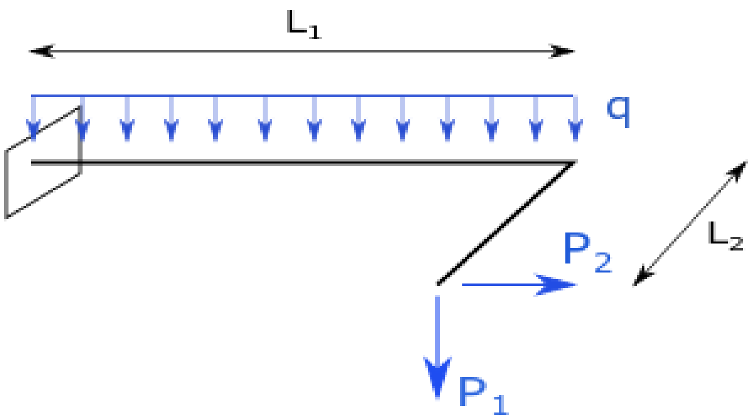
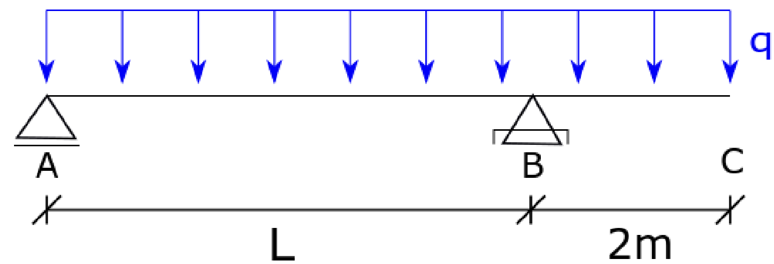
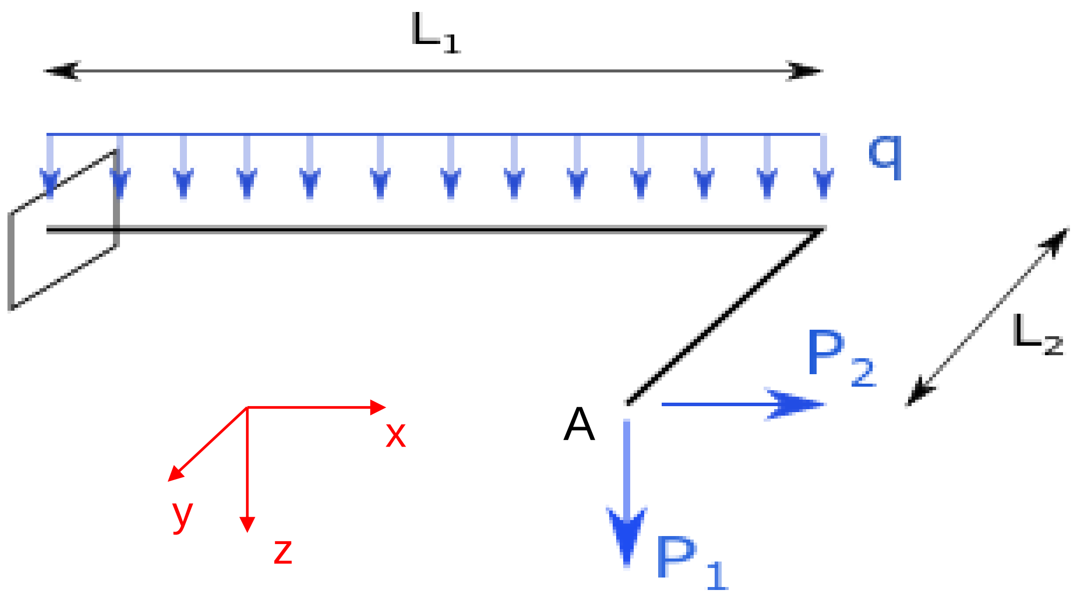
Cálculo de desplazamientos

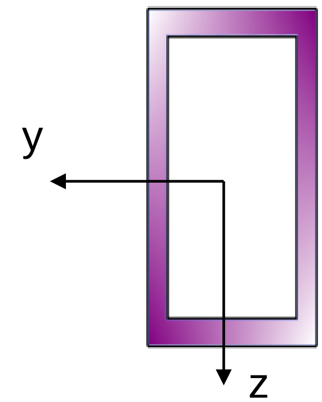




Tania Poletilo - Constanza Ruffinelli - Manuela Medina

Ejercicio 1: Calcular el desplazamiento vertical en A y θ_{Ax}.





$Perfil: 100\ x\ 250\ x\ 10\ mm$

$J_y = 4515\ cm^4$

$J_z = 1040\ cm^4$

$J_t = 2777\ cm^4$

Cargas:

$P_1 = 5\ kN$ $\quad L_1 = 4\ m$

$P_2 = 10\ kN$ $\quad L_2 = 1\ m$

$q = 5\ \frac{kN}{m}$

$E = 21000\ \frac{kN}{cm^2}$ $\quad G = 8000\ \frac{kN}{cm^2}$

Resolución:

Calculamos cada desplazamiento con TTV

$$W_E = W_i$$

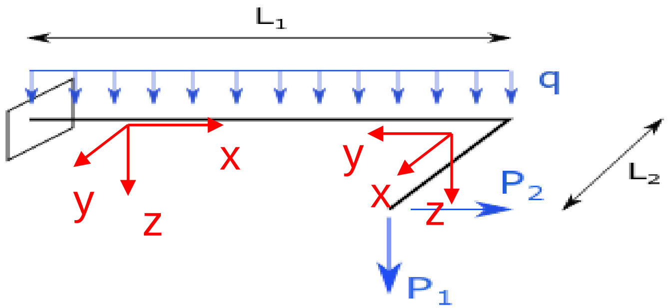
$$(+1) \cdot \eta = \int_L N^{se} \cdot \frac{N^{dv}}{E \cdot A}\, dx + \int_L M_{en}^{se} \cdot \frac{M_{en}^{dv}}{J_{en} \cdot E}\, dx + \int_L M_t^{se} \cdot \frac{M_t^{dv}}{J_t \cdot G}\, dx$$

Por lo que necesitamos:

- Diagramas del DV
- Diagramas de cada SE

Diagramas de características

DV

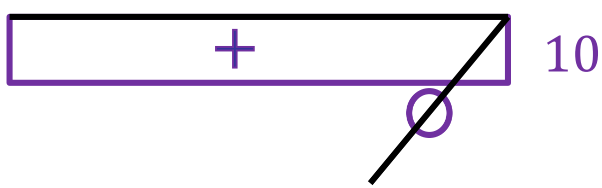


$P_1 = 5\ kN$ $\quad L_1 = 4\ m$

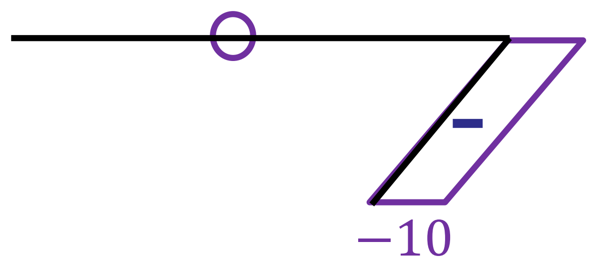
$P_2 = 10\ kN$ $\quad L_2 = 1\ m$

$q = 5\ \frac{kN}{m}$

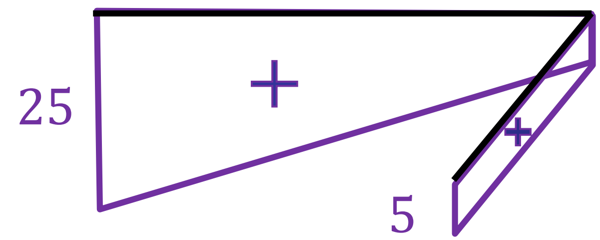
$N[kN]$



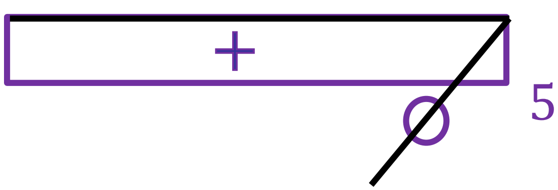
$Q_y[kN]$



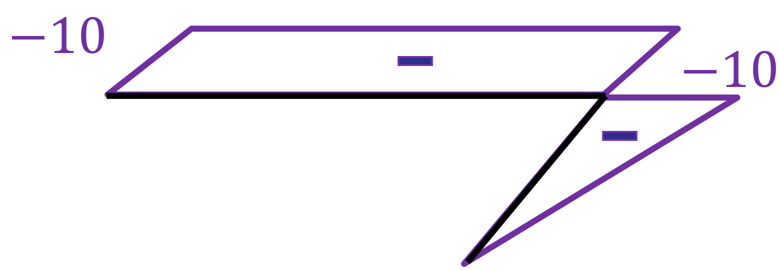
$Q_z[kN]$



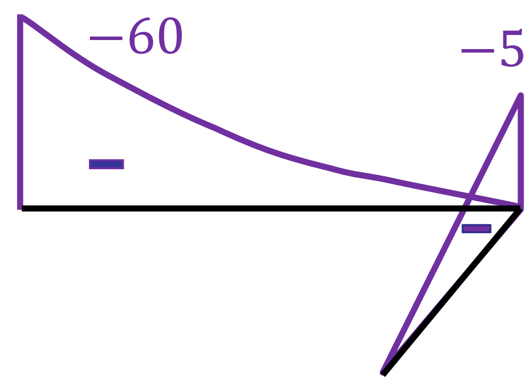
$M_t[kN\ m]$



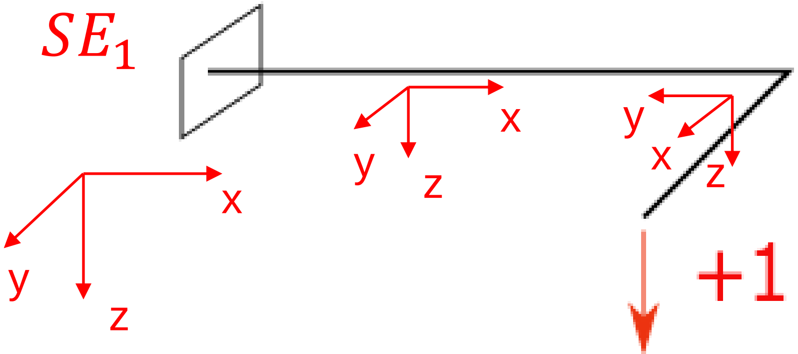
$M_z[kN\ m]$



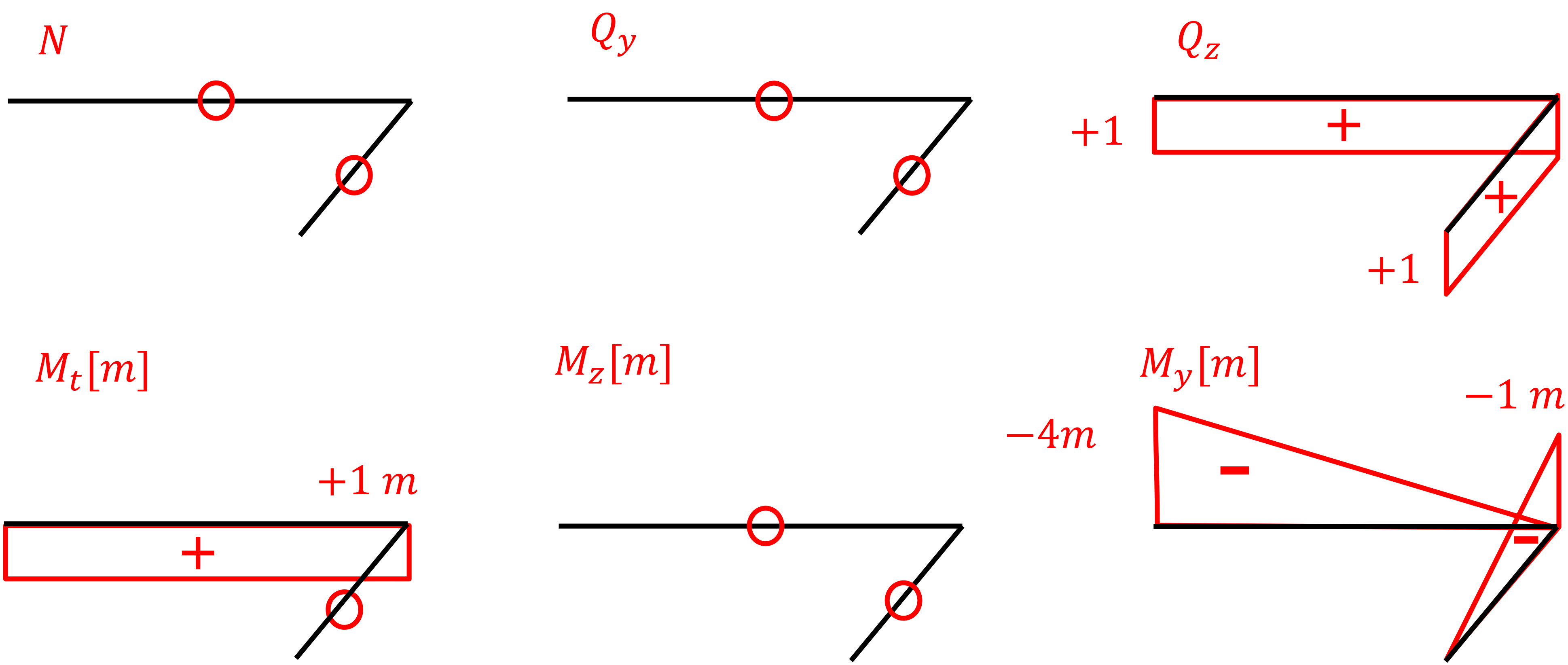
$M_y[kN\ m]$



Desplazamiento vertical: η_A

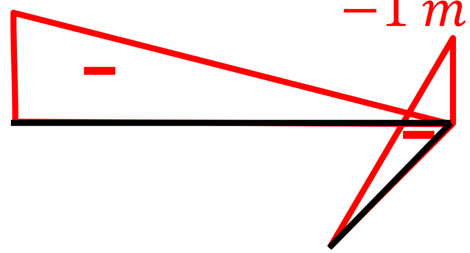


Quiero un desplazamiento vertical, por lo que en el SE debo poner una fuerza unitaria vertical en A

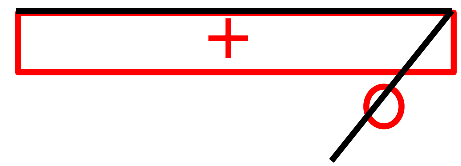


SE_1

$M_y[m]$

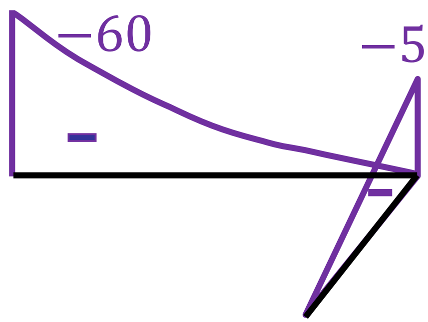


$M_t[m]$

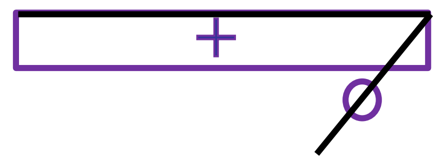


DV

$M_y[kN\ m]$

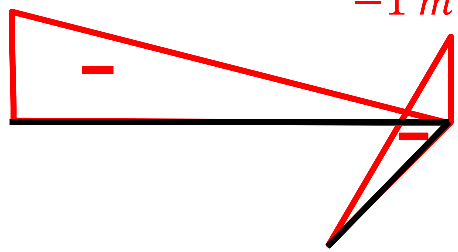


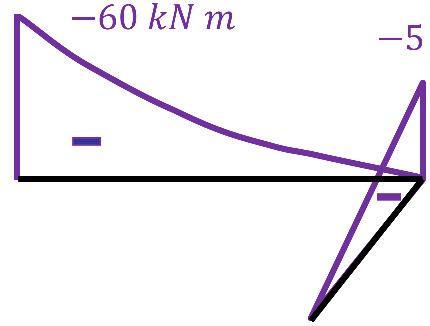
$M_t[kN\ m]$



$$\eta_A = \int_L M_{se} \cdot \frac{M_{dv}}{E \cdot J}\, dx + \int_L Mt_{se} \cdot \frac{Mt_{dv}}{G \cdot J_t}\, dx = \eta_A^{M_y} + \eta_A^{M_t}$$

