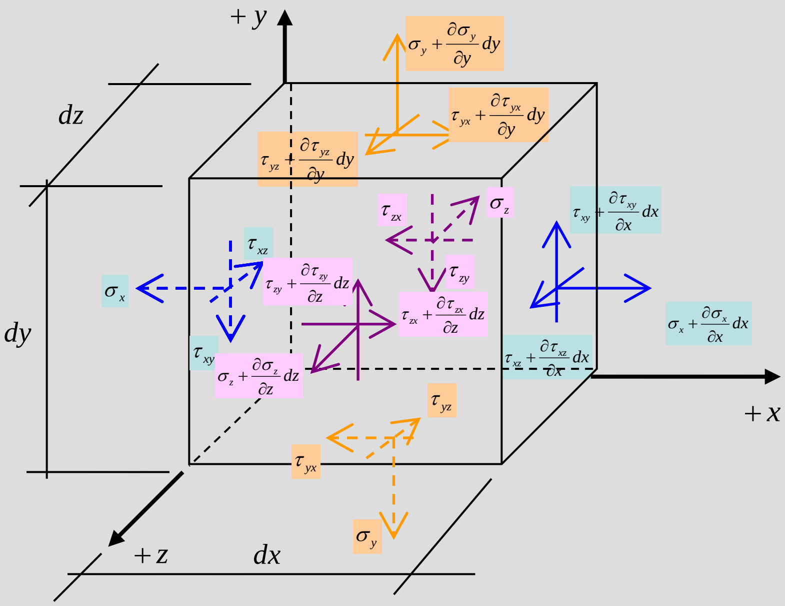
$$|\vec{\sigma}_n| = |\vec{\rho}_n| \cos\varphi \qquad |\vec{\sigma}_n| = \vec{\rho}_n \bullet \breve{n} \qquad \vec{\sigma}_n = (\vec{\rho}_n \bullet \breve{n})\breve{n}$$

$\vec{\tau}_n$: *tensión* tangencial *ascociada al plano n*

$$|\vec{\tau}_n| = |\vec{\rho}_n| \operatorname{sen}\varphi \qquad |\vec{\tau}_n| = \sqrt{|\vec{\rho}_n|^2 - |\vec{\sigma}_n|^2} \qquad \vec{\tau}_n = \breve{n} \times \vec{\rho}_n \times \breve{n}$$



$+y$
$+x$
$+z$
dx
dy
dz
$\sigma_y + \frac{\partial \sigma_y}{\partial y} dy$
$\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy$
$\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y} dy$
$\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx$
$\sigma_x + \frac{\partial \sigma_x}{\partial x} dx$
$\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x} dx$
$\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z} dz$
$\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz$
$\sigma_z + \frac{\partial \sigma_z}{\partial z} dz$
τ_{zx}
σ_z
τ_{zy}
τ_{xz}
σ_x
τ_{xy}
τ_{yz}
τ_{yx}
σ_y



