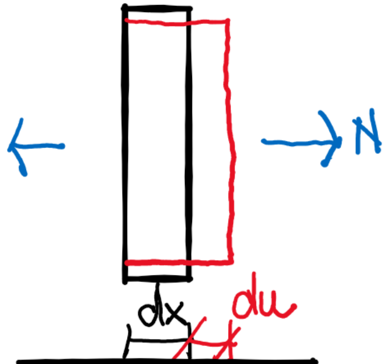
CÁLCULO DE DESPLAZAMIENTOS

Deformaciones

AXIL

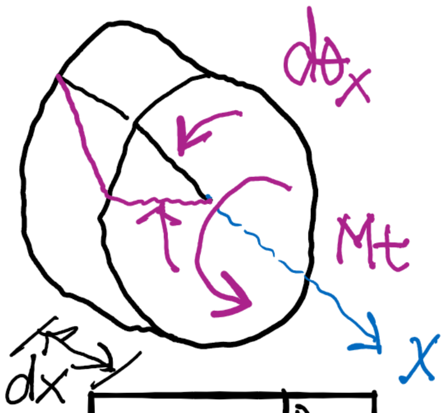


$$\boxed{\varepsilon_x = du/dx}$$

$$\sigma_x = N/A = \varepsilon_x . E$$

$$\boxed{\varepsilon_x = \frac{N}{E.A}}$$

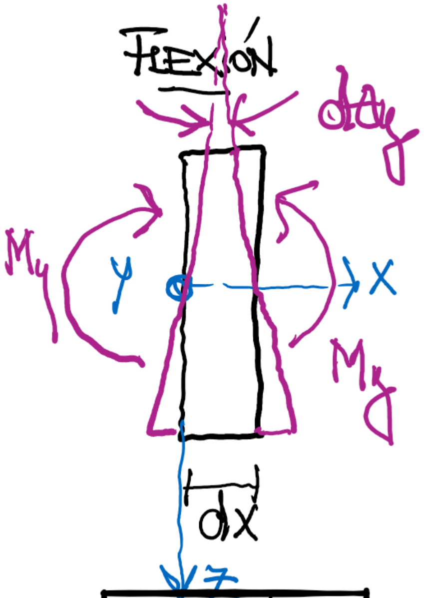
TORSOR



$$\boxed{\chi_x = \frac{d\theta_x}{dx}}$$

$$\boxed{\chi_x = \frac{M_t}{G.J_p}}$$

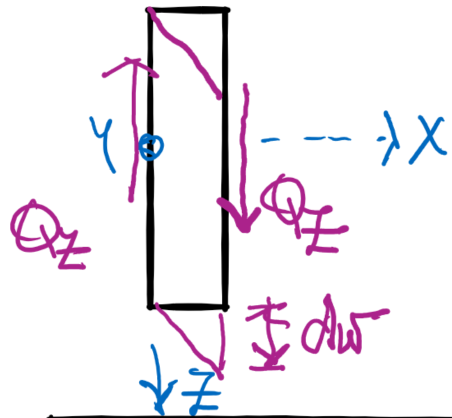
FLEXIÓN



$$\boxed{\chi_y = \frac{d\theta_y}{dx}}$$

$$\boxed{\chi_y = \frac{M_y}{E.J_y}}$$

CORTE



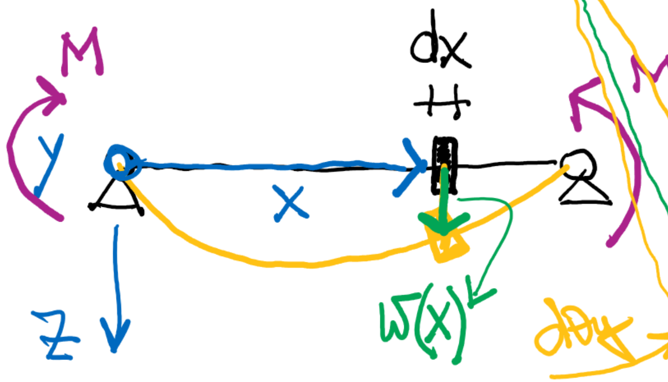
$$\boxed{\frac{dw}{dx} = \frac{Q_z}{G.A} \cdot K_z}$$