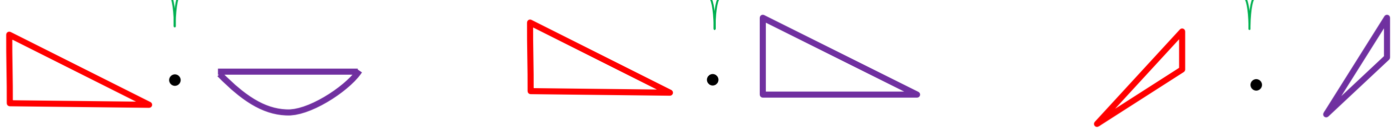
$$\eta_A^{M_y}$$





Cuidado!
La tangente no es nula,
por lo tanto el coeficiente
no esta en tabla.

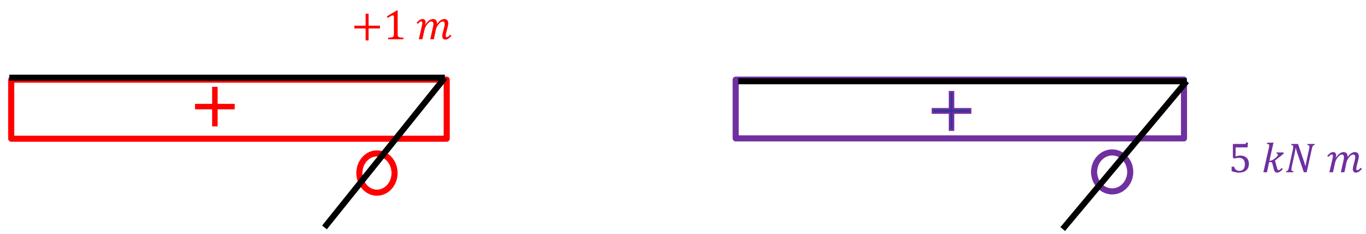
$$\int_L M_{se} \cdot \frac{M_{dv}}{E \cdot J} dx = \frac{1}{3} \cdot (-4m) \cdot \frac{10kNm}{E \cdot J_y} \cdot 4m + \frac{1}{3} \cdot (-4m) \cdot \frac{(-60kNm)}{E \cdot J_y} \cdot 4m + \frac{1}{3} \cdot (-1m) \cdot \frac{(-5kNm)}{E \cdot J_y} \cdot 1m$$



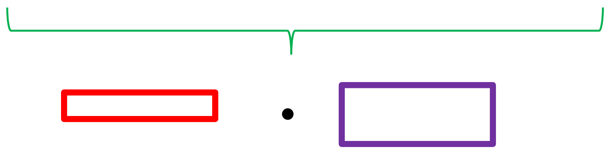
$$\eta_A^{M_y} = -0{,}562\ cm + 3{,}375\ cm + 0{,}018\ cm$$

$$\eta_A^{M_y} = 2{,}831\ cm$$

$\eta_A^{M_t}$



$$\int_L Mt_{se} \cdot \frac{Mt_{dv}}{G \cdot J_t} dx = 1 \cdot (+1\ m) \cdot \frac{(+5\ kN\ m)}{G \cdot J_t} \cdot 4\ m$$



$$\eta_A^{M_t} = +0{,}900\ cm$$

Desplazamiento total:

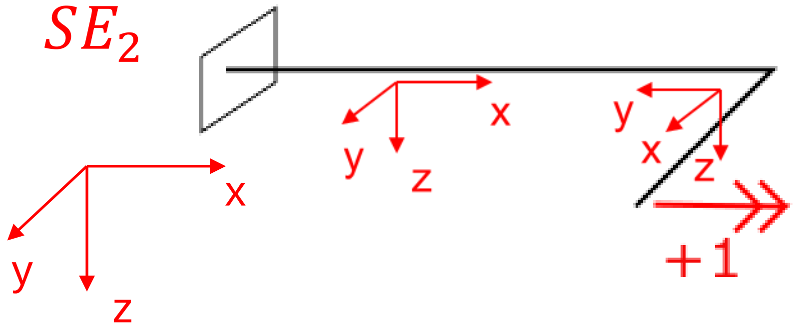
$$\eta_A = \eta_A^{M_y} + \eta_A^{M_t}$$

