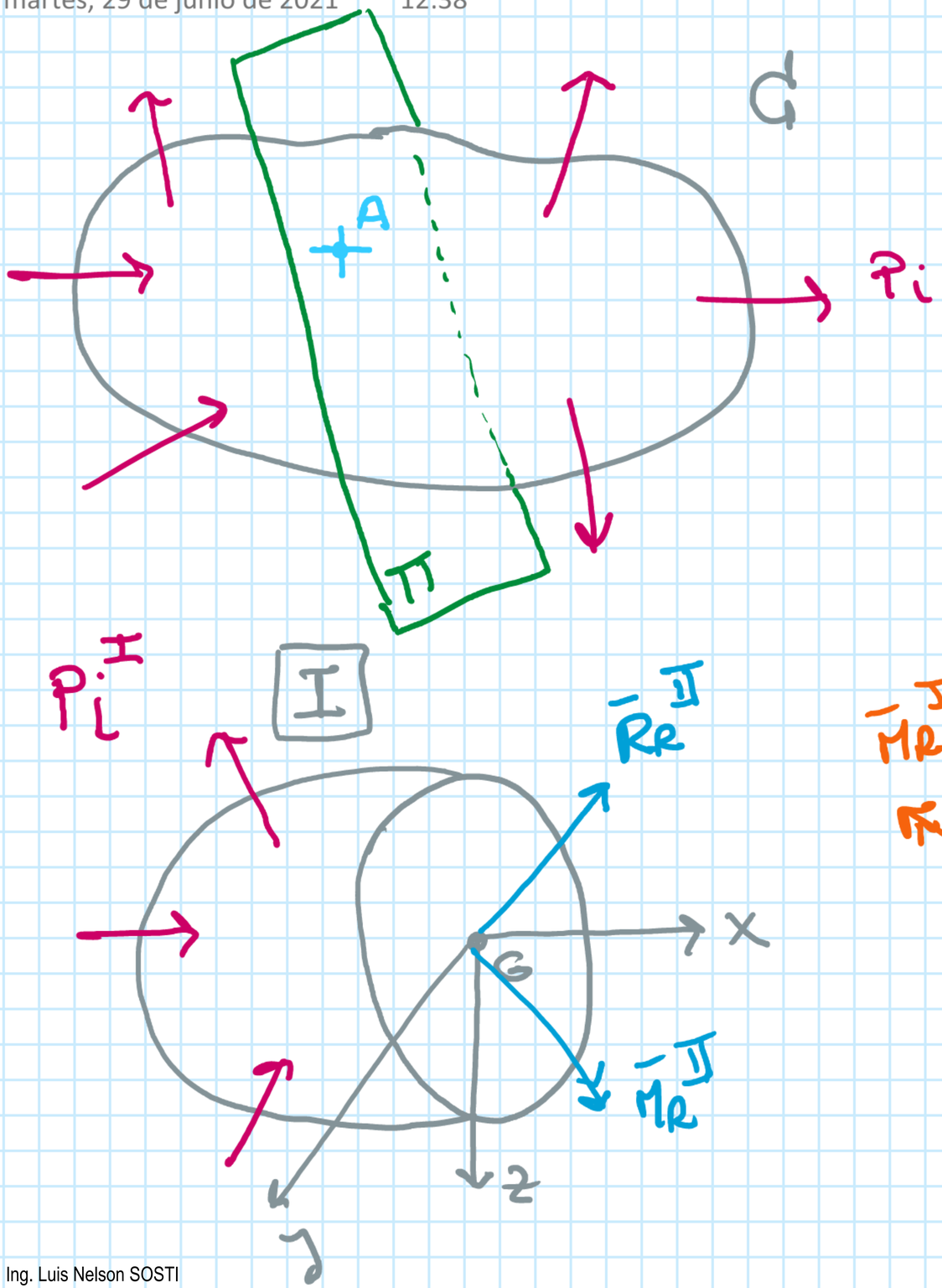
01 - INTRODUCCIÓN:

martes, 29 de junio de 2021 12:38



$FE = FA + FR = 0$

PUNTO 'A': PTO BAJO ESTUDIO O ANÁLISIS.

02 - DEFINICIONES:

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1) → Vector tensión $\bar{p}_A^{\pi} = \lim_{\Delta A \to 0} \frac{\Delta \bar{P}}{\Delta A} = \frac{d\bar{P}}{dA}$.

