

MA-08 - DIC - 2020:

MA-15 - DIC - 2020:

1) ET: → CIRCUNFERENCIA DE MOHR.

2) ED:

3) RT y D ≡ RC.

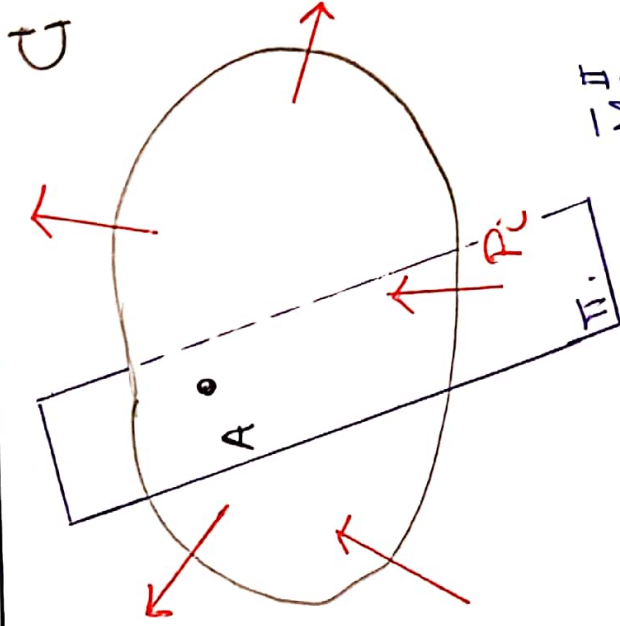
RELACIONES CONSTITUTIVAS.

4) TEL: TEORIAS DE LOS ESTADOS LÍMITES O DE FALLA

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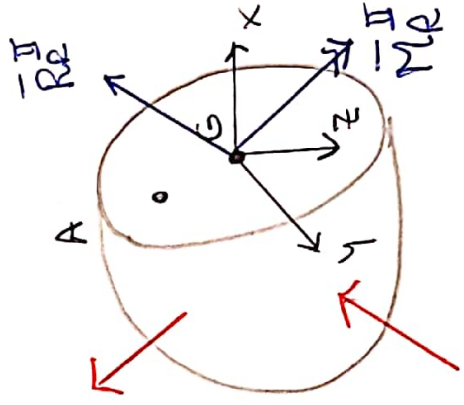
ET: "CIRCUNFERENCIA DE MOHR".

01 - INTRODUCCIÓN:

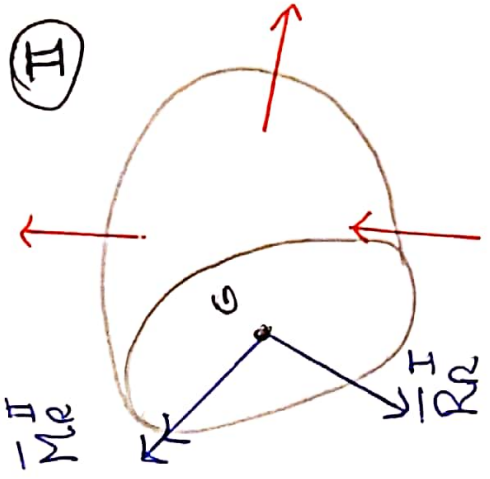


G: cuerpo
 P: FAS EN EQUILIBRIO
 $F_E = F_A + F_R$
 $F_E = 0.$

I



II



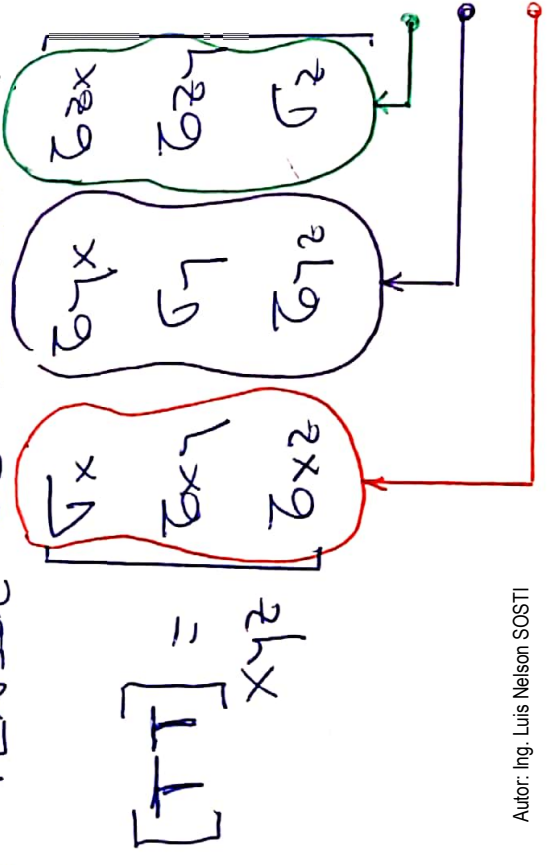
02 - DEFINICIONES:

• VECTOR TENSION

$$\bar{\sigma}^n = \lim_{\Delta A \rightarrow 0} \frac{\Delta \bar{P}}{\Delta A} = \frac{d\bar{P}}{dA}$$

• ESTADO DE TENSION EN UN PUNTO.

• TENSOR DE TENSIONES:



→ EL $[TT]$ ES UN ENTE MATEMÁTICO

que:

- REPRESENTA
- CARACTERIZA.
- Y PERMITE DETERMINAR

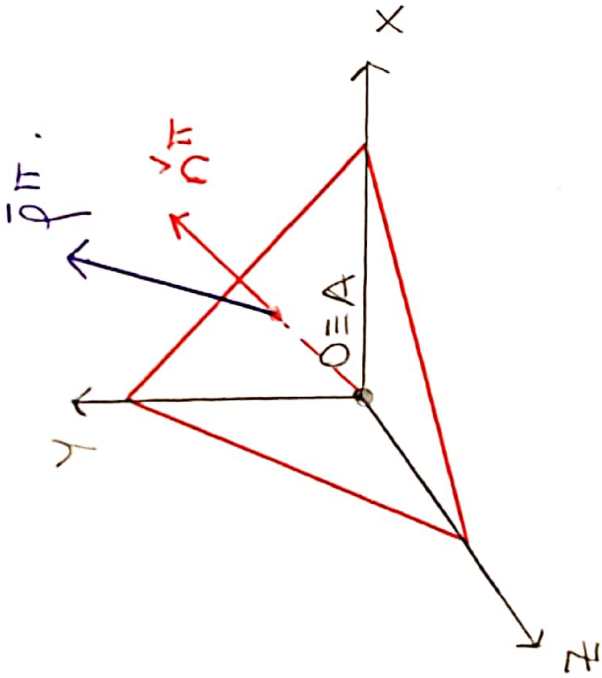
AL ESTADO DE TENSION EN UN PUNTO DEL CUERPO.

$$[TT] \{ \bar{n}^n \} = \{ \bar{p}^n \}$$

→ SE CONSTRUYE CONOCIENDO LAS DIRECCIONES TENSION ASOCIADAS A 3 PLANOS PERPENDICULARES.

