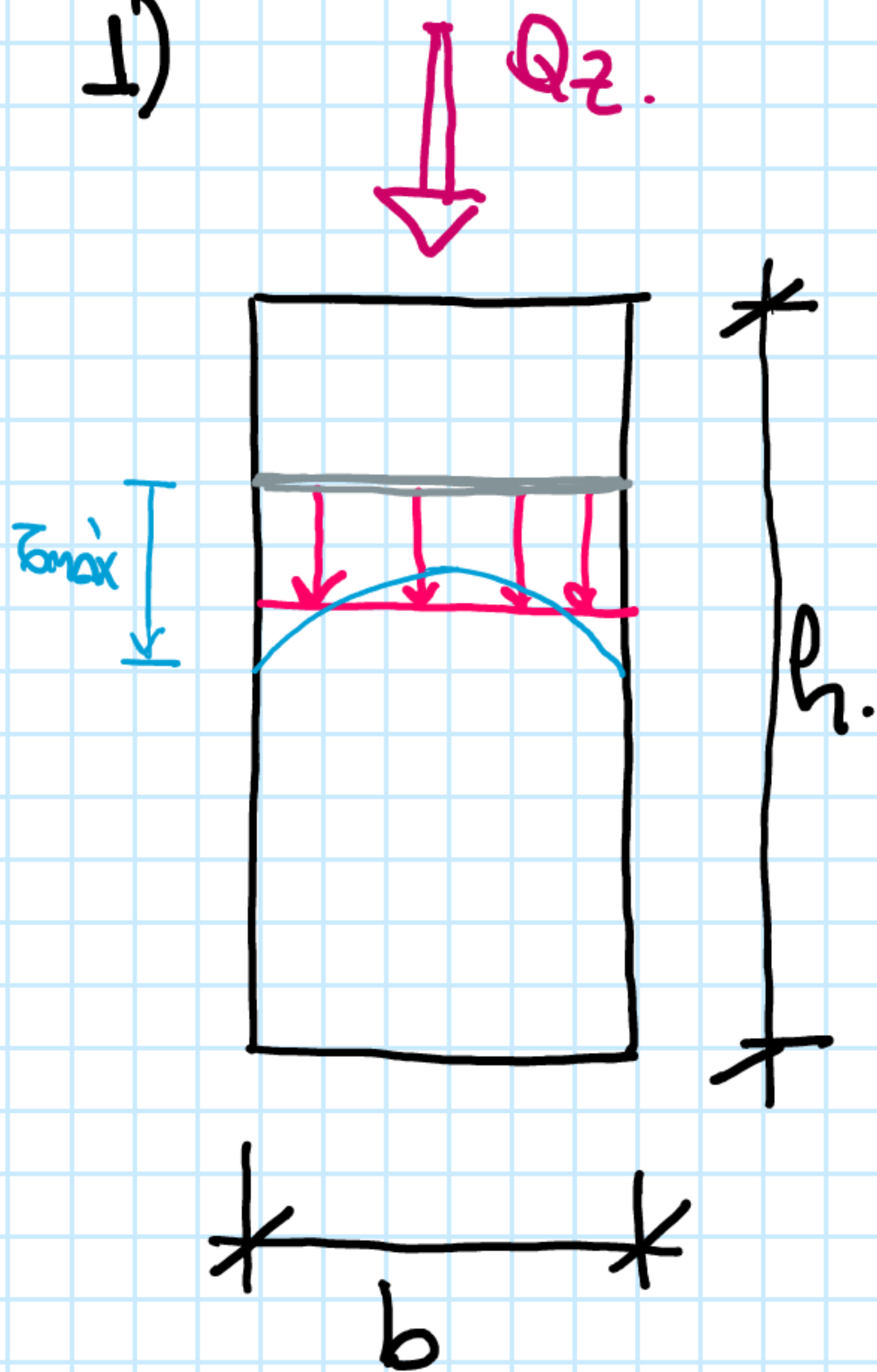


07 - LIMITACIONES AL USO DE LA FÓRMULA DE JOURAVSKY-COLLIGNON

martes, 8 de junio de 2021 09:26

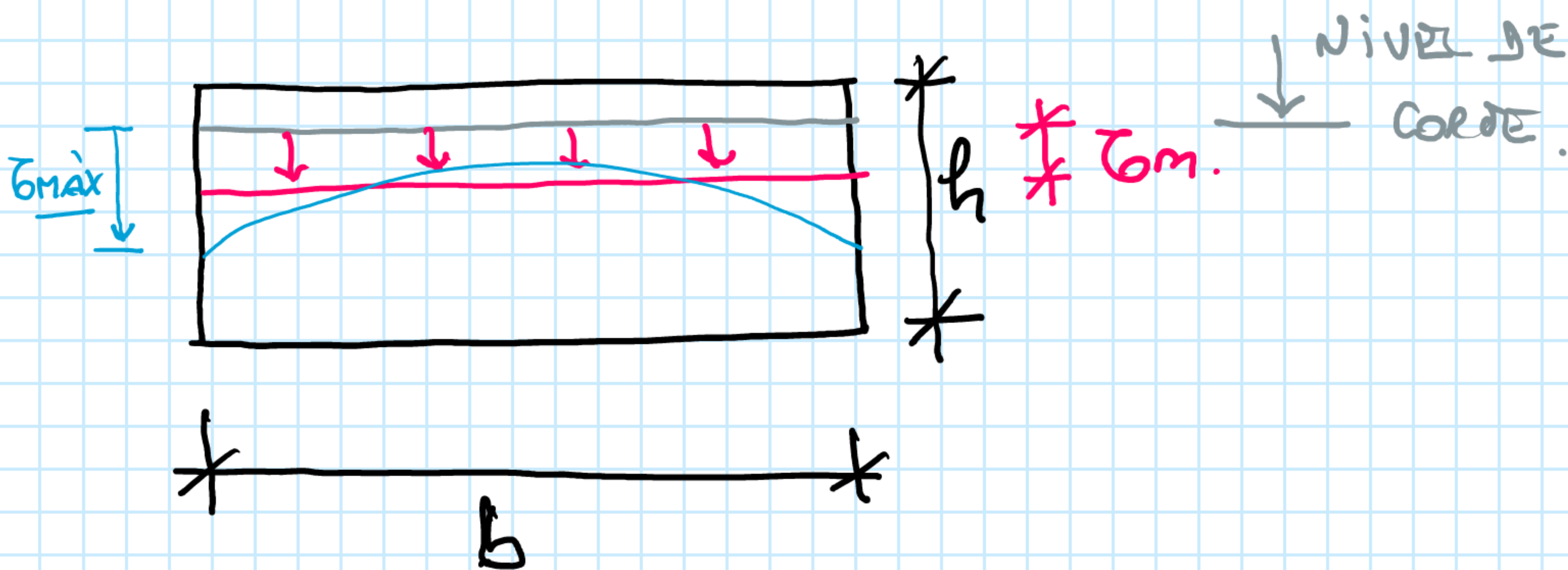
1)



NIVEL DE CORTE.

τ_m ES CONSTANTE EN TODO EL ANCHO DE CORTE.

• si $b = 0,5h \rightarrow \tau_{max} \approx 1,03 \tau_m$.



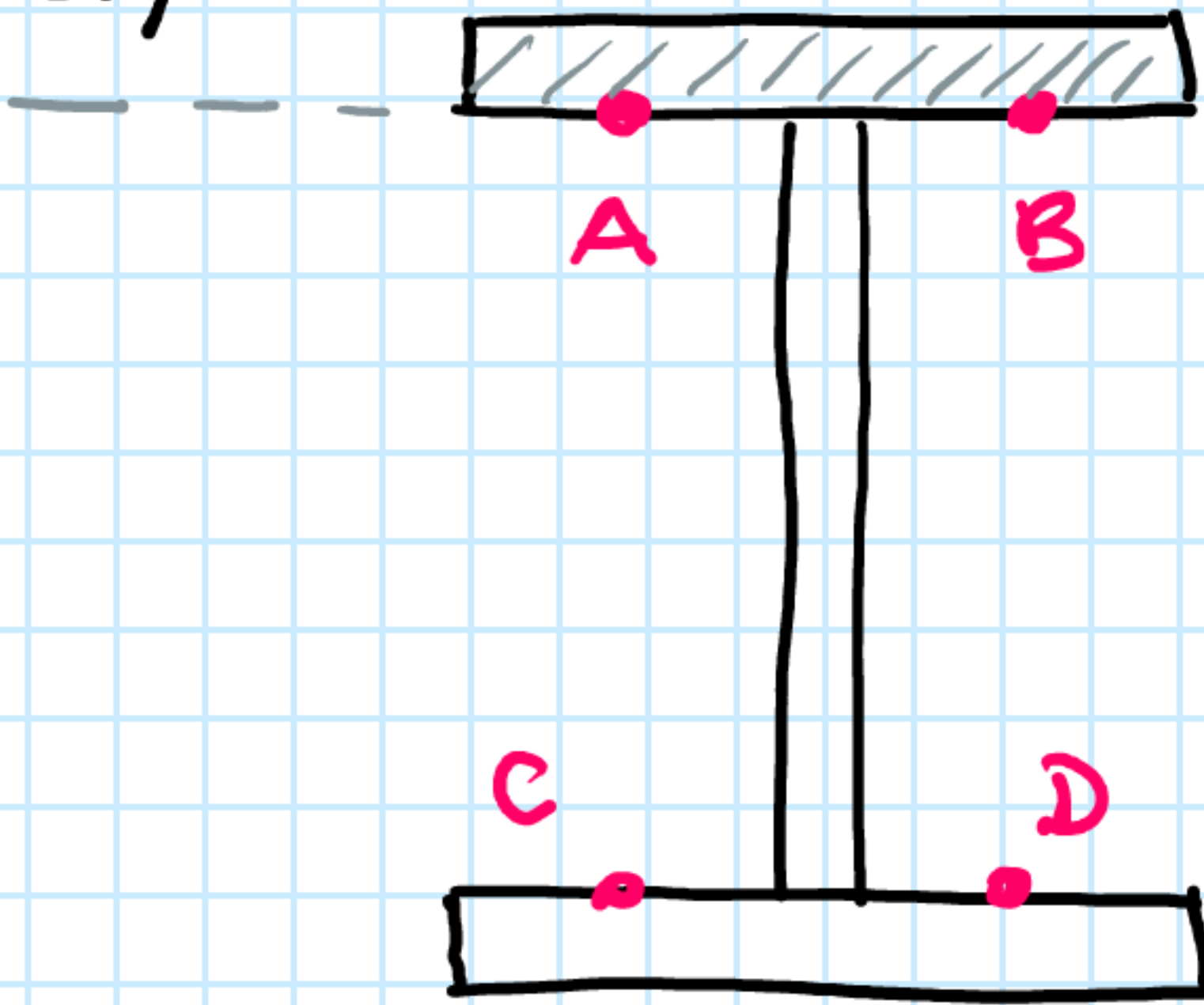
• si $b = 2h \rightarrow \tau_{max} \approx 1,40 \tau_m$

$\frac{b}{h} \uparrow \rightarrow \frac{\tau_{max}}{\tau_m} \uparrow$

07 - LIMITACIONES

martes, 8 de junio de 2021 09:39

2)

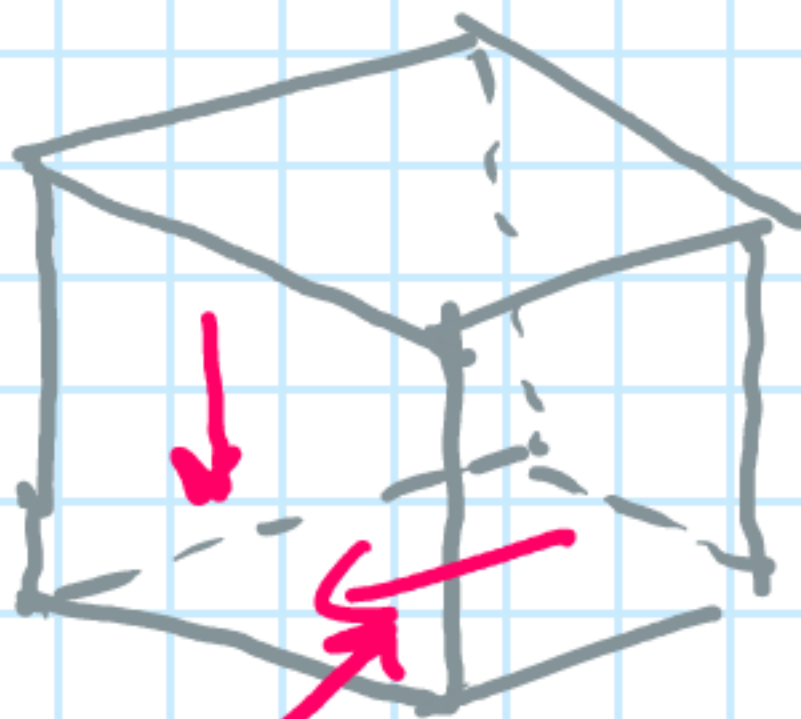


Q_z S^*
 I_y h

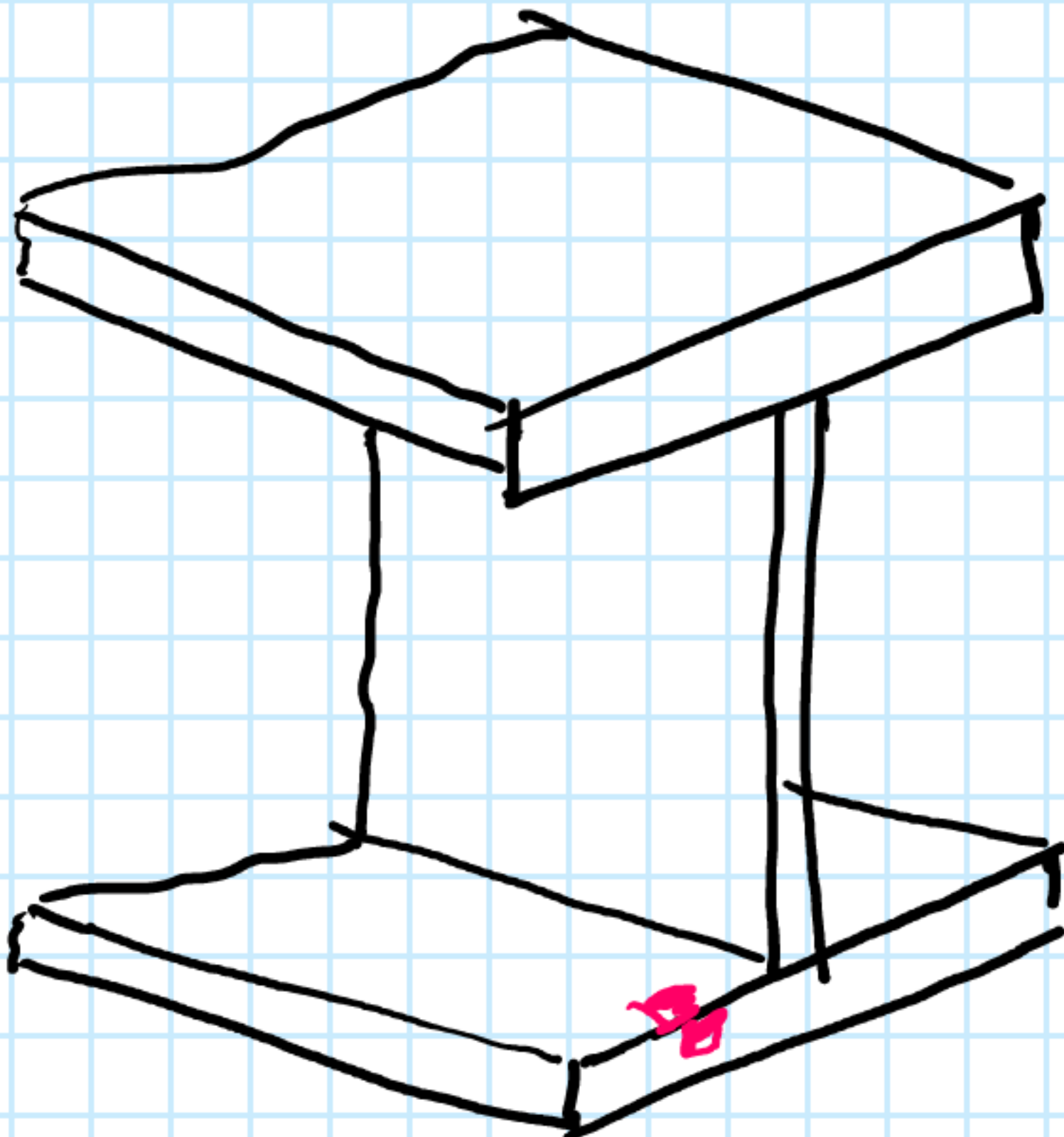
SEGUN STURANSKY

$\tau_A \neq 0$ $\tau_B \neq 0$ $\tau_C \neq 0$
 $\tau_D \neq 0$

A



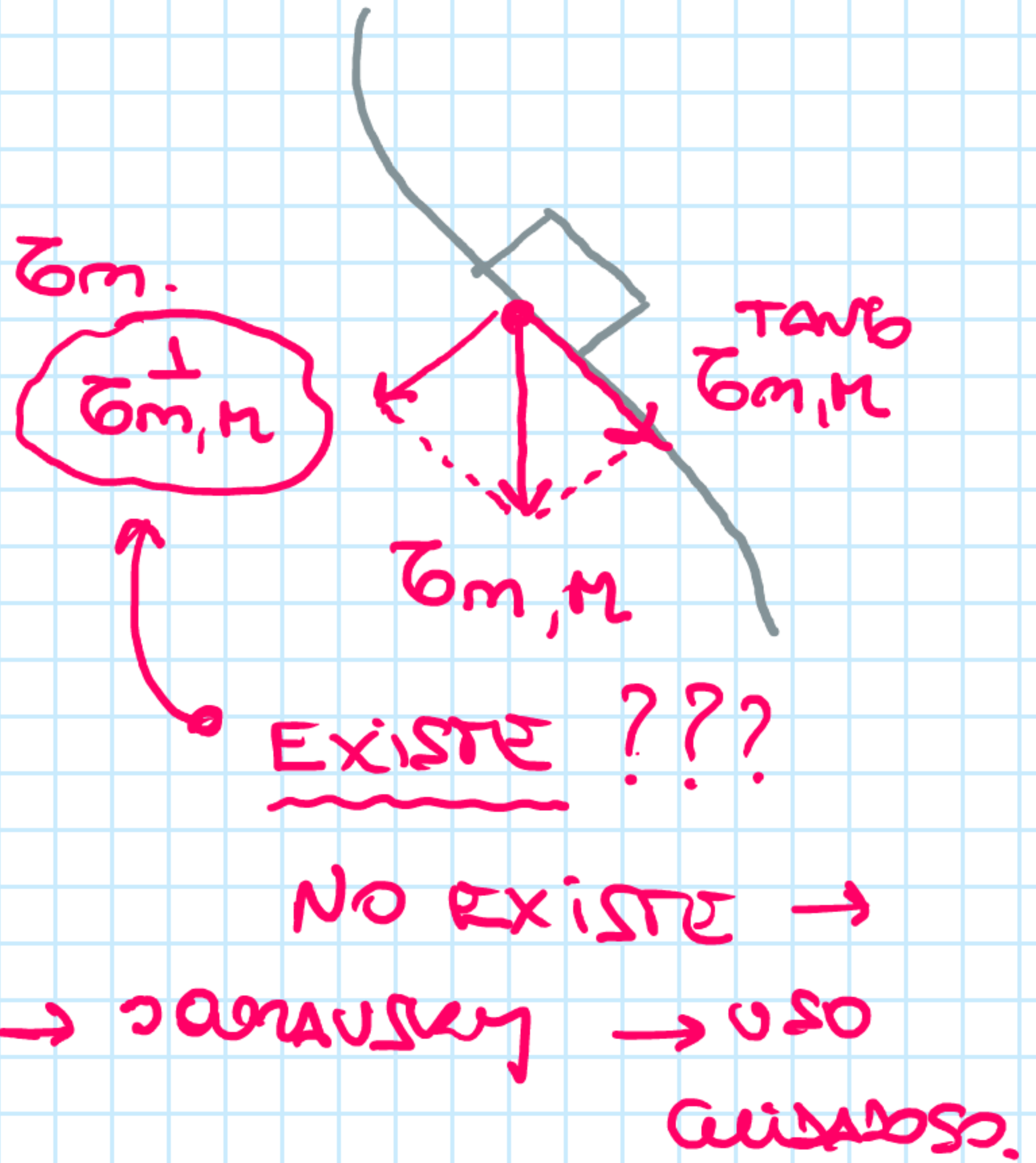
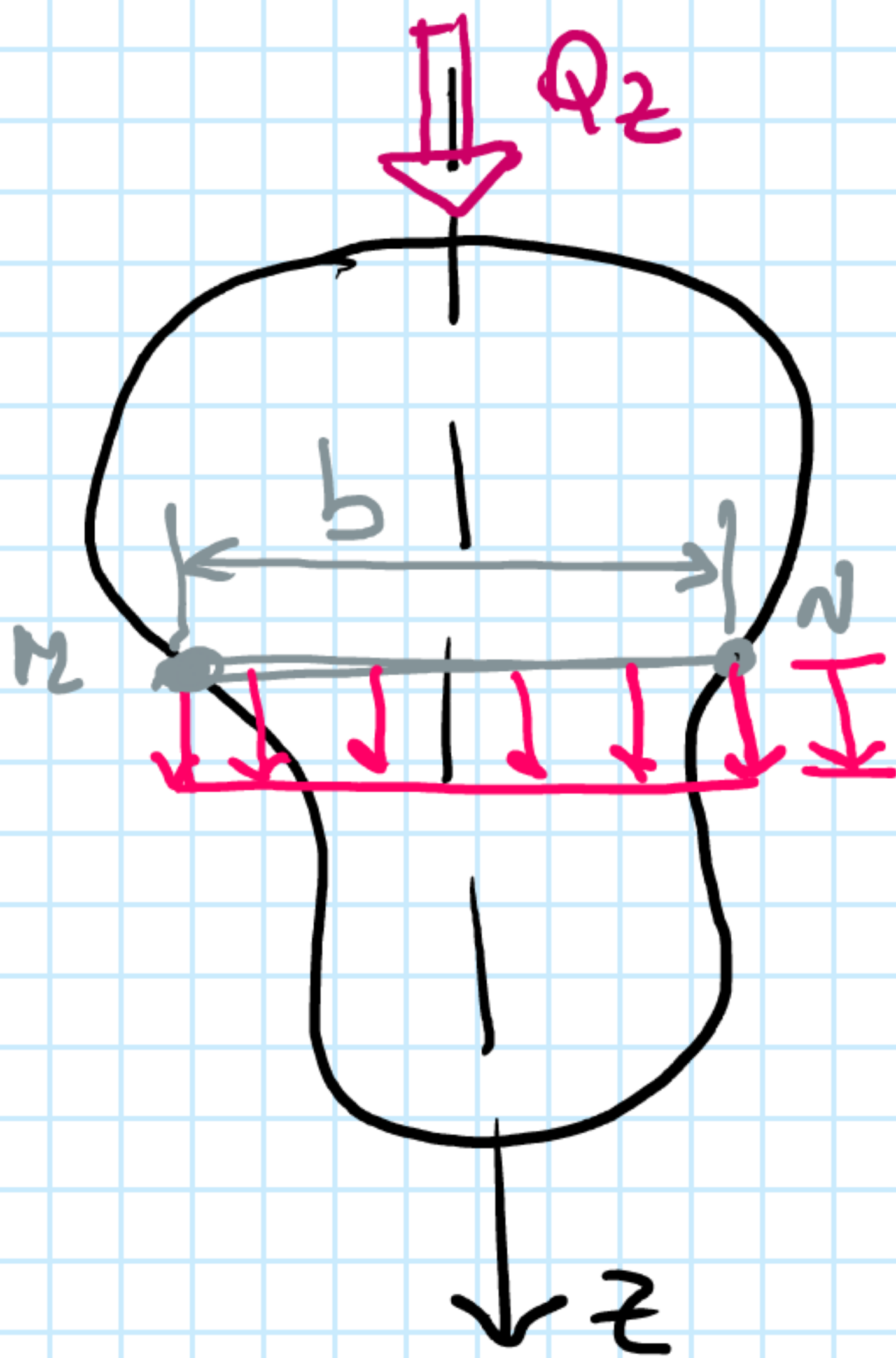
NO EXISTE LA τ QUE SE VE EN LA CARA INFERIOR.



07 - LIMITACIONES

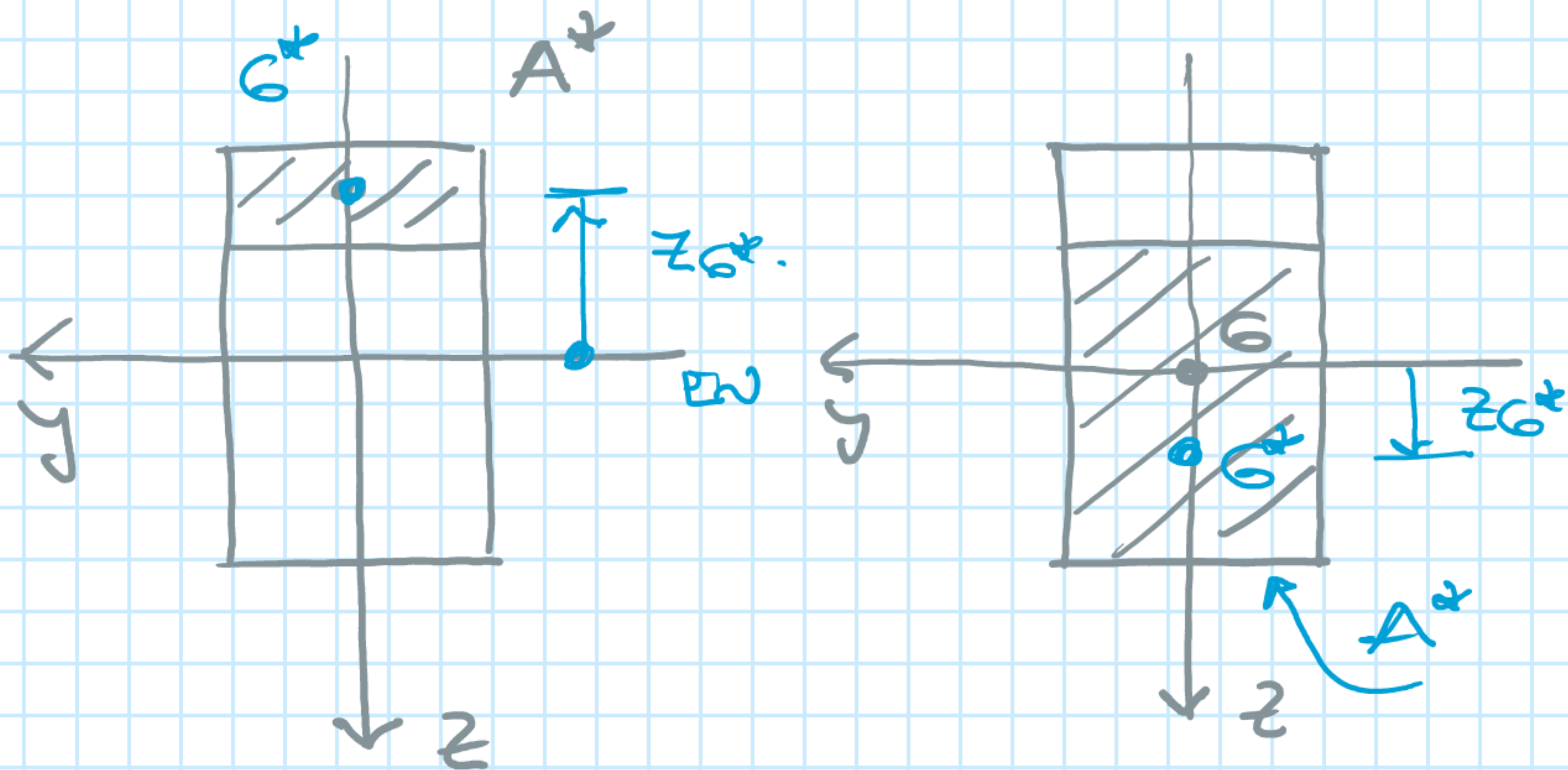
martes, 8 de junio de 2021 09:45

3)