2) → Estado de tensión en un punto.

3) → Tensor de tensiones:

$$[TT]_{xyz} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

componentes del vector tensión asociado al plano z

" " " " " " " y

" " " " " " " x.

→ El [TT] es un ente matemático que:

- Representa
- Caracteriza.
- y permite determinar

al estado de tensión en un punto.

$$[TT]\{\vec{n}^{\pi}\} = \{\bar{p}^{\pi}\}.$$

03 - DESARROLLOS PRELIMINARES:

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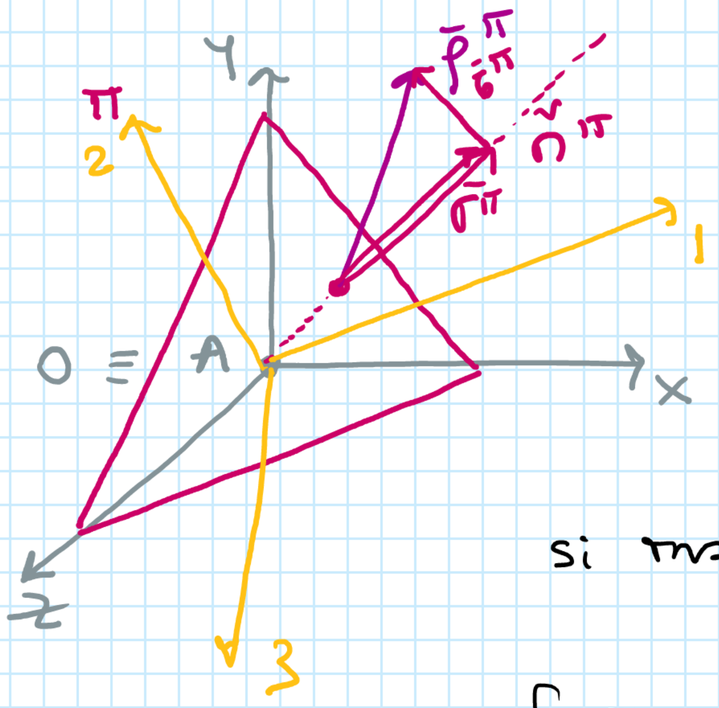

$$\{\bar{\rho}^{\pi}\} = [\pi\tau]\{\breve{n}^{\pi}\} \qquad (1a)$$

$$\begin{cases} \rho_x^{\pi} = \sigma_x l + \sigma_{yx} m + \sigma_{zx} n \\ \rho_y^{\pi} = \sigma_{xy} l + \sigma_y m + \sigma_{zy} n \\ \rho_z^{\pi} = \sigma_{xz} l + \sigma_{yz} m + \sigma_z n \end{cases} \qquad (1b)$$

si trabajamos con la terna ppal:

$(0; 1; 2; 3)$.

$$\begin{cases} \rho_1^{\pi} = \sigma_1 . l^* \\ \rho_2^{\pi} = \sigma_2 . m^* \\ \rho_3^{\pi} = \sigma_3 . n^* \end{cases} \qquad (1c) \leftarrow$$

$$[\vec{\tau}\tau]_{123} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\{\bar{\rho}^{\pi}\}_{123} = [\pi\tau]_{123}\{\breve{n}^{\pi}\}_{123}$$

$$\rho^{\pi}_{x7z} = \left|\bar{\rho}^{\pi}\right|_{x7z} = \sqrt{\rho_x^2 + \rho_7^2 + \rho_z^2} \qquad (2a)$$

$$\rho^{\pi}_{123} = \left|\bar{\rho}^{\pi}\right|_{123} = \sqrt{\rho_1^2 + \rho_2^2 + \rho_3^2} \qquad (2b)$$

$$\rho^{\pi}_{123} = \left|\bar{\rho}^{\pi}\right|_{123} = \sqrt{\sigma_1^2 (l^*)^2 + \sigma_2^2 (m^*)^2 + \sigma_3^2 (n^*)^2} \qquad (3).$$

$$\rho^{\pi} = \sqrt{(\sigma^{\pi})^2 + (\tau^{\pi})^2} \qquad (4)$$

→ combinando (3) y (4):

$$(\sigma^{\pi})^2 + (\tau^{\pi})^2 = \sigma_1^2 (l^*)^2 + \sigma_2^2 (m^*)^2 + \sigma_3^2 (n^*)^2 \qquad (5)$$