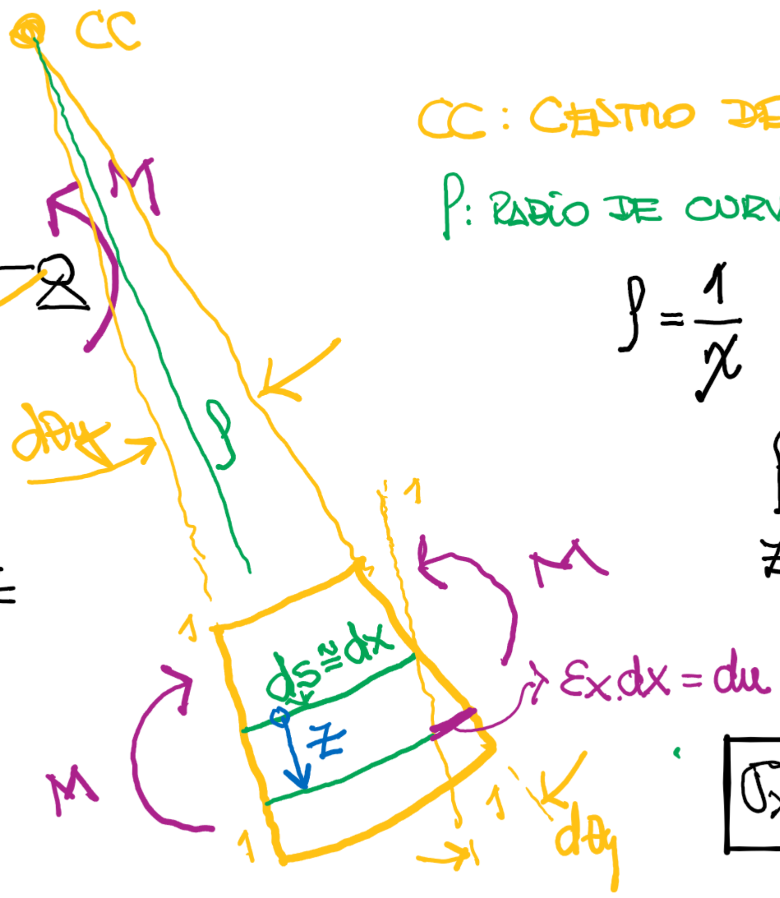
$$A_R = \frac{A}{K_z}$$

FACTOR DE FORMA

Se deforma conservando la direccion del eje

Se deforma en la dirección transversal al eje de barra - EJE CURVO





CC : CENTRO DE CURVATURA

ρ: RADIO DE CURVATURA

$$\rho = \frac{1}{\chi} \rightarrow \chi_y = \frac{d\theta_y}{dx} = \frac{1}{\rho}$$

$$\left.\begin{aligned} \rho \cdot d\theta_y &= dx \\ z \cdot d\theta_y &= \varepsilon_x \cdot dx \end{aligned}\right\}$$

