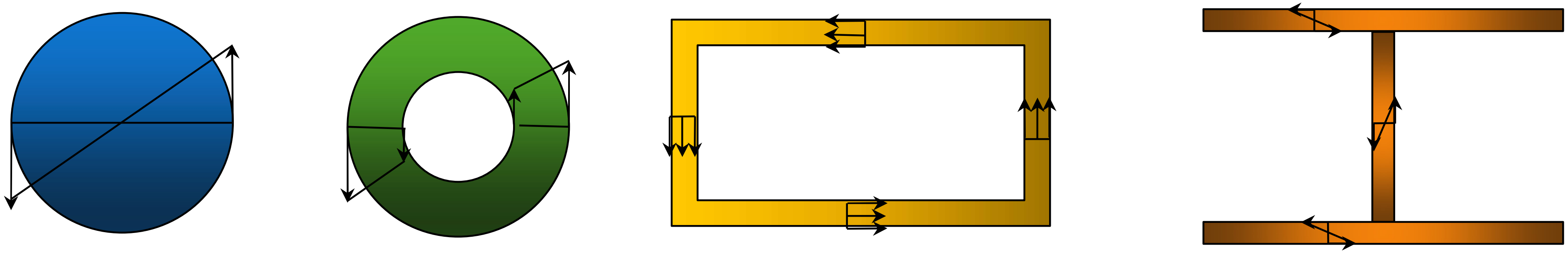
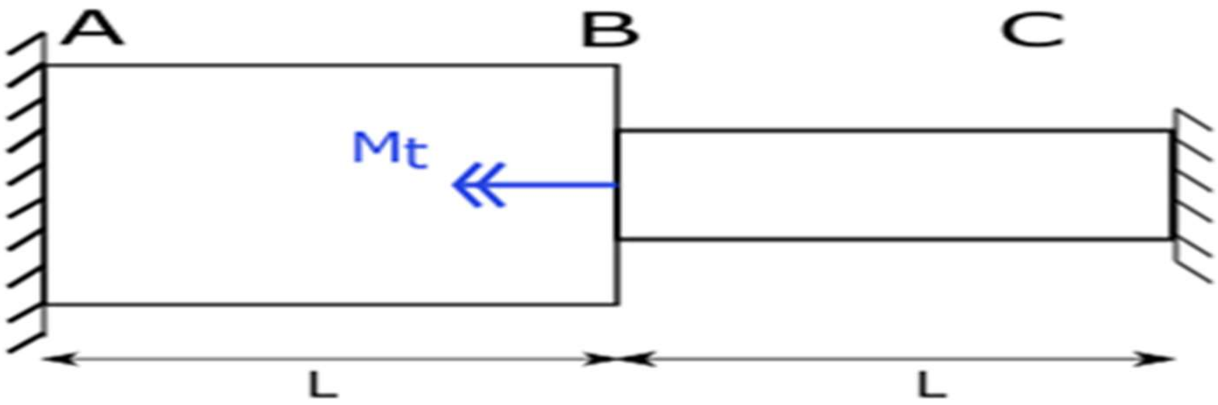
Solicitación por Torsión en Régimen Elástico



Constanza Ruffinelli – Tania Poletilo – Manuela Medina

Ejercicio 1:

1. Determinar M_t admisible
2. Trazar diagramas de M_t, curvatura y giros
3. Tensiones tangenciales para cada sección



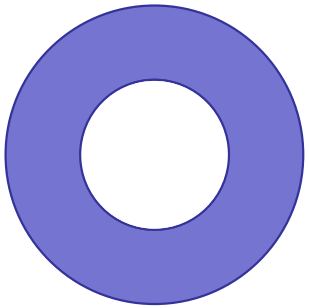
Datos:

$L = 4\ m$

$G = 85\ GPa$

$\tau_{adm} = 50\ MPa$

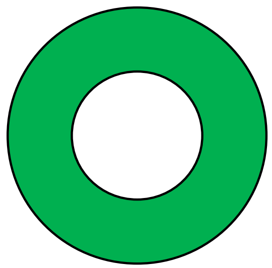
Sección AB:



$R_1 = 6\ cm$

$r_1 = 5{,}5\ cm$

Sección BC:

