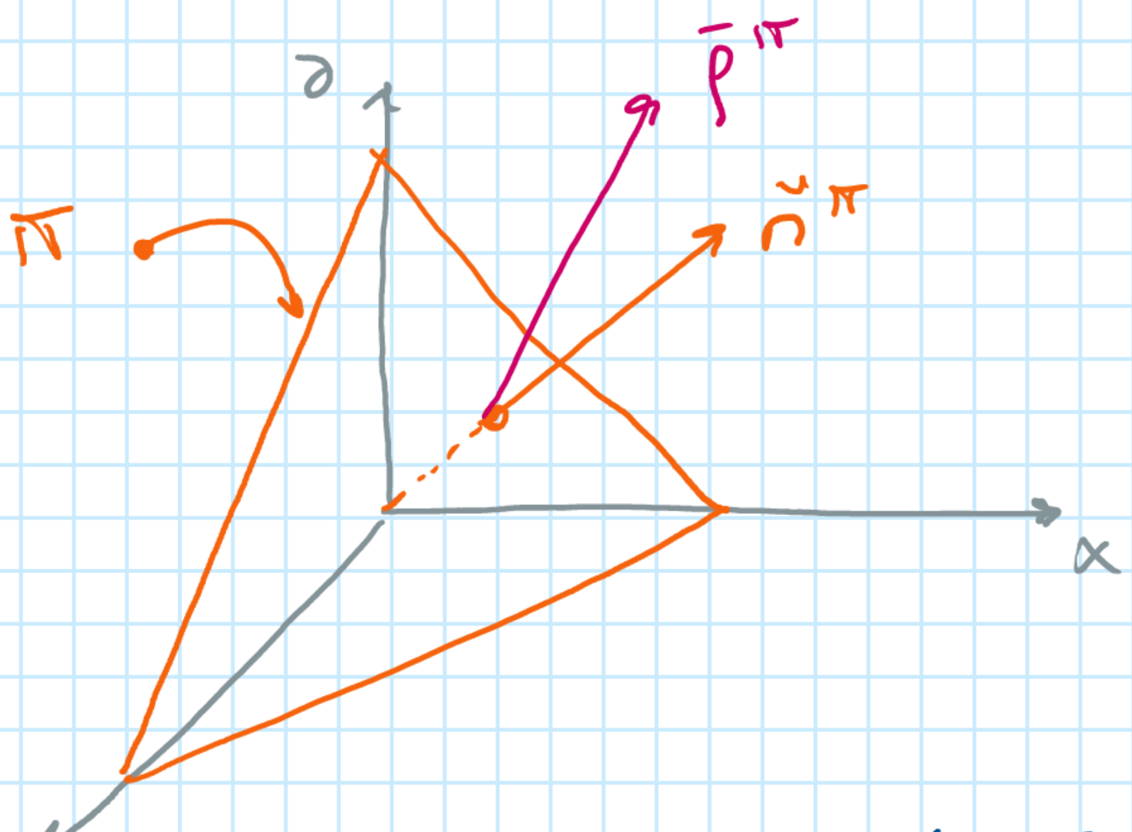
08.01 - INTRODUCCIÓN:

martes, 23 de noviembre de 2021 11:48



$$\{\bar{p}^{\pi}\} = [T\pi]\{\breve{n}^{\pi}\} \quad (1a).$$

$$\begin{cases} p_x^{\pi} = \sigma_x l + \tau_{yx} m + \tau_{zx} n \\ p_y^{\pi} = \tau_{xy} l + \sigma_y m + \tau_{zy} n \\ p_z^{\pi} = \tau_{xz} l + \tau_{yz} m + \sigma_z n \end{cases} \quad (1b)$$

EN LA FORMA (0,1,2,3)

$$\begin{cases} p_1^{\pi} = \sigma_1 l \\ p_2^{\pi} = \sigma_2 m \\ p_3^{\pi} = \sigma_3 n \end{cases} \quad (1c).$$

$$p^{\pi} = |\bar{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2} \quad (2a)$$

$$p^{\pi} = |\bar{p}| = \sqrt{p_1^2 + p_2^2 + p_3^2} \quad (2b)$$

$$p^{\pi} = \sqrt{\sigma_1^2 l^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2} \quad (3)$$

$$p^{\pi} = \sqrt{(\sigma^{\pi})^2 + (\tau^{\pi})^2} \quad (4)$$

(3) y (4):

$$\boxed{\sigma^2 + \tau^2 = \sigma_1^2 l^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2} \quad (5)$$

—o—o—

$$\{\bar{\sigma}^{\pi}\} = \underbrace{\left[\{\bar{p}^{\pi}\}^T \cdot \{\breve{n}^{\pi}\}\right]}_{\text{PSEUDO-MÓDULO}} \cdot \{\breve{n}^{\pi}\} \quad (6).$$

$$\sigma^{\pi} = \sigma = (\sigma_1 l\, ; \sigma_2 m\, ; \sigma_3 n) \cdot (l, m, n) \quad (7)$$

$$\boxed{\sigma^{\pi} = \sigma = \sigma_1 l^2 + \sigma_2 m^2 + \sigma_3 n^2} \quad (8)$$