$$\eta_A = 2{,}831\ cm + 0{,}900\ cm$$

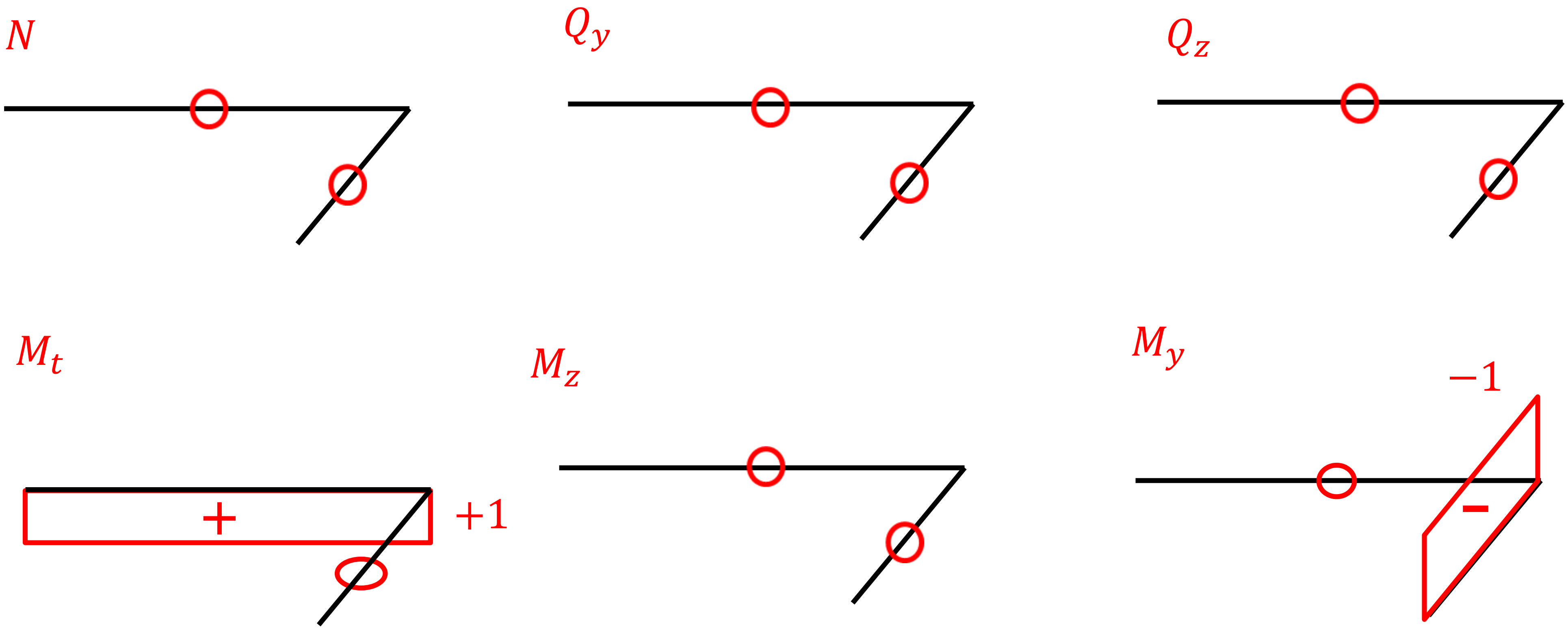
$$\boxed{\eta_A = 3{,}731\ cm}$$

Giro en A: θ_{xA}

SE_2

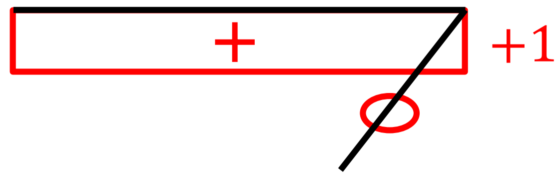


Quiero un giro, por lo que en el SE debo poner un momento unitario en A

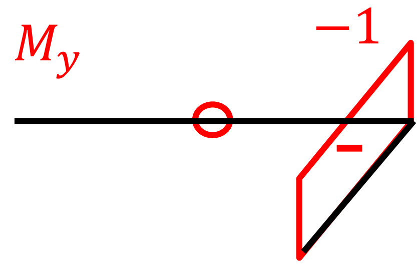


SE_1

M_t

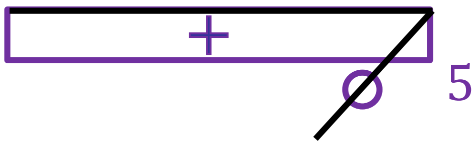


M_y

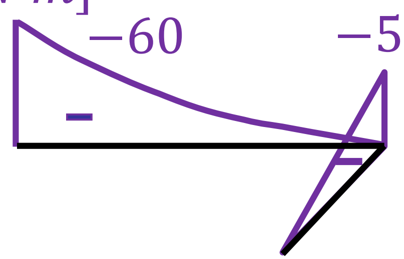


$M_t[kN\ m]$

DV

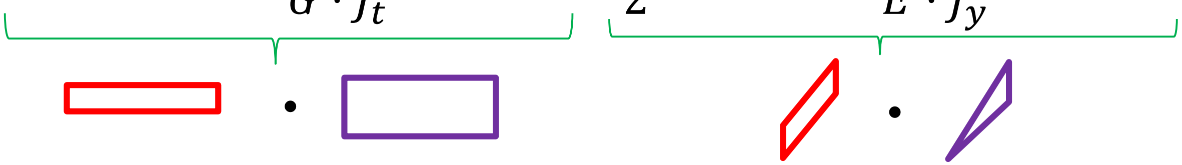


$M_y[kN\ m]$



$$\theta_{xA} = \int_L Mt_{se} \cdot \frac{Mt_{dv}}{G \cdot J_t} dx + \int_L M_{se} \cdot \frac{M_{dv}}{E \cdot J} dx$$

$$\theta_{xA} = 1 \cdot (+1) \cdot \frac{(+5\ kN\ m)}{G \cdot J_t} \cdot 4\ m + \frac{1}{2} \cdot (-1) \cdot \frac{(-5\ kN\ m)}{E \cdot J_y} \cdot 1\ m$$



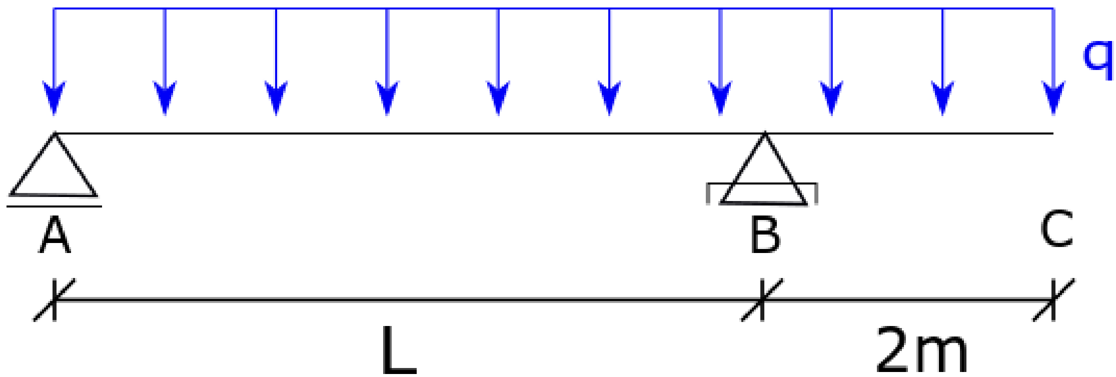
$$\theta_{xA} = 9{,}00 \cdot 10^{-3}\ rad + 2{,}63 \cdot 10^{-4}\ rad$$

$$\theta_{xA} = 9{,}263 \cdot 10^{-3}\ \text{rad}$$

Ejercicio 2:

a) Calcular la longitud "L" tal que el giro en la sección "B" sea nulo

b) Calcular el desplazamiento en "C"



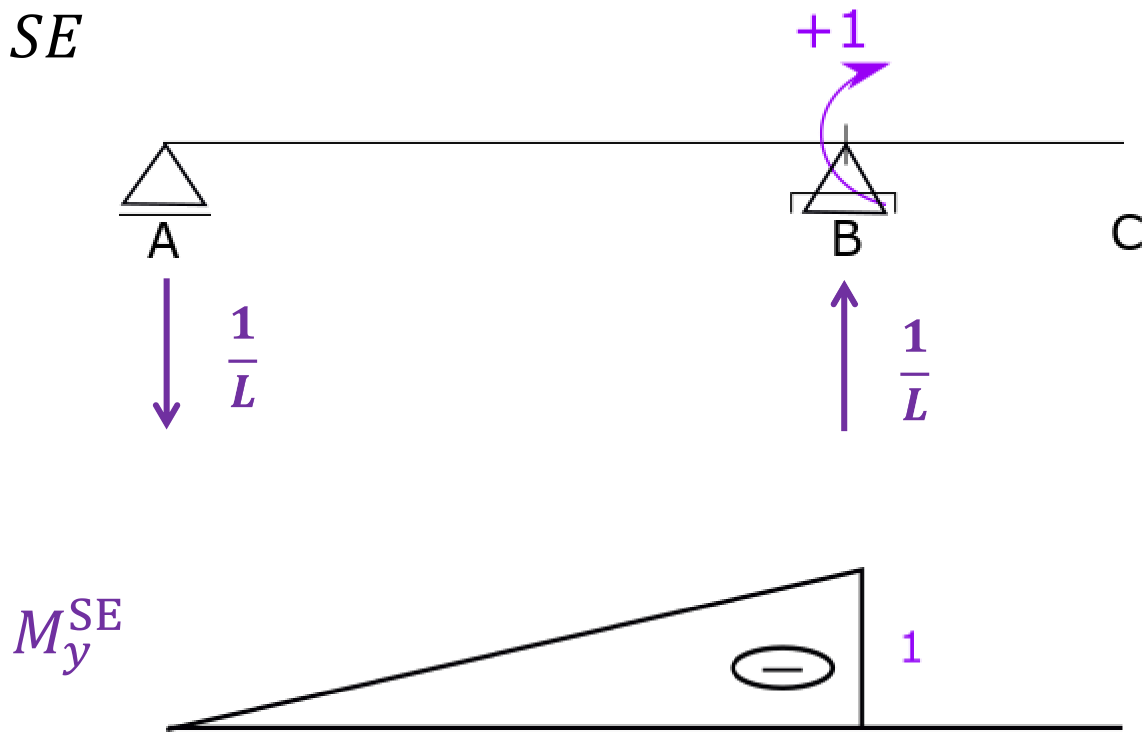
Datos:

$$E \cdot J = cte$$

$$q = 10 \ \frac{kN}{m}$$

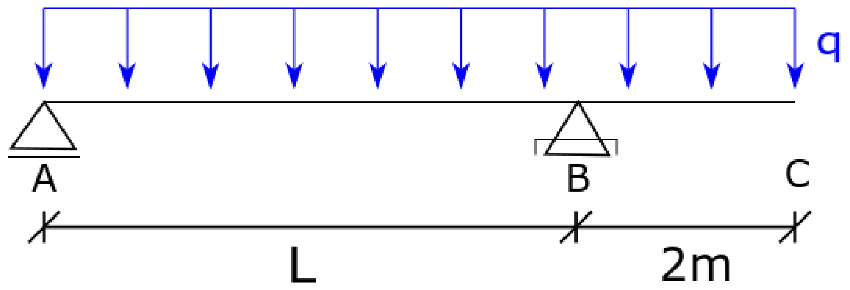
Calculo el giro utilizando TTV

Quiero calcular el giro, por lo tanto en el sistema equilibrado tengo que poner un momento unitario en “B”