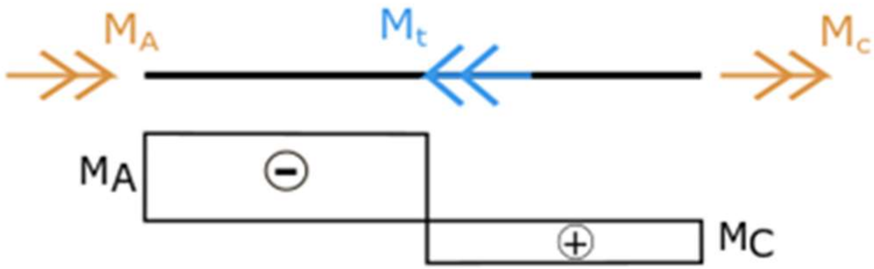


$R_2 = 4\ cm$

$r_2 = 3{,}5\ cm$

Resolución por inspección

A) Ecuación de equilibrio $M_t = M_A + M_C$



Usamos la teoría de Coulomb

$$J_p = \frac{\pi}{32} \cdot ({D_e}^4 - {D_i}^4)$$

$$\boldsymbol{J_{AB} = 598,37\ cm^4}$$

$$\boldsymbol{J_{BC} = 166,41\ cm^4}$$

B) Ecuación de compatibilidad $\theta_A = 0$ $\theta_C = 0$

$$\theta_B = \theta_A + \int_A^B \chi\, dx = -\frac{M_a}{(G \cdot J)_{AB}} \cdot L_{AB}$$

$M_c = M_t - M_A$ ⟹

$$\frac{M_a}{(G \cdot J)_{AB}} \cdot L_{AB} = \frac{M_t - M_a}{(G \cdot J)_{BC}} \cdot L_{BC}$$

$$\theta_c = \theta_B + \int_B^C \chi\, dx = -\frac{M_a}{(G \cdot J)_{AB}} \cdot L_{AB} + \frac{M_c}{(G \cdot J)_{BC}} \cdot L_{BC} = 0$$

Despejando: $M_A = \dfrac{M_t \cdot J_{AB}}{J_{AB} + J_{BC}}$ ⟹

$$M_A = 0{,}782\ M_t$$

$$M_C = 0{,}218\ M_t$$

1. $M_{t\,adm}$: verificamos ambas secciones y tomamos el menor M_t

Sección 1

$$\tau_{adm} = \frac{M_A}{J_{AB}} \cdot R_1 = \frac{0{,}782 \cdot M_t}{J_{AB}} \cdot R_1$$

$$M_t = \frac{\tau_{adm} \cdot J_{AB}}{R_1 \cdot 0{,}782} = 637{,}3\ kN\ cm$$

Sección 2

$$\tau_{adm} = \frac{M_c}{J_{BC}} \cdot R_2 = \frac{0{,}218 \cdot M_t}{J_{BC}} \cdot R_2$$