RESOLVIENDO:

$$\left.\begin{array}{lrcl} (5) & \sigma^2 + \tau^2 = p^2 & = & \sigma_1^2 l^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2 . \\ (8) & \sigma & = & \sigma_1 l^2 + \sigma_2 m^2 + \sigma_3 n^2 . \\ (9) & 1 & = & l^2 + m^2 + n^2 \end{array}\right\} \longrightarrow \text{MOHR}$$

08.02 - EXPRESIONES DE LAS CIRCUNFERENCIAS DE MOHR:

martes, 23 de noviembre de 2021 12:00

Mohr tomó las expresiones (5); (8) y (9):

$$\tau^2 + \left(\sigma - \underbrace{\frac{\sigma_2+\sigma_3}{2}}_{a}\right)^2 = \left(\sigma_1 - \frac{\sigma_2+\sigma_3}{2}\right)^2 l^2 + \left(\frac{\sigma_2-\sigma_3}{2}\right)^2 (1-l^2) \quad (10a)$$

$$\tau^2 + \left(\sigma - \underbrace{\frac{\sigma_1+\sigma_3}{2}}_{a}\right)^2 = \left(\sigma_2 - \frac{\sigma_1+\sigma_3}{2}\right)^2 m^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 (1-m^2) \quad (10b)$$

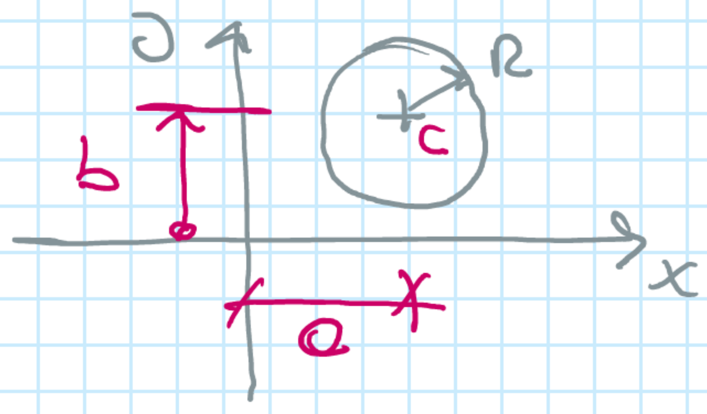
$$\tau^2 + \left(\sigma - \underbrace{\frac{\sigma_1+\sigma_2}{2}}_{a}\right)^2 = \left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right)^2 n^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2 (1-n^2) \quad (10c)$$

(I) → Datos: $\sigma_1, \sigma_2, \sigma_3$ → tensiones ppal.

(II) → Incógnitas: → II.1 → σ, τ → ρ
II.2 → l, m, n → $\check{n}$

(III) → Expresión de una circunferencia:

$$(x-a)^2 + (y-b)^2 = R^2$$



$x \to \sigma$
$y \to \tau$

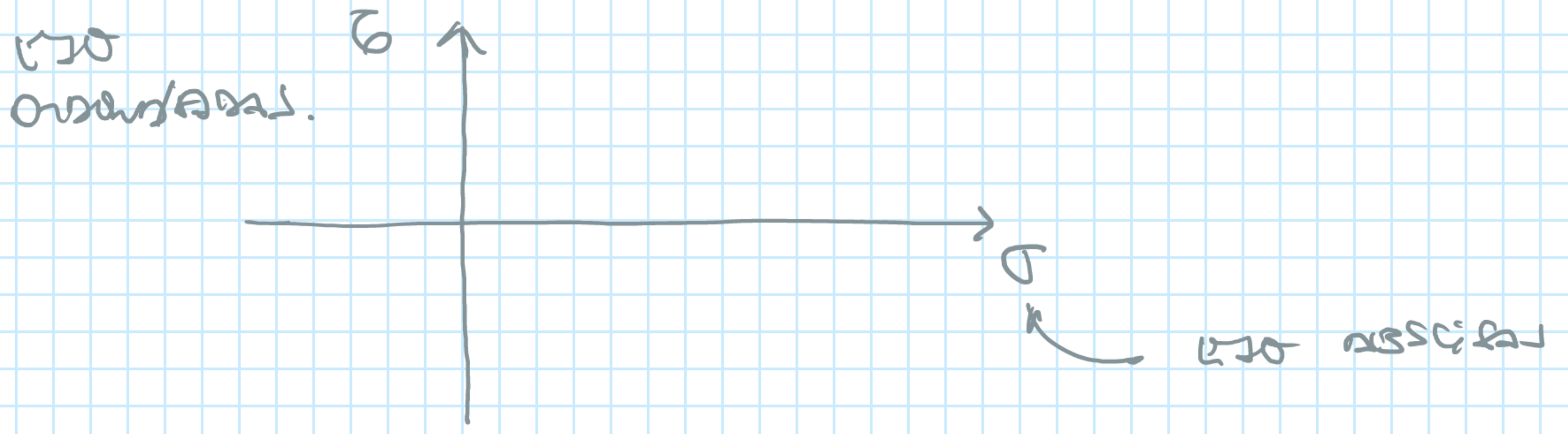
(IV) Las expresiones (10) representan familias de circunferencias.