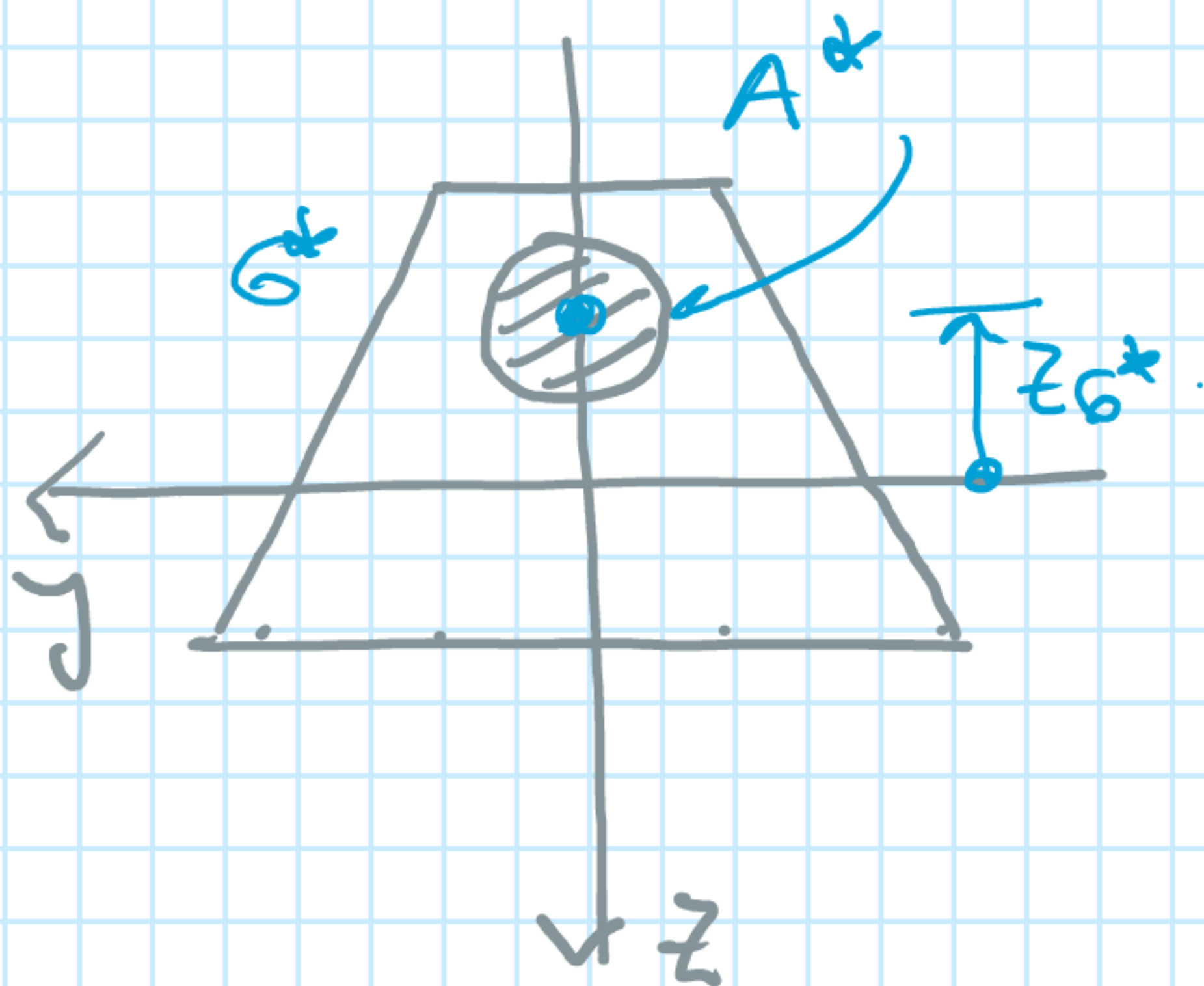
08 - CONSIDERACIONES PARA EL CÁLCULO DEL MOMENTO ESTÁTICO

martes, 8 de junio de 2021 09:50



$$S_{A^*} = A^* \cdot z_{G^*}$$

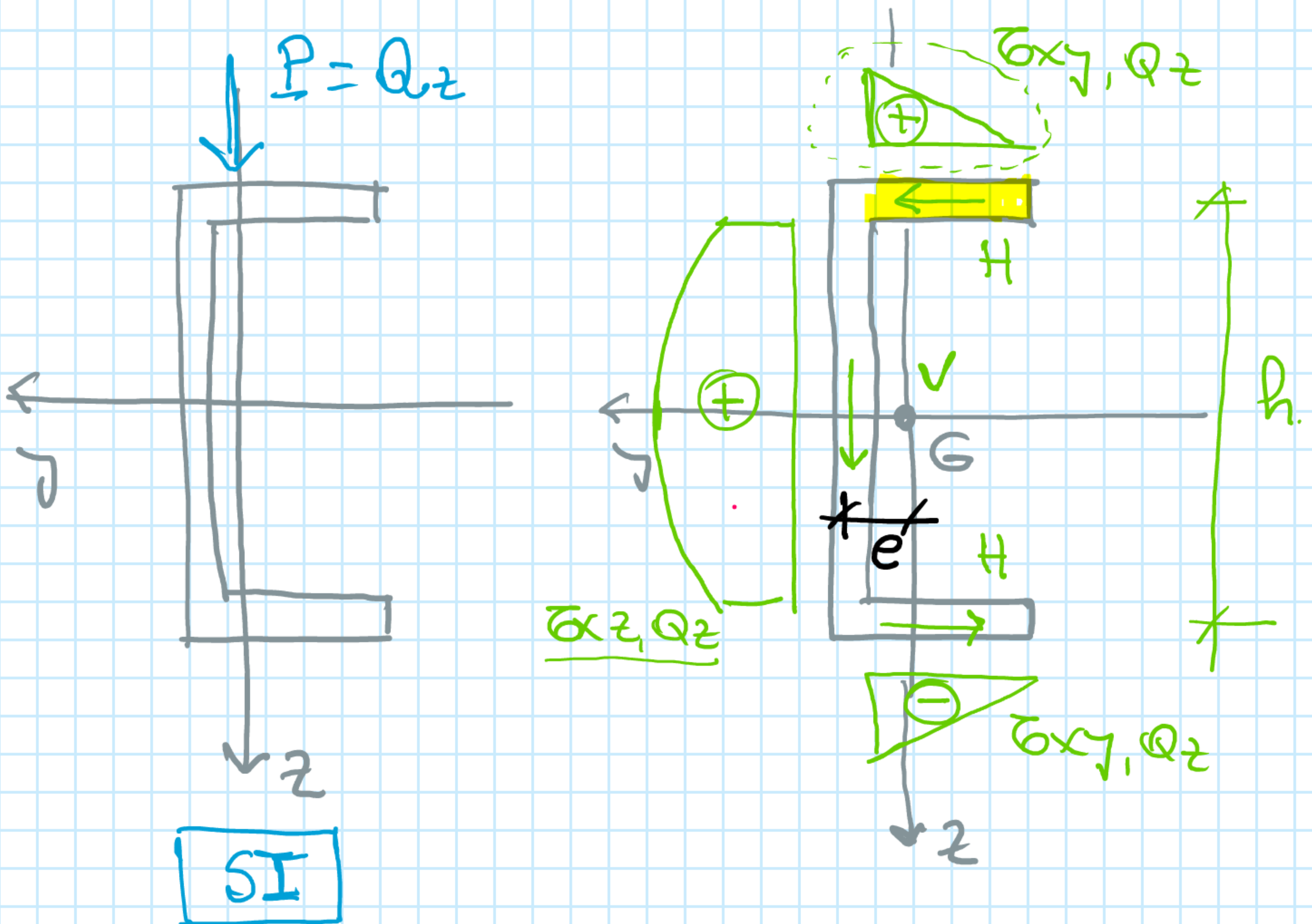
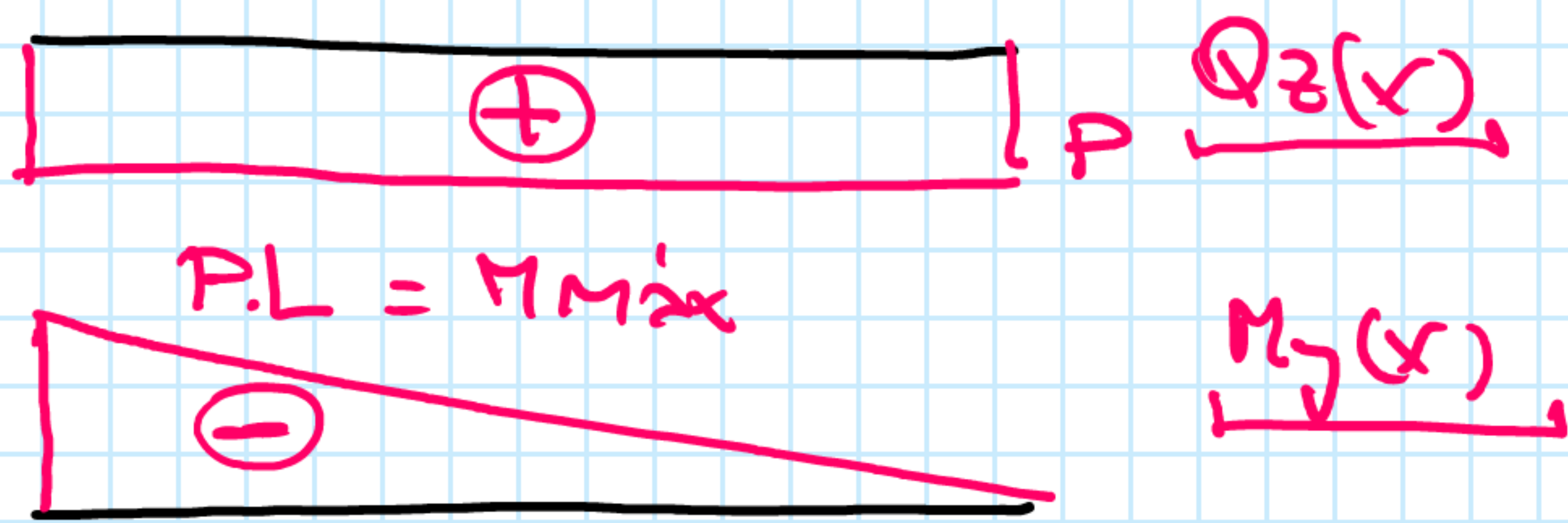
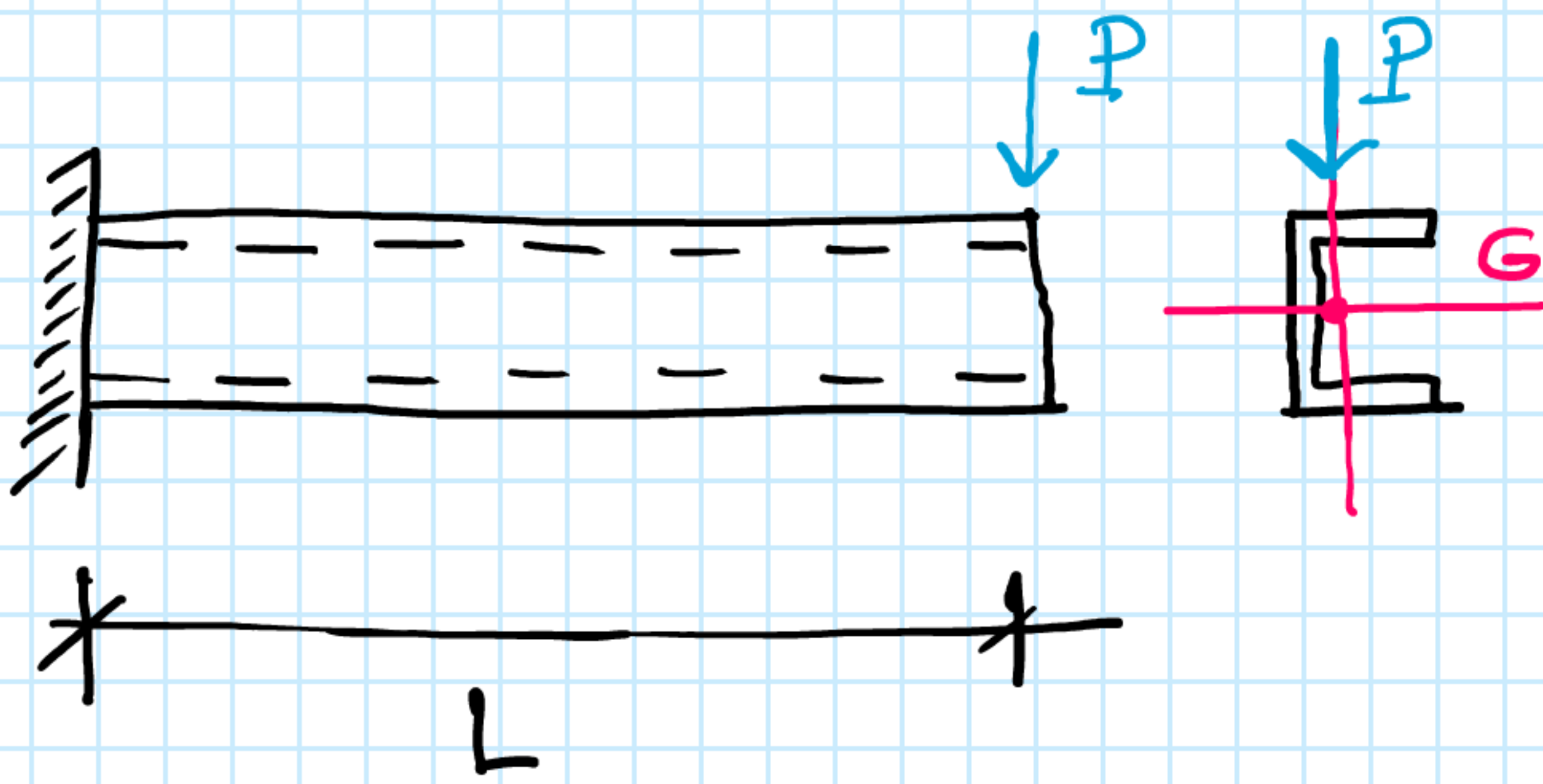
$$S_{A^*} = A^* \cdot z_{G^*}$$



$$S_{A^*} = A^* \cdot z_{G^*}$$

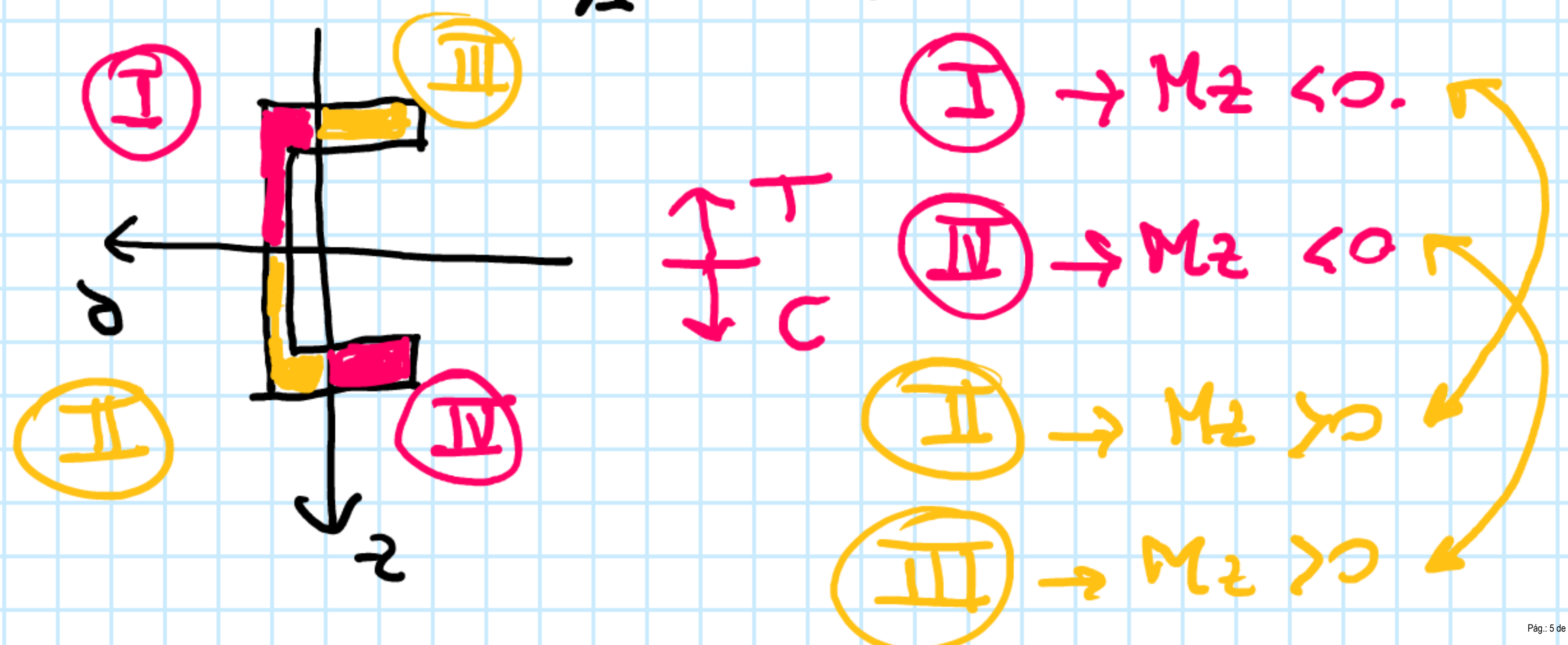
09 - CENTRO DE CORTE:

martes, 8 de junio de 2021 09:55



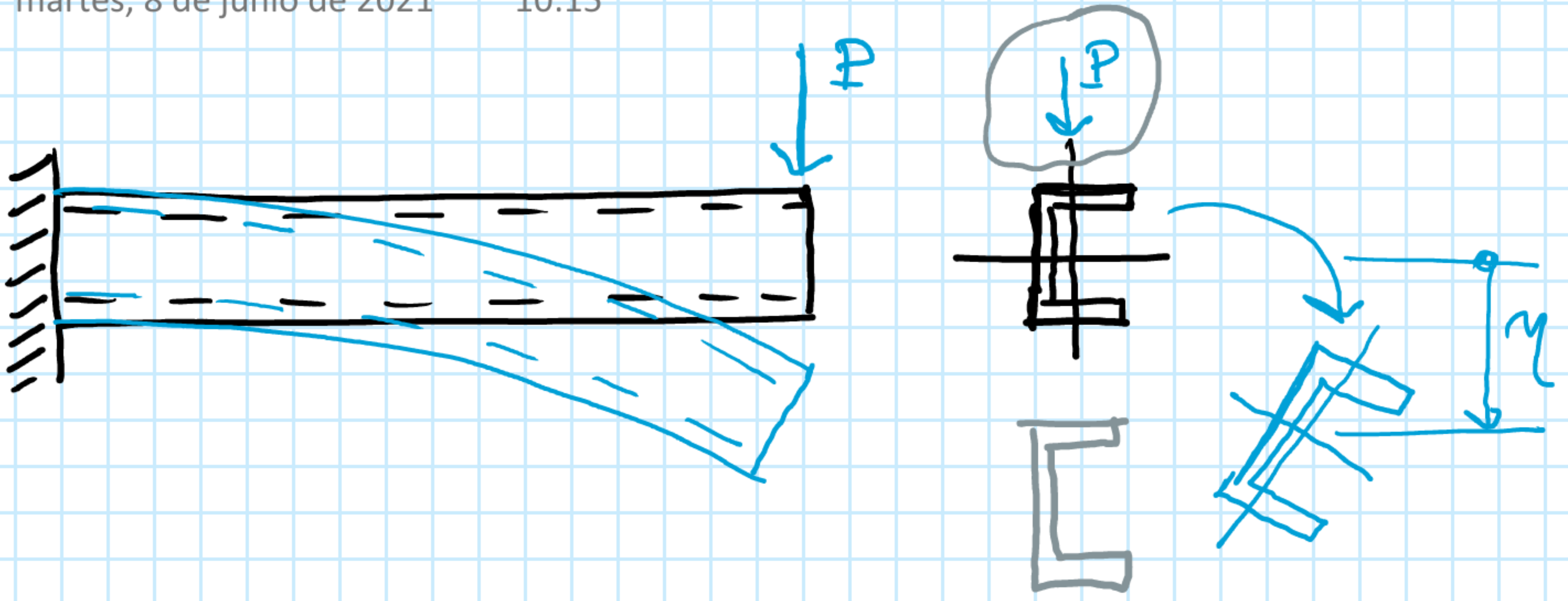
ECS. DE EQUIVALENCIA:

- ① $N = 0 = \int_A \sigma_x dA.$
 $0 = T - C \quad \checkmark$
- ② $Q_y = 0 = \int_A \tau_{xy} \cdot dA$
 $0 = H - H \quad \checkmark$
- ③ $Q_z = \int_A \tau_{xz} dA.$
 $Q_z = V \quad \checkmark$
- ④ $M_x = 0 = \int_A (-\tau_{xy} \cdot z + \tau_{xz} \cdot y) dA.$
 $0 \neq \underline{Hh} + \underline{V \cdot e} \quad \checkmark$ X
- ⑤ $M_y = \int_A \sigma_x \cdot z \cdot dA.$
 $M_y = T \cdot z = C \cdot z \quad \checkmark$
- ⑥ $M_z = 0 = \int_A -\sigma_x \cdot y \cdot dA. \quad \checkmark$



09 - CENTRO DE CORTE:

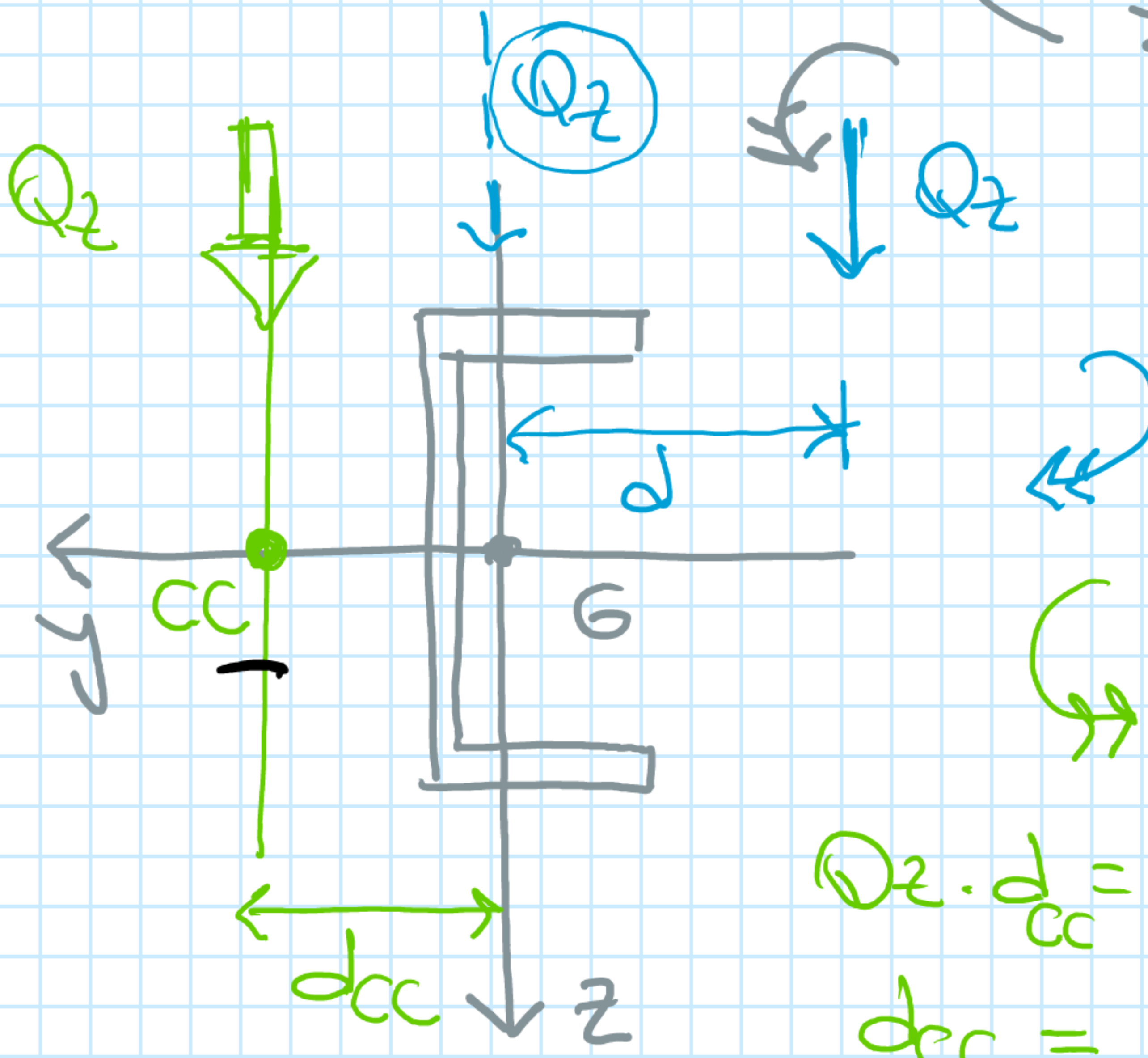
martes, 8 de junio de 2021 10:15



④ $M_x = 0 \neq H \cdot h + V \cdot e$

SI TENSIONES

JOHNSONSKY CORTE.



$$Q_z \cdot d_{cc} = H \cdot h + V \cdot e$$

$$d_{cc} = \frac{H \cdot h + V \cdot e}{Q_z}$$

$$Q_z \cdot d_{cc} = H \cdot h + V \cdot e$$

H y $V \rightarrow$ A PARTIR DE Q_z $f(Q_z)$.

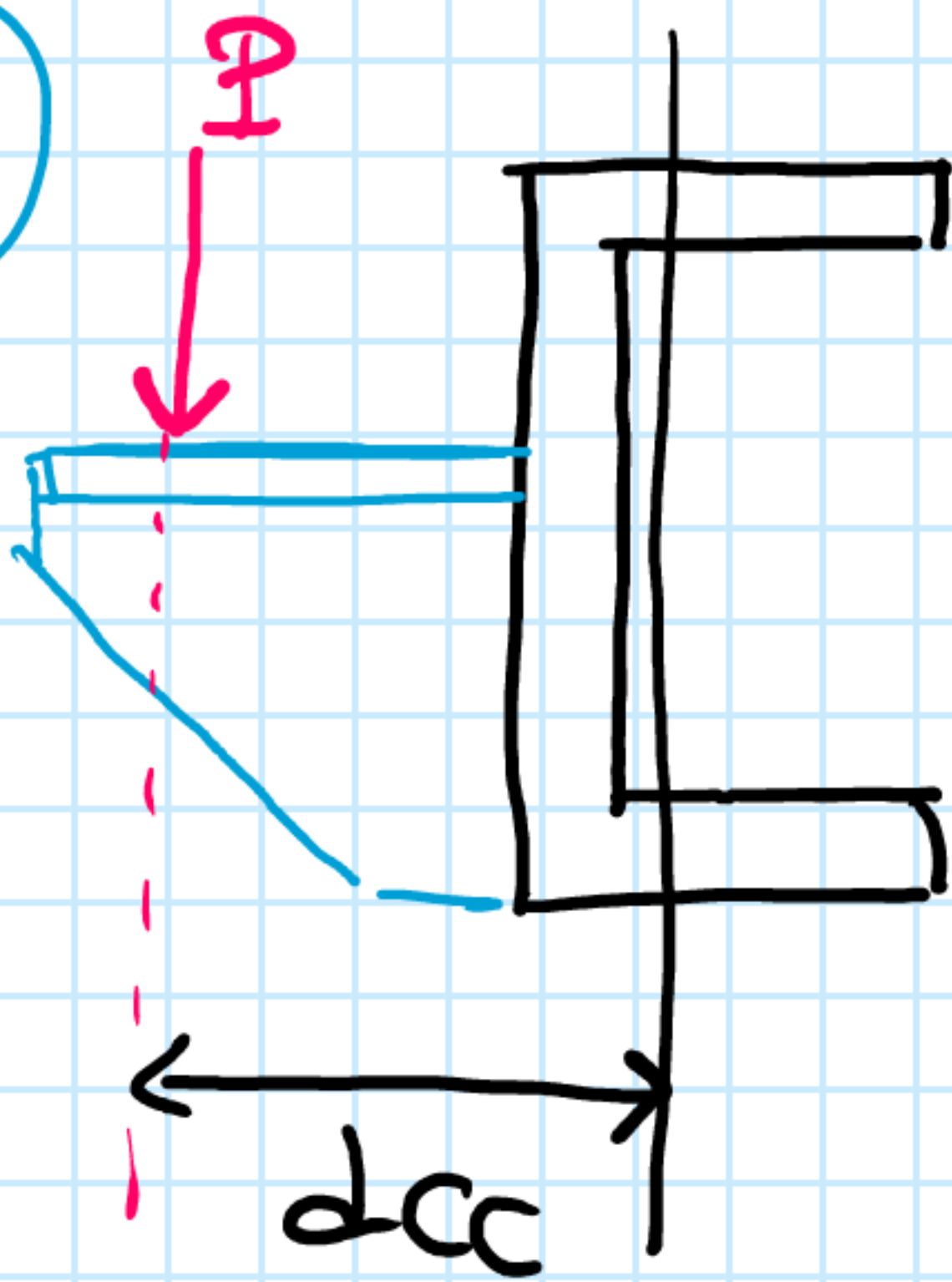
$d_{cc} =$ NO DEPENDE DEL ESTRANSO DE CORTE \rightarrow DEPENDE DE LAS CARACTERÍSTICAS GEOMÉTRICAS DE LA SECCIÓN.