$$M_t = \frac{\tau_{adm} . J_{BC}}{R_2 . 0{,}218} = 956\ kN\ cm$$

El momento máximo admisible es el menor de los dos

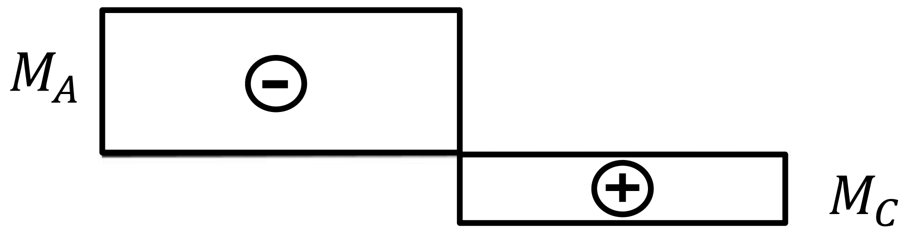
$$M_t = 637{,}3\ kN\ cm \Rightarrow$$

$$M_A = 498{,}6\ kN\ cm$$

$$M_C = 138{,}7\ kN\ cm$$

2. Diagramas

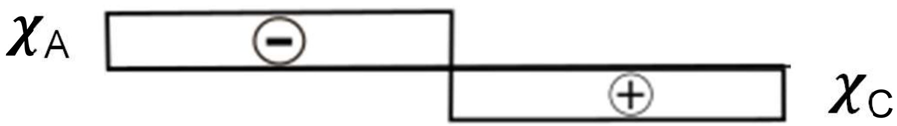
- Momento



$$M_A = 498{,}6\ kN\ cm$$

$$M_C = 138{,}7\ kN\ cm$$

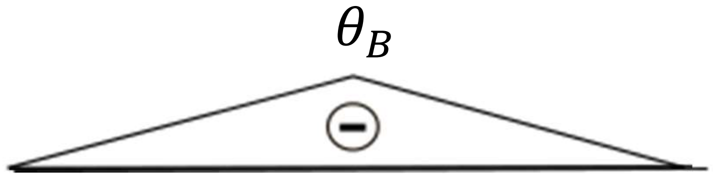
- Curvatura



$$\chi_A = \frac{M_A}{G \cdot J_{AB}} = 9{,}8 \cdot 10^{-5}\,\frac{1}{cm}$$

$$\chi_C = \frac{M_C}{G \cdot J_{BC}} = 9{,}8 \cdot 10^{-5}\,\frac{1}{cm}$$

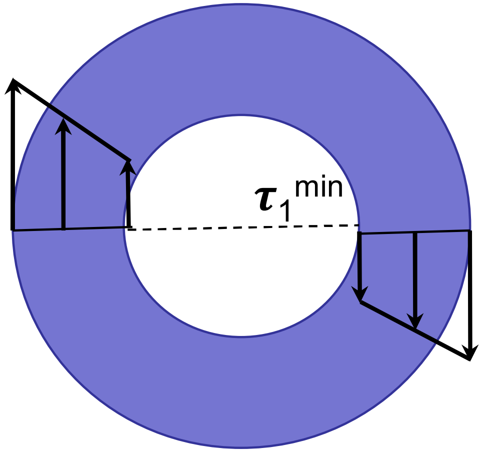
- Giro



$$\theta_B = \int \chi\, dx = \frac{M_A \cdot L}{G \cdot J_{AB}} = 0{,}039\ rad$$