ECUACIONES DIFERENCIALES DE EQUILIBRIO DEL CONTINUO

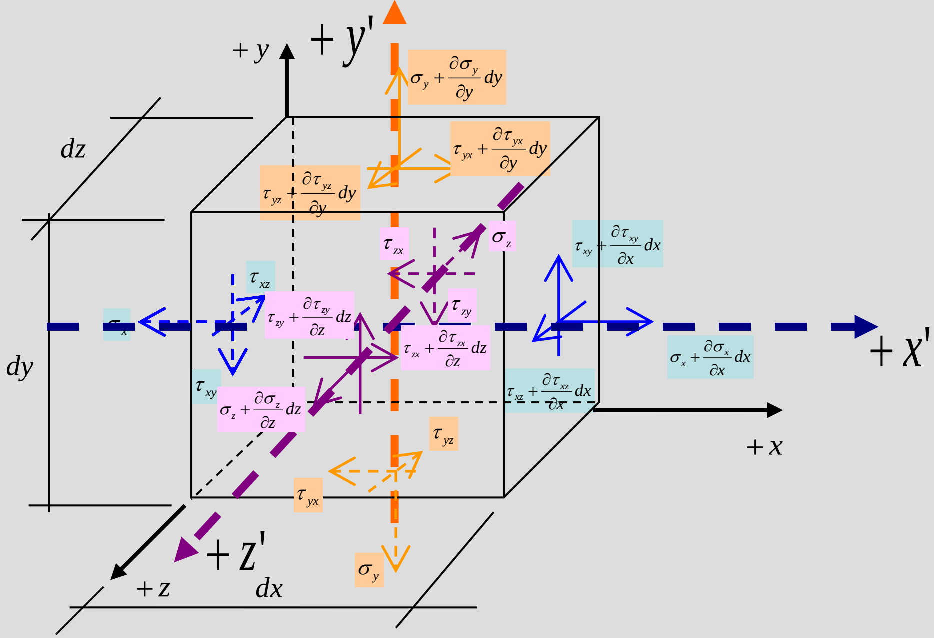
Cargas cuasiestáticas

$$\sum F_x = \left[-\sigma_x + \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right)\right] dydz + \left[-\tau_{yx} + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right)\right] dxdz + \left[-\tau_{zx} + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz\right)\right] dxdy + \gamma_x dxdydz - \frac{\partial^2 u}{\partial t^2}\frac{\gamma}{g} dxdydz = 0$$

$$\frac{\partial \sigma_x}{\partial x} dxdydz + \frac{\partial \tau_{yx}}{\partial y} dxdydz + \frac{\partial \tau_{zx}}{\partial z} dxdydz + \gamma_x dxdydz = 0 \Rightarrow \boxed{\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \gamma_x = 0}$$

$$\sum \Rightarrow F_y = 0 \quad \Rightarrow \boxed{\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \gamma_y = 0}$$

$$\sum \Rightarrow F_z = 0 \quad \Rightarrow \boxed{\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + \gamma_z = 0}$$



TEOREMA DE CAUCHY

$$\sum M_{x'} = +\left(\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y} dy\right) dxdz \frac{dy}{2} + \tau_{yz} dxdz \frac{dy}{2} - \left(\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z} dz\right) dxdy \frac{dz}{2} - \tau_{zy} dxdy \frac{dz}{2} = 0 \Rightarrow$$