09 - CENTRO DE CORTE:

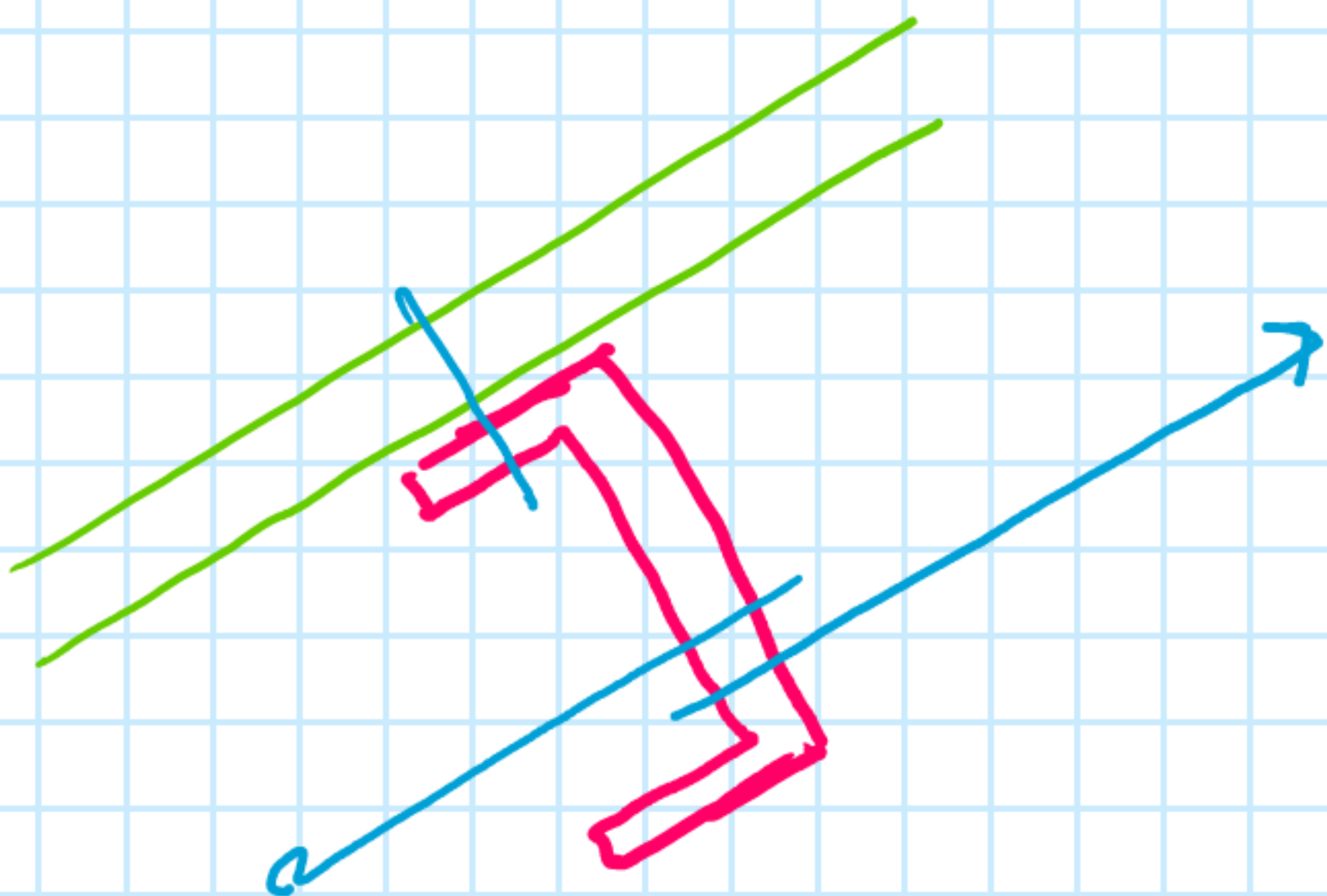
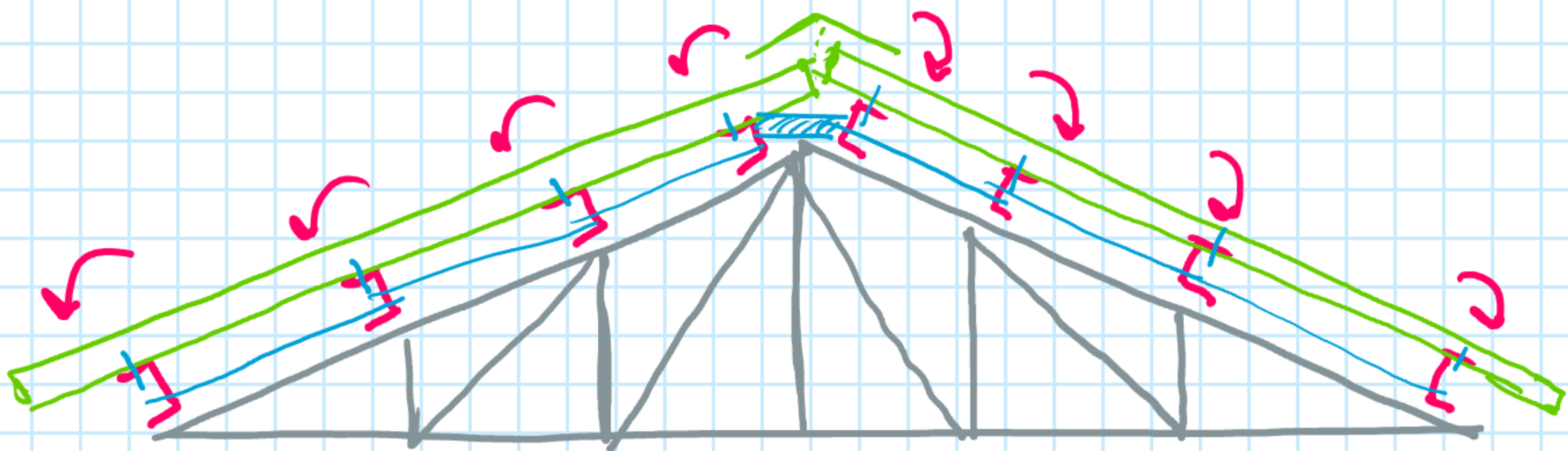
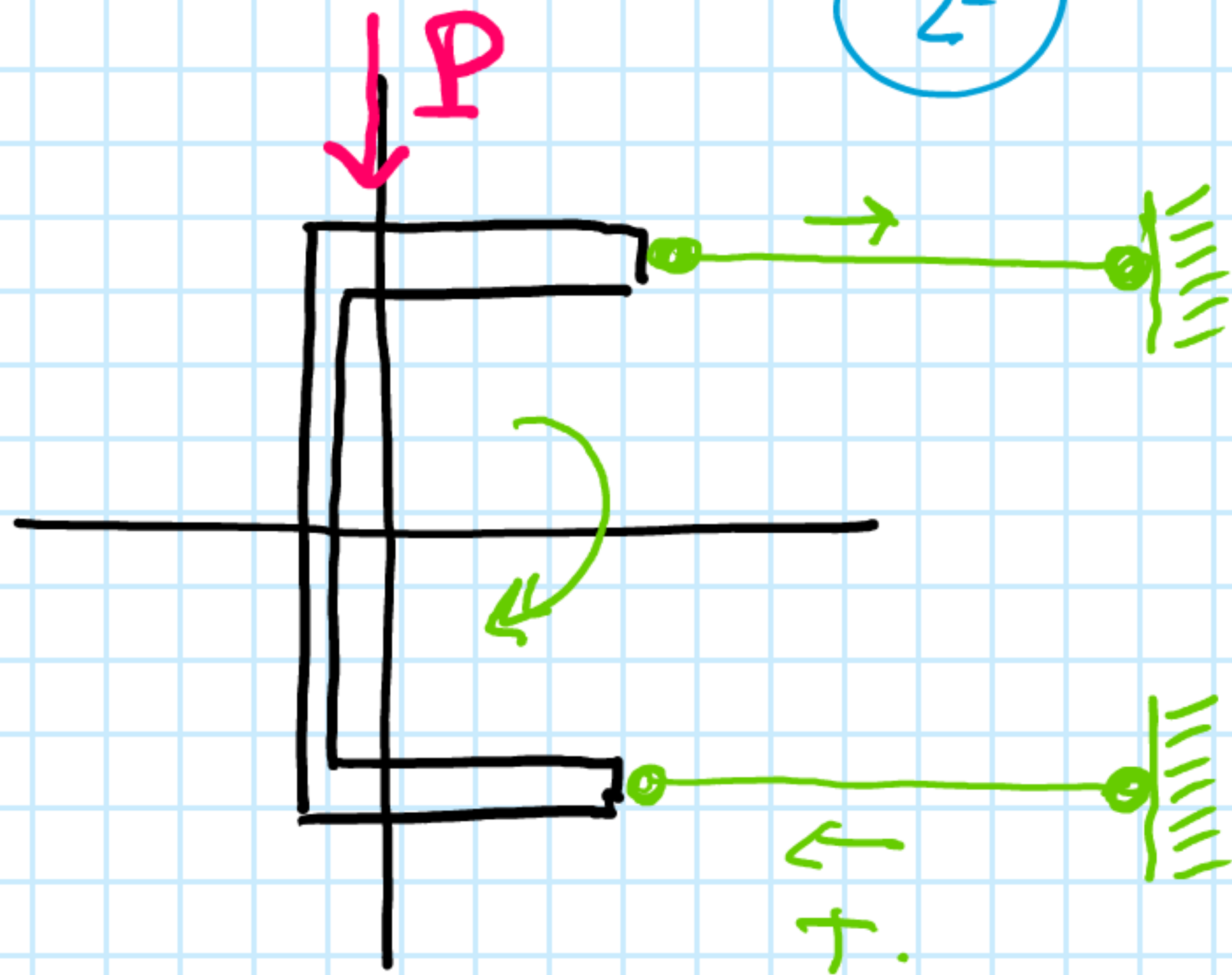
martes, 8 de junio de 2021

10:28

1°

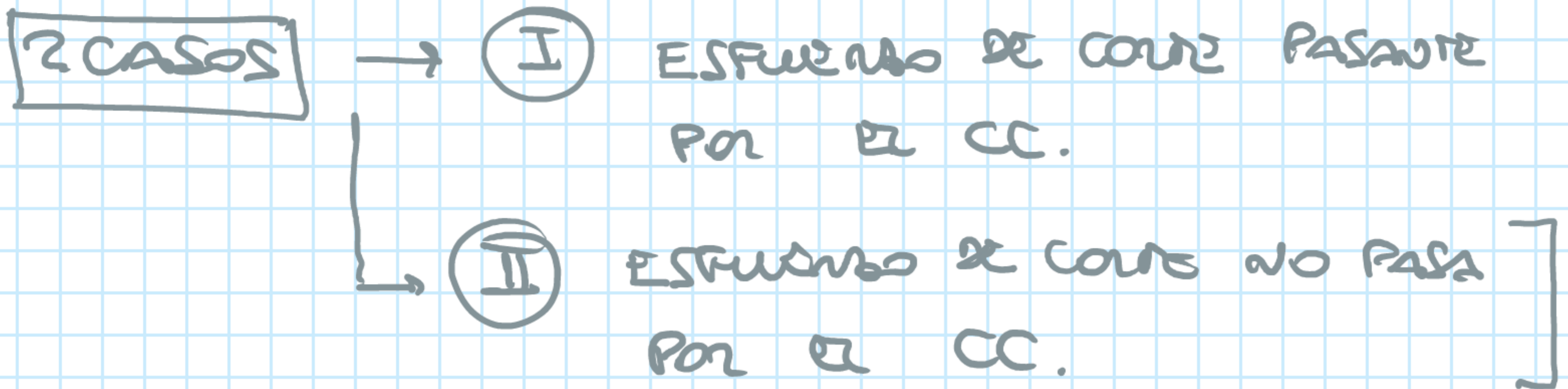


2°

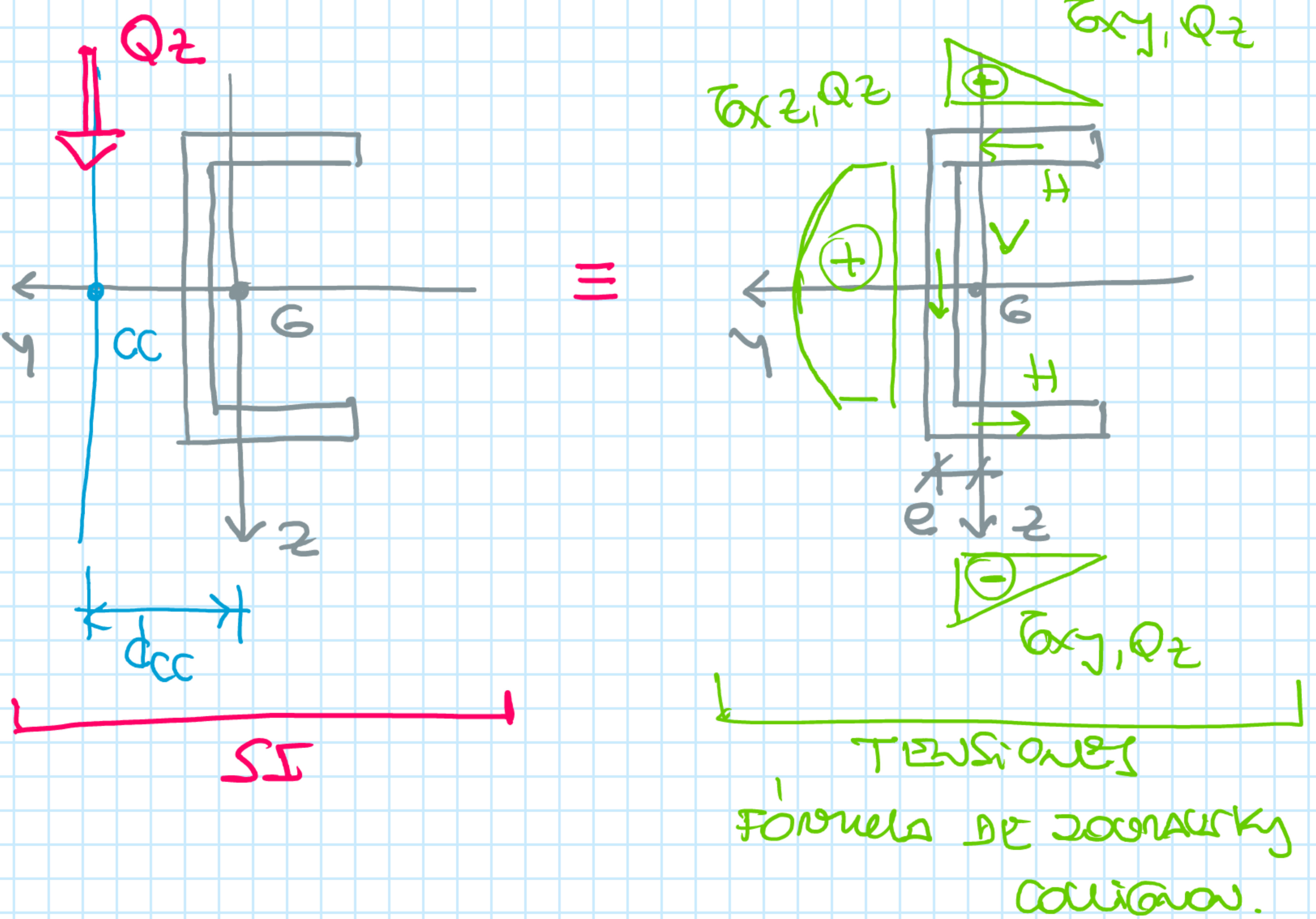


09 - CENTRO DE CORTE:

martes, 8 de junio de 2021 10:37



CASO Ⓘ:



• LA BARRA SE FLEXIONA PERO NO SE TORSIONA.

• LAS TENSIONES SE CALCULAN:

$$\sigma_x = \frac{M_y \cdot z}{I_y} ; \quad \tau = \frac{Q \cdot s^*}{I_y \cdot b^*}$$

I.1 MOMENTOS RESPECTO DEL 'G':

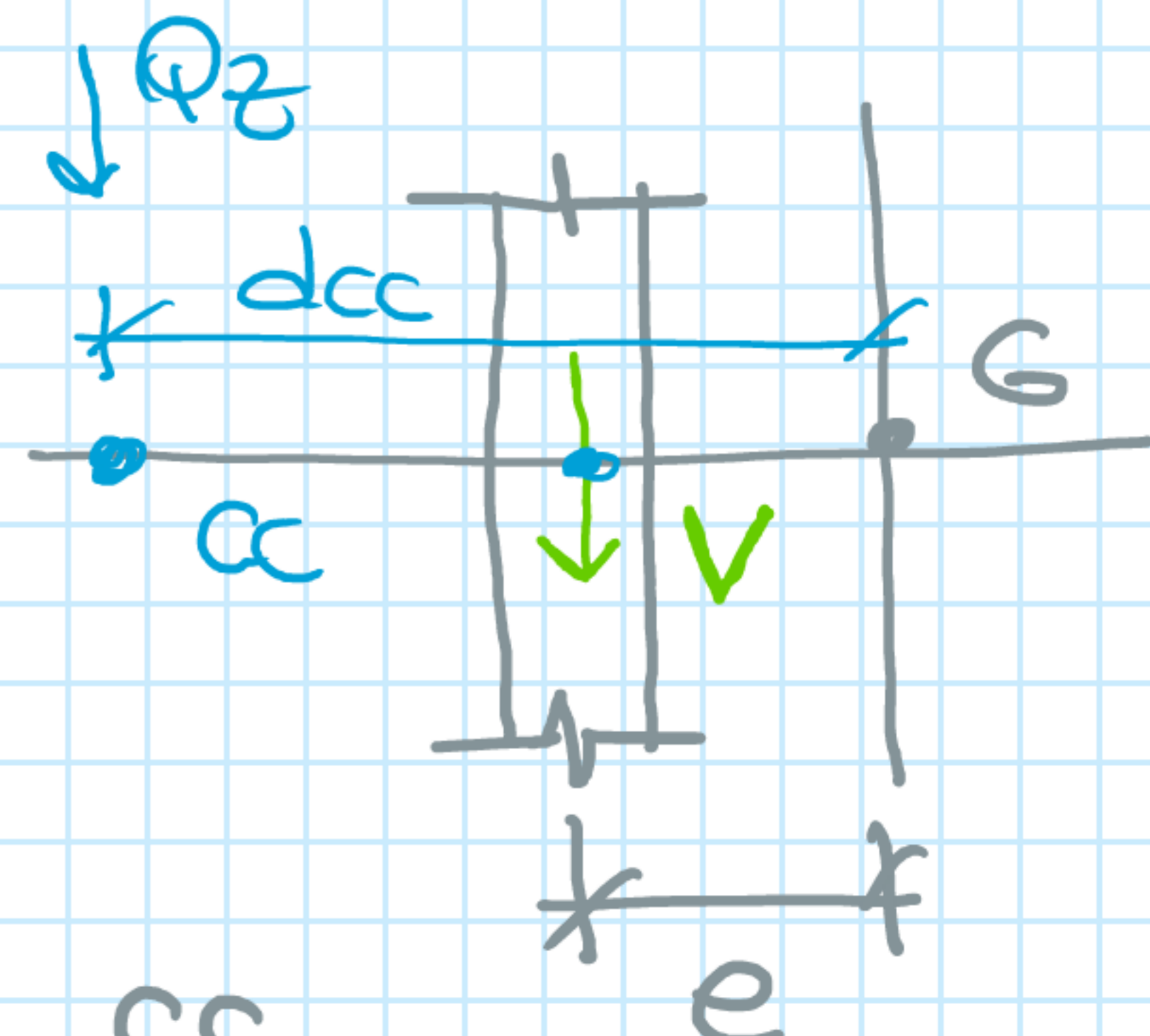
$$Q_z \cdot d_{cc} = H \cdot h + V \cdot e$$

SI
TENS.
↪
↪

I.2 - MOMENTOS RESPECTO DE LA LÍNEA MEDIA DEL ALMA.

$$Q_z \cdot (d_{cc} - e) = H \cdot h$$

↪
↪



I.3 - MOMENTOS RESPECTO POR CC.

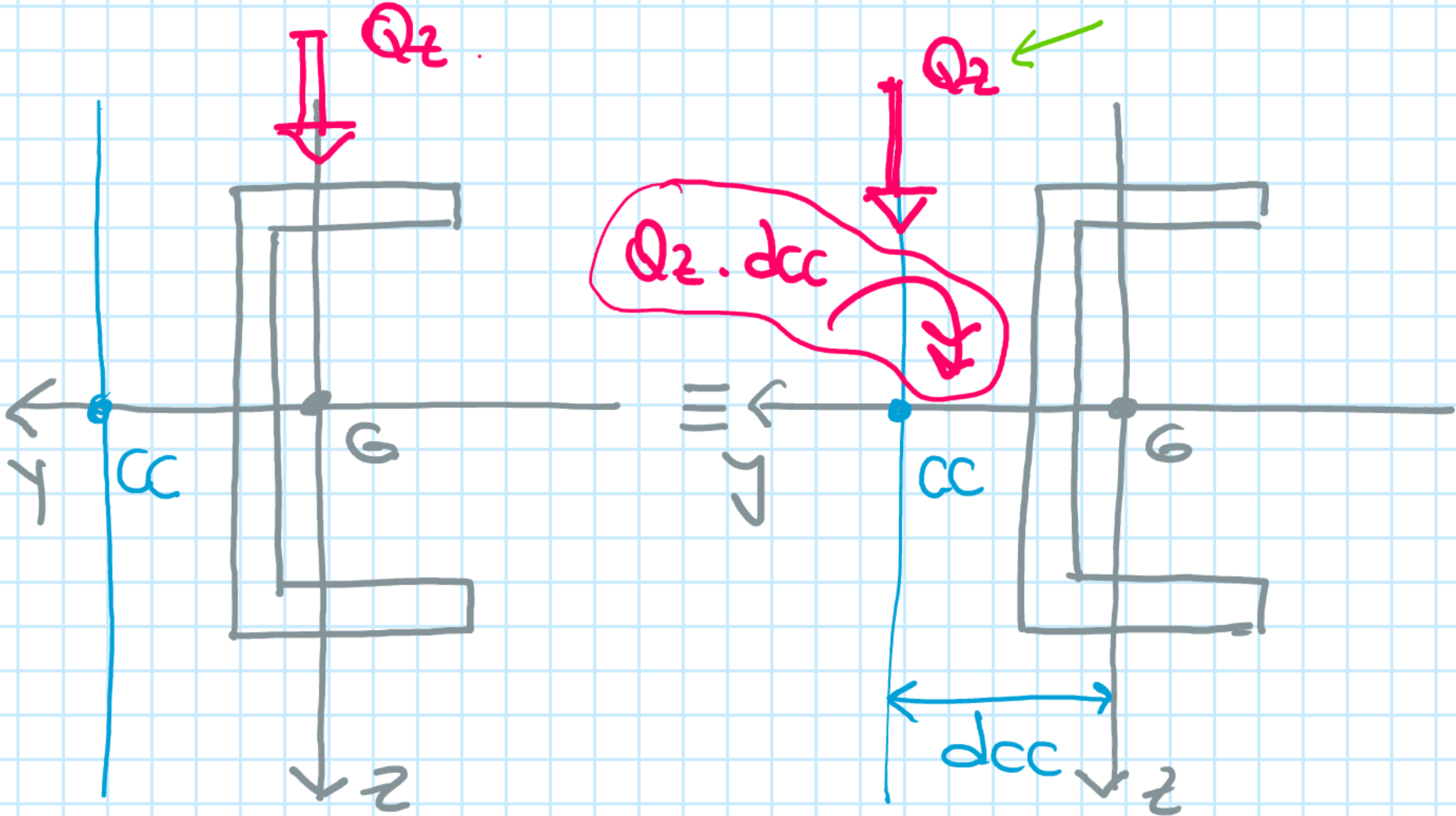
$$Q_z \cdot 0 = H \cdot h - V \cdot (d_{cc} - e)$$

↪
↪

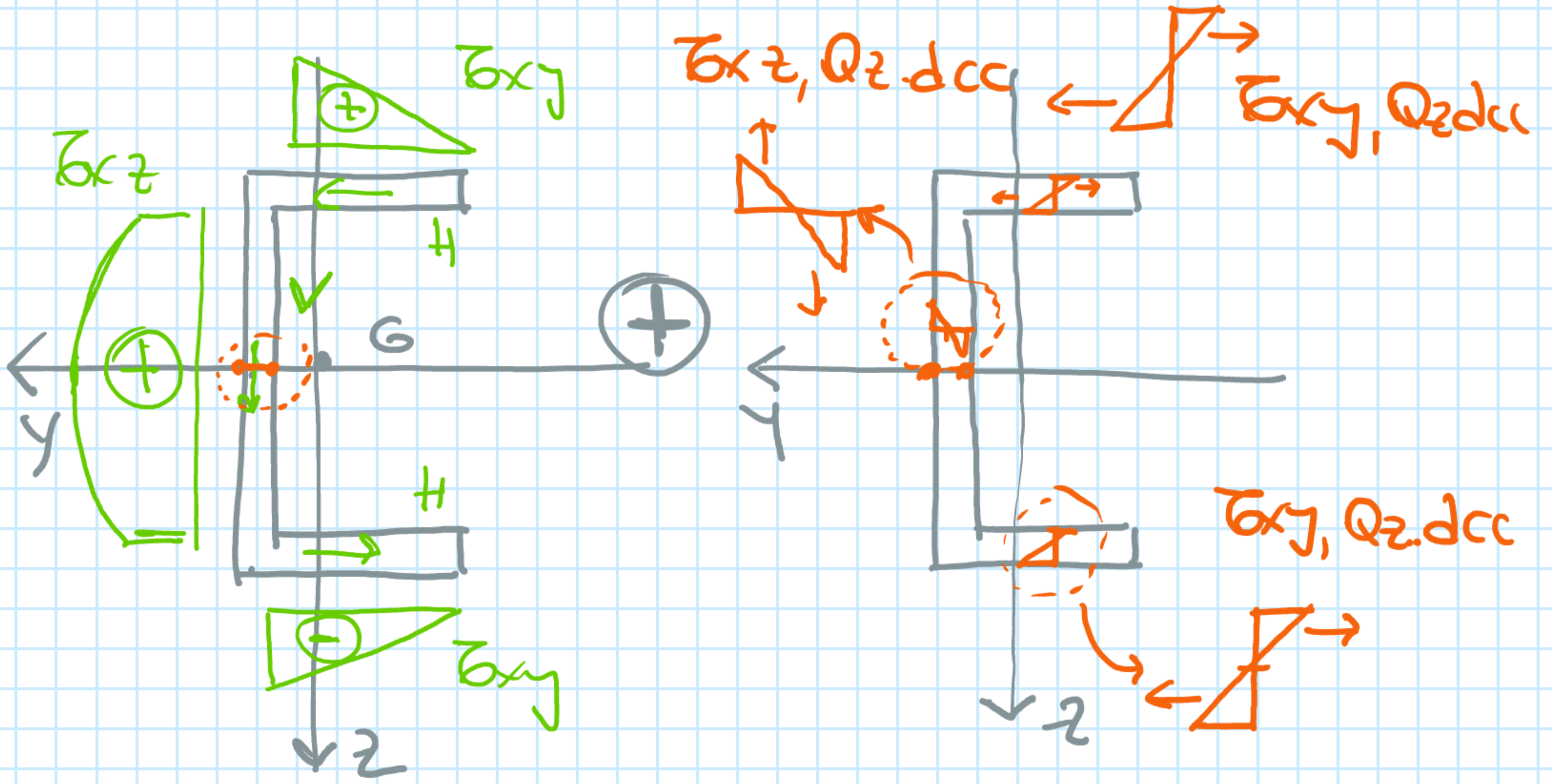
09 - CENTRO DE CORTE:

martes, 8 de junio de 2021 11:14

II ESFUERZO DE CORTE → NO PASANTE POR EL CC.



SI



CALCULADOS POR
SANTOVINCENTI.

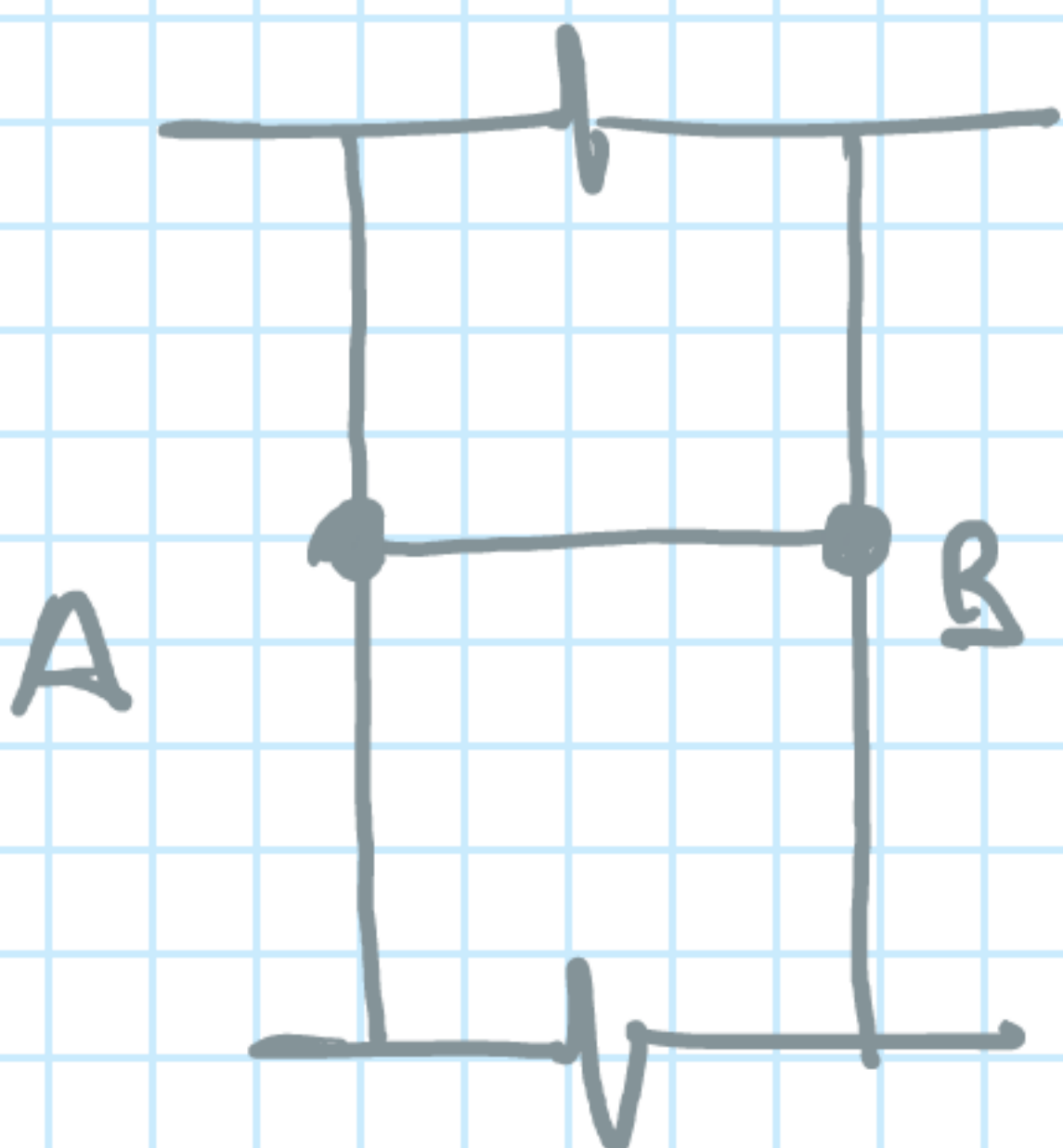
TENSIONES CALCULADAS
POR TEORÍA DE
SAINT VENANT.

$$\tau = \frac{M_T}{I_T} \cdot t$$

ALMA

A $\tau_{xz}, Q_z \downarrow + \tau_{xz}, Q_z dcc \uparrow$

B $\tau_{xz}, Q_z \downarrow + \tau_{xz}, Q_z dcc \downarrow$



09 - Centro de Corte:

martes, 8 de junio de 2021

11:29

1.- En secciones doblemente simétricas, la barra se flexiona y no se torsión. Se aplican las expresiones de flexión y de Jouravsky.

$$\sigma_x = \frac{M \cdot z}{I_y} \quad ; \quad \tau = \frac{Q \cdot S^*}{I_{Gz} \cdot b^*}$$

2.- En secciones con un eje de simetría pueden pasar 2 situaciones:

2.1- Si la carga está aplicada sobre el eje de simetría, la barra se flexiona pero no se torsiona;

2.2- Si la carga se aplica sobre el eje no simétrico, y pasante por G; la barra se flexiona y torsión. Con lo cual, para que esto último no ocurra, la carga debiera pasar por el CC.

3.- Siempre el CC se ubicará sobre los ejes de simetría.

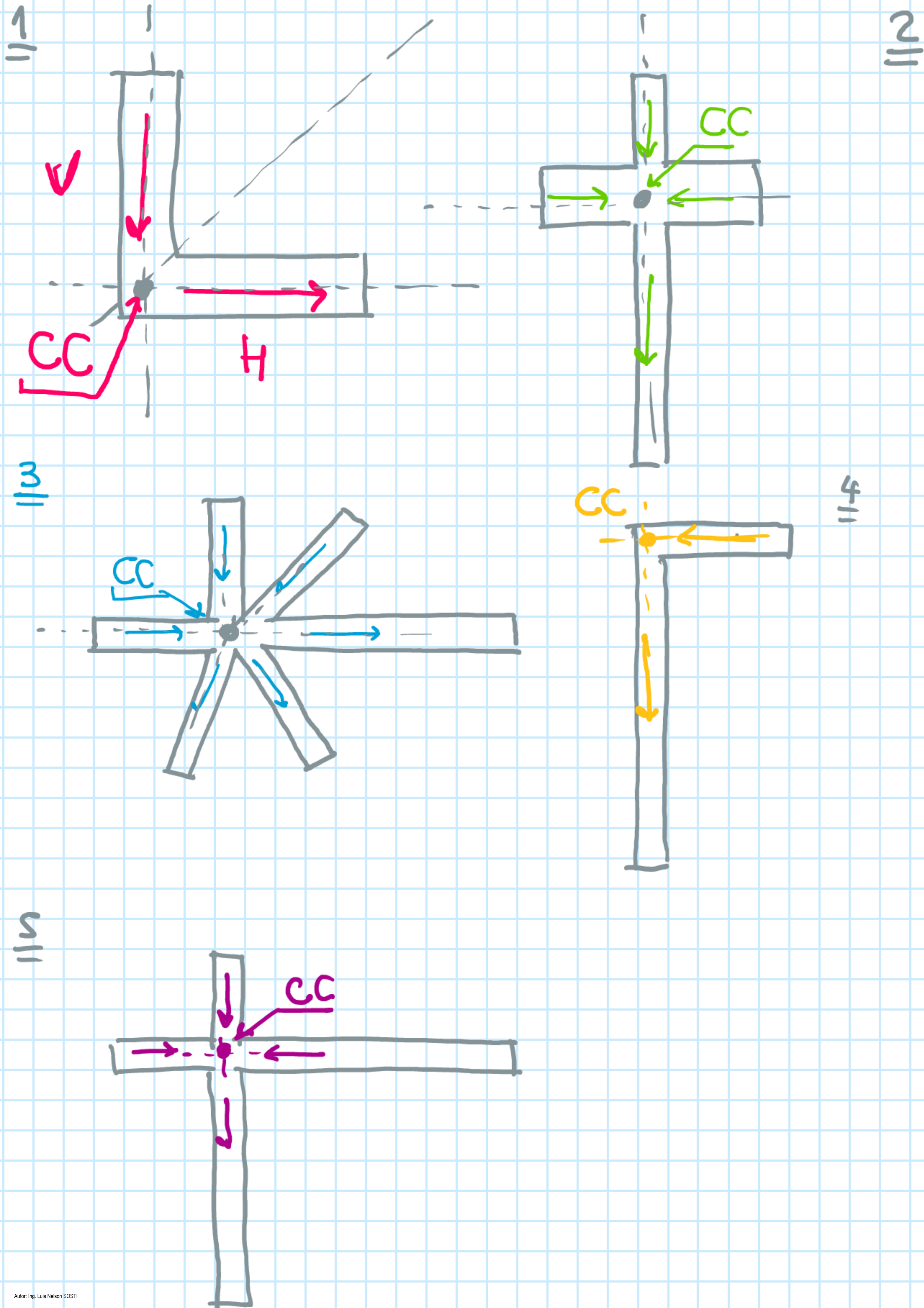
En secciones con doble eje de simetría, el G coincide con el CC.