03 - DESARROLLOS

PRELIMINARES:



• SI SE TRABAJA CON LA TERNAS

PRINCIPAL:

(0; 1; 2; 3).

$$\left\{ \begin{aligned} \rho_1^{II} &= \sigma_1 \cdot \lambda^* \\ \rho_2^{II} &= \sigma_2 \cdot m^* \\ \rho_3^{II} &= \sigma_3 \cdot n^* \end{aligned} \right\} \quad (1c)$$

$$\left\{ \bar{\rho}^{II} \right\} = [T T T] \left\{ \check{\rho}^{II} \right\} \quad (1a)$$

$$\left\{ \begin{aligned} \rho_x^{II} &= \sigma_x \cdot \lambda + \sigma_{yx} \cdot m + \sigma_{zx} \cdot n \\ \rho_y^{II} &= \sigma_{xy} \cdot \lambda + \sigma_y \cdot m + \sigma_{zy} \cdot n \\ \rho_z^{II} &= \sigma_{xz} \cdot \lambda + \sigma_{yz} \cdot m + \sigma_z \cdot n \end{aligned} \right\} \quad (1b)$$

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$$\rho_{x^2}^\pi = |\bar{\rho}^\pi| = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2} \quad (2a)$$

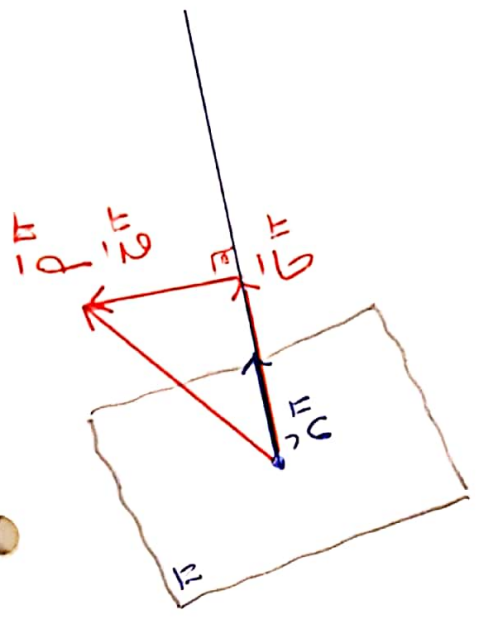
$$\rho_{123}^\pi = |\bar{\rho}_{123}^\pi| = \sqrt{\rho_1^2 + \rho_2^2 + \rho_3^2} \quad (2b)$$

$$\rho^\pi = \sqrt{\sigma_1^2 (\rho^*)^2 + \sigma_2^2 (m^*)^2 + \sigma_3^2 (n^*)^2} \quad (3)$$

$$\rho^\pi = \sqrt{(\sigma^\pi)^2 + (\bar{\rho}^\pi)^2} \quad (4)$$

→ combinando (3) y (4):

$$\sqrt{\sigma^2 + \bar{\rho}^2} = \sigma_1^2 (\rho^*)^2 + \sigma_2^2 (m^*)^2 + \sigma_3^2 (n^*)^2 \quad (5)$$



$$\{\bar{\rho}^\pi\} = \left[\{\bar{\rho}^\pi\} \cdot \{\dot{\rho}^\pi\} \right] \cdot \{\dot{\rho}^\pi\} \quad (6)$$

PSEUDO - MÓDULO .

$$\sigma^\pi = \sigma = \underbrace{(\sigma_1 \rho^*, \sigma_2 m^*, \sigma_3 n^*)}_{(1c)} \cdot (\dot{\rho}^*, m^*, n^*) \quad (7)$$

$$\sigma^\pi = \sigma = \sqrt{\sigma_1^2 (\rho^*)^2 + \sigma_2^2 (m^*)^2 + \sigma_3^2 (n^*)^2} \quad (8)$$

04) - construcción de MOHR:

$$\begin{aligned}
 (5) \rightarrow \sigma^2 + \tau^2 &= \sigma_1^2 \cdot \rho^2 + \sigma_2^2 \cdot m^2 + \sigma_3^2 \cdot n^2. \\
 (6) \rightarrow \sigma &= \sigma_1 \cdot \rho^2 + \sigma_2 \cdot m^2 + \sigma_3 \cdot n^2. \\
 (9) \rightarrow \rho &= \rho^2 + m^2 + n^2.
 \end{aligned}$$

→ MOHR

$$\begin{aligned}
 (10a) \quad \tau^2 + \left(\sigma - \frac{\sigma_2 + \sigma_3}{2} \right)^2 &= \left(\sigma_1 - \frac{\sigma_2 + \sigma_3}{2} \right)^2 \rho^2 + \left(\frac{\sigma_2 - \sigma_3}{2} \right)^2 (1 - \rho^2). \\
 (10b) \quad \tau^2 + \left(\sigma - \frac{\sigma_1 + \sigma_3}{2} \right)^2 &= \left(\sigma_2 - \frac{\sigma_1 + \sigma_3}{2} \right)^2 m^2 + \left(\frac{\sigma_1 - \sigma_3}{2} \right)^2 (1 - m^2). \\
 (10c) \quad \tau^2 + \left(\sigma - \frac{\sigma_1 + \sigma_2}{2} \right)^2 &= \left(\sigma_3 - \frac{\sigma_1 + \sigma_2}{2} \right)^2 n^2 + \left(\frac{\sigma_1 - \sigma_2}{2} \right)^2 (1 - n^2).
 \end{aligned}$$

OS - COMENTARIOS A LAS EXPRESIONES DE MOHR (10)

I DATOS: $\sigma_1; \sigma_2; \sigma_3$
 → SE MARCA EN TERNA PRAL.

II INCÓGNITAS:

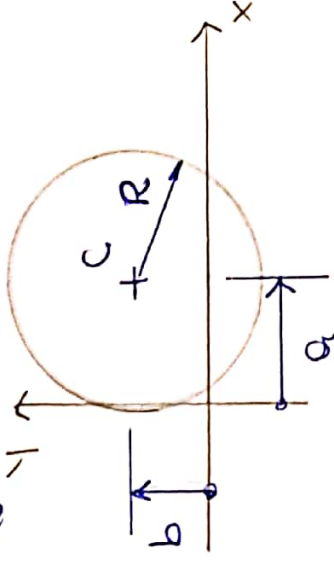
1) → σ y τ . ← COMO DATO ADICIONAL CONOCE EL PLANO (α, m, n) .

