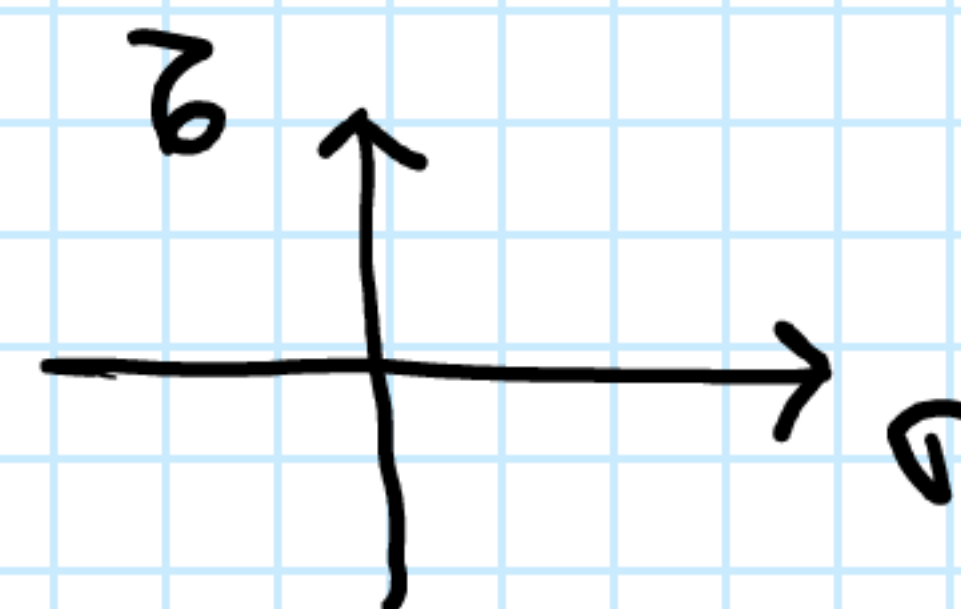


05 - COMENTARIOS A LAS EXPRESIONES (10):

martes, 6 de julio de 2021 09:14

VII) LAS FLUJAS DE CIRCUNFERENCIAS DEPENDEN DE LAS TENSIONES PRINCIPALES.

VIII) $(x-a)^2 + (y-b)^2 = R^2$
 $(r-a)^2 + (\theta-b)^2 = R^2$



$b = 0 \rightarrow$ EL CENTRO SE UBICA A TODAS LAS CIRCUNFERENCIAS SOBRE EL EJE DE ABSCISAS O 'r'!

IX) CADA FLUJA DEPENDE DE '1' SOLO PARAMETRO.

X) " " DE CIRCUNFERENCIAS VA A ESTAR LIMITADA POR 2 CIRCUNFERENCIAS EXTREMAS.

$$\begin{cases} 0 \leq l \leq 1 \\ 0 \leq m \leq 1 \\ 0 \leq n \leq 1 \end{cases}$$

XI) EN CADA EXPRESION (10) \rightarrow CADA VALOR DE LAS TENS. PRINCIPALES SE INTRODUCE CON SU SIGNO.

06 - REPRESENTACIONES:

martes, 6 de julio de 2021 09:23

1º FAMILIA: $(\text{lo a}) \quad \sigma^2 + \left(\sigma - \frac{\sigma_2 + \sigma_3}{2}\right)^2 = \underbrace{\left(\sigma_1 - \frac{\sigma_2 + \sigma_3}{2}\right)^2}_{l^2} + \underbrace{\left(\frac{\sigma_2 - \sigma_3}{2}\right)^2}_{(1-l^2)}$

$C_1 \equiv \left(\frac{\sigma_2 + \sigma_3}{2}; 0\right)$

$l = 0 \rightarrow R_{1,0} = \frac{\sigma_2 - \sigma_3}{2}$

$l = 1 \rightarrow R_{1,1} = \frac{\sigma_1 - \sigma_2 + \sigma_3}{2}$

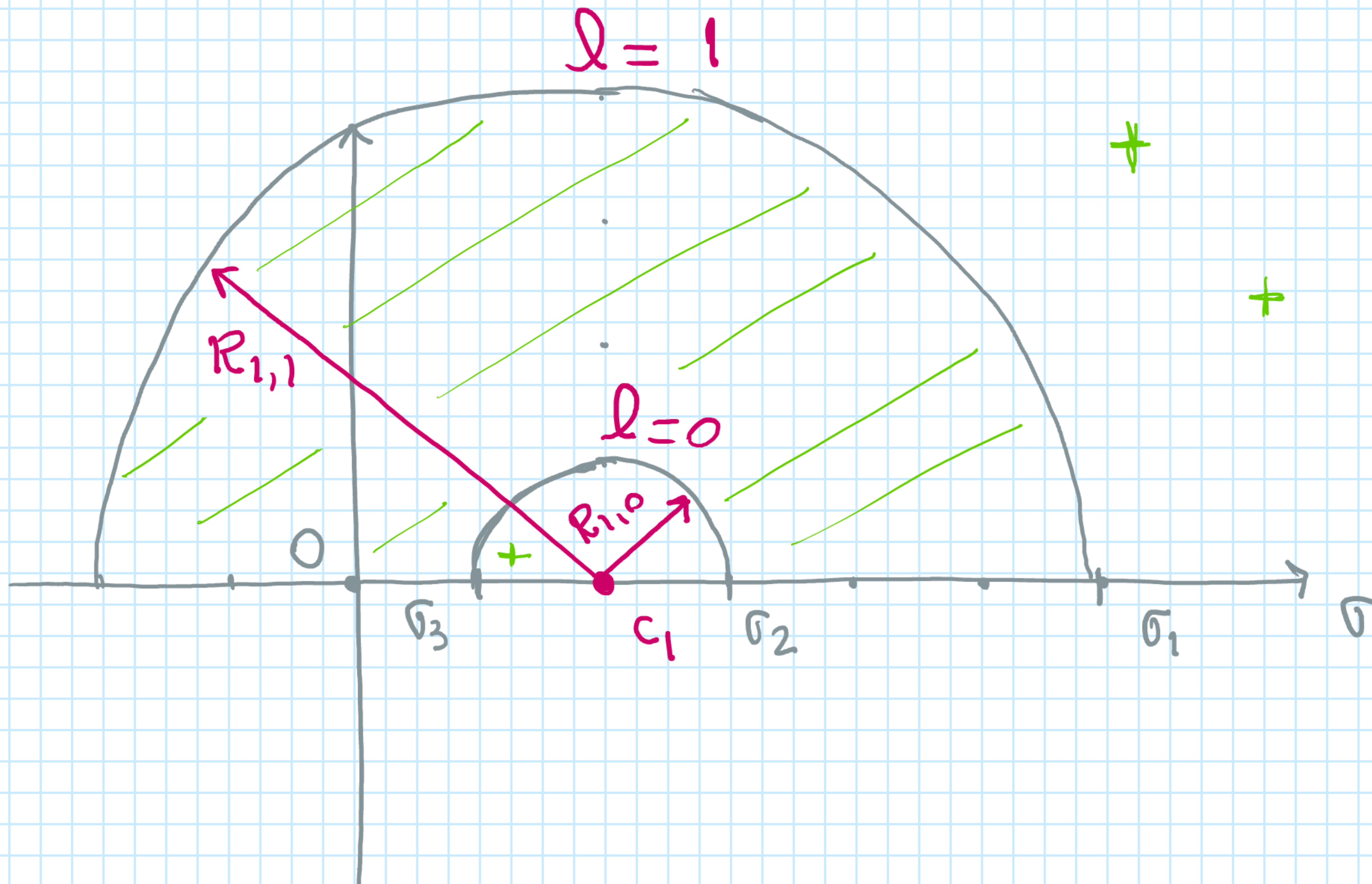
$\sigma_1 = 24 \text{ kw/cm}^2$

$\sigma_2 = 12 \text{ "}$

$\sigma_3 = 4 \text{ "}$

$R_{1,0} = \frac{12 - 4}{2} = 4$

$R_{1,1} = 24 - \frac{12 + 4}{2} = 16$



06 - REPRESENTACIONES:

martes, 6 de julio de 2021 09:23

2º FAMILIA:
$$\sigma^2 + \left(\sigma - \frac{\sigma_1 + \sigma_3}{2} \right)^2 = \left(\sigma_2 - \frac{\sigma_1 + \sigma_3}{2} \right)^2 m^2 + \left(\frac{\sigma_1 - \sigma_3}{2} \right)^2 (1 - m^2)$$

$$C_2 \equiv \left(\frac{\sigma_1 + \sigma_3}{2}, 0 \right)$$

$$m = 0 \rightarrow R_{2,0} = \frac{\sigma_1 - \sigma_3}{2}$$

$$m = 1 \rightarrow \frac{R_{2,1}}{\sigma} = \sigma_2 - \frac{\sigma_1 + \sigma_3}{2}$$

$$\sigma_1 = 24 \text{ kw/cm}^2$$

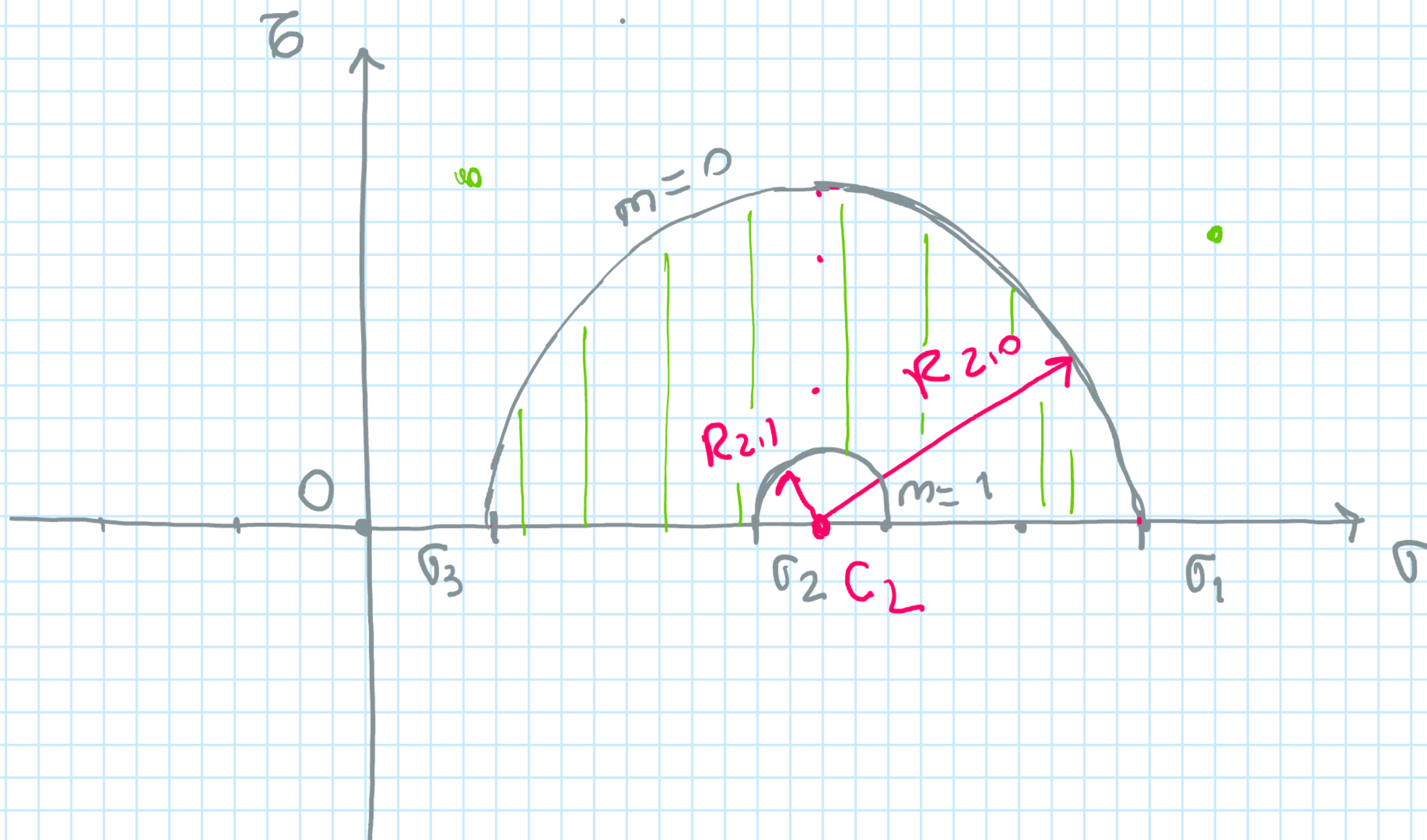
$$\sigma_2 = 12 \text{ "}$$

$$\sigma_3 = 4 \text{ "}$$

$$C_2 = \frac{24 + 4}{2} = 14$$

$$R_{2,0} = \frac{24 - 4}{2} = 10$$

$$R_{2,1} = 12 - \frac{24 + 4}{2} = -2$$

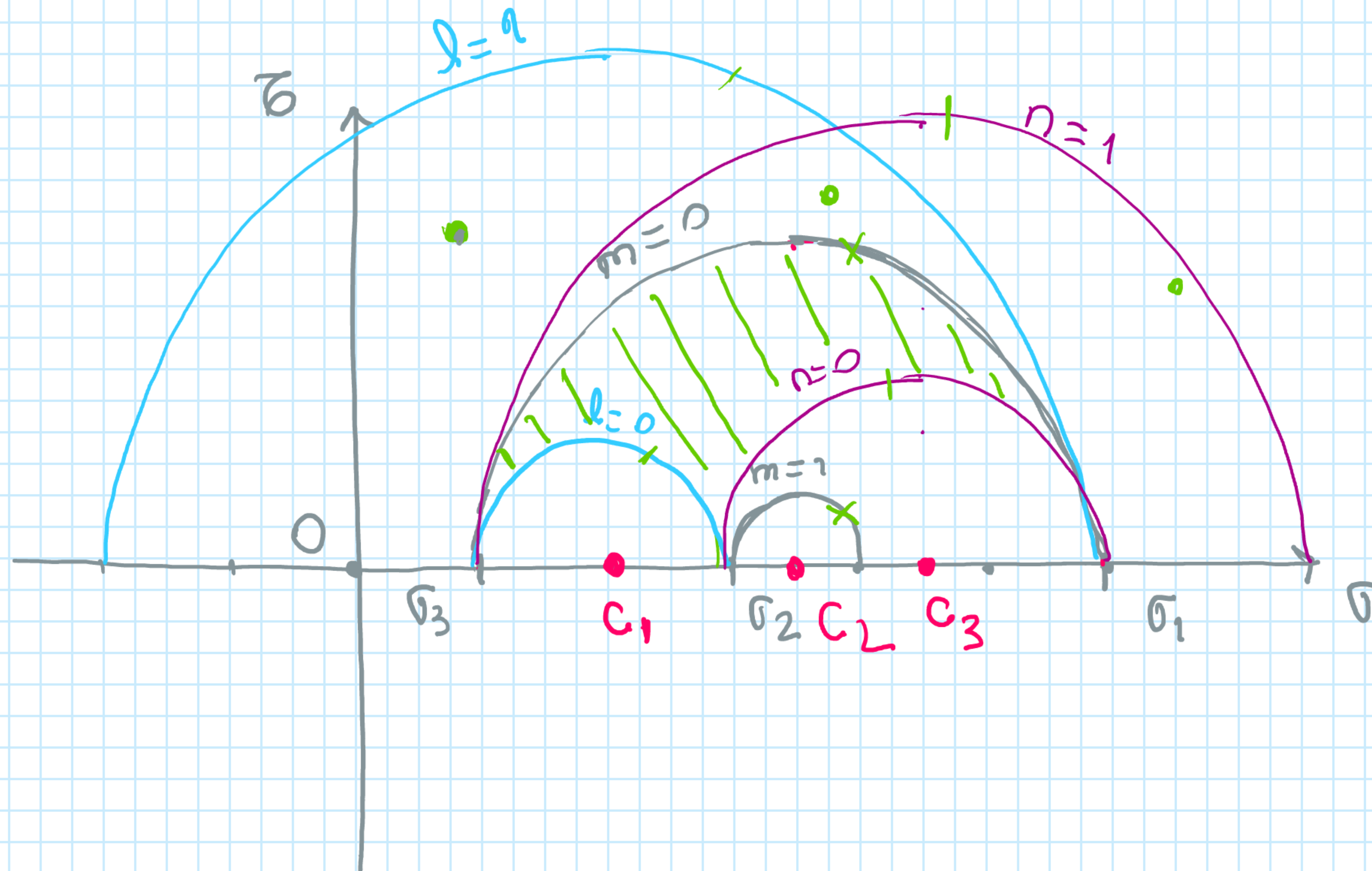


06 - REPRESENTACIONES:

martes, 6 de julio de 2021 09:23

$\text{PLANO } \pi \rightarrow \sigma_r \rightarrow \begin{matrix} \alpha = 1 \\ \beta = 2 \\ \gamma = 3 \end{matrix} \rightarrow \text{COS. DIRECCION} \left\{ \begin{matrix} 0 \leq l \leq 1 \\ 0 \leq m \leq 1 \\ 0 \leq n \leq 1 \end{matrix} \right.$

$\sigma_1 = 24 \text{ kw/cm}^2$
 $\sigma_2 = 12 \text{ ''}$
 $\sigma_3 = 4 \text{ ''}$



07 - OBJETIVO:

