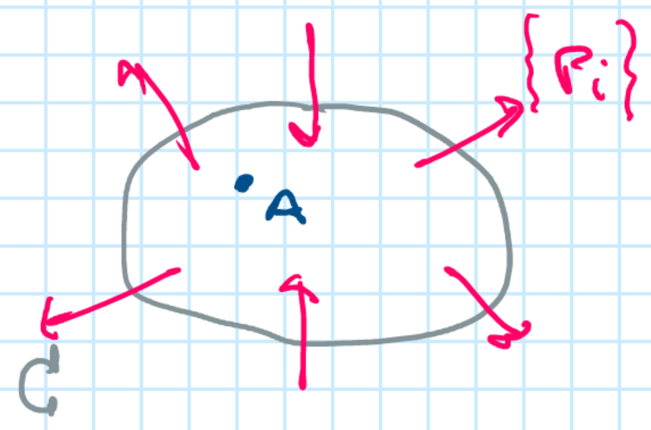
06.17 - DESCOMPOSICIÓN DEL TT:

martes, 23 de noviembre de 2021 09:09

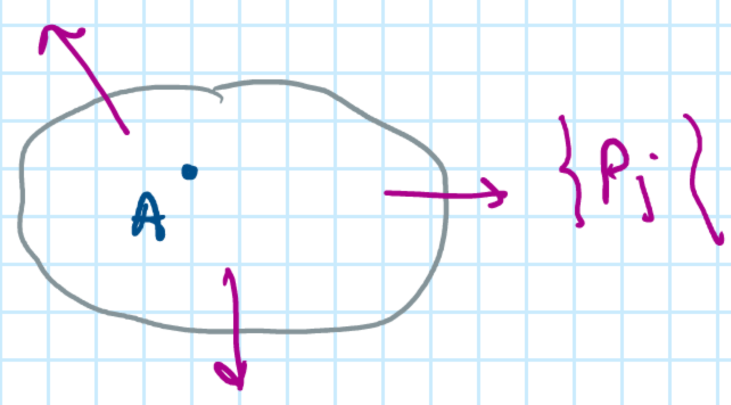
$[TT] \longrightarrow$ Matriz 3×3.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

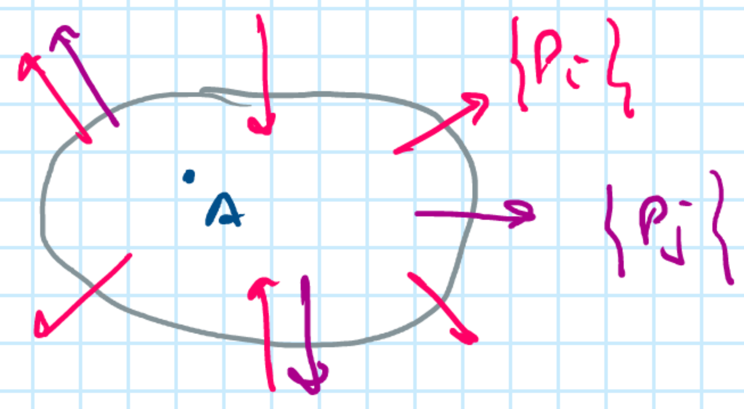
1) $[TT_1] + [TT_2] = [TT_{TOT}]$



$\{FE\} = \{P_i\} \rightarrow [TT^A] = [TT_1^A]_{xyz}$



$\{FE\} = \{P_j\} \rightarrow [TT_2^A]_{xyz}$



$\{FE\} = \{P_i\} + \{P_j\} \rightarrow [TT_{TOT}^A]_{xyz} = [TT_1^A]_{xyz} + [TT_2^A]_{xyz}$

2)
$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{[A]} = \underbrace{\begin{bmatrix} a_{11} & \frac{1}{2}(a_{12}+a_{21}) & \frac{1}{2}(a_{13}+a_{31}) \\ \frac{1}{2}(a_{12}+a_{21}) & a_{22} & \frac{1}{2}(a_{23}+a_{32}) \\ \frac{1}{2}(a_{13}+a_{31}) & \frac{1}{2}(a_{23}+a_{32}) & a_{33} \end{bmatrix}}_{[S_A]} +$$

$$+ \underbrace{\begin{bmatrix} 0 & \frac{1}{2}(a_{12}-a_{21}) & \frac{1}{2}(a_{13}-a_{31}) \\ -\frac{1}{2}(a_{12}-a_{21}) & 0 & \frac{1}{2}(a_{23}-a_{32}) \\ -\frac{1}{2}(a_{13}-a_{31}) & -\frac{1}{2}(a_{23}-a_{32}) & 0 \end{bmatrix}}_{[A_A]}$$

$$[A] = \underbrace{[S_A]}_{\text{tensor simétrico}} + \underbrace{[A_A]}_{\text{tensor antisimétrico}} \longrightarrow [TT] = [TT_S] + [TT_A]$$