$$\tau_{yz} = \tau_{zy}$$

Infinitésimo de orden superior

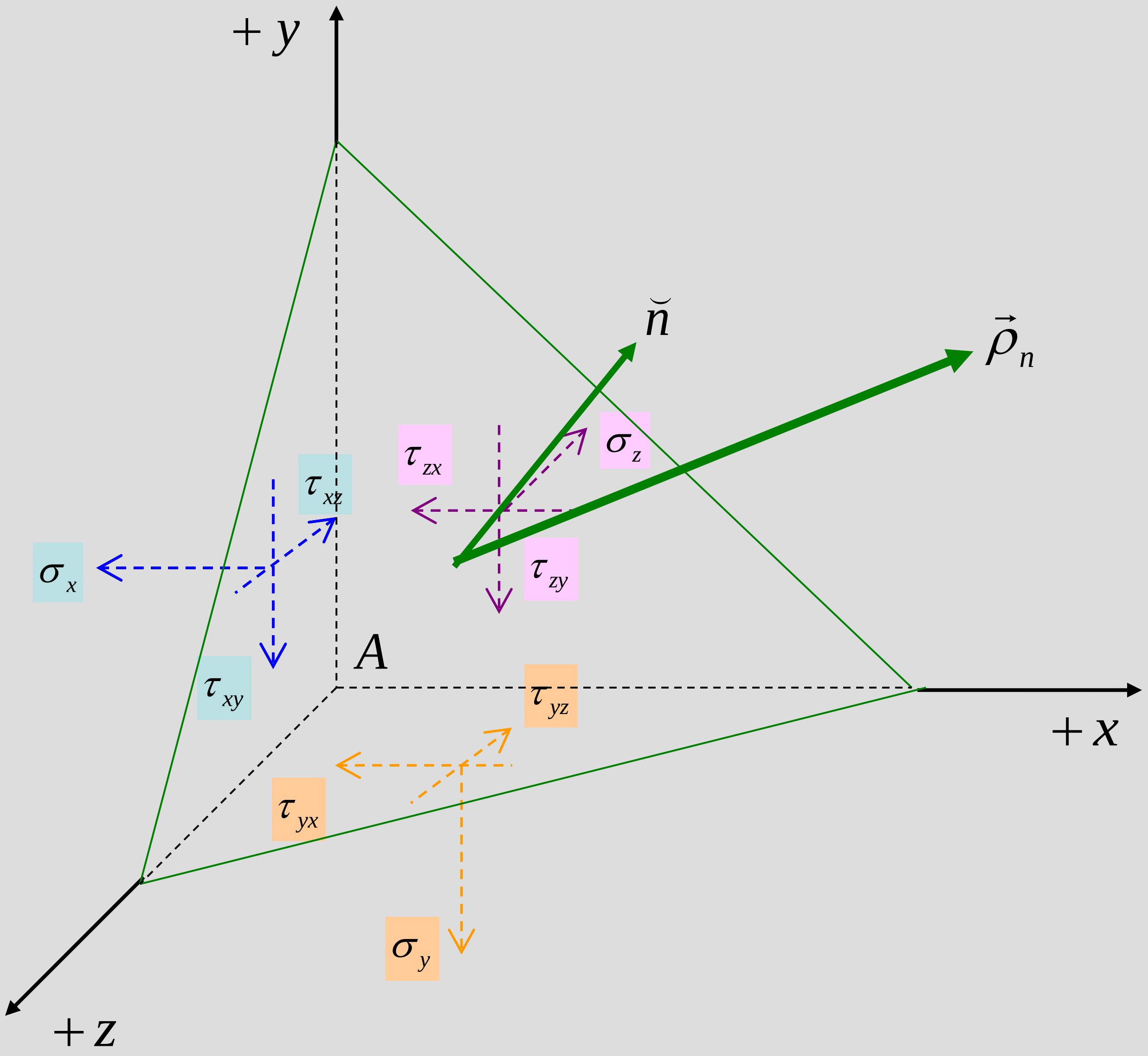
Infinitésimo de orden superior

$$\sum M_{y'} = 0 \Rightarrow$$

$$\tau_{xz} = \tau_{zx}$$

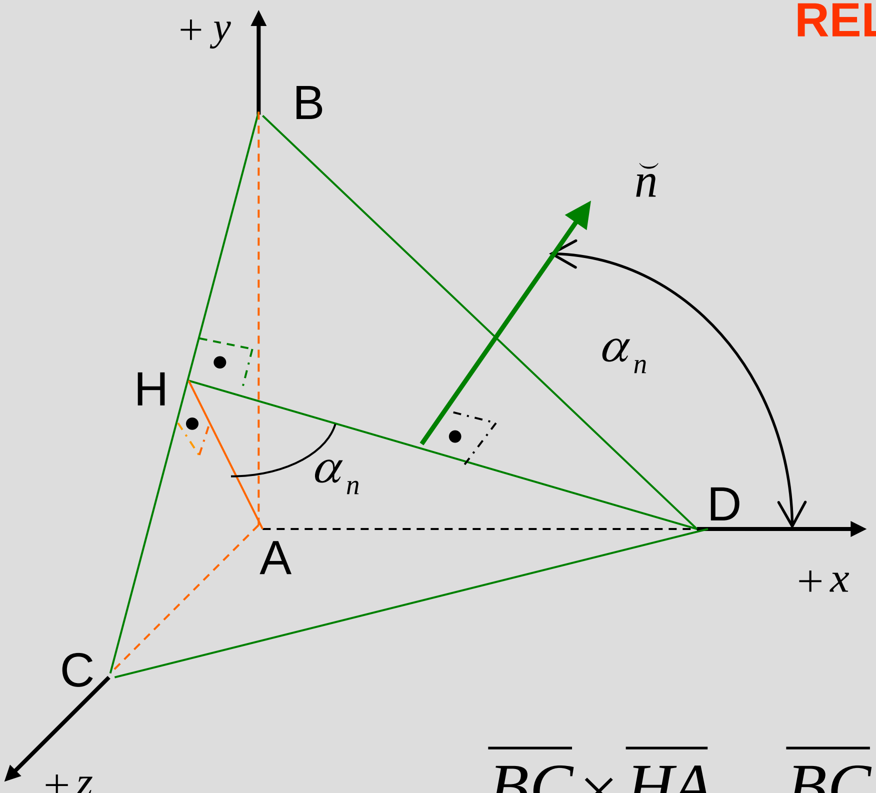
$$\sum M_{z'} = 0 \Rightarrow$$

$$\tau_{xy} = \tau_{yx}$$



$+y$
$\breve{n}$
$\vec{\rho}_n$
τ_{zx}
σ_z
τ_{xz}
σ_x
τ_{zy}
A
τ_{xy}
τ_{yz}
$+x$
τ_{yx}
σ_y
$+z$

RELACION DE AREAS DEL TETRAEDRO



$$\breve{n} = n_x \breve{i} + n_y \breve{j} + n_z \breve{k}$$

$$n_x = \cos\alpha_n$$

$$n_y = \cos\beta_n$$

$$n_z = \cos\gamma_n$$

Cosenos directores

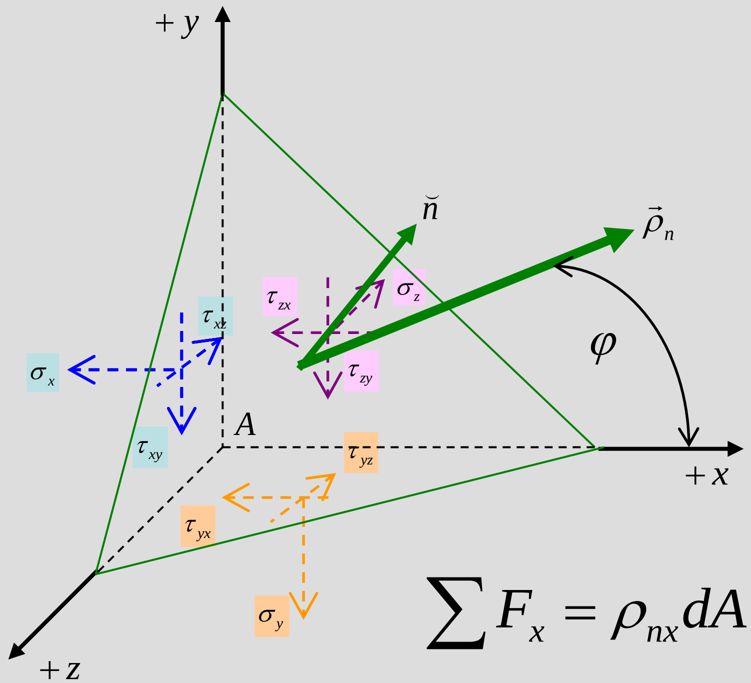
$$AreaBCD = \frac{\overline{BC} \times \overline{HD}}{2} = dA$$

$$AreaACB = \frac{\overline{BC} \times \overline{HA}}{2} = \frac{\overline{BC} \times \overline{HD} \cos\alpha}{2} = dA \cos\alpha_n = dA \times n_x$$

$$AreaACB = dA \times n_x$$

$$AreaACD = dA \times n_y$$

$$AreaABD = dA \times n_z$$



$$\breve{n} = n_x \breve{i} + n_y \breve{j} + n_z \breve{k}$$

$$\vec{\rho}_n = \rho_{nx} \breve{i} + \rho_{ny} \breve{j} + \rho_{nz} \breve{k}$$

$$\rho_n \times dA \times \cos\varphi = \rho_{nx} dA$$

$$\sum F_x = \rho_{nx} dA - \sigma_x dA \times n_x - \tau_{yx} dA \times n_y - \tau_{zx} dA \times n_z = 0$$

$$\rho_{nx} = \sigma_x \times n_x + \tau_{yx} \times n_y + \tau_{zx} \times n_z$$

$$\rho_{ny} = \tau_{xy} \times n_x + \sigma_y \times n_y + \tau_{zy} \times n_z$$

$$\rho_{nz} = \tau_{xz} \times n_x + \tau_{yz} \times n_y + \sigma_z \times n_z$$

$$\Rightarrow \begin{Bmatrix} \rho_{nx} \\ \rho_{ny} \\ \rho_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

$$\begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

TENSOR DE TENSIONES

•Caracteriza al Estado de Tensión (permite conocer el vector tensión asociado a cada plano pasante por el punto)

