EJERCICIO de TORSIÓN en RÉG. ANELÁSTICO. — Clase Virtual del 12/06/2024

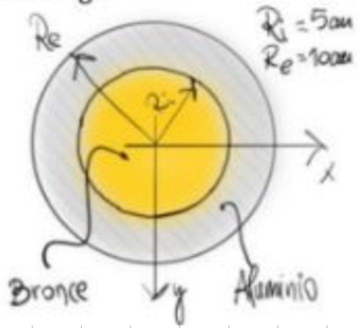
Ejercicio 2

Para la sección compuesta por dos materiales concéntricos que actúan solidariamente, donde el núcleo interior está hecho de bronce ($\tau_{Br\,F}$ = 10 kN/cm2, G_{Br} = 3600 kN/cm2) y la corona circular exterior está hecha de aluminio ($\tau_{Al\,F}$ = 4 kN/cm2, G_{Al} = 2800 kN/cm2), tal como se muestra en la figura, se pide:

a) Determinar el momento torsor elástico y trazar los diagramas de tensiones y deformaciones cuando actúa dicho esfuerzo.

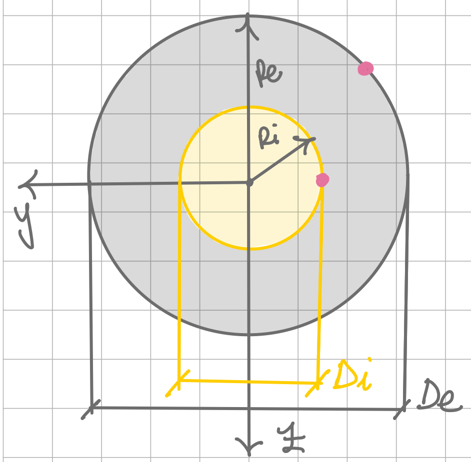
b) Determinar el momento torsor de plastificación total y trazar los diagramas de tensiones y deformaciones cuando actúa dicho esfuerzo.

c) Si se carga la sección con un momento torsor tal que la deformación máxima del bronce es igual a su deformación de fluencia y luego se procede a su descarga, determinar las tensiones y deformaciones residuales en la sección. Deberán dejar asentados los diagramas de tensiones y deformaciones que se producen en la carga y en la descarga.



DATOS $D_i = 10cm$, $R_i = 5cm$ $\tau_{Fl,Br} = 10 \frac{kN}{cm^2}$, $G_{Br} = 3600\ kN/cm^2$

$D_e = 20\ cm$, $R_e = 10cm$ $\tau_{Fl,Al} = 4 \frac{kN}{cm^2}$, $G_{Al} = 2800 \frac{kN}{cm^2}$



$$J_{P,Br} = \frac{\pi}{32} \cdot D_i^4 = \frac{\pi}{2} R_i^4 = \frac{\pi}{32} \cdot (10cm)^4 \Rightarrow J_{P,Br} = 981,75 cm^4$$

$$J_{P,Al} = \frac{\pi}{32}(D_e^4 - D_i^4) = \frac{\pi}{32}(20^4 - 10^4)cm^4 \Rightarrow J_{P,Al} = 14726,22 cm^4$$

a) Mt_e ⟶ la primera fibra llega a la fluencia ⟶ Vale la linealidad mecánica

es el límite entre Rég. Elástico y Rég. Plástico

Vale Coulomb ⟶ $\tau = \frac{Mt \cdot R}{J_P}$

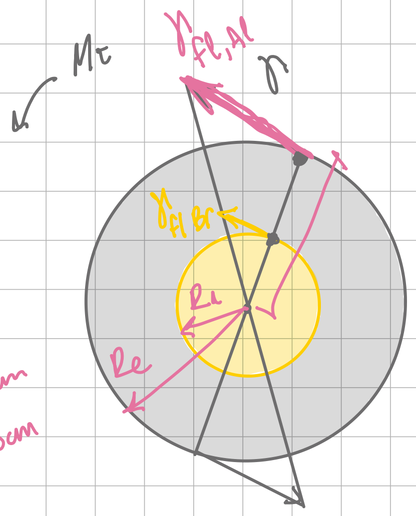
$\chi = \frac{Mt}{G\,J_P}$ ⇒ Ec. de Compatibilidad (Para resolver el Hiperestático Interno).