03 - DESARROLLOS:

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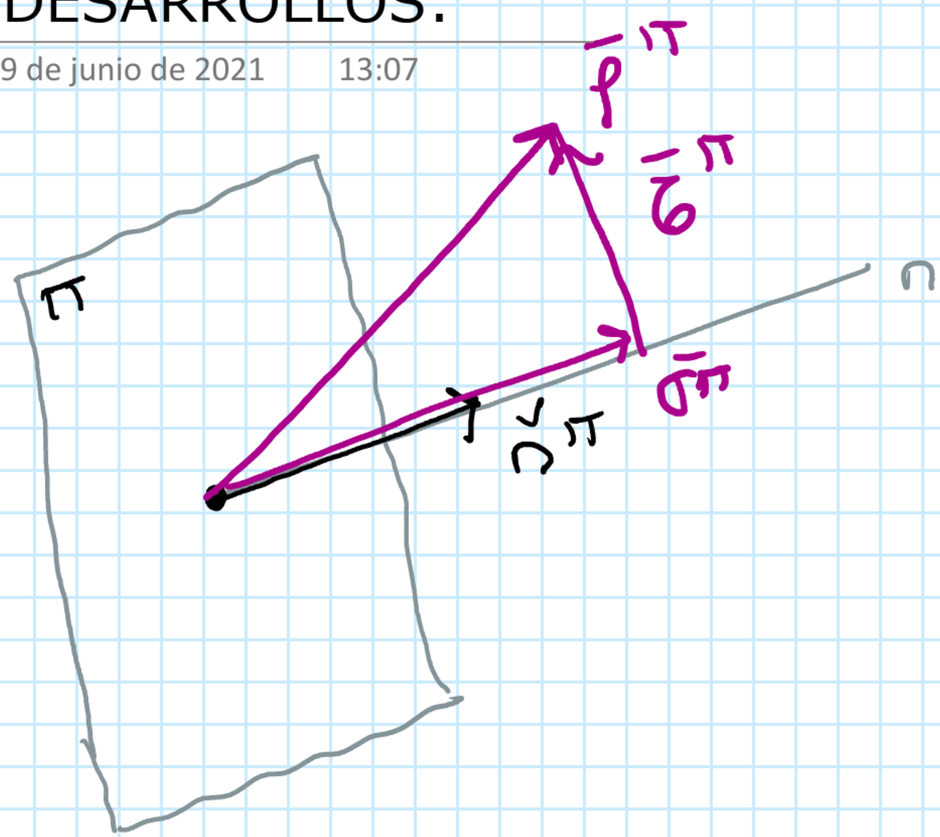

$$\{\bar{\sigma}^{\pi}\} = \underbrace{\left[\{\bar{p}^{\pi}\}^{T} \cdot \{\bar{n}^{\pi}\}\right]}_{\text{Pseudo - módulo}} \{\bar{n}^{\pi}\} \quad (6)$$

$$\sigma^{\pi} = \sigma = (\sigma_1 l^*;\ \sigma_2 m^*;\ \sigma_3 . n^*)(l^*;\ m^*;\ n^*) \quad (7)$$

$$\sigma^{\pi} = \sigma = \sigma_1 (l^*)^2 + \sigma_2 (m^*)^2 + \sigma_3 (n^*)^2 \quad (8)$$

04 - CONSTRUCCIÓN DE MOHR:

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$$(5)\quad \sigma^2 + \tau^2 = \sigma_1^2 l^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2$$

$$(8)\quad \sigma = \sigma_1 l^2 + \sigma_2 m^2 + \sigma_3 n^2.$$

$$(9)\quad 1 = l^2 + m^2 + n^2$$

} → MOHR

$$\tau^2 + \left(\sigma - \frac{\sigma_2+\sigma_3}{2}\right)^2 = \left(\sigma_1 - \frac{\sigma_2+\sigma_3}{2}\right)^2 l^2 + \left(\frac{\sigma_2-\sigma_3}{2}\right)^2 (1-l^2) \quad (10a)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_3}{2}\right)^2 = \left(\sigma_2 - \frac{\sigma_1+\sigma_3}{2}\right)^2 m^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 (1-m^2) \quad (10b)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 = \left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right)^2 n^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2 (1-n^2) \quad (10c)$$

05 - COMENTARIOS A LAS EXPRESIONES (10):