4.- En secciones sin ejes de simetría, se deberán determinar las coordenadas del CC mediante las tensiones tangeaciales de Jouravsky, planteando momentos respecto de 2 puntos.

09 - CENTRO DE CORTE: Secciones Cruciformes

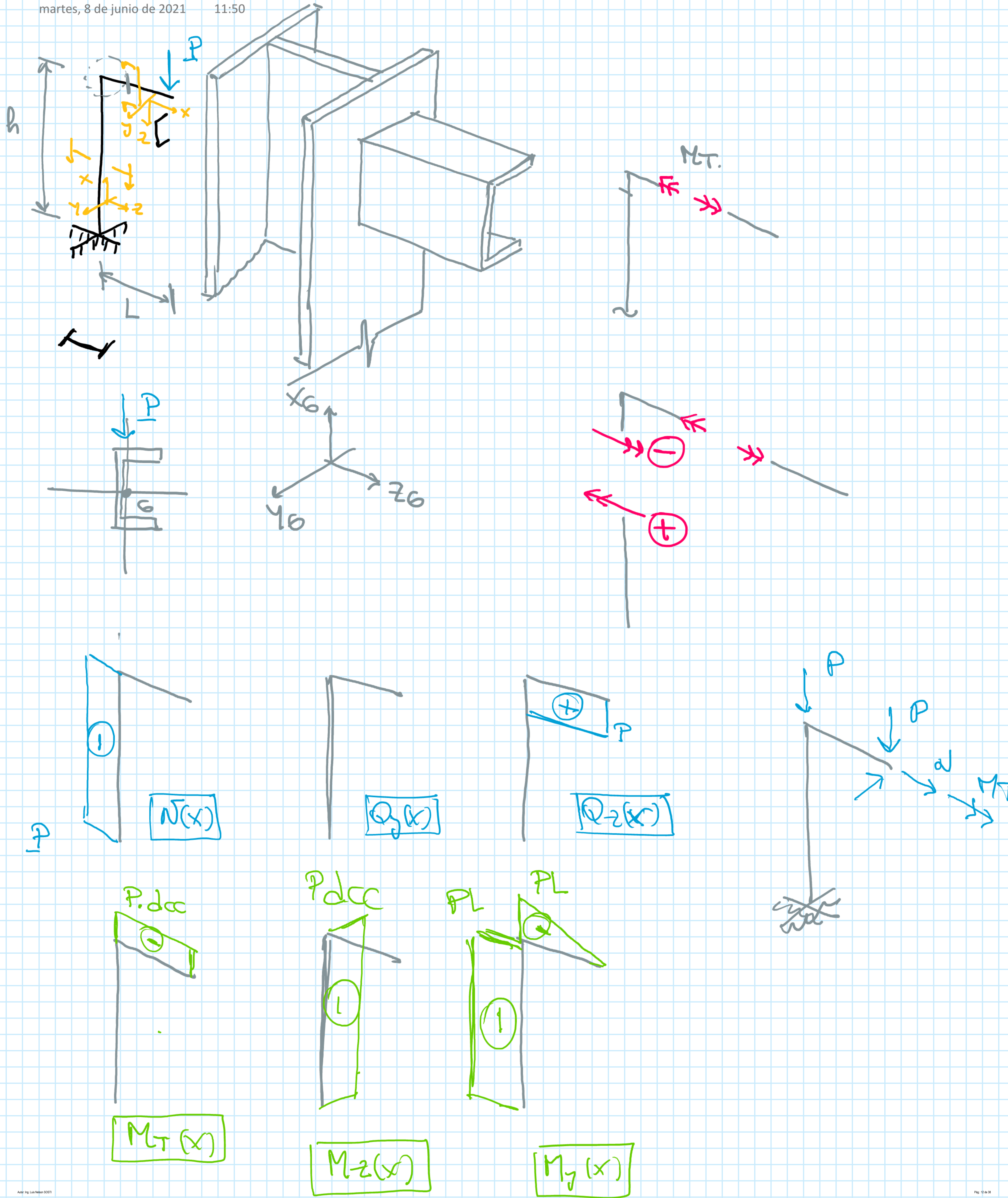
martes, 8 de junio de 2021

11:40



09 - CENTRO DE CORTE: Ejemplo

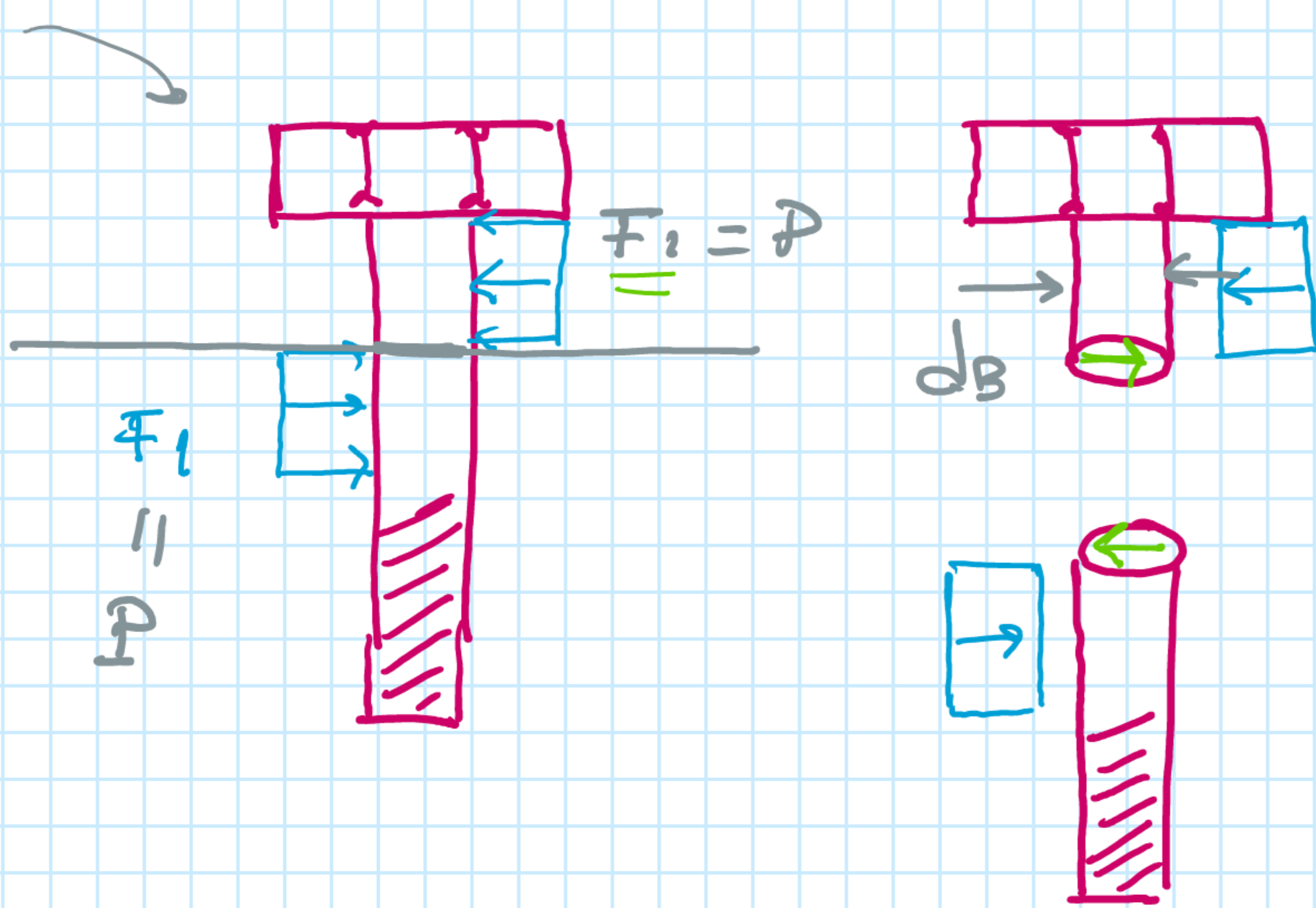
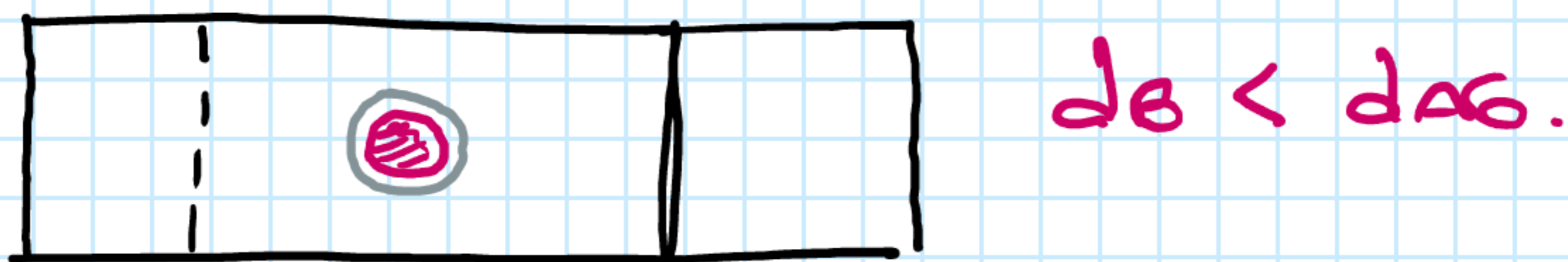
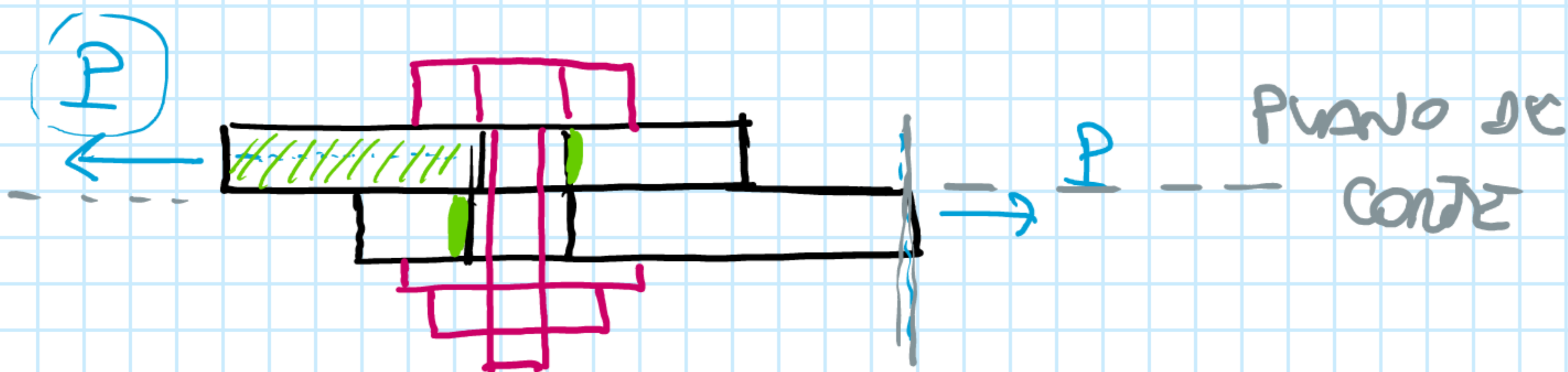
martes, 8 de junio de 2021 11:50



10 - UNIONES:

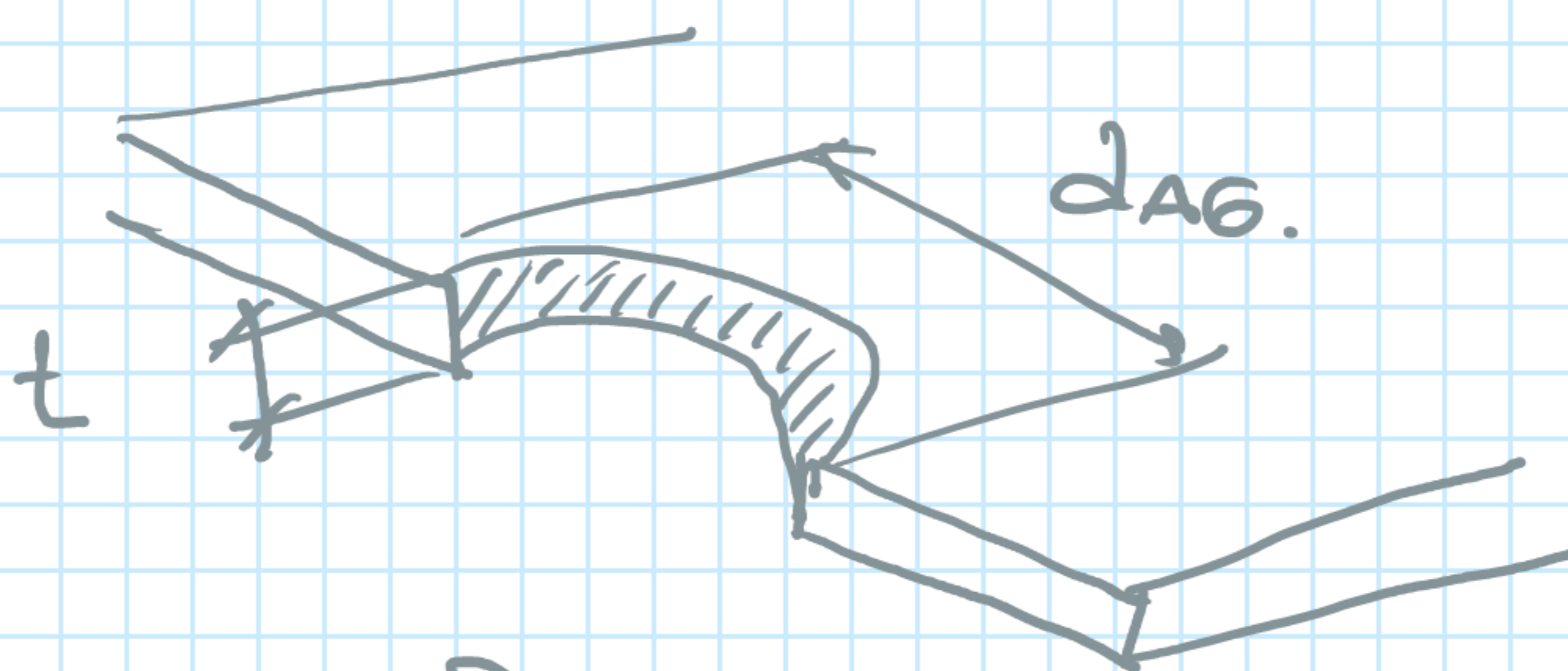
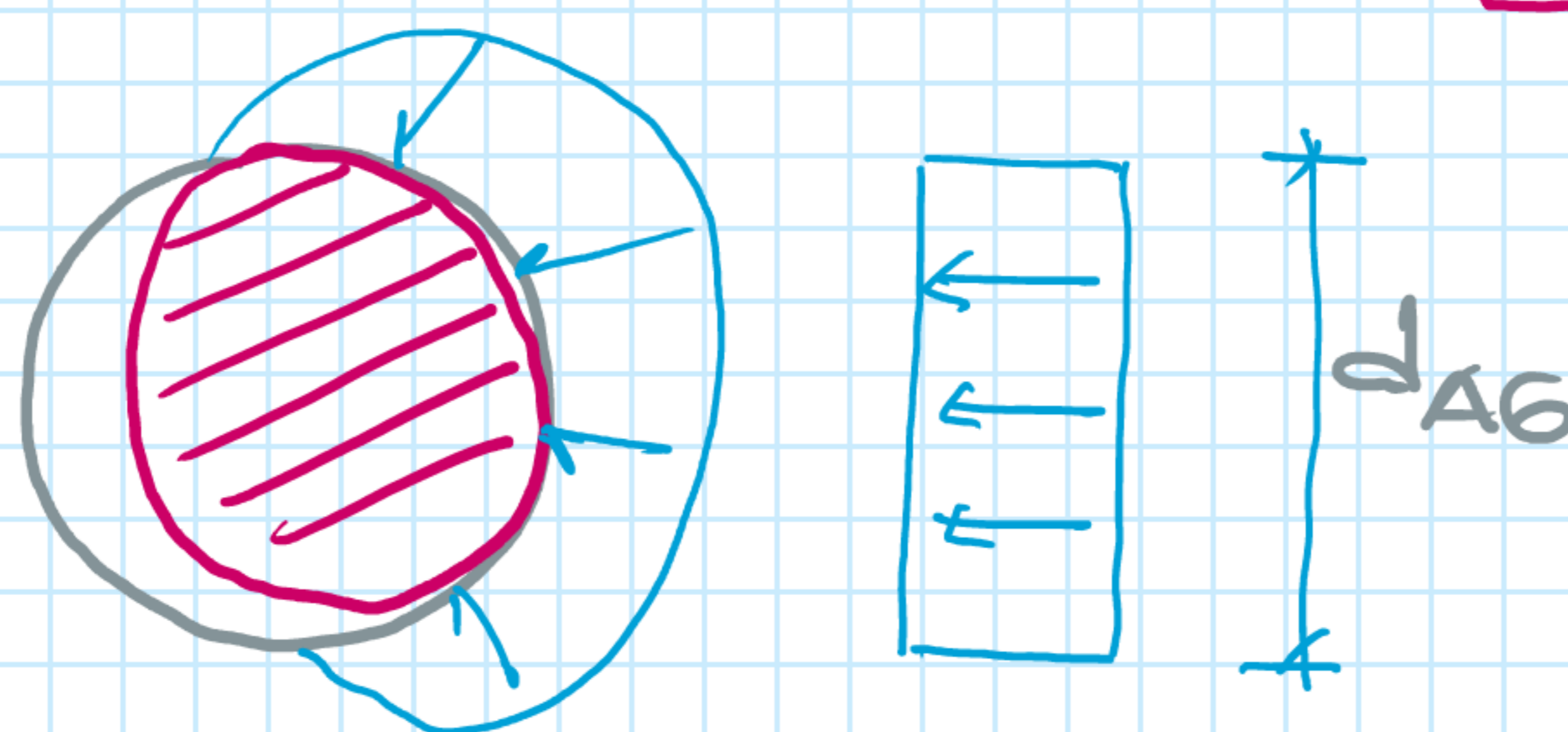
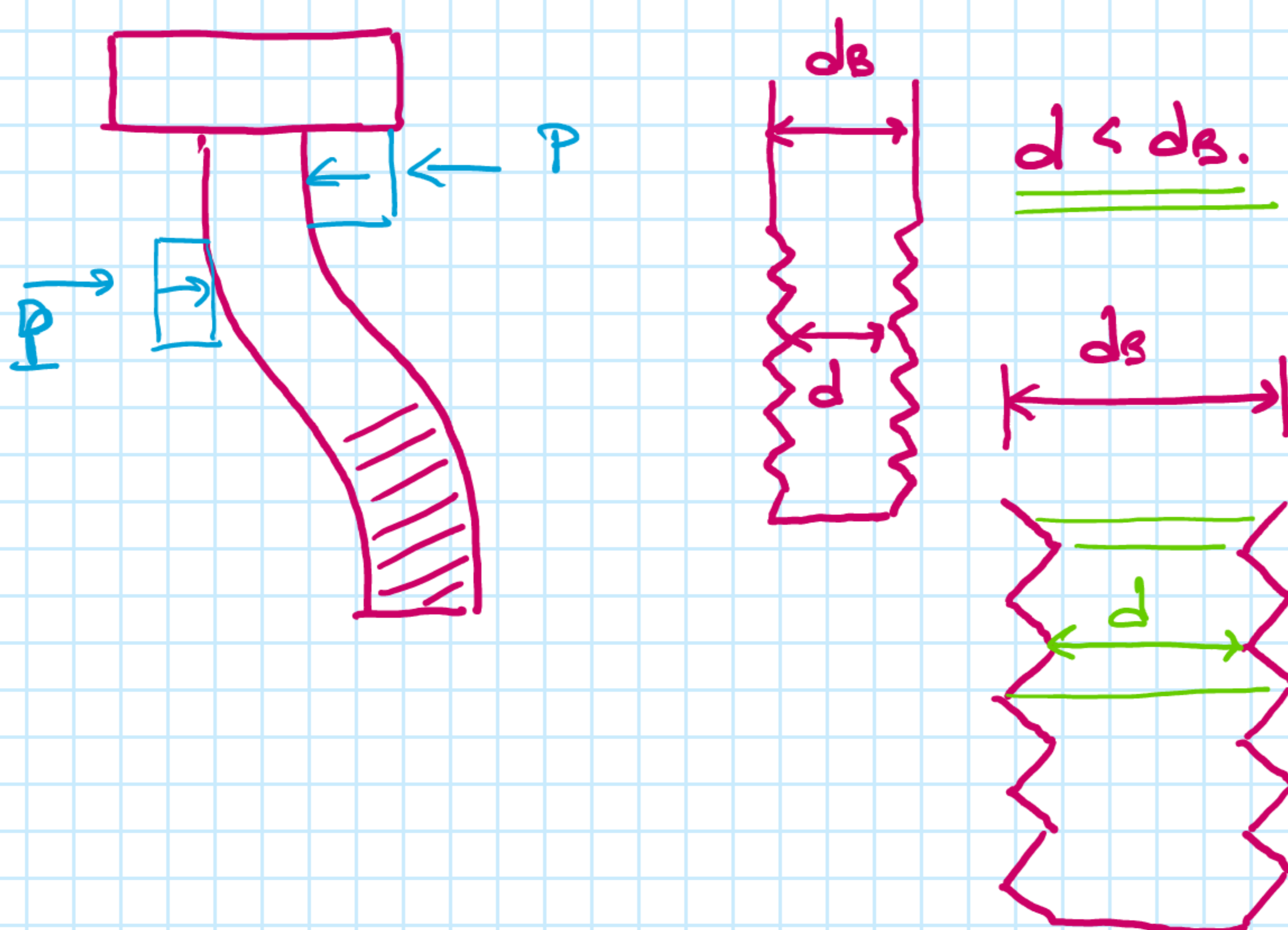
martes, 8 de junio de 2021 12:08

FUBA - E18472021-10-1010-CLT-FV2 - PV-02 - 2021-06-08



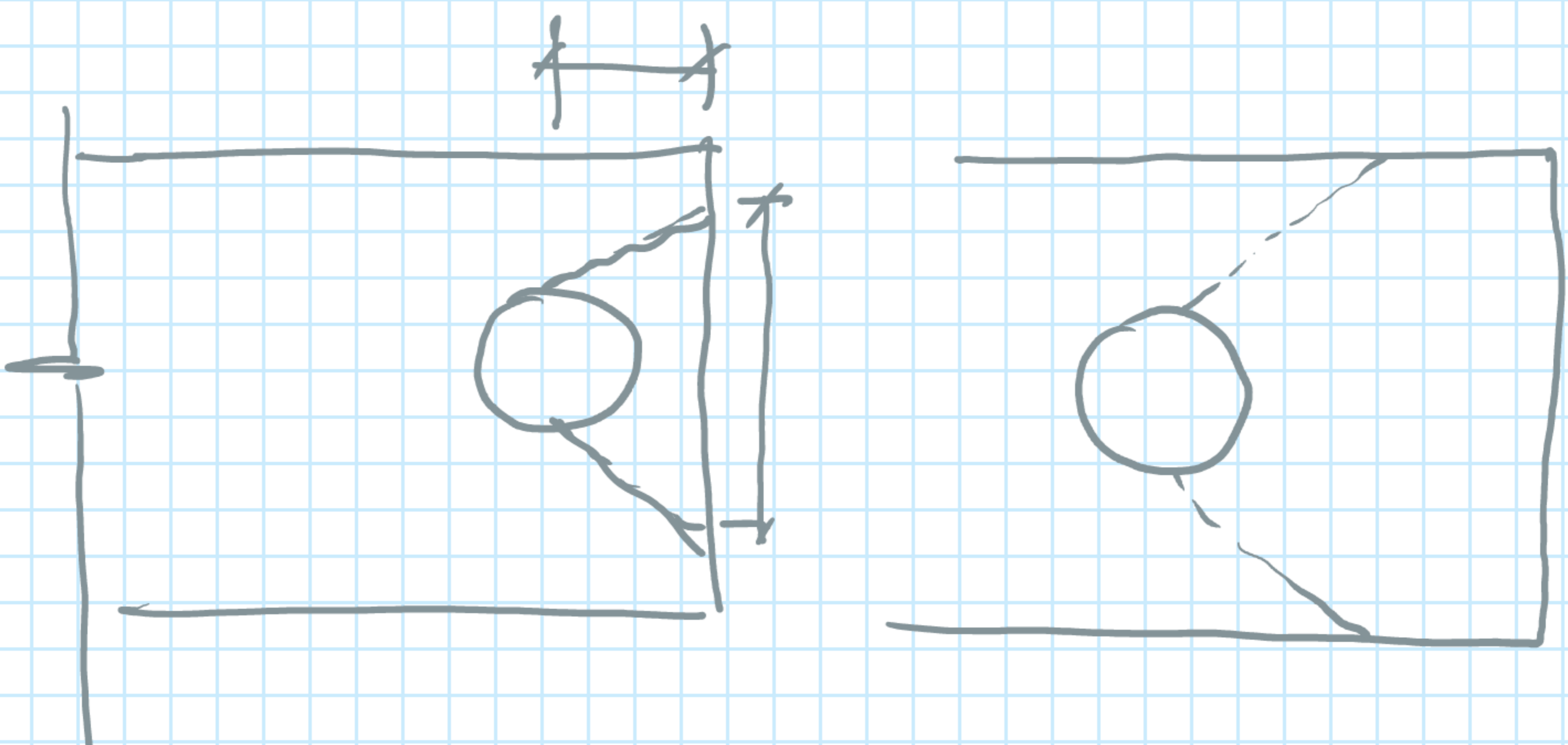
SECCION DEL BULON $A_B = \frac{\pi}{4} d_B^2$

$$\frac{P}{A_B} = \sigma_B \leq \sigma_{ADM, B.}$$



$$\sigma_{APLAST} = \frac{P}{d_{AG} \cdot t}$$

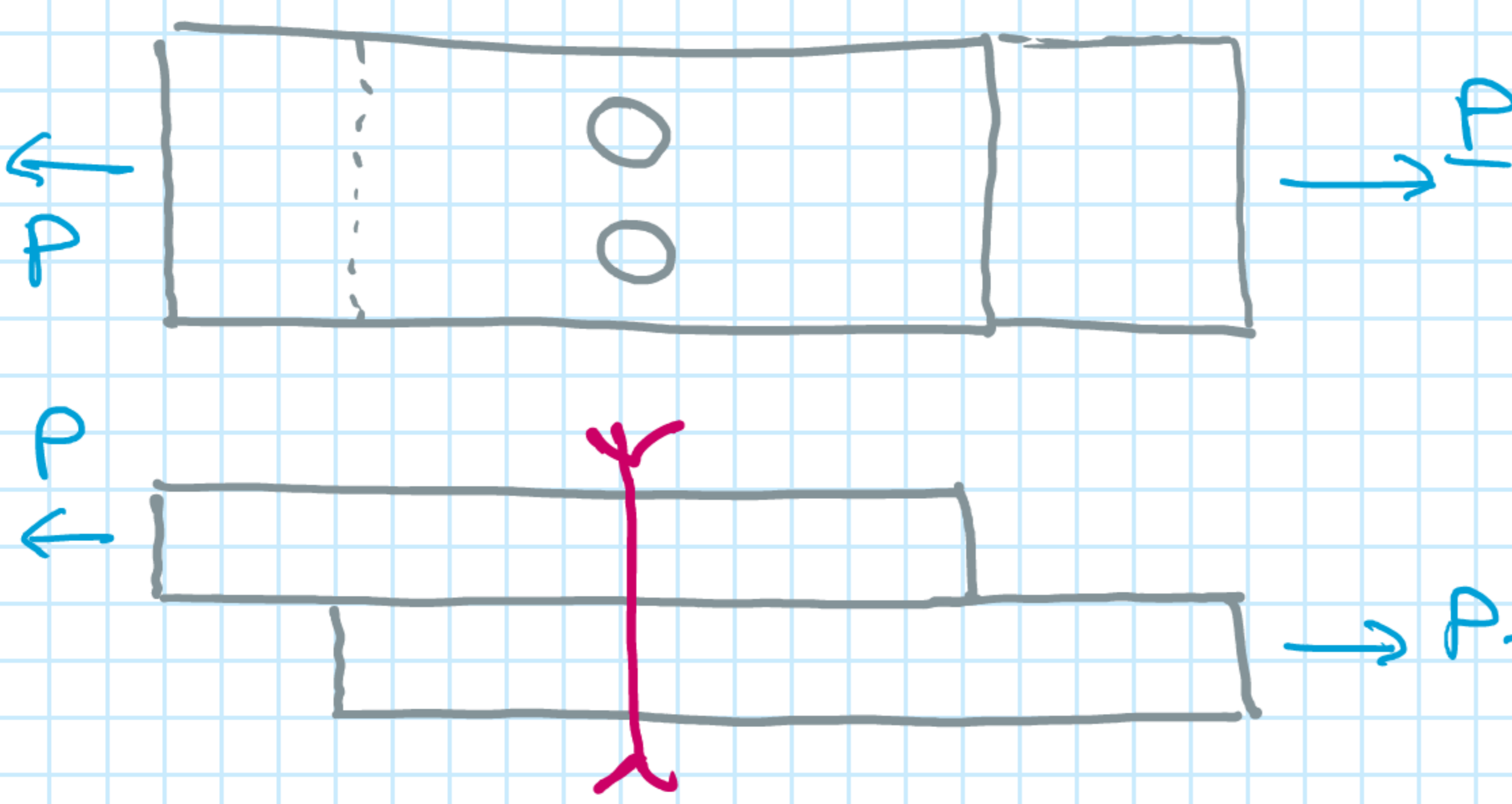
UNIONES POR CORTE Y APLASTAMIENTO.