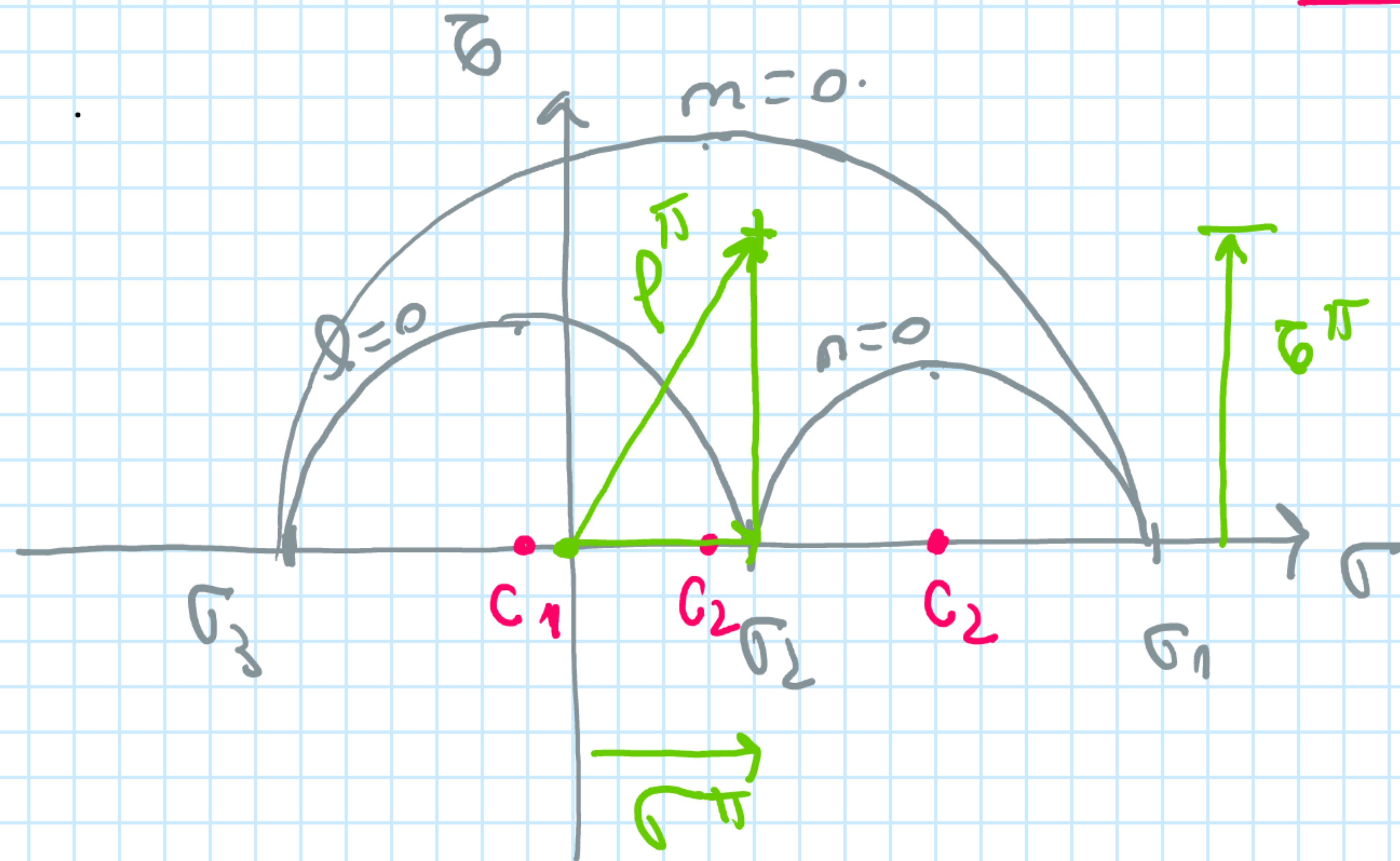
martes, 6 de julio de 2021 09:54

1º) **DATOS**

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} + (l, m, n) \xrightarrow{\text{¿qu? ¿observamos?}}$$

$$\begin{Bmatrix} p^\pi \\ \sigma^\pi \\ \tau^\pi \end{Bmatrix}$$

$$\begin{Bmatrix} e_{11} \\ e_{22} \\ e_{33} \end{Bmatrix} + (l, m, n) \rightarrow \begin{Bmatrix} e_{11}^\pi \\ e_{22}^\pi \\ e_{33}^\pi \end{Bmatrix}$$



2º) **DATOS**

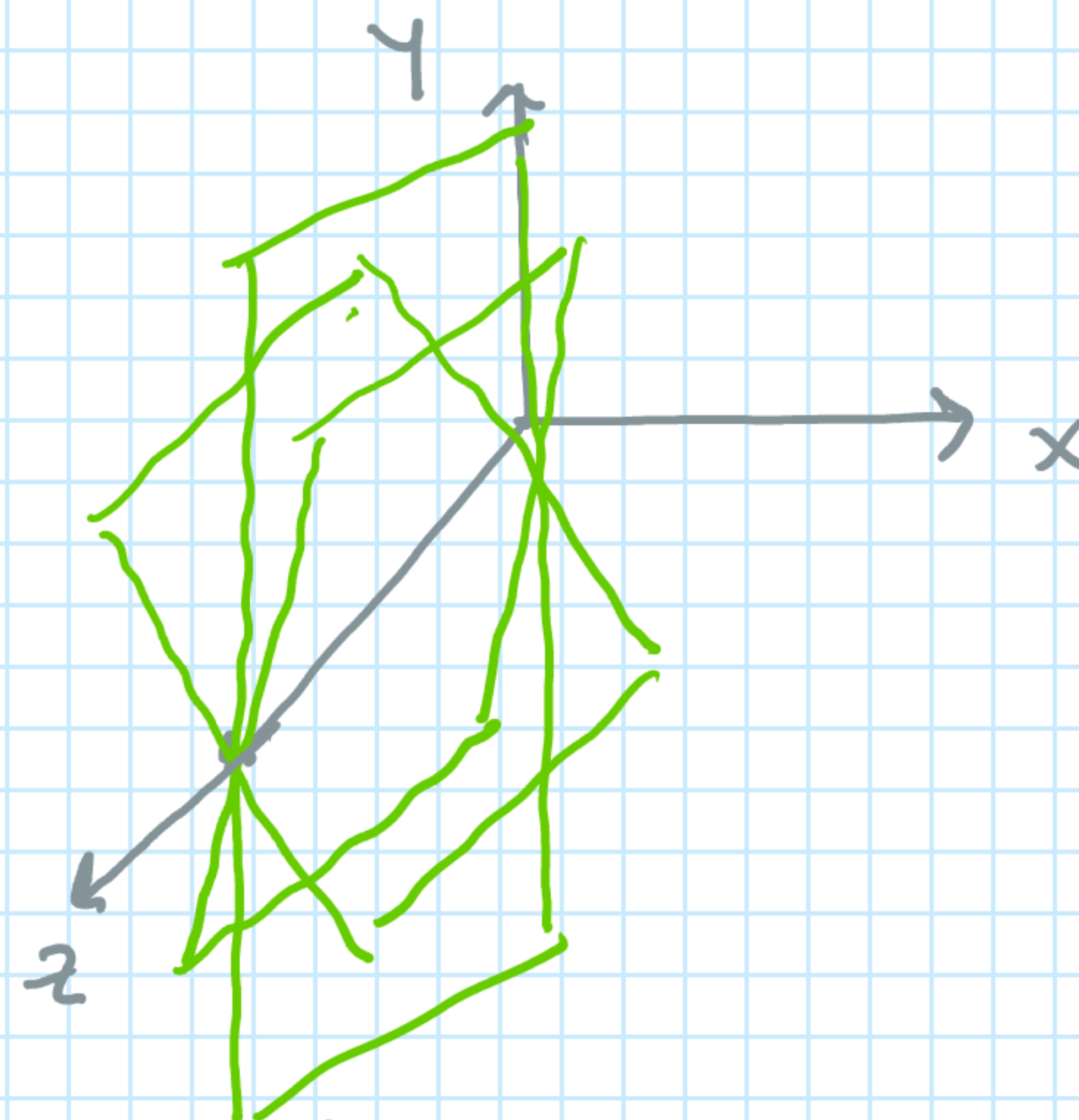
$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} + (\sigma^\pi; \tau^\pi) \xrightarrow{\text{Plano } \pi \rightarrow (l, m, n) \rightarrow (\alpha, \beta, \gamma)}$$

LAS CIRCULAS FUERZAS DE MEMBR EQUIVALENTE \rightarrow CONSTRUCCION CRITICA \rightarrow
 \rightarrow EQUIVALENTE AL $[\pi]$
 $e_1; e_2; e_3 \rightarrow [\pi]$

08 - ESTADO PLANO Y EJE DIRECTOR:

martes, 6 de julio de 2021 10:04

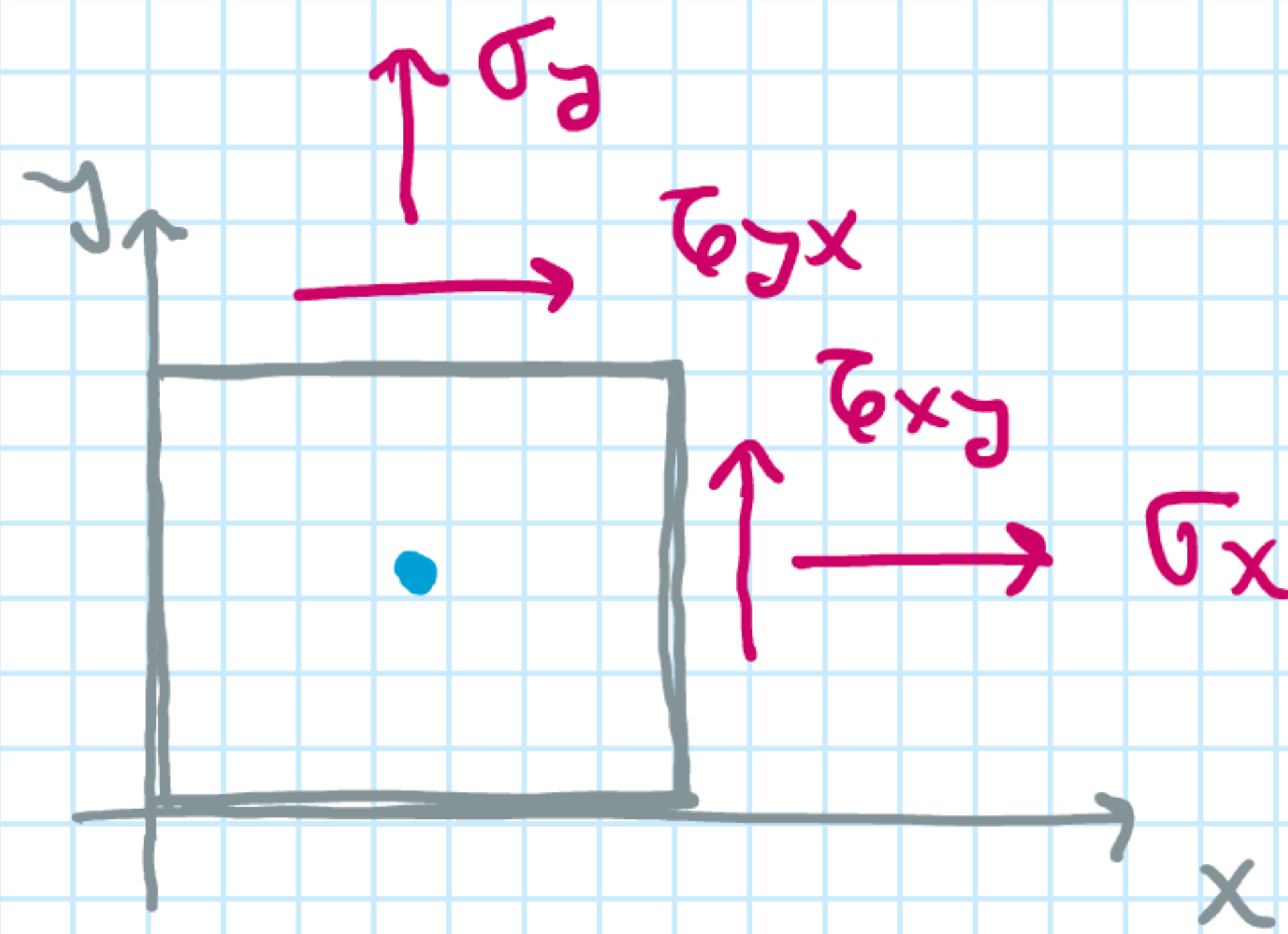
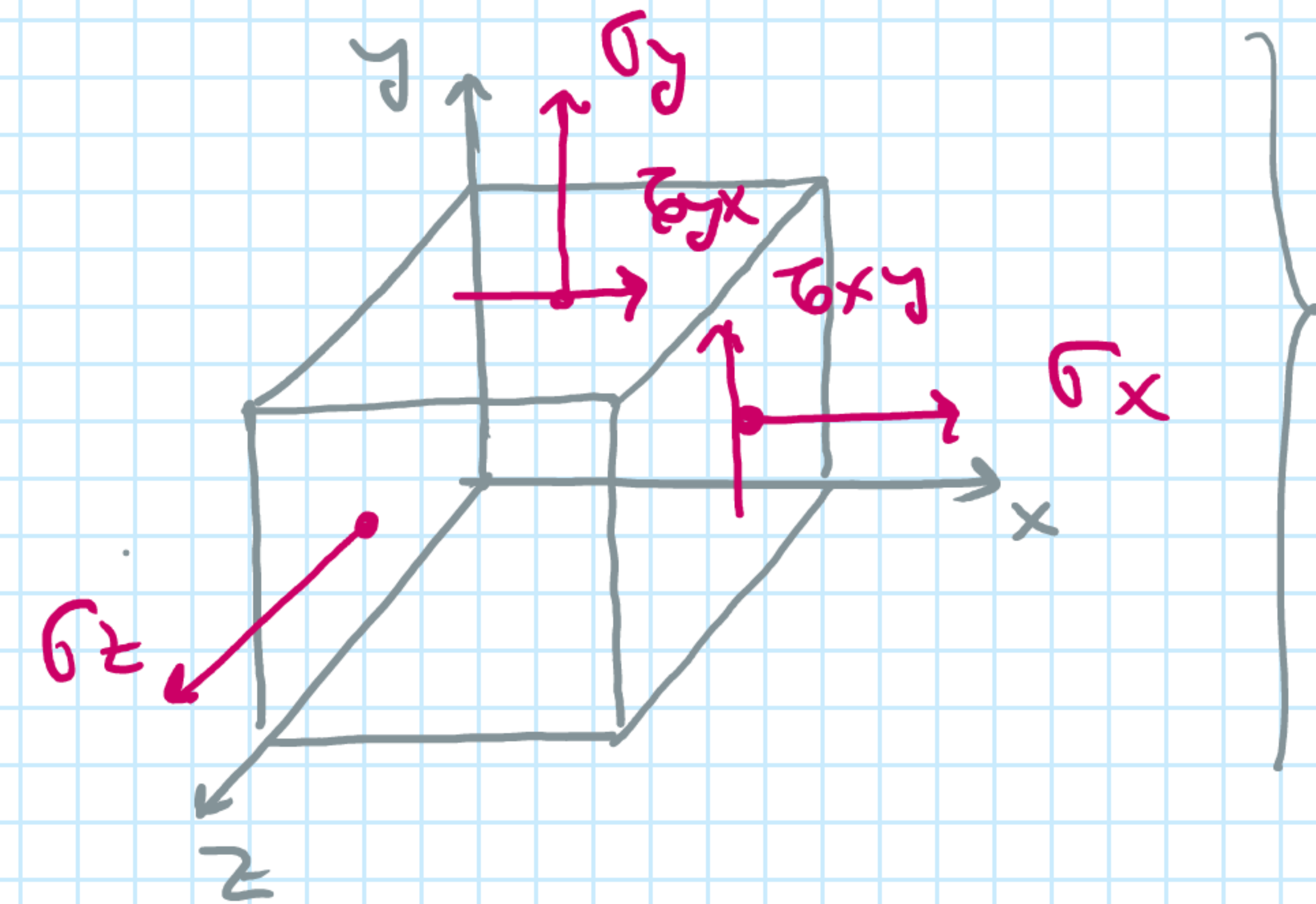
$$[\underline{T}] = \begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$



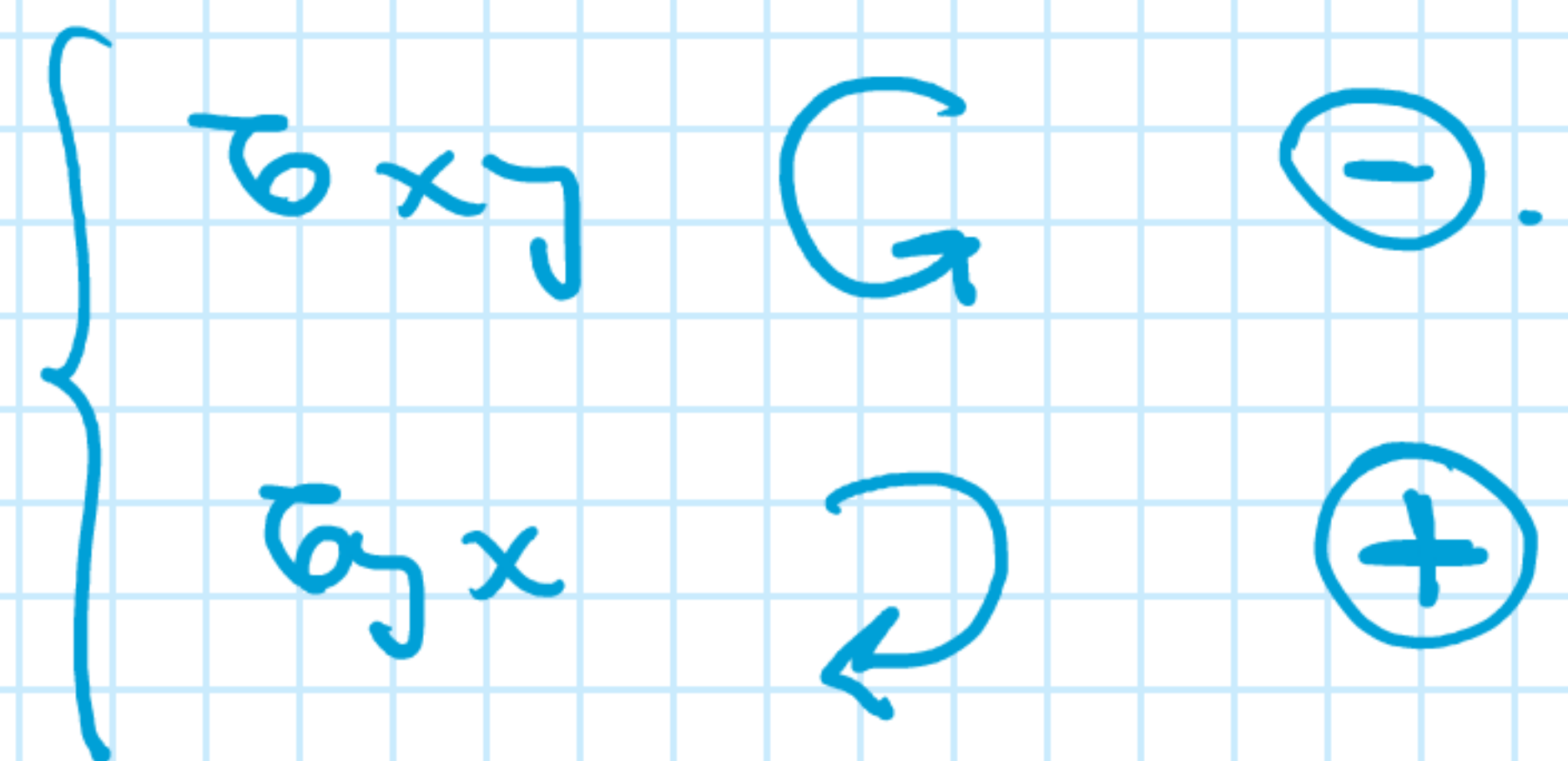
$$\alpha \rightarrow \underline{\sigma} = \begin{Bmatrix} \sigma \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

PLANO.



