

Análisis Numérico / Métodos Matemáticos y Numéricos (75.12/95.04/95.13)

Minimización Numérica

Minimización

Bisección

Dada la siguiente función, hallar una aproximación del mínimo en el intervalo [0; 4]

$$f(x) = \frac{x^2}{10} - 2 \operatorname{sen}(x)$$

k	a	m _i	b	m _d	c	f(a)	f(m _i)	f(b)	f(m _d)	f(c)	delta μ
1	0	1	2	3	4	0,00000	-1,58294	-1,41859	0,61776	3,11360	
2	0	0,5	1	1,5	2	0,00000	-0,93385	-1,58294	-1,76999	-1,41859	0,50000
3	1	1,25	1,5	1,75	2	-1,58294	-1,74172	-1,76999	-1,66172	-1,41859	0,25000
4	1,25	1,375	1,5	1,625	1,75	-1,74172	-1,77272	-1,76999	-1,73300	-1,66172	0,12500
5	1,25	1,3125	1,375	1,4375	1,5	-1,74172	-1,76139	-1,77272	-1,77562	-1,76999	0,06250
6	1,375	1,40625	1,4375	1,46875	1,5	-1,77272	-1,77523	-1,77562	-1,77387	-1,76999	0,03125
7	1,40625	1,421875	1,4375	1,453125	1,46875	-1,77523	-1,77569	-1,77562	-1,77501	-1,77387	0,01563

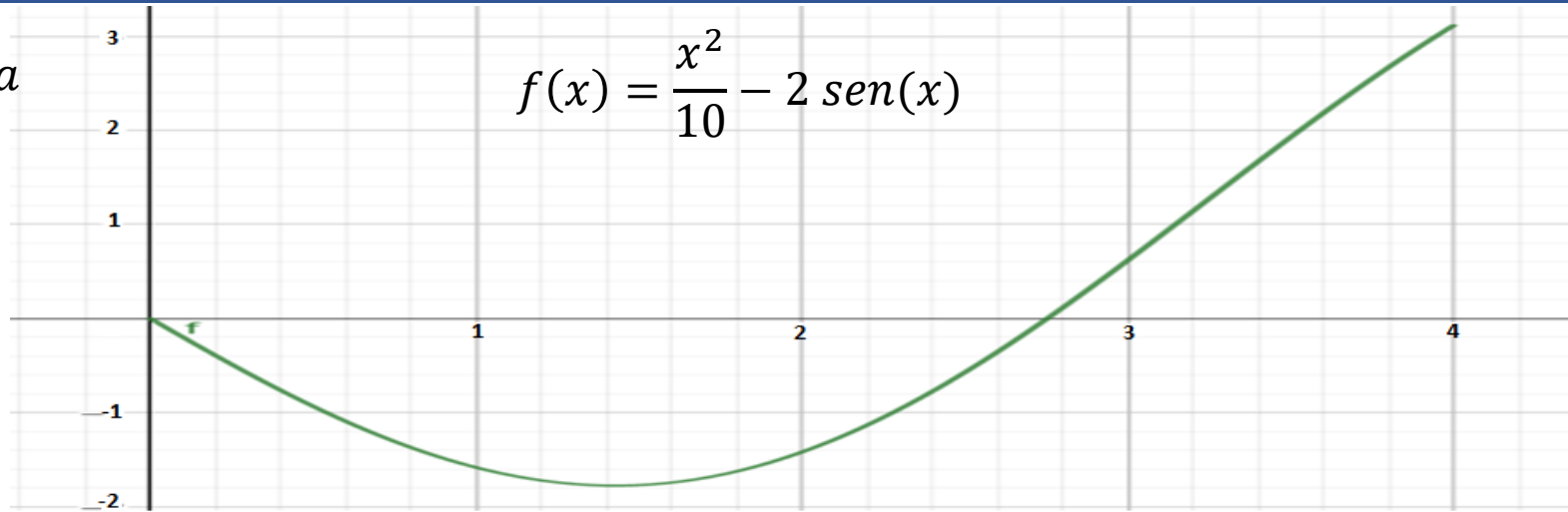
Sección Dorada

Dada la siguiente función, hallar una aproximación del mínimo en el intervalo $[0; 4]$