10 - UNIONES

martes, 8 de junio de 2021 12:26

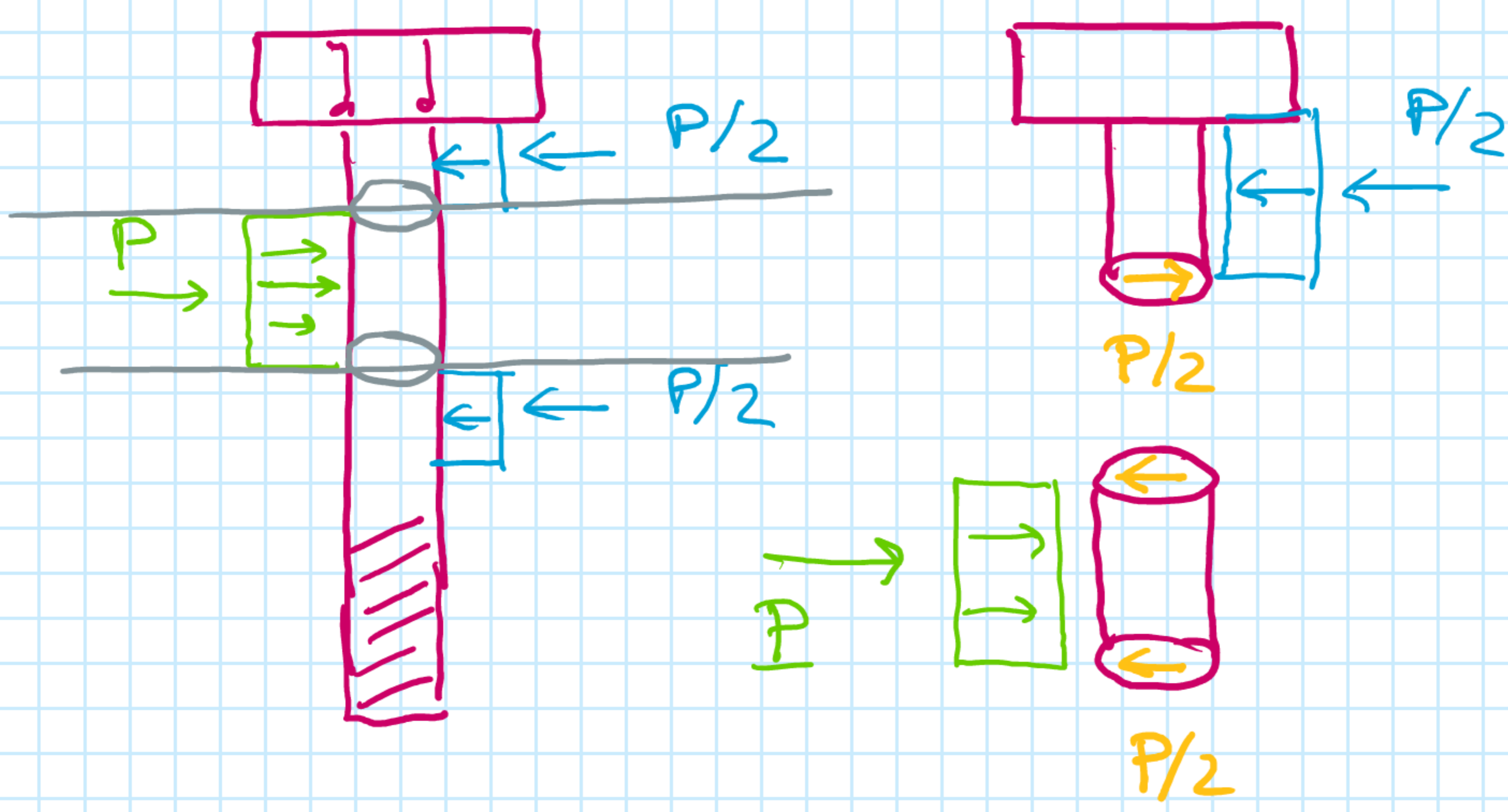
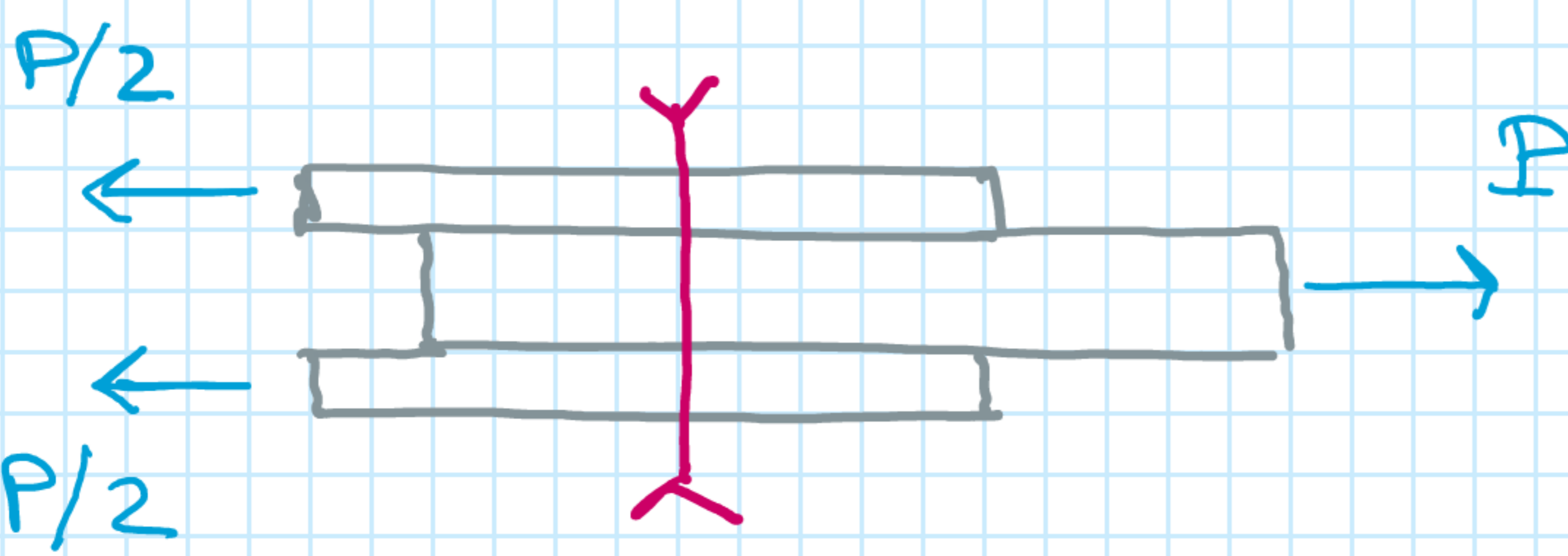
2 CHAPAS Y 2 BULONES:



$$\tau_B = \frac{P}{n \cdot A_B} \leq \tau_{ADM, B}$$

$\left\{ \begin{array}{l} n = \text{n}^\circ \text{ DE BULONES.} \\ n = 2 \end{array} \right.$

3 CHAPAS Y 1 BULÓN



$$\tau_B = \frac{P}{(2) A_B} \leq \tau_{ADM, B}$$

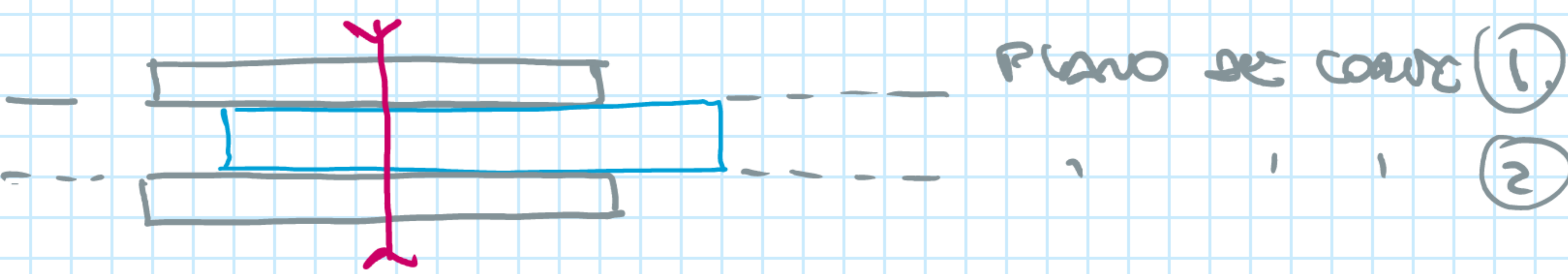
\swarrow $\text{n}^\circ \text{ DE PLANOS DE CORTE. 'm'}$

$$\tau_B = \frac{P}{m A_B} \leq \tau_{ADM, B}$$

$$\tau_B = \frac{P}{n \cdot m \cdot A_B} \leq \tau_{ADM, B}$$

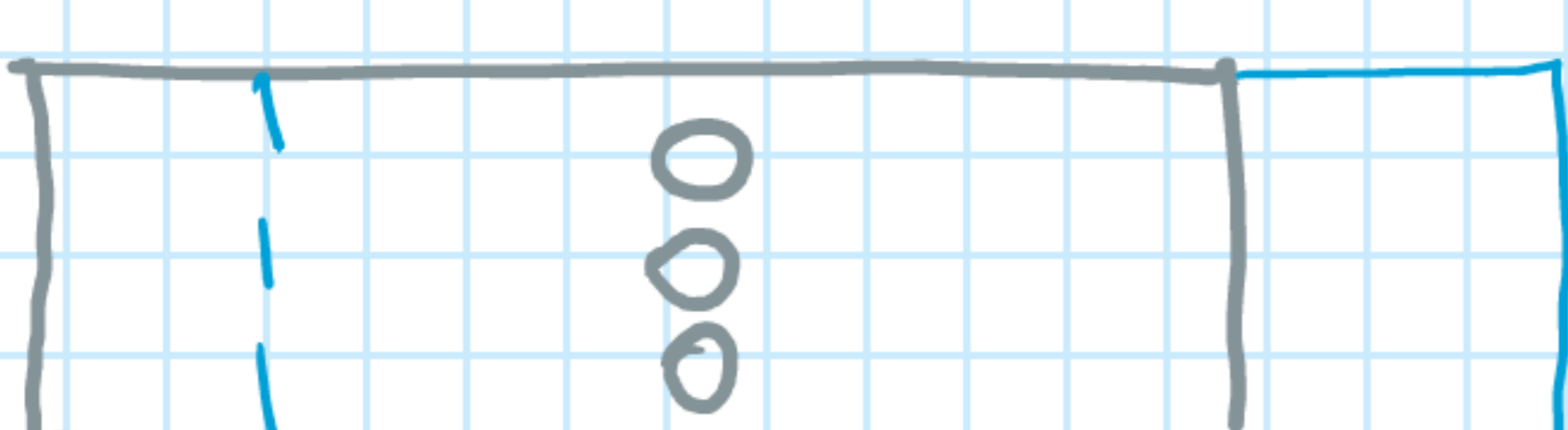
$n = \text{n}^\circ \text{ DE BULONES}$

$m = \text{n}^\circ \text{ DE PLANOS DE CORTE}$



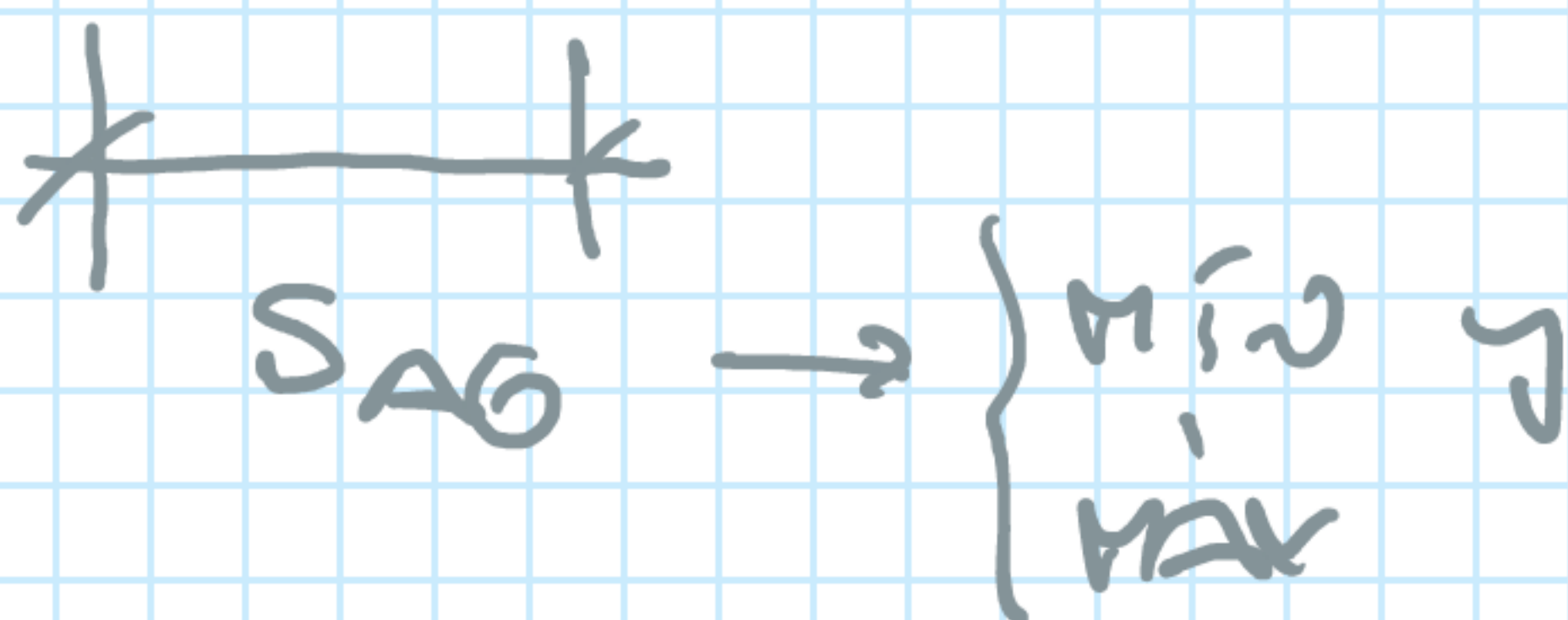
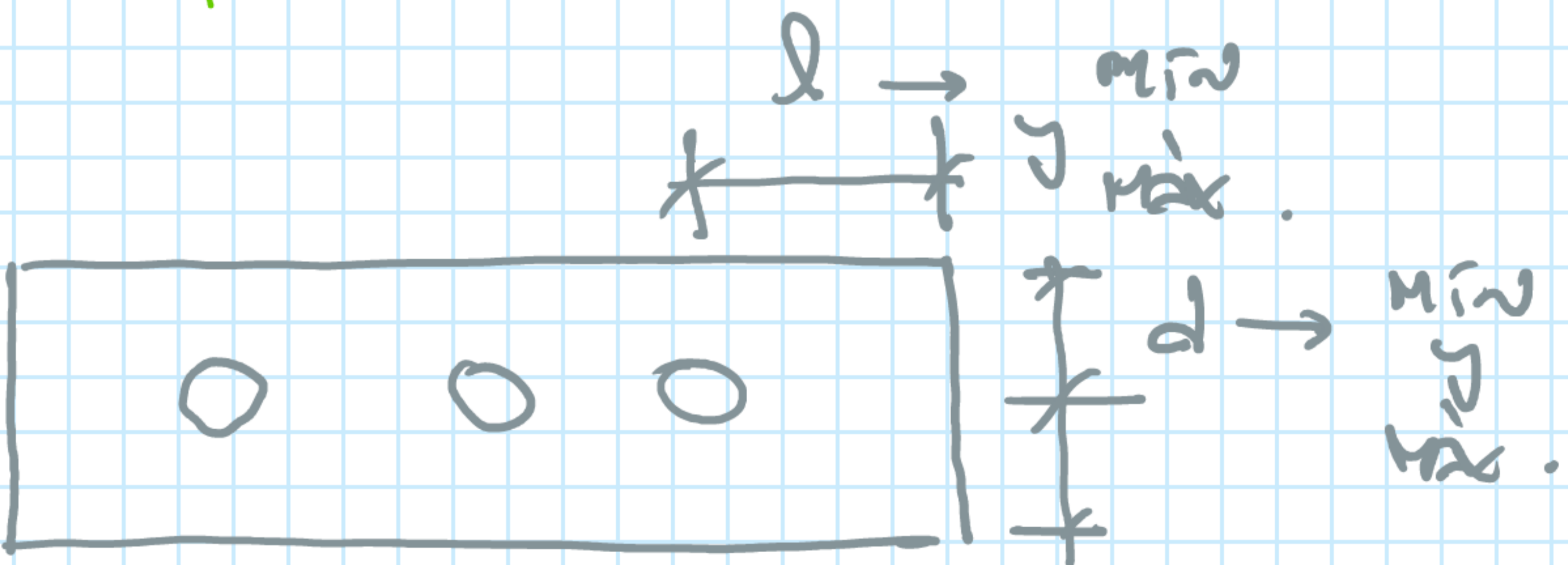
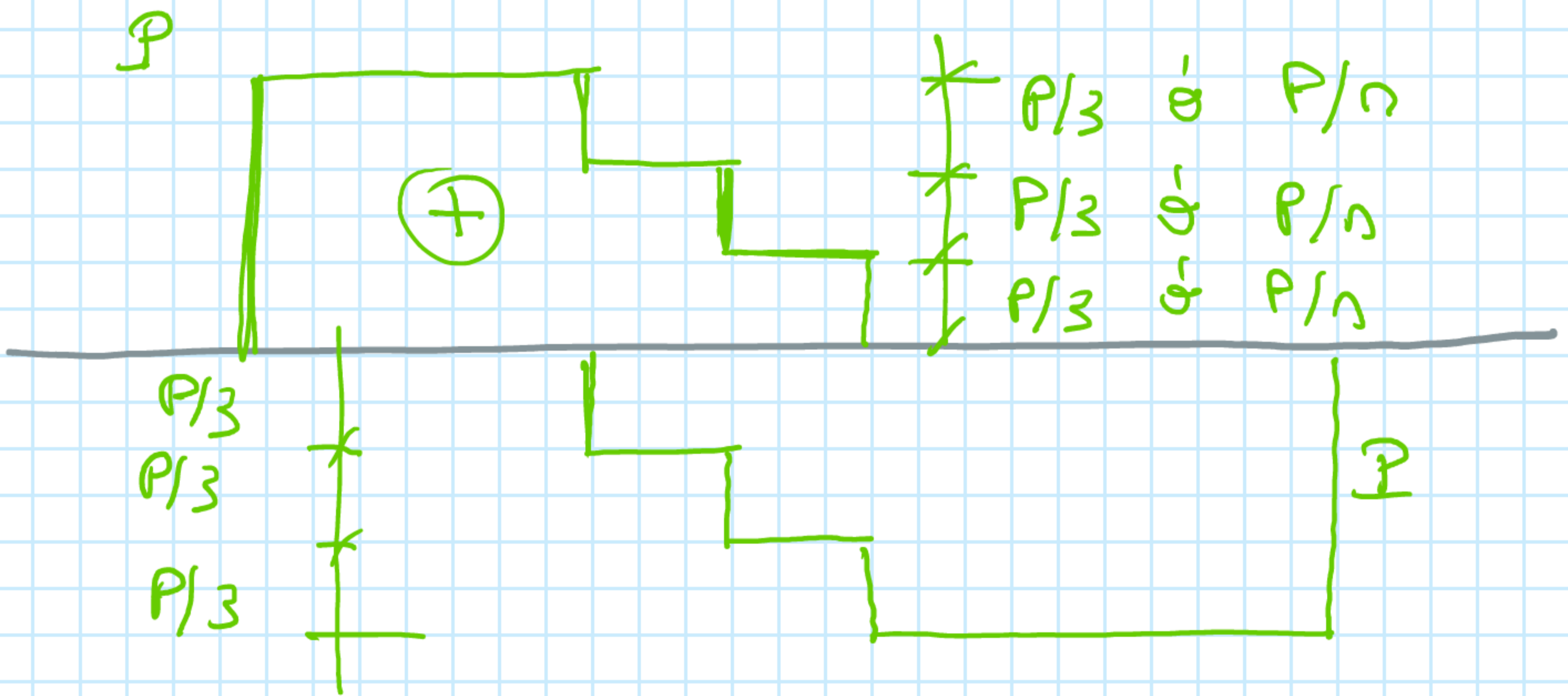
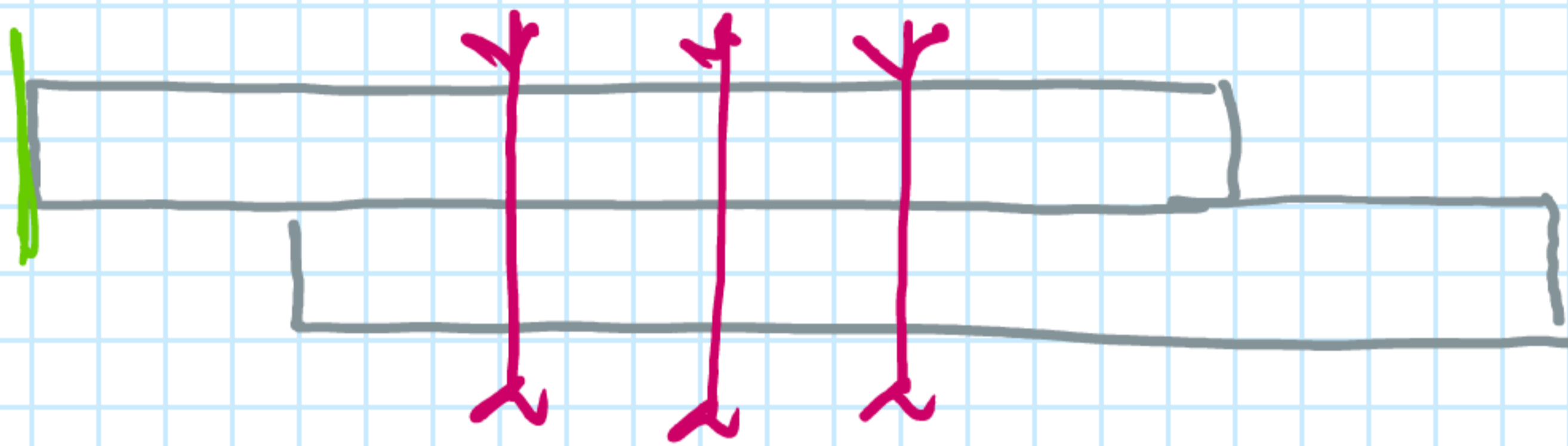
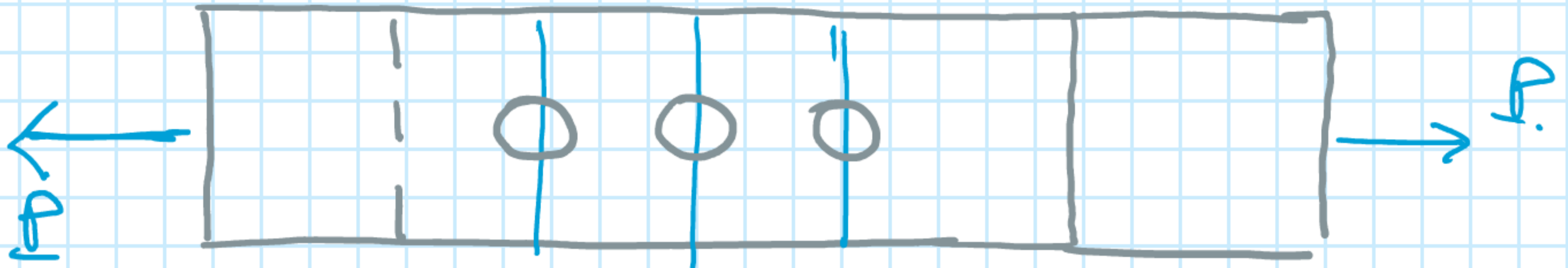
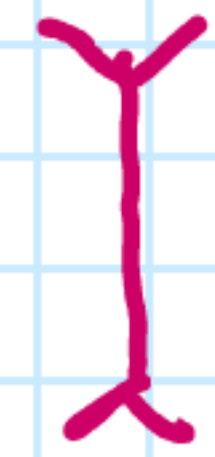
$n = 3$

$m = 2$



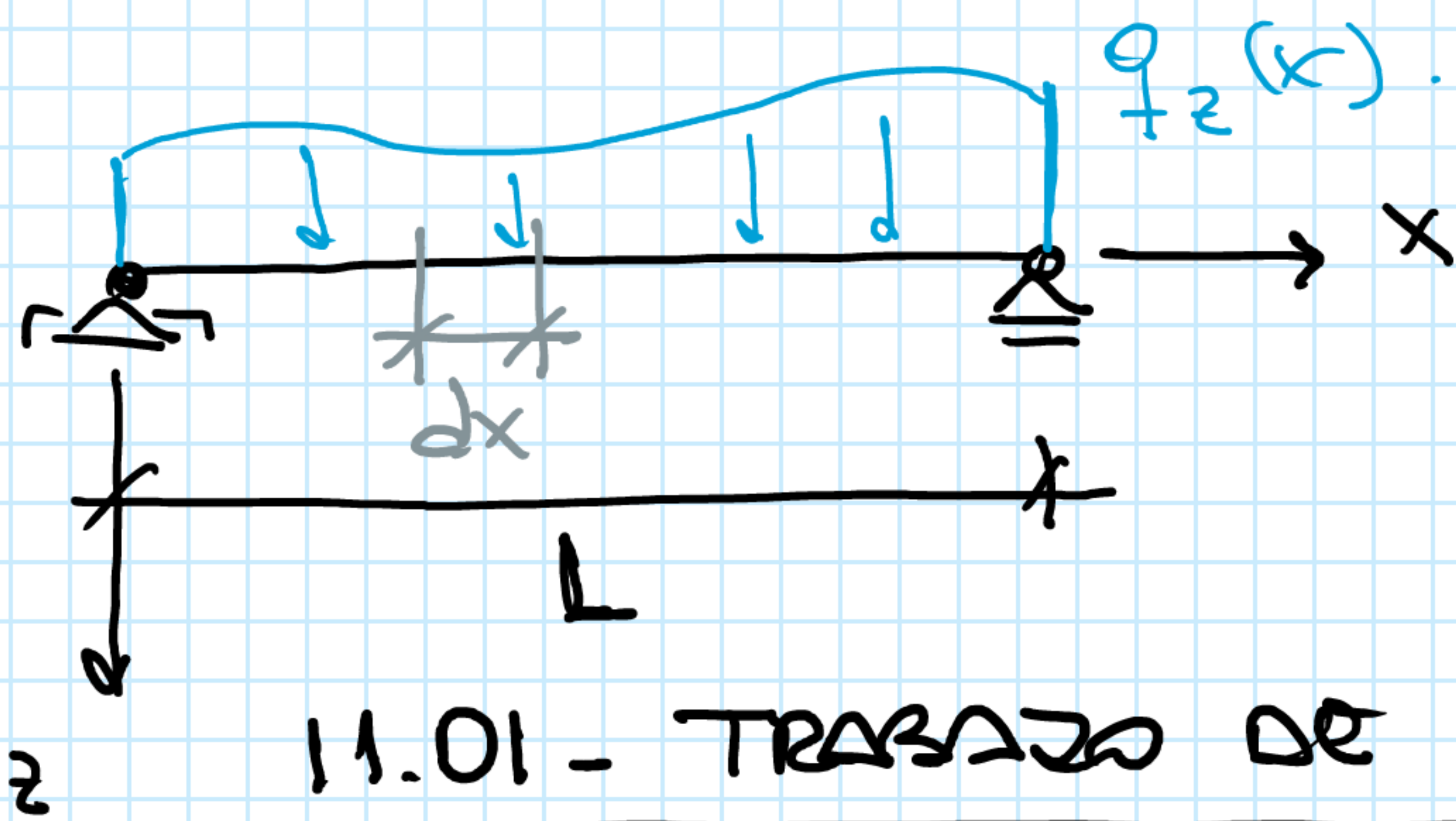
10 - UNIONES:

martes, 8 de junio de 2021 12:47

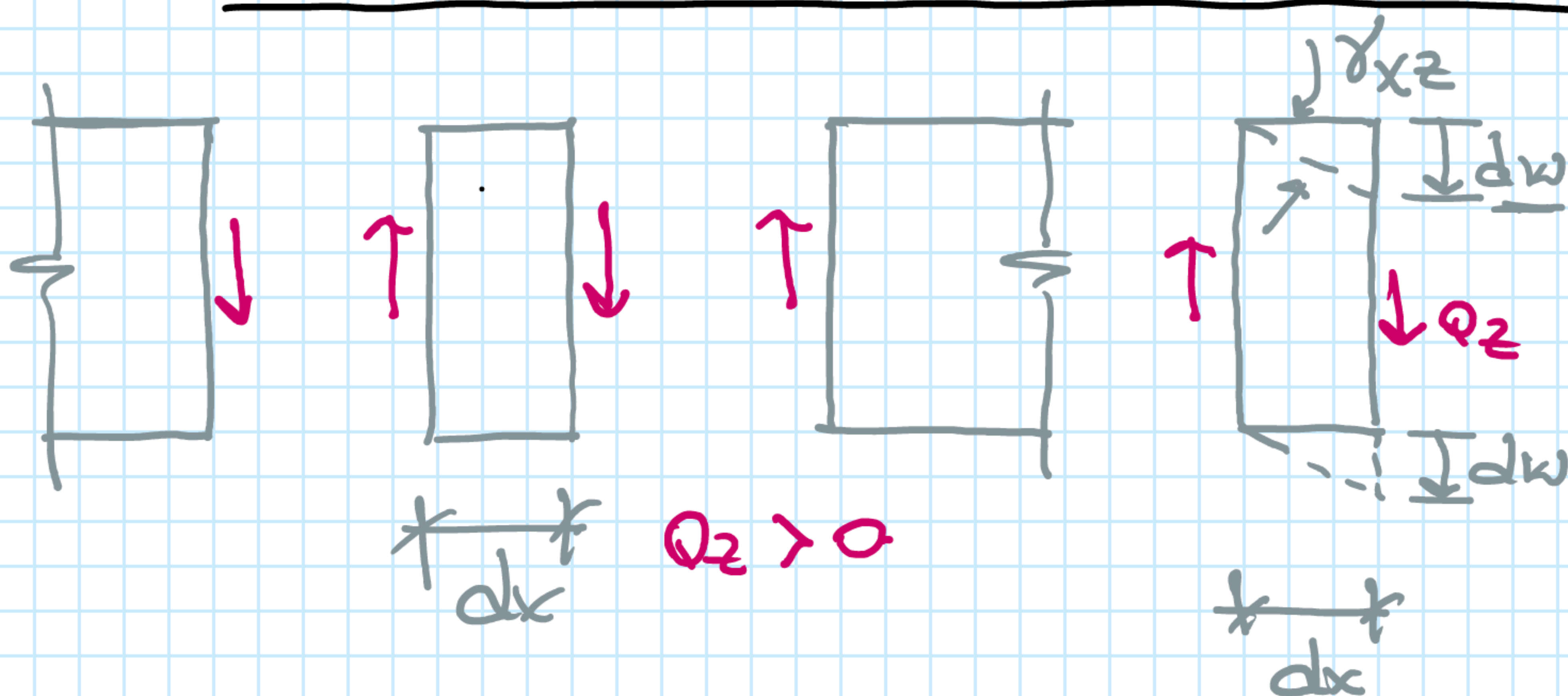


11 - DEFORMACIÓN POR CORTE:

martes, 8 de junio de 2021 13:01

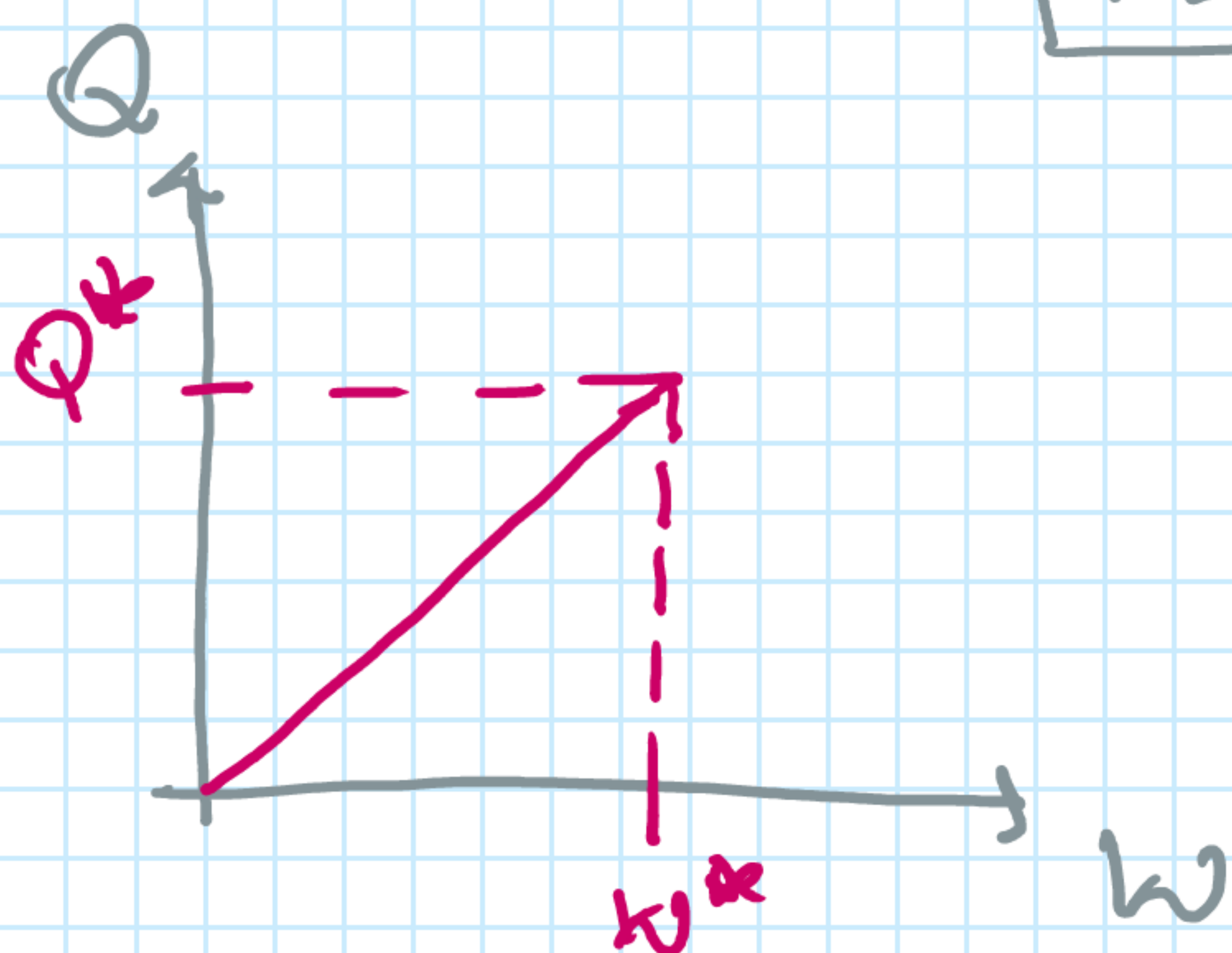


11.01 - TRABAJO DE FZAS EXTERNAS



HIPOTESIS: RELACION LINEAL ENTRE ESFUERZOS y DESPLAZAMIENTOS

HLM



$$\gamma_{xz} \cdot dx = dw$$

$$\Delta dw = Q \cdot \Delta dw$$

Es lo mismo trabajar con valores finitos que infinitesimal Tanto del esfuerzo como de los desplazamientos

$$Q = k \cdot w$$

$$\Delta dw = Q \cdot \Delta dw = k \cdot dw \Delta dw$$

↑ trabajo ↑ esfuerzo

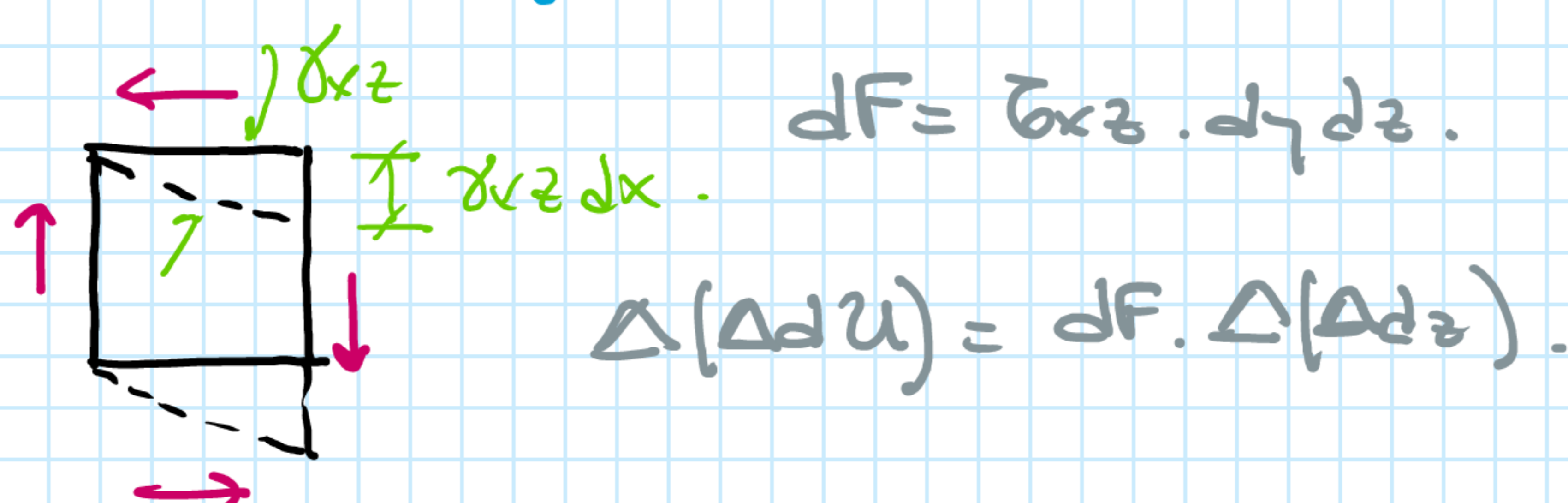
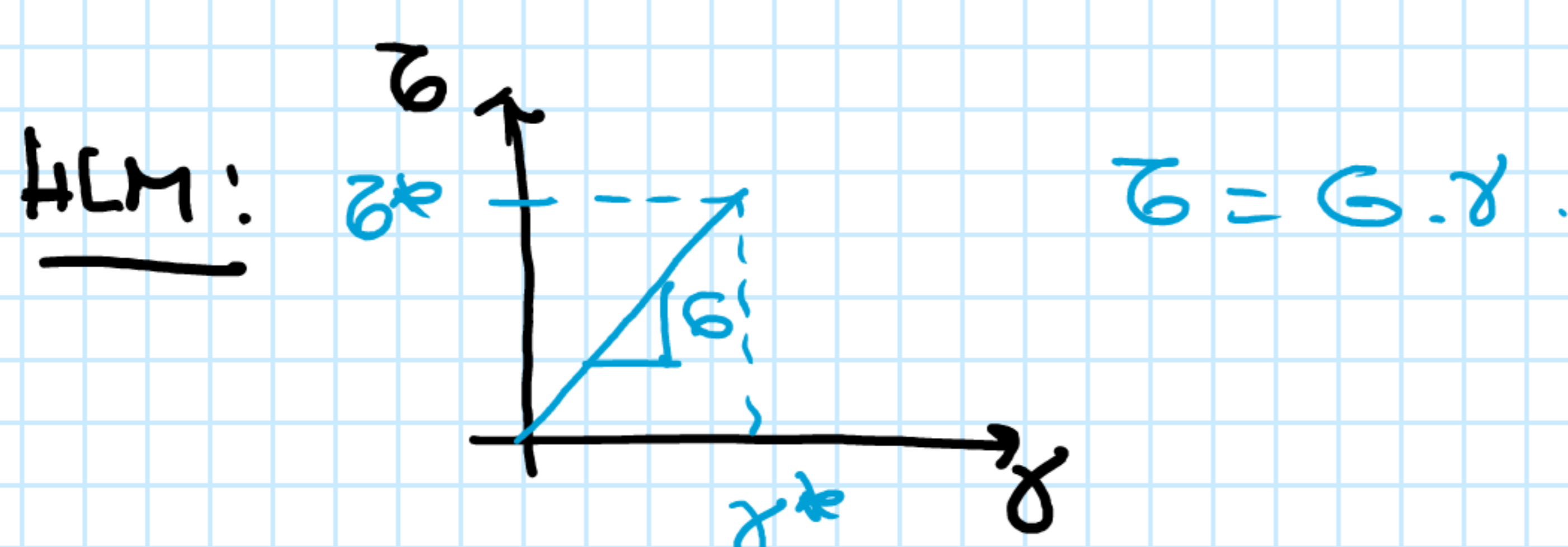
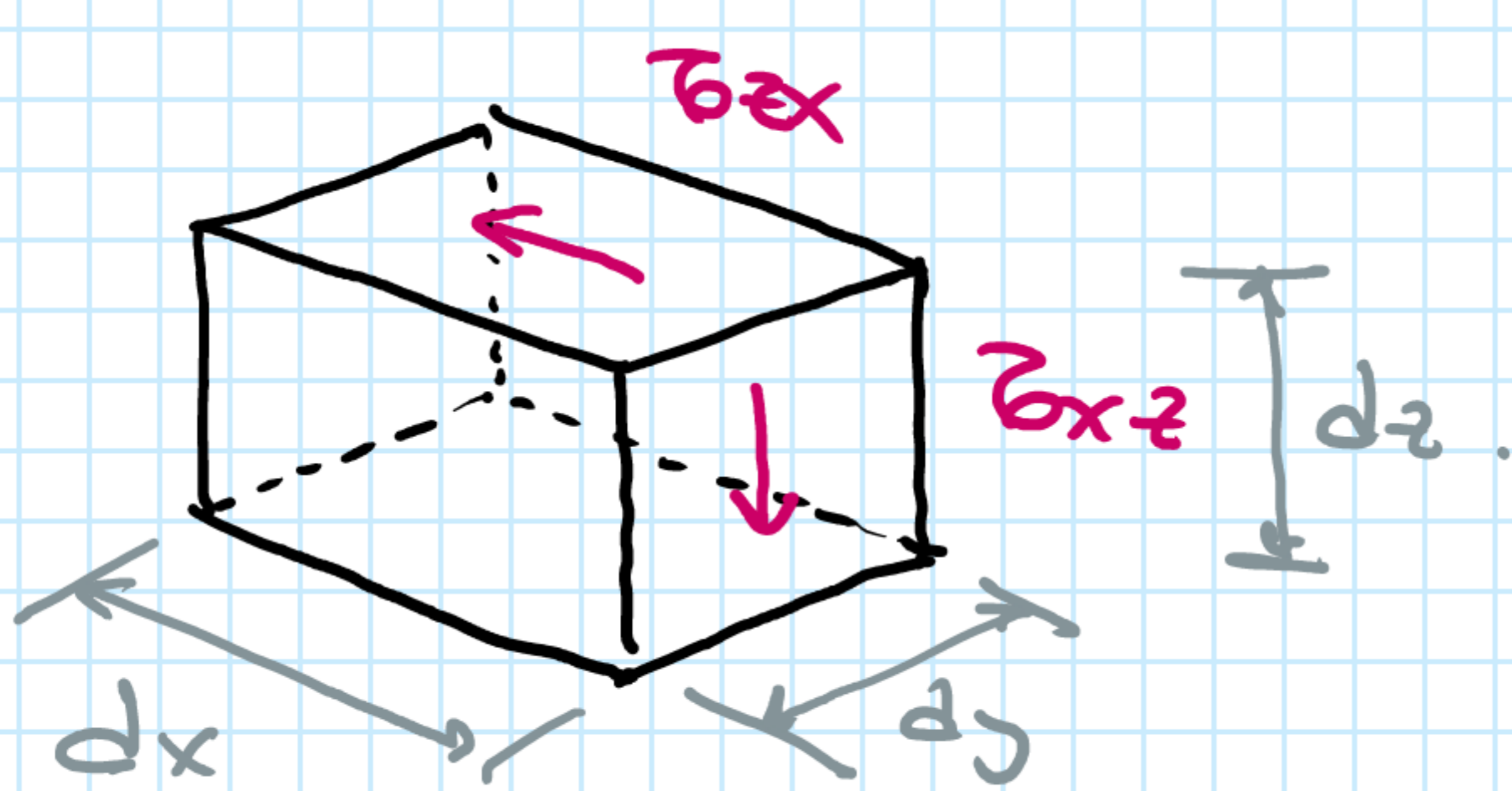
$$dW = \int \Delta dW = \int Q \cdot \Delta dw = \int_0^{dw^*} k \cdot dw \Delta dw$$

$$dW = \frac{1}{2} k (dw^*)^2 = \frac{1}{2} \underbrace{k}_{Q^*} \cdot dw^*$$

$$dW = \frac{1}{2} Q^* dw^*$$

$$dW = \frac{1}{2} Q dw$$

11.02 - EN ENERGÍA INTERNA DE DEFORMACIÓN



$\Delta u = \tau_{xz} \cdot dy \cdot dz \cdot \Delta(\Delta z)$

$\Delta u = \tau_{xz} \cdot dy \cdot dz \cdot \gamma_{xz} \cdot dx$

$dU = \int \Delta u = \int dF \cdot \Delta z = \int \underbrace{\tau_{xz} \cdot dy \cdot dz}_{dF} \cdot \underbrace{\gamma_{xz} \cdot dx}_{\Delta z}$

$\Delta u = \int dF \cdot \Delta(\Delta z) = \int_0^{\gamma^*} \tau_{xz} \cdot d\gamma_{xz} \cdot dV$

$= \frac{1}{2} G \gamma_{xz}^2 \cdot dV$

$dU = \int \Delta u = \int \frac{1}{2} \tau_{xz} \gamma_{xz} \cdot dV =$

$dW = dU$

$\frac{1}{2} Q \cdot dW = \int \frac{1}{2} \tau_{xz} \cdot \gamma_{xz} \cdot dx \cdot \underbrace{dy \cdot dz}_{dA}$

$Q \frac{dW}{dx} = \int \tau_{xz} \cdot \gamma_{xz} \cdot dA$

$\gamma_{xz} = \tau_{xz} / G$

$Q \frac{dW}{dx} = \int \frac{\tau_{xz}^2}{G} \cdot dA = \int \frac{Q^2 (s^*)^2}{I_N^2 b^2 G} \cdot \frac{1}{6} \cdot dA$

$I_N = A \cdot r_0^2$

$Q \frac{dW}{dx} = \int \frac{Q^2 \cdot (s^*)^2}{A^2 r_0^4 b^2 G} \cdot dA$

$Q \frac{dW}{dx} = \frac{Q^2}{r_0^4 \cdot GA} \int \frac{(s^*)^2}{b^2} \frac{dA}{A}$

$dW = \frac{Q}{GA} \left[\int \frac{(s^*)^2}{r_0^4 b^2} \frac{dA}{A} \right] dx$

$dC = \frac{Q}{GA} \left[K_\gamma \right] dx$

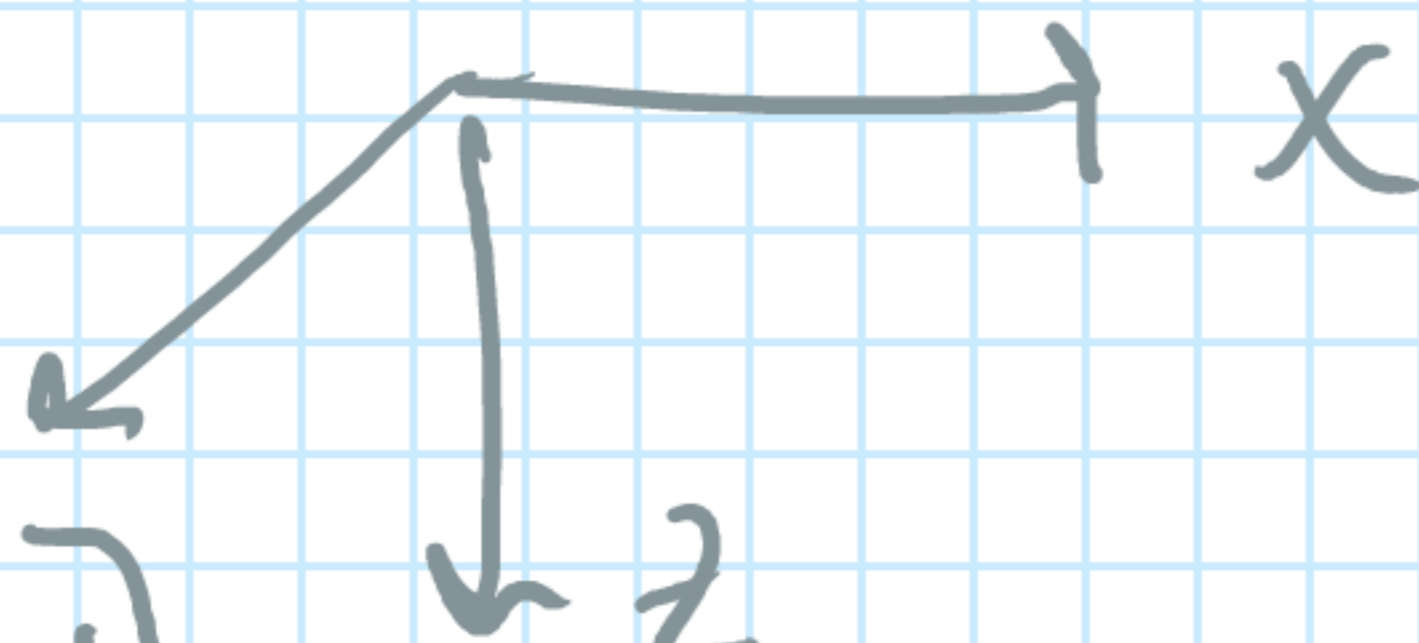
FACTOR DE FORMA

$dW = dC$

$x \rightarrow u$

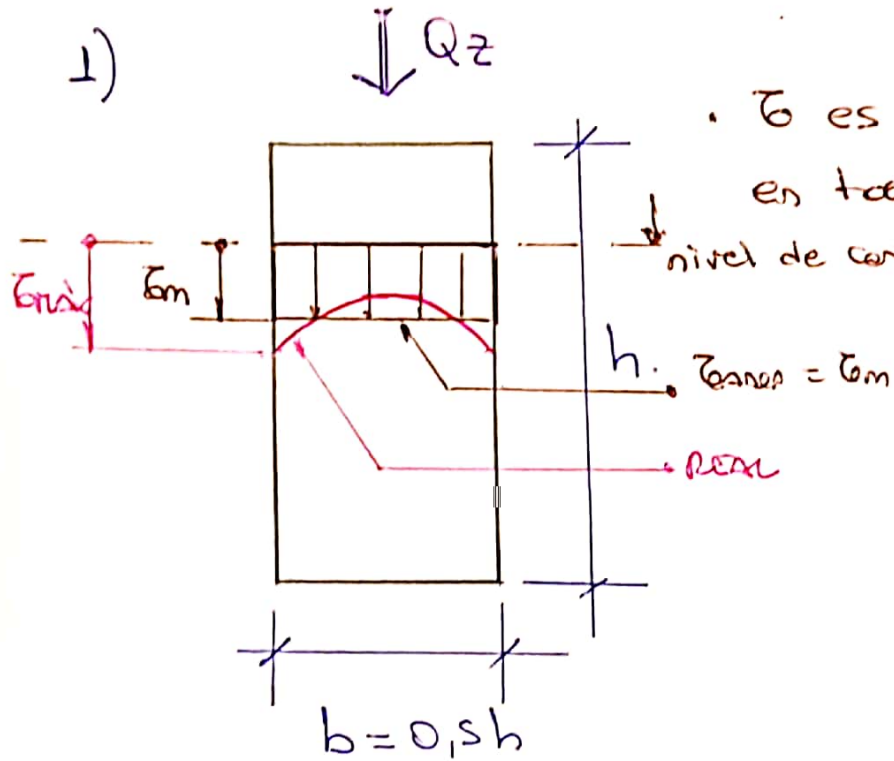
$y \rightarrow v$

$z \rightarrow w$

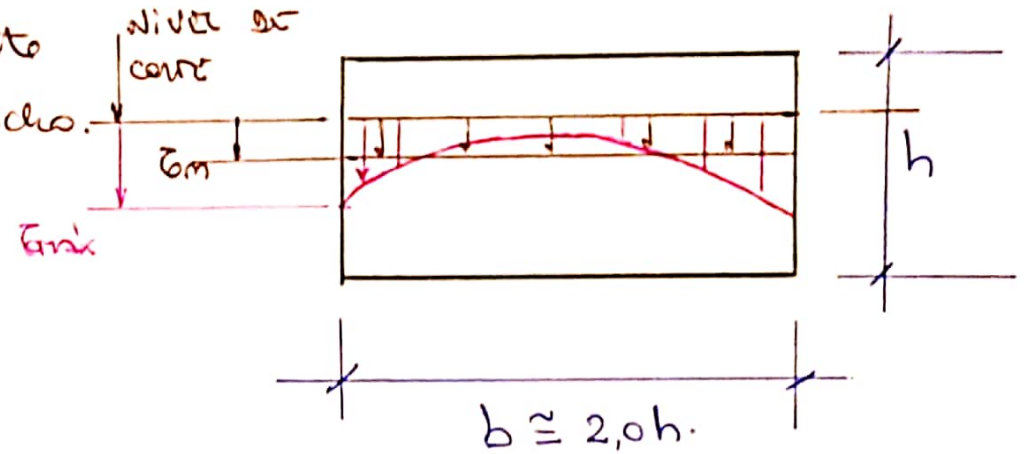


LIMITACIONES AL USO DE LA FÓRMULA DE JOHNSON:

1)



• τ es constante en todo el ancho.

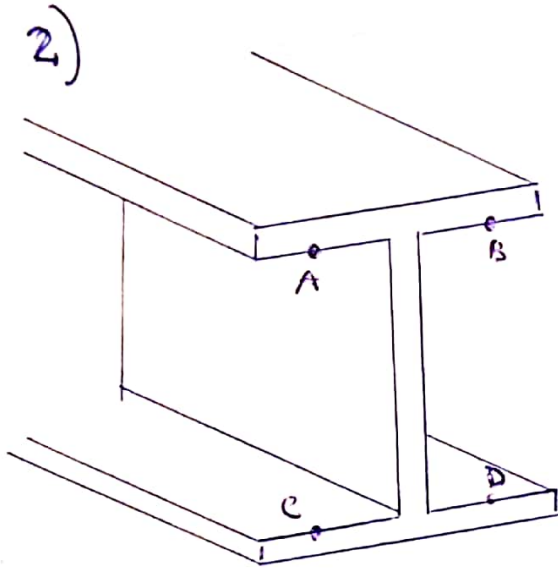


→ si $b \approx 2,0h$ →

$\tau_{max} \approx 1,40 \tau_m$

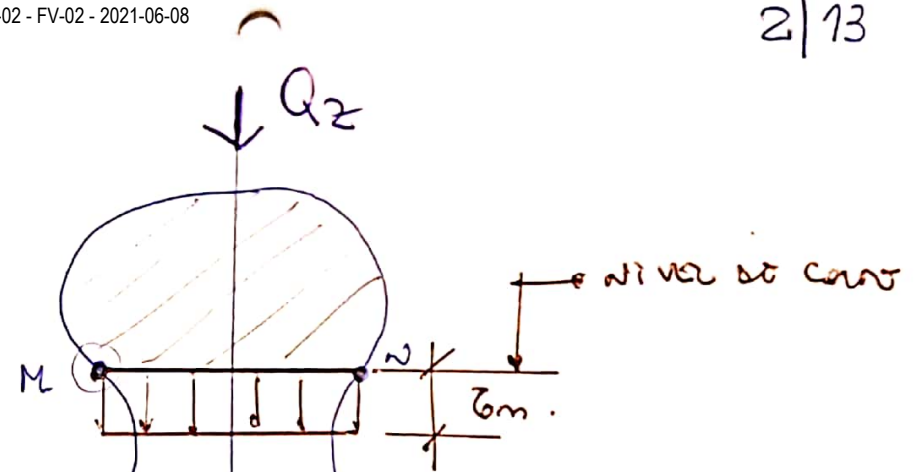
→ $\frac{b}{h} \uparrow \rightarrow \frac{\tau_{max}}{\tau_m} \uparrow$

→ si $b \approx 0,5h$ → $\tau_{max} \approx 1,03 \tau_m$.

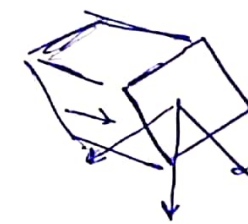


$$\left\{ \begin{array}{l} \tau_A = 0 \\ \tau_B = 0 \\ \tau_C = 0 \\ \tau_D = 0 \end{array} \right\} \rightarrow \begin{array}{l} \text{La fórmula de} \\ \text{Jourawski} \rightarrow \\ \rightarrow \text{Error} \end{array}$$

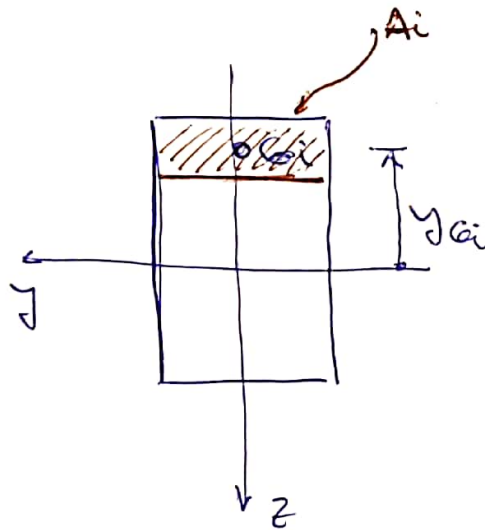
3)



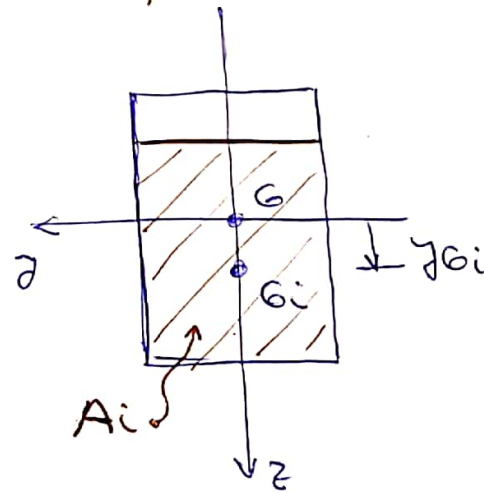
LA FÓRMULA DE
JOURAWSKY
↓
CONDUCE A
UN CIERTO
ERROR.



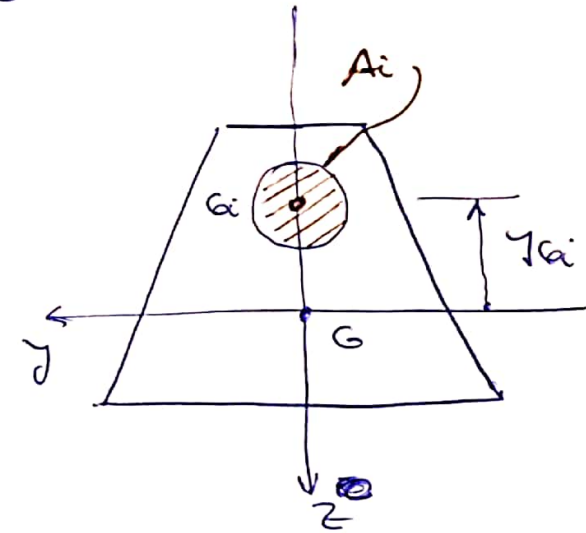
COMENTARIO SOBRE LA DETERMINACIÓN DEL MOMENTO ESTÁTICO.