$$\chi_{Al} = \chi_{Br}$$

$$\frac{Mt_{Al}}{G_{Al}\,J_{P_{Al}}} = \frac{Mt_{Br}}{G_{Br}\,J_{P_{Br}}}$$

$$\frac{Mt_{Al}}{G_{Al}\,Jp_{Al}} = \frac{Mt_{Br}}{G_{Br}\,Jp_{Br}} \longrightarrow Mt_{Al} = Mt_{Br}\,\frac{G_{Al}\,Jp_{Al}}{G_{Br}\,Jp_{Br}}$$

(1) Analizo las distorsiones → qué fibra llega primero a la fluencia?



Ri = 5cm
Re = 10cm

$$\gamma_{fl,Al} = \frac{\tau_{fl,Al}}{G_{Al}} = \frac{4\ kN/cm^2}{2800\ kN/cm^2} = \frac{1,43\times10^{-3}}{2}$$

$$\gamma_{fl,Br} = \frac{\tau_{fl,Br}}{G_{Br}} = \frac{10\ kN/cm^2}{3600\ kN/cm^2} = \underbrace{2,78\times10^{-3}\ 2}_{5,5\times10^{-3}}$$

⇒ el Al es el 1er material que llega a la fluencia

$$\tau = \frac{Mt^{Al}\cdot R}{Jp}$$

fl, Al ; Al ; Re = 10cm

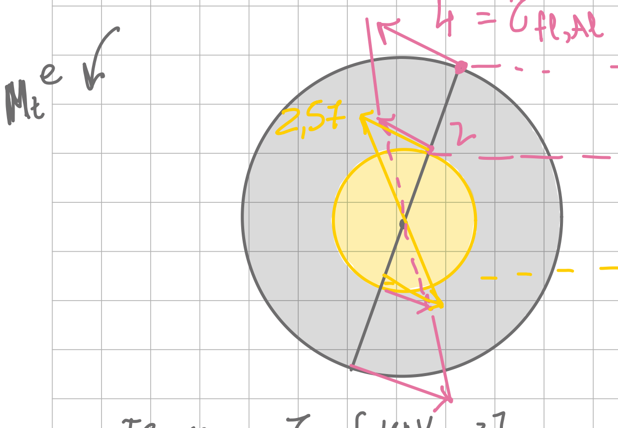
$$Mt^e \neq \boxed{Mt,_{Al}} = \frac{\tau_{fl,Al}\cdot Jp_{Al}}{Re} = \frac{4\ kN/cm^2 \quad 14726,22\ cm^4}{10\ cm} = \boxed{58,9\ kNm}$$

$$Mt^e = Mt,_{Al} + Mt,_{Br}$$

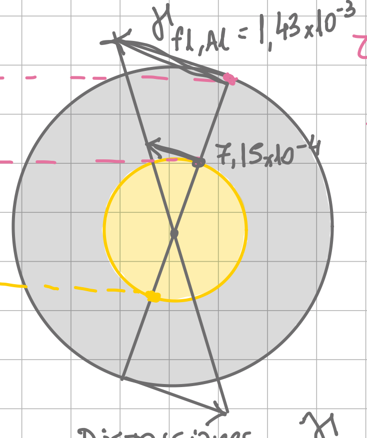
$$Mt_{Br} = Mt_{Al}\,\frac{G_{Br}\,Jp_{Br}}{G_{Al}\,Jp_{Al}} = 58,9\ kN\cdot m\ \frac{3600\ kN/cm^2 \cdot 981,75\ cm^4}{2800\ kN/cm^2 \quad 14726,22\ cm^4}$$

$$\boxed{Mt_{Br} = 5,05\ kN\cdot m}$$

$$Mt^e = 58,9\ kNm + 5,05\ kN\cdot m \rightarrow \boxed{Mt^e = 63,95\ kN\cdot m}$$ ← Rta (a)