2) → (α, m, n) ← COMO DATO ADICIONAL CONOCE (σ, τ)

III LAS EXPRESIONES (10) REPRESENTAN "CIRCUNFERENCIAS"

$$(x - a)^2 + (y - b)^2 = R^2$$

$x \rightarrow \sigma$
 $y \rightarrow \tau$



IV LAS EXPRESIONES (10) REPRESENTAN

MAN →

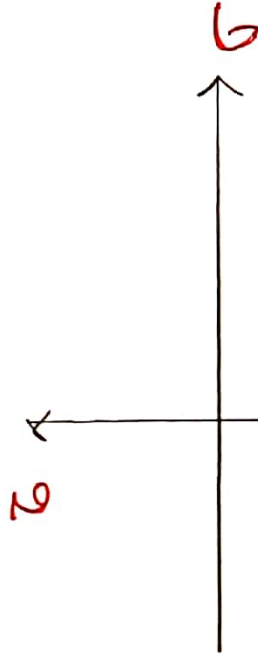
"FAMILIAS DE CIRCUNFERENCIAS"

$$\left[\begin{array}{l} 0 \leq \alpha \leq \pi \\ 0 \leq m \leq \pi \\ 0 \leq n \leq \pi \end{array} \right]$$

IV) con σ y τ \rightarrow ρ .

$$\rho = \sqrt{\sigma^2 + \tau^2}$$

VI) \rightarrow LAS EXPRESIONES (10) SE REPRESENTAN GRÁFICAMENTE;



VII) LAS FUIAS DE CIRCUNFERENCIAL DEPENDE DE LAS TENSIONES PPAVES.

$$\text{VIII) } (x - a)^2 + (y - b)^2 = R^2$$

$$(\sigma - a)^2 + (\tau - b)^2 = R^2$$

\rightarrow LOS CENTROS DE LAS 3 FUIAS DE CIRCUNFERENCIAL SE UBICAN SOBRE EL EJE DE ABSISAS (EJE X \equiv EJE σ).

IX) \rightarrow CADA FUIA DEPENDE DE "1" SOLO PARAMETRO.

X) \rightarrow CADA FUIA DE CIRCUNFERENCIAL VA A ESTAR LIMITADA POR 2 CIRCUNFERENCIALS EXTREMAS.

XI) \rightarrow EN LAS EXPRESIONES (10) \rightarrow \rightarrow CADA VALOR DE LAS TENS. PPAVES SE INTRODUCE CON SU SIGNO.

06 - REPRESENTACIONES:

1ª FAMILIA:

$$C_1 = \left(\frac{\sigma_2 + \sigma_3}{2}; 0 \right)$$

$$\lambda = 0 \rightarrow R_{1,0} = \frac{\sigma_2 - \sigma_3}{2}$$

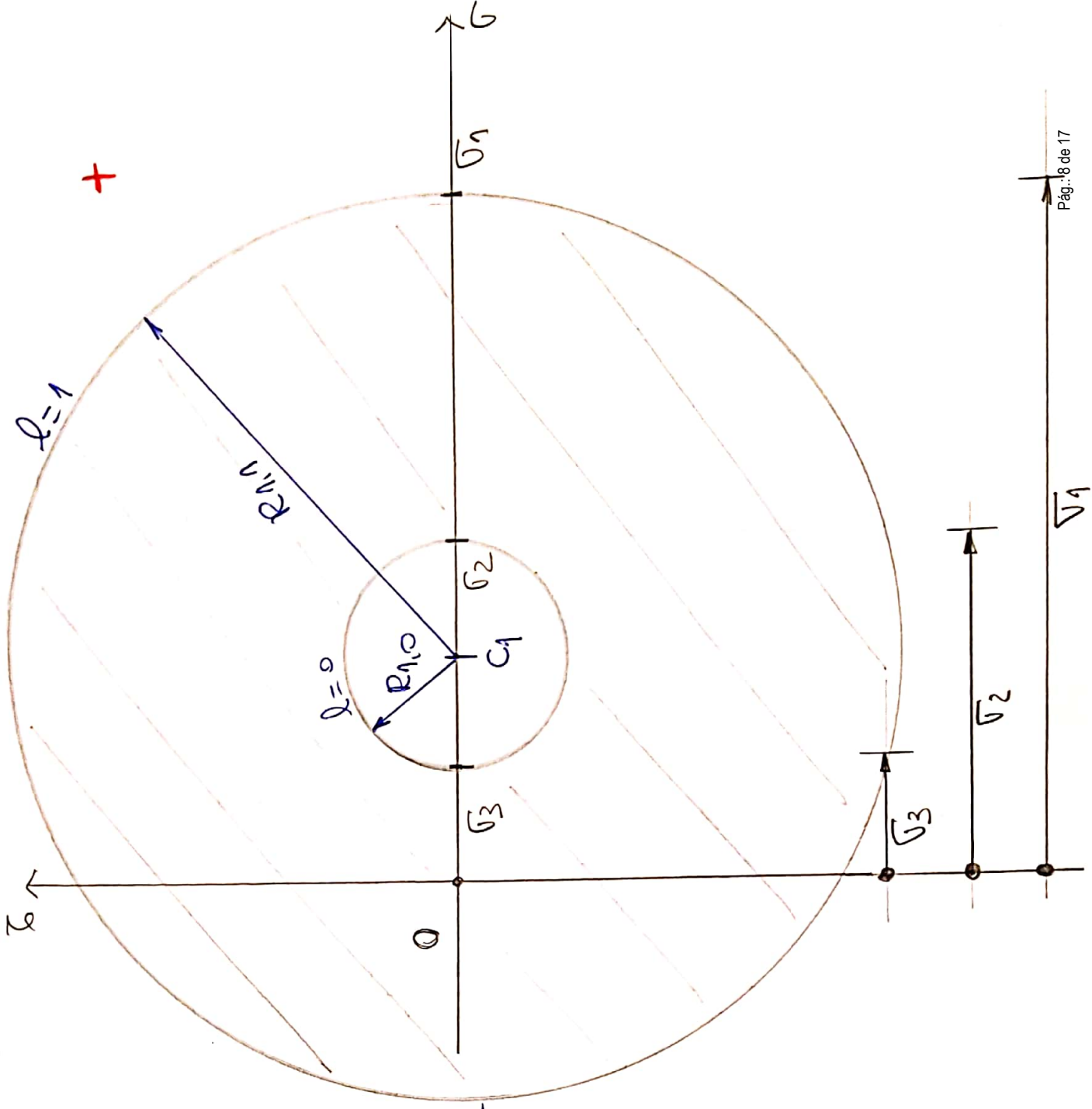
$$\lambda = 1 \rightarrow R_{1,1} = \frac{\sigma_1 - \sigma_2 + \sigma_3}{2}$$