3. Tensiones tangenciales

Sección 1 - AB

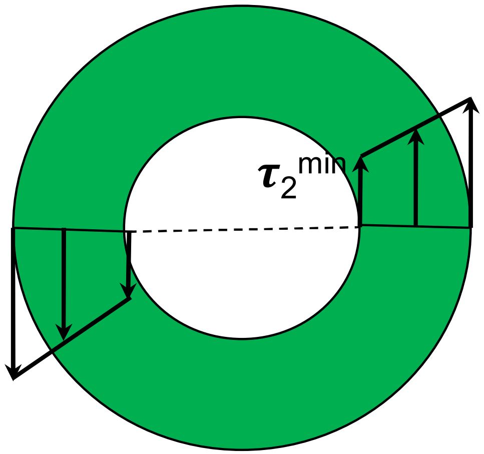


$$M_A = 498{,}6\ kN\ cm \qquad J_{AB} = 598{,}37\ cm^4$$

$$\tau_1^{mín} = \frac{M_A}{J_{AB}} \cdot r_1 = 4{,}58\,\frac{kN}{cm^2}$$

$$\tau_1^{máx} = \frac{M_A}{J_{AB}} \cdot R_1 = 5\,\frac{kN}{cm^2}$$

Sección 2 - BC

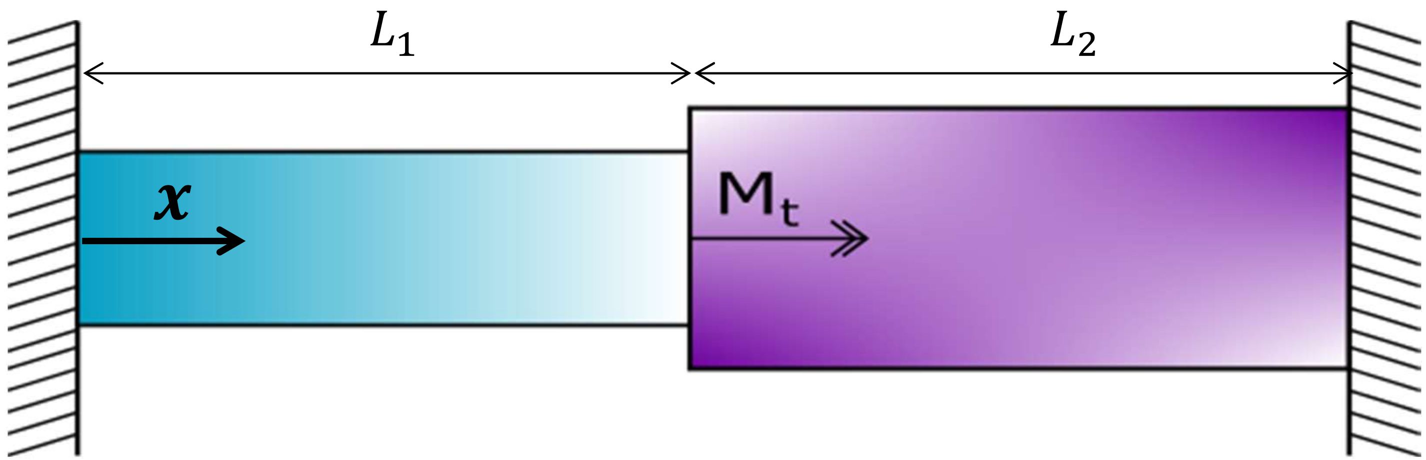


$$M_C = 138{,}7\ kN\ cm \qquad J_{BC} = 166{,}41\ cm^4$$

$$\tau_2^{mín} = \frac{M_C}{J_{BC}} \cdot r_2 = 2{,}92\,\frac{kN}{cm^2}$$

$$\tau_2^{máx} = \frac{M_C}{J_{BC}} \cdot R_2 = 3{,}33\,\frac{kN}{cm^2}$$

Ejercicio 2: Calcular las reacciones de vínculo

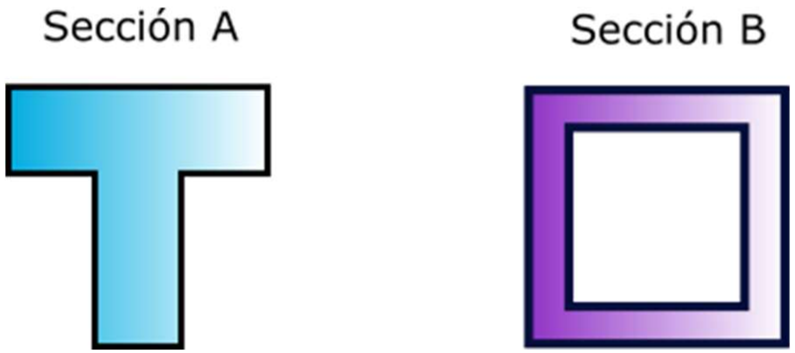


Datos:

$$L_1 = 2\ m$$

$$L_2 = 3\ m$$

$$M_t = 10\ kN\ m$$

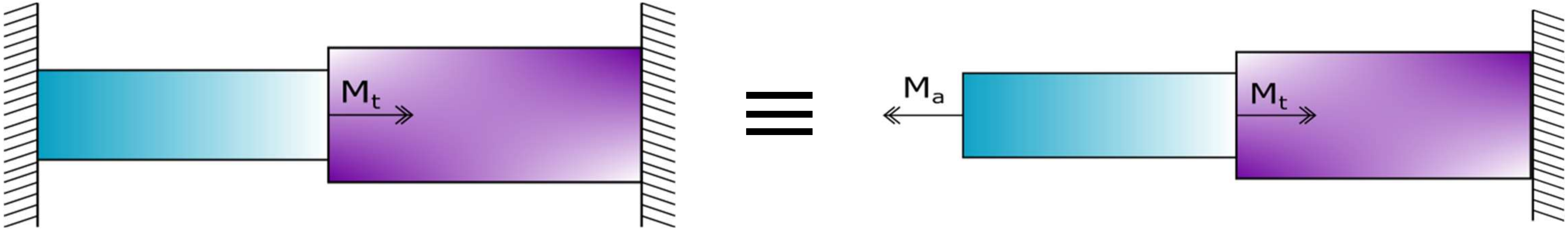


$$G_a = 8000 \frac{kN}{cm^2} \qquad G_b = 6000 \frac{kN}{cm^2}$$

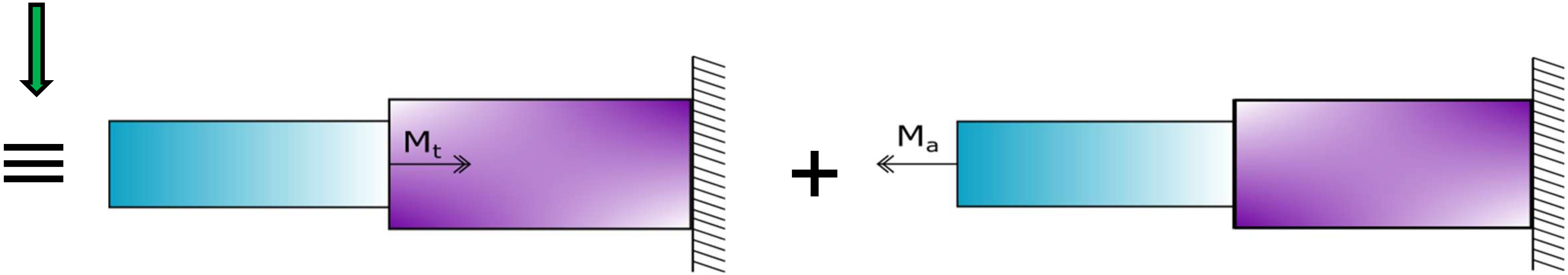
$$J_{ta} = 115\ cm^4 \qquad J_{tb} = 800\ cm^4$$

