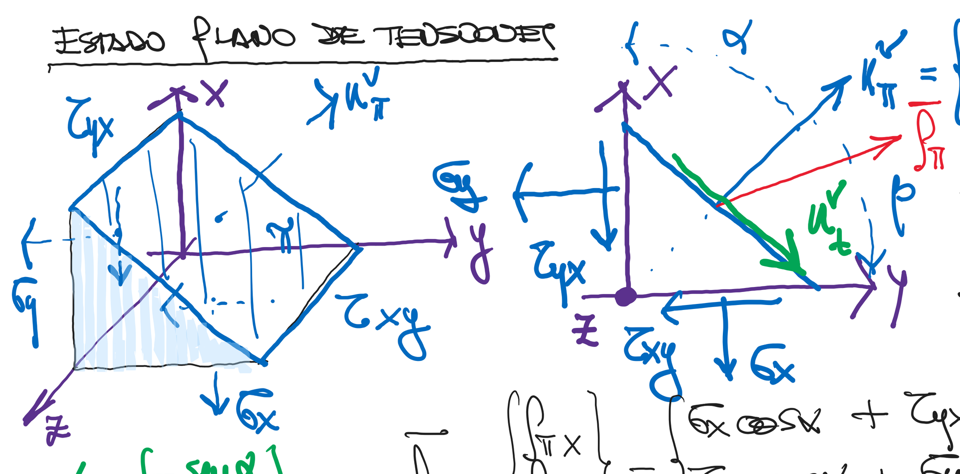
ESTADO PLANO DE TENSÕES



$$\vec{n}_\pi^{\,v} = \begin{Bmatrix} \cos\alpha \\ \cos\beta \\ 0 \end{Bmatrix} = \begin{Bmatrix} \cos\alpha \\ \operatorname{sen}\alpha \\ 0 \end{Bmatrix}$$

$$\vec{\rho}_\pi = \begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} \cos\alpha \\ \operatorname{sen}\alpha \\ 0 \end{Bmatrix}$$

$$\vec{n}_t^{\,v} = \begin{Bmatrix} -\operatorname{sen}\alpha \\ \cos\alpha \\ 0 \end{Bmatrix}$$

$$\vec{\rho}_\pi = \begin{Bmatrix} \rho_{\pi x} \\ \rho_{\pi y} \\ \rho_{\pi z} \end{Bmatrix} = \begin{Bmatrix} \sigma_x \cos\alpha + \tau_{yx} \operatorname{sen}\alpha \\ \tau_{xy} \cos\alpha + \sigma_y \operatorname{sen}\alpha \\ 0 \end{Bmatrix}$$

$\sigma_\pi = \vec{\rho}_\pi \cdot \vec{n}_\pi^{\,v}$

$\tau_\pi = \vec{\rho}_\pi \cdot \vec{n}_t^{\,v}$

$$\sigma_\pi = \sigma_x \cdot \cos\alpha \cdot \cos\alpha + \tau_{yx} \operatorname{sen}\alpha \cdot \cos\alpha + \tau_{xy} \cos\alpha \operatorname{sen}\alpha + \sigma_y \operatorname{sen}\alpha \operatorname{sen}\alpha$$

$$\sigma_\pi = \sigma_x \cos^2\alpha + \tau_{xy} \cdot 2 \operatorname{sen}\alpha \cos\alpha + \sigma_y \cdot \operatorname{sen}^2\alpha$$

$$\boxed{\sigma_\pi = \sigma_x \cos^2\alpha + \tau_{xy} \cdot \operatorname{sen} 2\alpha + \sigma_y \operatorname{sen}^2\alpha}$$

$$\boxed{2 \operatorname{sen}\alpha \cdot \cos\alpha = \operatorname{sen} 2\alpha}$$

$$\tau_\pi = \sigma_x \cos\alpha(-\operatorname{sen}\alpha) + \tau_{yx} \cdot \operatorname{sen}\alpha(-\operatorname{sen}\alpha) + \tau_{xy} \cos\alpha \cdot \cos\alpha + \sigma_y \operatorname{sen}\alpha \cos\alpha$$

$$\tau_\pi = \frac{\sigma_y - \sigma_x}{2} \cdot \operatorname{sen} 2\alpha + \tau_{xy}(-\operatorname{sen}^2\alpha + \cos^2\alpha)$$

$$\boxed{\cos^2\alpha - \operatorname{sen}^2\alpha = \cos 2\alpha}$$

$$\boxed{\tau_\pi = \frac{\sigma_y - \sigma_x}{2} \operatorname{sen} 2\alpha + \tau_{xy} \cdot \cos 2\alpha}$$