Tensiones τ [kN/cm²]



Distorsiones γ [-]

$$\tau_{Al} = \frac{Mt_{,Al}\cdot Ri}{Jp_{,Al}}$$

$$\tau_{Al} = \frac{58,9\ kNm\ 5cm}{14726,22\ cm^4}$$

$$\tau_{Al} = 2\ kN/cm^2$$

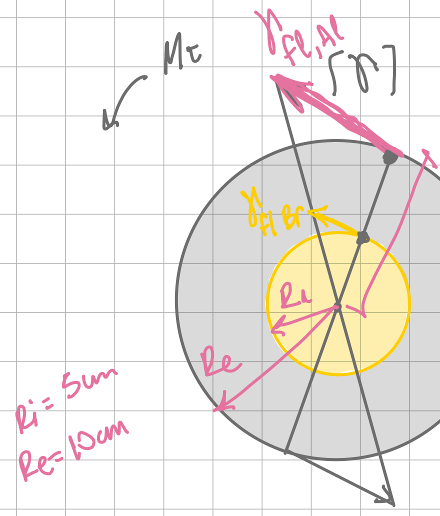
$$\tau_{Br} = \frac{M_{t,Br}\cdot R_i}{J_{p,Br}} = \frac{5,05\ kN\cdot \cancel{m}\ \ 5\cancel{cm}}{981,75\ cm^{\cancel{4}\,3}} \cdot \left[\frac{100\ cm}{\cancel{m}}\right] = 2,57\ kN/cm^2 < \tau_{fl,Bs} = 10\ \frac{kN}{cm^2}$$

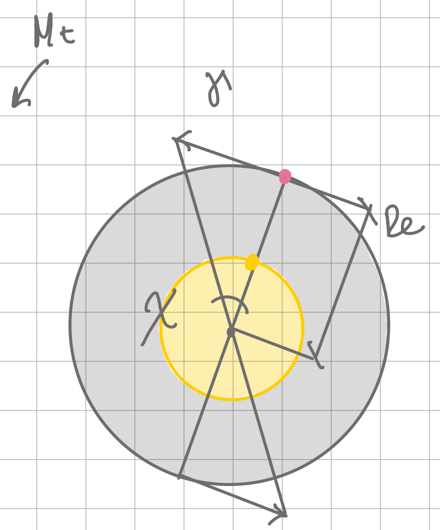
$$= \frac{505\ kN\cdot \cancel{cm}\cdot 5\cancel{cm}}{981,75\ cm^{\cancel{4}\,2}}$$

$$\gamma_{Al}(R_i) = \frac{\tau_{Al}}{G_{Al}}$$

$$\gamma_{Br}(R_i) = \frac{\tau_{Br}}{G_{Br}}$$

BONUS TRACK → usando las curvaturas (χ) en lugar de las distorsiones (γ)



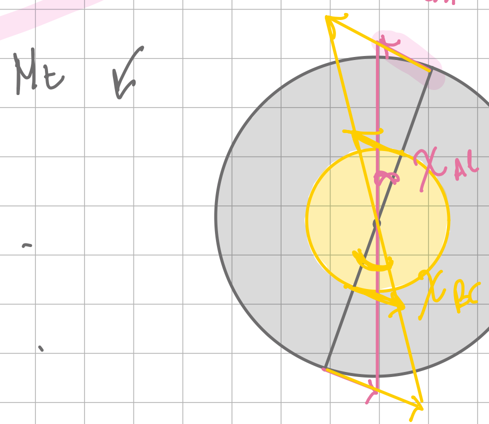


$$\chi = \frac{\gamma}{R_e}$$

$$\chi^e_{Al} = \frac{\gamma_{fl,Al}}{R_e}\ ; \quad \chi^e_{Br} = \frac{\gamma_{fl,Br}}{R_i}\ .$$

$$\chi^e_{Al} = 1,43\times10^{-4}\ \frac{1}{cm}\ ; \quad \chi^e_{Br} = 5,56\times10^{-4}\ \frac{1}{cm}$$