NUEVA CONVENCION P/MATER



09 - EJEMPLO:

martes, 6 de julio de 2021 10:15

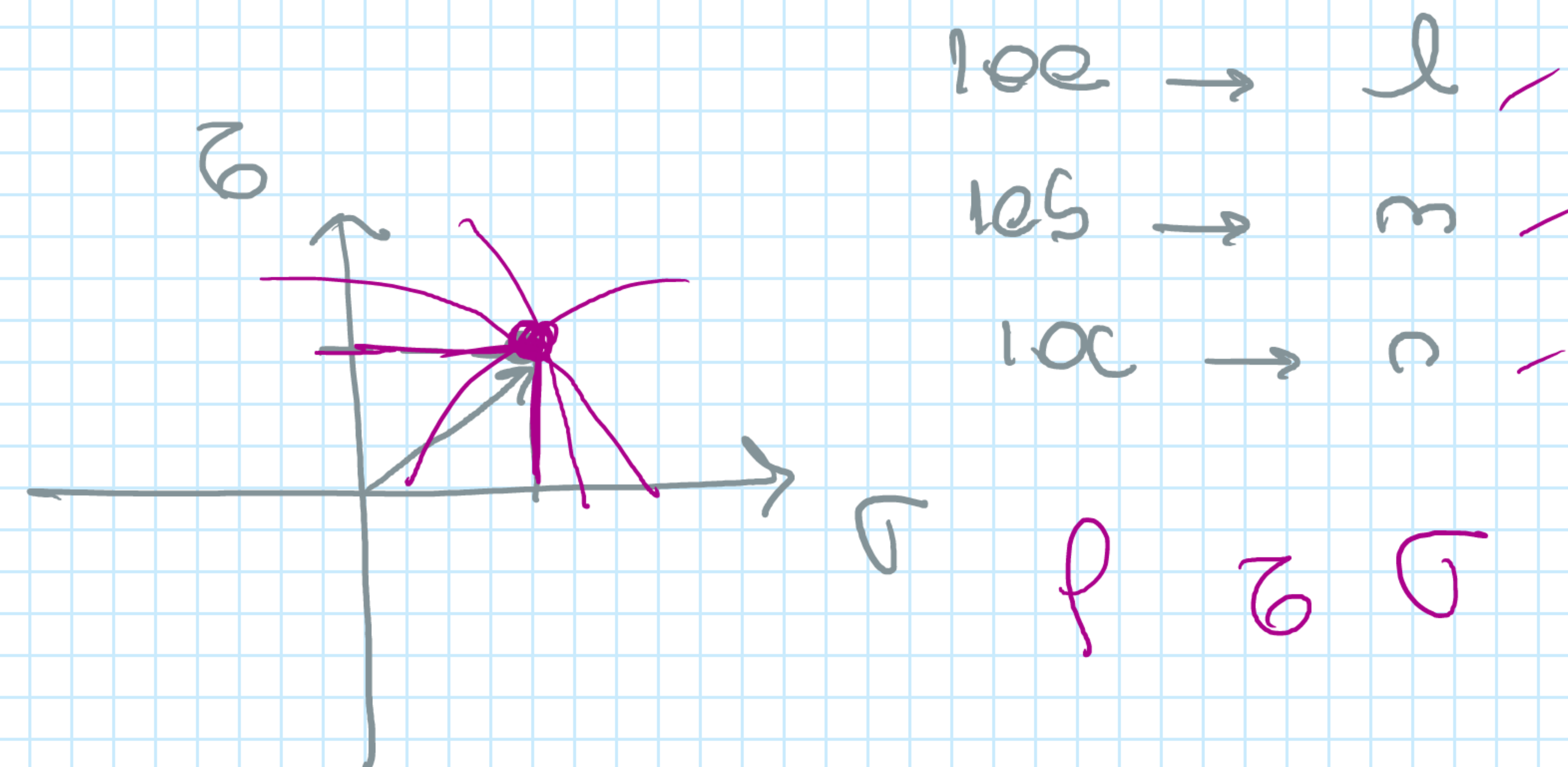
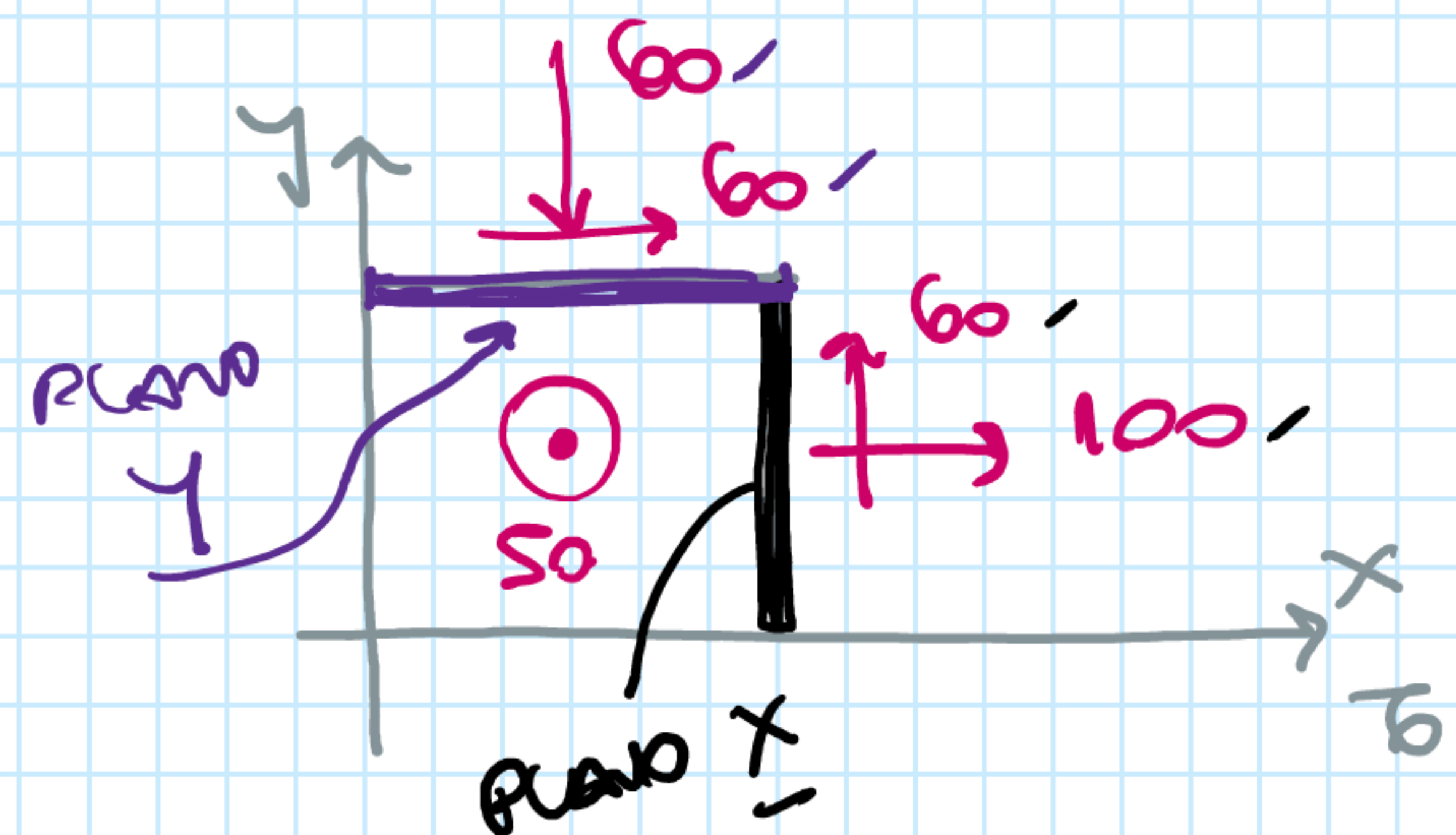
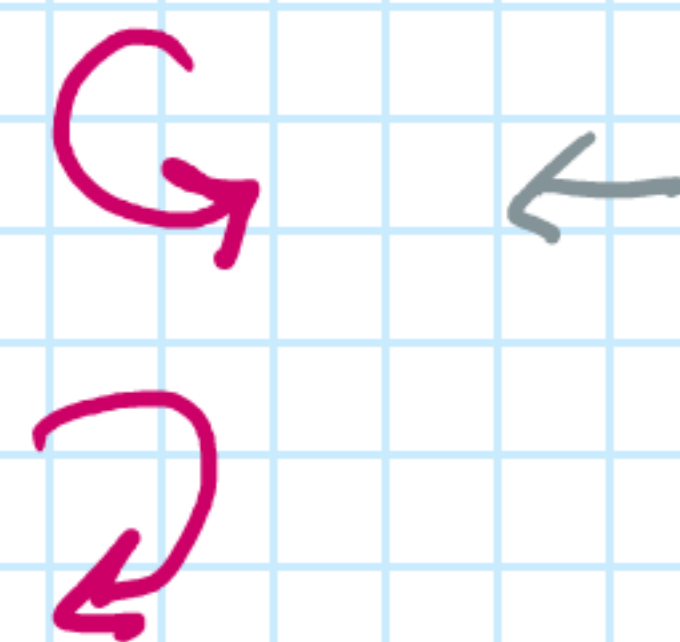
$\sigma_x = 100 \text{ MPa}$
 $\sigma_y = -60 \text{ MPa}$
 $\tau_{xy} = +60 \text{ MPa}$
 $\sigma_z = +50$

$$[TT] = \begin{bmatrix} 100 & 60 & 0 \\ 60 & -60 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

PLANOS cuyo eje director es el eje e'_3

$n = 0$

$\tau_{xy} < 0$
 $\tau_{yx} > 0$



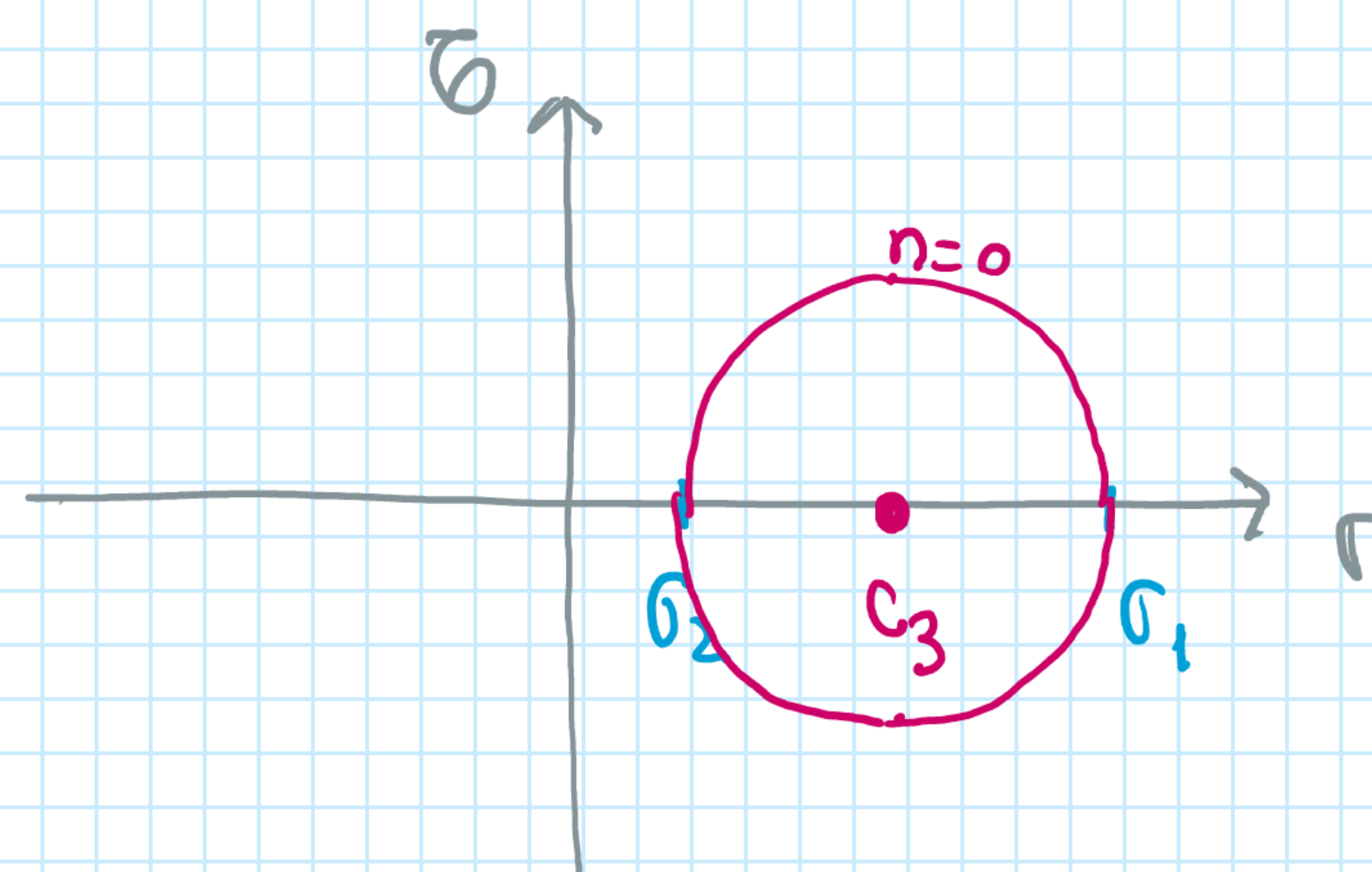
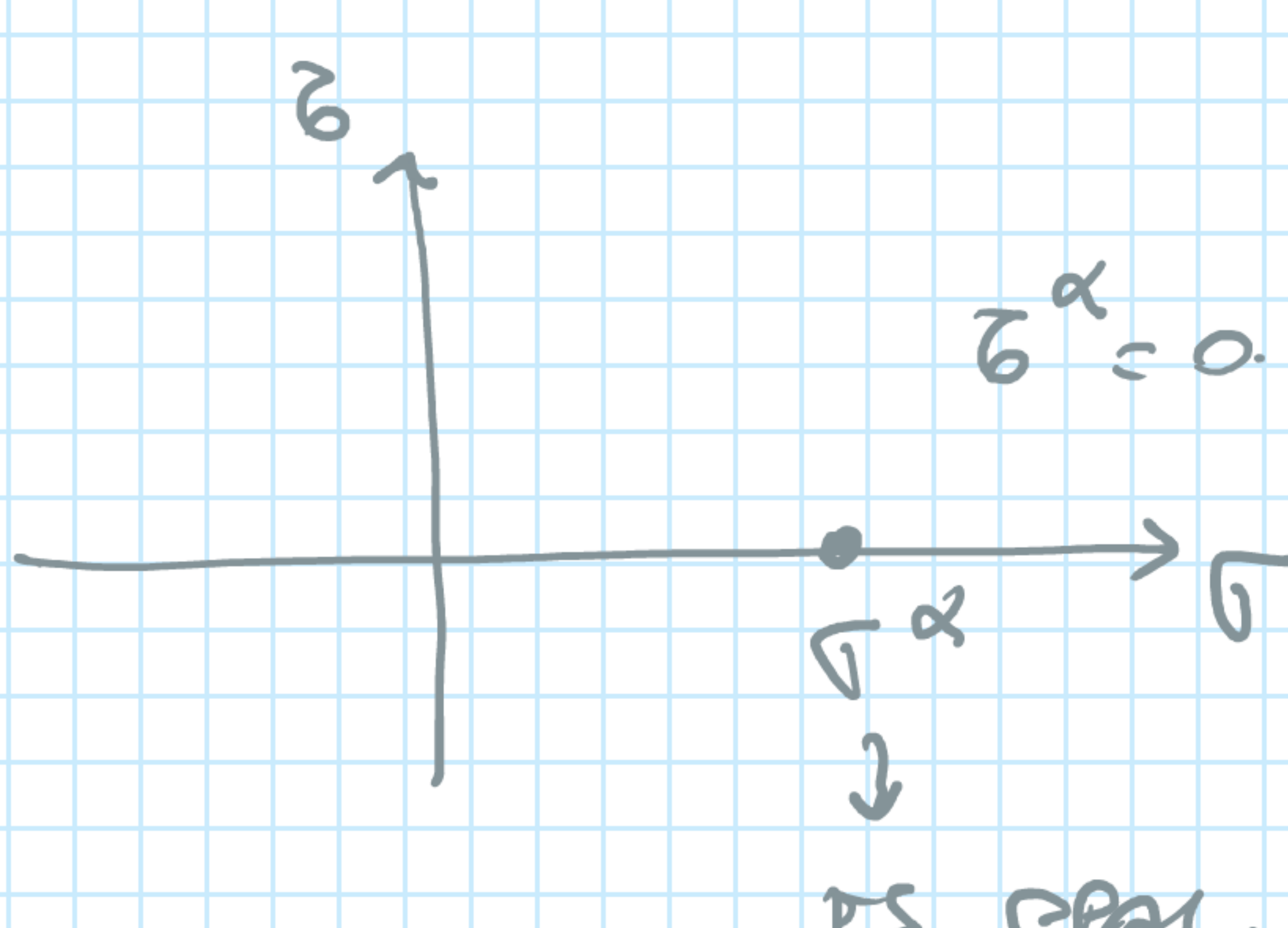
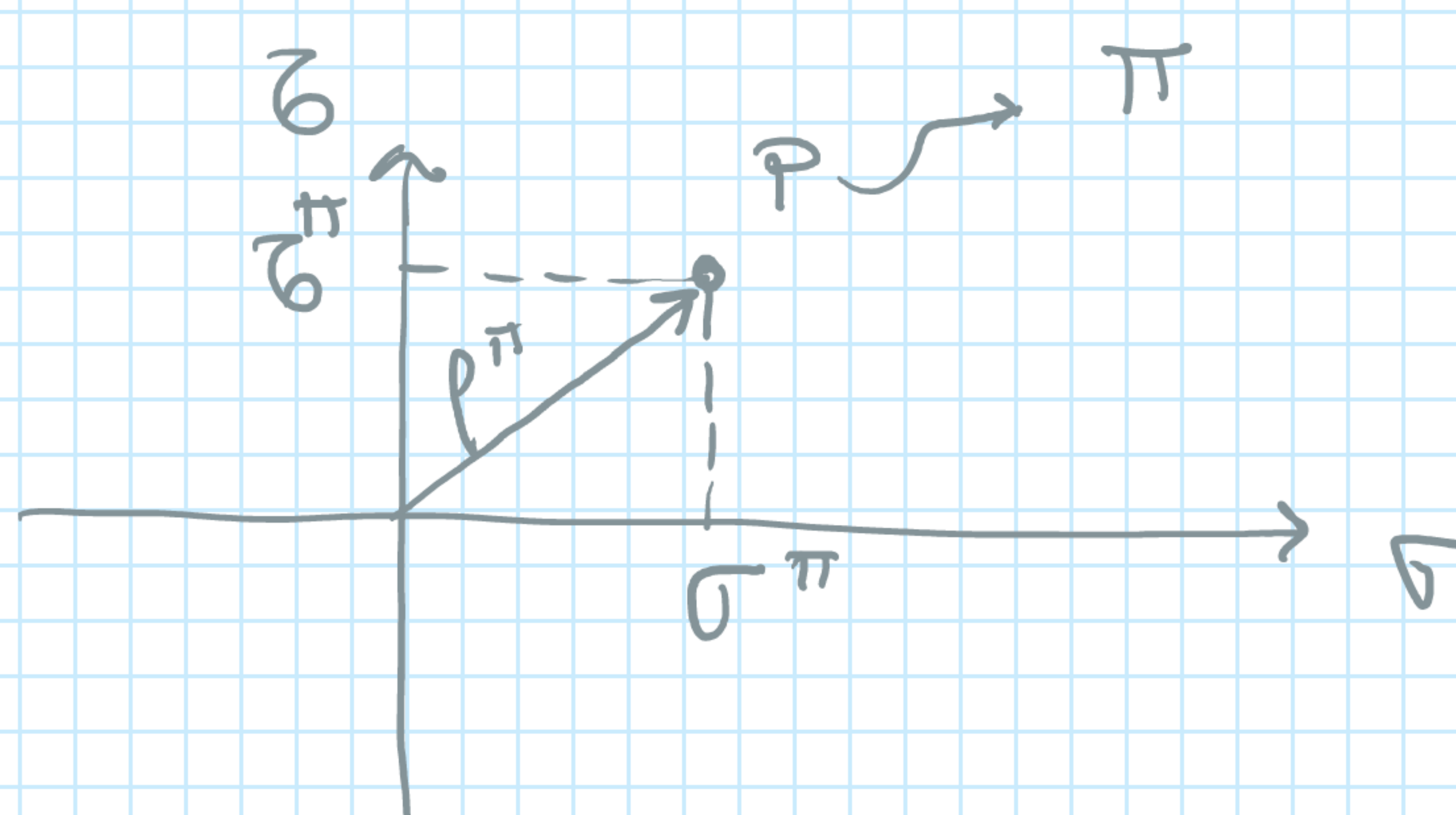
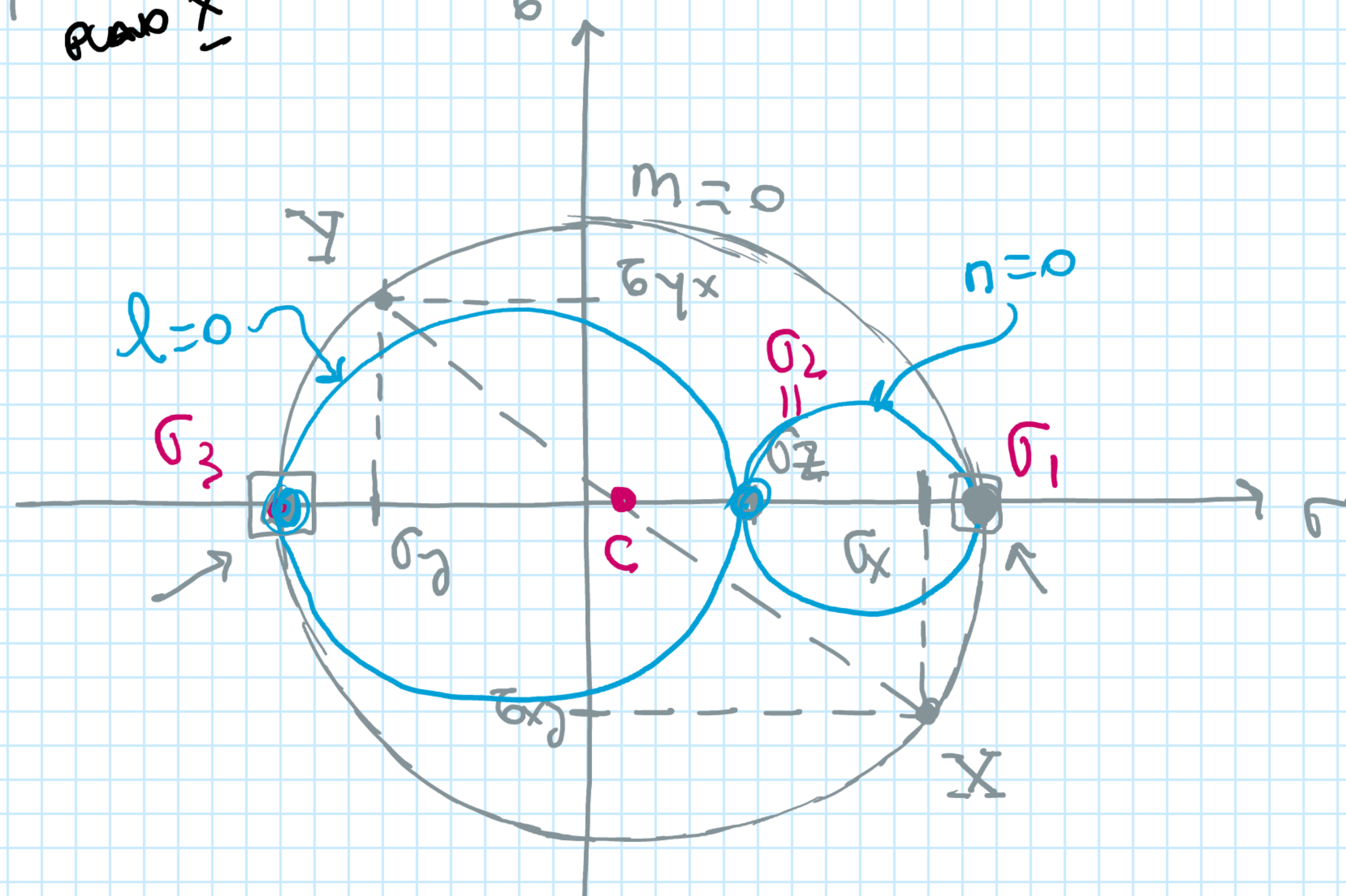
- 100 → l ✓
- 105 → m ✓
- 100 → n ✓