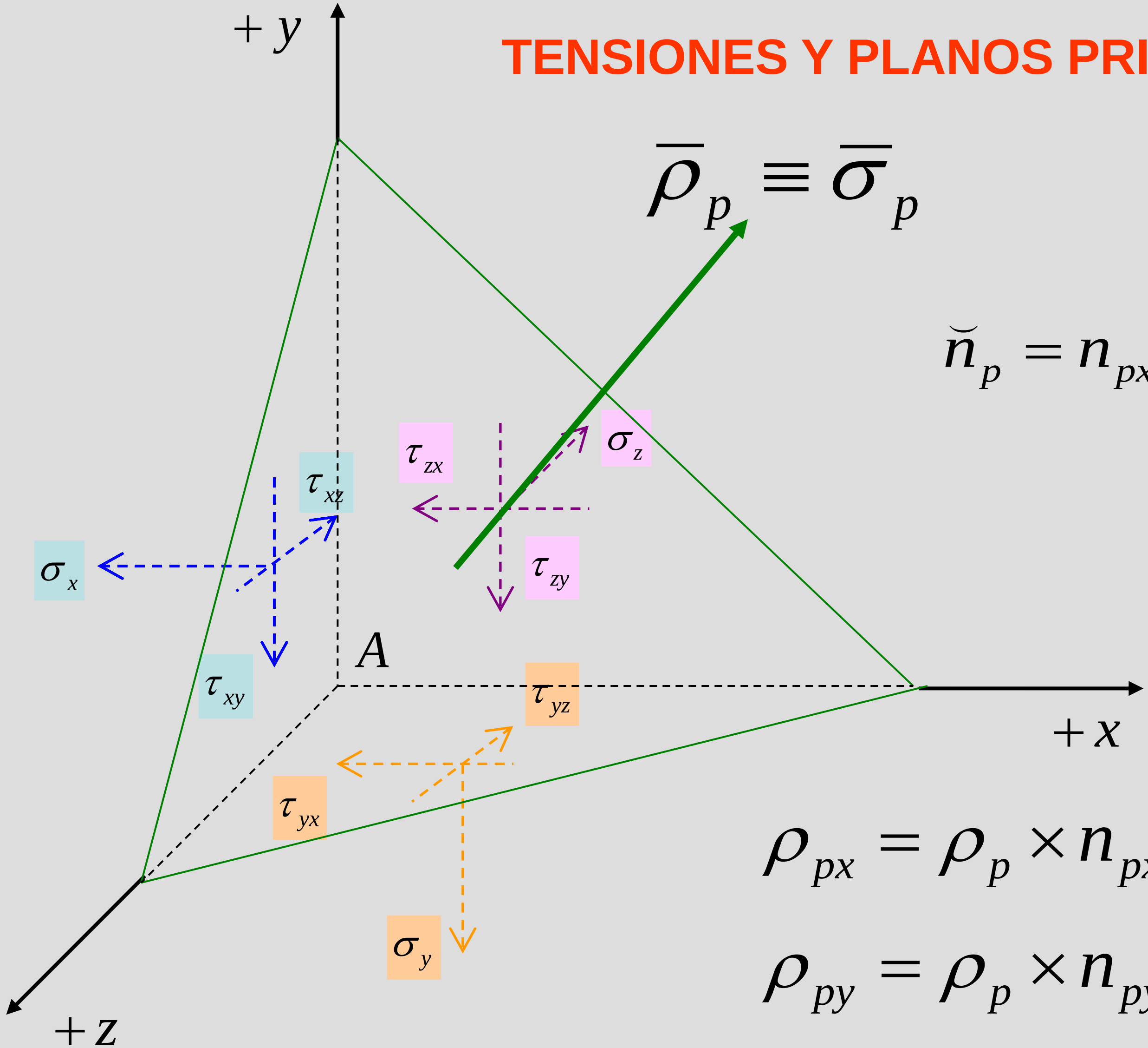
Si se cambian los ejes de referencia

•Al cambiar los planos coordenados (planos con que se caracteriza al estado de tensión), varía el tensor de tensiones