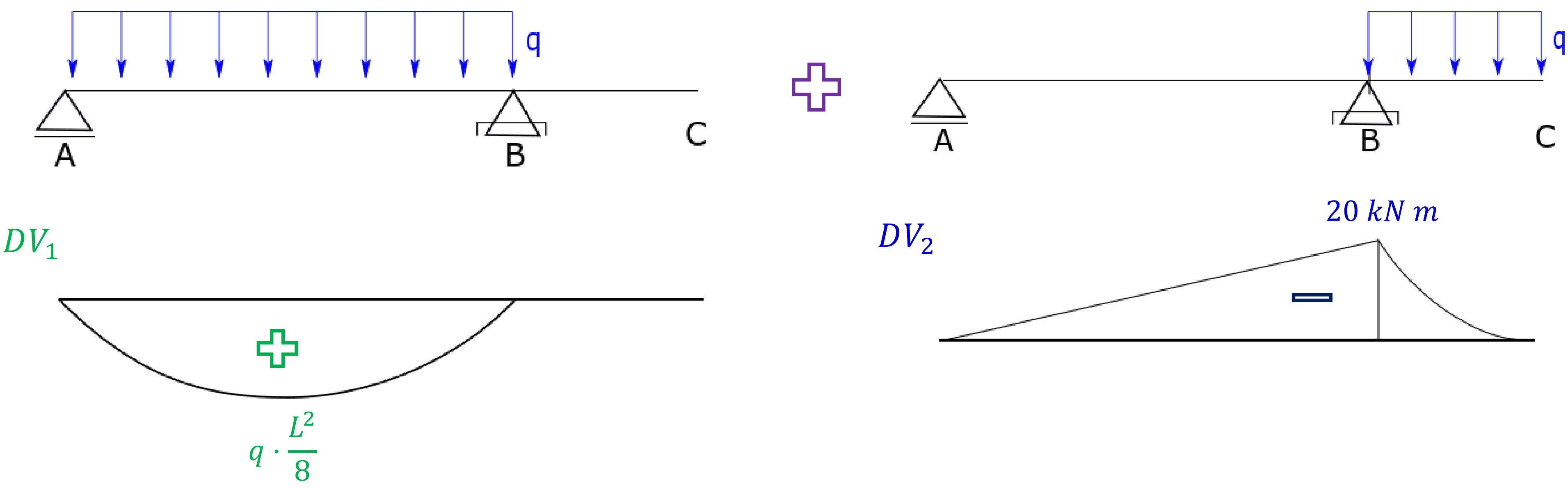


DV

Opción 1: Realizo los diagramas de características directamente de la estructura completa



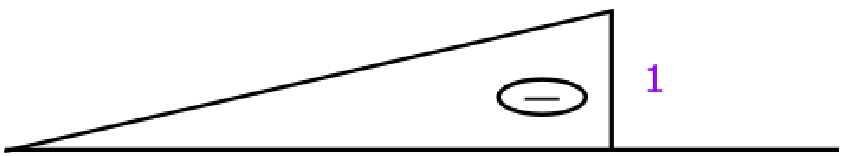
Opción 2: Utilizo superposición de efectos



$$\theta_B = \int_L M^{SE} \cdot \frac{M^{DV}}{E \cdot J}\, dx = \int M^{SE} \cdot \frac{M^{DV_1}}{E \cdot J}\, dx + \int M^{SE} \cdot \frac{M^{DV_2}}{E \cdot J}\, dx$$

$$\int M^{SE} \cdot \frac{M^{DV_1}}{E \cdot J}\, dx \Longrightarrow$$

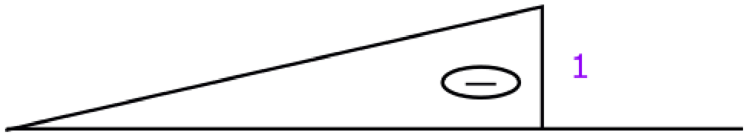
α				
	1	$^1/_2$	$^1/_2$	$^2/_3$
	$^1/_2$	$^1/_3$	$^1/_6$	$^1/_3$

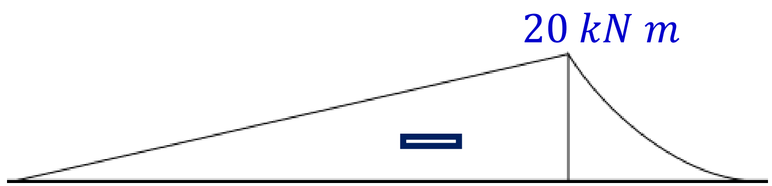


$$\int M^{SE} \cdot \frac{M^{DV_1}}{E \cdot J}\, dx = \frac{1}{3} \cdot \frac{(-1) \cdot \left(10\, \frac{kN}{m} \cdot \frac{L^2}{8}\right) \cdot L}{E \cdot J} = -\frac{5}{12}\, \frac{kN}{m} \cdot \frac{L^3}{E \cdot J}$$

$$\int M^{SE} \cdot \frac{M^{DV_2}}{E \cdot J}\, dx$$

α				
	1	$^1/_2$	$^1/_2$	$^2/_3$
	$^1/_2$	$^1/_3$	$^1/_6$	$^1/_3$





$$\int M^{SE} \cdot \frac{M^{DV_2}}{E \cdot J}\, dx = \frac{1}{3} \cdot \frac{(-1) \cdot (-20\ kN\ m) \cdot L}{E \cdot J} = \frac{20}{3} \cdot \frac{L}{E \cdot J}\ kN\ m$$

$$\theta_B = \int M^{SE} \cdot \frac{M^{DV_1}}{E \cdot J}\, dx + \int M^{SE} \cdot \frac{M^{DV_2}}{E \cdot J}\, dx$$

Reemplazando y recordando que $\theta_B = 0$

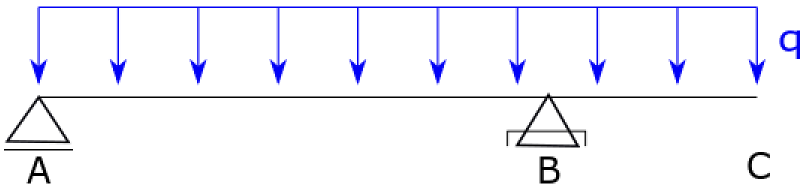
$$0 = -\frac{5}{12}\frac{kN}{m} \cdot \frac{L^3}{E \cdot J} + \frac{20}{3} \cdot \frac{L}{E \cdot J}\, kN\, m$$

$$\frac{5}{12}\frac{kN}{m} \cdot \frac{L^3}{E \cdot J} = \frac{20}{3} \cdot \frac{L}{E \cdot J}\, kN\, m \rightarrow \frac{5}{12} \cdot L^2 = \frac{20}{3}\, m^2$$

$$\boxed{L = 4\, m}$$

b) Desplazamiento en “C”