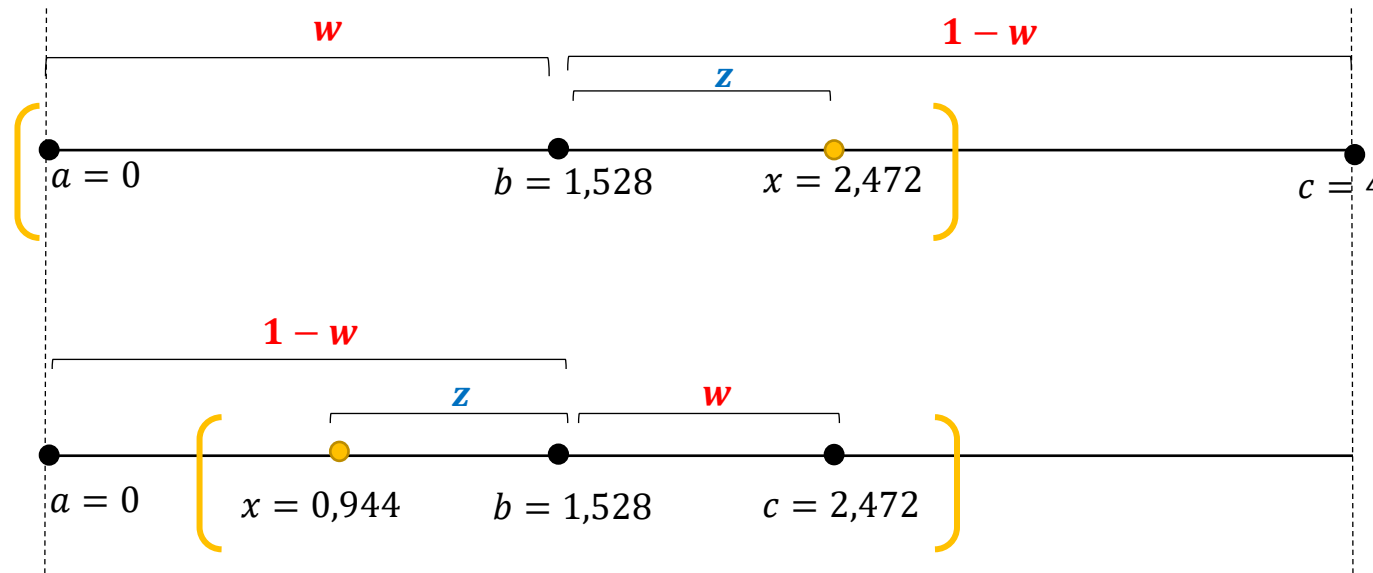
$$f(x) = \frac{x^2}{10} - 2 \operatorname{sen}(x)$$

Minimización

Sección Dorada



$$\frac{w}{ac} = \frac{z}{bc} ; w + z = 1 - w$$



$$\begin{aligned} f(a) &= 0 \\ f(b) &= -1,76 \\ f(x) &= -0,629 \\ f(c) &= 3,11 \end{aligned}$$

$$\begin{aligned} f(a) &= 0 \\ f(x) &= -1,53 \\ f(b) &= -1,76 \\ f(c) &= -0,629 \end{aligned}$$