DADO EL EJEMPLO:

$$\sigma_1 = 12$$

$$\sigma_2 = 6$$

$$\sigma_3 = 2$$



ACLARACIONES:

$$\sigma_1 > \sigma_2 > \sigma_3.$$

EJ: $\sigma_1 = 20 \text{ kN/cm}^2$

$\sigma_2 = 8 \text{ "}$

$\sigma_3 = -4 \text{ "}$

EJ: $\sigma_2 = -25 \text{ "}$

$\sigma_3 = -50 \text{ "}$

$\sigma_1 = -10 \text{ "}$

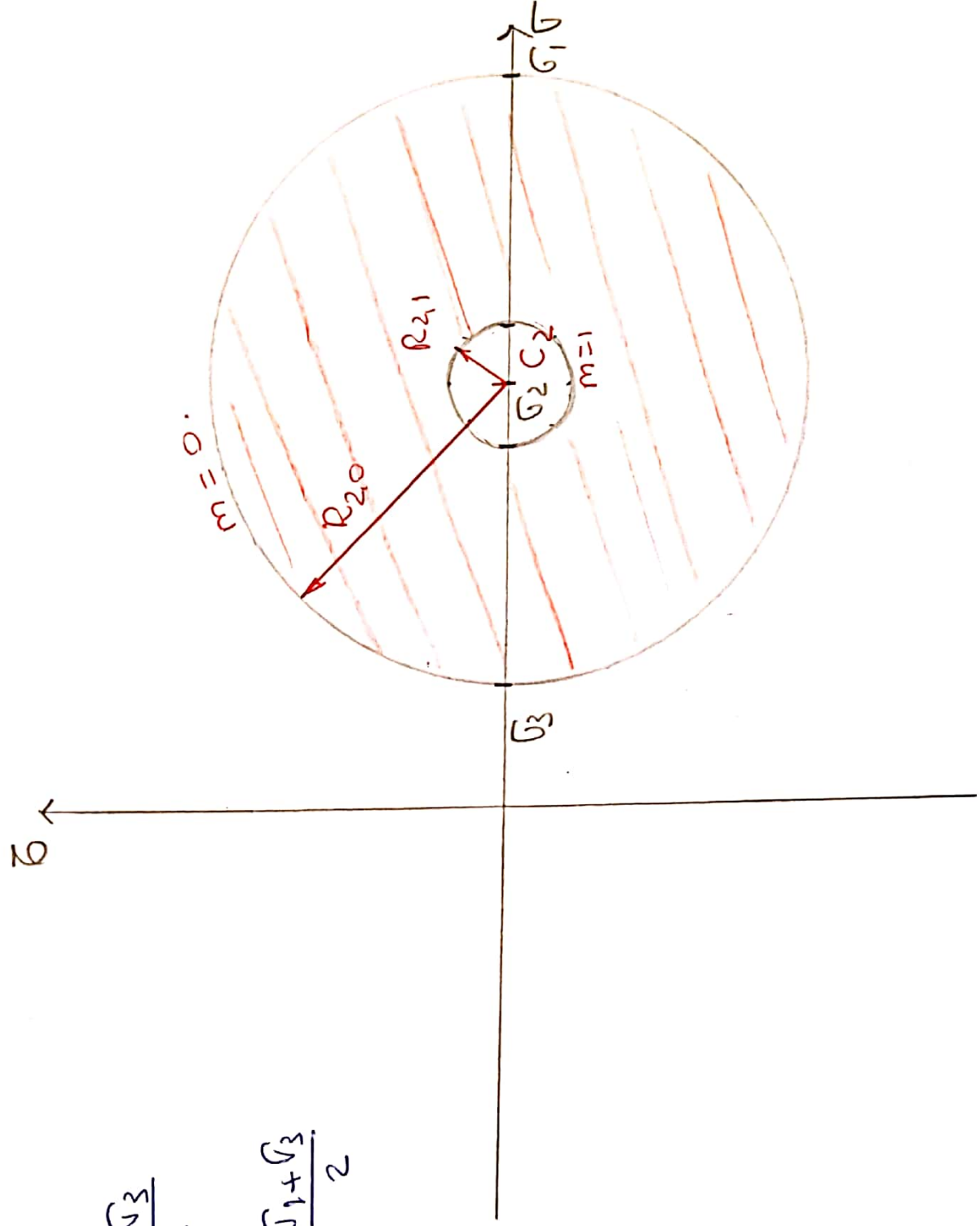
2e familia:

$$C_2 = \left(\frac{\sigma_1 + \sigma_3}{2}, 0 \right)$$

$$m=0 \rightarrow R_{2,0} = \frac{\sigma_1 - \sigma_3}{2}$$

$$m=1 \rightarrow R_{2,1} = \sigma_2 - \frac{\sigma_1 + \sigma_3}{2}$$

$$\left. \begin{aligned} \sigma_1 &= 12 \\ \sigma_2 &= 6 \\ \sigma_3 &= 2 \end{aligned} \right\}$$

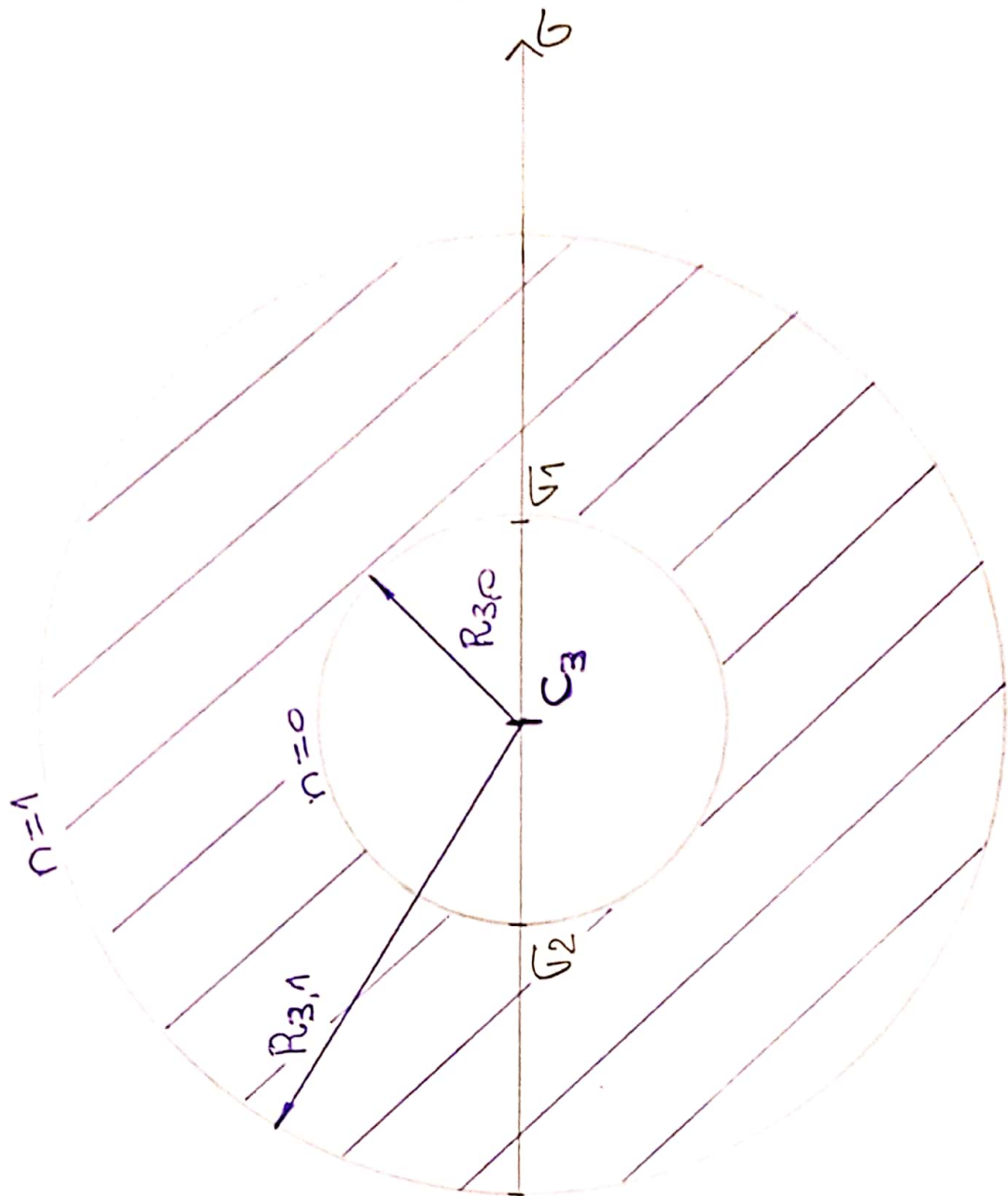
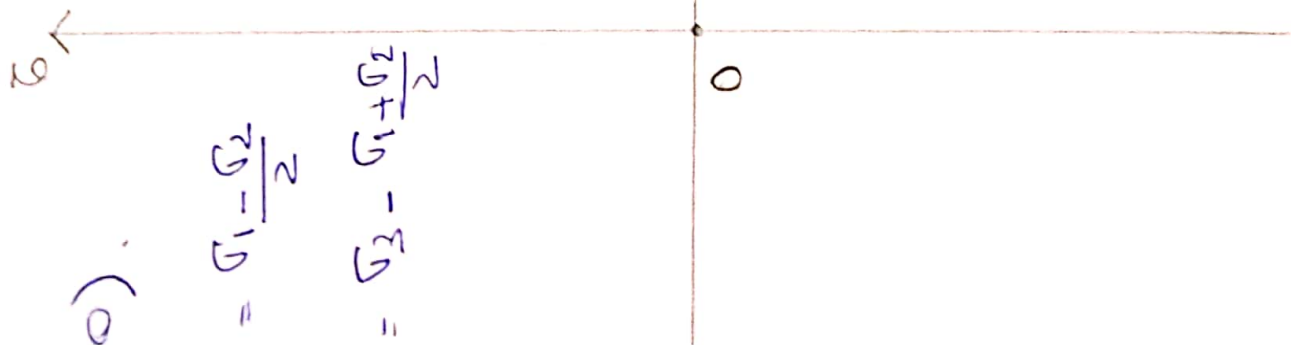


