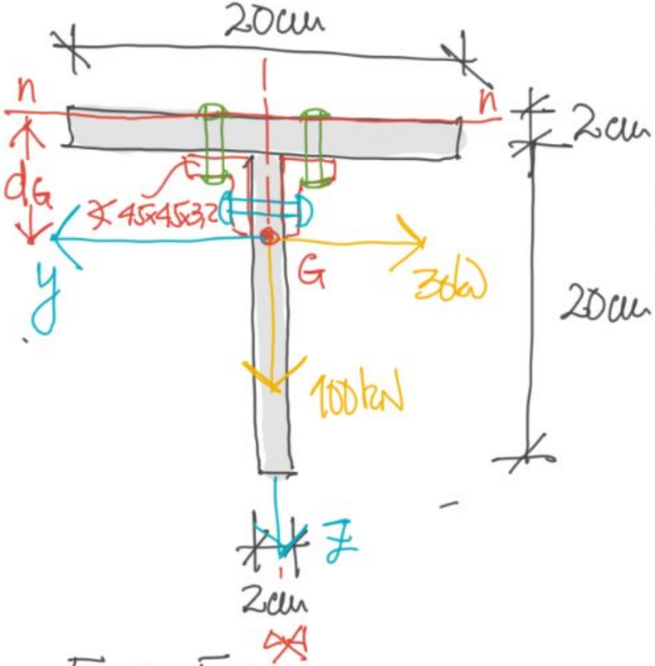
Ejercicio 4



$$\tau_{adm}^{bulón} = 10\,kN/cm^2$$
$$\tau_{adm}^{acero} = 8\,kN/cm^2$$
$$sep = 20\,cm$$
Perfiles ángulo 45x45x3,2 mm
$$A = 2{,}83\,cm^2$$
$$J_z = J_y = 5{,}24\,cm^4$$

$$S_{nn} = A_{tot} \cdot d_G = (20\times2)\,cm^2 \times 1\,cm + 2\times 2{,}83\,cm^2 \times (2\,cm + 1{,}19\,cm) + (20\times2)\,cm^2 \times 12\,cm$$

$$85{,}66\,cm^2 \cdot d_G = 538{,}055\,cm^3 \rightarrow d_G = 6{,}28\,cm$$

$$J_y = \frac{20\times2^3}{12}\,cm^4 + 40\,cm^2 \times (6{,}28-1)^2\,cm^2 + 2\times5{,}24\,cm^4 + 2\times2{,}83\,cm^2 \times (6{,}28-2-1{,}19)^2\,cm^2 + \frac{2\times20^3}{12}\,cm^4 + 40\,cm^2 \times (12-6{,}28)^2\,cm^2$$

$$J_y = 3835{,}06\,cm^4$$

$$J_z = \frac{2\times20^3}{12}\,cm^4 + \frac{20\times2^3}{12}\,cm^4 + 2\times5{,}24\,cm^4 + 2\times2{,}83\,cm^2 \times (1{,}19+1)^2\,cm^2$$

$$J_z = 1384{,}3\,cm^4$$

$$F_b \geq F_{resb}$$

$$\tau_{adm}^{bulón} \cdot A_{bulón} \cdot n \geq \tau_{resb} \cdot b \cdot sep$$

$$\tau_{adm}^{bulón} \cdot A_{bulón} \cdot n \geq \frac{\phi \cdot S_w^*}{J_w \cdot b} \cdot b \cdot sep$$