DV

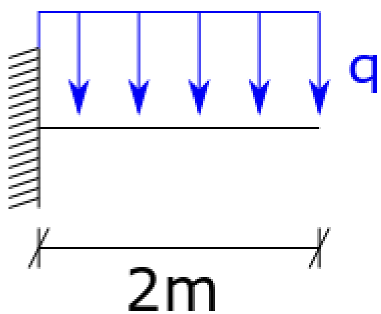


SE

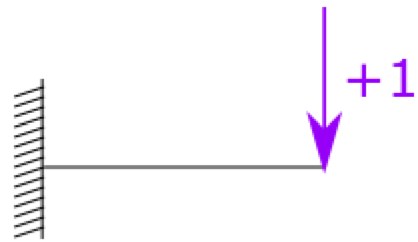
Pero sabemos que en el punto B no hay giros ni desplazamientos

El punto B funciona como un empotramiento

DV

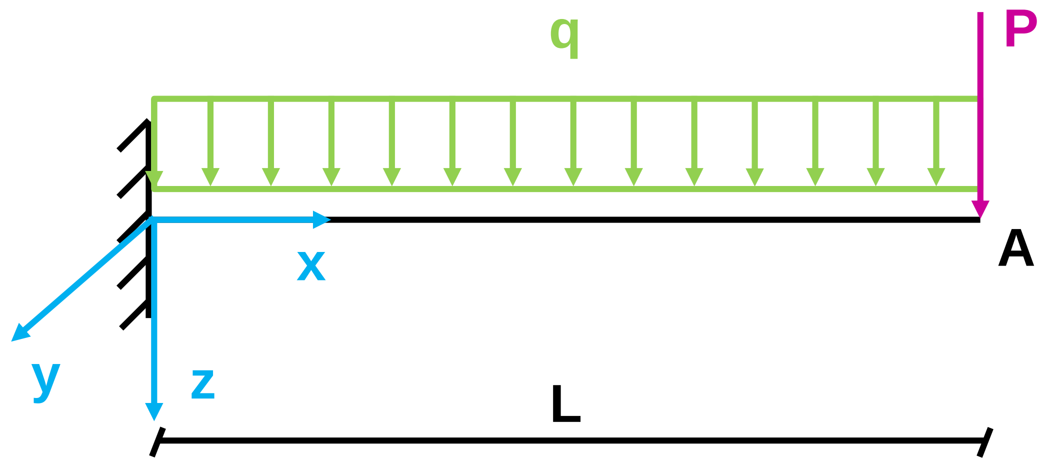


SE

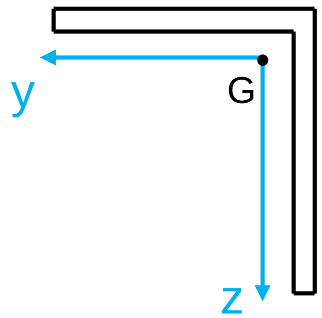


Ejercicio 3:

a) Calcular el desplazamiento máximo del punto A



Datos



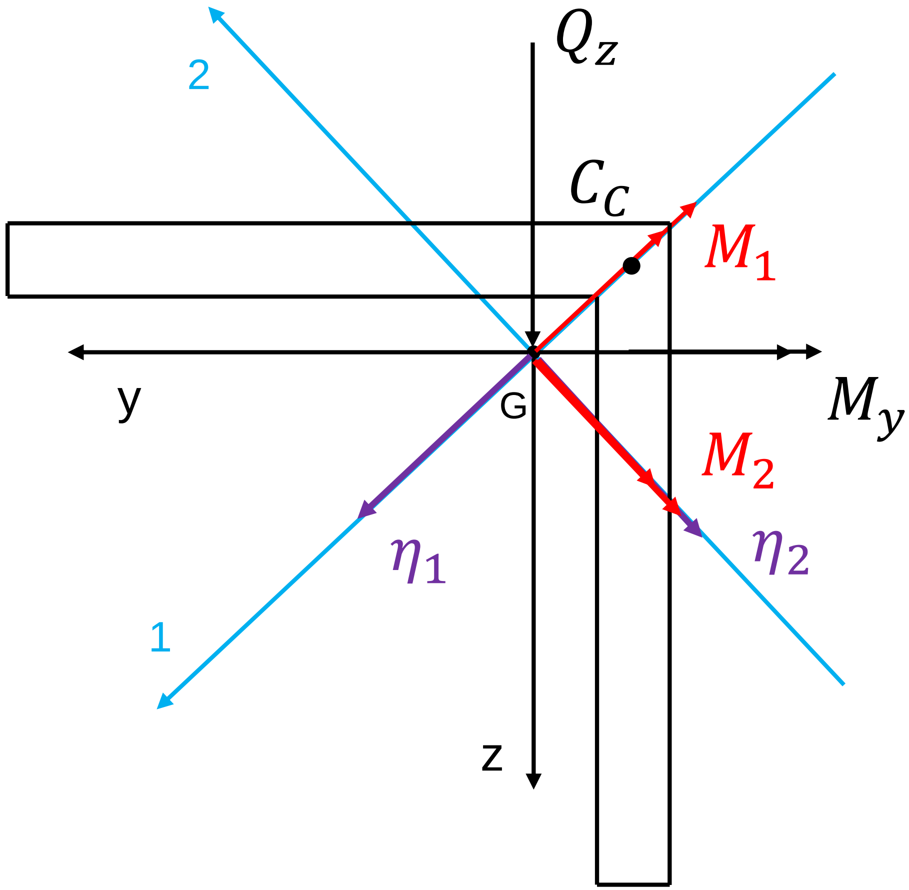
L

P, q

E

$Perfil\ angulo\ de\ alas\ iguales$

- Analizamos la sección



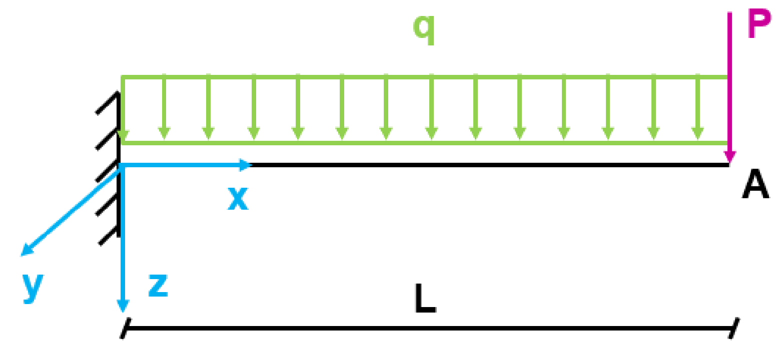
¡Observación!
Tenemos que trabajar con los ejes principales de inercia

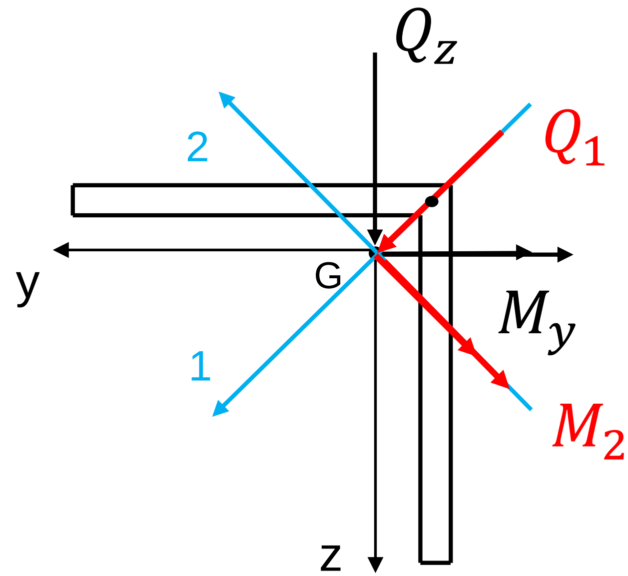
$$M_1 = M_y \cdot \cos 45^0 = M_2$$

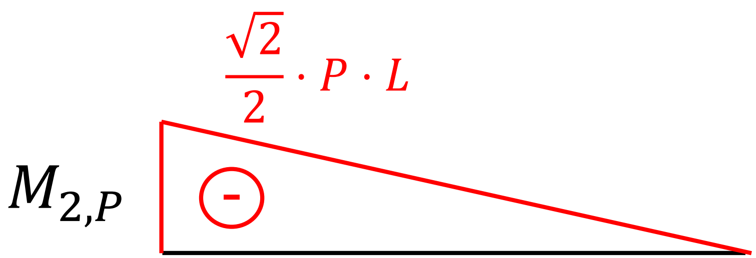
¡Observación!
Aparece un efecto de torsión debido a que Q no pasa por el centro de corte