3º FAMILIA:

$$C_3 = \left(\frac{\sigma_1 + \sigma_2}{2}, 0 \right)$$

$$n=0 \rightarrow R_{3,0} = \frac{\sigma_1 - \sigma_2}{2}$$

$$n=1 \rightarrow R_{3,1} = \frac{\sigma_3 - \sigma_1 + \sigma_2}{2}$$

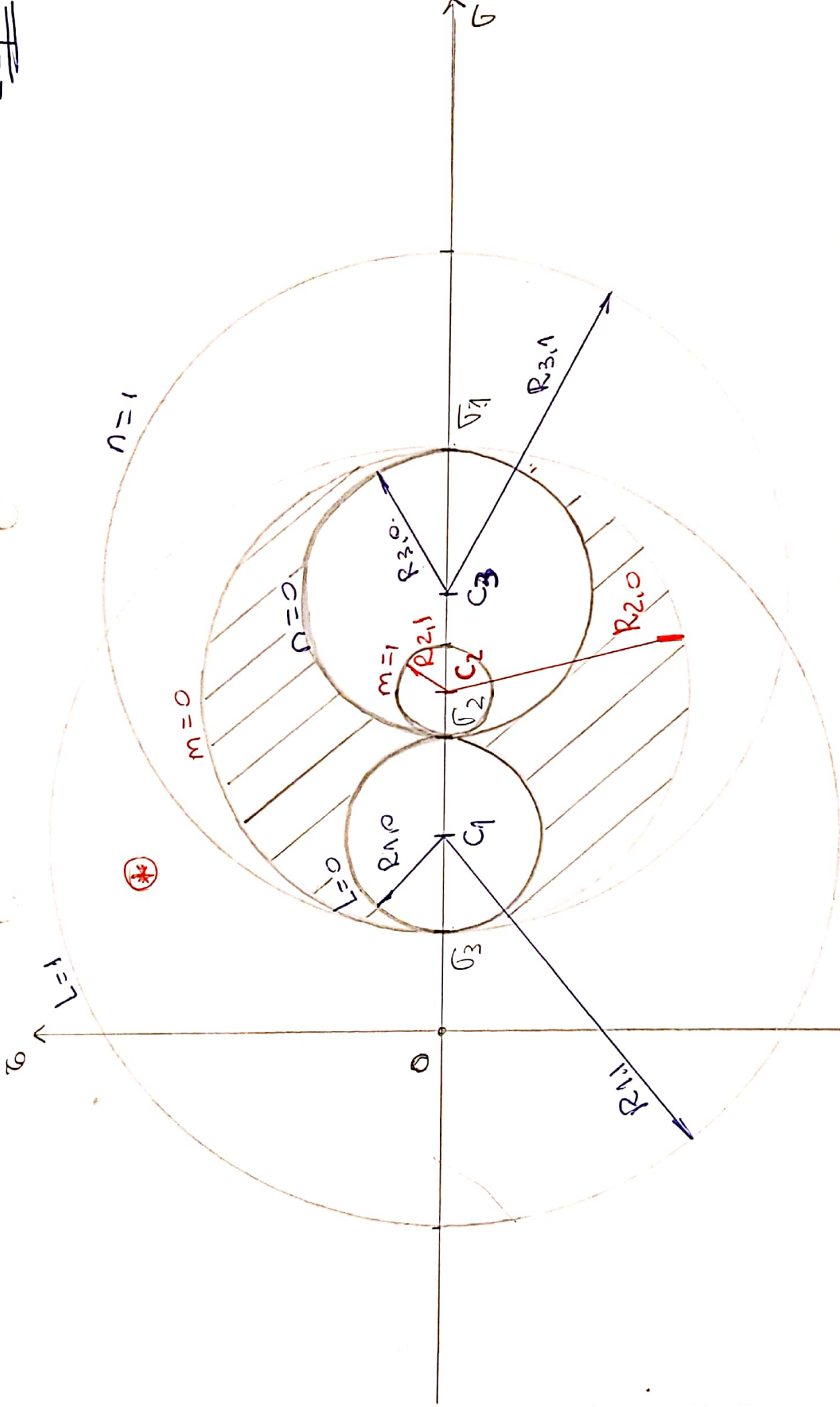


DATOS:

$$\sigma_1 = 12$$

$$\sigma_2 = 6$$

$$\sigma_3 = 2$$



EN ESTE ESQUEMA, SE ~~SE~~ DIBUJAN LOS 3 PARES DE CIRCULOS FONDOS CUAL EXISTEN DE CADA FAMILIA Y SE SUPERPONEN BUSCANDO EL MAYOR PG INTERSECCION DE LAS FAMILIAS 3 FAMILIAS.