CIRCUNFERENCIA

- l = 0 → 1
- m = 0 → 2
- n = 0 → 3

- σ_2 y σ_3
- σ_1 y σ_3
- σ_1 y σ_2

$\sigma_x = 100$
 $\sigma_y = -60$



09 - EJEMPLO:

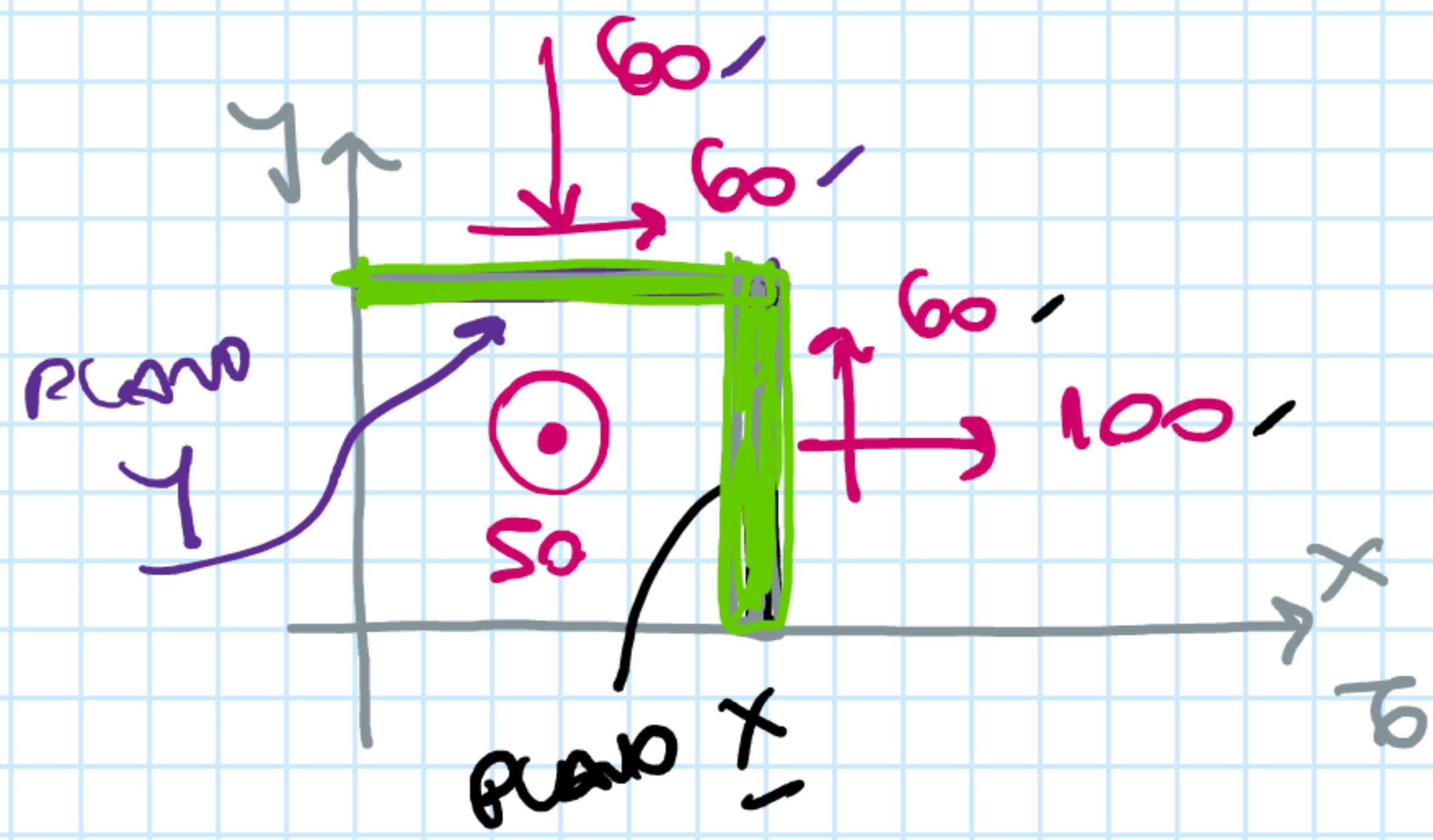
martes, 6 de julio de 2021 10:15

$$\begin{aligned} \sigma_x &= 100 \text{ MPa} \\ \sigma_y &= -60 \text{ MPa} \\ \tau_{xy} &= +60 \text{ MPa} \\ \sigma_z &= +50 \end{aligned}$$

$$[TT] = \begin{bmatrix} 100 & 60 & 0 \\ 60 & -60 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

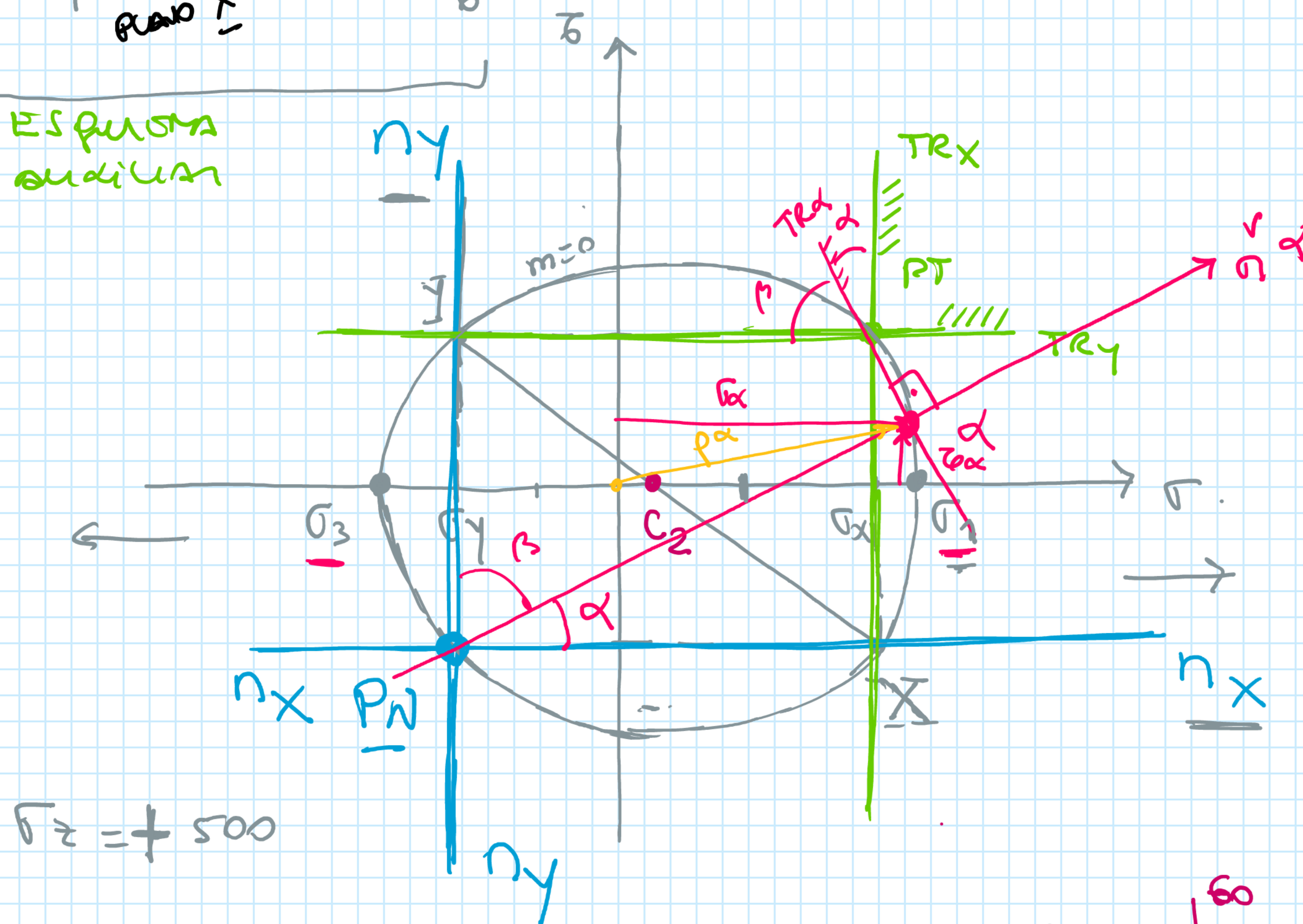
PLANOS cuyo eje director es el eje z'