Para resolver éste ejercicio utilizaremos el Método de las Incógnitas Estáticas

Método de las incógnitas estáticas



Superposición
de Efectos

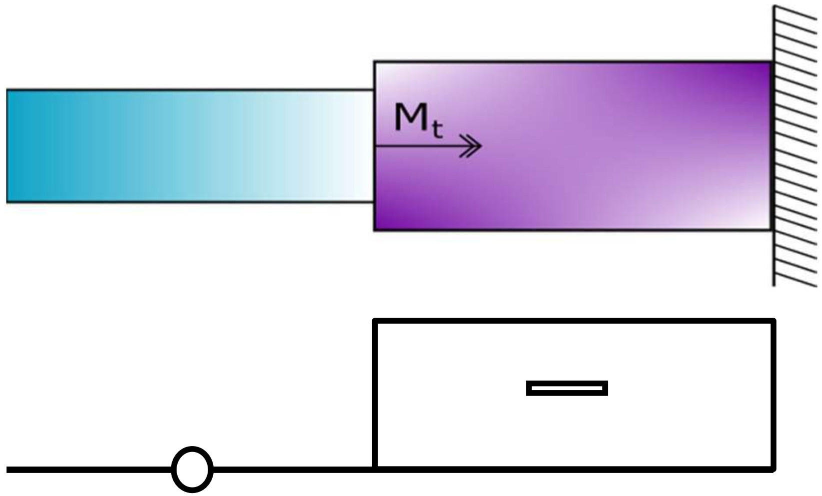


$$\theta_A^H = \theta_A^{M_A} + \theta_A^{M_t} = 0$$

Resolución por Teorema de los Trabajos Virtuales

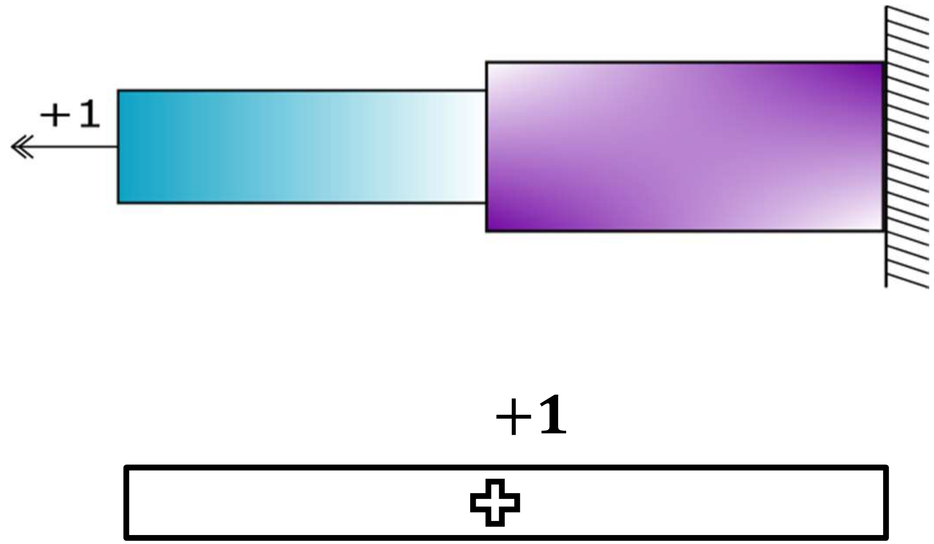
Calculo el giro $\theta_A^{M_t}$

DV_1



M_t

SE

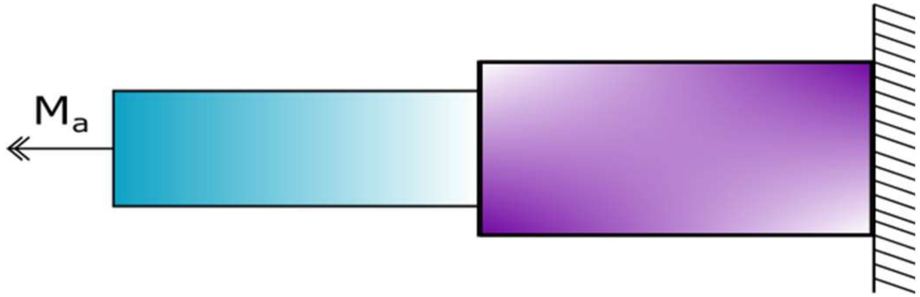


$$\theta_A^{M_t} = \int_l M_{SE}\, d\theta = \int_l M_{SE} \cdot \frac{M_{DV}}{G \cdot J}\, dx = M_{SE} \cdot \frac{M_{DV_1} \cdot L_2}{G_b \cdot J_b} = +1 \cdot \frac{-1000\ kN\ cm \cdot 300\ cm}{6000\,\frac{kN}{cm^2} \cdot 800\ cm^4}$$

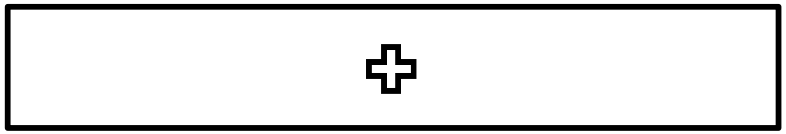
$$\theta_A^{M_t} = -0{,}0625\ rad$$

Calculo el giro $\theta_A^{M_A}$

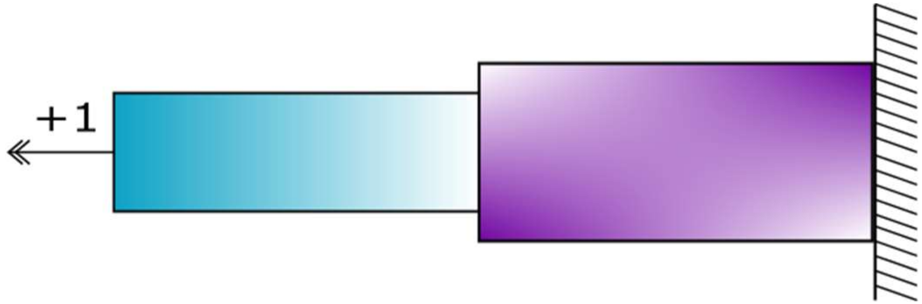
$\boldsymbol{DV_2}$



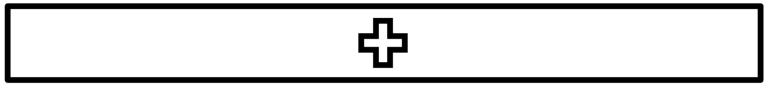
$\boldsymbol{M_a}$