$$z\,\frac{d\theta_y}{dx} = \boxed{\varepsilon_x = z \cdot \chi_y}$$

$$\boxed{\sigma_x = E\,\varepsilon_x = E \cdot \chi_y \cdot z}$$

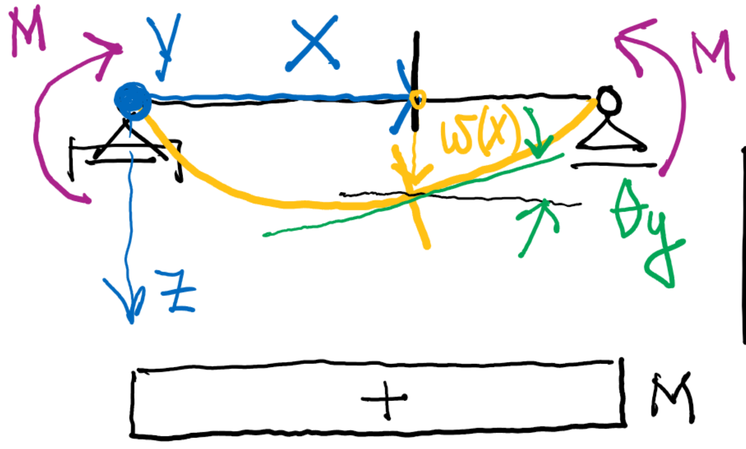
$$\sigma_x = \frac{M_y}{J_y} \cdot z = E \cdot \chi_y \cdot z$$

$$\boxed{\chi_y = \frac{d\theta_y}{dx} = \frac{M_y}{E J_y}}$$

EXPRESION MATEMATICA DE LA CURVATURA

$$\chi = \frac{dw^2/dx^2}{\left[1 + \left(\frac{dw}{dx}\right)^2\right]^{3/2}}$$

$$\frac{dw}{dx} = w' \lll 1 \rightarrow w' = 0 \qquad \chi = d^2w/dx^2$$



$$\theta_y \cong tg\,\theta_y = \frac{dw(x)}{dx}$$

$$\boxed{\chi_y = \frac{d\theta_y}{dx} = \frac{d^2w}{dx}}$$

$$\boxed{\chi_y = \frac{M_y}{E.J_y}}$$

$$\boxed{\frac{dw^2}{dx^2} = -\frac{M_y}{EJ_y}}$$

CONDICIONES DE BORDE

$X=0 \rightarrow w(0)=0 \rightarrow \boxed{C_2=0}$

$X=L \rightarrow w(L)=0 \rightarrow \boxed{C_1 = \frac{M_y.L}{2EJ_y}}$

$$w' = -\frac{M_y}{EJ_y}\cdot x + C_1$$

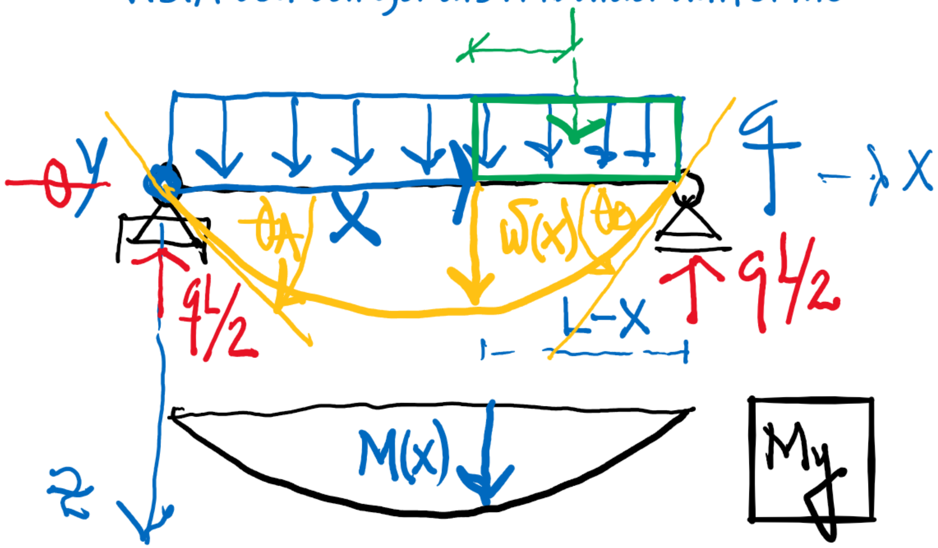
$$\boxed{w = -\frac{M_y}{EJ_y}\cdot\frac{x^2}{2} + C_1\cdot x + C_2}$$

$$\boxed{w(x) = -\frac{M_y}{EJ_y}\cdot\frac{x^2}{2} + \frac{M_y.L}{2EJ_y}\cdot x}$$

