Tema 3

$\sigma_{adm1} := 16 \ \frac{kN}{cm^2}$ $E_1 := 20000 \ \frac{kN}{cm^2} = 200 \ GPa$ $A_1 := 25 \ cm^2$ $\lambda_1 := 1.2 \cdot 10^{-5}$ $L := 800 \ cm$

$\sigma_{adm2} := 8 \ \frac{kN}{cm^2}$ $E_2 := 12500 \ \frac{kN}{cm^2}$ $A_2 := 15 \ cm^2$ $\lambda_2 := 1.4 \cdot 10^{-5}$ $\Delta T := -50$

Ecuaciones de equilibrio

$N_{biela} = -\frac{P}{2}$ $N_{biela} = N_1 + N_2$

Ecuación de compatibilidad de desplazamientos

$$\Delta L_2 = \Delta L_1$$

$$\frac{N_2 \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L = \frac{N_1 \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L$$

$$N_2 = \frac{E_2 \cdot A_2}{E_1 \cdot A_1} \cdot N_1 + \left(\lambda_1 - \lambda_2\right) \cdot \Delta T \cdot E_2 \cdot A_2$$

$N_{biela} = N_1 + N_2$ Se reescribe como $N_{biela} = N_1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1} \cdot N_1 + \left(\lambda_1 - \lambda_2\right) \cdot \Delta T \cdot E_2 \cdot A_2$

Entonces $$N_1 = \frac{N_{biela} - \left(\lambda_1 - \lambda_2\right) \cdot \Delta T \cdot E_2 \cdot A_2}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)}$$

El desplazamiento del punto D se obtiene como

$\Delta L_D = \Delta L_{1.\Delta T} + \Delta L_{1.N}$ donde $\Delta L_D = 0 \ cm$

Cálculo por Δ T

$N_{1.T} = -N_{2.T}$ $\Delta L_{1.\Delta T} = \Delta L_{2.\Delta T}$

$$\frac{N_{1.T} \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L = \frac{N_{2.T} \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L$$

$$\frac{N_{1.T} \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L = \frac{-N_{1.T} \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L$$

$$N_{1.T}\left(\frac{1}{E_1 \cdot A_1} + \frac{1}{E_2 \cdot A_2}\right) = \left(\lambda_2 - \lambda_1\right) \cdot \Delta T$$

$$N_{1.T}\left(\frac{1}{E_1 \cdot A_1} + \frac{1}{E_2 \cdot A_2}\right) = \left(\lambda_2 - \lambda_1\right) \cdot \Delta T$$

$N_{1.T} := \frac{\left(\lambda_2 - \lambda_1\right) \cdot \Delta T}{\frac{1}{E_1 \cdot A_1} + \frac{1}{E_2 \cdot A_2}} = -13.6364 \ kN$ $N_{2.T} := -N_{1.T} = 13.6364 \ kN$

$$\Delta L_{1.\Delta T} := \frac{N_{1.T} \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L = -0.5018 \text{ cm}$$

$$\Delta L_{2.\Delta T} := \frac{N_{2.T} \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L = -0.5018 \text{ cm}$$

Cálculo por P

$$N_{2.P} = -\frac{P}{2} - N_{1.P}$$

$$\Delta L_{1.P} = \Delta L_{2.P}$$

$$\frac{N_{1.P} \cdot L}{E_1 \cdot A_1} = \frac{N_{2.P} \cdot L}{E_2 \cdot A_2}$$

$$\frac{N_{1.P}}{E_1 \cdot A_1} = \frac{\left(-\frac{P}{2} - N_{1.P}\right)}{E_2 \cdot A_2}$$

$$\frac{E_2 \cdot A_2}{E_1 \cdot A_1} \cdot N_{1.P} + N_{1.P} = -\frac{P}{2}$$

$$N_{1.P} = \frac{-\frac{P}{2}}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)}$$