TENSÕES PRINCIPAIS

$$\left\{[T] - \sigma_i [I]\right\} \cdot \vec{n}_i^{\,v} = 0$$

$$\begin{vmatrix} \sigma_x - \sigma_i & \tau_{xy} \\ \tau_{yx} & \sigma_y - \sigma_i \end{vmatrix} = 0$$

$$\boxed{\sigma_i^3 - I_1 \sigma_i^2 + I_2 \sigma_i - I_3 = 0}$$

$$(\sigma_x - \sigma_i)(\sigma_y - \sigma_i) - \tau_{xy}^2 = 0$$

$$\sigma_x \sigma_y - \sigma_x \sigma_i - \sigma_y \sigma_i + \sigma_i^2 - \tau_{xy}^2 = 0$$

$$\sigma_i^2 - (\sigma_x + \sigma_y)\sigma_i + \sigma_x \sigma_y - \tau_{xy}^2 = 0$$

$$\boxed{\sigma_i} = \frac{(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x + \sigma_y)^2 - (\sigma_x \sigma_y - \tau_{xy}^2) \cdot 4}}{2} = \boxed{\frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\sigma_x^2 + 2\sigma_x\sigma_y + \sigma_y^2 - 4\sigma_x\sigma_y + 4\tau_{xy}^2$$

$$\sigma_x^2 - 2\sigma_x\sigma_y + \sigma_y^2 + 4\tau_{xy}^2$$

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2$$

$$\frac{d\sigma_n}{d\alpha} = 0 \rightarrow \sigma_{n\,max}$$

$$\sigma_n = \sigma_x \cos^2\alpha + \tau_{xy} \operatorname{sen} 2\alpha + \sigma_y \cdot \operatorname{sen}^2\alpha$$

$$\frac{\partial \sigma_n}{\partial \alpha} = -\sigma_x \cos\alpha \cdot 2 \operatorname{sen}\alpha + \tau_{xy}\, 2\cos 2\alpha + \sigma_y \operatorname{sen}\alpha \cdot 2\cos\alpha = 0$$

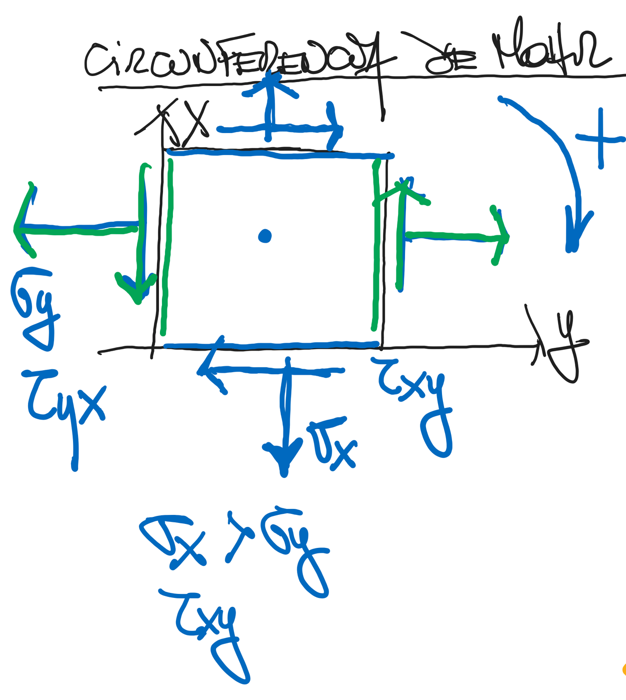
$$(\sigma_y - \sigma_x) \operatorname{sen} 2\alpha + \tau_{xy}\, 2\cos 2\alpha = 0$$