$0 \le l \le 1$
$0 \le m \le 1$
$0 \le n \le 1$

(V) Obtengo σ, τ → ρ.

$$\rho = \sqrt{\sigma^2 + \tau^2}$$

(VI) A estas familias de circunferencias → las vamos a representar en un gráfico (σ, τ)



(VII) Las familias de circunferencias dependen de $\sigma_1, \sigma_2, \sigma_3$, de las tensiones ppales.

(VIII) Los centros de las 3 familias de circunferencias se ubican sobre el eje de abscisas.

(IX) Cada familia depende de 1 sólo parámetro → $\{l, m, n\}$

(X) Cada familia va a estar limitada por 2 circunf. extremas.

(XI) Cada valor de tensión ppal entra con su signo en las expresiones (10).

08.03 - REPRESENTACIÓN DE LAS CIRCUNFERENCIAS DE MOHR:

martes, 23 de noviembre de 2021 12:00

Mohr para las condiciones (5) ; (8) y (9) :

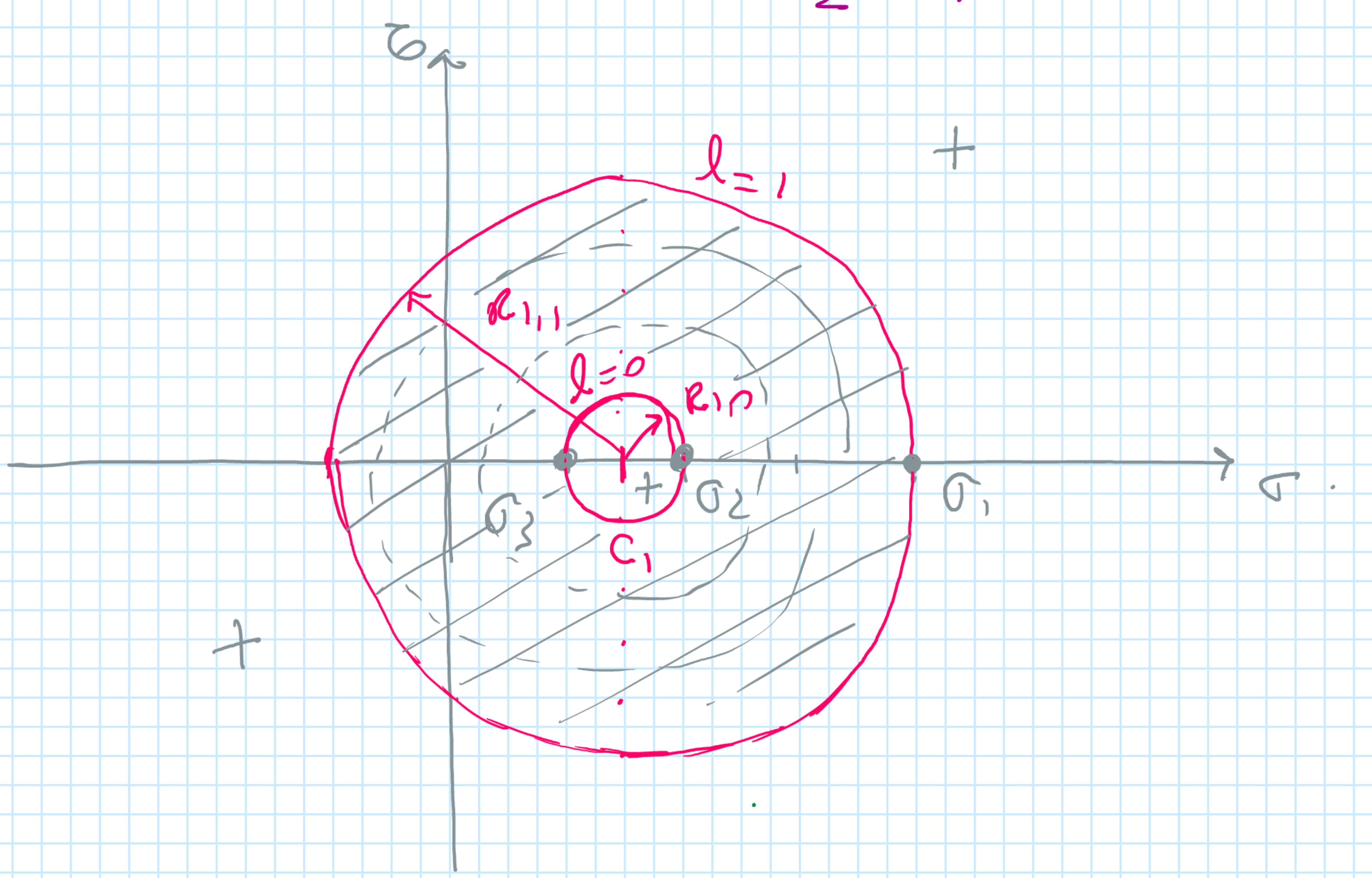
$$\tau^2 + \left(\sigma - \frac{\sigma_2+\sigma_3}{2}\right)^2 = \left(\sigma_1 - \frac{\sigma_2+\sigma_3}{2}\right)^2 l^2 + \left(\frac{\sigma_2-\sigma_3}{2}\right)^2 (1-l^2) \quad (10a)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_3}{2}\right)^2 = \left(\sigma_2 - \frac{\sigma_1+\sigma_3}{2}\right)^2 m^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 (1-m^2) \quad (10b)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 = \left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right)^2 n^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2 (1-n^2) \quad (10c)$$