$$\chi^e = \frac{M_t}{G\ J_p}$$

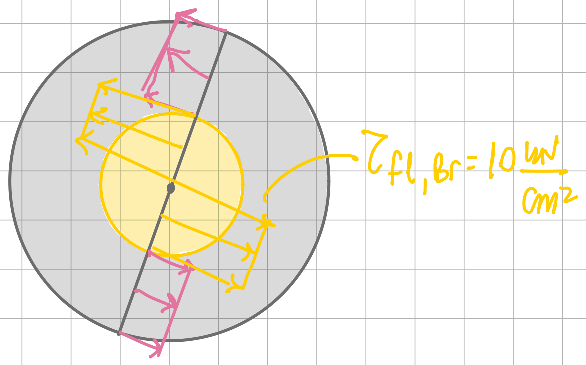


$$\Rightarrow \chi^e = \min\begin{cases}\chi^e_{Al}\\ \chi^e_{Bs}\end{cases}$$

$$\begin{cases} M_{t\,Al}^{e} = \chi^e_{Al}\cdot G_{Al}\cdot J_{P\,Al}\ . \\ M_{t\,Br}^{e} = \chi^e_{Al}\cdot G_{Br}\cdot J_{P\,Br}\ . \end{cases}$$

b) M_t de plastificación total
M_t^{p}
(todas las fibras tienen τ_{fl})

$\tau_{fl,Al} = 4\ kN/cm^2$

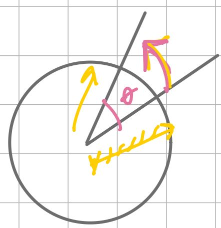


inciso (a)

$$M_t^{e} < M_t^{p}$$

$$M_t = \int_A \tau \cdot r\, dA$$

$$dA = r\,d\theta\, dr$$



$$M_t^{p} = \int\int \tau\, r\, r\, d\theta\, dr$$

$$M_t^{p} = \int\int \tau\, r^2\, dr\, d\theta$$

$$M_t^{p} = \int_0^{2\pi}\int_0^{Ri} \tau^{Br}\, r^2\, dr\, d\theta + \int_0^{2\pi}\int_{Ri}^{Re} \tau^{Al}\, r^2\, dr\, d\theta$$

$$M_t^{p} = \tau_{fl,Br} \cdot 2\pi \cdot \frac{Ri^3}{3} + \boxed{\tau_{fl,Al} \cdot 2\pi \cdot \left(\frac{Re^3}{3} - \frac{Ri^3}{3}\right)}$$

$$= 10\ \frac{kN}{cm^2} \cdot 2\pi \cdot \frac{(5\ cm)^3}{3} + 4\ \frac{kN}{cm^2} \cdot 2\pi \left(\frac{10^3}{3} - \frac{5^3}{3}\right) cm^3$$

$$M_t^{p} = 2\,618{,}0\ kN\cdot cm + \boxed{7330{,}4\ kNcm} \Rightarrow M_t^{p} = 9948{,}38\ kNcm$$

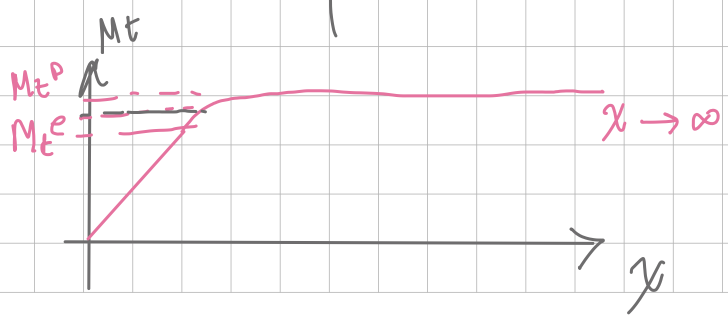
Rta (b) → $\boxed{M_t^{p} = 99{,}5\ kNm}$