Calculamos los diagramas de τ

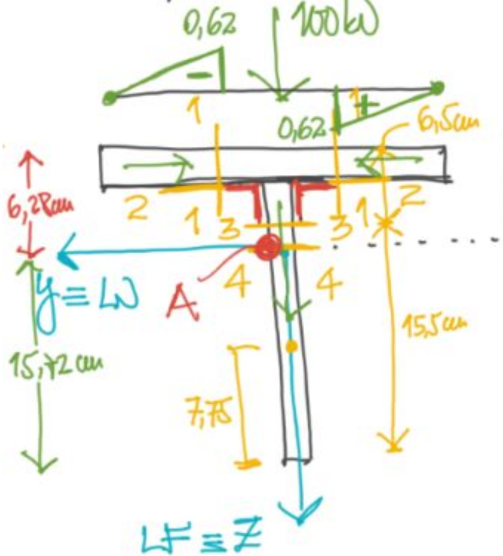
Para Q_z

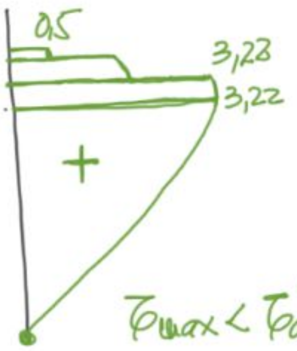
$\tau_{11} = 0{,}62\ \text{kN/cm}^2$

$\tau_{22} = 0{,}5\ \text{kN/cm}^2$

$\tau_{33} = 3{,}22\ \text{kN/cm}^2$

$\tau_{44} = 3{,}22\ \text{kN/cm}^2 = \tau_{max}$





$\tau_{max} < \tau_{adm}^{acero}$ ✓

$$\tau_{tg}\big|_{Q_z} = \frac{Q_z\, S_y^*}{J_y\, b}$$

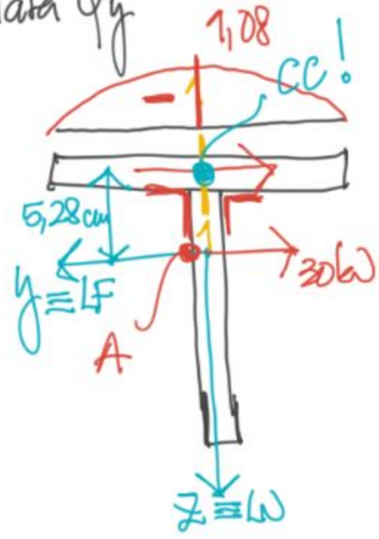
$S_y^{*11} = 4{,}5\text{cm} \times 2\text{cm} \times 5{,}28\text{cm} = 47{,}52\text{cm}^3$

$S_y^{*22} = 20\text{cm} \times 2\text{cm} \times 5{,}28\text{cm} = 211{,}2\text{cm}^3$

$S_y^{*33} = 15{,}5\text{cm} \times 2\text{cm} \times (22\text{cm} - 7{,}75\text{cm} - 6{,}28\text{cm}) = 247{,}07\text{cm}^3$

$S_y^{*44} = 15{,}72\text{cm} \times 2\text{cm} \times \frac{15{,}72\text{cm}}{2} = 247{,}12\text{cm}^3$

Para Q_y



$$\tau_{tg}\big|_{Q_y} = \frac{Q_y\, S_z^*}{J_z\, b}$$

$S_z^{*11} = 10\text{cm} \times 2\text{cm} \times 5\text{cm} = 100\text{cm}^3$

$$\tau_{11} = \frac{30\text{kN} \times 100\text{cm}^3}{1384{,}3\text{cm}^4 \times 2\text{cm}} = 1{,}08\ \text{kN/cm}^2$$

$M_t\big|_{Q_y} = 30\text{kN} \times 5{,}28\text{cm}$

OJO!

$M_t\big|_{Q_y} = 158{,}4\ \text{kNcm}$

$J_T = \frac{1}{3}\, 2\, (2\text{cm})^3\, 20\text{cm}$

$J_T = 106{,}67\text{cm}^4$

$\tau\big|_{M_t} = \frac{M_T}{J_T}\, e$

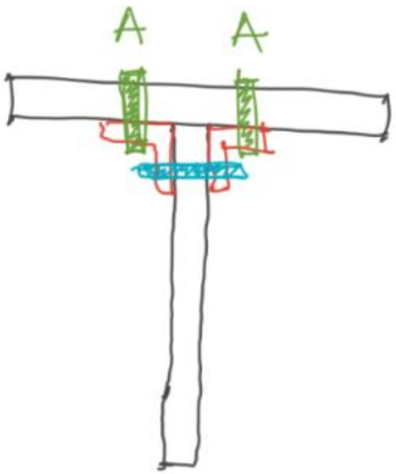
$\tau\big|_{M_t} = 2{,}97\ \text{kN/cm}^2$