1ª Familia: (10a).

- Ubicación del centro: $C_1 = \left(\frac{\sigma_2+\sigma_3}{2} ; 0\right)$
- $R_{1,0} \rightarrow l = 0. \rightarrow R_{1,0} = \frac{\sigma_2-\sigma_3}{2}$; $\tau^2 + \left(\sigma - \frac{\sigma_2+\sigma_3}{2}\right)^2 = \left(\frac{\sigma_2-\sigma_3}{2}\right)^2$
- $l = 1 \rightarrow R_{1,1} = \sigma_1 - \frac{\sigma_2+\sigma_3}{2}$; $\tau^2 + \left(\sigma - \frac{\sigma_2+\sigma_3}{2}\right)^2 = \left(\sigma_1 - \frac{\sigma_2+\sigma_3}{2}\right)^2$



$\sigma_1 = 16$

$\sigma_2 = 8$

$\sigma_3 = 4$

$C_1 = \left(\frac{4+8}{2}, 0\right) = (6,0)$

$R_{1,0} = \frac{8-4}{2} = 2$

$R_{1,1} = 16 - \frac{8+4}{2} = 16 - 6 = 10.$

08.03 - REPRESENTACIÓN DE LAS CIRCUNFERENCIAS DE MOHR:

martes, 23 de noviembre de 2021 12:00

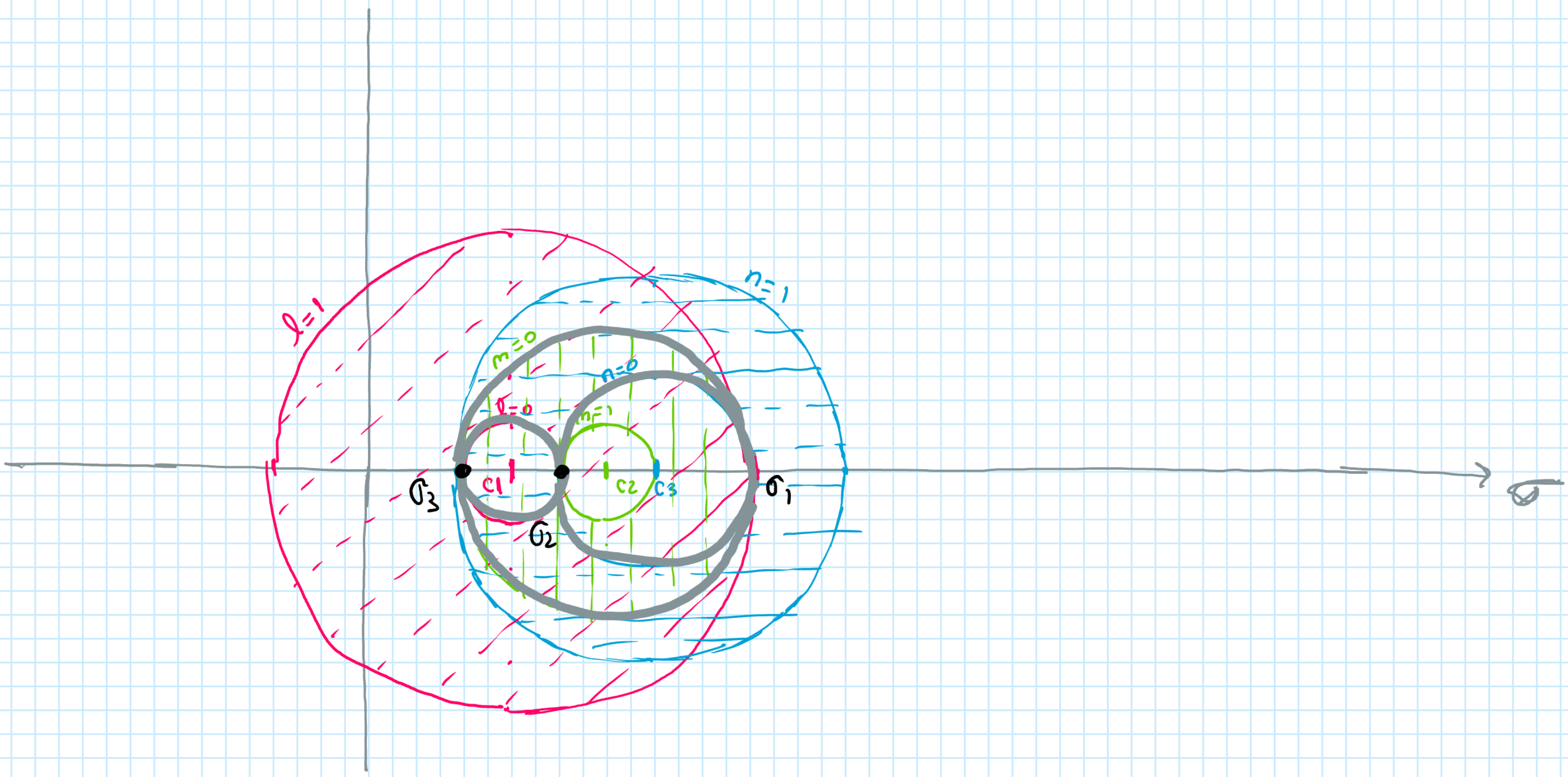
Mohr para las ecuaciones (5); (8) y (9):