•El versor normal a un plano determinado tendrá componentes distintos

•El vector tensión asociado a ese plano, tendrá componentes distintos, pues es el mismo vector físico, representado en otra terna

TENSIONES Y PLANOS PRINCIPALES



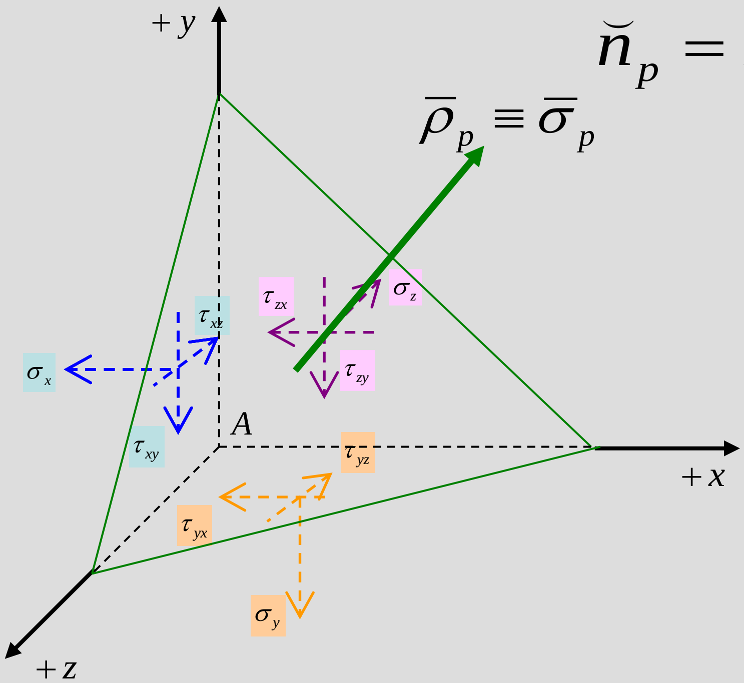
$$\bar{\rho}_p \equiv \bar{\sigma}_p$$

$$\breve{n}_p = n_{px}\breve{i} + n_{py}\breve{j} + n_{pz}\breve{k}$$

$$\rho_{px} = \rho_p \times n_{px} = \sigma_p \times n_{px}$$

$$\rho_{py} = \rho_p \times n_{py} = \sigma_p \times n_{py}$$

$$\rho_{pz} = \rho_p \times n_{pz} = \sigma_p \times n_{pz}$$



$$\breve{n}_p = n_{px}\breve{i} + n_{py}\breve{j} + n_{pz}\breve{k}$$

$$\rho_{px} = \rho_p \times n_{px} = \sigma_p \times n_{px}$$

$$\rho_{py} = \rho_p \times n_{py} = \sigma_p \times n_{py}$$

$$\rho_{pz} = \rho_p \times n_{pz} = \sigma_p \times n_{pz}$$

$$\begin{Bmatrix} \rho_{px} \\ \rho_{py} \\ \rho_{pz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{Bmatrix} n_{px} \\ n_{py} \\ n_{pz} \end{Bmatrix}$$

$$\sigma_p \times n_{px} = \sigma_x \times n_{px} + \tau_{yx} \times n_{py} + \tau_{zx} \times n_{pz} \quad 0 = (\sigma_x - \sigma_p) \times n_{px} + \tau_{yx} \times n_{py} + \tau_{zx} \times n_{pz}$$

$$\sigma_p \times n_{py} = \tau_{xy} \times n_{px} + \sigma_y \times n_{py} + \tau_{zy} \times n_{pz} \quad 0 = \tau_{xy} \times n_{px} + (\sigma_y - \sigma_p) \times n_{py} + \tau_{zy} \times n_{pz}$$

$$\sigma_p \times n_{pz} = \tau_{xz} \times n_{px} + \tau_{yz} \times n_{py} + \sigma_z \times n_{pz} \quad 0 = \tau_{xz} \times n_{px} + \tau_{yz} \times n_{py} + (\sigma_z - \sigma_p) \times n_{pz}$$

$$0 = (\sigma_x - \sigma_p) \times n_{px} + \tau_{yx} \times n_{py} + \tau_{zx} \times n_{pz}$$