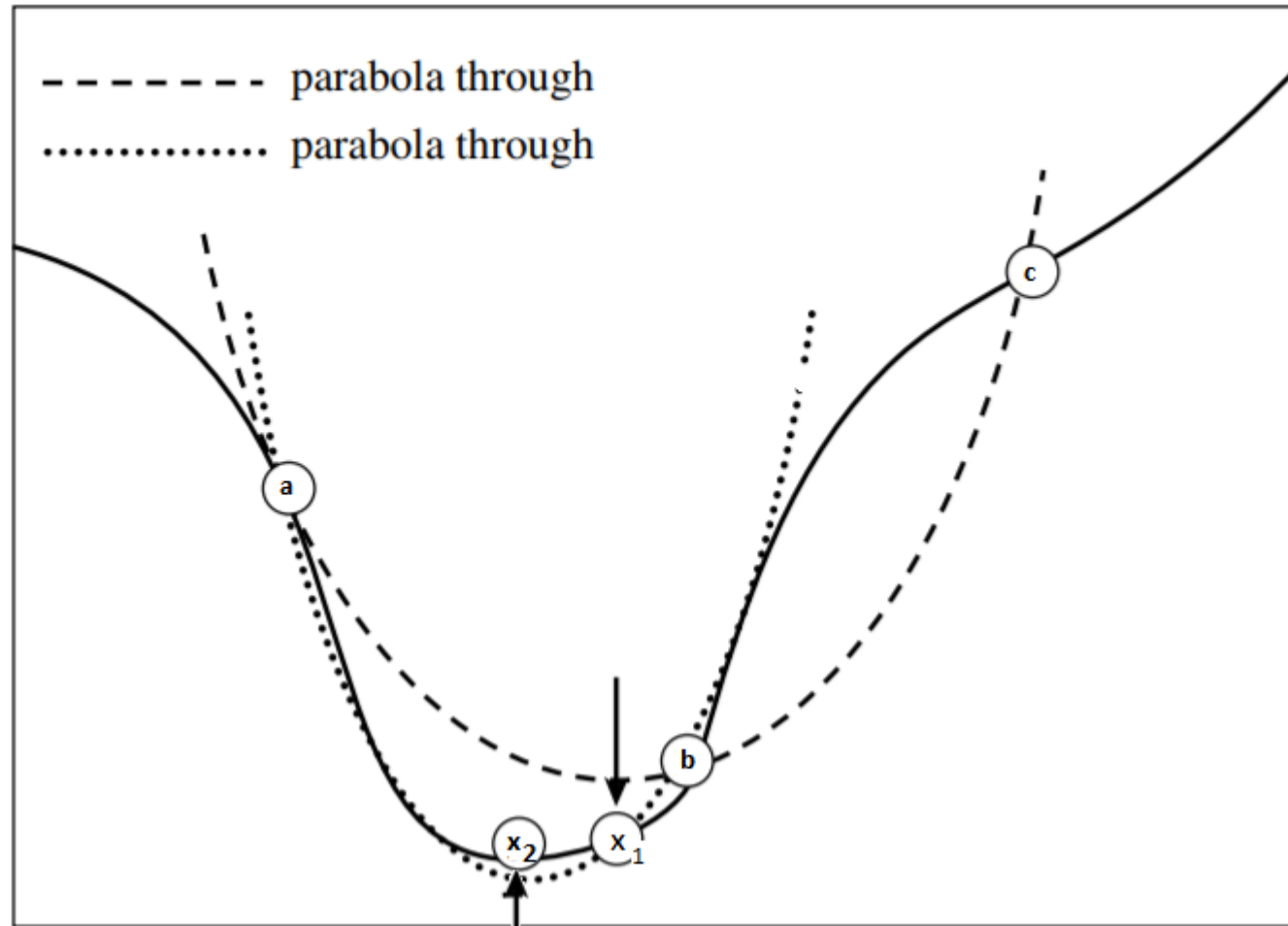
Minimización

k	a	b	x	c	f(a)	f(b)	f(x)	f(c)
1	0	1,52788	2,47215	4	0	-1,76472	-0,62995	3,11360
2	0	0,94427	1,52788	2,47215	0	-1,53097	-1,76472	-0,62995
3	0,94427	1,52788	1,88856	2,47215	-1,53097	-1,76472	-1,54321	-0,62995
4	0,94427	1,30495	1,52788	1,88856	-1,53097	-1,75945	-1,76472	-1,54321
5	1,304952	1,52788	1,66562	1,888529	-1,75945	-1,76472	-1,71359	-1,54324
6	1,304952	1,442717	1,52788	1,66562	-1,75945	-1,77547	-1,76472	-1,71359

Minimización

Interpolación Parabolica Inversa

$$f(x) = \frac{x^2}{10} - 2 \operatorname{sen}(x)$$



Minimización

Interpolación Parabolica Inversa

$$f(x) = \frac{x^2}{10} - 2 \operatorname{sen}(x)$$

$$a = 0,5$$

$$b = 1,5$$

$$c = 2,5$$

3 nodos

grado 2 del polinomio

$$P_2(x) = f(a) L_a(x) + f(b) L_b(x) + f(c) L_c(x)$$

$$L_a(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)} = \frac{(x-1,5)(x-2,5)}{(0,5-1,5)(0,5-2,5)}$$

$$f(a) = -0,934$$

$$f(b) = -1,770$$

$$f(c) = -0,572$$

$$L_b(x) = \frac{(x-a)(x-c)}{(b-a)(b-c)} = \frac{(x-0,5)(x-2,5)}{(1,5-0,5)(1,5-2,5)}$$