$\boxed{M_t^{e} = 63{,}95\ kN\cdot m}$ ← Rta (a)

$$M_t^{p} > M_t^{e}$$

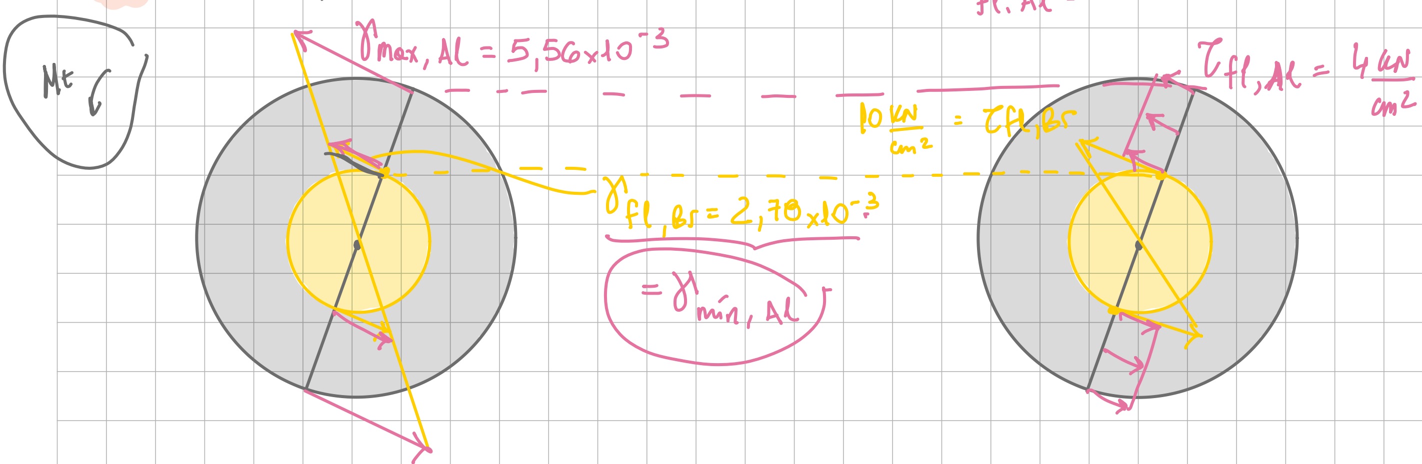
χ^{e}

$\chi^{p} \to \infty$



c) M_t tal que $\gamma_{max\ Br} = \gamma_{fl,Br}$

CARGA



[γ]

$$\frac{\gamma_{fl,Br}}{R_i} = \frac{\gamma_{max\ Al}}{R_e}$$

$$\gamma_{max,Al} = \gamma_{fl,Br} \left(\frac{R_e}{R_i}\right) \quad \frac{10}{5} = 2$$

[τ]

$$M_t = \underbrace{M_t^{Br}} + M_t^{Al} \longrightarrow \text{Ec. de Equivalencia}$$

vale Coulomb

$$M_t^{Br} = \frac{\tau_{fl,Br} \cdot J_{p,Br}}{R_i}$$

$$M_t^{Al} = \tau_{fl,Al} \cdot 2\pi \cdot \left(\frac{R_e^3}{3} - \frac{R_i^3}{3} \right)$$