$$0 = \tau_{xy} \times n_{px} + (\sigma_y - \sigma_p) \times n_{py} + \tau_{zy} \times n_{pz}$$

$$0 = \tau_{xz} \times n_{px} + \tau_{yz} \times n_{py} + (\sigma_z - \sigma_p) \times n_{pz}$$

$$\begin{Vmatrix} \sigma_x - \sigma_p & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - \sigma_p & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_p \end{Vmatrix} = 0$$

$$-\sigma_p^3 + I_1 \sigma_p^2 - I_2 \sigma_p + I_3 = 0$$

ECUACION DE LAGRANGE

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$I_3 = \begin{Vmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{Vmatrix} = \lVert T.T \rVert$$

$$\begin{Bmatrix} \rho_{nx} \\ \rho_{ny} \\ \rho_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} \qquad \begin{Bmatrix} \rho_{n1} \\ \rho_{n2} \\ \rho_{n3} \end{Bmatrix} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix}$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$

$$I_3 = \|T.T\| = \sigma_1 \times \sigma_2 \times \sigma_3$$

Si las tres tensiones principales son distintas de cero

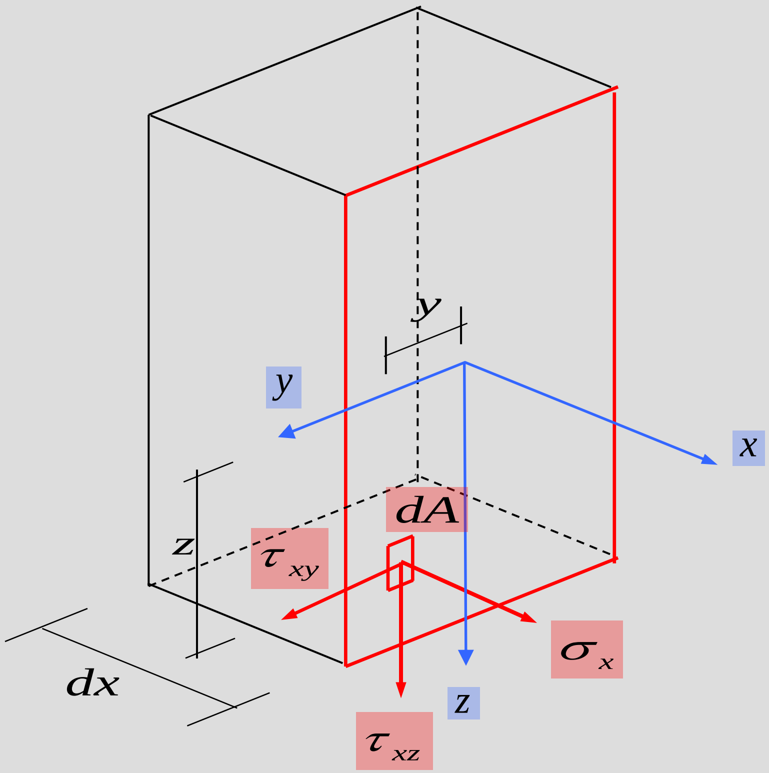
ESTADO TRIPLE O ESPACIAL $\Rightarrow I_3 \neq 0$

Si sólo una tensión principal. es nula

ESTADO DOBLE O PLANO $\Rightarrow I_3 = 0 \qquad I_2 \neq 0$

Si dos tensiones principales son nulas

ESTADO SIMPLE O LINEAL $\Rightarrow I_3 = 0 \qquad I_2 = 0$



Ecuaciones de equivalencia

$$N_x = \int_A dN_x = \int_A \sigma_x dA$$

$$Q_y = \int_A dQ_y = \int_A \tau_{xy} dA$$

$$Q_z = \int_A dQ_z = \int_A \tau_{xz} dA$$

$$M_x = \int_A dM_x = \int_A \left(y\,\tau_{xz} - z\,\tau_{xy}\right) dA$$

$$M_y = \int_A dM_y = \int_A z\,\sigma_x\, dA$$

$$M_z = \int_A dM_z = -\int_A y\,\sigma_x\, dA$$