$\tau_{max} = \tau_A = 2{,}97 + 3{,}22$

$\tau_{max} = 6{,}19\ \text{kN/cm}^2 < \tau_{adm}^{acero}$ $(8\ \text{kN/cm}^2)$

CUMPLE

Calculo los bulones



$$\underbrace{\tau_{adm}^{bulón}}_{10\,kN/cm^2} \times A_{bulón} \times 2 \geq 0{,}5\,kN/cm^2 \times \underbrace{11\,cm}_{\text{éste fue el "b" que usamos p/calc. } \tau_{zz}/Q_z} \times 20\,cm$$

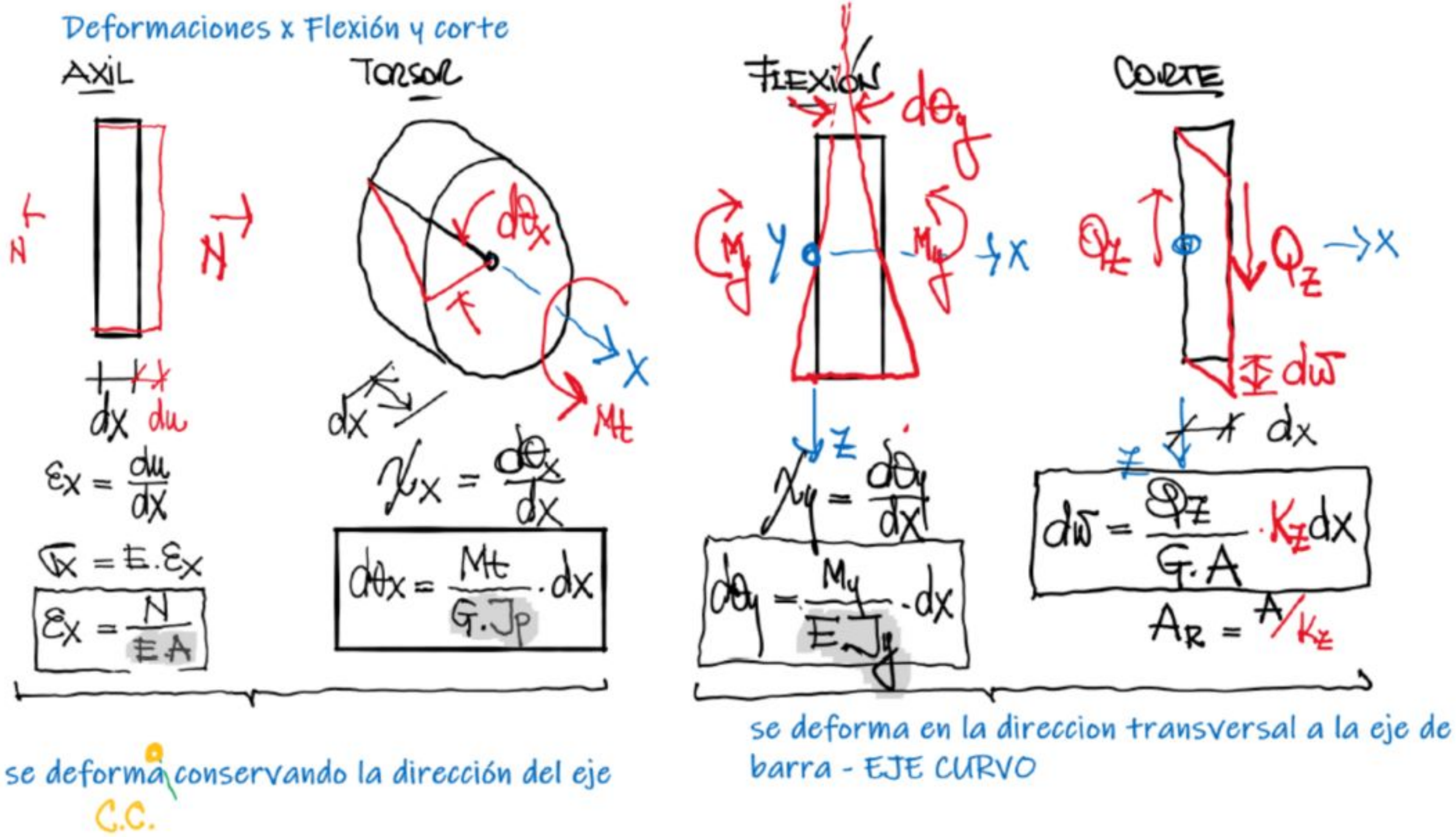
$$\textcircled{1}\; A_{bulón} = 5{,}5\,cm^2$$