Entonces

$$\Delta L_D = \Delta L_{1.\Delta T} + \Delta L_{1.P}$$

$$0 \text{ cm} = \Delta L_{1.\Delta T} + \frac{N_{1.P} \cdot L}{E_1 \cdot A_1}$$

$$0 \text{ cm} = \Delta L_{1.\Delta T} + \frac{\frac{-\frac{P}{2}}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)} \cdot L}{E_1 \cdot A_1}$$

$$P := \Delta L_{1.\Delta T} \cdot \frac{E_1 \cdot A_1}{L} \cdot \left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right) \cdot 2 = -862.5 \text{ kN}$$

Como P comprimía, negativo indica que la P tiene sentido opuesto para verificar la condición

Resulta

$$N_{1.P} := \frac{-\frac{P}{2}}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)} = 313.6364 \text{ kN}$$

$$N_{2.P} := -\frac{P}{2} - N_{1.P} = 117.6136 \text{ kN}$$

$$\Delta L_{1.P} := \frac{N_{1.P} \cdot L}{E_1 \cdot A_1} = 0.5018 \text{ cm}$$

$$\Delta L_{2.P} := \frac{N_{2.P} \cdot L}{E_2 \cdot A_2} = 0.5018 \text{ cm}$$

$$N_1 := N_{1.T} + N_{1.P} = 300 \text{ kN}$$

$$N_2 := N_{2.T} + N_{2.P} = 131.25 \text{ kN}$$

$$\sigma_1 := \frac{N_1}{A_1} = 120 \text{ MPa}$$

$$\sigma_2 := \frac{N_2}{A_2} = 87.5 \text{ MPa}$$

$$\varepsilon_1 := \frac{\sigma_1}{E_1} + \lambda_1 \cdot \Delta T = 0$$

$$\varepsilon_2 := \frac{\sigma_2}{E_2} + \lambda_2 \cdot \Delta T = 0$$

$$\Delta L_1 := \varepsilon_1 \cdot L = 0 \text{ mm}$$

$$\Delta L_2 := \varepsilon_2 \cdot L = 0 \text{ mm}$$

Tema 4

$\sigma_{adm1} := 15 \ \frac{kN}{cm^2}$ $\quad E_1 := 20000 \ \frac{kN}{cm^2} = 200 \ GPa$ $\quad A_1 := 20 \ cm^2$ $\quad \lambda_1 := 1.5 \cdot 10^{-5}$ $\quad L := 600 \ cm$

$\sigma_{adm2} := 10 \ \frac{kN}{cm^2}$ $\quad E_2 := 10000 \ \frac{kN}{cm^2}$ $\quad A_2 := 10 \ cm^2$ $\quad \lambda_2 := 1.2 \cdot 10^{-5}$ $\quad \Delta T := -60$

Ecuaciones de equilibrio

$N_{biela} = -\frac{P}{2}$ $\quad N_{biela} = N_1 + N_2$

Ecuación de compatibilidad de desplazamientos

$$\Delta L_2 = \Delta L_1$$

$$\frac{N_2 \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L = \frac{N_1 \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L$$

$$N_2 = \frac{E_2 \cdot A_2}{E_1 \cdot A_1} \cdot N_1 + \left(\lambda_1 - \lambda_2\right) \cdot \Delta T \cdot E_2 \cdot A_2$$

$N_{biela} = N_1 + N_2$ Se reescribe como $N_{biela} = N_1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1} \cdot N_1 + \left(\lambda_1 - \lambda_2\right) \cdot \Delta T \cdot E_2 \cdot A_2$

Entonces $N_1 = \frac{N_{biela} - \left(\lambda_1 - \lambda_2\right) \cdot \Delta T \cdot E_2 \cdot A_2}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)}$

El desplazamiento del punto D se obtiene como

$\Delta L_D = \Delta L_{1.\Delta T} + \Delta L_{1.N}$ donde $\Delta L_D = 0 \ cm$