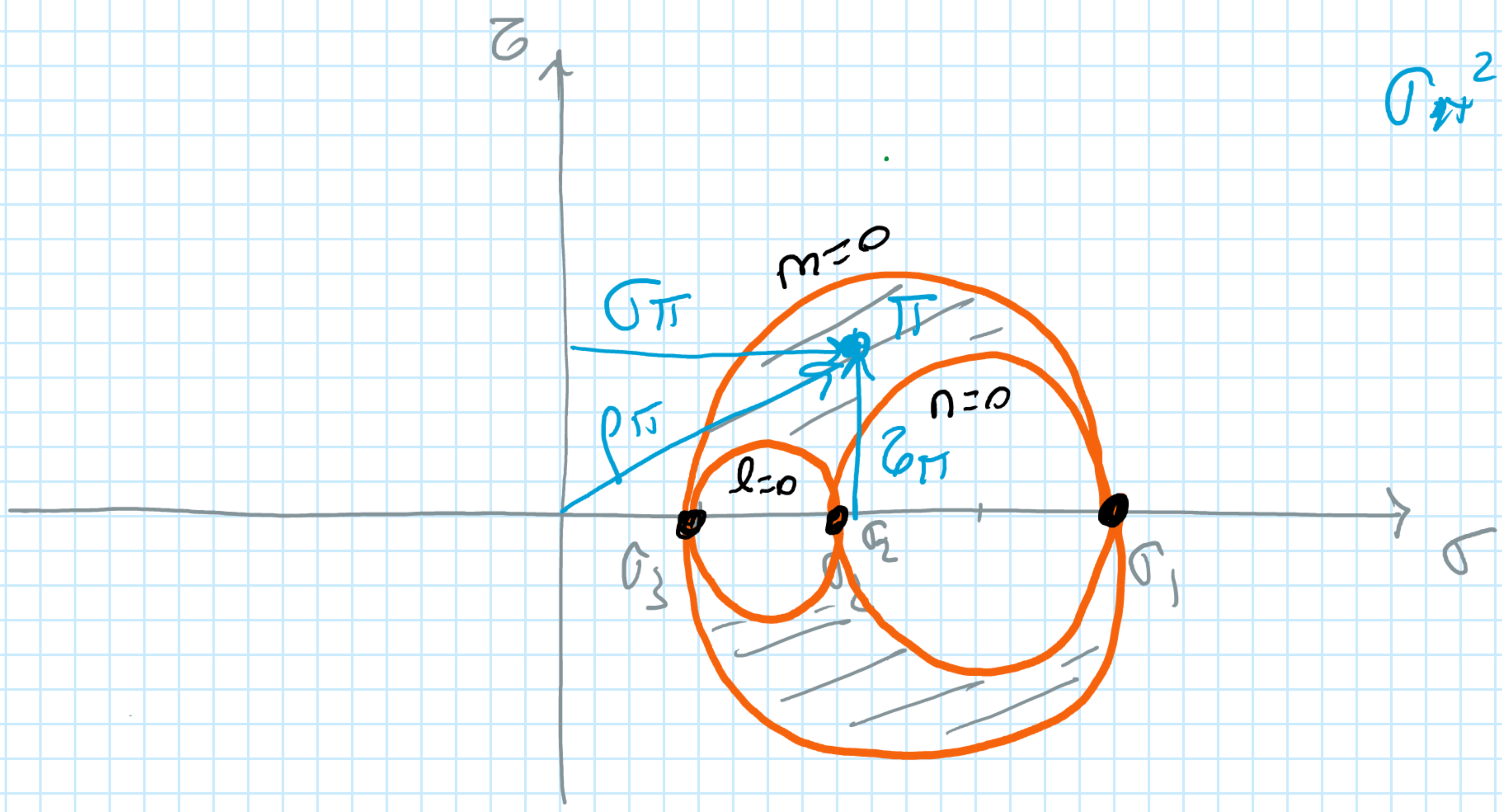
$$\tau^2 + \left(\sigma - \frac{\sigma_2+\sigma_3}{2}\right)^2 = \left(\sigma_1 - \frac{\sigma_2+\sigma_3}{2}\right)^2 l^2 + \left(\frac{\sigma_2-\sigma_3}{2}\right)^2 (1-l^2) \quad (10a)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_3}{2}\right)^2 = \left(\sigma_2 - \frac{\sigma_1+\sigma_3}{2}\right)^2 m^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 (1-m^2) \quad (10b)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 = \left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right)^2 n^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2 (1-n^2) \quad (10c)$$