$$w_{max} \rightarrow \frac{dw(x)}{dx} = 0 \rightarrow x = L/2$$

$$\boxed{w_{max} = \frac{M.L^2}{8EJ_y}}$$

V.S.A con carga distribuida uniforme



$\boxed{M_y}$

$$\frac{d^2 w}{dx^2} = -\frac{M(x)}{EJ_y}$$

$$M(x) = \frac{qL}{2}\cdot(L-x) - q\cdot\frac{(L-x)^2}{2}$$

$$M(x) = \frac{q}{2}(L.x - x^2)$$

$$w'(x) = -\frac{q}{2EJ_y}\cdot\left(L\frac{x^2}{2} - \frac{x^3}{3}\right) + C_1$$

$$\boxed{w(x) = -\frac{q}{2EJ_y}\left(L\frac{x^3}{6} - \frac{x^4}{12}\right) + C_1.x + C_2}$$

CONDICIONES DE BORDE

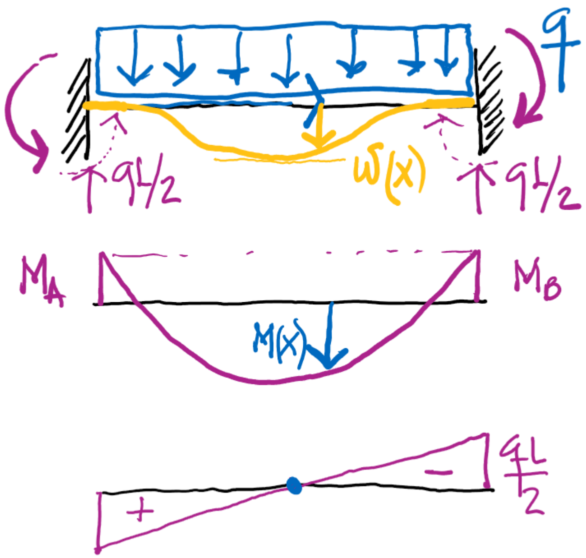
$x=0 \rightarrow w(0)=0 \rightarrow \boxed{C_2=0}$

$x=L \rightarrow w(L)=0 \rightarrow C_1$

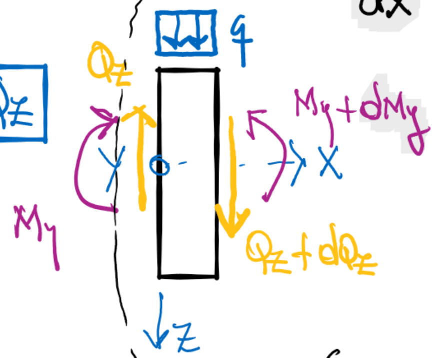
$$\boxed{w(x) = \frac{q}{EJ_y}\frac{x^4}{24} - \frac{q}{EJ_y}\cdot L\cdot\frac{x^3}{12} + \frac{q.x.L^3}{24.EJ_y}}$$

$$\boxed{w_{max} = w(L/2) = \frac{5}{384}\frac{q.L^4}{EJ_y}} \leq L/300 \text{ ó } L/500$$

Ej 3 Viga doblemente empotrada con carga uniformemente distribuida



$\boxed{M_y}$ $\boxed{Q_z}$



$$w'' = -M(x)/EJ_y$$

$$M(x) = ?$$

$$w''' = -\frac{dM(x)}{dx}\cdot\frac{1}{EJ_y} = -\frac{Q_z}{EJ_y}$$

CARGA $$w^{IV} = -\frac{dQ_z}{dx}\cdot\frac{1}{EJ_y} = +\frac{q}{EJ_y}$$

$$q\cdot dx + dQ_z = 0$$

$$\frac{dQ_z}{dx} = -q$$

CORTE $$w''' = +\frac{q}{EJ_y}\cdot x + C_1$$

MOMENTO $$w'' = +\frac{q}{EJ_y}\cdot\frac{x^2}{2} + C_1\cdot x + C_2$$

GIRO $$w' = +\frac{q}{EJ_y}\cdot\frac{x^3}{6} + C_1\cdot\frac{x^2}{2} + C_2 x + C_3$$