Cálculo por Δ T

$N_{1.T} = -N_{2.T}$ $\quad \Delta L_{1.\Delta T} = \Delta L_{2.\Delta T}$

$$\frac{N_{1.T} \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L = \frac{N_{2.T} \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L$$

$$\frac{N_{1.T} \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L = \frac{-N_{1.T} \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L$$

$$N_{1.T} \left(\frac{1}{E_1 \cdot A_1} + \frac{1}{E_2 \cdot A_2}\right) = \left(\lambda_2 - \lambda_1\right) \cdot \Delta T$$

$$N_{1.T} \left(\frac{1}{E_1 \cdot A_1} + \frac{1}{E_2 \cdot A_2}\right) = \left(\lambda_2 - \lambda_1\right) \cdot \Delta T$$

$N_{1.T} := \frac{\left(\lambda_2 - \lambda_1\right) \cdot \Delta T}{\frac{1}{E_1 \cdot A_1} + \frac{1}{E_2 \cdot A_2}} = 14.4 \ kN$ $\quad N_{2.T} := -N_{1.T} = -14.4 \ kN$

$\Delta L_{1.\Delta T} := \frac{N_{1.T} \cdot L}{E_1 \cdot A_1} + \lambda_1 \cdot \Delta T \cdot L = -0.5184 \text{ cm}$

$\Delta L_{2.\Delta T} := \frac{N_{2.T} \cdot L}{E_2 \cdot A_2} + \lambda_2 \cdot \Delta T \cdot L = -0.5184 \text{ cm}$

Cálculo por P

$N_{2.P} = -\frac{P}{2} - N_{1.P}$

$$\Delta L_{1.P} = \Delta L_{2.P}$$

$$\frac{N_{1.P} \cdot L}{E_1 \cdot A_1} = \frac{N_{2.P} \cdot L}{E_2 \cdot A_2}$$

$$\frac{N_{1.P}}{E_1 \cdot A_1} = \frac{\left(-\frac{P}{2} - N_{1.P}\right)}{E_2 \cdot A_2}$$

$$\frac{E_2 \cdot A_2}{E_1 \cdot A_1} \cdot N_{1.P} + N_{1.P} = -\frac{P}{2}$$

$$N_{1.P} = \frac{-\frac{P}{2}}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)}$$

Entonces

$$\Delta L_D = \Delta L_{1.\Delta T} + \Delta L_{1.P}$$

$$0 \text{ cm} = \Delta L_{1.\Delta T} + \frac{N_{1.P} \cdot L}{E_1 \cdot A_1}$$

$$0 \text{ cm} = \Delta L_{1.\Delta T} + \frac{\frac{-\frac{P}{2}}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)} \cdot L}{E_1 \cdot A_1}$$

$$P := \Delta L_{1.\Delta T} \cdot \frac{E_1 \cdot A_1}{L} \cdot \left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right) \cdot 2 = -864 \text{ kN}$$

Como P comprimía, negativo indica que la P tiene sentido opuesto para verificar la condición

Resulta

$N_{1.P} := \frac{-\frac{P}{2}}{\left(1 + \frac{E_2 \cdot A_2}{E_1 \cdot A_1}\right)} = 345.6 \text{ kN}$

$N_{2.P} := -\frac{P}{2} - N_{1.P} = 86.4 \text{ kN}$

$\Delta L_{1.P} := \frac{N_{1.P} \cdot L}{E_1 \cdot A_1} = 0.5184 \text{ cm}$

$\Delta L_{2.P} := \frac{N_{2.P} \cdot L}{E_2 \cdot A_2} = 0.5184 \text{ cm}$

$N_1 := N_{1.T} + N_{1.P} = 360 \text{ kN}$

$N_2 := N_{2.T} + N_{2.P} = 72 \text{ kN}$

$\sigma_1 := \frac{N_1}{A_1} = 180 \text{ MPa}$

$\sigma_2 := \frac{N_2}{A_2} = 72 \text{ MPa}$

$\varepsilon_1 := \frac{\sigma_1}{E_1} + \lambda_1 \cdot \Delta T = 0$

$\varepsilon_2 := \frac{\sigma_2}{E_2} + \lambda_2 \cdot \Delta T = 0$

$\Delta L_1 := \varepsilon_1 \cdot L = 0 \text{ mm}$

$\Delta L_2 := \varepsilon_2 \cdot L = 0 \text{ mm}$