OBJETIVO:

1) $\overbrace{\text{DATOS}}^{\text{que?}}$ $\left\{ \begin{array}{l} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{array} \right\}$

$\xrightarrow{\text{que?}}$ $\left\{ \begin{array}{l} \sigma^I \\ \sigma^{II} \\ \sigma^{III} \end{array} \right\}$

(λ, m, n)

2) $\overbrace{\text{DATOS}}^{\text{que?}}$ $\left\{ \begin{array}{l} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{array} \right\}$

$\xrightarrow{\text{que?}}$

(σ^I, σ^{II})

(λ, m, n)

ESTADOS PRINCIPALES

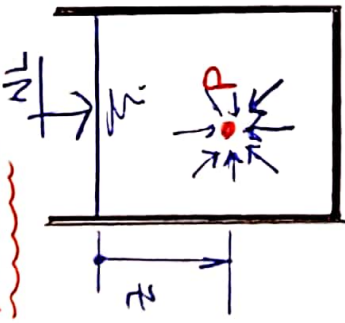
• LAS COMPONENTES DE MOHR ES LA CONSTRUCCIÓN CONJUNTA EQUIVALENTE AL [F.T.].

• A LAS CONCENTRACIONES DE $\lambda = 0$; $m = 0$; $n = 0 \rightarrow$ SE VA DENOMINA "CIRCUNFERENCIAS FUNDAMENTALES DE MOHR".

07 - APLICACIONES PARTICULARES

RES.

EJ01:



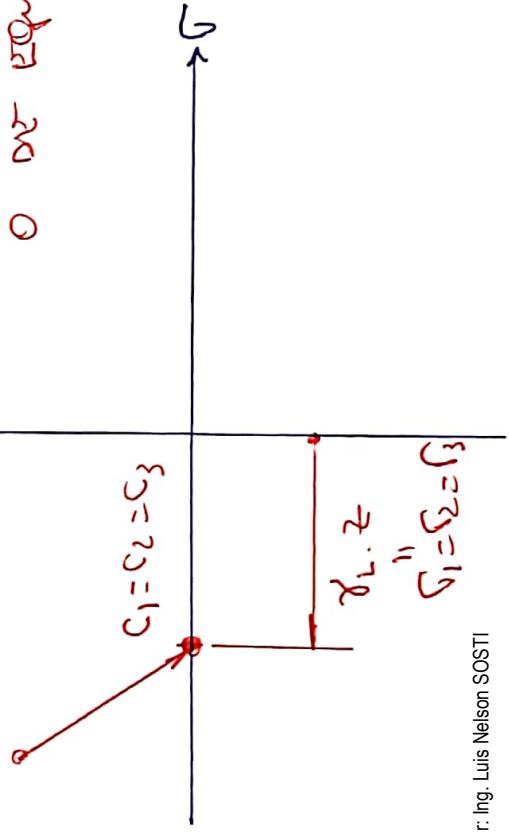
$P = \delta_{Lz}$

$$[T\tau] = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = \begin{bmatrix} \delta_{Lz} & 0 & 0 \\ 0 & \delta_{Lz} & 0 \\ 0 & 0 & \delta_{Lz} \end{bmatrix}$$

EN ESTE PUNTO
TENEMOS LAS 3 COMPONENTES

ESTADO HIDROSTATICO.

O DE EQUICOMPRESION.



EJ02 $[T\tau] = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix}$

"EQUI TRACCION"

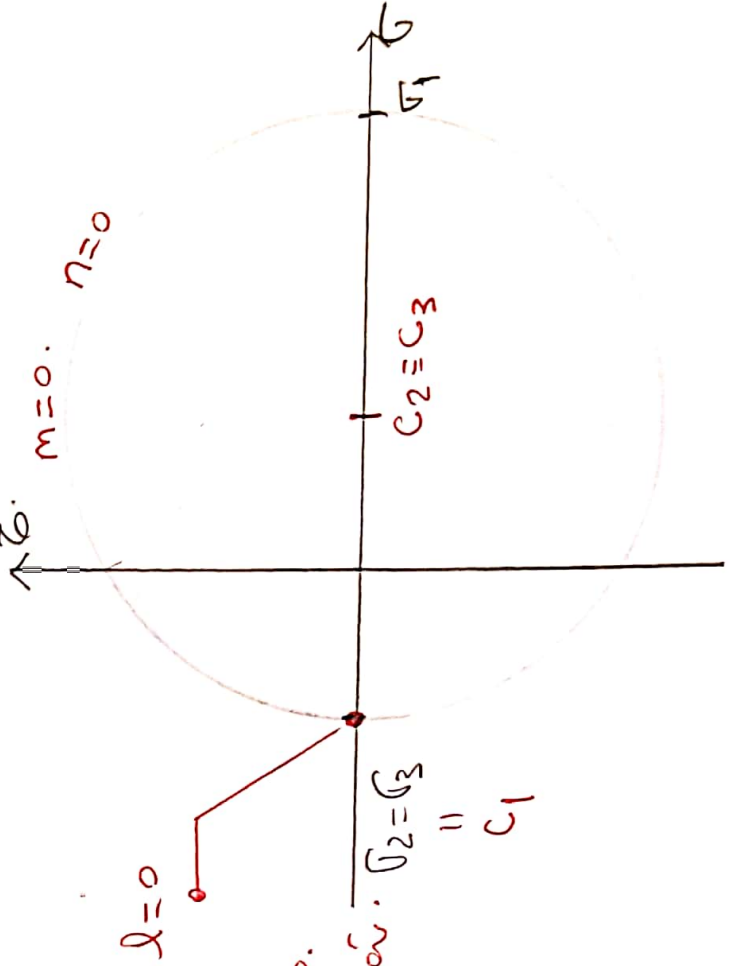


EJ03 $[T\tau] = \begin{bmatrix} 60 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & -20 \end{bmatrix}$

