OBJETO:

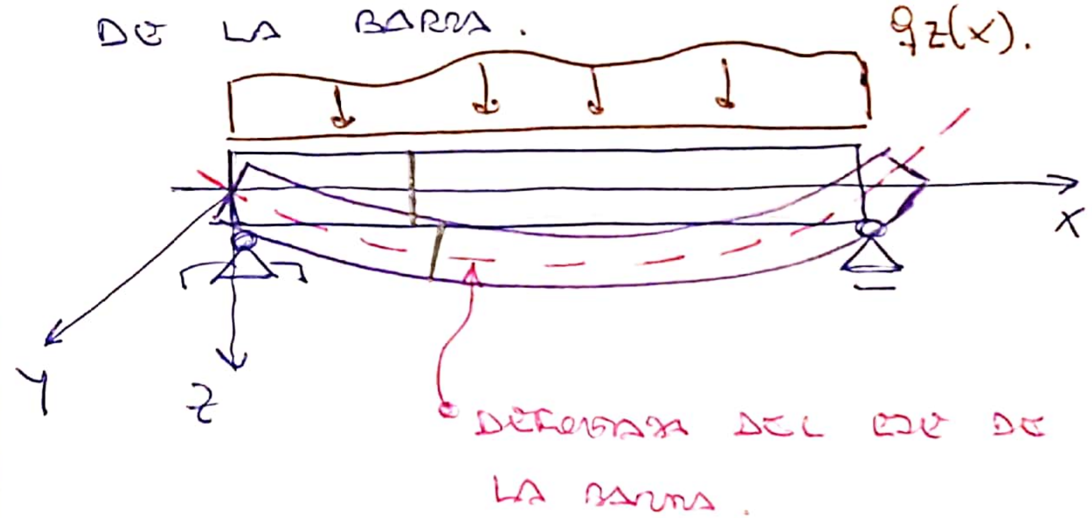
1º) CÁLCULO DE DESPLAZAMIENTOS EN ELEMENTOS ESTRUCTURALES SOLICITADOS A FLEXIÓN:

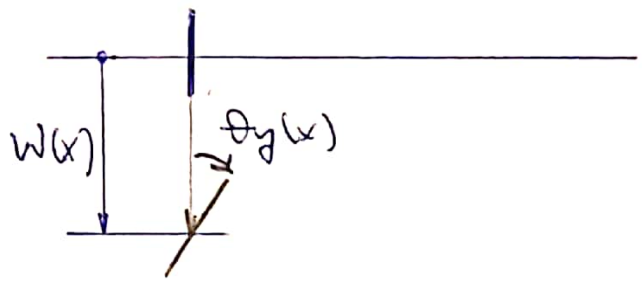
- SA → $\delta(x) = \Delta L(x)$
- SF → X.
- ST → $\phi(x)$
- FU → ✓ dc

2º) **CdD** → ELEMENTO ESTRUCTURAL. → SOLICITADO POR UNA COMBINACIÓN DE ESFUERZOS.

DEFINICIÓN DE
- LÍNEA ELÁSTICA.
- CURVA ELÁSTICA.
- ELÁSTICA

→ DEFORMADA DEL EJE DE LA PIEZA O DE LA BARRA.





NOMENCLATURA:

$P_z(x) = q_z(x)$ = FUNCIÓN FZAS ACTIVAS EN LA DIRECCIÓN 'z'.

$P_y(x) = q_y(x)$ = " EN LA DIRECCIÓN 'y'.

$M_y(x) \longleftrightarrow Q_z(x)$

$M_z(x) \longleftrightarrow Q_y(x)$

$W_z(x) = W(x)$: FUNCIÓN DESPLAZAMIENTO EN LA DIRECCIÓN 'z'.

$V_y(x) = V(x)$: FUNCIÓN DESPLAZAMIENTO EN LA DIRECCIÓN 'y'.

$\theta_y(x)$: FUNCIÓN GIROS ABSOLUTOS ALREDEDOR DEL EJE 'y'

$\chi_y(x)$: FUNCIÓN CURVATURA DE FLEXIÓN ALREDEDOR DEL EJE 'y'

$\theta_z(x)$: FUNCIÓN "GIROS ABSOLUTOS ALREDEDOR DEL EJE 'z'".

$\chi_z(x)$: FUNCIÓN CURVATURA DE FLEXIÓN ALREDEDOR DEL EJE 'z'.

B_y: RIGIDEZ A FLEXIÓN DE LA BARRA RESPECTO DEL EJE 'y'.

$B_y = E \cdot I_y$.

B_z: IDEM RESPECTO DEL EJE 'z'.

$B_z = E \cdot I_z$

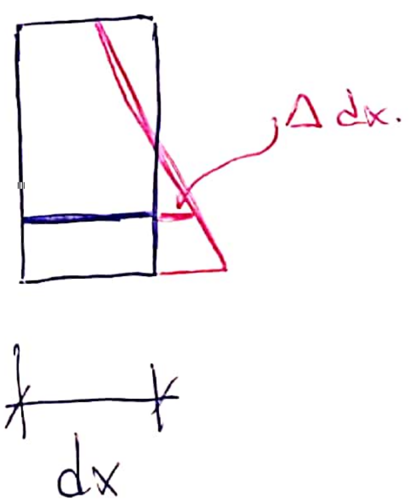
C: CENTRO DE CURVATURA.

ρ: RADIO DE CURVATURA.

dx: DIFERENCIAL DE LONGITUD 'INICIAL' DE UNA PARTE DE LA BARRA.

ds: DIFERENCIAL DEL ARCO DE LA ELÁSTICA DE LA BARRA.

$d\theta_y$: VARIACIÓN DE LOS ÁNGULOS ENTRE 2 SECCIONES ALREDEDOR DEL EJE 'Y' O
$d\theta_z$: 'Z' RESPECTIVAMENTE



$$\begin{cases} L_0 = dx. \\ L_f = dx + \Delta dx \end{cases}$$

HIPÓTESIS:

- MATERIAL → CONTINUO, HOMOGÉNEO, ISÓTROPO
- HLG: HIPÓTESIS DE LOS PEQUEÑOS DESPLAZAMIENTOS
  - → HLE.
  - → HLC.
- HLM: → LEY DE HOOKE.
- CARGAS CUASIESTÁTICAS.
- BARRAS DE EJE RECTO.

I) FLEXIÓN SIMPLE.