$\sigma_1 = 16$; $\sigma_2 = 8$; $\sigma_3 = 4$.

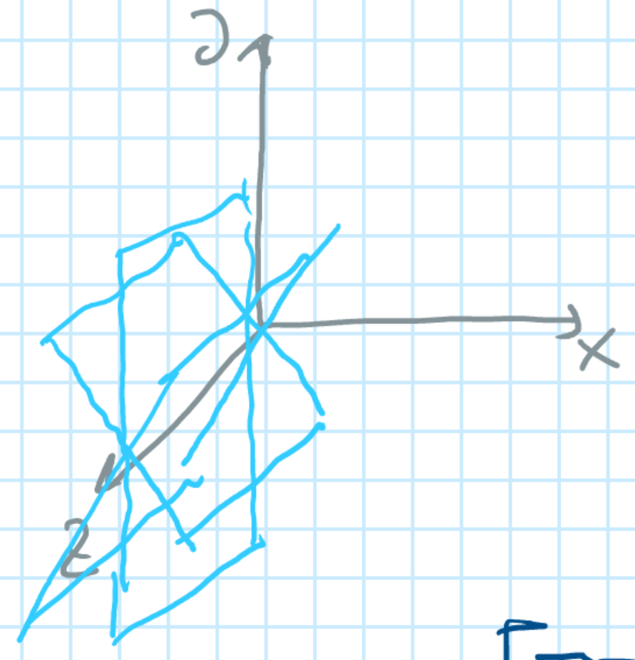




$$\sigma_\pi^2 + \tau_\pi^2 = \rho_\pi^2$$

08.04 - CIRCUNFERENCIA DE MOHR APLICADA A ESTADOS PLANOS Y/O EJE DIRECTOR PPAL:

martes, 23 de noviembre de 2021 12:44



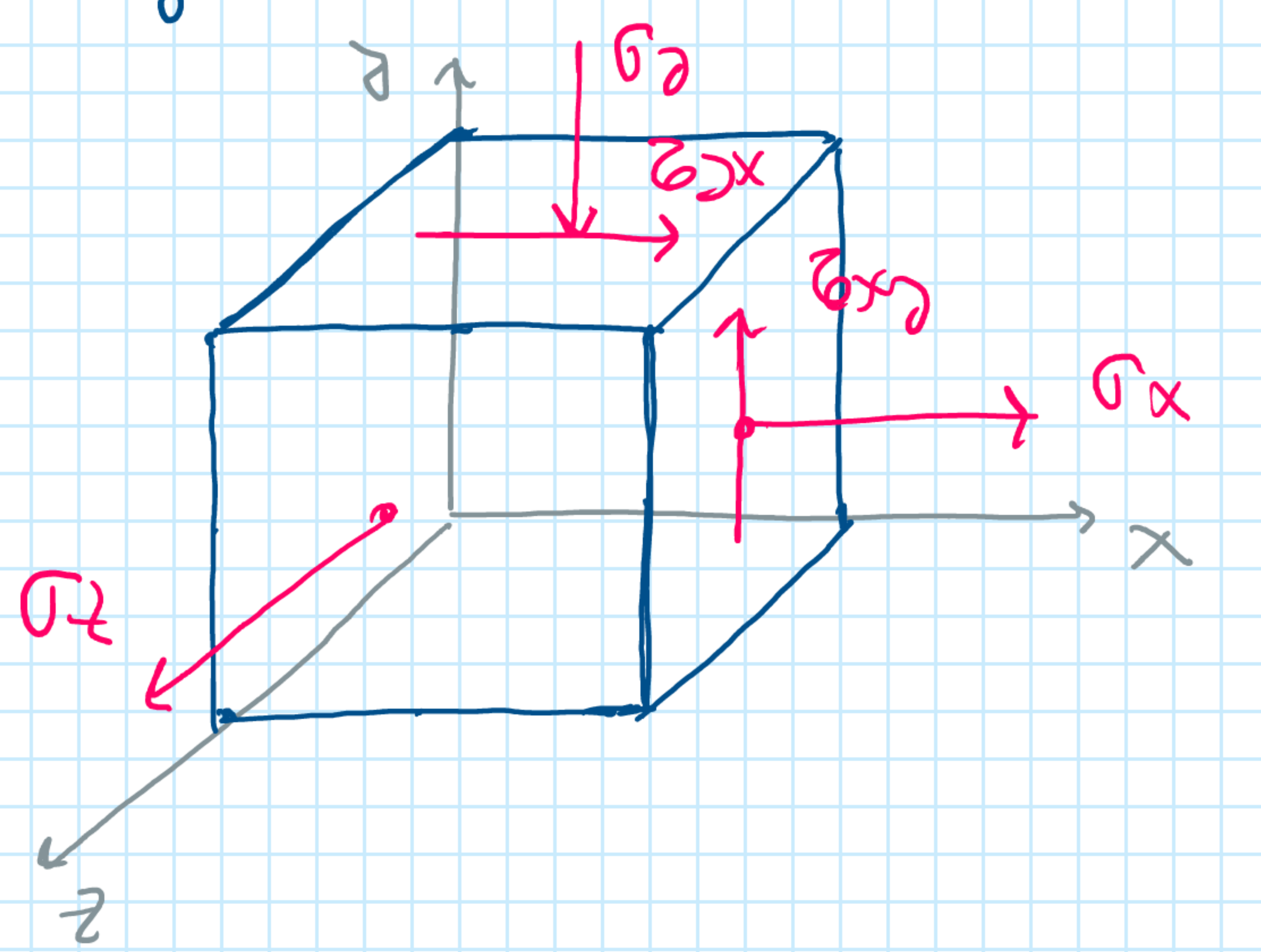
Eje 'z' es eje director.