$$M_t^{Al} = 7330,4 \text{ kNcm} = 73,30 \text{ kNm}$$

$$M_t^{Br} = 1963,5 \text{ kN cm} = 19,64 \text{ kNm}$$

$$M_t = 19,64 \text{ kNm} + 73,30 \text{ kNm} = 92,94 \text{ kNm}$$

$$M_t > M_t^e$$

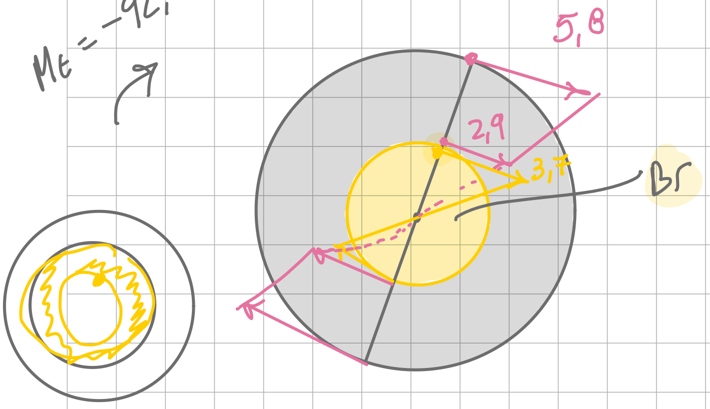
$$M_t < M_t^p$$

La descarga SIEMPRE es LINEAL ⟺ ELÁSTICA

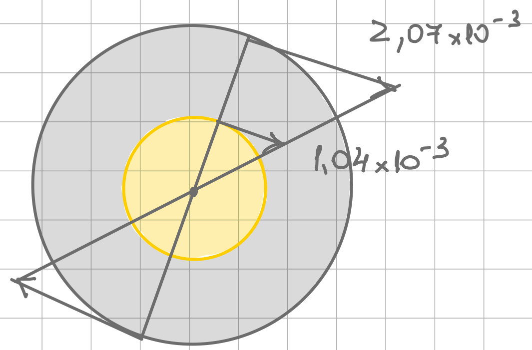
DESCARGA

$$M_t^D = M_{t\,Br}^D + M_{t\,Al}^D$$

$M_t = -92{,}94\ kNm$



[τ]



[γ]

$$\chi_{Br} = \chi_{Al}$$

$$\frac{M_{t\,Br}^D}{G_{Br}\,J_{p\,Br}} = \frac{M_{t\,Al}^D}{G_{Al}\,J_{p\,Al}} \longrightarrow M_{t\,Br}^D = M_{t\,Al}^D \frac{G_{Br}\,J_{p\,Br}}{G_{Al}\,J_{p\,Al}} \Rightarrow M_{t\,Br}^D = 0{,}0857\,M_{t\,Al}^D$$

$$M_{t\,Br}^D = 0{,}0857\,M_{t\,Al}^D$$

$$M_t^D = M_{t\,Br}^D + M_{t\,Al}^D = 0{,}0857\,M_{t\,Al}^D + M_{t\,Al}^D = 1{,}0875\,M_{t\,Al}^D$$

$$M_{t\,Al}^D = 85{,}5\ kNm \longrightarrow M_{t\,Br}^D = 7{,}4\ kNm$$

TENSIONES

$$\tau = \frac{M_t \cdot R}{J_p}$$

$$\tau_{max,Al}^D = \frac{M_{t\,Al}^D \cdot R_e}{J_{p,Al}} = 5{,}8\ \frac{kN}{cm^2}$$

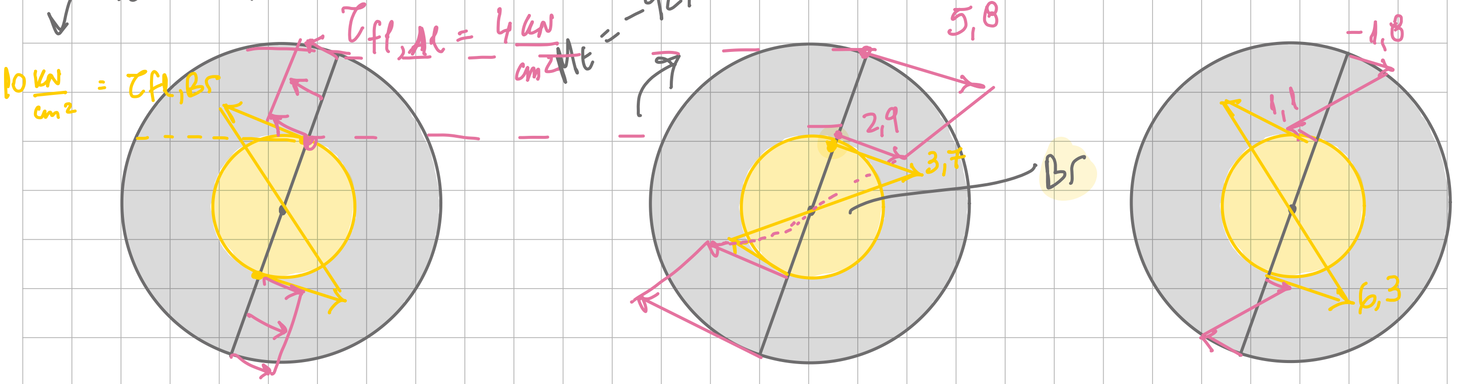
$$\tau_{min,Al}^D = \frac{M_{t\,Al}^D \cdot R_i}{J_{p,Al}} = 2{,}9\ \frac{kN}{cm^2}$$