II) BARRAS "ESBELTAS." $\frac{L}{d} \geq 10$

MÉTODOS P/ EL CÁLCULO DE DEFORMACIONES:

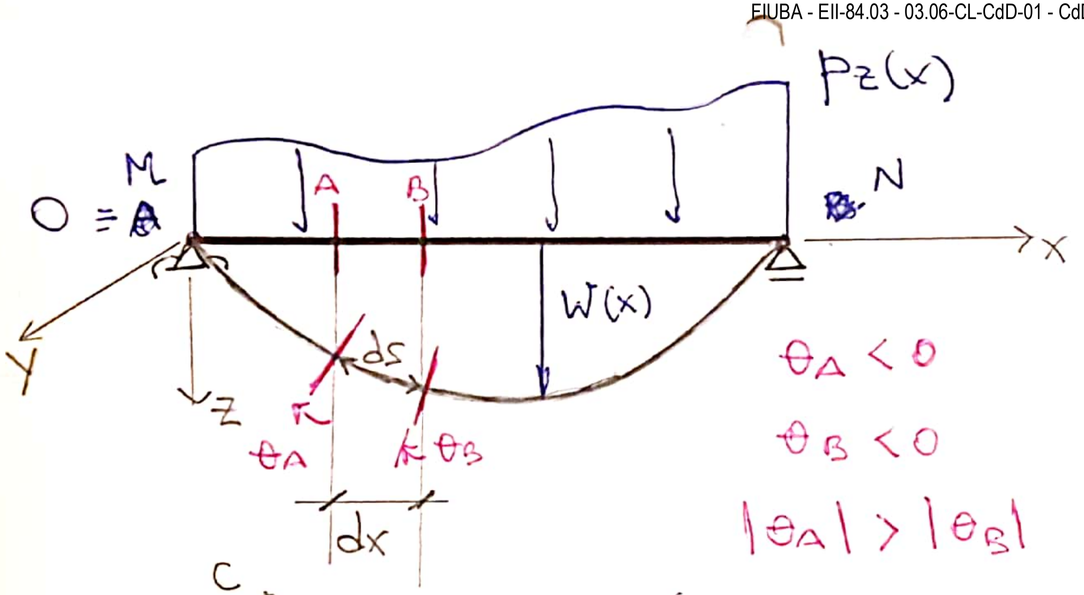
1) MÉTODO EC. DIFERENCIAL DE LA LÍNEA ELÁSTICA.

2) " " UNIVERSAL ' ' ' '

3) " 'TEOREMAS DE MOHR'

4) " ENERGÉTICO BASADO EN EL ~~TEOREMA~~ TEOREMA. DE ~~CASTIGLIANO~~ CASTIGLIANO.

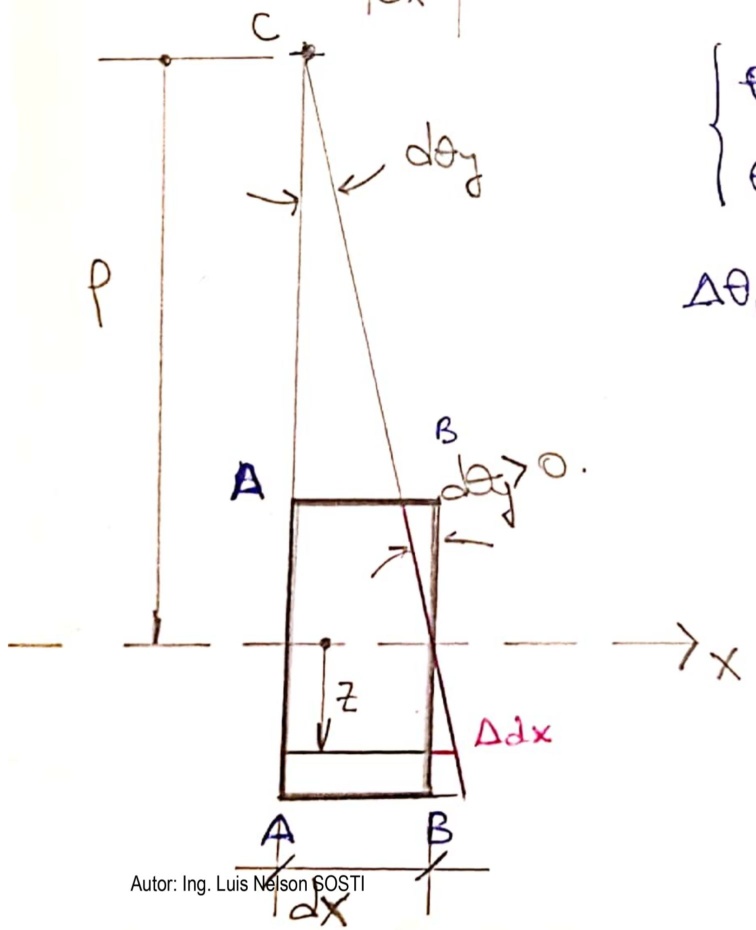
5) " " " EN TTV.



$\theta_A < 0$

$\theta_B < 0$

$|\theta_A| > |\theta_B|$



$$\begin{cases} \theta_A = \theta. \\ \theta_B = \theta + d\theta. \end{cases}$$

$$\Delta\theta_{BA} = \theta_B - \theta_A = d\theta.$$

$$d\theta > 0$$

$$\theta_A = -5$$

$$\theta_B = -3$$

$$\theta_B - \theta_A = -3 - (-5)$$

$$\theta_B - \theta_A = +2 = d\theta.$$

$$\rho \cdot d\theta_y = dx \quad (1a)$$

$$\frac{1}{\rho} = \frac{d\theta_y}{dx} \quad (1b).$$

$$z \cdot d\theta_y = \Delta dx\Big]_z \quad (2a)$$

$$d\theta_y = \frac{\Delta dx\big]_z}{z} \quad (2b).$$

Divido por '$\underline{dx}$' a la expresión (2b).

$$\frac{d\theta_y}{dx} = \underbrace{\frac{\Delta dx\big]_z}{dx}}_{\varepsilon_x} \cdot \frac{1}{z} = \frac{\varepsilon_x}{z} \quad (3).$$

$$\boxed{\frac{1}{\rho} = \frac{d\theta_y}{dx} = \frac{\varepsilon_x}{z} = \theta'_y} \quad (4).$$

Es válida p/ cualquier tipo de comportamiento del material ya sea elástico o anelástico.

→ ELÁSTICO → "LEY DE HOOKE".

$$\sigma_x = E \cdot \varepsilon_x \rightarrow \varepsilon_x = \frac{\sigma_x}{E} \qquad (5)$$

$$\frac{1}{\rho} = \frac{d\theta_y}{dx} = \theta'_y = \frac{\sigma_x}{E \cdot z} \qquad (6)$$

→ FLEXIÓN SIMPLE ⟶ VIMOS QUE:

$$\sigma_x = \frac{M_y(x)}{I_y} \cdot z \qquad (7)$$

$$\rightarrow \frac{1}{\rho} = \frac{d\theta_y}{dx} = \theta'_y = \frac{M_y(x)}{E I_y} \cdot \frac{\cancel{z}}{\cancel{z}}$$

$$\rightarrow \frac{1}{\rho} = \frac{d\theta_y}{dx} = \theta'_y = \frac{M_y(x)}{E I_y} = \frac{M_y(x)}{B_y} \qquad (8)$$

→ RIGIDEZ A LA FLEXIÓN ~~EN LA~~ RESPECTO DEL EJE 'y'.

COMENTARIOS:

I) EL MATERIAL ES EL MISMO P/ TODA LA BARRA → HOMOGÉNEA →

$$E = cte = E(x)$$

II) LA INERCIA DE LA BARRA ES CONS_ TANTO:

$$I_y(x) = I_y = cte.$$

III) $E \cdot I_y = B_y$ = RIGIDEZ A LA FLEXIÓN RESPECTO DEL EJE 'y'.