$$n = 0$$



$$\begin{aligned} \tau_{xy} &< 0. \\ \tau_{yx} &> 0. \end{aligned}$$

ES QUE SON
ORTOGONALES

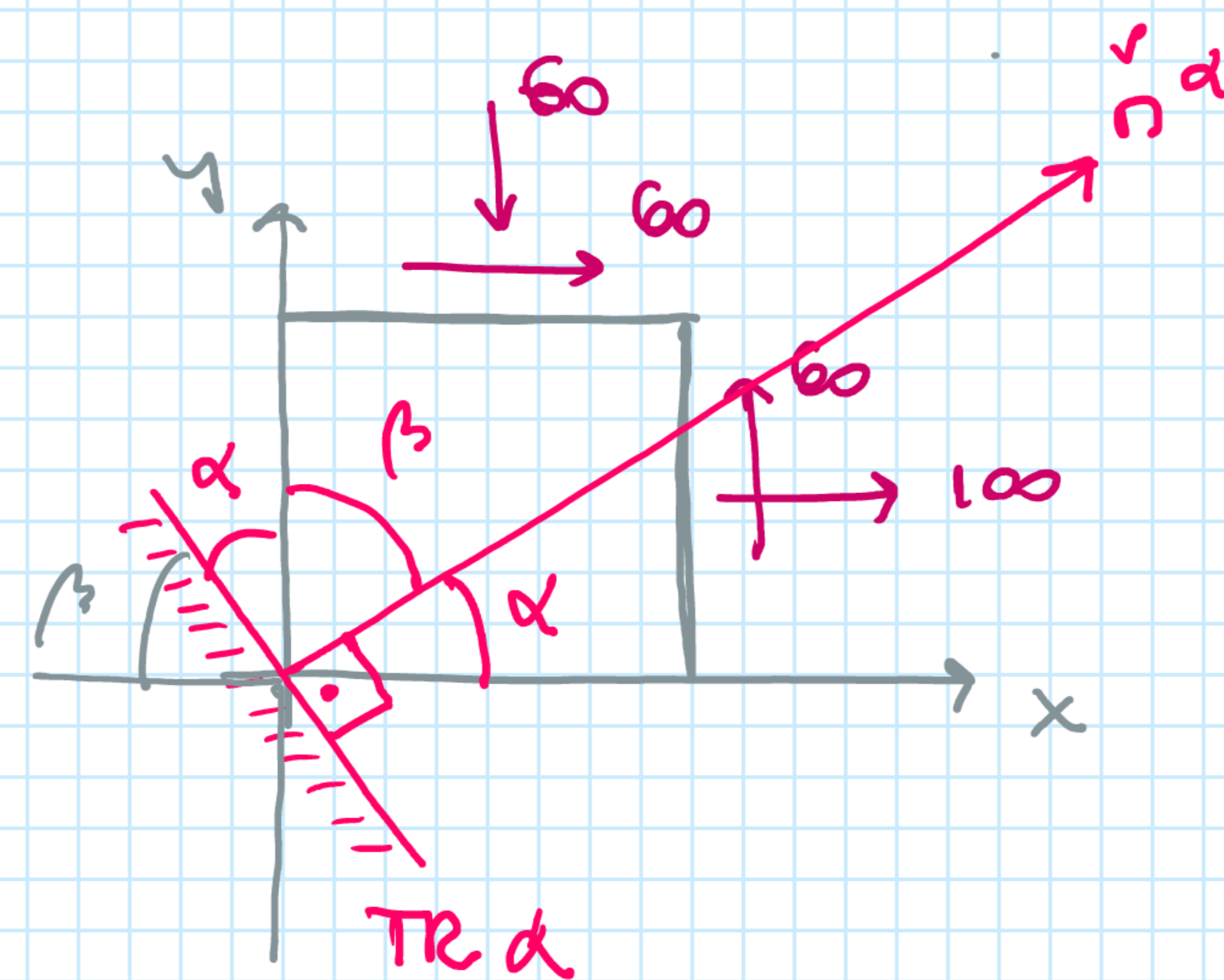
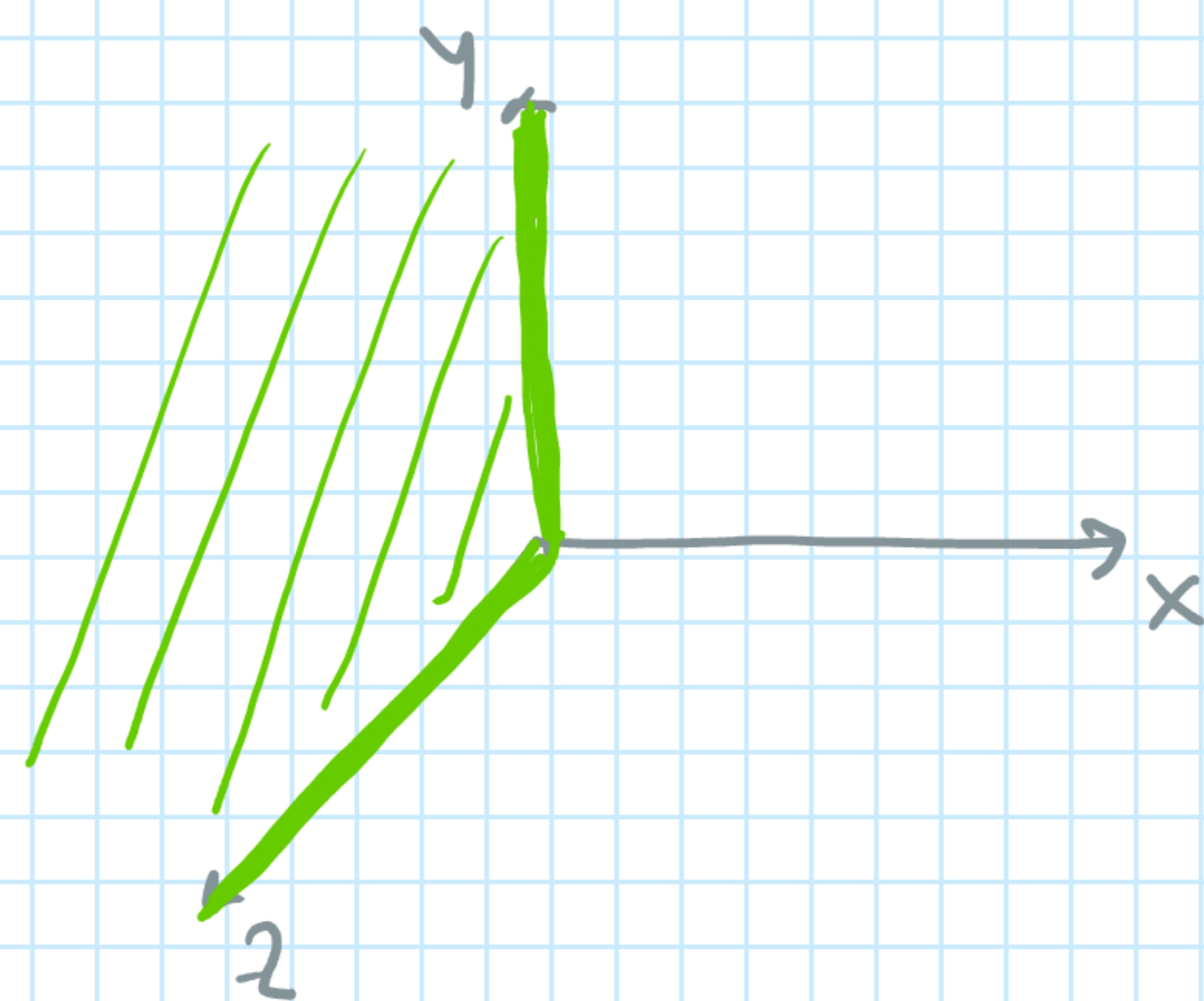


- ELEMENTOS
- NORMALES ✓
 - TRAZAS ✓
 - POLOS ✓

$P_N \equiv$ POLO DE
NORMALES

$P_T \equiv$ POLO DE
TRAZAS

$$\sigma_z = +50$$



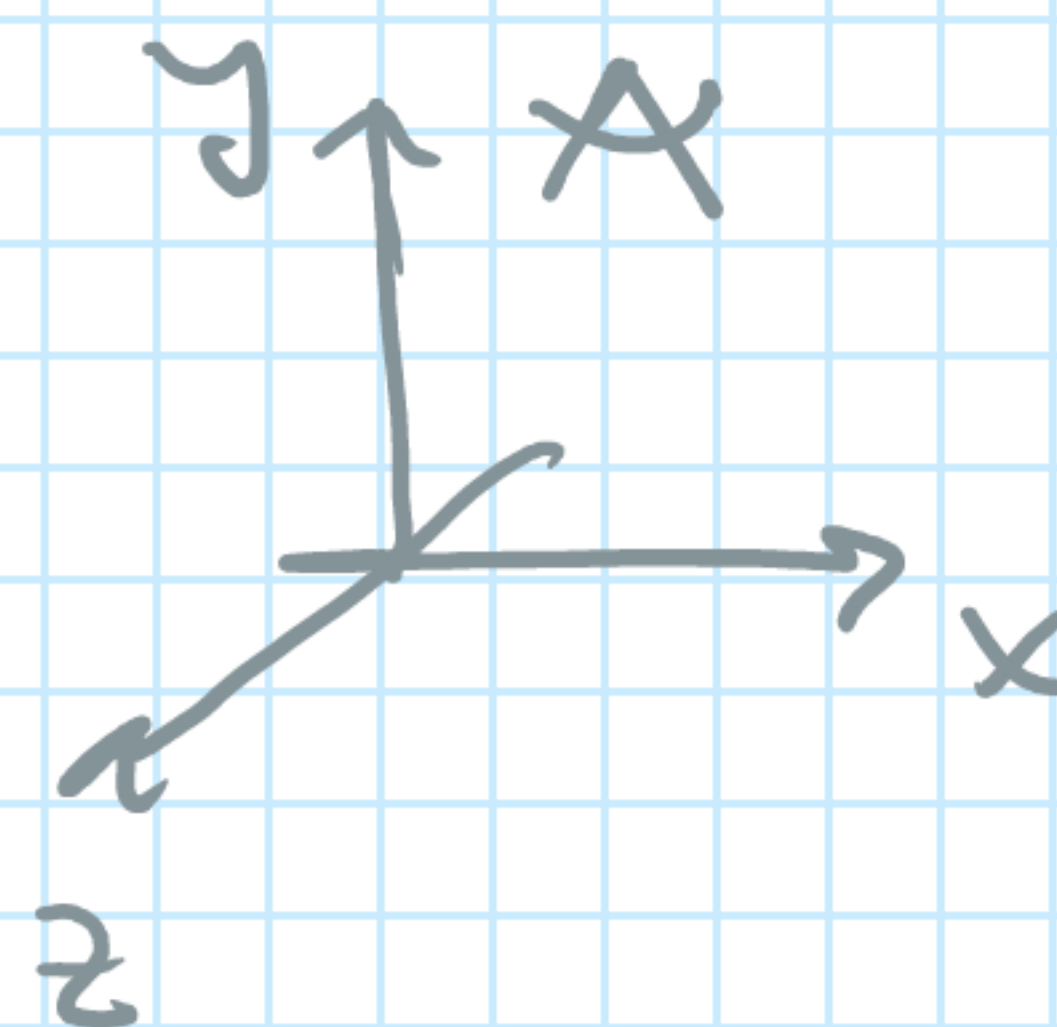
PLANO α
 $n = 0 \rightarrow \alpha = 90^\circ$

$$\begin{aligned} \alpha = 30^\circ &\rightarrow l = 0,866 \\ \beta = 60^\circ &\rightarrow m = 0,50. \end{aligned}$$

10 - ACLARACIÓN:

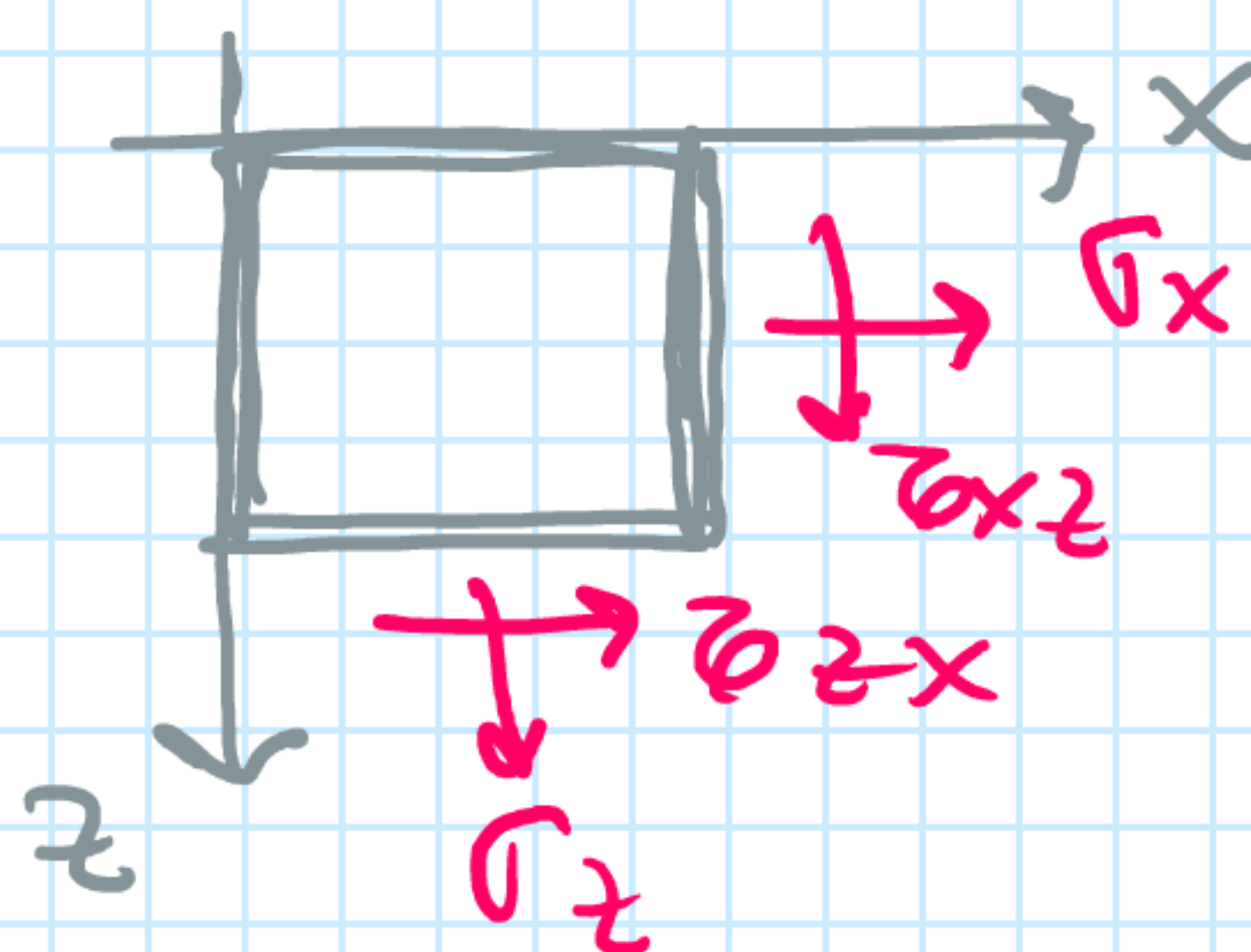
martes, 6 de julio de 2021 11:06

II) EJE DIRECTOR O SISTEMAS 'y'

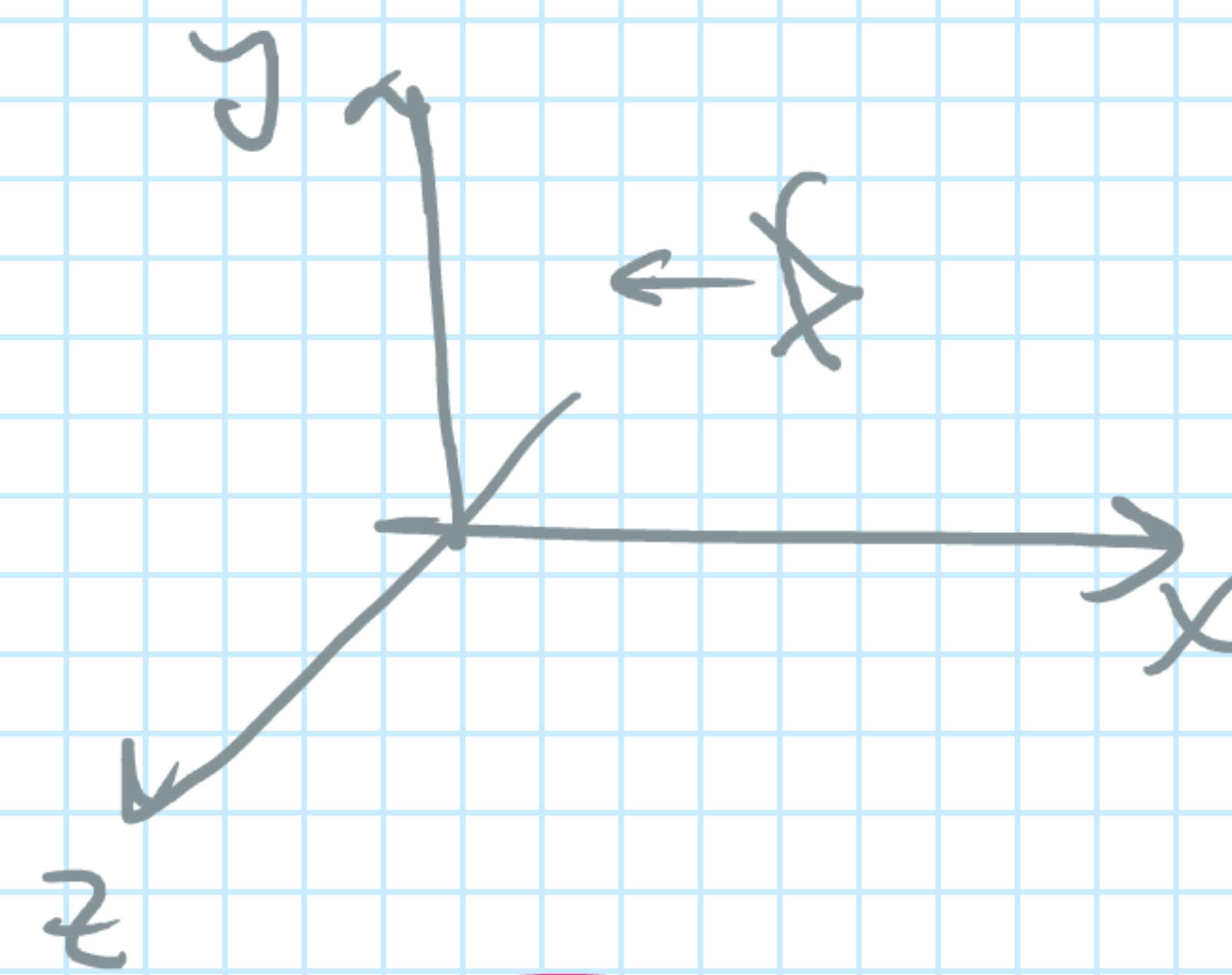


$$[TII] = \begin{bmatrix} \sigma_x & 0 & \tau_{zx} \\ 0 & \sigma_y & 0 \\ \tau_{xz} & 0 & \sigma_z \end{bmatrix}$$

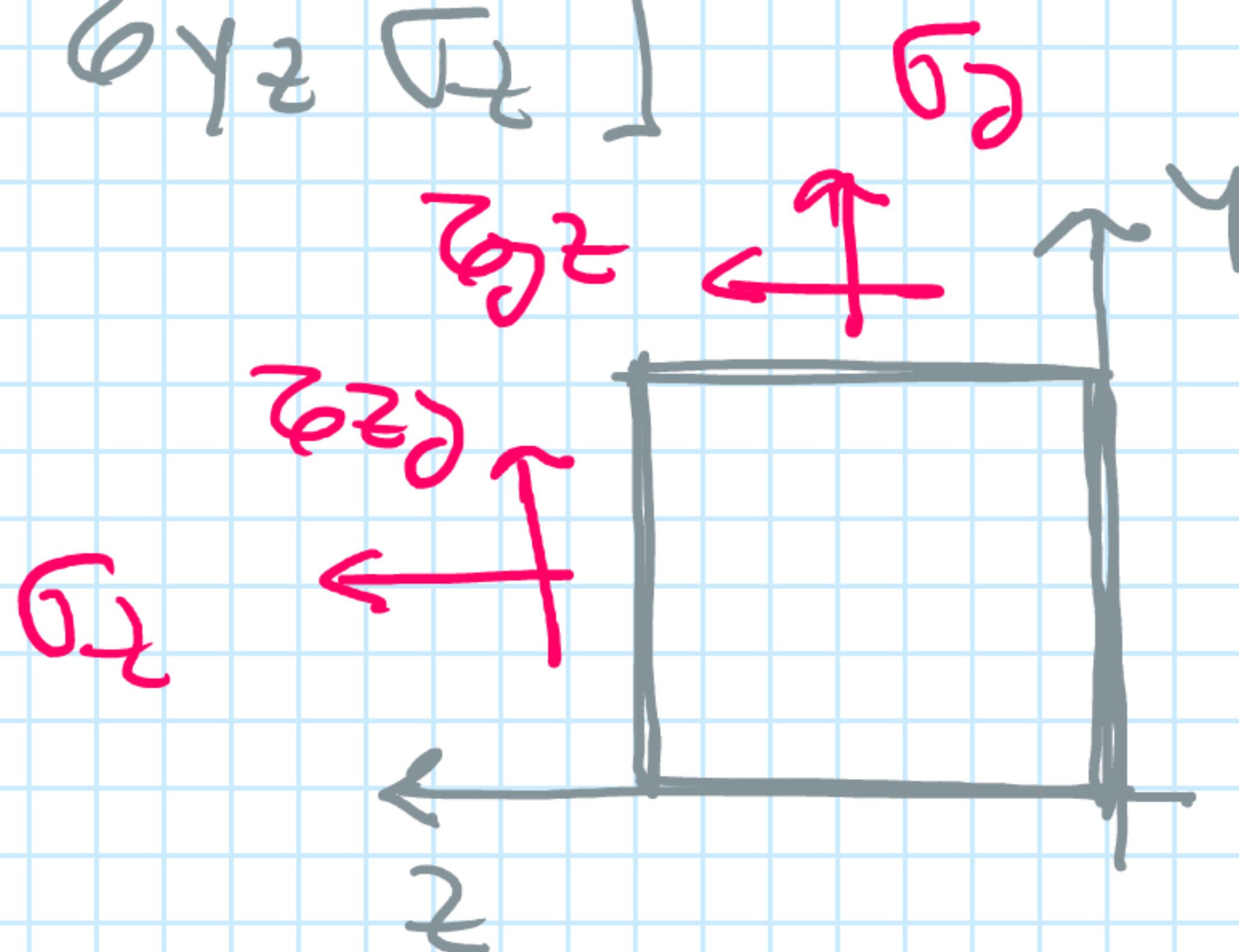
$l \neq 0; m = 0; n \neq 0.$



III) EJE DIRECTOR O SISTEMAS 'x'



$$[TIII] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & \tau_{zy} \\ 0 & \tau_{yz} & \sigma_z \end{bmatrix}$$