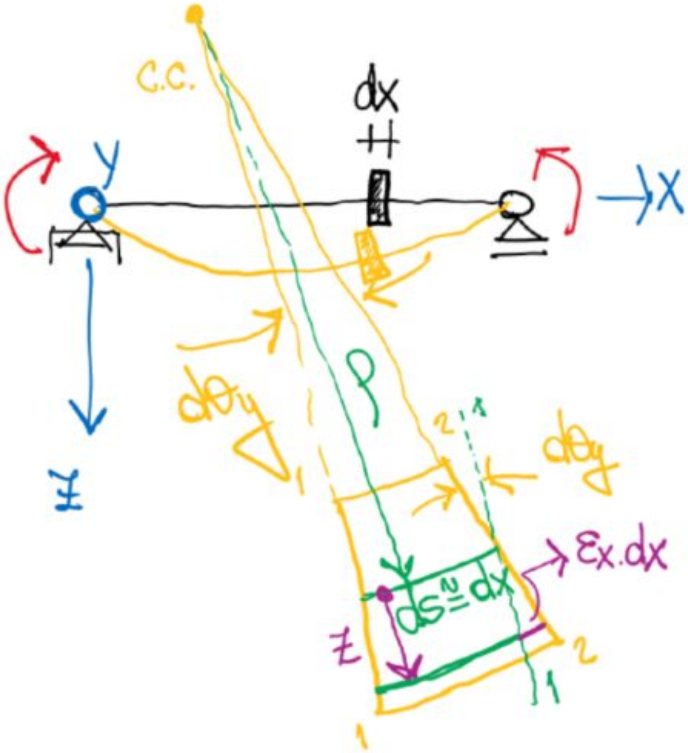
$$\underbrace{\tau_{adm}^{bulón}}_{10\,kN/cm^2} \times A_{bulón} \times 2 \geq \underbrace{\frac{100\,kN \times [40\,cm^2 \times (22\,cm - 10\,cm - 6{,}28\,cm)]}{3835{,}06\,cm^4 \; \cancel{b}}}_{5{,}97\,kN/cm} \cancel{b} \; 20\,cm$$

$$\textcircled{2}\; A_{bulón} = 5{,}97\,cm^2$$

$d_{bulón} \geq 2{,}76\,cm \rightarrow$ debería achicar la separación y/o usar bulones + resistentes

Deformación por flexión





EXPRESION MATEMATICA DE LA CURVATURA

$$\rho = \frac{1}{\chi} \rightarrow \chi_y = \frac{d\theta_y}{dx} = \frac{1}{\rho}$$

$$\rho \cdot d\theta_y = dx$$

$$z \cdot d\theta_y = \varepsilon_x \cdot dx \rightarrow \varepsilon_x = \chi_y \cdot z$$

$$\sigma_x = E\,\varepsilon_x = E\,\chi_y \cdot z$$

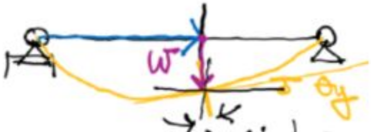
$$\sigma_x = \frac{M_y}{J_y} \cdot z = E\,\chi_y \cdot z \Rightarrow \boxed{\chi_y = \frac{M_y}{E \cdot J_y}}$$