ELÁSTICA $$\boxed{w = +\frac{q}{EJ_y}\cdot\frac{x^4}{24} + C_1\cdot\frac{x^3}{6} + C_2\cdot\frac{x^2}{2} + C_3\cdot x + C_4}$$

POLINOMIO DE 4to GRADO

CONDICIONES DE VÍNCULO

$x=0 \rightarrow w(0) = 0 \rightarrow w'(0) = 0$

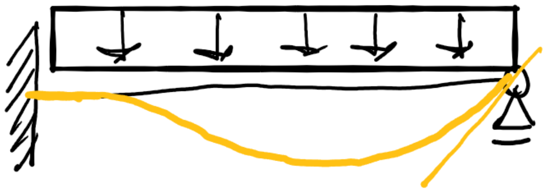
$x=L \rightarrow w(L) = 0 \rightarrow w'(L) = 0$

$x = L/2 \rightarrow w'(L/2) = 0$

$\rightarrow w'''(L/2) = 0$ CORTE

$$\boxed{w_{max} = \frac{1}{384}\,\frac{qL^4}{EJ_y}}$$

Ej 4 Viga Empotrada / Simplemente apoyada con carga uniforme

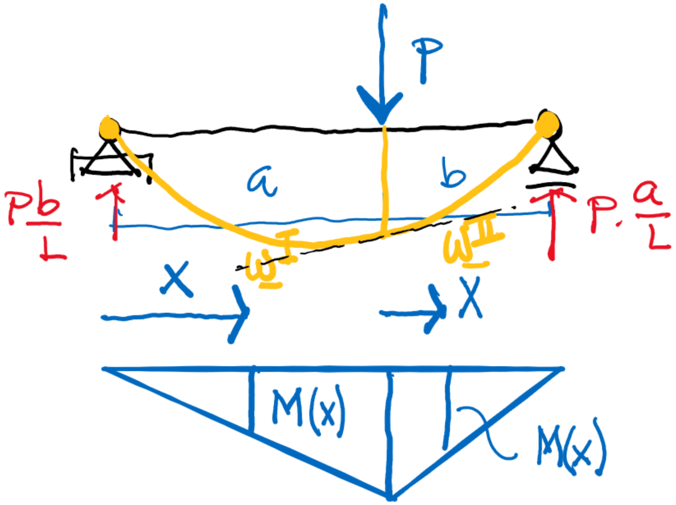


$w(0) = 0 \qquad w'(0) = 0$

$w(L) = 0 \qquad w''(L) = 0$

Ej VSA con una carga concentrada

$$w''(x) = -\frac{M(x)}{EJ_y}$$



I $\quad 0 \leq x \leq a \qquad M(x) = P \cdot \frac{b}{L} \cdot x \to w^{I} \quad \left\{ \begin{matrix} C_1 \\ C_2 \end{matrix} \right\}$

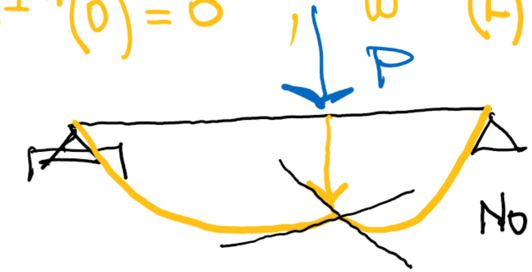
II $\quad a \leq x \leq L \qquad M(x) = P \cdot \frac{a}{L}(L - x) \to w^{II} \quad \left\{ \begin{matrix} C_3 \\ C_4 \end{matrix} \right\}$

CONDICIONES DE BORDE

$w^{I}(0) = 0 \quad ; \quad w^{II}(L) = 0$

$w^{I''}(0) = 0 \quad ; \quad w^{II''}(L) = 0$

$w^{I}(a) = w^{II}(a)$



$w^{I'}(a) = w^{II'}(a)$