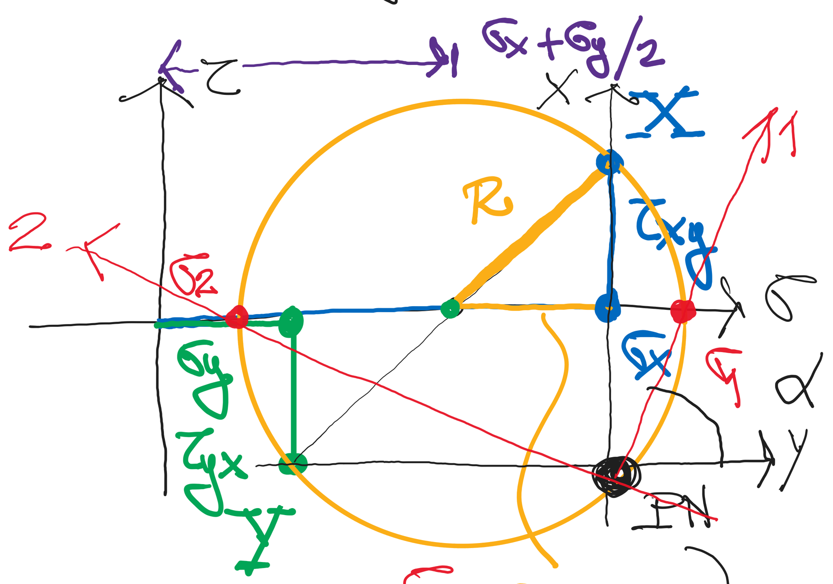
$$\boxed{\operatorname{tg} 2\alpha} = \frac{\operatorname{sen} 2\alpha}{\cos 2\alpha} = \boxed{\frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

$\rightarrow \alpha_1$

$\rightarrow \alpha_2 = \alpha_1 + 90°$

CIRCUNFERÊNCIA DE MOHR





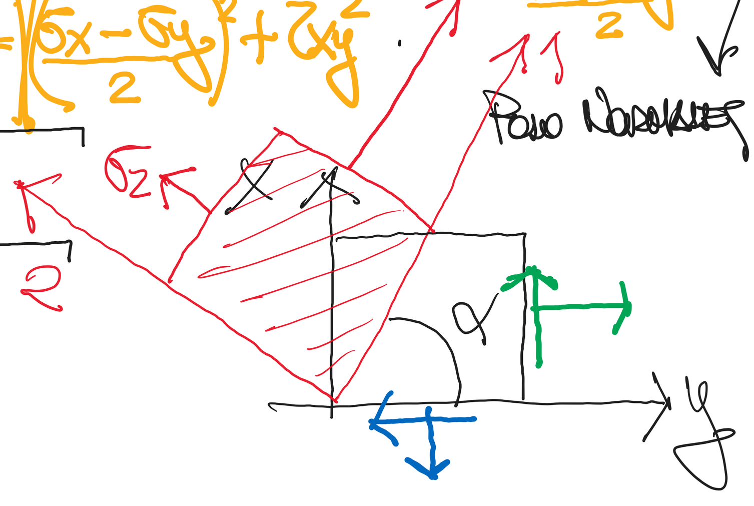
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\frac{\sigma_x - \sigma_y}{2}$

Polo Normales

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\quad}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\quad}$$



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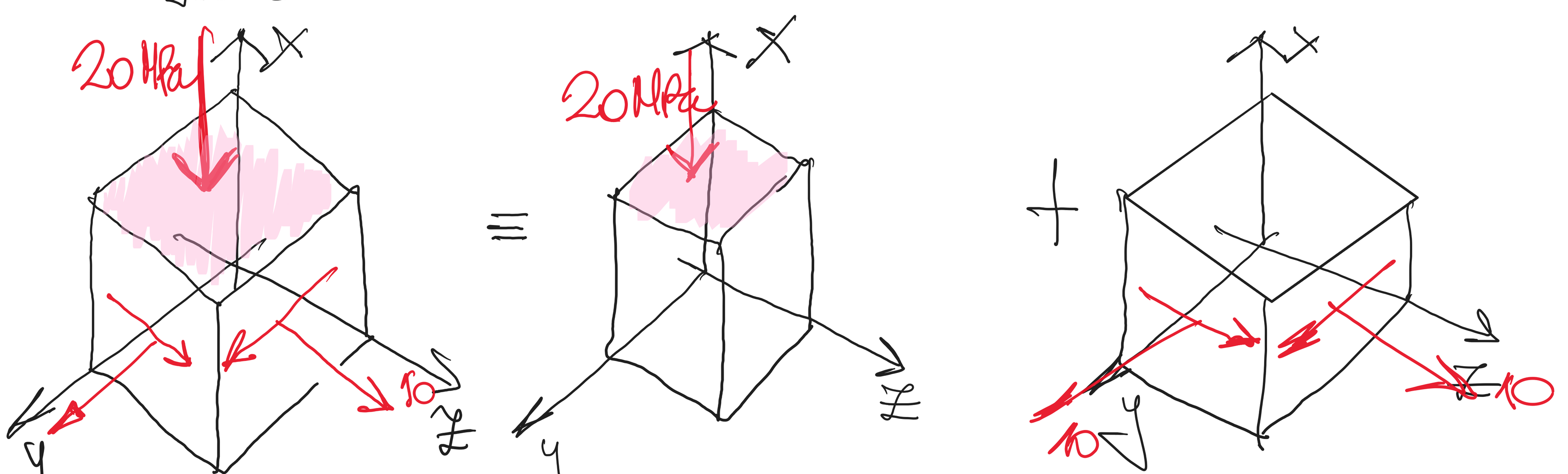