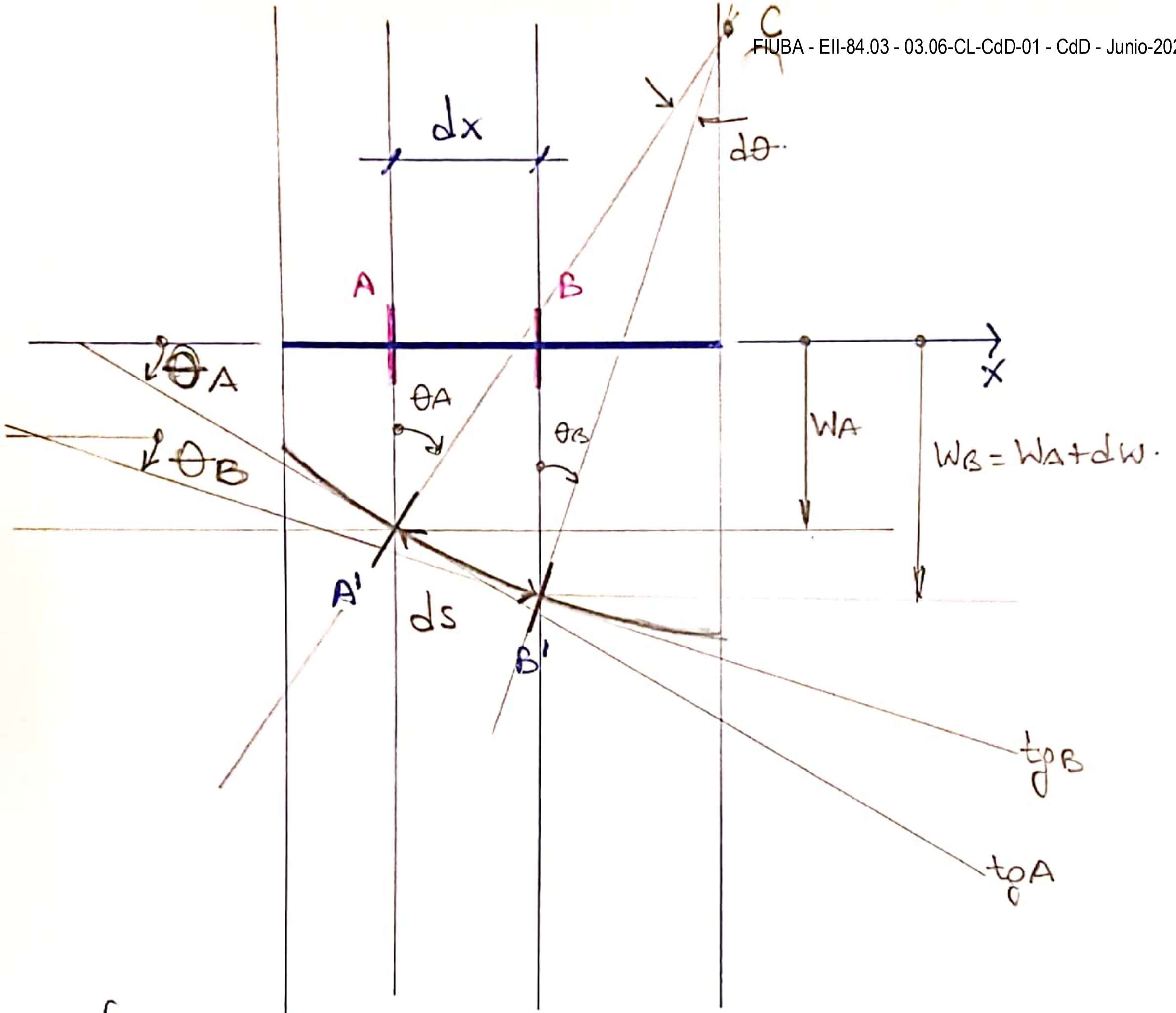
IV) $$\rho = \frac{E \cdot I_y}{M_y(x)} = \frac{B_y}{M_y(x)}$$

$$B_y \uparrow \longrightarrow \rho \uparrow$$

Si $\left.\begin{matrix} E = cte \\ I_y = cte \end{matrix}\right\} \rightarrow B_y = cte \rightarrow \rho = \rho[M_y(x)]$

V) $M_y(x) = cte \longrightarrow \rho = cte \longrightarrow$

⟶ LA DEFORMADA → ARCO DE CIRCUNFE_ RENCIA.



$$\begin{cases} \theta_A = \theta \\ \theta_B = \theta + d\theta \\ \theta_B - \theta_A = d\theta \end{cases}$$

$$ds = \rho \cdot d\theta \rightarrow \frac{1}{\rho} = \frac{d\theta_y}{ds} \quad (9).$$

- Por hip. pequeños desplazamientos
  (+)
  Hip. pequeñas deformaciones.

$$ds \cong dx$$

- $$\frac{1}{\rho} = \frac{d\theta_y}{dx}. \quad (10).$$

- $$\tan\theta_y = \frac{dW}{dx}. \quad (11).$$

- Hip. peq. desplazam. $\rightarrow$

$$\rightarrow \tan\theta_y = \theta_y. \rightarrow$$

$$\theta_y(x) = \frac{dW(x)}{dx} = W'(x). \quad (12).$$

- Derivo la (12)

$$\theta'_y(x) = \frac{d\theta_y(x)}{dx} = \frac{d^2 w(x)}{dx^2} = W''(x) \quad (13)$$

- COMBINO LA (13) CON LA (8).

$$\theta'_y(x) = \frac{d\theta_y(x)}{dx} = \frac{d^2 w(x)}{dx^2} = w''(x) = \frac{1}{\rho} = \underbrace{\frac{M_y(x)}{E I_y}}_{B_y}. \quad (14).$$

- RETOMO LA (12) PARA VER LOS SIGNOS DE LA EXPRESIÓN (14)

$$\theta_y(x) = - \frac{dw(x)}{dx} \rightarrow \theta'_y(x) = - \frac{d^2 w(x)}{dx^2}$$