$l = 0$
 $m \neq 0$
 $n \neq 0$

MA-08-DIC-2020:

1/17

MA-15-DIC-2020:

ET: "CIRCUNFERENCIA DE MOHR"

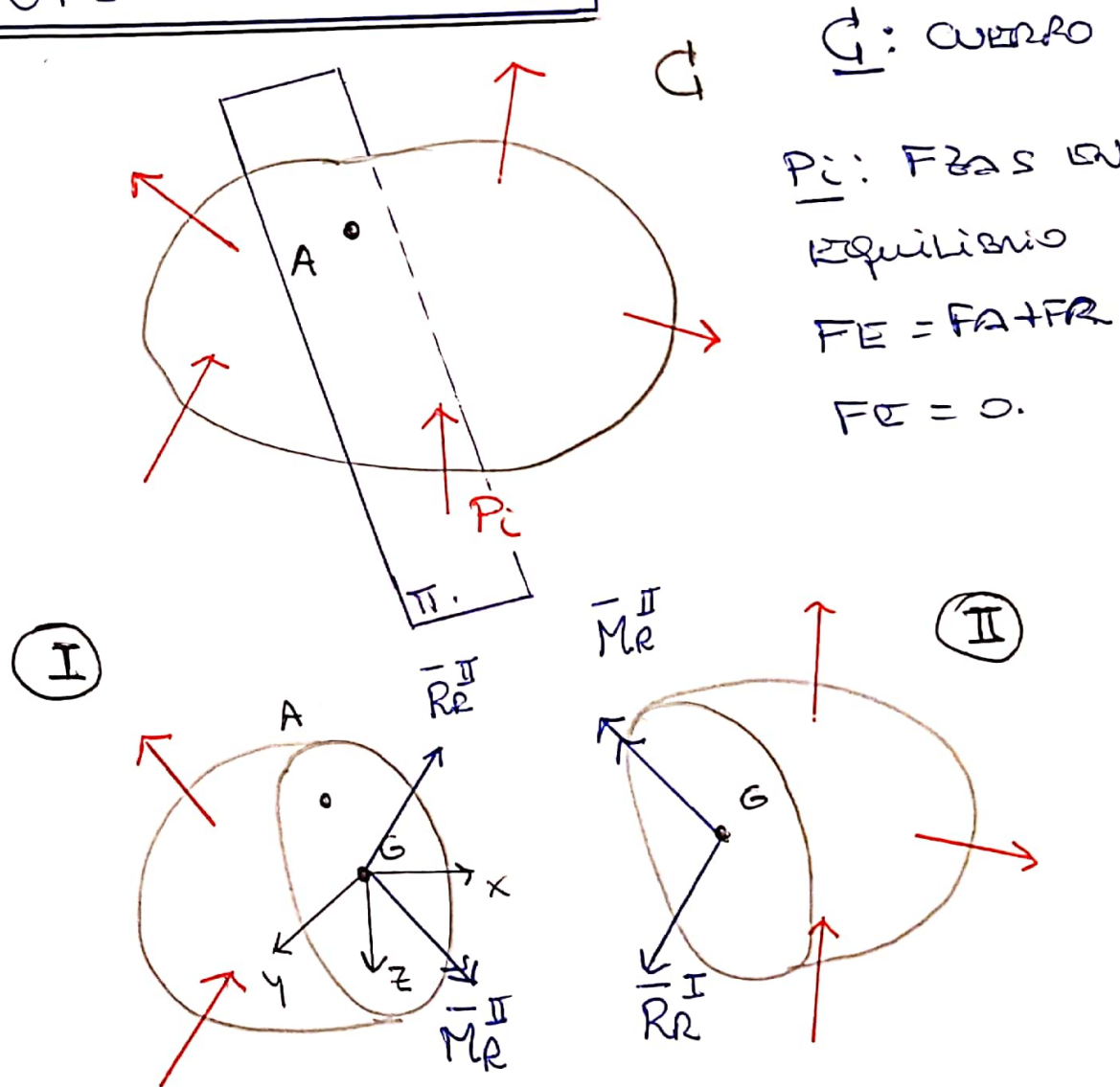
1) ET: → CIRCUNFERENCIA DE MOHR.

2) ED:

3) $RT_{yD} \equiv RC$.
RELACIONES CONSTITUTIVAS.

4) TEL: TEORIAS DE LOS ESTADOS LÍMITES O DE FALLA

01 - INTRODUCCIÓN:



02 - DEFINICIONES :

- VECTOR TENSION

$$\bar{p}^\pi = \lim_{\Delta A \rightarrow 0} \frac{\Delta \bar{P}}{\Delta A} = \frac{d\bar{P}}{dA}$$

→ EL $[TT]$ ES UN ENTE MATEMÁTICO QUE :

- REPRESENTA

→ CARACTERIZADA.

- Y PERMITE DETERMINAR

AL ESTADO DE TENSION EN UN PUNTO DEL CUERPO.

- ESTADO DE TENSION EN UN PUNTO.

- TENSOR DE TENSIONES :

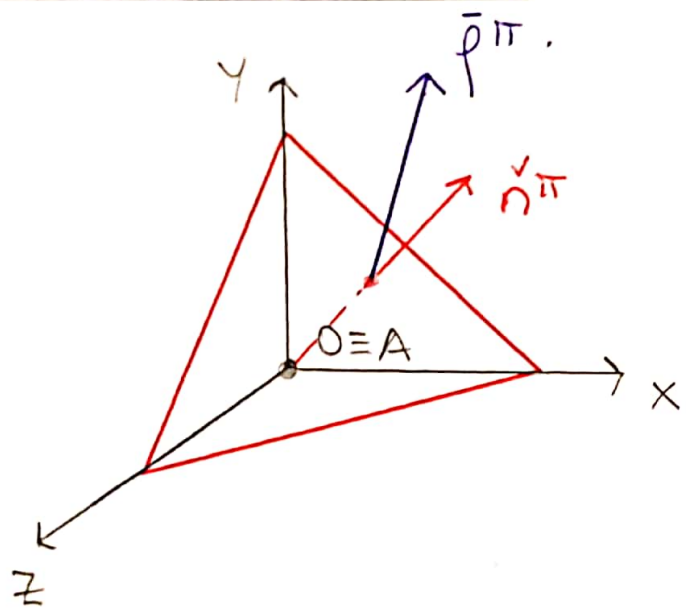
$$[TT] = \begin{matrix} & \begin{matrix} \sigma_x \\ \sigma_{xy} \\ \sigma_{xz} \end{matrix} & \begin{matrix} \sigma_{yx} \\ \sigma_y \\ \sigma_{yz} \end{matrix} & \begin{matrix} \sigma_{zx} \\ \sigma_{zy} \\ \sigma_z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \end{matrix}$$

Plano 'z'
Plano 'y'
Plano 'x'

$$[TT] \{ \bar{n}^\pi \} = \{ \bar{p}^\pi \}$$

→ SE CONSTRUYE CONOCIENDO LOS VECTORES TENSION ASOCIADOS A 3 PLANOS PERPENDICULARES.

03 - DESARROLLOS PRELIMINARES:



- SI SE TRABAJA CON LA TERNIA PRINCIPAL:

$(0; 1; 2; 3).$

$$\left\{ \begin{array}{l} p_1^{\pi} = \sigma_1 \cdot l^* \\ p_2^{\pi} = \sigma_2 \cdot m^* \\ p_3^{\pi} = \sigma_3 \cdot n^* \end{array} \right\} \quad (1c)$$

$$\{ \bar{p}^{\pi} \} = [T T] \{ \check{n}^{\pi} \}. \quad (1a)$$

$$\left\{ \begin{array}{l} p_x^{\pi} = \sigma_x \cdot l + \sigma_{yx} \cdot m + \sigma_{zx} \cdot n. \\ p_y^{\pi} = \sigma_{xy} \cdot l + \sigma_y \cdot m + \sigma_{zy} \cdot n. \\ p_z^{\pi} = \sigma_{xz} \cdot l + \sigma_{yz} \cdot m + \sigma_z \cdot n. \end{array} \right\} \quad (1b).$$

$$\rho_{x_{72}}^{\pi} = |\vec{\rho}^{\pi}| = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2} \quad (2a)$$

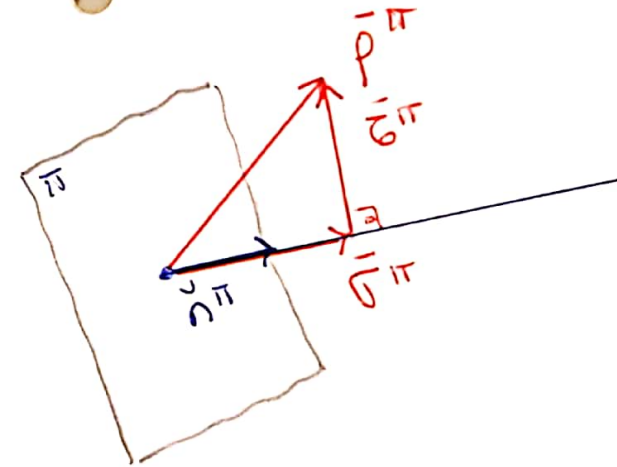
$$\rho_{123}^{\pi} = |\vec{\rho}_{123}^{\pi}| = \sqrt{\rho_1^2 + \rho_2^2 + \rho_3^2} \quad (2b)$$

$$\rho^{\pi} = \sqrt{\sigma_1^2 (\rho^*)^2 + \sigma_2^2 (m^*)^2 + \sigma_3^2 (n^*)^2} \quad (3)$$

$$\rho^{\pi} = \sqrt{(\sigma^{\pi})^2 + (\tau^{\pi})^2} \quad (4)$$

→ COMBINANDO (3) y (4):

$$\sigma^2 + \tau^2 = \sigma_1^2 (\rho^*)^2 + \sigma_2^2 (m^*)^2 + \sigma_3^2 (n^*)^2 \quad (5)$$



$$\{\vec{\sigma}^{\pi}\} = \left[\{\vec{\rho}^{\pi}\} \cdot \{\vec{n}^{\pi}\} \right] \cdot \{\vec{n}^{\pi}\} \quad (6)$$

PSEUDO-MÓDULO.

$$\sigma^{\pi} = \sigma = \underbrace{(\sigma_1 \rho^*, \sigma_2 m^*, \sigma_3 n^*)}_{(1c)} \cdot (\rho^*, m^*, n^*) \quad (7)$$

$$\sigma^{\pi} = \sigma = \sigma_1 (\rho^*)^2 + \sigma_2 (m^*)^2 + \sigma_3 (n^*)^2 \quad (8)$$

04) - CONSTRUCCIÓN DE MOHR:

$$(5) \rightarrow \sigma^2 + \tau^2 = \sigma_1^2 \cdot l^2 + \sigma_2^2 \cdot m^2 + \sigma_3^2 \cdot n^2.$$

$$(8) \rightarrow \sigma = \sigma_1 \cdot l^2 + \sigma_2 \cdot m^2 + \sigma_3 \cdot n^2.$$

$$(9) \rightarrow 1 = l^2 + m^2 + n^2.$$

\rightarrow MOHR

$$(10a) \left[\tau^2 + \left(\sigma - \frac{\sigma_2 + \sigma_3}{2} \right)^2 = \left(\sigma_1 - \frac{\sigma_2 + \sigma_3}{2} \right)^2 l^2 + \left(\frac{\sigma_2 - \sigma_3}{2} \right)^2 (1 - l^2). \right]$$

$$(10b) \left[\tau^2 + \left(\sigma - \frac{\sigma_1 + \sigma_3}{2} \right)^2 = \left(\sigma_2 - \frac{\sigma_1 + \sigma_3}{2} \right)^2 m^2 + \left(\frac{\sigma_1 - \sigma_3}{2} \right)^2 (1 - m^2). \right]$$

$$(10c) \left[\tau^2 + \left(\sigma - \frac{\sigma_1 + \sigma_2}{2} \right)^2 = \left(\sigma_3 - \frac{\sigma_1 + \sigma_2}{2} \right)^2 n^2 + \left(\frac{\sigma_1 - \sigma_2}{2} \right)^2 (1 - n^2). \right]$$

OS - COMPONENTES A LAS EXPRESIONES DE MOHR (10)

I DATOS: $\sigma_1; \sigma_2; \sigma_3$
 → SE TRABAJA E/ TERNA PRAL.

II INCÓGNITAS:

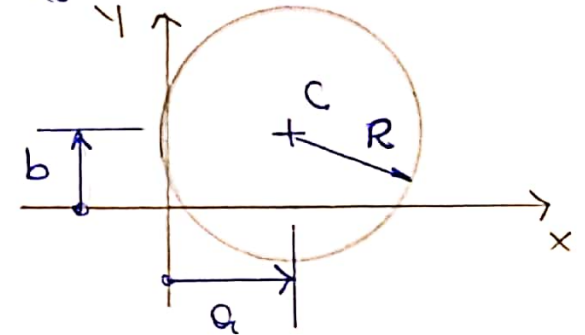
1) → $\sigma_y \bar{\sigma}$. ← COMO DATO ADICIONAL CONOCIDO EL PLANO (l, m, n) .

2) → (l, m, n) ← COMO DATO ADICIONAL CONOCIDO $(\sigma_y \bar{\sigma})$

III LAS EXPRESIONES (10) REPRESENTAN "CIRCUNFERENCIAS"

$$(x - a)^2 + (y - b)^2 = R^2$$

$x \rightarrow \sigma$
 $y \rightarrow \bar{\sigma}$



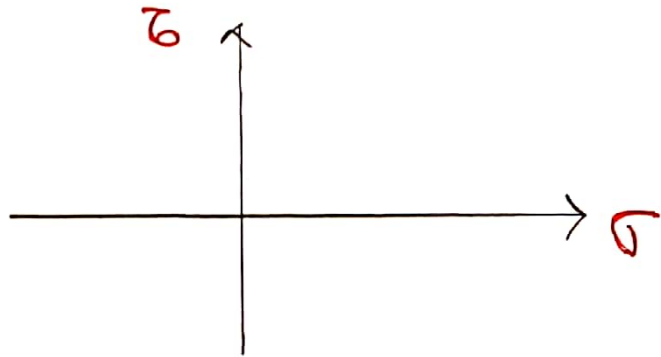
IV LAS EXPRESIONES (10) REPRESENTAN → "FAMILIAS DE CIRCUNFERENCIAS"

$$\left[\begin{array}{l} 0 \leq l \leq 1 \\ 0 \leq m \leq 1 \\ 0 \leq n \leq 1 \end{array} \right.$$

VI) con $\sigma, \tau \rightarrow \rho$.

$$\rho = \sqrt{\sigma^2 + \tau^2}$$

VII) \rightarrow LAS EXPRESIONES (10) SE REPRESENTAN GRÁFICAMENTE;



VIII) LAS FLIJAS DE CIRCUNFERENCIA DEPENDEN DE LAS TENSIONES PROPIAS.

IX) $(x - a)^2 + (y - b)^2 = R^2$
 $(\sigma - a)^2 + (\tau - b)^2 = R^2$

\rightarrow LOS CENTROS DE LAS 3 FLIJAS DE CIRCUNFERENCIALES SE UBICAN SOBRE EL EJE DE ABSISAS (EJE X = EJE σ).

X) \rightarrow CADA FLIJA DEPENDE DE "1" SOLO PARAMETRO.

XI) \rightarrow CADA FLIJA DE CIRCUNFERENCIAS VA A ESTAR LIMITADA POR 2 CIRCUNFERENCIAS EXTREMAS.

XII) \rightarrow EN LAS EXPRESIONES (10) \rightarrow CADA VALOR DE LAS TENS. PROPIAS SE INTRODUCEN CON SU SIGNO.