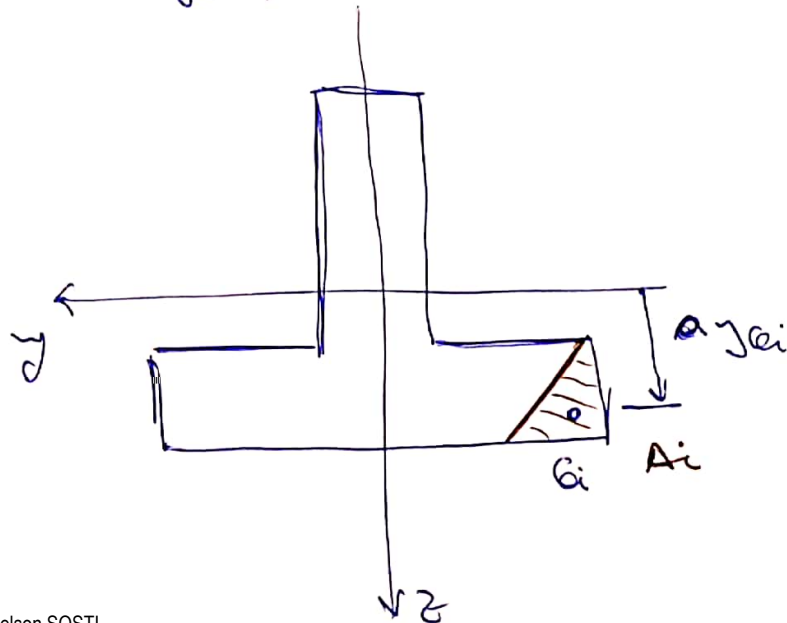
$$S_{A_i} = A_i \cdot y_{G_i}$$



$$S_{A_i} = A_i \cdot y_{G_i}$$

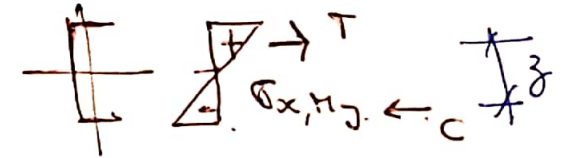
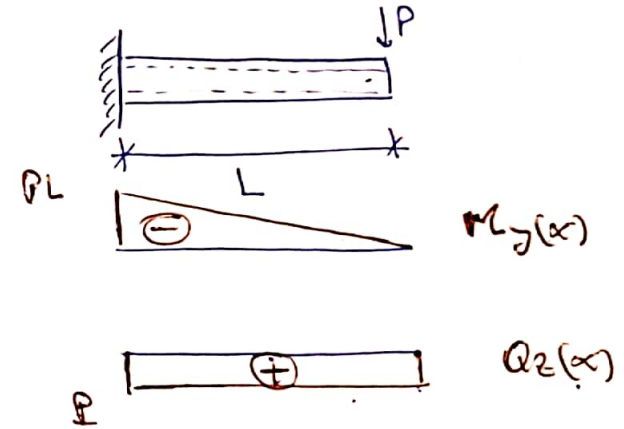
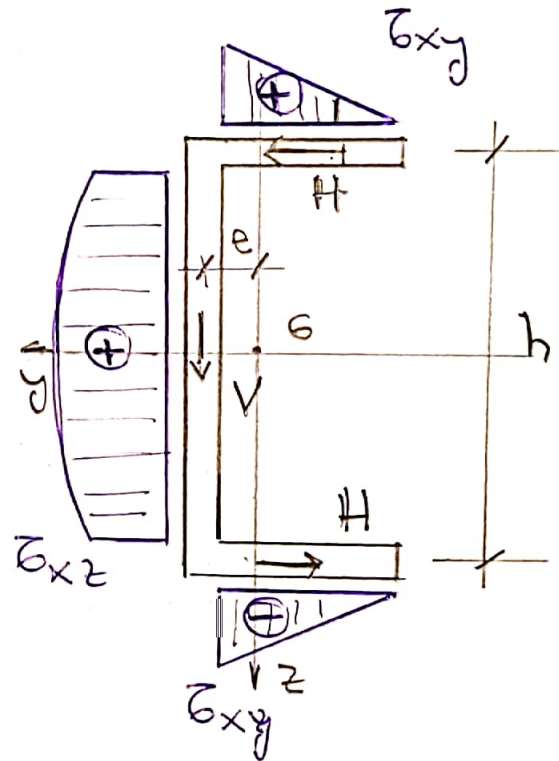
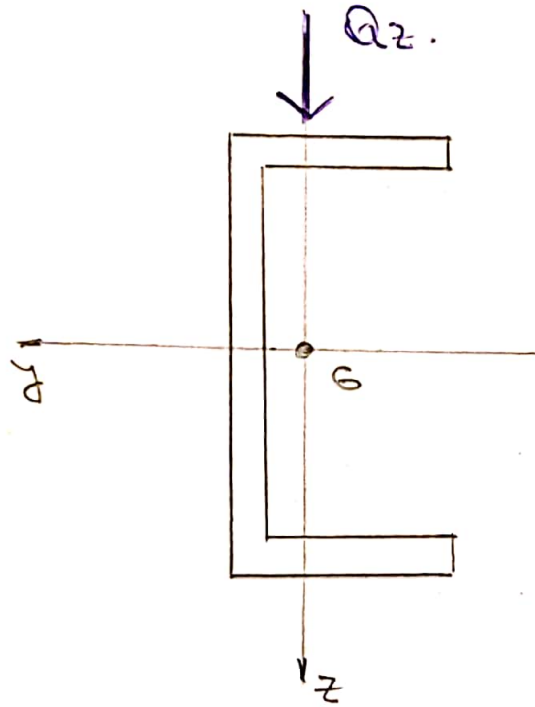


$$S_{A_i} = A_i \cdot y_{G_i}$$



$$S_{A_i} = A_i \cdot y_{G_i}$$

CENTRO DE CORTE:



$$\textcircled{X} \left\{ \begin{aligned} 0 &= \int_A \sigma \cdot dA \\ 0 &= T - C \end{aligned} \right. \checkmark$$

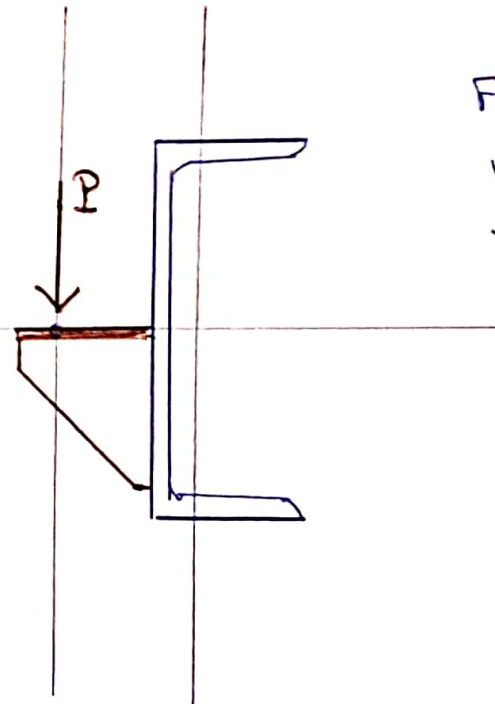
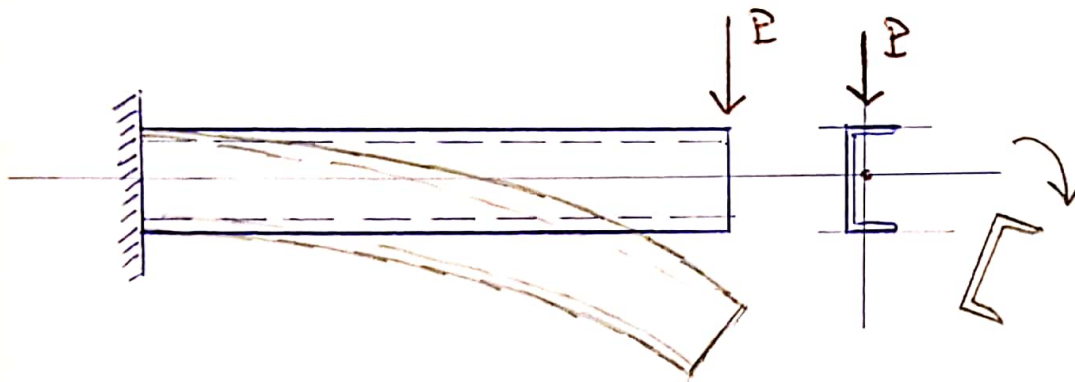
$$\left\{ \begin{aligned} M_x = 0 &= \int_A (\sigma_{xy} \cdot z + \sigma_{xz} \cdot y) dA = \\ &= H \cdot h + v \cdot e \end{aligned} \right. \checkmark$$

$$\textcircled{Y} \left\{ \begin{aligned} 0 &= \int_A \sigma_{xy} \cdot dA \\ 0 &= H - H \end{aligned} \right. \checkmark$$

$$\left\{ \begin{aligned} M_y = \int_A \sigma \cdot z \cdot dA = T \cdot z = C \cdot z \end{aligned} \right. \checkmark$$

$$\textcircled{Z} \left\{ \begin{aligned} Q_z = \int_A \sigma_{xz} \cdot dA = v \end{aligned} \right. \checkmark$$

$$\left\{ \begin{aligned} M_z = 0 = - \int_A \sigma \cdot y \cdot dA \end{aligned} \right. \checkmark$$

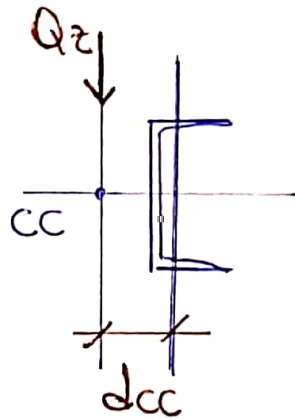


FORMAS DE
EVITAR LA
TELURACION.

SI.

$$Q_z \cdot d_{cc} = \underbrace{H \cdot h + V \cdot e}_{\text{TENSIONES}}$$

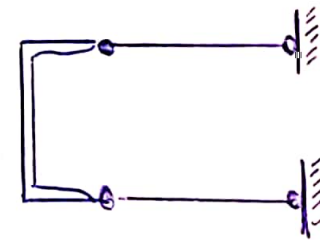
$$d_{cc} = \frac{Hh + Ve}{Q_z}$$

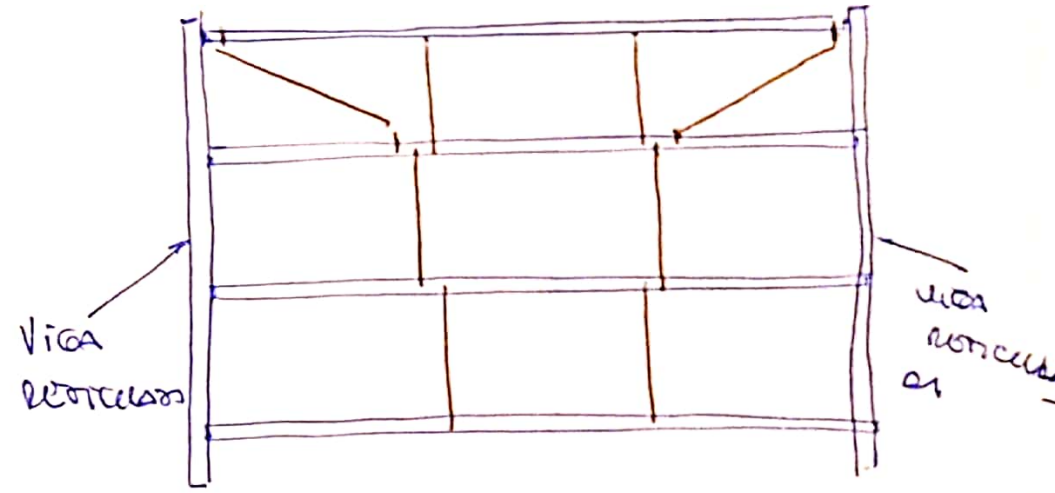
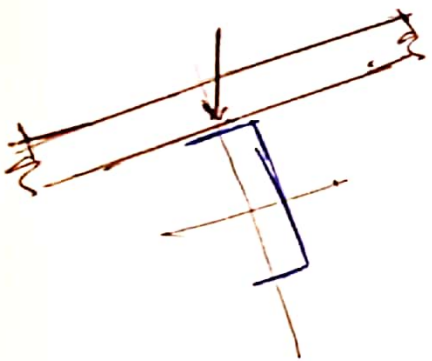
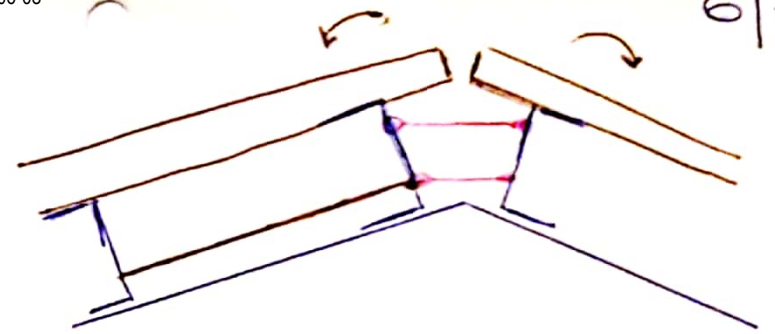
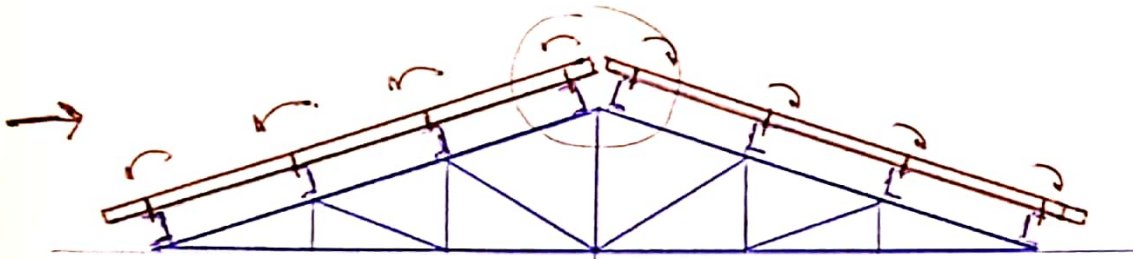


$$Q \cdot d_{cc} \Downarrow$$

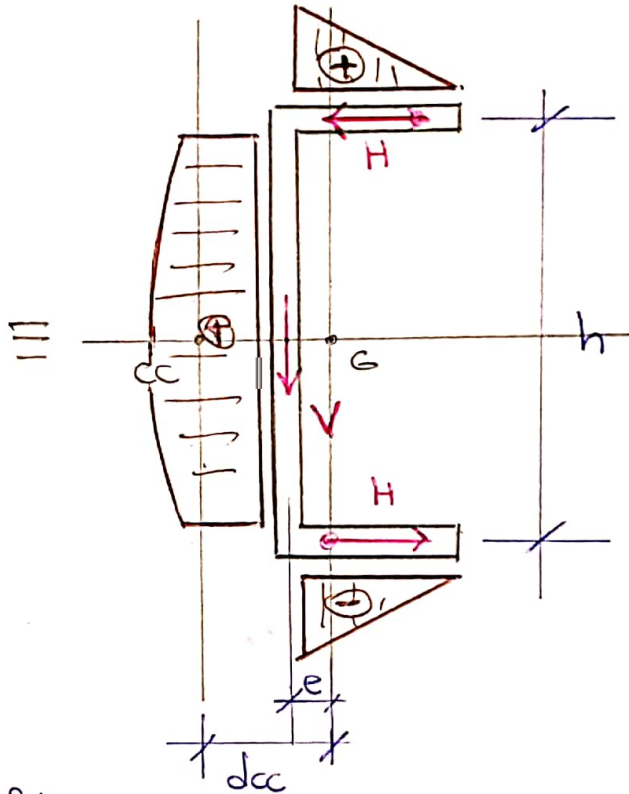
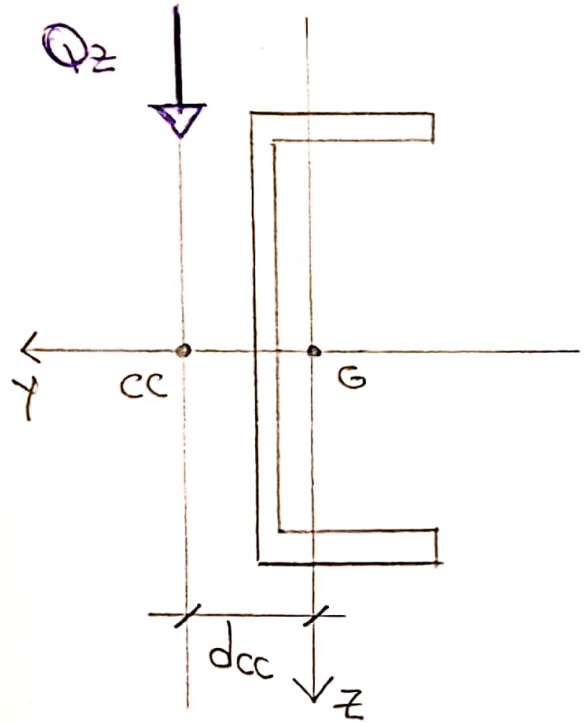
$d_{cc} \nrightarrow$ NO DEPENDE DE LAS
SOLICITACIONES, SÓLO
DEPENDE DE LAS CARACTERÍSTI-
CAS GEOMÉTRICAS DEL PERFIL.

$$d_{cc} = f(\text{geometría del perfil}).$$





1º CASO: ESFUERZO DE CORTE



SI TENS

I

$$Q_z \cdot d_{CC} = V \cdot e + H \cdot h$$

↓ ↪

TOMA MOMENTOS RESPECTO DEL BARRICENTRO

II RESPECTO DEL CENTRO DEL ALTA

$$Q_z (d_{CC} - e) = V \cdot 0 + H \cdot h$$

↪ ↪

→ LA BARRA SE FLEXIONA PERO NO SE TORSIONA.

→ LAS TENSIONES SE CALCULAN

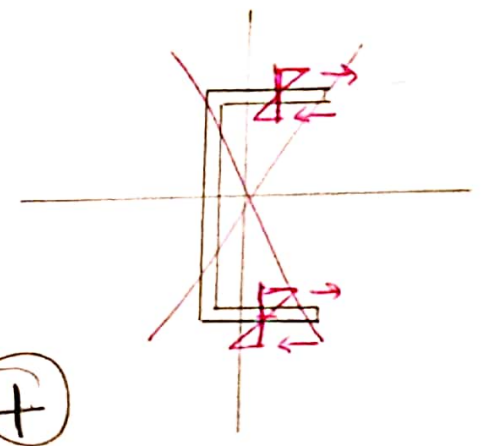
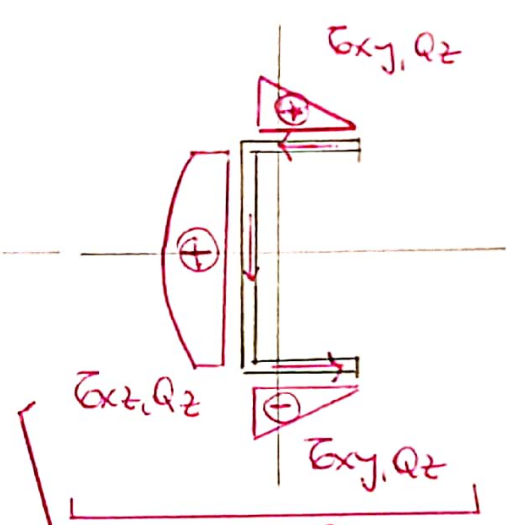
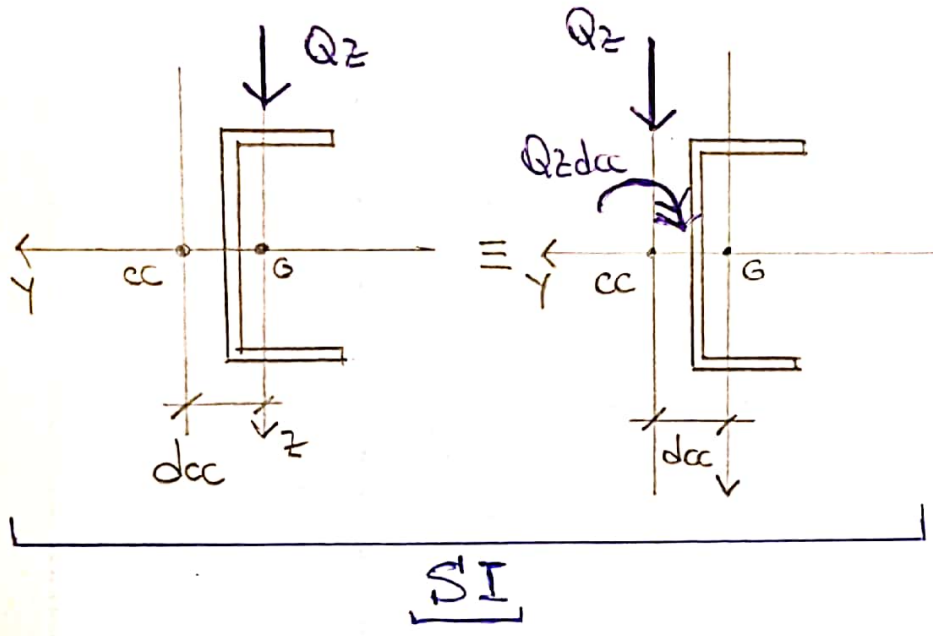
$$\sigma_x = \frac{M_y}{I_y} \cdot z \quad ; \quad \tau = \frac{Q \cdot S^*}{I_{ow} \cdot b^*}$$

III RESPECTO DEL CC:

$$Q_z \cdot 0 = -V \cdot (d_{CC} - e) + H \cdot h$$

= 0 ↪ ↪

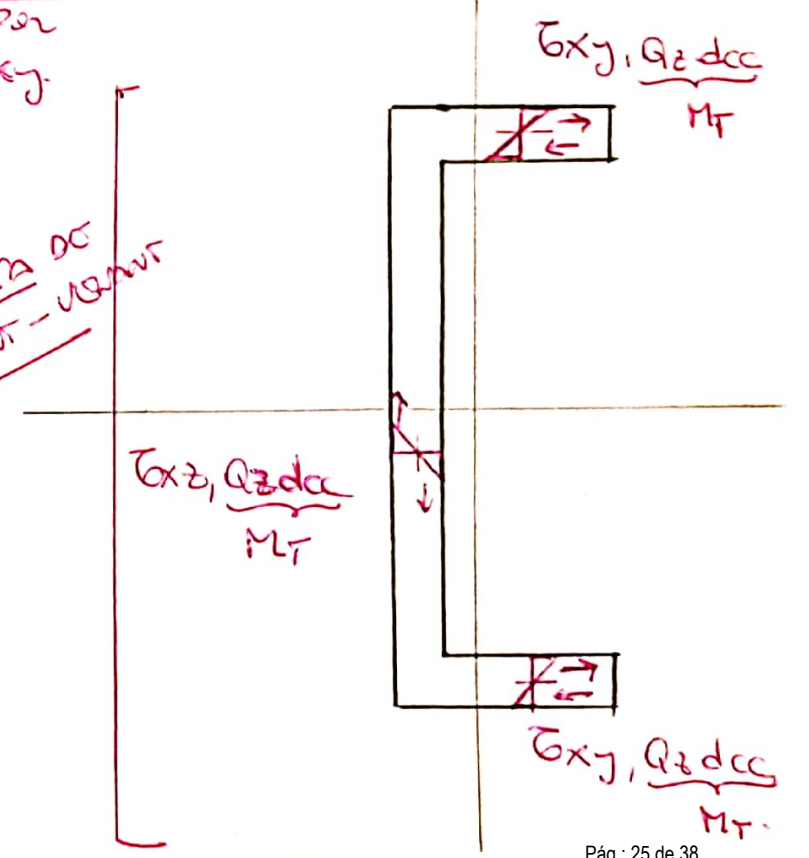
2º CASO: ESFUERZO DE CORTE



CALCULOS POR
JOURAUSKY.

TEORIA DE
SAINT-VENANT

TENSIONES



→ LA BARRA SE FLEXIONA Y SE
TORSIONA.

I) → SECCIONES DOBLEMENTE SÍMETRICAS → LA ^{BARRA} ~~SECCION~~ SE FLEXIONA Y NO SE TORSIONA.

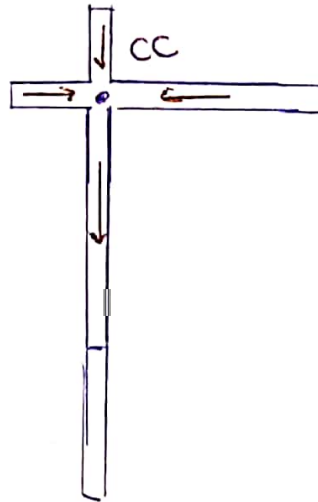
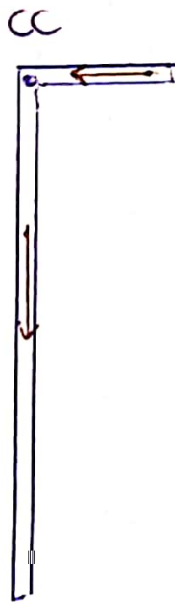
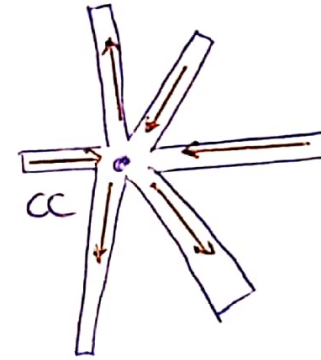
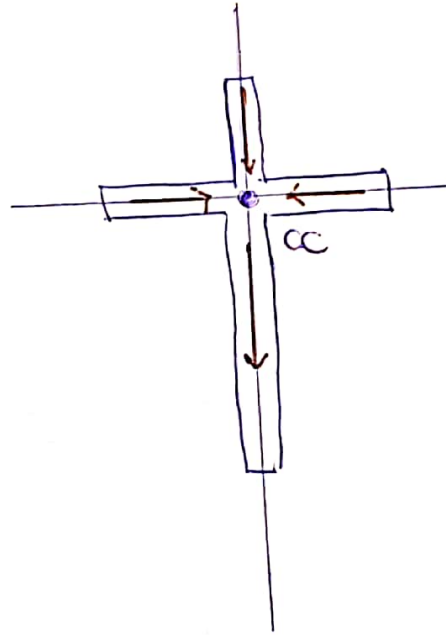
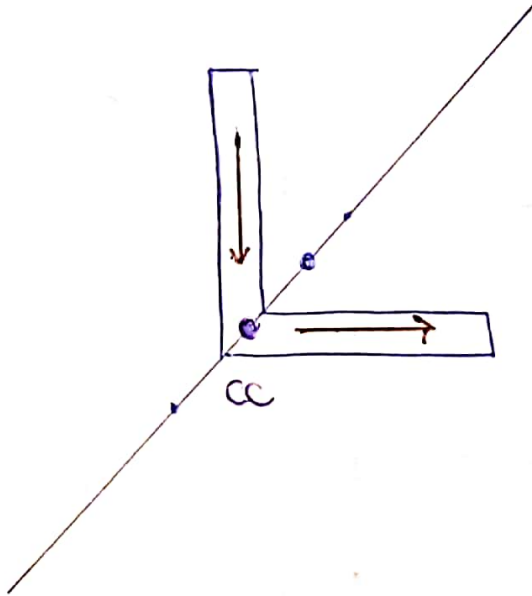
$$\rightarrow \sigma_x = \frac{M_y \cdot z}{I_y} \quad ; \quad \tau = \frac{Q \cdot S^*}{I_{xy} \cdot b}$$

(Vale también p/ secciones de simetría axial).

II) → SECCIONES C/ SIMPLI SÍMETRÍA :

II.1) → LA CARGA SE APLICA EN EL EJE DE SÍMETRÍA → LA ~~SECCION~~ BARRA SE FLEXIONA ~~PERO~~ Y NO SE TORSIONA.

II.2) → LA CARGA SE APLICA EN EL EJE NO SÍMETRICO → SE OCSIONA! PERO LA BARRA SE FLEXIONA PERO AHORA SE TORSIONA.

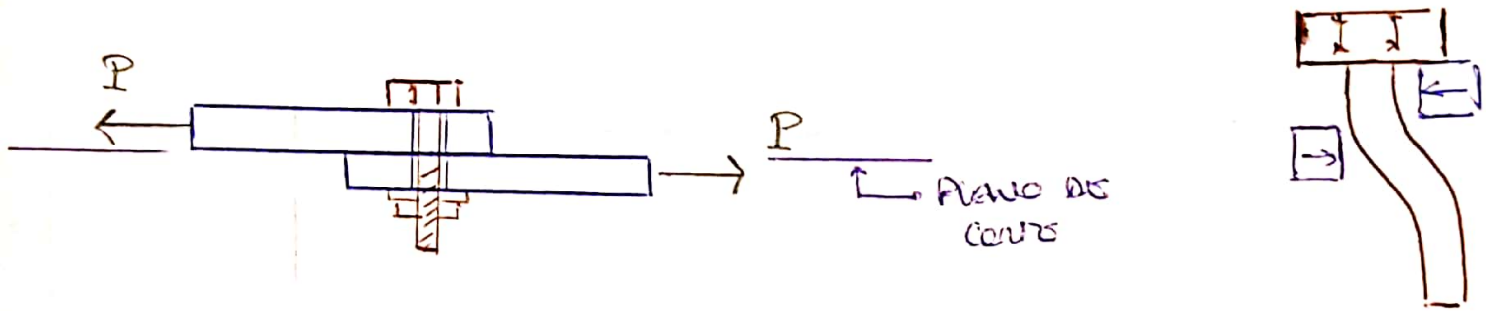


$$Q \cdot d_{cc} = H \cdot h + V \cdot e$$

Si Q pase por el $cc \rightarrow d_{cc} = 0$
 el 1º miembro es nulo.

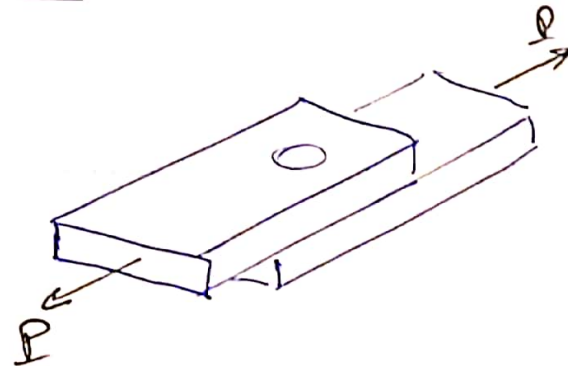
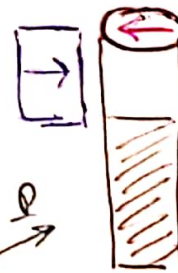
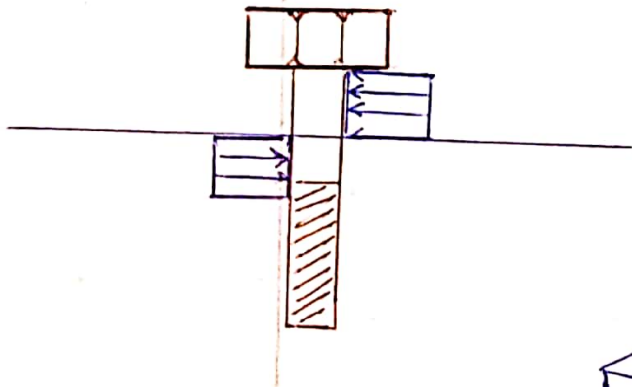
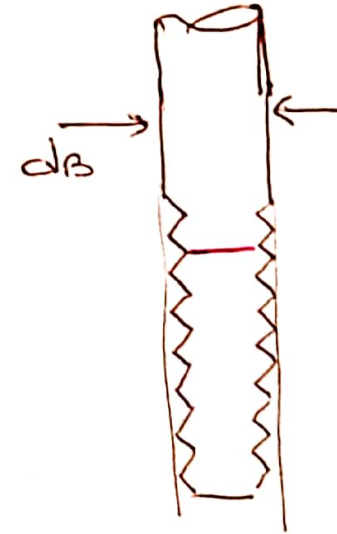
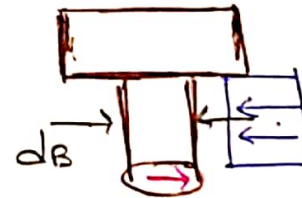
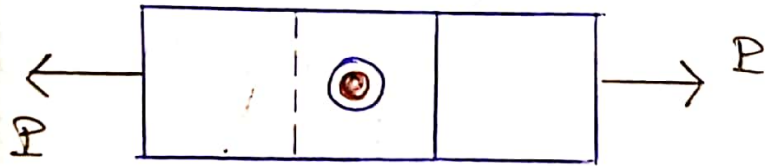
$$Q \cdot d_{cc} = 0$$

Uniques:

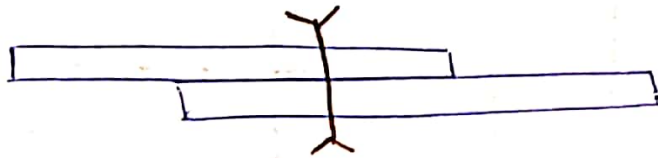
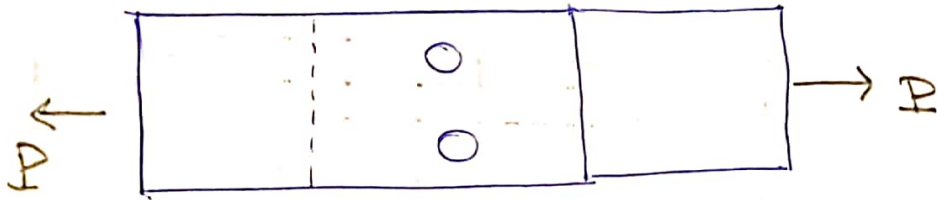


$$\frac{P}{A_B} = \tau_B \leq \tau_{adm_B}$$

$$\hookrightarrow \frac{\pi \cdot d_B^2}{4}$$



2 CHAPAS Y 2 BULONES



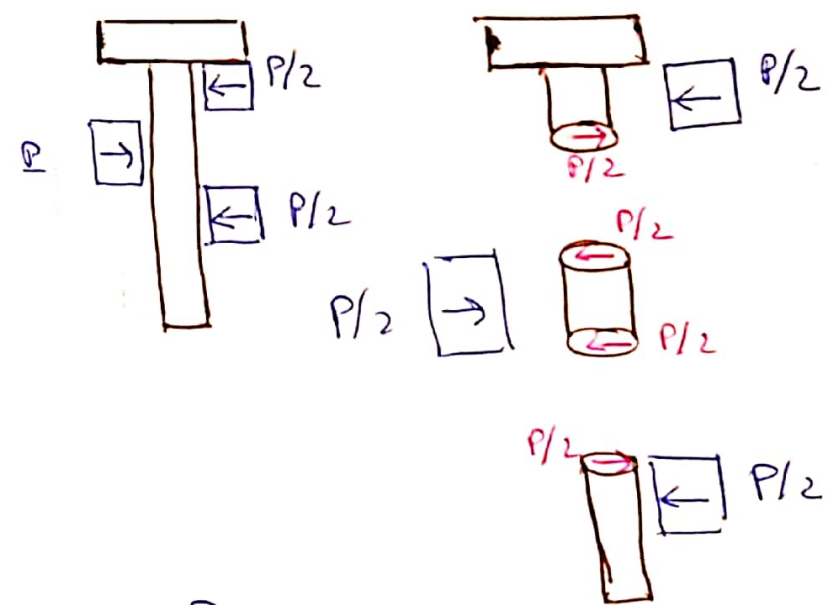
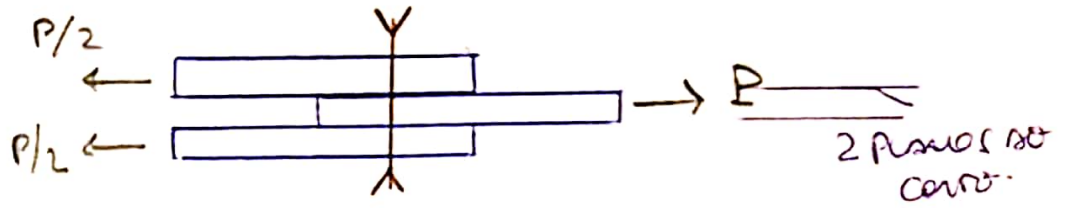
2 BULONES.

$$\tau_B = \frac{P}{n \cdot A_B} \leq \tau_{adm, B}$$

\swarrow
 n : nº de Bulones

$n = 2$

3 CHAPAS Y 2 BULONES



$$\tau_B = \frac{P}{m \cdot A_B}$$

\swarrow
 m : nº de placas de Corte.