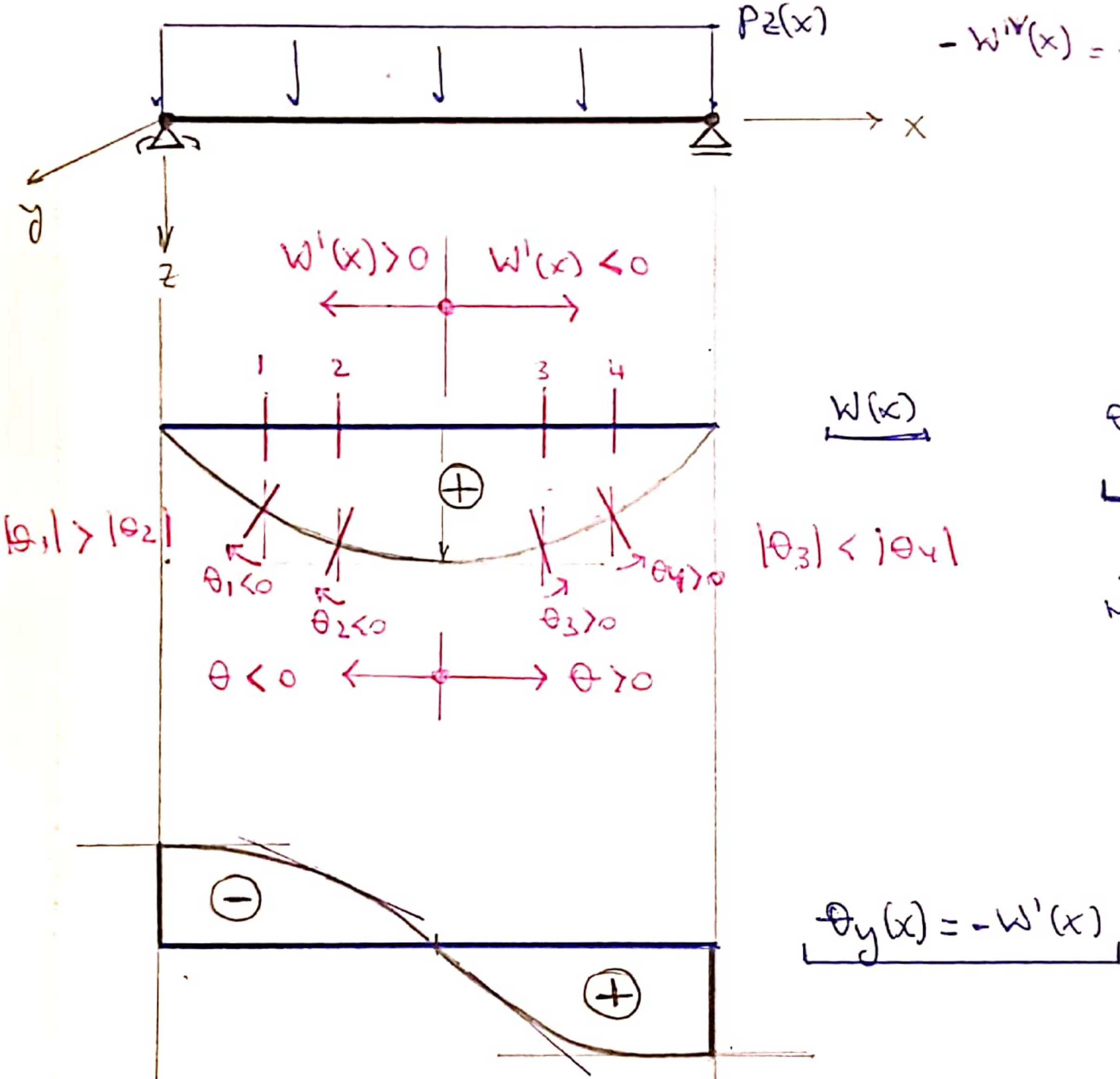
$dx > 0.$

$dw > 0$

$\theta < 0.$

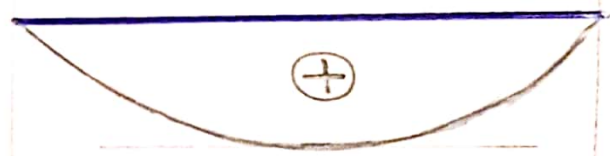
$$\boxed{\frac{d^2 w(x)}{dx^2} = -\theta'_y(x) = -\frac{d\theta_y(x)}{dx} = -\frac{M_y(x)}{E I_y}} \quad (15)$$

$$-W^{IV}(x) = +\theta_y'''(x) = +\frac{M_y''(x)}{B_y} = +\frac{Q_z'(x)}{B_y} = -\frac{P_z(x)}{B_y}$$

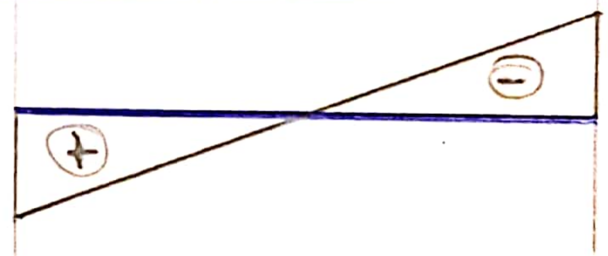


$$\theta_y' = \frac{M_y}{B_y}$$

$$= -W''(x)$$

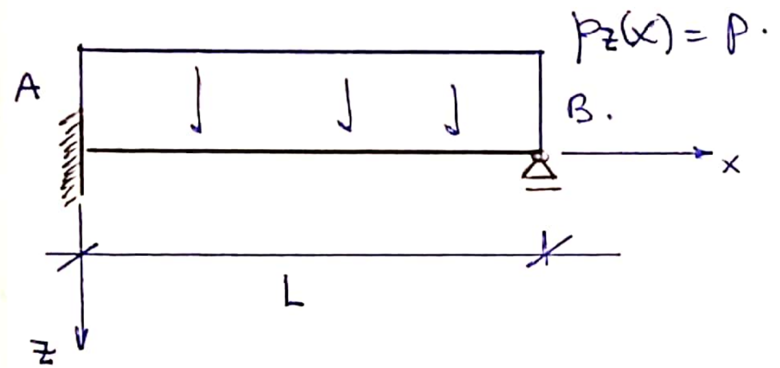


$$-W'''(x) = +\theta_y''(x) = +\frac{M_y'(x)}{B_y} = +\frac{Q_z(x)}{B_y}$$



$$\cancel{0 = -W'''(x) = +\theta_y''(x) = +\frac{M_y'(x)}{B_y} = +\frac{Q_z(x)}{B_y}}$$

EJERCICIO:



IPN 200 $\quad \sigma_{ADM} = 15\ \text{kN/cm}^2$.
$E = 21.000\ \text{kN/cm}^2$.
$L = 4\ \text{m}$

1) RELACIONES BÁSICAS

$$W_z(x) = W(x)$$

$$W'(x) = -\theta_y(x)$$

$$W''(x) = -\theta'_y(x) = -\frac{M_y(x)}{B_y}.$$

$$W'''(x) = -\theta''_y(x) = -\frac{M'_y(x)}{B_y} = -\frac{Q_z(x)}{B_y}.$$

$$W^{IV}(x) = -\theta'''_y(x) = -\frac{M''_y(x)}{B_y} = -\frac{Q'_z(x)}{B_y} = +\frac{P_z(x)}{B_y}$$

2) CONDICIONES DE BORDE:

VARIABLE	TIPO 'CB'	$X = 0$	$X = L$
DESPLAZAMIENTO	CBC	$W_0(0) = 0$	$W(L) = 0.$
GIROS	CBC	$W'(0) = 0$	
MOMENTO	CBE		$W''(L) = 0.$
CORTE.	CBE.		

