Tema 2:

Ejercicio 1:

$L := 4\ \text{m}$

$h := 45\ \text{cm}$

$b := 15\ \text{cm}$

$\sigma_{fl} := 24\ \frac{\text{kN}}{\text{cm}^2} = 2.4 \cdot 10^8\ \text{Pa}$

$J := \frac{b \cdot h^3}{12} = 1.1391 \cdot 10^5\ \text{cm}^4$

$A := b \cdot h = 675\ \text{cm}^2$

$E := 20000\ \frac{\text{kN}}{\text{cm}^2}$

$M = P \cdot \frac{L}{2}$ $\quad$ $N = P$

Calcular la P de encuentro plástico

$\sigma_{fl} = \frac{M}{J} \cdot \frac{h}{2} + \frac{N}{A}$

$P_e := \frac{\sigma_{fl}}{\left(\frac{L}{J} \cdot \frac{h}{2} + \frac{1}{A}\right)} = 298.1595\ \text{kN}$

Calcular la P de colapso

$\int \sigma \, \mathrm{d}A = N$ $\quad$ $\int \sigma \cdot \mathbf{e} \, \mathrm{d}A = M$

$T - C = P$ $\quad$ $T \cdot \left(\frac{h - x}{2}\right) + C \cdot \left(\frac{x}{2}\right) = M$

$\sigma_{fl} \cdot x \cdot b - \sigma_{fl} \cdot (h - x) \cdot b = P$ $\quad$ $\sigma_{fl} \cdot x \cdot b \cdot \left(\frac{h - x}{2}\right) + \sigma_{fl} \cdot (h - x) \cdot b \cdot \frac{x}{2} = P \cdot L$

$$\begin{bmatrix} P_c \\ x \end{bmatrix} := \left(\text{roots}\left(\begin{bmatrix} \sigma_{fl} \cdot x\ \text{cm} \cdot b - \sigma_{fl} \cdot (h - x\ \text{cm}) \cdot b - P\ \text{kN} \\ \sigma_{fl} \cdot x\ \text{cm} \cdot b \cdot \left(\frac{h - x\ \text{cm}}{2}\right) + \sigma_{fl} \cdot (h - x\ \text{cm}) \cdot b \cdot \frac{x\ \text{cm}}{2} - P\ \text{kN} \cdot L \end{bmatrix}; \begin{bmatrix} P \\ x \end{bmatrix}\right)\right) = \begin{bmatrix} 455.2652 \\ 23.1323 \end{bmatrix}$$

$P_c := P_c\ \text{kN}$ $\quad$ $x := x\ \text{cm}$

$P_c = 455.2652\ \text{kN}$ $\quad$ $M_P := P_c \cdot L = 1821.0607\ \text{kN m}$

Longitud de plastificación:

$\sigma_{fl} = \frac{P_c}{A} + \frac{M_f}{J} \cdot \frac{h}{2}$ $\quad$ $M_f := \left(\sigma_{fl} - \frac{P_c}{A}\right) \cdot J \cdot \frac{2}{h} = 1180.8551\ \text{kN m}$ $\quad$ $L_p := L \cdot \left(1 - \frac{M_f}{M_P}\right) = 1.4062\ \text{m}$

Ejercicio 2:

IPN 220

$h := 220\ \text{mm}$

$b := 98\ \text{mm}$

$A := 39.5\ \text{cm}^2$

$S_y := 278\ \text{cm}^3$

$J_y := 3060\ \text{cm}^4$

$t_{ala} := 12.2\ \text{mm}$

$t_{alma} := 8.1\ \text{mm}$

$J_z := 162\ \text{cm}^4$

$N_x := 200\ \text{kN}$

$e_y := -5\ \text{mm}$

$e_z := -60\ \text{mm}$

$Q_z := 10\ \text{kN}$

$M_t := 0.5\ \text{kN m}$

$E := 20000\ \frac{\text{kN}}{\text{cm}^2}$ $\mu := 0.25$ $G := \frac{E}{2\cdot(1+\mu)} = 8000\ \frac{\text{kN}}{\text{cm}^2}$

$\sigma_{fl} := 24\ \frac{\text{kN}}{\text{cm}^2}$ $CS := 1.6$ $\sigma_{adm} := \frac{\sigma_{fl}}{CS} = 15\ \frac{\text{kN}}{\text{cm}^2}$