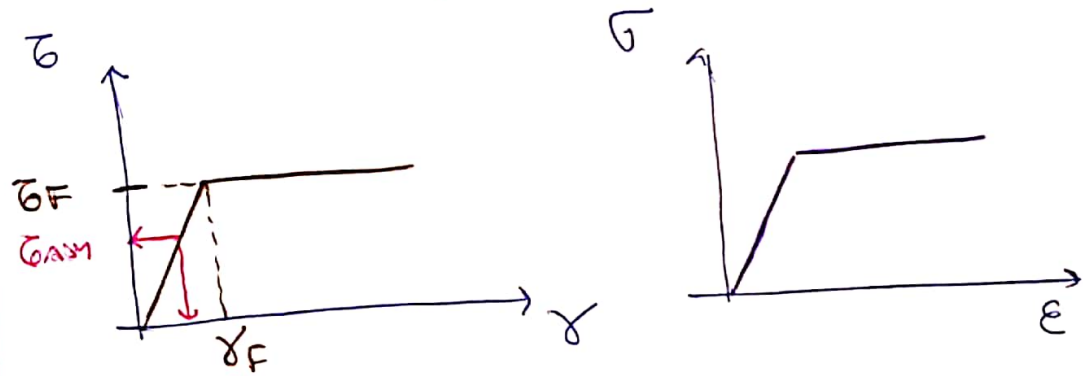
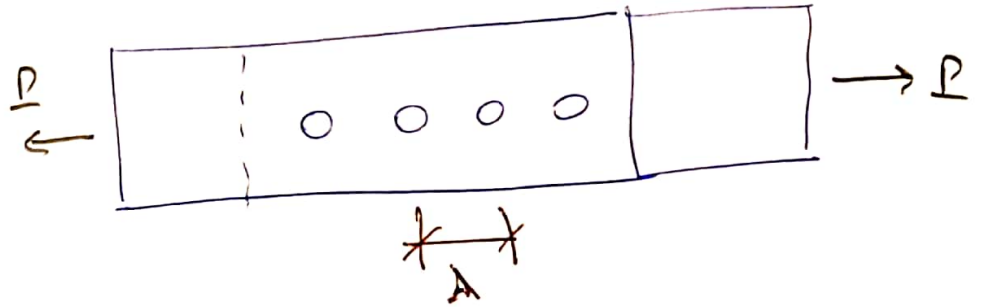
$m = 2$

3 CHAPAS Y 2 BULONES:

$$\tau_B = \frac{P}{n \cdot m \cdot A_B} \leq \tau_{ADM.B.}$$

n: n° de Bulones

m: n° de placas de corte.



$$\frac{\tau_F}{\tau_{ADM}}$$

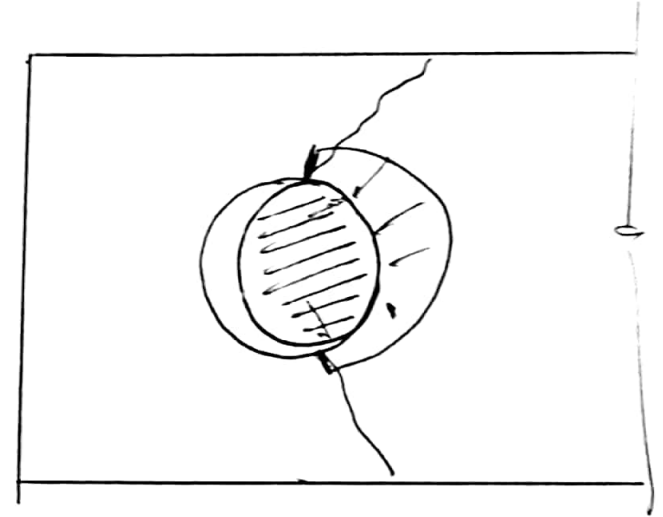
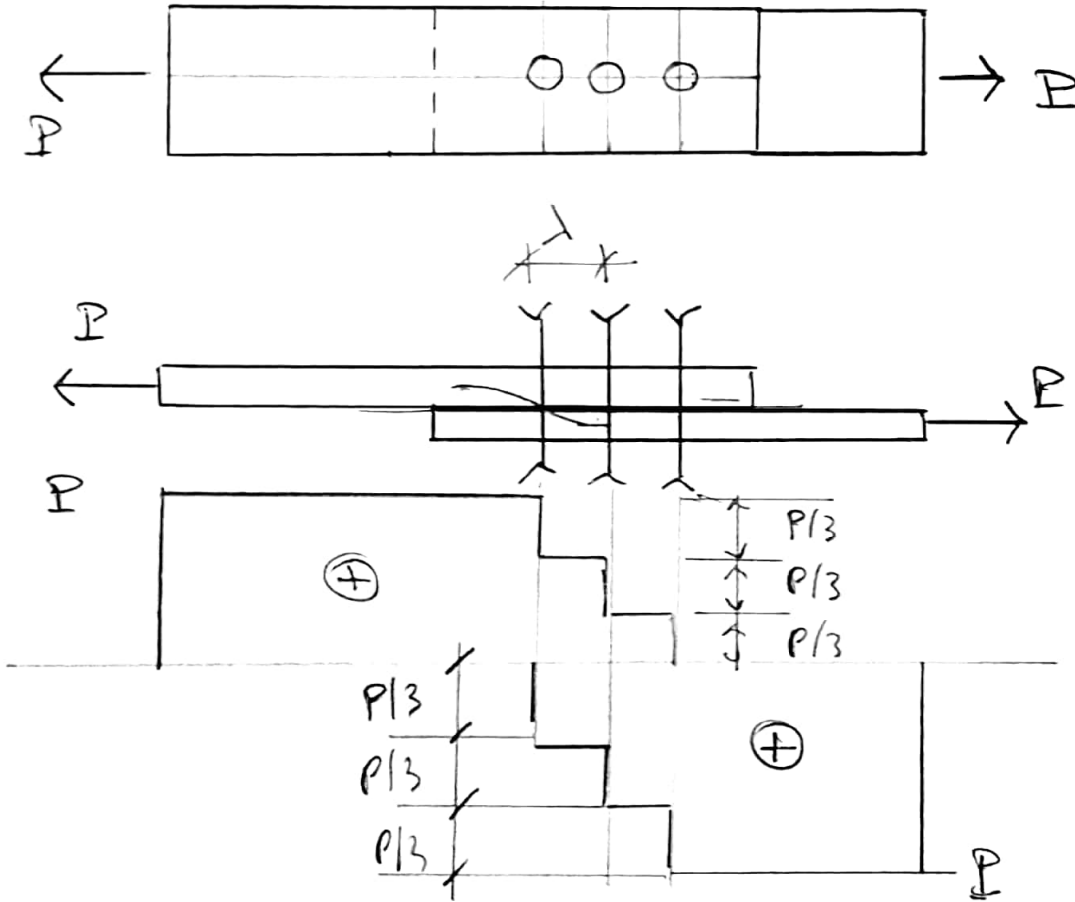
$$\tau_{ADM} = \tau_{TE} \leq \tau_{ADM}$$

$$\tau_{max} = \frac{M_T}{I_P} \cdot R \leq \tau_{ADM}$$

$$M_T \rightarrow \tau = \tau_F \rightarrow M_{TE} = M_{TF}$$

$$\tau_F = \frac{M_{TE}}{I_P} \cdot R \rightarrow M_{TE} = \frac{\tau_F \cdot I_P}{R}$$

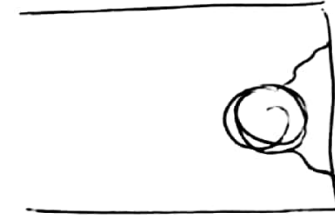
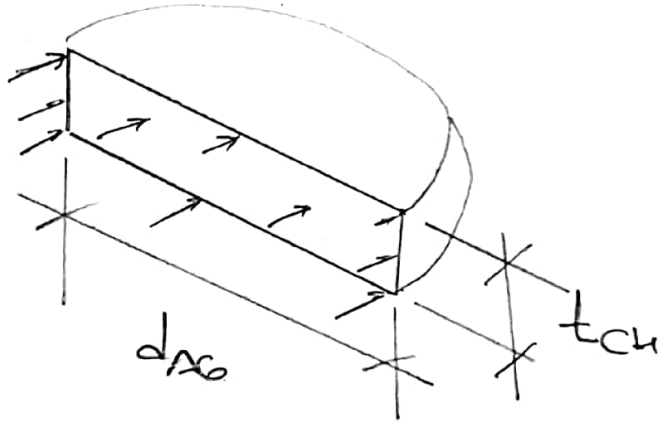
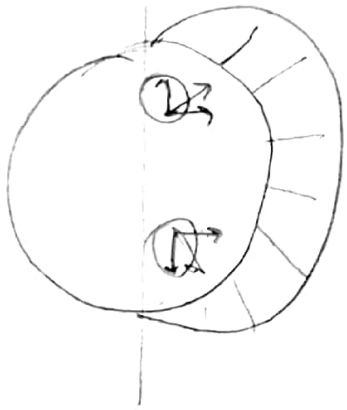
APLASTAMIENTO:



$$A_{\text{area}} = d_{\text{AG}} \cdot t$$

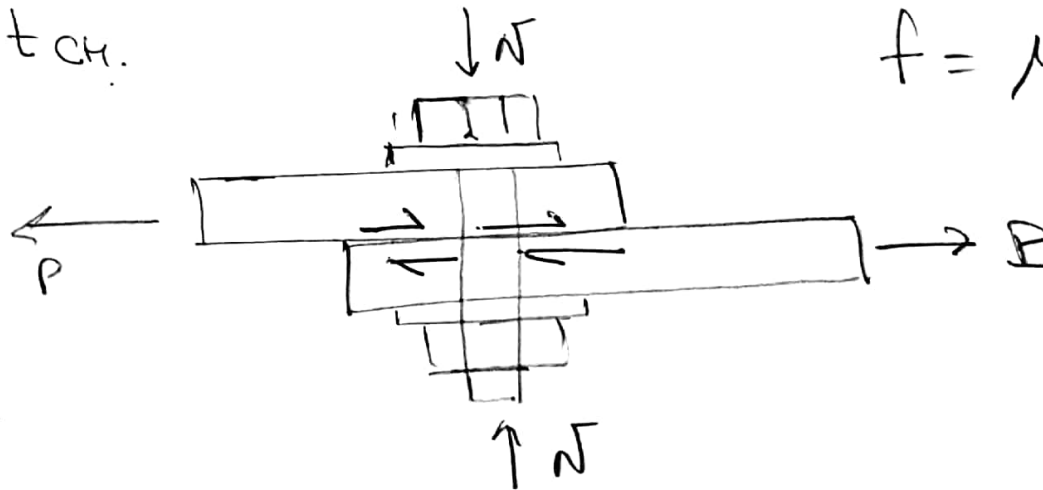
$$\frac{P}{3 \cdot A_{\text{area}}} \leq \sigma_{\text{ADM, APLAST}}$$

$$\frac{P}{n \cdot A_{\text{AG}}} \leq \sigma_{\text{ADM, APLAST}}$$



$$A^* = d_{AG} \cdot t_{CH}$$

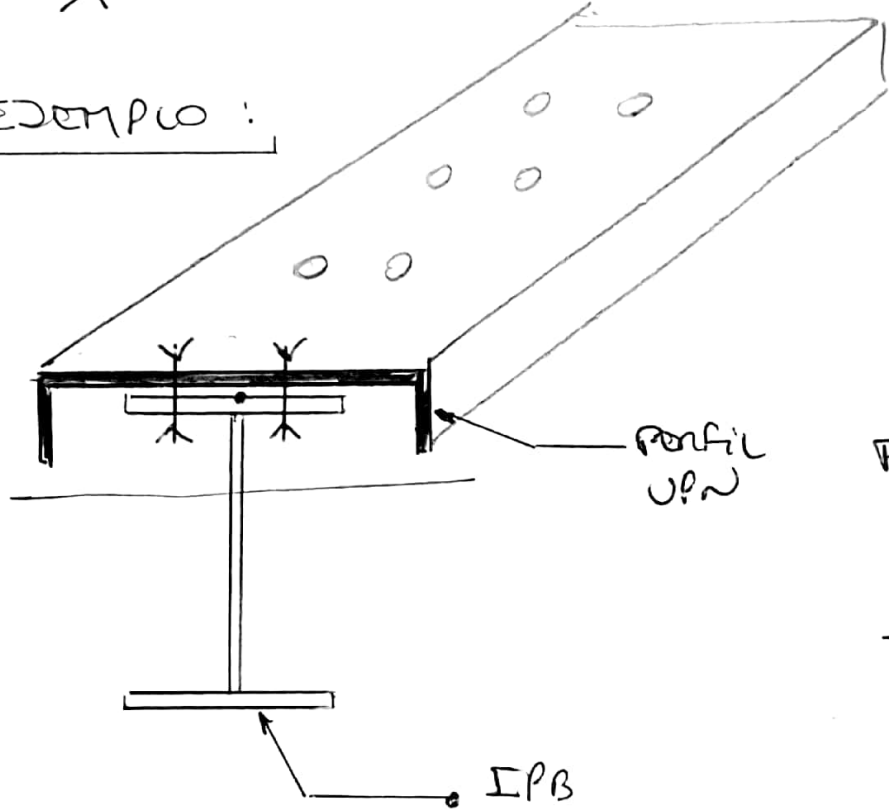
$$f = \mu N$$



$$\frac{P_{Bi}}{\lambda} = F_{Bz} \text{ Por Unidad de Longitud} = f = q.$$



EXAMPLE:



$$F_{Bi} = A_{Bi} \cdot \tau_{adm, B}$$

$$F_{B, tot} = n \cdot A_{Bi} \tau_{adm, B} \quad (n=2)$$

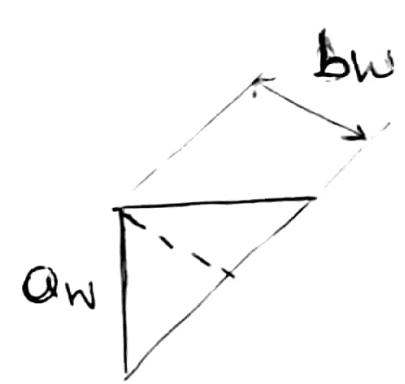
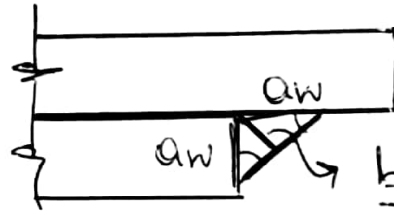
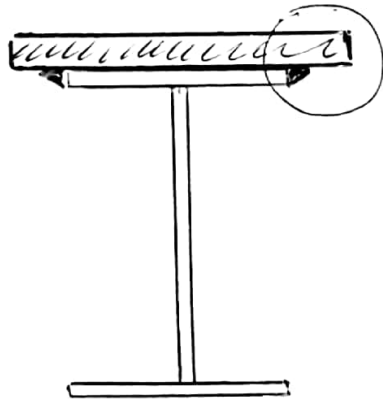
Flujo de corte = $\frac{F_{B, tot}}{\lambda} = \frac{F_{B, tot}}{\lambda}$

$$\tau_{cm} = \frac{Q \cdot S^*}{I_{ov} \cdot b} \rightarrow b \tau_{cm} = \frac{Q S^*}{I_{ov}}$$

Betón:
 d_B
 espesor
 superficie

$$\frac{Q \cdot S^*}{I_{ov}} = \frac{F_{B, tot}}{sep.} = \frac{F_{B, tot}}{\lambda}$$

$$S^* = A_{UPN} \cdot (z_{G, UPN} - z_{G, SC})$$



$$\frac{bw}{Aw} = \cos 45^\circ$$

$$bw = Aw \cdot \cos 45^\circ$$

$$Aw = bw \cdot \sin 45^\circ \cdot n_{clw}$$

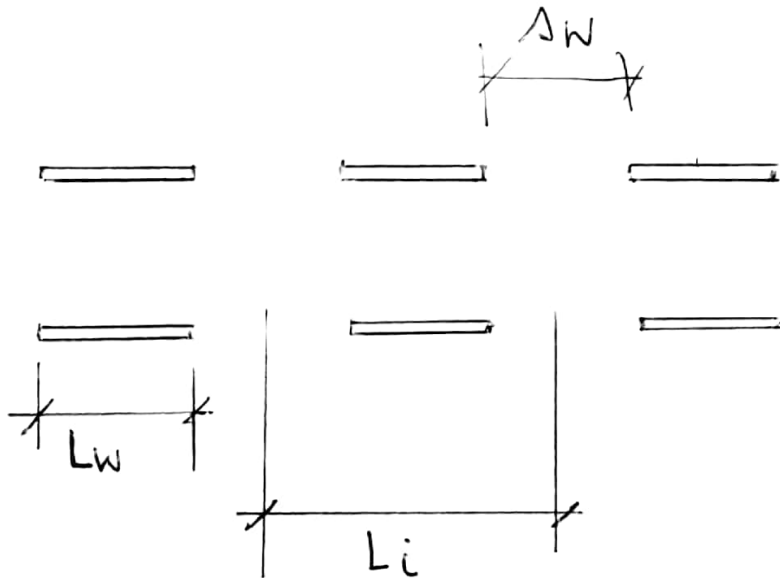
$\tau_{clw, w}$

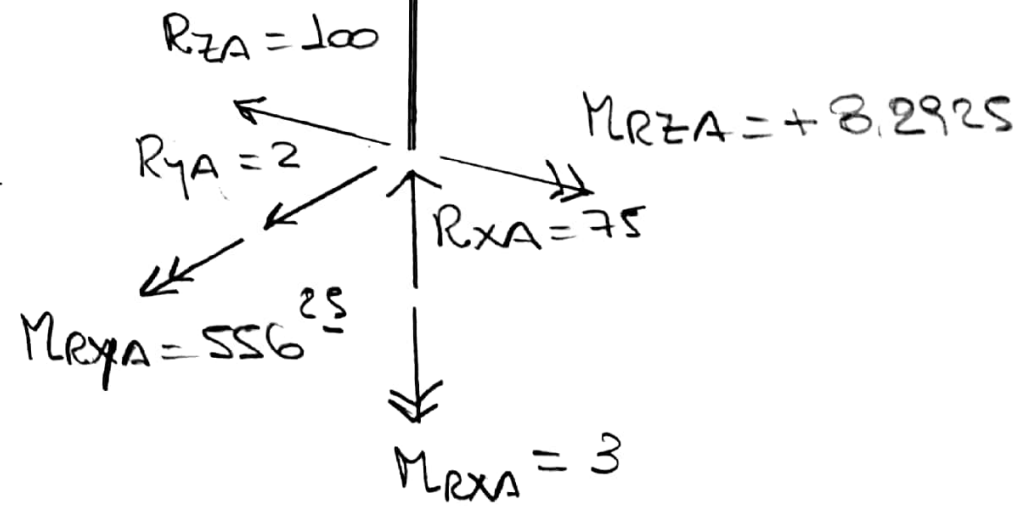
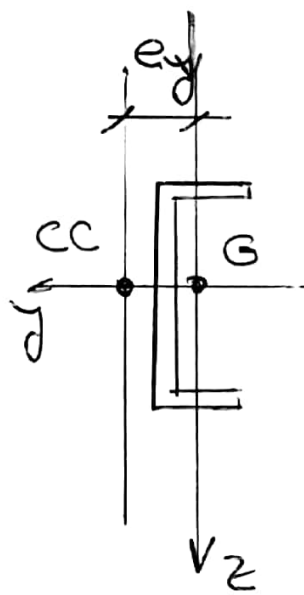
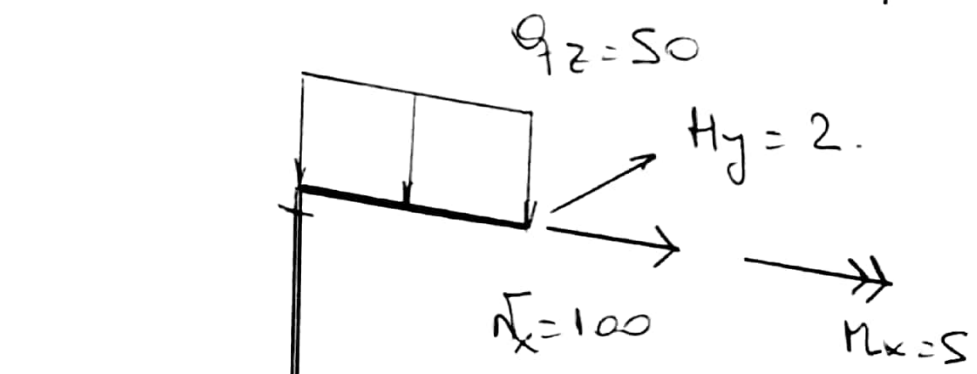
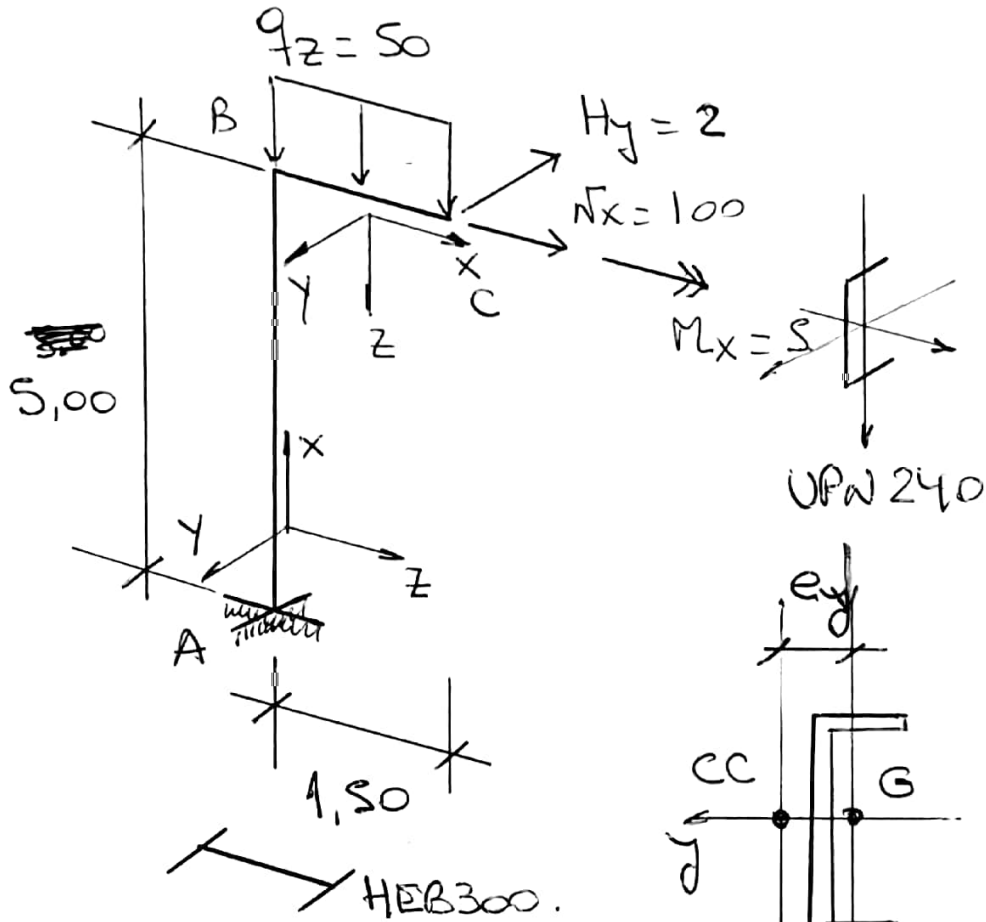
$$Aw \cdot \tau_{clw, w} = F_w$$

$$q = \frac{F_w}{L_i}$$

$$L_i = 2 \cdot \frac{bw}{2} + L_w$$

$$q = \frac{q \cdot S^*}{I_{clw}}$$



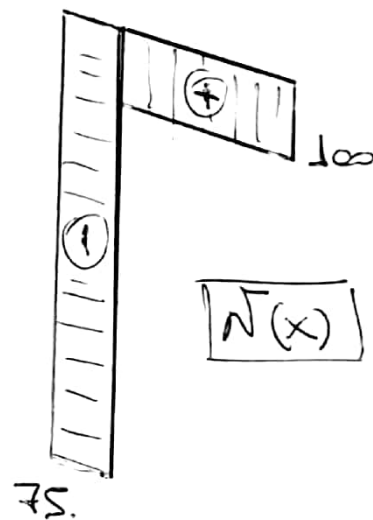
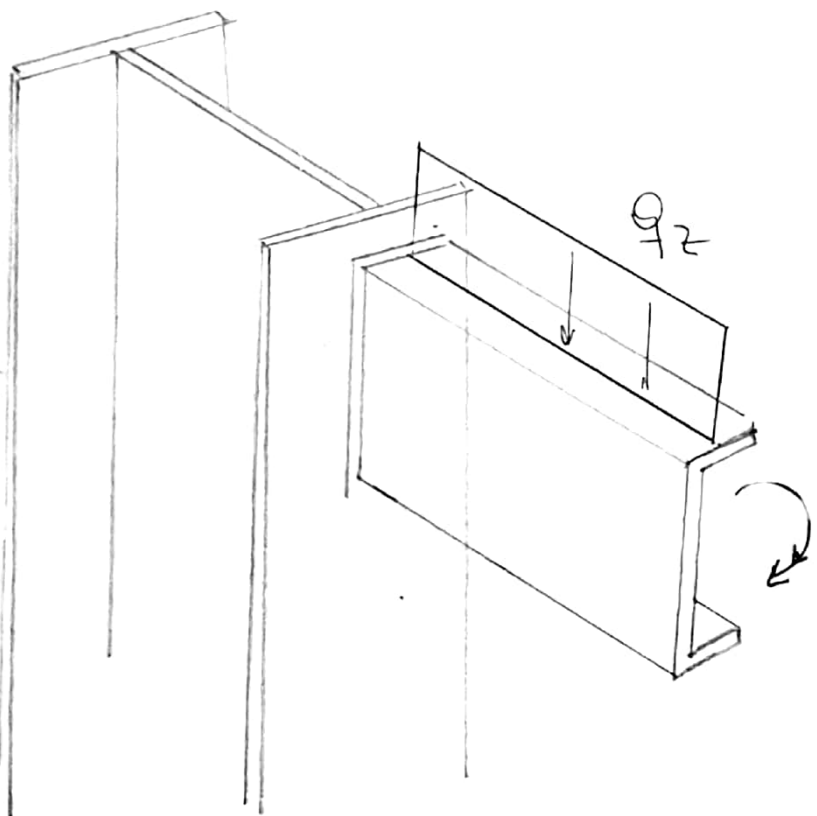


[L] = m [M] = kNm
 [P] = kW
 [q] = kW/m

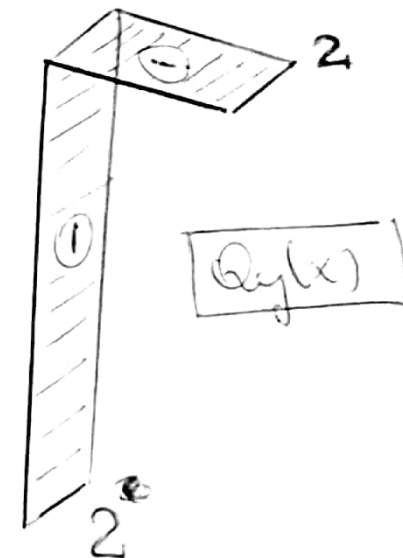
$e_y = 4,39 \text{ cm}$

$M_{RYA} = -(-50 \cdot 1,50 \cdot 0,75 - 100 \cdot 5) = +556 \text{ }^{\text{25}}$

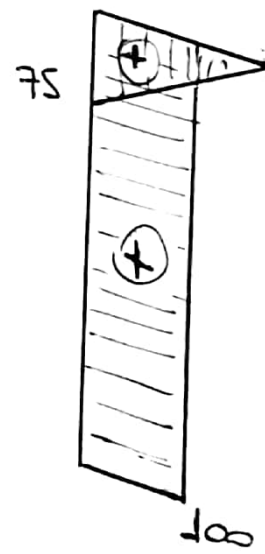
$M_{RZA} = -\left[-2 \cdot 5 + 5 - 50 \cdot 1,50 \cdot \frac{4,39}{100}\right] = +8,2925$



$$N(x)$$



$$Q_y(x)$$



$$Q_z(x)$$