3) EXPRESIONES BÁSICAS

$$W^{IV}(x) = \frac{p_z(x)}{B_y}$$

$$W'''(x) = \frac{p_z(x)}{B_y} \cdot x + C_1.$$

$$W''(x) = \frac{p_z(x)}{2B_y} \cdot x^2 + C_1 \cdot x + C_2$$

$$W'(x) = \frac{p_z(x)}{6B_y} \cdot x^3 + \frac{1}{2} C_1 x^2 + C_2 x + C_3.$$

$$W(x) = \frac{p_z(x)}{24B_y} \cdot x^4 + \frac{1}{6} C_1 x^3 + \frac{1}{2} C_2 x^2 + C_3 x + C_4.$$

4) DESARROLLO - DET. CONSTANTES:

- $x = 0 \rightarrow W(0) = 0.$

$$W(x=0) = 0 + C_4 \rightarrow \boxed{C_4 = 0}$$

- $x = 0 \rightarrow W'(0) = 0.$

$$W'(x=0) = 0 = C_3 \rightarrow \boxed{C_3 = 0}$$

- $x = L \rightarrow W(L) = 0.$

$$\boxed{0 = \frac{pL^4}{24B_y} + \frac{1}{6} C_1 L^3 + \frac{1}{2} C_2 L^2} \quad \text{I}$$

- $x = L \rightarrow W''(L) = 0.$

$$0 = \frac{pL^2}{2B_y} + C_1 L + C_2$$

$$\boxed{C_2 = -\frac{pL^2}{2B_y} - C_1 L.} \quad \text{II}$$

DE (I) y (II) OBTENEMOS:

$$\boxed{\begin{aligned} C_1 &= -\frac{5}{8}\frac{p}{B_y}L \\ C_2 &= +\frac{1}{8}\frac{p}{B_y}L^2 \end{aligned}}$$

$$W^{IV}(x) = \frac{p}{B_y}.$$

$$W'''(x) = \frac{p}{B_y}\cdot x - \frac{5}{8}\frac{p}{B_y}L = -\frac{Q_z(x)}{B_y}$$

$$W''(x) = \frac{p}{2B_y}x^2 - \frac{5}{8}\frac{p}{B_y}L\cdot x + \frac{1}{8}\frac{p}{B_y}L^2 = -\frac{M_y(x)}{B_y}$$

$$W'(x) = \frac{p}{6B_y}\cdot x^3 - \frac{5}{16}\frac{p}{B_y}Lx^2 + \frac{1}{8}\frac{p}{B_y}L^2 x = -\theta_y(x)$$

$$\boxed{W(x) = \frac{p}{24B_y}\cdot x^4 - \frac{5}{48}\frac{p}{B_y}L\cdot x^3 + \frac{1}{16}\frac{p}{B_y}L^2x^2} \quad \text{(I)}$$

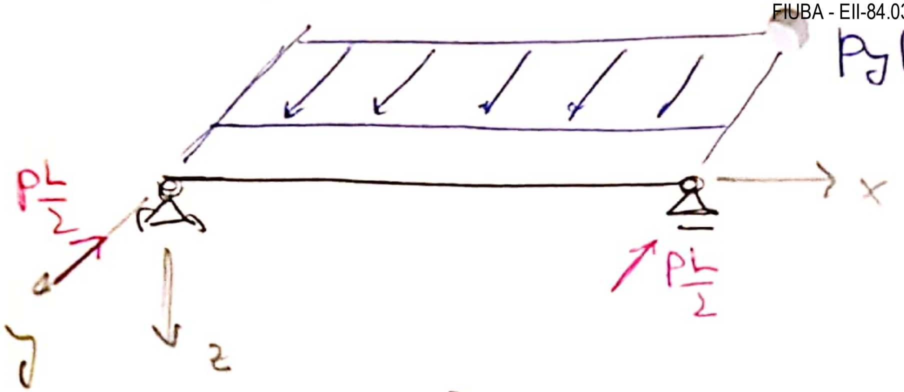
EC. DE LA ELÁSTICA.

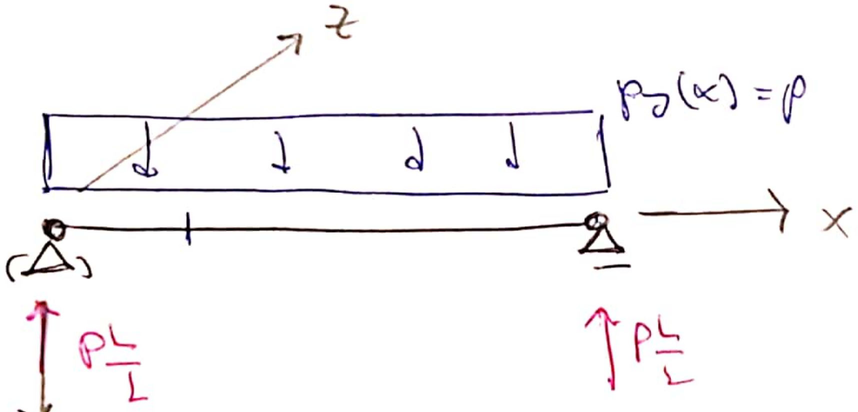
(II) $\theta_y(x) = -\frac{p}{6B_y}x^3 + \frac{5}{16}\frac{p}{B_y}Lx^2 - \frac{1}{8}\frac{p}{B_y}L^2x.$

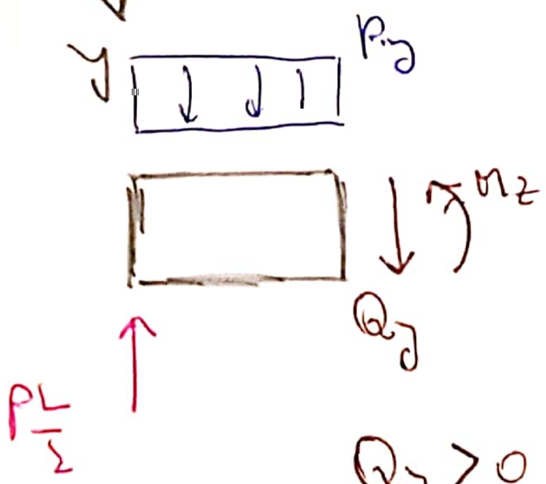
(III) $M_y(x) = -\frac{p}{2}x^2 + \frac{5}{8}pLx - \frac{1}{8}pL^2$

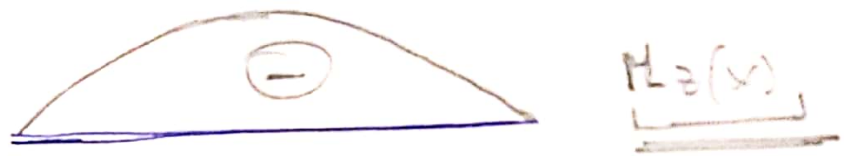
(IV) $Q_z(x) = -px + \frac{5}{8}pL.$

$$\left\{\begin{aligned} p_z(x) &= -\frac{dQ_z(x)}{dx}. \\ Q_z(x) &= \frac{dM_y(x)}{dx} \end{aligned}\right.$$


$p_y(x) = p$.
$\frac{pL}{2}$
$\frac{pL}{2}$
x
y
z


z
$p_y(x) = p$
x
$\frac{pL}{2}$
$\frac{pL}{2}$


y
p_y
M_z
Q_y
$\frac{pL}{2}$


$M_z(x)$

$Q_y > 0$

$M_z < 0$

CÁLCULO DE DESPLAZAMIENTOS POR MEDIO DEL TTV

$$W = U$$

$$U = \int_b \overset{SE}{N} \cdot \overset{DV}{dn} = \int_b \overset{SE}{N} \cdot \underbrace{\frac{\overset{DV}{N}}{EA} \cdot dx}_{dn}$$