$\boldsymbol{SE}$



$\boldsymbol{+1}$



$$\theta_A^{M_A} = \int_l M_{SE} \cdot \frac{M_{DV}}{G \cdot J}\, dx = 1 \cdot \frac{M_a \cdot L_1}{G_a \cdot J_a} + 1 \cdot \frac{M_a \cdot L_2}{G_b \cdot J_b}$$

$$\theta_A^{M_A} = M_a \cdot \left(\frac{200\ cm}{8000\,\frac{kN}{cm^2} \cdot 115\ cm^4} + \frac{300\ cm}{6000\,\frac{kN}{cm^2} \cdot 800\ cm^4} \right)$$

$$\boxed{\theta_A^{M_A} = 2{,}799 \cdot \frac{10^{-4}}{kN\ cm} \cdot M_a}$$

Reemplazando en la ecuación de compatibilidad

$$\theta_A^H = \theta_A^{M_A} + \theta_A^{M_t} = 0$$

$$\theta_A^{M_A} + \theta_A^{M_t} = -0{,}0625\, rad + 2{,}799 \cdot \frac{10^{-4}\, rad}{kN\ cm} \cdot M_a = 0$$

$$M_a = \frac{0{,}0625\, kN\ cm}{2{,}799\ 10^{-4}} \quad \Rightarrow \quad \boxed{M_a = 223{,}3\, kN\ cm}$$

Aclaración

No suele ser conveniente usar TTV para torsión, es recomendable plantearlo directamente por inspección (o MIE pero calculando el giro directamente)