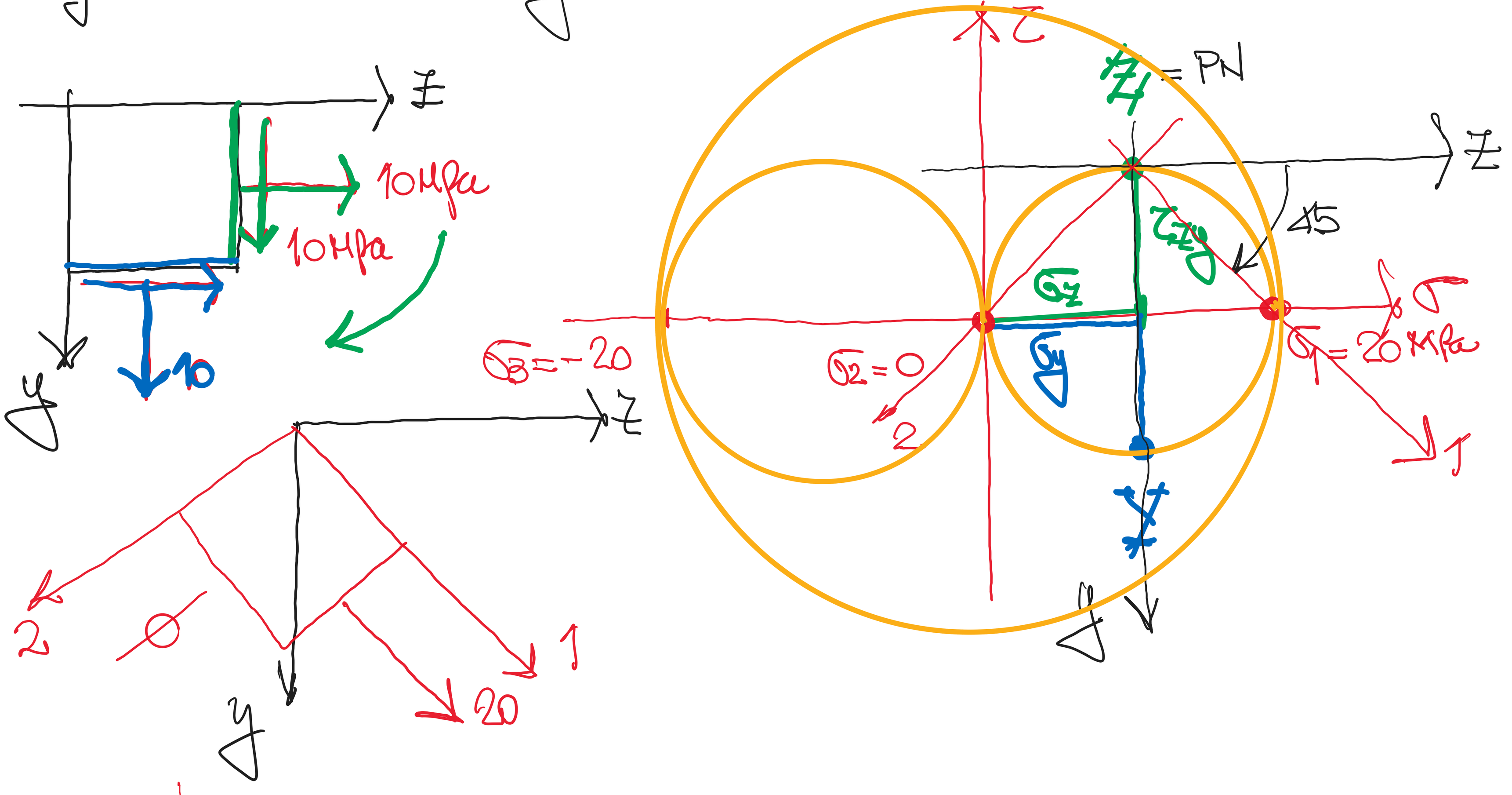

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