ELÁSTICO Y PROPORCIONAL. (Ley de Hooke)

$$U = \sum_{i=1}^{n} \left[\int_{b_i} \overset{SE}{N} \cdot \overset{DV}{dn} + \int_{b_i} \overset{SE}{Q_z} \cdot \overset{DV}{dc_z} + \int_{b_i} \overset{SE}{Q_y} \cdot \overset{DV}{dc_y} + \int_{b_i} \overset{SE}{M_T} \cdot \overset{DV}{d\phi} + \int_{b_i} \overset{SE}{M_y} \, \overset{DV}{d\theta_y} + \int_{b_i} \overset{SE}{M_z} \, \overset{DV}{d\theta_z} \right]$$

- SI LOS SISTEMAS SON ELÁSTICOS Y PROPORCIONALES:

$$\overset{DV}{dn} = \frac{\overset{DV}{N}(x)}{EA} \cdot dx. \qquad \overset{DV}{dc_y} = \frac{K_y \, \overset{DV}{Q_y}(x)}{G \cdot A} \cdot dx. \qquad \overset{DV}{d\theta_z} = \frac{\overset{DV}{M_z}(x)}{E I_z} \cdot dx$$

FACTOR DE FORMA

$$\overset{DV}{dc_z} = \frac{K_z \, \overset{DV}{Q_z}(x)}{G \cdot A} \cdot dx \qquad \overset{DV}{d\theta_y} = \frac{\overset{DV}{M_y}(x)}{E I_y} \cdot dx$$

$d\phi^{DV} = \dfrac{M_T^{DV}(x)}{G\, I_P} \cdot dx$ → SECCIONES CIRCULARES (MACIZAS y HUECAS). T. DE COULOMB.

$d\phi^{DV} = \dfrac{M_T^{DV}(x)}{4 G \Omega^2} \displaystyle\int \dfrac{ds}{e} \cdot dx.$ → SECCIONES SIMPLEMENTE CONEXAS → T. DE BREDT.

$d\phi^{DV} = \dfrac{M_T^{DV}(x)}{4 G \Omega^2} \displaystyle\sum_{j=1}^{m} \dfrac{\Delta s}{e_m} \cdot dx \quad \Bigg\| \quad d\phi = \dfrac{M_T^{DV}(x)}{4 G \Omega^2} \Delta s \sum_{j=1}^{m} \dfrac{1}{e_m} \, dx.$

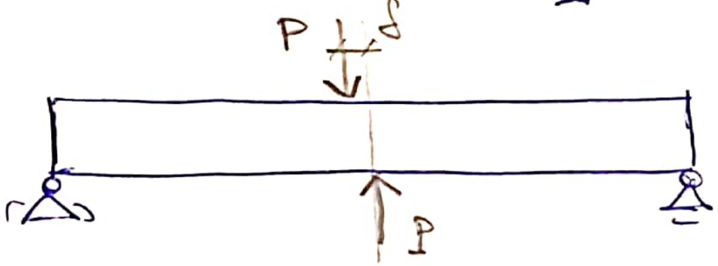
$d\phi^{DV} = \dfrac{M_T^{DV}(x)}{4 G \Omega^2} \cdot \dfrac{s}{e} \cdot dx.$

$d\phi^{DV} = \dfrac{M_T^{DV}(x)}{G\, I_T} \cdot dx$ → SECCIONES ABIERTAS → T. DE SAINT VENANT.

$$\mathcal{U} = \sum_{i=1}^{n} \left[\int_{b_i} \frac{\overset{SE}{N}\,\overset{DV}{N}}{EA}\,dx + \cancel{\int_{b_i} \frac{K_y \overset{SE}{Q_y}\,\overset{DV}{Q_y}}{GA}\,dx} + \cancel{\int_{b_i} \frac{K_z \overset{SE}{Q_z}\,\overset{DV}{Q_z}}{GA}\,dx} + \int_{b_i} \frac{\overset{SE}{M_T}\,\overset{DV}{M_T}}{G I_T}\,dx + \int_{b_i} \frac{\overset{SE}{M_y}\,\overset{DV}{M_y}}{E I_y}\,dx + \right.$$

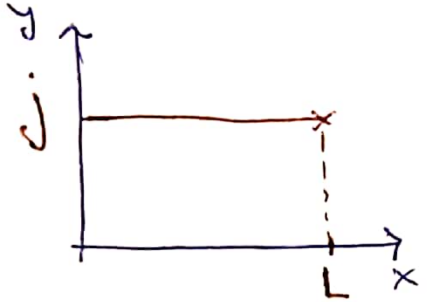
$$\left. + \int_{b_i} \frac{\overset{SE}{M_z}\,\overset{DV}{M_z}}{E I_z}\,dx \right]$$

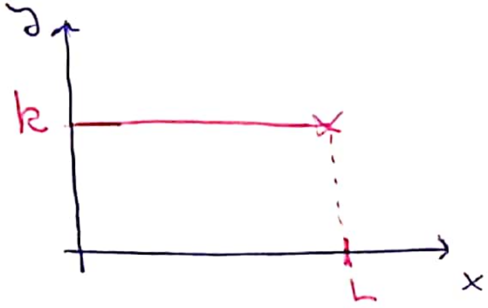
- BARRAS ESBELTAS $\frac{L}{d} \geq 10. \longrightarrow$ LAS DEFORMACIONES POR CORTE SON DESPRECIABLES



CONSTRUCCIÓN DE LAS TABLAS:

1)



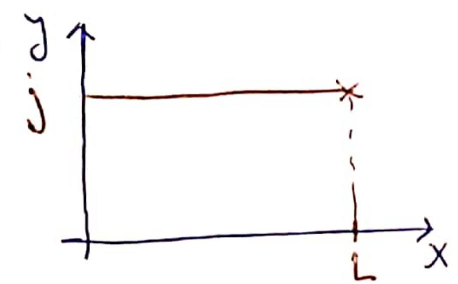


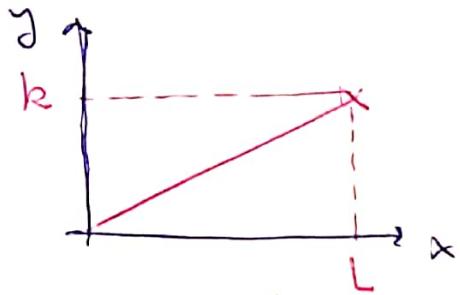
$$\int y_1(x) \cdot y_2(x)\, dx = \int j \cdot k \cdot dx =$$

$$= j \cdot k \cdot x \Big|_0^L = j \cdot k\, L$$

LO PROPORCIONADO POR LAS TABLAS.