OG - REPRESENTACIONES:

1ª FAMILIA:

$$C_1 = \left(\frac{\sigma_2 + \sigma_3}{2} ; 0 \right)$$

$$l=0 \rightarrow R_{1,0} = \frac{\sigma_2 - \sigma_3}{2}$$

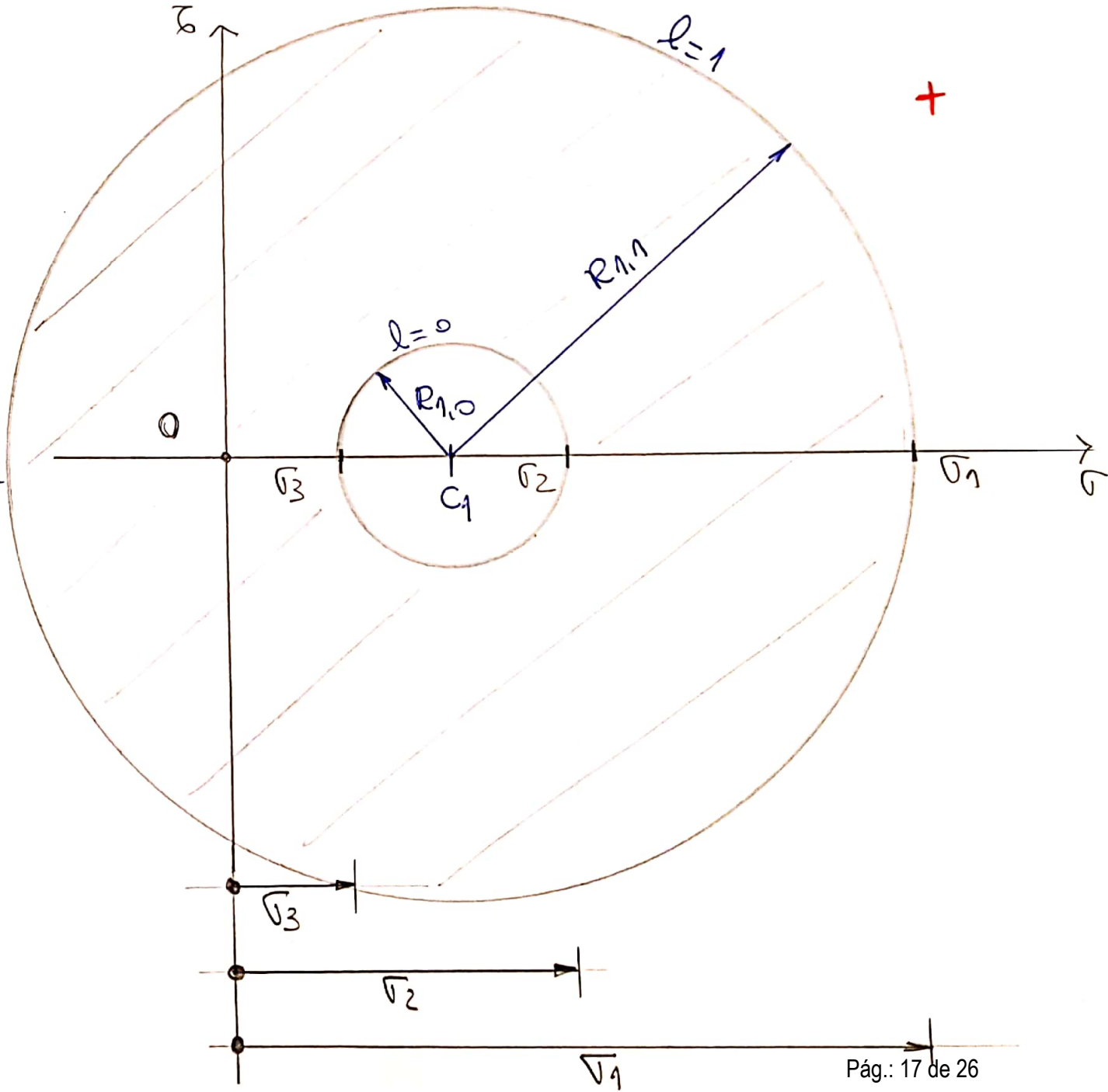
$$l=1 \rightarrow R_{1,1} = \sigma_1 - \frac{\sigma_2 + \sigma_3}{2}$$

DADO P/ EJEMPLO:

$$\sigma_1 = 12$$

$$\sigma_2 = 6$$

$$\sigma_3 = 2$$



ACLARACIONES:

$$\sigma_1 > \sigma_2 > \sigma_3.$$

EJ: $\sigma_1 = 20 \text{ kw/cm}^2$

$$\sigma_2 = 8 \text{ "}$$

$$\sigma_3 = -4 \text{ "}$$

EJ: $\sigma_2 = -25 \text{ "}$

$$\sigma_3 = -50 \text{ "}$$

$$\sigma_1 = -10 \text{ "}$$

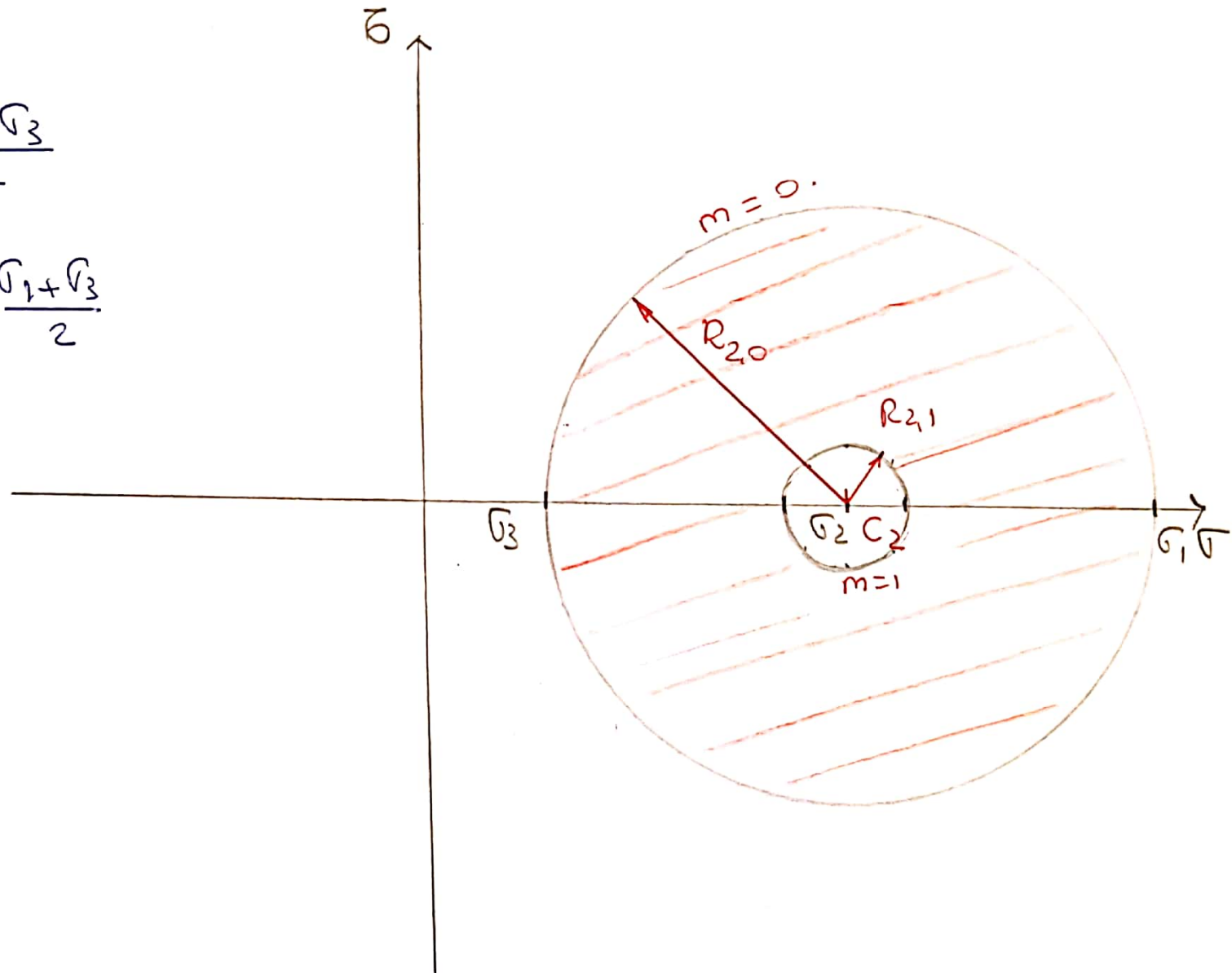
2ª familia:

$$C_2 \equiv \left(\frac{\sigma_1 + \sigma_3}{2} ; 0 \right)$$

$$m=0 \rightarrow R_{2,0} = \frac{\sigma_1 - \sigma_3}{2}$$

$$m=1 \rightarrow R_{2,1} = \sigma_2 - \frac{\sigma_1 + \sigma_3}{2}$$

$$\left. \begin{array}{l} \sigma_1 = 12 \\ \sigma_2 = 6 \\ \sigma_3 = 2 \end{array} \right\}$$



3º FAMILIA:

$$C_3 = \left(\frac{\sigma_1 + \sigma_2}{2}, 0 \right)$$

$$n=0 \rightarrow R_{3,0} = \frac{\sigma_1 - \sigma_2}{2}$$

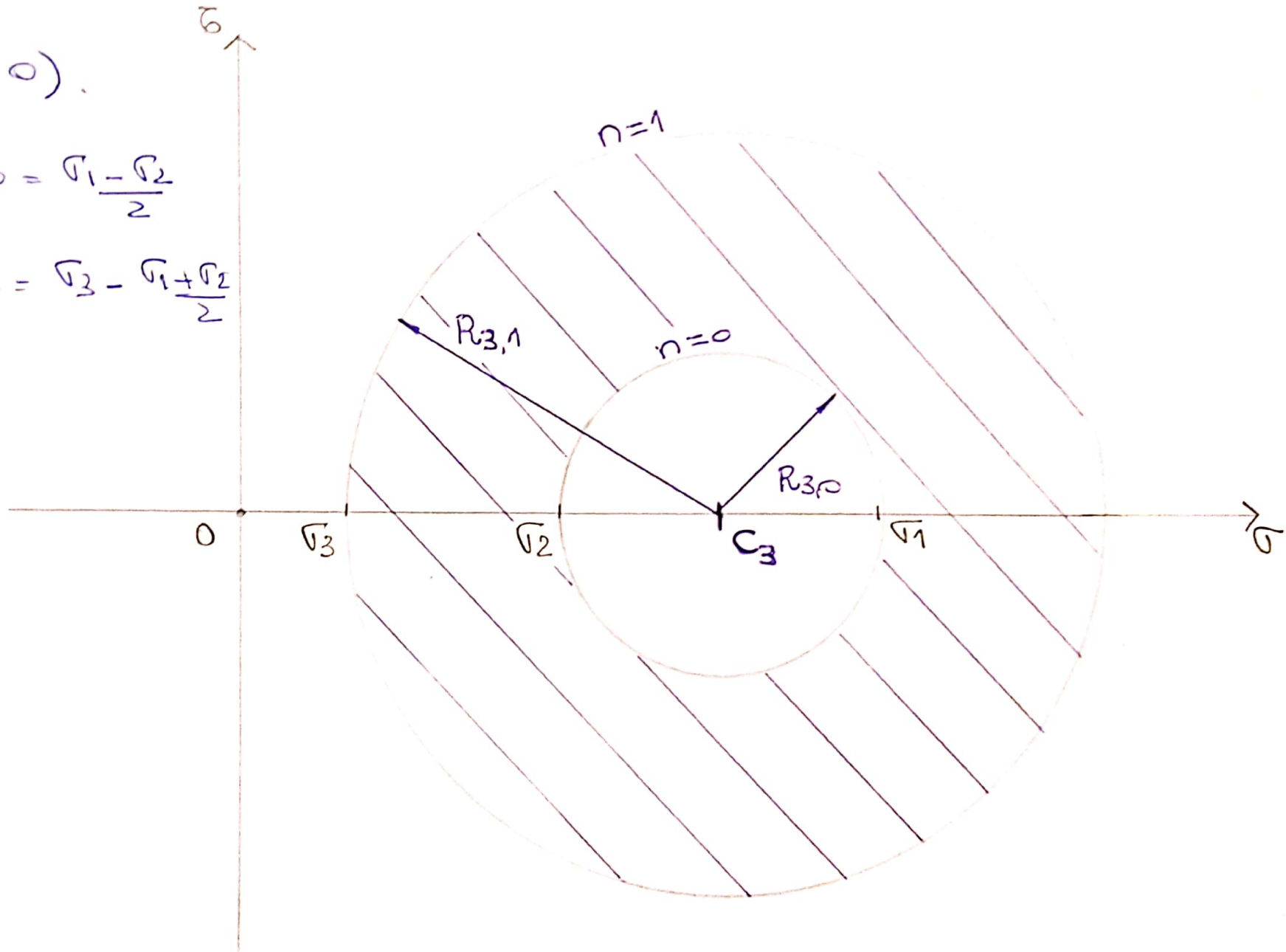
$$n=1 \rightarrow R_{3,1} = \sigma_3 - \frac{\sigma_1 + \sigma_2}{2}$$

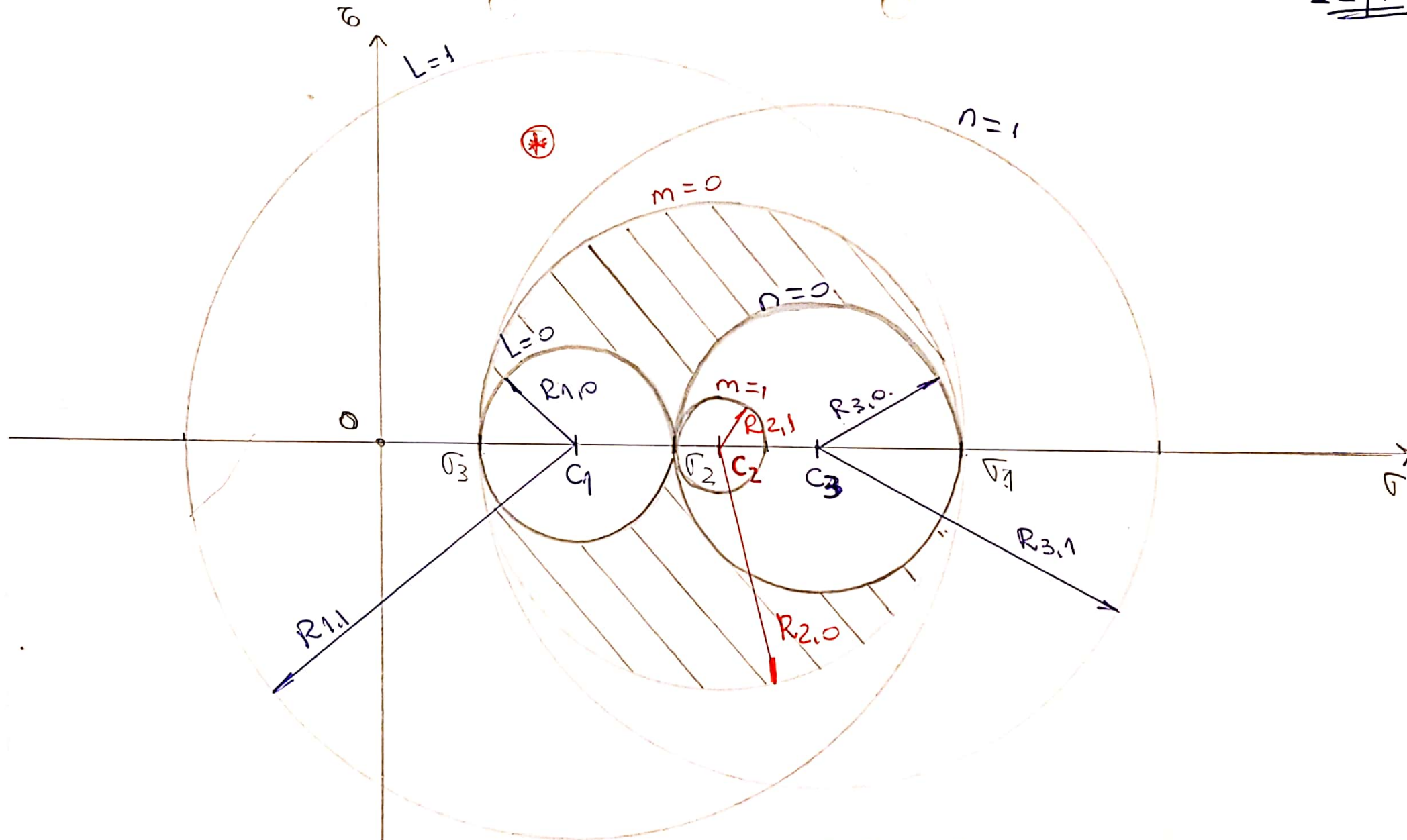
DATOS:

$$\sigma_1 = 12$$

$$\sigma_2 = 6$$

$$\sigma_3 = 2$$

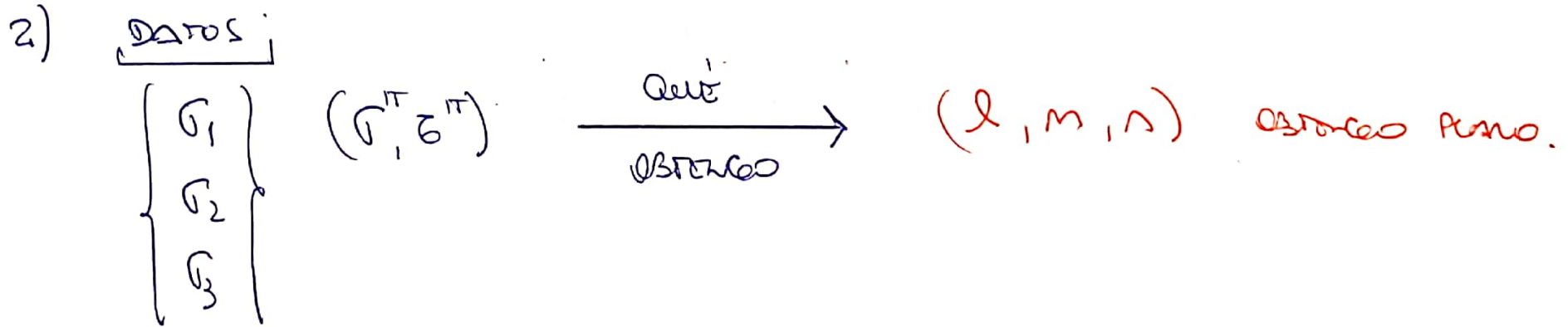
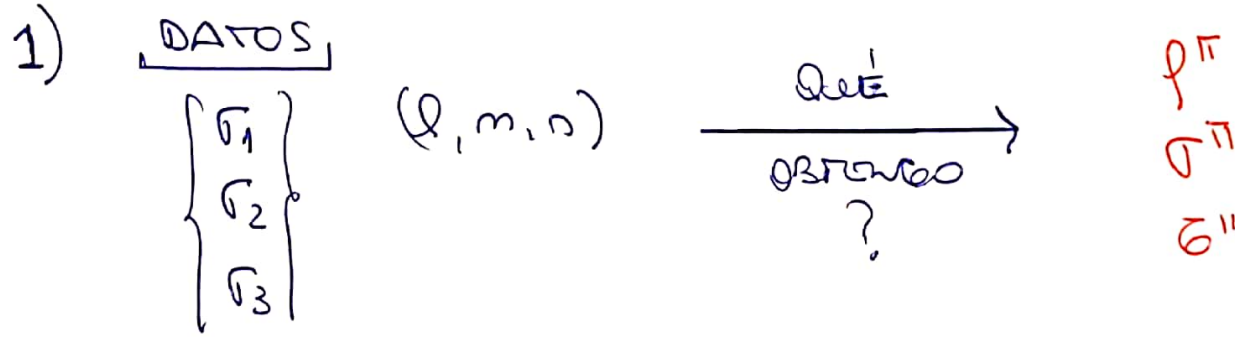




EN ESTE ESQUEMA, SE ~~SUPERPONEN~~ DIBUJAN LOS 3 PARES DE CIRCULO FONDA CIAL EXTERNAS DE CADA FAMILIA Y SE SUPERPONEN BUELO EL AREA DE INTERSECCION DE LAS 3 FAMILIAS.