Eje director : eje 'z'.
Eje 'z' es principal de tensiones

$$[T\sigma] = \begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

$\tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$

$\sigma_y < \sigma_z < \sigma_x$.



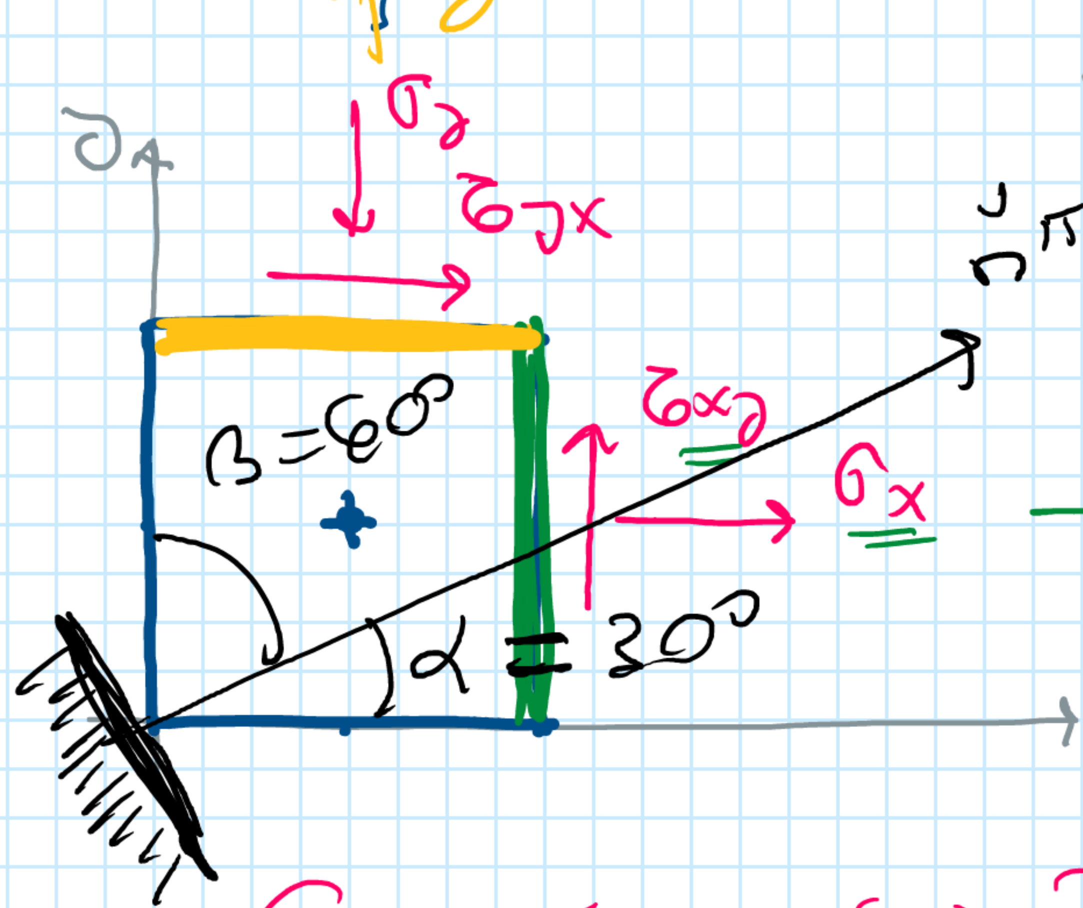


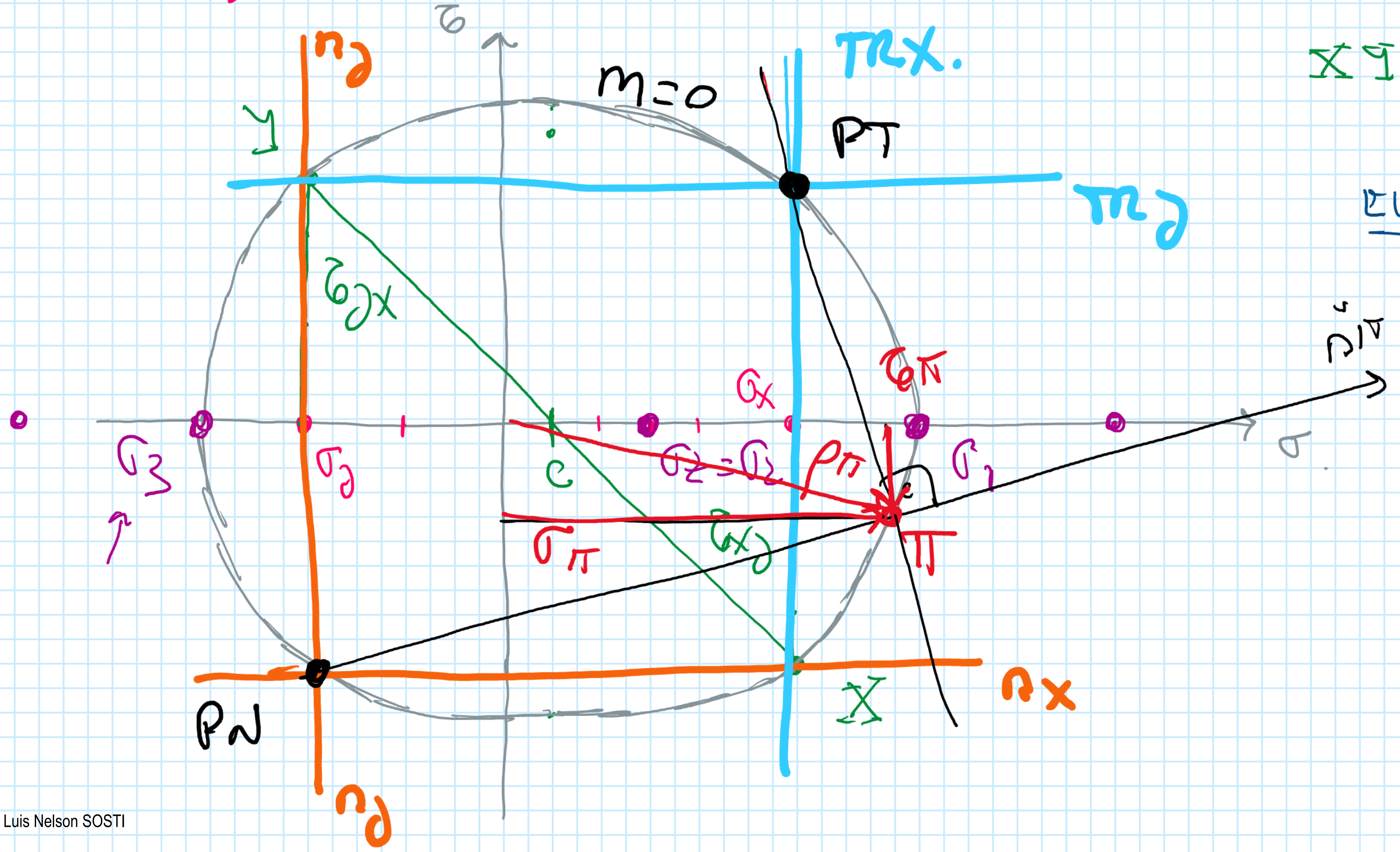
Figura de análisis complementaria

$\alpha = 30^\circ$ $l = 0{,}866$
$\beta = 60^\circ$ $m = 0{,}5$
$\gamma = 90^\circ$ $n = 0$.

τ_{xy} (horario) < 0 (−)
τ_{yx} (antihorario) > 0 (+).
Nueva convención de Mohr.

$\sigma_x = +60$ MPa ; $\sigma_y = -40$ MPa ; $\sigma_z = +30$ MPa ;

$\tau_{xy} = \tau_{yx} = +50$ MPa.



XY = Diámetro de la circunferencia Mohr

Elementos
- Normales
- Tangentes [illegible]
- Polos → normal PN
  → [illegible] PT.