2)

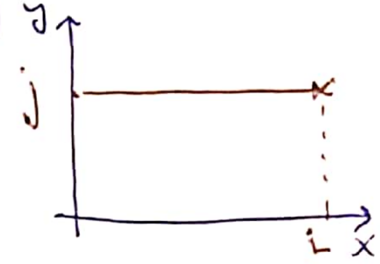


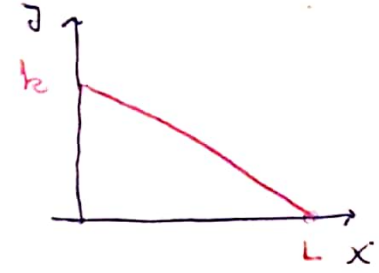


$y_1(x) = j$ $\qquad$ $y_2(x) = \frac{k}{L} x.$

$$\int y_1 \cdot y_2 \cdot dx = \int j \cdot \frac{k}{L} x\, dx = j \frac{k}{L} \cdot \frac{x^2}{2}\Big|_0^L = j \frac{k}{\cancel{L}} \frac{L^{\cancel{2}}}{2} = \frac{1}{2} j k\, L$$

3)





$y_1(x) = j$ $\qquad$ $y_2(x) = -\frac{k}{L} x + k$

$$y_2(x) = k\left(1 - \frac{x}{L}\right).$$

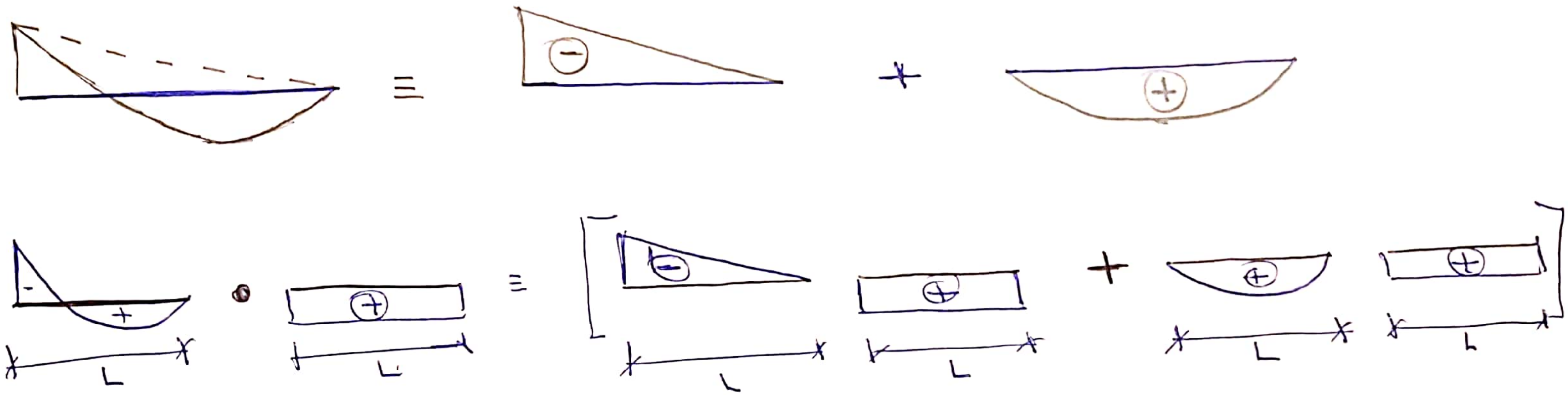
$$\int y_1 \cdot y_2\, dx = \int j \cdot k \left(1 - \frac{x}{L}\right) dx =$$

$$= jk \cdot x \Big|_0^L - jk \frac{1}{2} \frac{x^2}{L}\Big|_0^L =$$

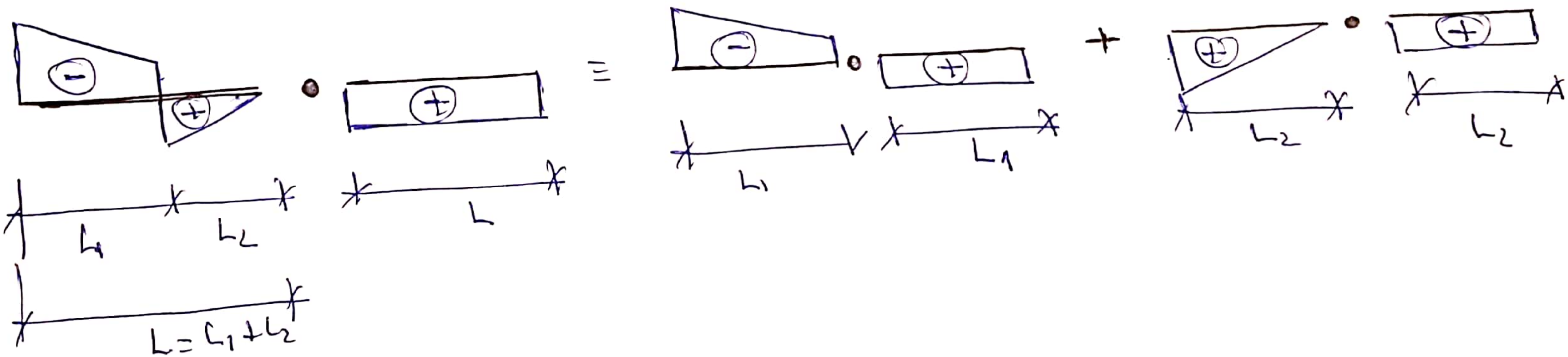
$$= jkL - \frac{1}{2} jkL = \boxed{\frac{1}{2} jk\, L}$$

ACLARACIONES:

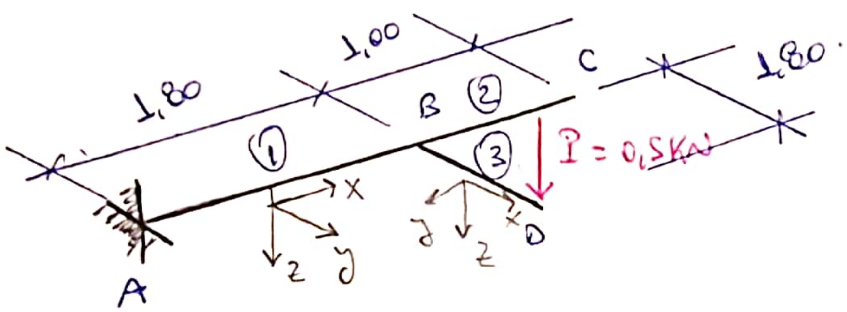
I)



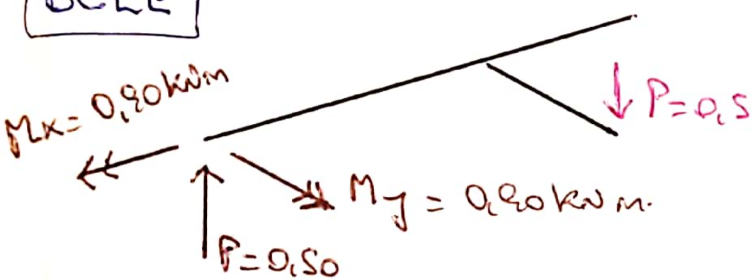
II)



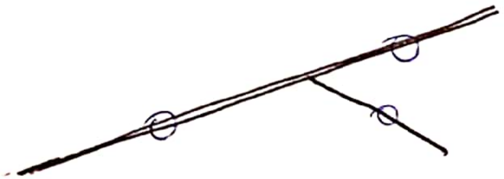
DV



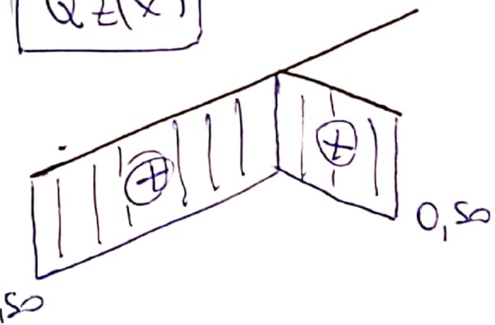
DCLE



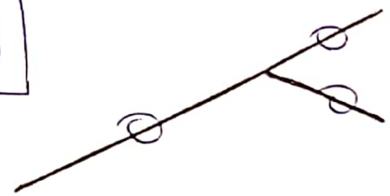
N(x)