$\sigma_1 = 60 \quad \sigma_2 = \sigma_3 = -20$

$m = 0.$

$n = 0$

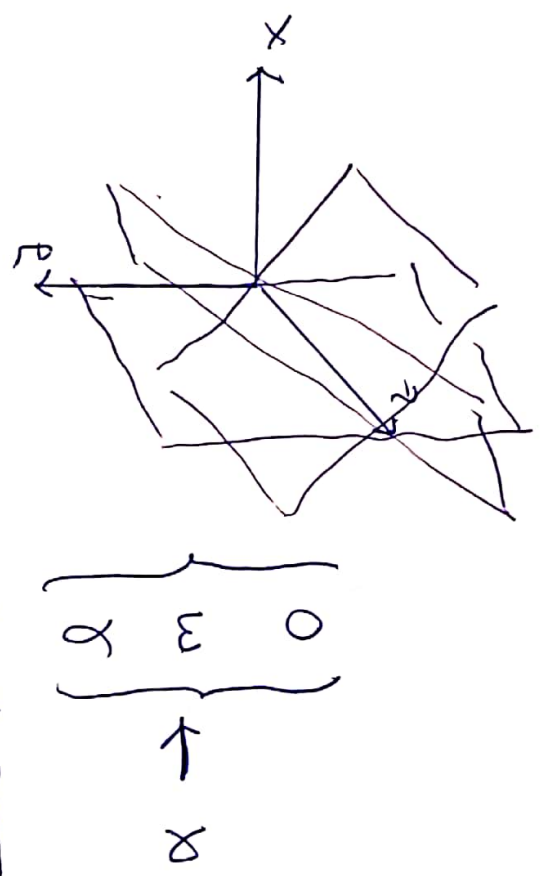


08 - Circunferencia

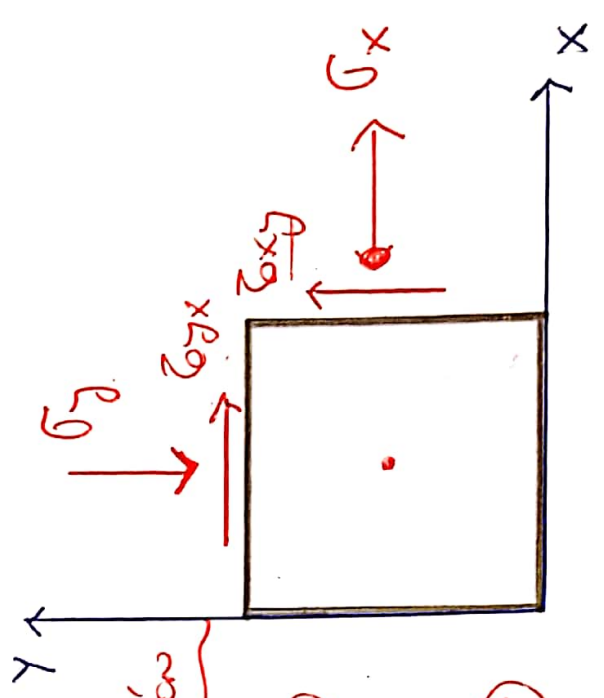
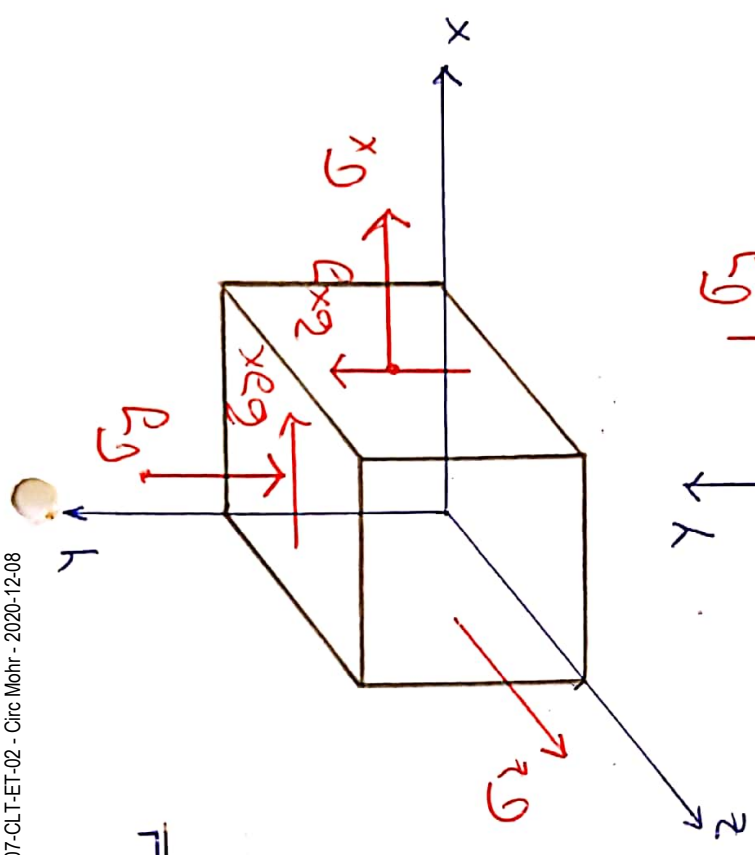
ESTADO PLANO Y EJE DIAGONAL:

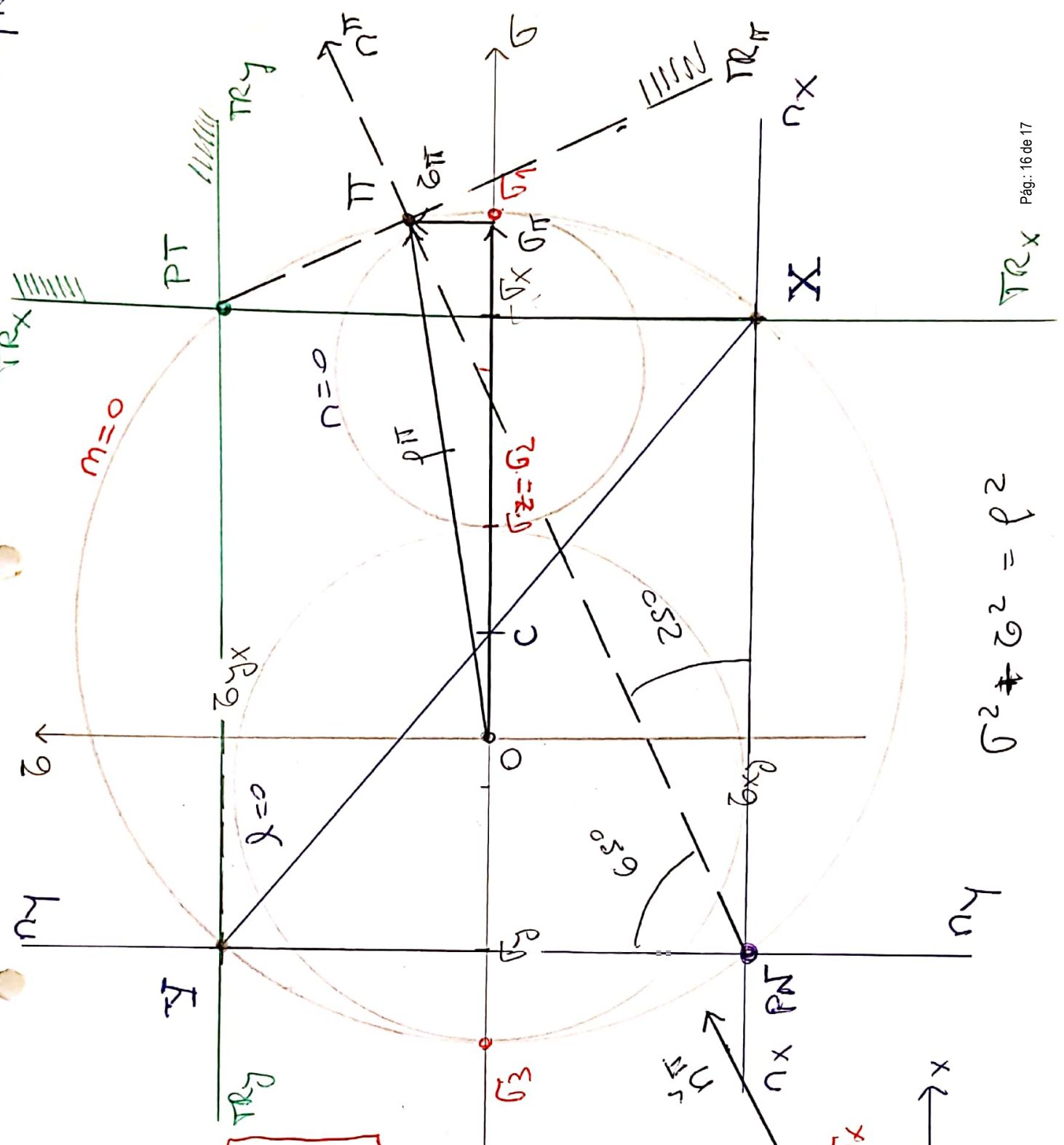
$$[T] = \begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