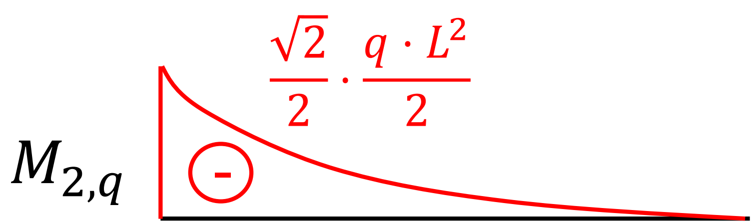
Desplazamiento $\eta_{A,1}$

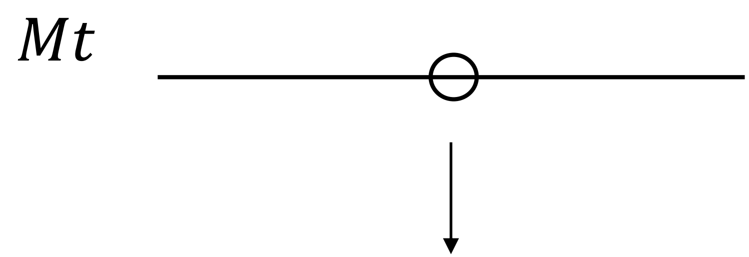
- Diagramas de la deformación virtual







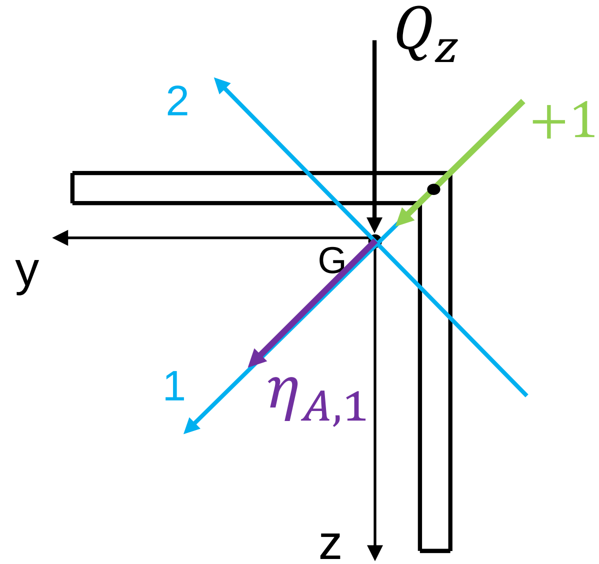


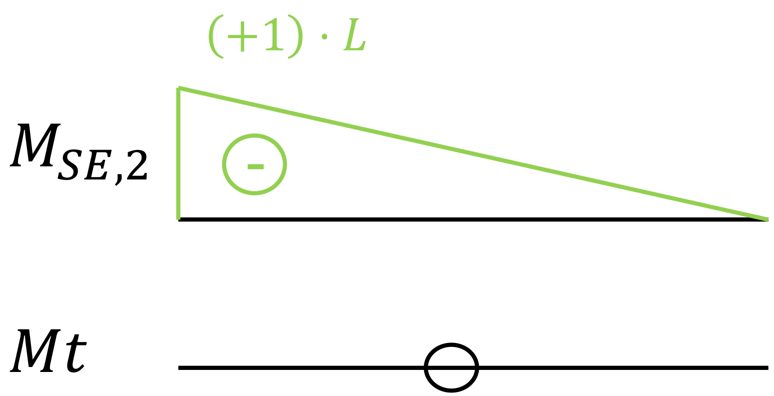


Q_1 pasa
por el centro de corte

- Sistema equilibrado → Aplicamos fuerza unitaria en **A** con dirección 1

  - *Sección A*

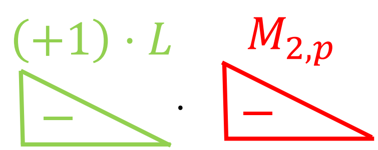




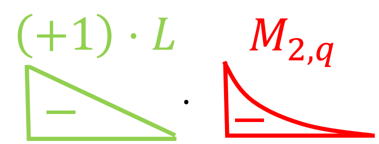
- Aplicamos el **Teorema de los trabajos virtuales**

$$(+1)\cdot\eta_{A,1}=\int\frac{M_2^{SE}\cdot M_2^{DV}}{E\cdot J_2}\,\mathrm{d}x+\underbrace{\int\frac{M_t^{SE}\cdot M_t^{DV}}{G\cdot J}\,dx}_{=0}$$

$(+1)\cdot\eta_{A,1}=\int_0^L$



$\dfrac{\ \cdot\ }{E\cdot J_2}\,\mathrm{d}x\;+\;\int_0^L$



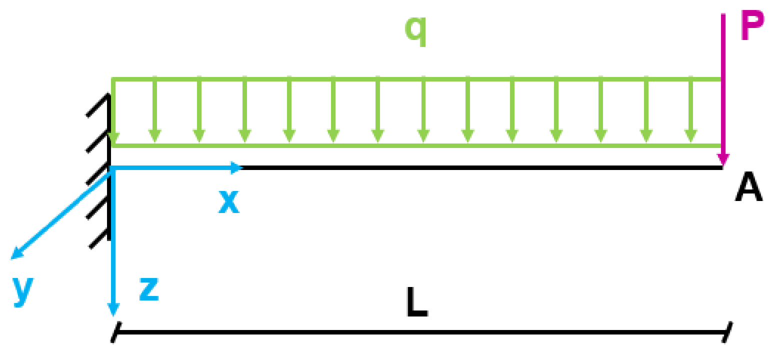
$\dfrac{\ \cdot\ }{E\cdot J_2}\,\mathrm{d}x$

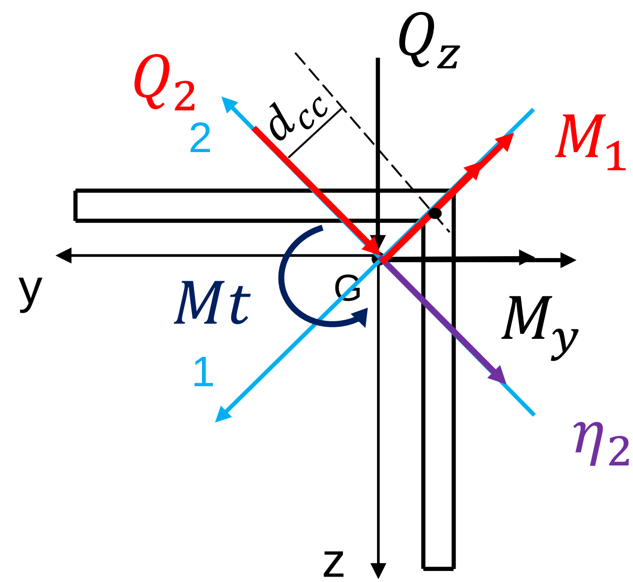
$$(+1)\cdot\eta_{A,1}=\frac{1}{3}\cdot\left(-(+1)\cdot L\right)\cdot\frac{\sqrt{2}}{2}\cdot\frac{(-P\cdot L)}{E\cdot J_2}\cdot L+\frac{1}{4}\cdot\left(-(+1)\cdot L\right)\cdot\frac{\sqrt{2}}{2}\cdot\frac{(-q\cdot L^2)}{2\cdot E\cdot J_2}\cdot L$$