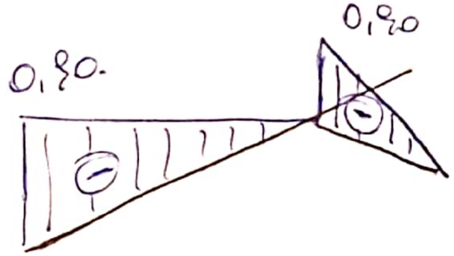
Qz(x)



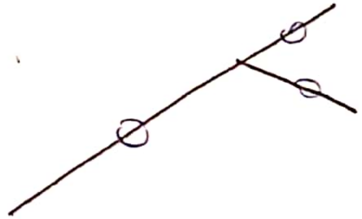
Qy(x)

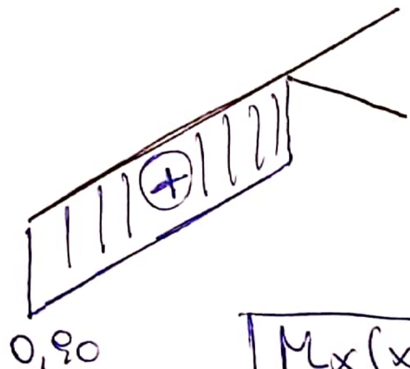


My(x)



Mz(x)





Mx(x)

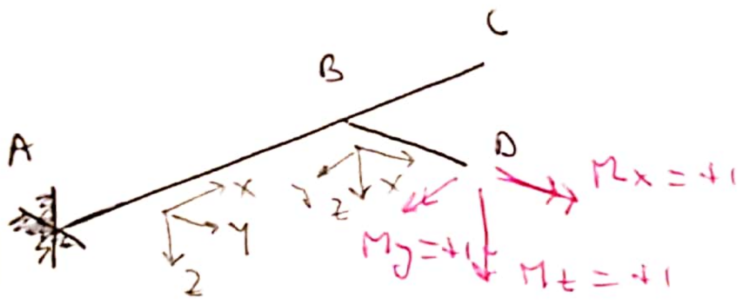
SE θ_y^D

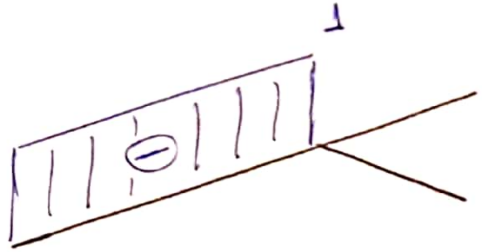
θ^D

θ_x^D

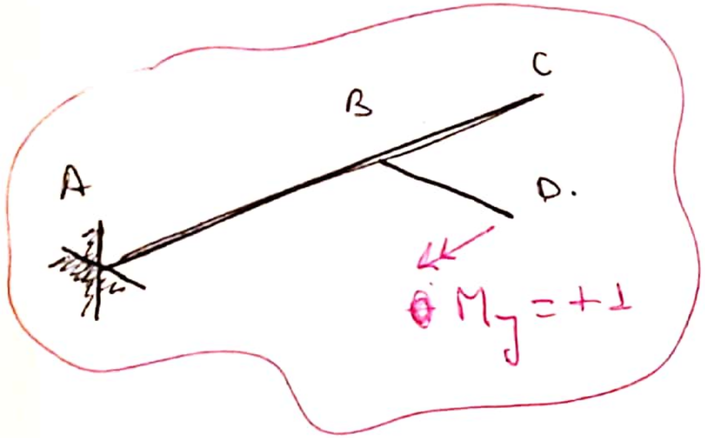
θ_y^D

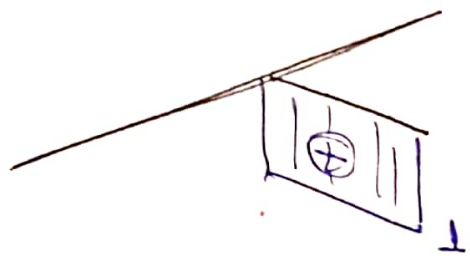
θ_z^D



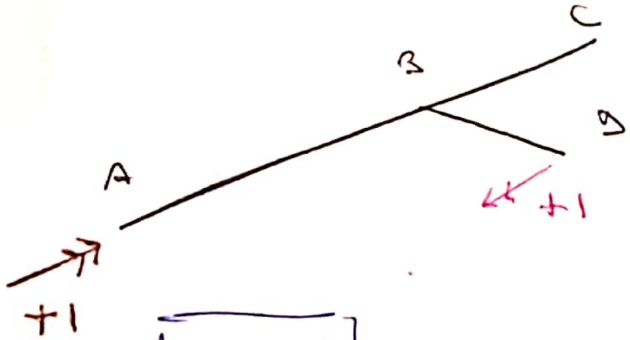


$\boxed{M_x(x)}$





$\boxed{M_y(x)}$



$\boxed{\text{DELE}}$

$N(x) = 0$

$Q_y(x) = 0$

$Q_z(x) = 0$

$M_z(x) = 0$

$E = 21.000\ KN/cm^2.$

$G = 8000\ KN/cm^2.$

$\cancel{A} = D = 7\,cm.$

$A = \frac{\pi D^2}{4} = 38,48\,cm^2.$

$I_y = I_z = \frac{\pi . D^4}{64} = 117,86\,cm^4$

$I_P = \frac{\pi D^4}{32} = 235,72\,cm^4$

$W = (+1)^{SE} . \theta_y^{D\ DV}$?

$N \begin{cases} SE \to 0 \\ DV \to 0. \end{cases}$

Q_y y Q_z → se desprecian.

$M_z \begin{cases} SE \to 0 \\ DV \to 0 \end{cases}$

$$\underbrace{(+1).\theta_y^D}_{W} = \underbrace{\int_{b1} \frac{(+90).(-1)}{8000 . 235,72}\, dx}_{M_x^{b1}} + \cancel{\underbrace{\int_{b2} \frac{0.0}{G.I_p}\, dx}_{M_x^{b2}}} +$$

$$+ \cancel{\underbrace{\int_{b3} \frac{0.0}{G.I_p} . dx}_{M_x^{b2}}} + \cancel{\underbrace{\int_{b1} \frac{(-90).(0)}{21000 . 117,86}\, dx}_{M_y^{b1}}} + \cancel{\underbrace{\int_{b2} \frac{0\ 0}{E I_y} . dx}_{M_y^{b2}}} +$$

$$+ \int_{b3} \frac{1}{2} . \frac{(-90)(+1)}{21000 . 117,86} . dx. =$$

$$= \frac{90\,KNcm . (-1)\, 180\,cm}{8000\,\frac{KN}{cm^2} . 235,72\,cm^4} + \frac{1}{2} \frac{(-90).(+1)\, 180}{21000 . 117,86} =$$

$$= \underbrace{-8,58 . 10^{-3}} - 3,27 . 10^{-3} = \boxed{-0,01186\,rad}$$

$$\boxed{\theta_y^D = -0,01186\,rad}$$