Autor: Ing. Luis Nelson SOSTI

OBJETIVO:

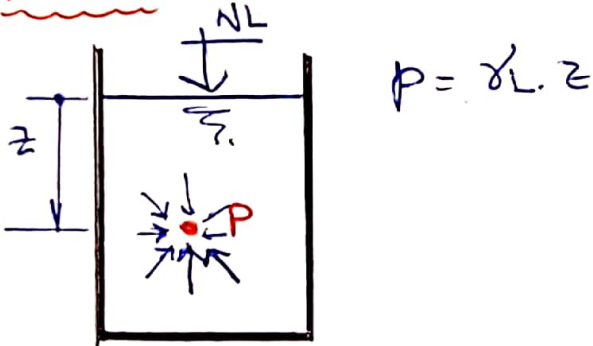


• LAS CIRCUNFERENCIAS DE MOMENTO EN LA CONSTRUCCIÓN DE UNAS EQUIVALENDO AL [TT].

• A LAS CIRCUNFERENCIAS DE $l=0$; $m=0$; $n=0$ → SE LAS DENOMINA "CIRCUNFERENCIAS FUNDAMENTALES DE MOMENTO".

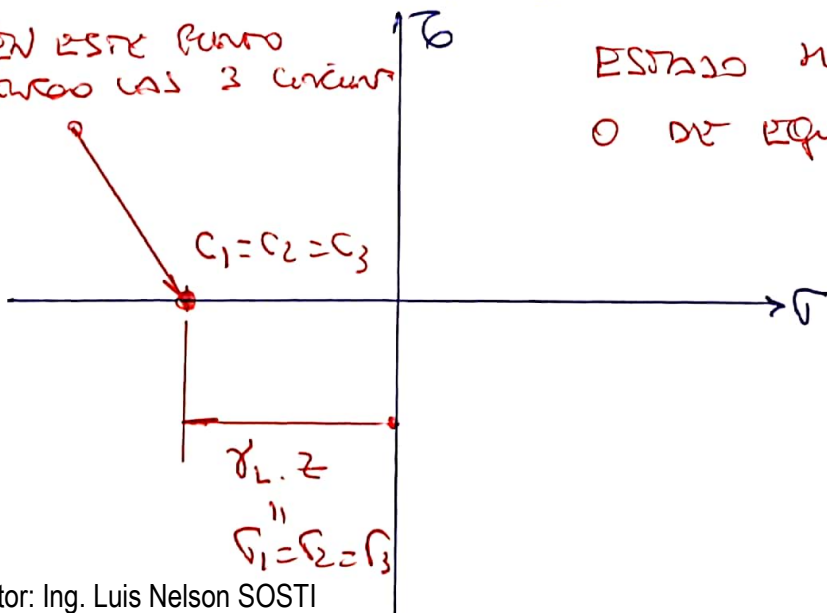
07 - APLICACIONES PARTICULARES
RES.

EJ 01:



$$[\sigma] = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = \begin{bmatrix} \gamma_L z & 0 & 0 \\ 0 & \gamma_L z & 0 \\ 0 & 0 & \gamma_L z \end{bmatrix}$$

EN ESTE PUNTO
RECORO LAS 3 CURVAS

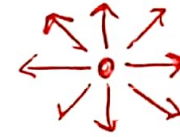


ESTADO HIDROSTÁTICO.

O DE EQUICOMPRESIÓN.

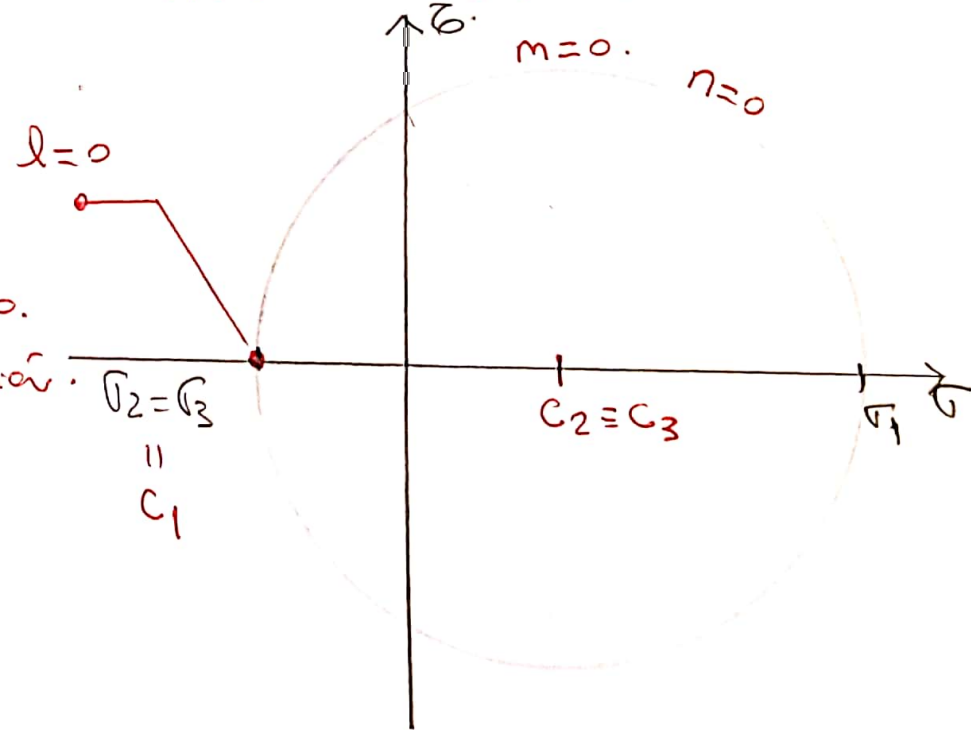
EJ 02 $[\sigma] = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix}$

"EQUITRACCIÓN"



EJ 03 $[\sigma] = \begin{bmatrix} 60 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & -20 \end{bmatrix}$

$\sigma_1 = 60 \quad \sigma_2 = \sigma_3 = -20$



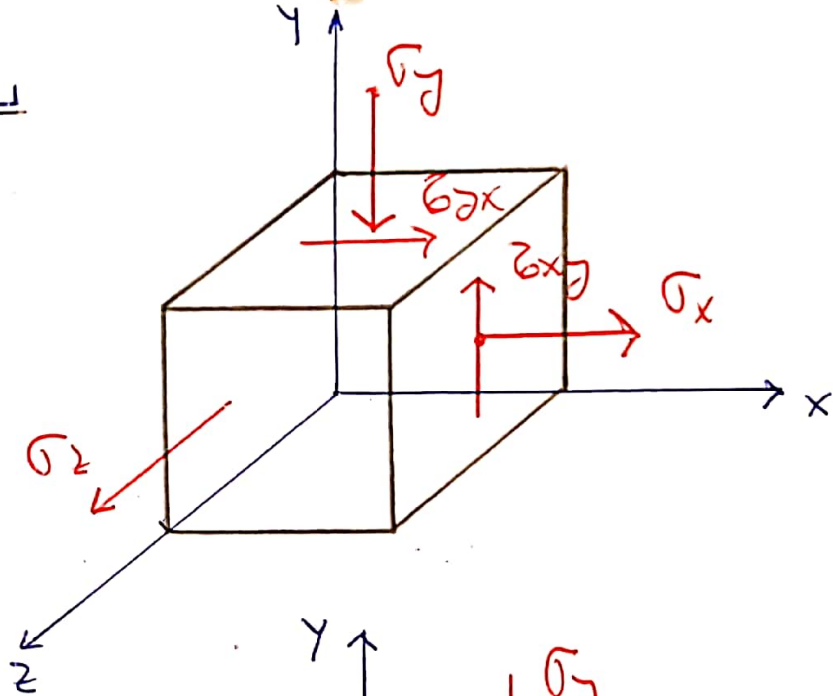
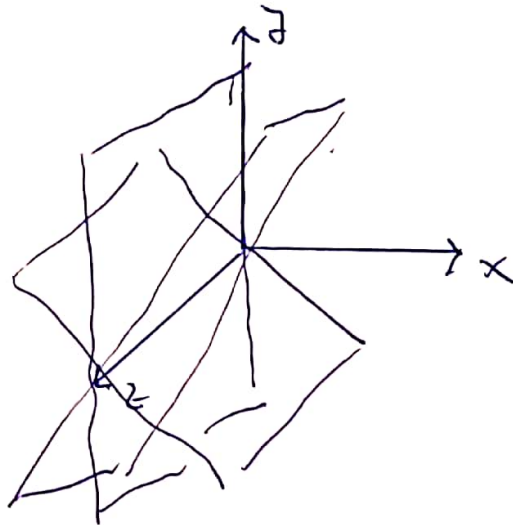
08 - CIRCUNFERENCIA

ESTADO PLANO Y ESTE DISEÑO.

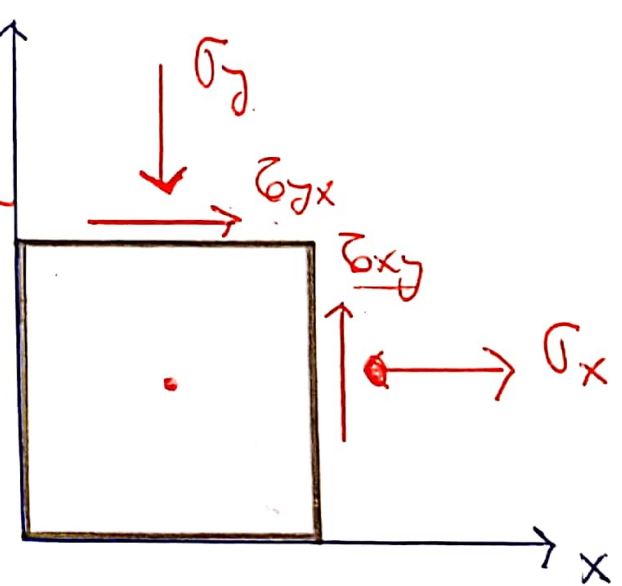
$$[TT] = \begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

TOMAR COMO ESTE DISEÑO → AL ESTE PPAE.

$$\alpha \rightarrow \begin{Bmatrix} l \\ m \\ 0 \end{Bmatrix}$$



NUOVA CONVENCIÓN



$\sigma_x = 80.$

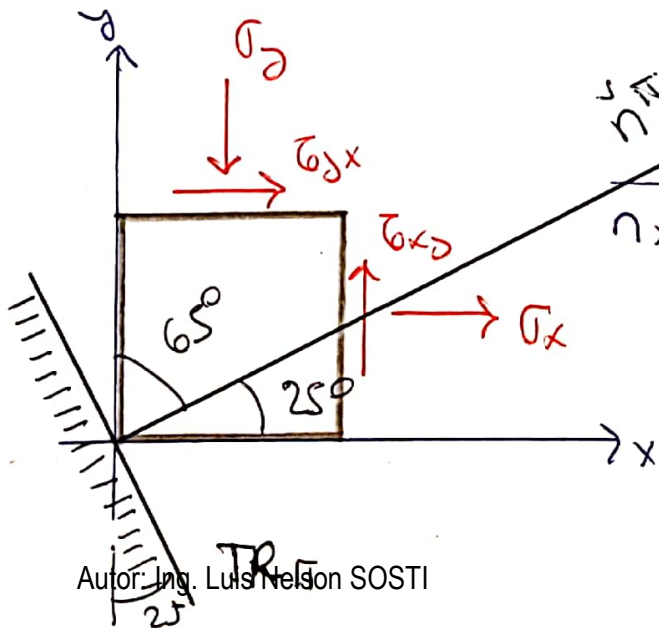
$\sigma_y = -40.$

$\tau_{xy} = +50$

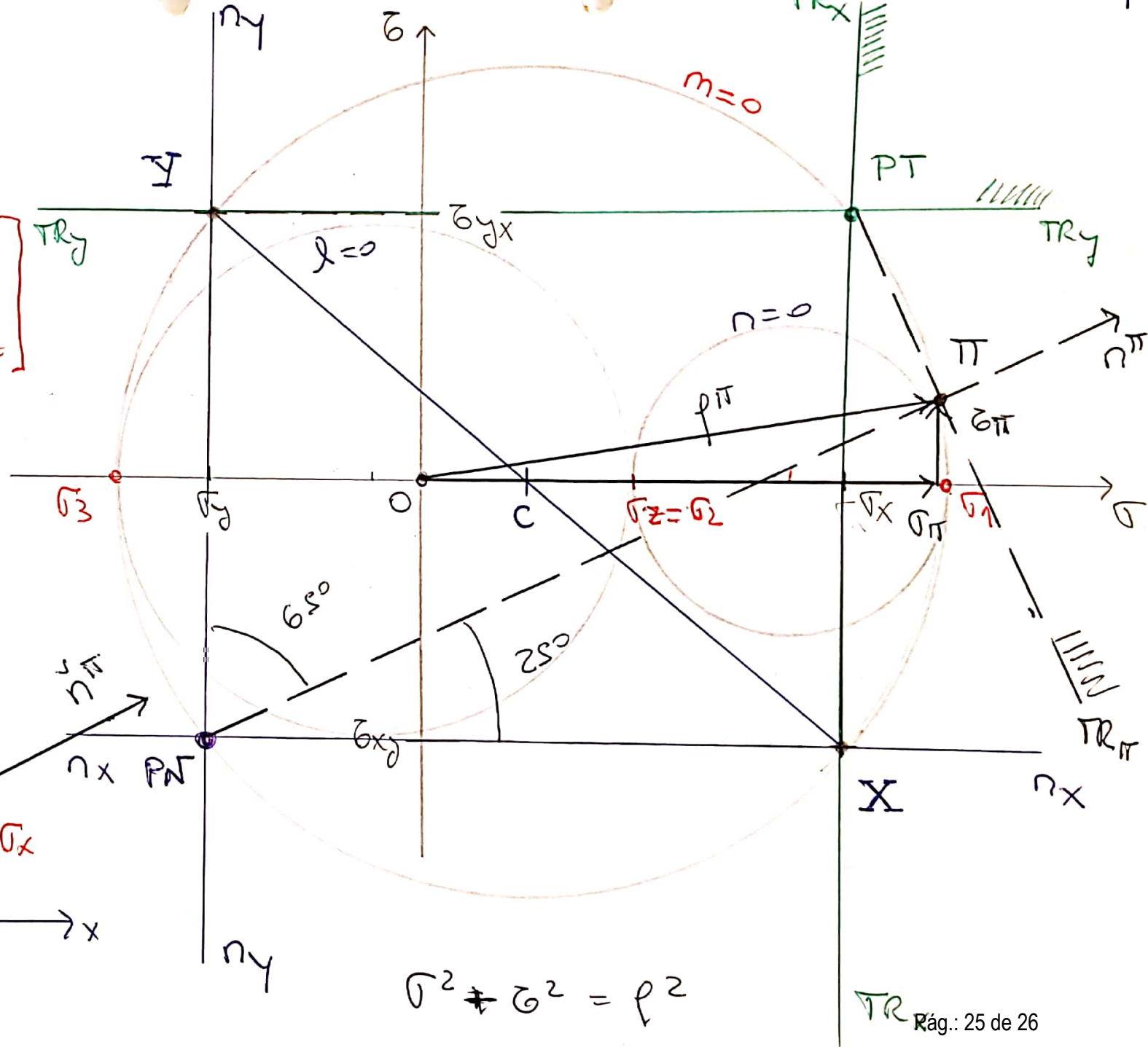
$$[T] = \begin{bmatrix} +80 & +50 & 0 \\ +50 & -40 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

$\sigma_z = +40$

~~$\sigma_z = 100$~~



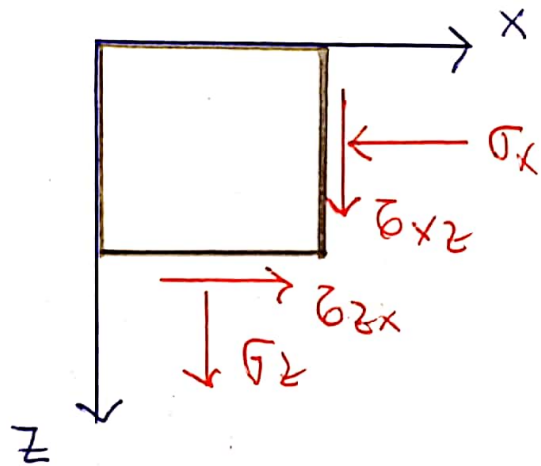
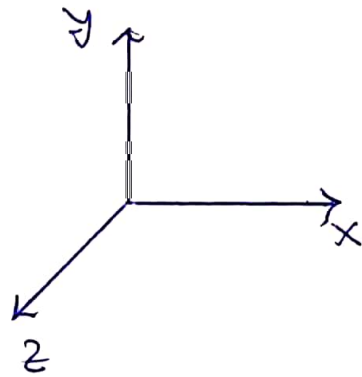
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$\sigma^2 + \tau^2 = \rho^2$

EJ: ESE DIRECCION O SOSTEN \rightarrow 'y'

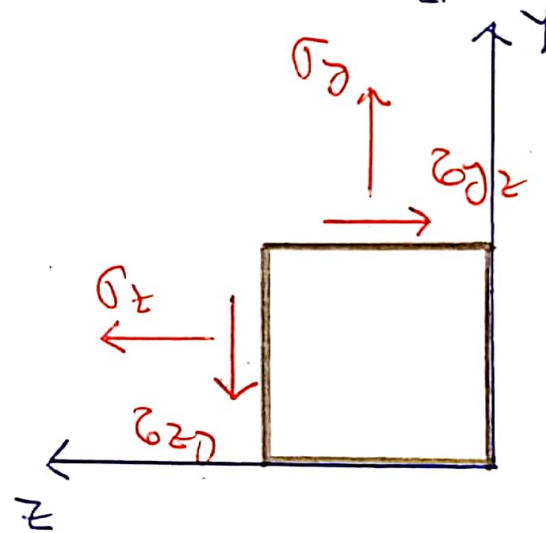
$$[\sigma] = \begin{bmatrix} \sigma_x & 0 & \tau_{zx} \\ 0 & \sigma_y & 0 \\ \tau_{xz} & 0 & \sigma_z \end{bmatrix}$$



$l \neq 0 ; m = 0 ; n \neq 0$

EJ: ESE DIRECCION O SOSTEN \rightarrow 'x' 17 | 17

$$[\sigma] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & \tau_{yz} \\ 0 & \tau_{zy} & \sigma_z \end{bmatrix}$$



$l = 0 ; m \neq 0 ; n \neq 0$