$M_y := N_x \cdot e_z = -12\ \text{kN m}$

$M_z := -N_x \cdot e_y = 1\ \text{kN m}$

$\sigma_n := \frac{N_x}{A} = 5.0633\ \frac{\text{kN}}{\text{cm}^2}$

$\sigma_{ymax} := \frac{M_y}{J_y}\cdot\frac{h}{2} = -4.3137\ \frac{\text{kN}}{\text{cm}^2}$ $\sigma_{yA} := \frac{M_y}{J_y}\cdot\left(\frac{h}{2}-t_{ala}\right) = -3.8353\ \frac{\text{kN}}{\text{cm}^2}$

$\sigma_{zmax} := \frac{M_z}{J_z}\cdot\frac{b}{2} = 3.0247\ \frac{\text{kN}}{\text{cm}^2}$ $\sigma_{zA} := \frac{M_z}{J_z}\cdot\left(\frac{-t_{alma}}{2}\right) = -0.25\ \frac{\text{kN}}{\text{cm}^2}$

$S_{alma} := b\cdot t_{ala}\cdot\frac{\left(h-t_{ala}\right)}{2} = 124.2228\ \text{cm}^3$

$\tau_{minalma} := \frac{Q_z\cdot S_{alma}}{J_y\cdot t_{alma}} = 0.5012\ \frac{\text{kN}}{\text{cm}^2}$ $J_t := \frac{2\cdot t_{ala}^{\,3}\cdot b}{3} + t_{alma}^{\,3}\cdot\frac{\left(h-2\cdot t_{ala}\right)}{3} = 15.3285\ \text{cm}^4$

$\tau_{Malma} := \frac{M_t\cdot t_{alma}}{J_t} = 2.6421\ \frac{\text{kN}}{\text{cm}^2}$

$\sigma_{xA} := \sigma_n + \sigma_{yA} + \sigma_{zA} = 0.978\ \frac{\text{kN}}{\text{cm}^2}$

$\tau_{xzA} := \tau_{Malma} + \tau_{minalma} = 3.1433\ \frac{\text{kN}}{\text{cm}^2}$

$\sqrt{\sigma_{xA}^{\,2} + 3\cdot\tau_{xzA}^{\,2}} = 5.5315\ \frac{\text{kN}}{\text{cm}^2}$

$\varepsilon_{xA} := \frac{\sigma_{xA}}{E} = 4.89\cdot10^{-5}$ $\varepsilon_{yA} := \mu\cdot\frac{-\sigma_{xA}}{E} = -1.2225\cdot10^{-5}$

$\varepsilon_{xzA} := \frac{\tau_{xzA}}{2\cdot G} = 0.0196\ \%$

Tema 1:

Ejercicio 1:

$L := 5\ \text{m}$

$h := 60\ \text{cm}$

$b := 20\ \text{cm}$

$\sigma_{fl} := 24\ \frac{\text{kN}}{\text{cm}^2} = 2.4 \cdot 10^8\ \text{Pa}$

$J := \frac{b \cdot h^3}{12} = 3.6 \cdot 10^5\ \text{cm}^4$

$A := b \cdot h = 1200\ \text{cm}^2$

$E := 20000\ \frac{\text{kN}}{\text{cm}^2}$

$M = P \cdot \frac{L}{2}$

$N = P$

Calcular la P de encuentro plástico

$\sigma_{fl} = \frac{M}{J} \cdot \frac{h}{2} + \frac{N}{A}$

$P_e := \frac{\sigma_{fl}}{\left(\frac{L}{J} \cdot \frac{h}{2} + \frac{1}{A}\right)} = 564.7059\ \text{kN}$

Calcular la P de colapso

$\int \sigma \, \mathrm{d}A = N$

$\int \sigma \cdot \mathbf{e} \, \mathrm{d}A = M$

$T - C = P$

$T \cdot \left(\frac{h - x}{2}\right) + C \cdot \left(\frac{x}{2}\right) = M$

$\sigma_{fl} \cdot x \cdot b - \sigma_{fl} \cdot (h - x) \cdot b = P$

$\sigma_{fl} \cdot x \cdot b \cdot \left(\frac{h - x}{2}\right) + \sigma_{fl} \cdot (h - x) \cdot b \cdot \frac{x}{2} = P \cdot L$

$$\begin{bmatrix} P_c \\ x \end{bmatrix} := \left(\text{roots}\left(\begin{bmatrix} \sigma_{fl} \cdot x\ \text{cm} \cdot b - \sigma_{fl} \cdot (h - x\ \text{cm}) \cdot b - P\ \text{kN} \\ \sigma_{fl} \cdot x\ \text{cm} \cdot b \cdot \left(\frac{h - x\ \text{cm}}{2}\right) + \sigma_{fl} \cdot (h - x\ \text{cm}) \cdot b \cdot \frac{x\ \text{cm}}{2} - P\ \text{kN} \cdot L \end{bmatrix}; \begin{bmatrix} P \\ x \end{bmatrix}\right)\right) = \begin{bmatrix} 863.2238 \\ 30.8992 \end{bmatrix}$$