$$\chi = \frac{dw^2/dx^2}{\left[1+\left(\frac{dw}{dx}\right)^2\right]^{3/2}}$$

$$\frac{dw}{dx} = w' <<<< 1 \rightarrow w' = 0$$

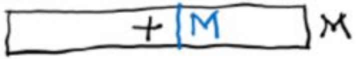
Ejercicio 1

$$\chi_y = \frac{d\omega^2}{dx^2}$$



$$\theta_y \cong tg\,\theta_y = \frac{d\omega}{dx}$$

$$\boxed{\chi_y = \frac{d\theta_y}{dx} = \frac{d^2\omega}{dx^2} = -\frac{M}{E.J_y}}$$



$$\omega' = -\frac{M}{EJ_y}.x + C_1$$

$$\boxed{\omega = -\frac{M}{EJ_y}\cdot\frac{x^2}{2} + C_1.x + C_2}$$

CONDICIONES DE BORDE

$$x = 0 \rightarrow \omega(0) = 0 \rightarrow \boxed{C_2 = 0}$$

$$x = L \rightarrow \omega(L) = 0 \rightarrow \boxed{C_1 = \frac{M.L}{2EJ_y}}$$

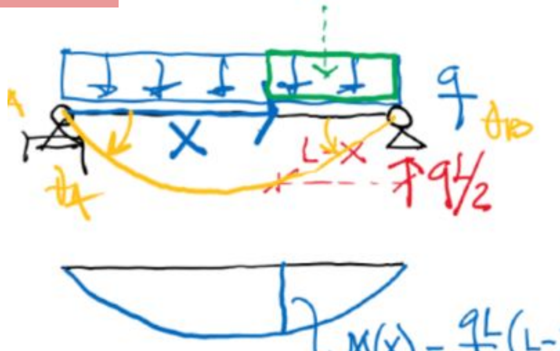
$$\boxed{\omega = -\frac{M}{EJ_y}\cdot\frac{x^2}{2} + \frac{M.L}{2EJ_y}\cdot x}$$

$$\omega_{max} \rightarrow \frac{d\omega}{dx} = 0 \rightarrow x = \frac{L}{2}$$

$$\boxed{\omega_{max} = \frac{M.L^2}{8E.J_y}}$$

Ejercicio 2

V.S.A. con carga distribuida uniforme



$$M(x) = \frac{qL}{2}(L-x) - q\frac{(L-x)^2}{2}$$

$$\boxed{M(x) = \frac{q}{2}(L.x - x^2)}$$

$$\frac{d^2 w}{dx^2} = -\frac{M}{EJ_y} = -\frac{\frac{q}{2}(Lx - x^2)}{EJ_y}$$

$$w'(x) = -\frac{q}{2EJ_y}\left(L.\frac{x^2}{2} - \frac{x^3}{3}\right) + C_1$$

$$\boxed{w(x) = -\frac{q}{2EJ_y}\left(L.\frac{x^3}{6} - \frac{x^4}{12}\right) + C_1.x + C_2}$$

$x=0 \rightarrow w(0) = 0 \rightarrow \boxed{C_2 = 0}$

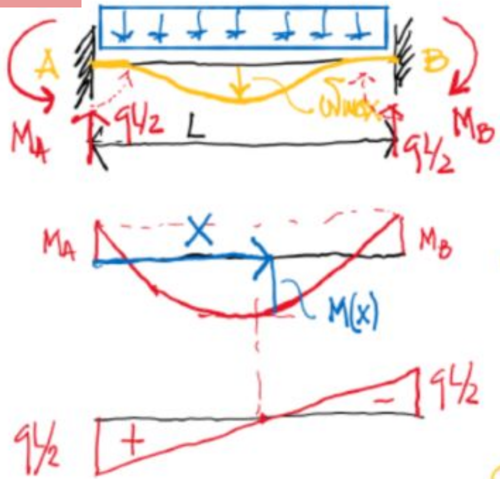
$x=L \rightarrow w(L) = 0$

$$\boxed{w(x) = \frac{q}{EJ_y}\frac{x^4}{24} - \frac{q}{EJ_y}\cdot L\frac{x^3}{12} + \frac{q.x.L^3}{24\, EJ_y}}$$