3)
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} a_0 & 0 & 0 \\ 0 & a_0 & 0 \\ 0 & 0 & a_0 \end{bmatrix}}_{\text{esférico}} + \underbrace{\begin{bmatrix} (a_{11}-a_0) & a_{12} & a_{13} \\ a_{21} & (a_{22}-a_0) & a_{23} \\ a_{31} & a_{32} & (a_{33}-a_0) \end{bmatrix}}_{\text{desviador.}}$$

$$a_0 = \frac{\overbrace{a_{11} + a_{22} + a_{33}}^{}}{3} \longrightarrow a_0 = \frac{I_1}{3}$$

$$\sigma_0 = \frac{\overbrace{\sigma_x + \sigma_y + \sigma_z}^{I_1}}{3} = \frac{I_1\{Tr[TT]\}}{3} = \frac{\overbrace{\sigma_1 + \sigma_2 + \sigma_3}^{I_1}}{3}$$

$$[TT] = \underbrace{[TT_{ESF}]}_{\text{Tensor de tensiones esférico}} + \underbrace{[TT_{DESV}]}_{\text{Tensor de tensiones desviador.}}$$

TT esférico $\longrightarrow$ involucra cambios de volumen.

TT desviador $\longrightarrow$ " " " forma.

$[TT_{ESF}] \longrightarrow \neq 0$ el cuerpo experimentará cambios de volumen.
$\quad \llcorner \longrightarrow = 0$ " " no " " " "

$[TT_{DESV}] \longrightarrow \neq 0$ " " experimentará " " forma
$\quad \llcorner \longrightarrow = 0$ " " no " " " "

06.18 - TENSIONES OCTAÉDRICAS:

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DEF.: Se denominan tensiones "octaédricas" a las tensiones correspondientes o asociadas a planos que forman los mismos ángulos con las direcciones principales.

1º) → TERNA PRINCIPAL. $(0; 1; 2; 3)$.

2º) → PLANO CUALQUIERA → $\breve{n}^{\pi}$

$$\left.\begin{array}{l} \alpha = \widehat{1n} \\ \beta = \widehat{2n} \\ \gamma = \widehat{3n} \end{array}\right\} \rightarrow \text{si } \alpha = \beta = \gamma \rightarrow \bar{\rho}^{\pi} \rightarrow$$

"TENSIÓN OCTAÉDRICA"

2) $\left.\begin{array}{l} \cos\alpha = l \\ \cos\beta = m \\ \cos\gamma = n \end{array}\right\}$ $\quad l^2 + m^2 + n^2 = 1 \; ; \; l = m = n = c$

$$c^2 + c^2 + c^2 = 1$$

$$3c^2 = 1 \rightarrow c = \pm \frac{1}{\sqrt{3}}$$

$$\boxed{c = \pm \frac{\sqrt{3}}{3}}$$

$$\alpha = \beta = \gamma \cong 54{,}736 = 54^\circ\, 44'\, 8{,}2''.$$

3) VECTOR TENSIÓN:

$$\{\bar{\rho}\} = [T\sigma]_{123} \{\breve{n}\}$$

$$\begin{Bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{Bmatrix} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{Bmatrix} l = c \\ m = c \\ n = c \end{Bmatrix} \rightarrow \begin{array}{l} \rho_1 = \sigma_1 c \\ \rho_2 = \sigma_2 c \\ \rho_3 = \sigma_3 c \end{array}$$

$$\{\bar{\rho}\} \wedge |\bar{\rho}| \; ; \; \{\bar{\sigma}\} \wedge |\bar{\sigma}| \; ; \; \{\bar{\tau}\} \wedge |\bar{\tau}|$$

4) ¿CUÁNTOS PLANOS HAY? → TENDREMOS 8 PLANOS OCTAÉDRICOS