$P_c := P_c\ \text{kN}$

$x := x\ \text{cm}$

$P_c = 863.2238\ \text{kN}$

$M_p := P_c \cdot L = 4316.119\ \text{kN m}$

Longitud de plastificación:

$\sigma_{fl} = \frac{P_c}{A} + \frac{M_f}{J} \cdot \frac{h}{2}$

$M_f := \left(\sigma_{fl} - \frac{P_c}{A}\right) \cdot J \cdot \frac{2}{h} = 2793.6776\ \text{kN m}$

$L_p := L \cdot \left(1 - \frac{M_f}{M_p}\right) = 1.7637\ \text{m}$

Ejercicio 2:

IPN 280

$h := 280\ mm$

$b := 119\ mm$

$A := 61\ cm^2$

$S_y := 542\ cm^3$

$J_y := 7590\ cm^4$

$t_{ala} := 15.2\ mm$

$t_{alma} := 10.1\ mm$

$J_z := 364\ cm^4$

$N_x := 250\ kN$

$e_y := -6\ mm$

$e_z := -50\ mm$

$Q_z := 20\ kN$

$M_t := 0.7\ kN\ m$

$E := 20000\ \frac{kN}{cm^2}$ $\mu := 0.25$ $G := \frac{E}{2\cdot(1+\mu)} = 8000\ \frac{kN}{cm^2}$

$\sigma_{fl} := 24\ \frac{kN}{cm^2}$ $CS := 1.6$ $\sigma_{adm} := \frac{\sigma_{fl}}{CS} = 15\ \frac{kN}{cm^2}$

$M_y := N_x \cdot e_z = -12.5\ kN\ m$

$M_z := -N_x \cdot e_y = 1.5\ kN\ m$

$\sigma_n := \frac{N_x}{A} = 4.0984\ \frac{kN}{cm^2}$

$\sigma_{ymax} := \frac{M_y}{J_y}\cdot\frac{h}{2} = -2.3057\ \frac{kN}{cm^2}$ $\sigma_{yA} := \frac{M_y}{J_y}\cdot\left(\frac{h}{2}-t_{ala}\right) = -2.0553\ \frac{kN}{cm^2}$

$\sigma_{zmax} := \frac{M_z}{J_z}\cdot\frac{b}{2} = 2.4519\ \frac{kN}{cm^2}$ $\sigma_{zA} := \frac{M_z}{J_z}\cdot\left(\frac{-t_{alma}}{2}\right) = -0.2081\ \frac{kN}{cm^2}$

$S_{alma} := b\cdot t_{ala}\cdot\frac{\left(h-t_{ala}\right)}{2} = 239.4851\ cm^3$

$\tau_{minalma} := \frac{Q_z\cdot S_{alma}}{J_y\cdot t_{alma}} = 0.6248\ \frac{kN}{cm^2}$ $J_t := \frac{2\cdot t_{ala}^{\ 3}\cdot b}{3} + t_{alma}^{\ 3}\cdot\frac{\left(h-2\cdot t_{ala}\right)}{3} = 36.4324\ cm^4$

$\tau_{Malma} := \frac{M_t\cdot t_{alma}}{J_t} = 1.9406\ \frac{kN}{cm^2}$

$\sigma_{xA} := \sigma_n + \sigma_{yA} + \sigma_{zA} = 1.8349\ \frac{kN}{cm^2}$ $\tau_{xzA} := \tau_{Malma} + \tau_{minalma} = 2.5654\ \frac{kN}{cm^2}$

$\sqrt{\sigma_{xA}^{\ 2} + 3\cdot\tau_{xzA}^{\ 2}} = 4.8073\ \frac{kN}{cm^2}$ $\varepsilon_{xA} := \frac{\sigma_{xA}}{E} = 9.1746\cdot10^{-5}$ $\varepsilon_{yA} := \mu\cdot\frac{-\sigma_{xA}}{E} = -2.2937\cdot10^{-5}$

$\varepsilon_{xzA} := \frac{\tau_{xzA}}{2\cdot G} = 0.016\ \%$