$$\boxed{w_{max}(L/2) = \frac{5}{384}\,\frac{q.L^4}{EJ_y}} \ll \frac{L}{300} \text{ ó } \frac{L}{500}$$

Ejercicio 3

Ej 3 viga doblemente empotrada con una carga uniformemente distribuida



$$w'' = -\frac{M}{EJ_y}$$

$$w''' = -\frac{dM_y}{dx}\,\frac{1}{EJ_y} = -\frac{Q_z}{EJ_y}$$

CARGA $\quad w^{IV} = -\frac{dQ_z}{dx}\,\frac{1}{EJ_y} = +\frac{q_z}{EJ_y}$

$$\frac{dM_y}{dx} = Q_z$$

$$dQ_z = -q\,dx$$

CORTE $\quad w''' = +\frac{q_z}{EJ_y}\cdot x + C_1$

MOMENTO $\quad w'' = +\frac{q_z}{EJ_y}\cdot\frac{x^2}{2} + C_1\cdot x + C_2$

GIRO $\quad w' = +\frac{q_z}{EJ_y}\,\frac{x^3}{6} + C_1\frac{x^2}{2} + C_2 x + C_3$

ELASTICA $\quad \boxed{w = +\frac{q_z}{EJ_y}\cdot\frac{x^4}{24} + C_1\cdot\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3\cdot x + C_4}$

POLINOMIO DE 4to GRADO

CONDICIONES DE BORDE

$x = 0 \rightarrow w(0) = 0 \rightarrow w'(0) = 0$

$x = L \rightarrow w(L) = 0 \rightarrow w'(L) = 0$

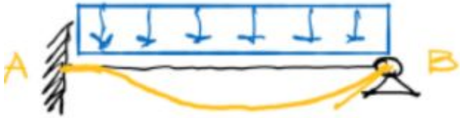
$x = L/2 \rightarrow w'(L/2) = 0$

$x = L/2 \rightarrow w'''(L/2) = 0$

$$\boxed{w_{max} = \frac{1}{384}\,\frac{qL^4}{EJ_y}}$$

Ejercicio 4

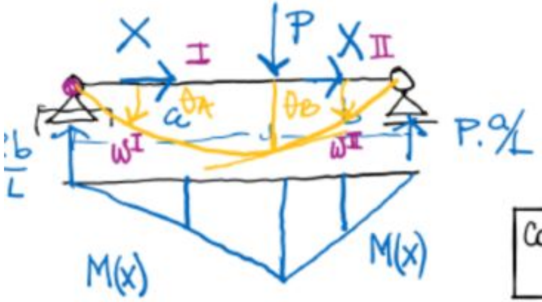
Ej 4 viga empotrada / Simple apoyada con carga uniforme



$w(0) = 0 \qquad w''(L) = 0$

$w(L) = 0 \qquad w'(0) = 0$

Ej 5 VSA con una concentrada



$0 \leq x \leq a \qquad M(x) = \frac{P.b}{L}\cdot x \rightarrow w^{I} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix}$

$a \leq x \leq L \qquad M(x) = \frac{P.a}{L}(L - x) \rightarrow w^{II} \begin{Bmatrix} C_3 \\ C_4 \end{Bmatrix}$

$$\frac{d^2 w}{dx^2} = -\frac{M(x)}{EJ_y}$$

$w''(0) = 0$

$w''(L) = 0$

CONDICIONES DE BORDE

$w^{I}(0) = 0 \qquad w^{II}(L) = 0$

$w^{I}(a) = w^{II}(a)$

$w^{I\prime}(a) = w^{II\prime}(a)$