$$L_c(x) = \frac{(x-a)(x-b)}{(c-a)(c-b)} = \frac{(x-0,5)(x-1,5)}{(2,5-0,5)(2,5-1,5)}$$

$$P_2(x) = -0,934 \frac{(x-1,5)(x-2,5)}{(0,5-1,5)(0,5-2,5)} - 1,770 \frac{(x-0,5)(x-2,5)}{(1,5-0,5)(1,5-2,5)} - 0,572 \frac{(x-0,5)(x-1,5)}{(2,5-0,5)(2,5-1,5)}$$

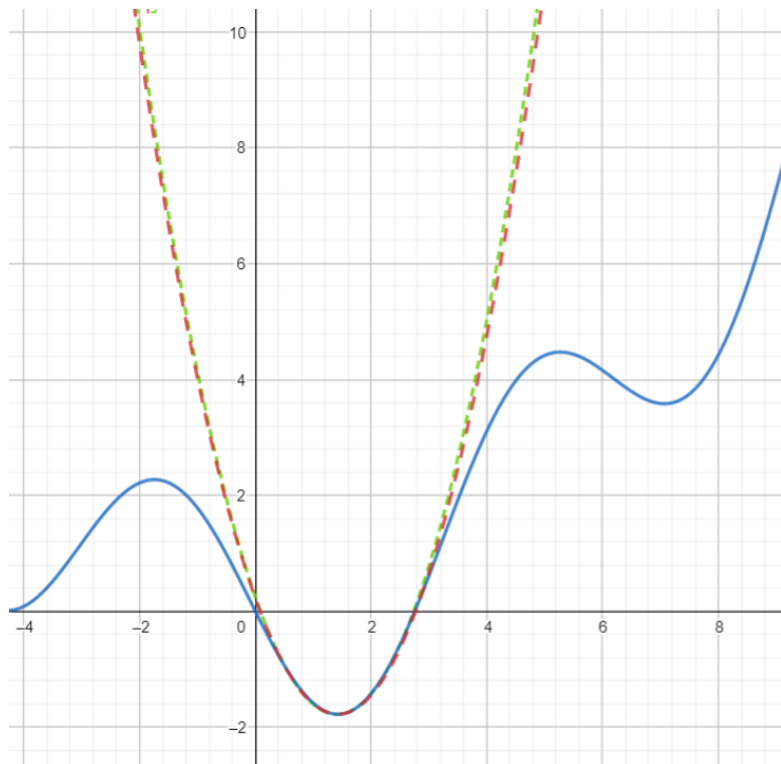
$$P'_2(x) = 0$$

$$x_{min} = 1,41104$$

Minimización

Interpolación Parabolica Inversa

a	b	c	P_2	$x (\rightarrow x_{\min})$	f(a)	f(b)	f(c)	f(x _{min})
0,5	1,5	2,5	$= -0,93385 \cdot \frac{(x-1,5)(x-2,5)}{(0,5-1,5)(0,5-2,5)} - 1,76999 \cdot \frac{(x-0,5)(x-2,5)}{(1,5-0,5)(1,5-2,5)} - 0,57194 \cdot \frac{(x-0,5)(x-1,5)}{(2,5-0,5)(2,5-1,5)}$	1,41104	-0,933851	-1,76999	-0,571944	-1,77543
0,5	1,41104	1,5	$= -0,93385 \cdot \frac{(x-1,41104)(x-1,5)}{(0,5-1,41104)(0,5-1,5)} - 1,775429 \cdot \frac{(x-0,5)(x-1,5)}{(1,41104-0,5)(1,41104-1,5)} - 1,76999 \cdot \frac{(x-0,5)(x-1,41104)}{(1,5-0,5)(1,5-1,41104)}$	1,424480	-0,933851	-1,77543	-1,76999	-1,77572



Minimización

Método de Interpolación de Orden Superior

$$f(x) = \frac{x^2}{10} - 2 \operatorname{sen}(x)$$

		f(x)	f'(x)
a	0,5	-0,93385	-1,65517
b	1,5	-1,76999	0,158526
c	2,5	-0,57194	2,102287

Interpol. Hermite →

$P_5(x)$

→

*Interpol.
Parab. Inversa*