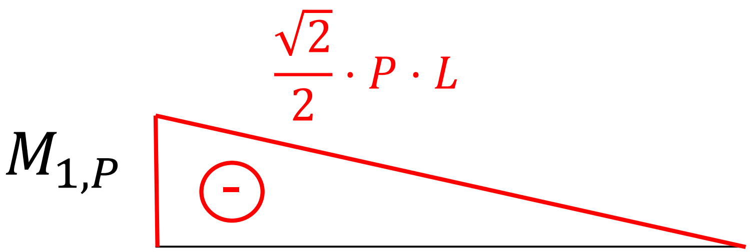
$$\boxed{\eta_{A,1}=\frac{\sqrt{2}}{2\cdot E\cdot J_2}\cdot\left(\frac{P\cdot L^3}{3}+\frac{q\cdot L^4}{8}\right)}$$

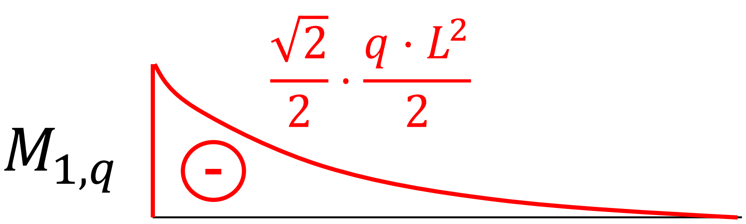
Desplazamiento η_2

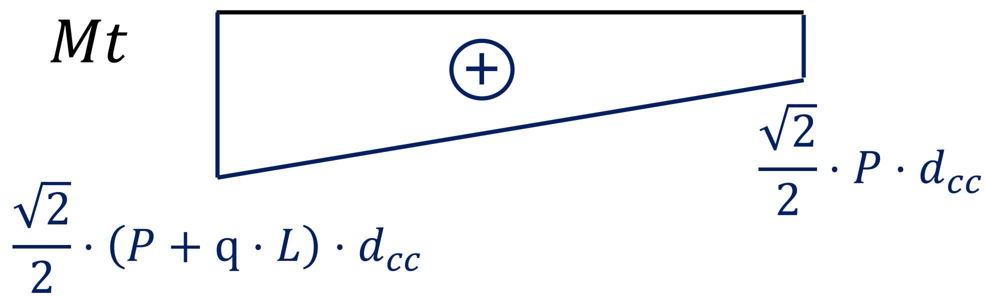
- Diagramas de deformación virtual





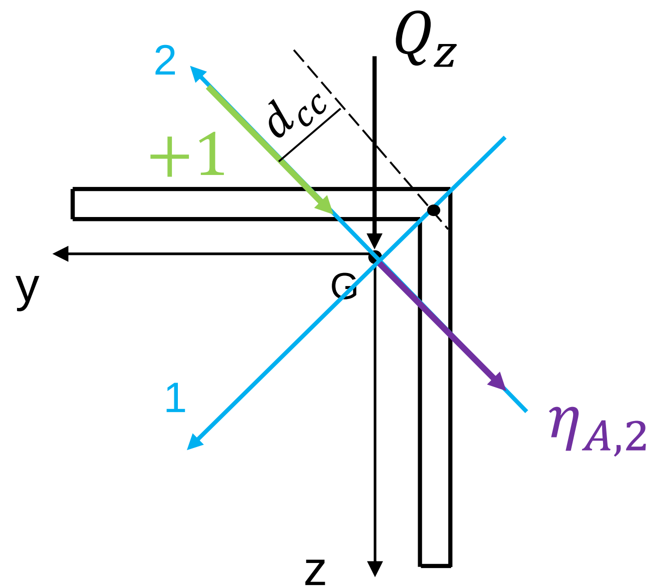






- Sistema equilibrado ⟶ Aplicamos fuerza unitaria en **A** con dirección 2

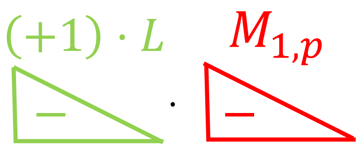
  - *Sección A*

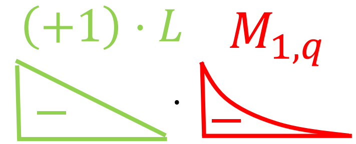


- Aplicamos el **Teorema de los trabajos virtuales**

$$(+1)\cdot \eta_{A,2} = \underbrace{\int \frac{M_1^{SE}\cdot M_1^{DV}}{E\cdot J_1}\,\mathrm{d}x}_{①} + \underbrace{\int \frac{M_t^{SE}\cdot M_t^{DV}}{G\cdot J}\,dx}_{②}$$

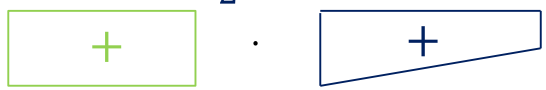
① $= \displaystyle\int_0^L \frac{[\text{diagrama}]\cdot[\text{diagrama}]}{E\cdot J_1}\,\mathrm{d}x + \int_0^L \frac{[\text{diagrama}]\cdot[\text{diagrama}]}{E\cdot J_1}\,\mathrm{d}x$



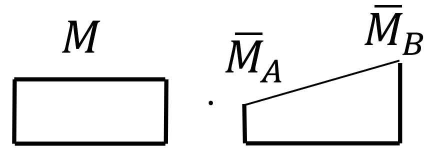


$$① = \frac{\sqrt{2}}{2\cdot E\cdot J_1}\cdot\left(\frac{P\cdot L^3}{3}+\frac{q\cdot L^4}{8}\right)$$

$$\textcircled{2} = \int_0^L \frac{\overset{(+1)\cdot d_{cc}}{[+]} \cdot \overset{\frac{\sqrt{2}}{2}\cdot(P+q\cdot L)\cdot d_{cc} \quad \frac{\sqrt{2}}{2}\cdot P\cdot d_{cc}}{[+]}}{G\cdot J}\,dx = \frac{\sqrt{2}}{4\cdot G\cdot J}\cdot d_{cc}^2\cdot L\cdot(2\cdot P+q\cdot L)$$



Aplicando la fórmula
y reordenando



$$= \frac{1}{2}\cdot M\cdot(\bar{M}_A+\bar{M}_B)\cdot L$$

- Entonces,

$$\eta_{A,2} = \textcircled{1} + \textcircled{2}$$

$$\eta_{A,2} = \frac{\sqrt{2}}{2\cdot E\cdot J_1}\cdot\left(\frac{P\cdot L^3}{3}+\frac{q\cdot L^4}{8}\right)+\frac{\sqrt{2}}{4\cdot G\cdot J}\cdot d_{cc}^2\cdot L\cdot(2\cdot P+q\cdot L)$$

- Cual es el **desplazamiento máximo** del punto A?

$$\eta_{A,1} = \frac{\sqrt{2}}{2 \cdot E \cdot J_2} \cdot \left(\frac{P \cdot L^3}{3} + \frac{q \cdot L^4}{8} \right)$$

$$\eta_{A,2} = \frac{\sqrt{2}}{2 \cdot E \cdot J_1} \cdot \left(\frac{P \cdot L^3}{3} + \frac{q \cdot L^4}{8} \right) + \frac{\sqrt{2}}{4 \cdot \mathrm{G} \cdot \mathrm{J}} \cdot d_{cc}^2 \cdot L \cdot (2 \cdot P + q \cdot L)$$

- El **desplazamiento máximo** del punto A, considerando esta sección, va a ser la composición de los calculados en los ejes 1 y 2

$$\eta_A = \sqrt{\left(\eta_{A,1}\right)^2 + \left(\eta_{A,2}\right)^2}$$

¡Observación!
La dirección del desplazamiento no es z