TOCAN COMO ESTE DISEÑO → AL ESTE PRAT.



NUEVA CONVENCION



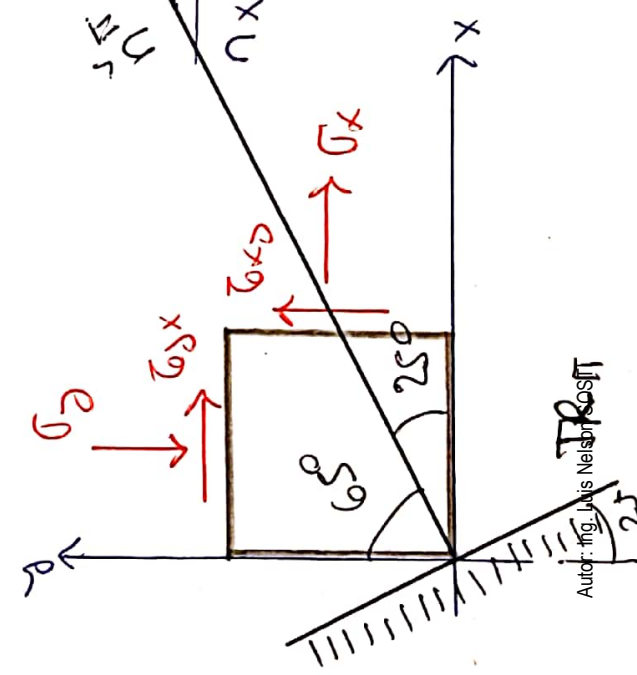


$\sigma_x = 80.$
 $\sigma_y = -40.$
 $\tau_{xy} = 60$

$$[\sigma] = \begin{bmatrix} +80 & 0 & 0 \\ 0 & -40 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

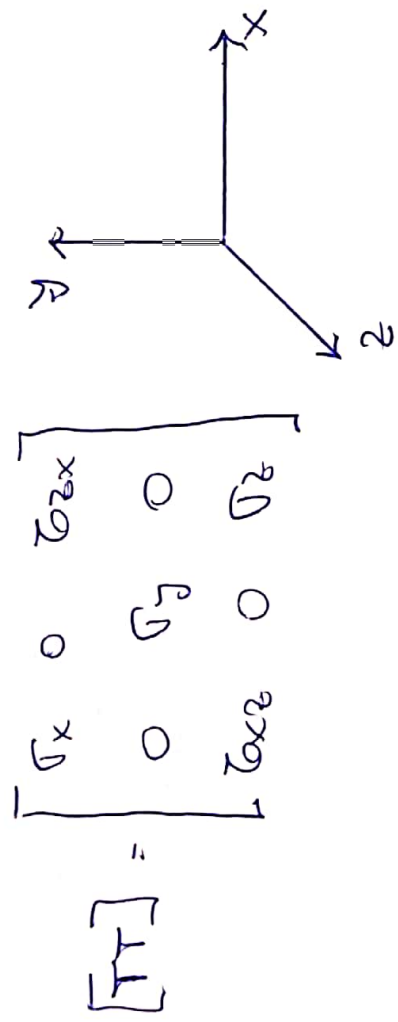
$\sigma_z = 40$

~~σ_z = 40~~



$\sigma^2 + \tau^2 = \rho^2$

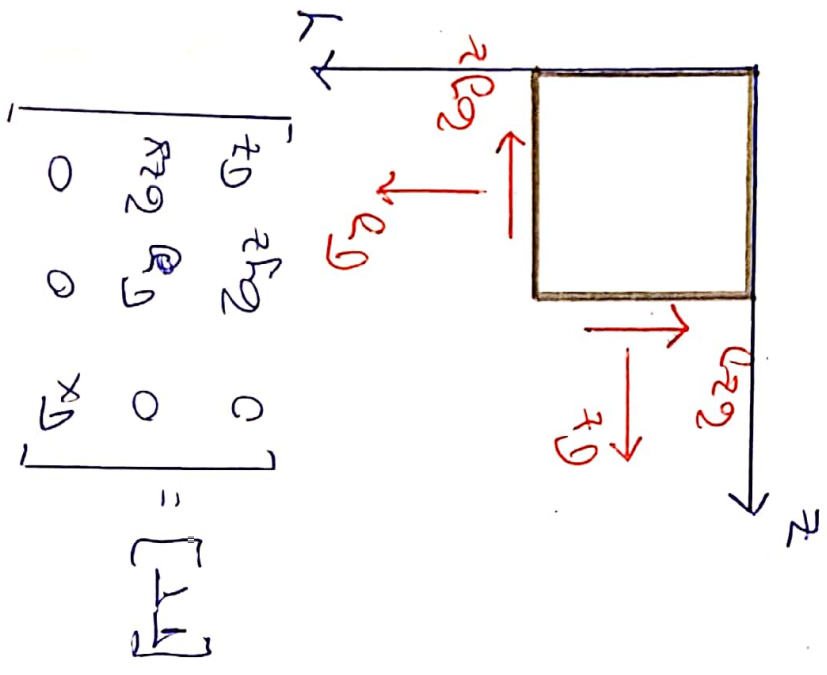
ED: EJE DIRECCION O SOSTEN → 'y'



$$[T] = \begin{bmatrix} \sigma_x & 0 & \tau_{zx} \\ 0 & \sigma_y & 0 \\ \tau_{xz} & 0 & \sigma_z \end{bmatrix}$$

$\lambda \neq 0 ; m = 0 ; n \neq 0$

ED: EJE DIRECCION O SOSTEN → 'x'



$$[T] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & \tau_{zy} \\ 0 & \tau_{yz} & \sigma_z \end{bmatrix}$$

$\lambda = 0 ; m \neq 0 ; n \neq 0$