$$\tau_{max,Br}^D = \frac{M_{t\,Br}^D \cdot R_i}{J_{p,Br}} = 3{,}7\ \frac{kN}{cm^2}$$

RESIDUALES

[τ]

$M_t^C = 92,94 \text{ kNm}$

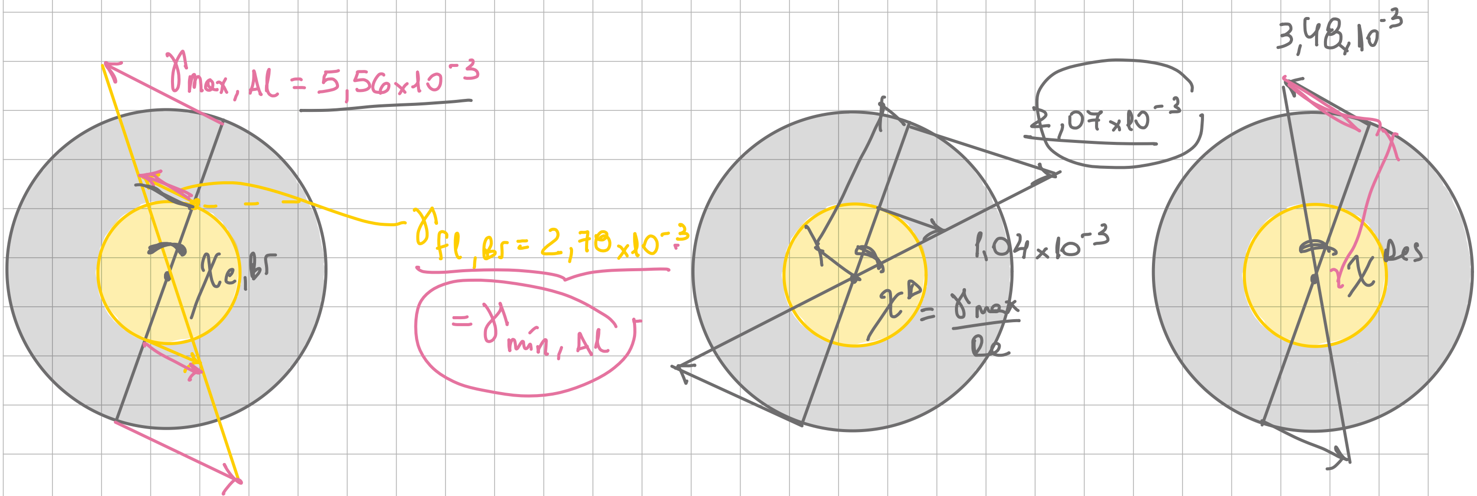


1) $\tau_{Al}^{Res} = +4 \frac{kN}{cm^2} - 5,8 \frac{kN}{cm^2} = -1,8 \frac{kN}{cm^2}$

2) $\tau_{Al}^{Res} = +4 \frac{kN}{cm^2} - 2,9 \frac{kN}{cm^2} = 1,1 \frac{kN}{cm^2}$

$\tau_{Br}^{Res} = +10 \frac{kN}{cm^2} - 3,7 \frac{kN}{cm^2} = 6,3 \frac{kN}{cm^2}$

[γ]



[γ]

$\chi^{res} = \chi^C + \chi^D$

$\gamma_{res} = 5,56 \times 10^{-3} - 2,07 \times 10^{-3} = 3,48 \times 10^{-3}$