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$$\tau^2 + \left(\sigma - \frac{\sigma_2+\sigma_3}{2}\right)^2 = \left(\sigma_1 - \frac{\sigma_2+\sigma_3}{2}\right)^2 l^2 + \left(\frac{\sigma_2-\sigma_3}{2}\right)^2 (1-l^2) \quad (10a)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_3}{2}\right)^2 = \left(\sigma_2 - \frac{\sigma_1+\sigma_3}{2}\right)^2 m^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 (1-m^2) \quad (10b)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 = \left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right)^2 n^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2 (1-n^2) \quad (10c)$$

(I) DATOS: σ_1 ; σ_2 ; σ_3 TRABAJAN CON LA TERNA PAL.

(II) INCÓGNITAS:

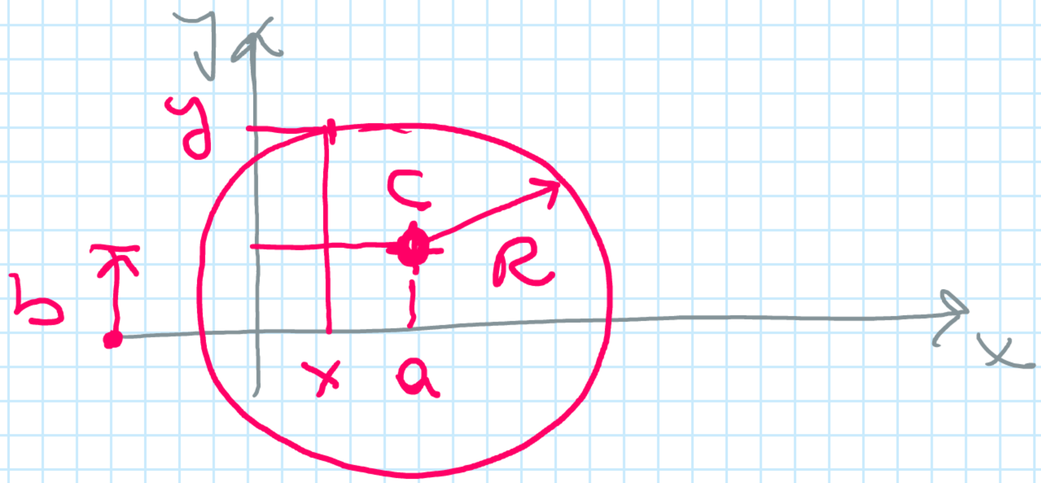
↳ 1) σ y τ → como a A P

(l, m, n)

↳ 2) (l, m, n) → como dato así nal cenal (σ, τ).

(III) LAS EXPRESIONES (10) REPRESENTAN "CIRCUNFERENCIAS"

$$(x-a)^2 + (y-b)^2 = R^2$$



$\begin{cases} x \rightarrow \sigma \rightarrow \text{EJE DE ABSCISAS} \\ y \rightarrow \tau \rightarrow \text{" " ORDENADAS} \end{cases}$

05 - COMENTARIOS A LAS EXPRESIONES (10):

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$$\tau^2 + \left(\sigma - \frac{\sigma_2+\sigma_3}{2}\right)^2 = \left(\sigma_1 - \frac{\sigma_2+\sigma_3}{2}\right)^2 l^2 + \left(\frac{\sigma_2-\sigma_3}{2}\right)^2 (1-l^2) \quad (10a)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_3}{2}\right)^2 = \left(\sigma_2 - \frac{\sigma_1+\sigma_3}{2}\right)^2 m^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 (1-m^2) \quad (10b)$$

$$\tau^2 + \left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 = \left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right)^2 n^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2 (1-n^2) \quad (10c)$$

(IV) LAS EXPRESIONES (10) REPRESENTAN → "FAMILIAS DE CIRCUNFERENCIAS"

$$\begin{cases} 0 \le l \le 1 \\ 0 \le m \le 1 \\ 0 \le n \le 1 \end{cases}$$

$$1 = l^2 + m^2 + n^2$$

(V) CONOCIDOS (l, m, n); ES DECIR, CONOCIDO EL PLANO → DETERMINO σ_0 Y τ_0 DEL PLANO →

$$\rho = \sqrt{\sigma^2 + \tau^2}$$

(VI) LAS EXPRESIONES (10) SE REPRESENTAN GRÁFICAMENTE: