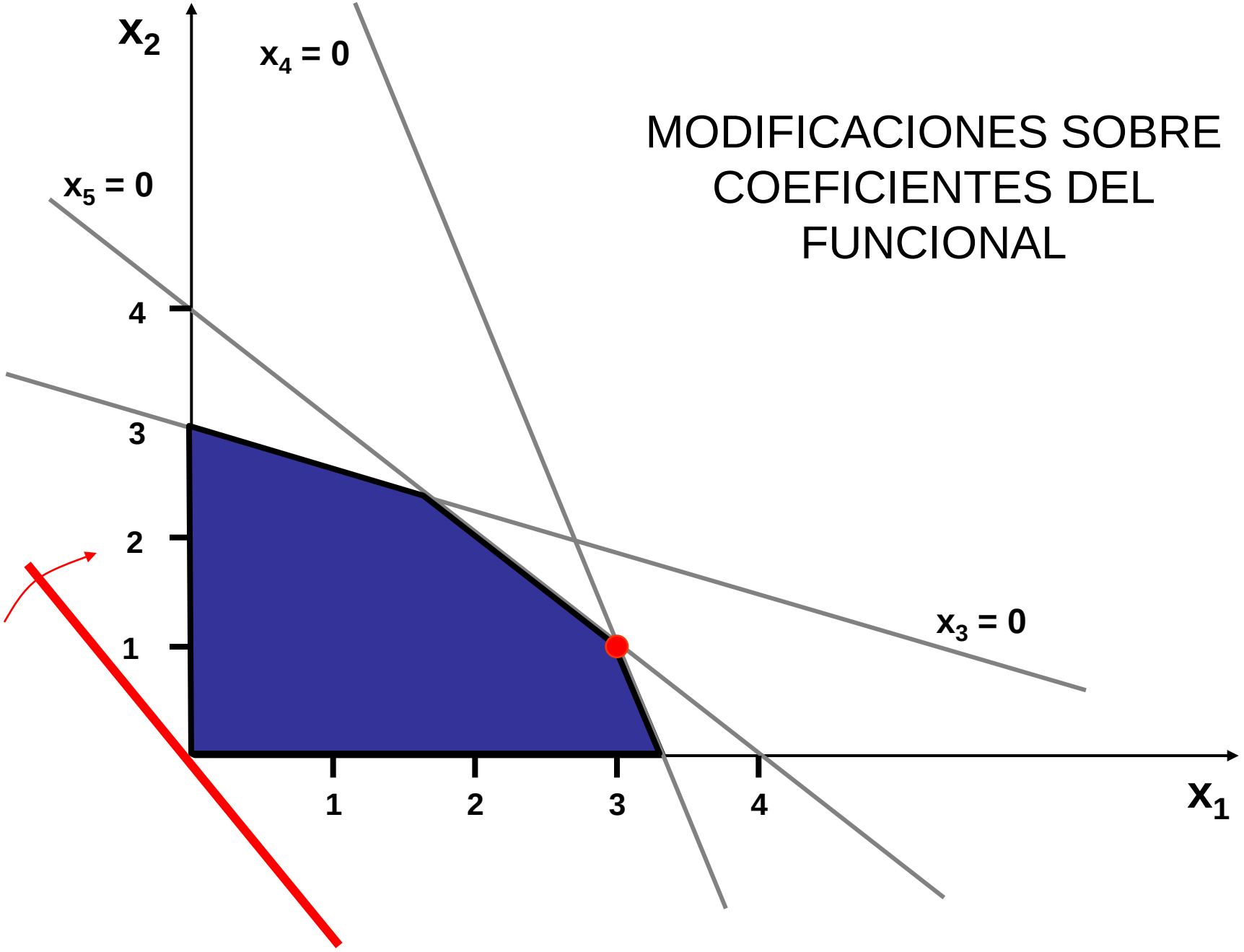
ANÁLISIS DE SENSIBILIDAD

- MODIFICACIONES SOBRE
  - COEFICIENTES DE EFICIENCIA (c_j)
  - TÉRMINOS INDEPENDIENTES (b_i)

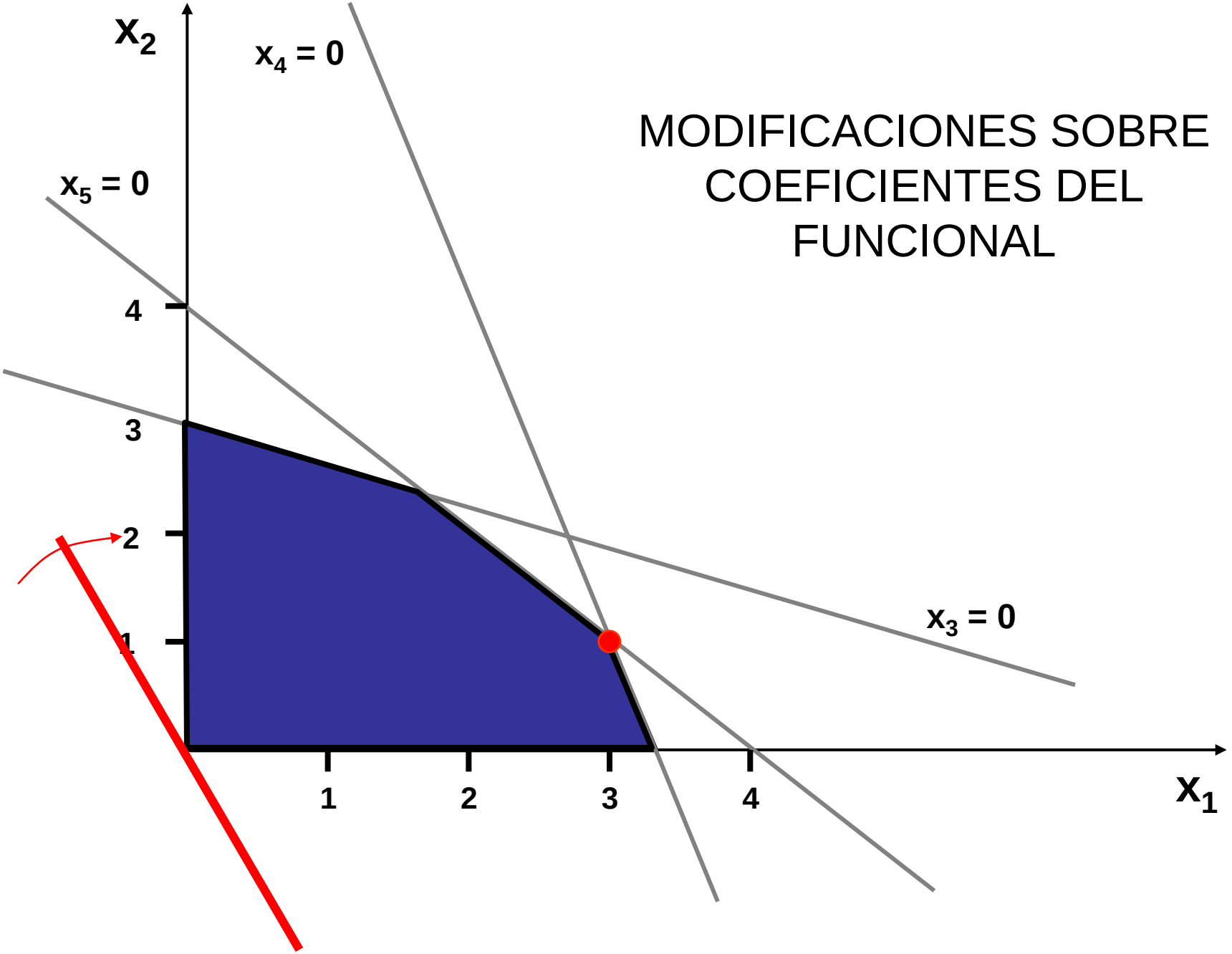
- AGREGADO DE
  - NUEVOS PRODUCTOS
  - NUEVAS RESTRICCIONES

- RANGOS DE VALIDEZ DE LA SOLUCIÓN ÓPTIMA
  - DIRECTA
  - DUAL

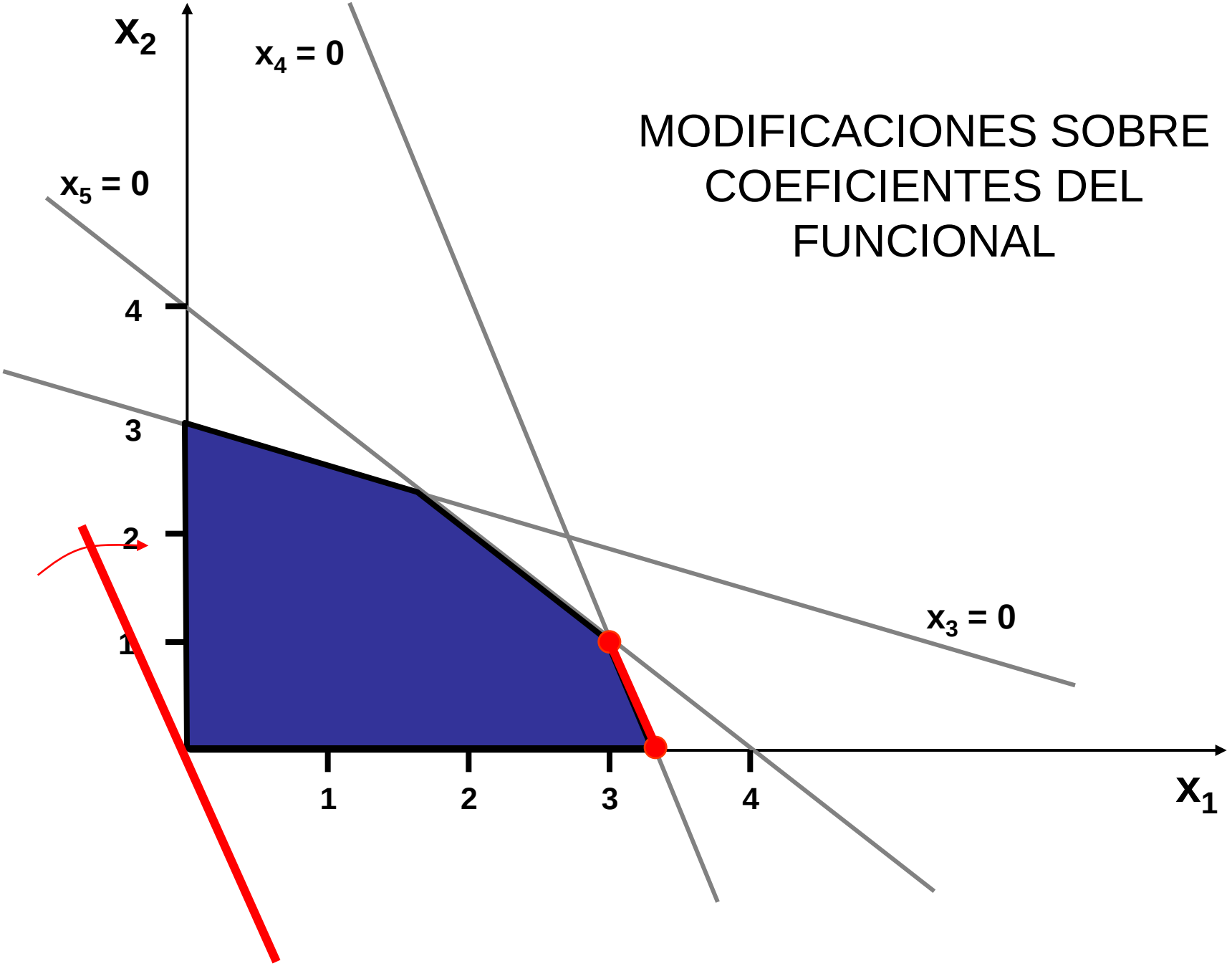
MODIFICACIONES SOBRE COEFICIENTES DEL FUNCIONAL



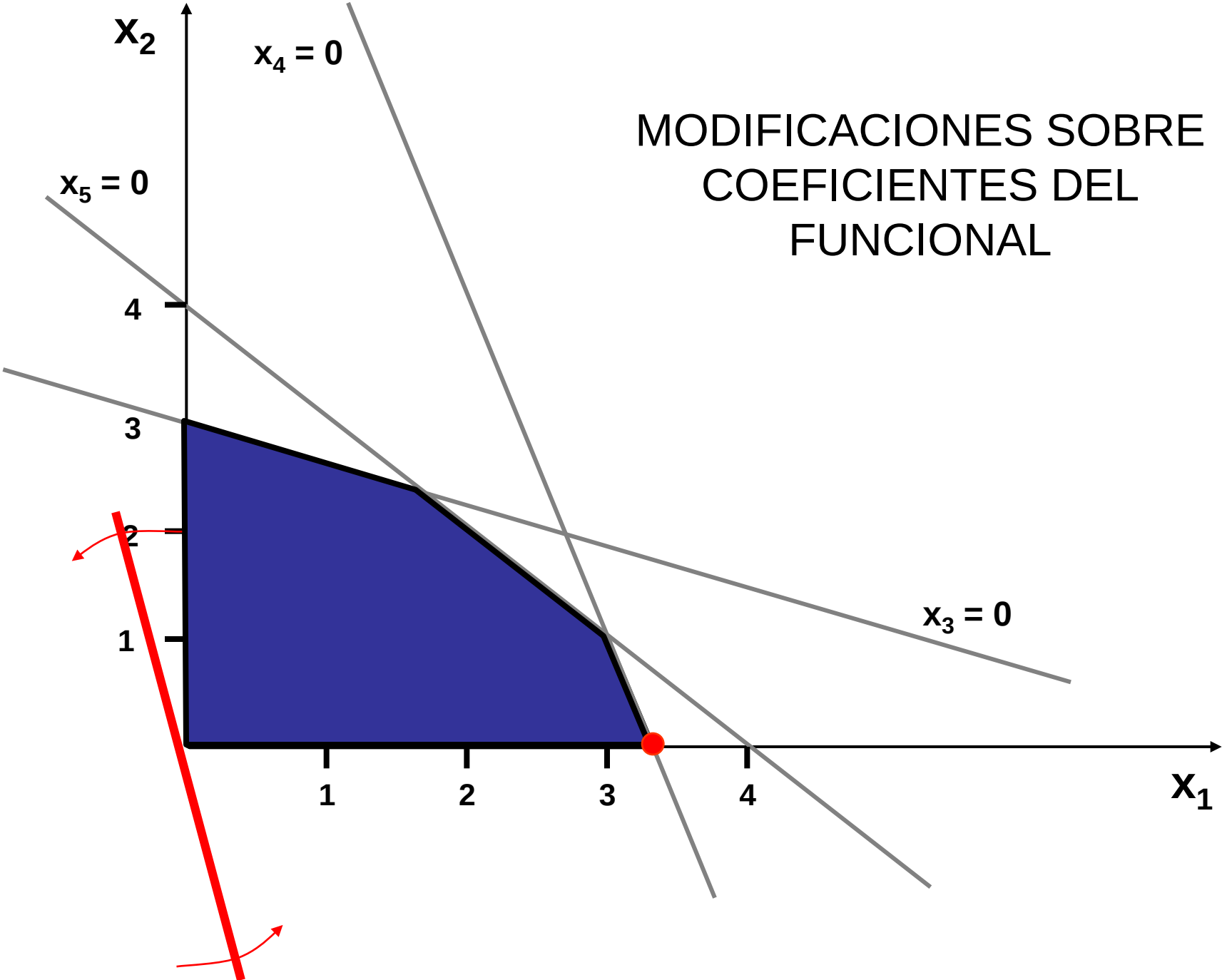
MODIFICACIONES SOBRE COEFICIENTES DEL FUNCIONAL



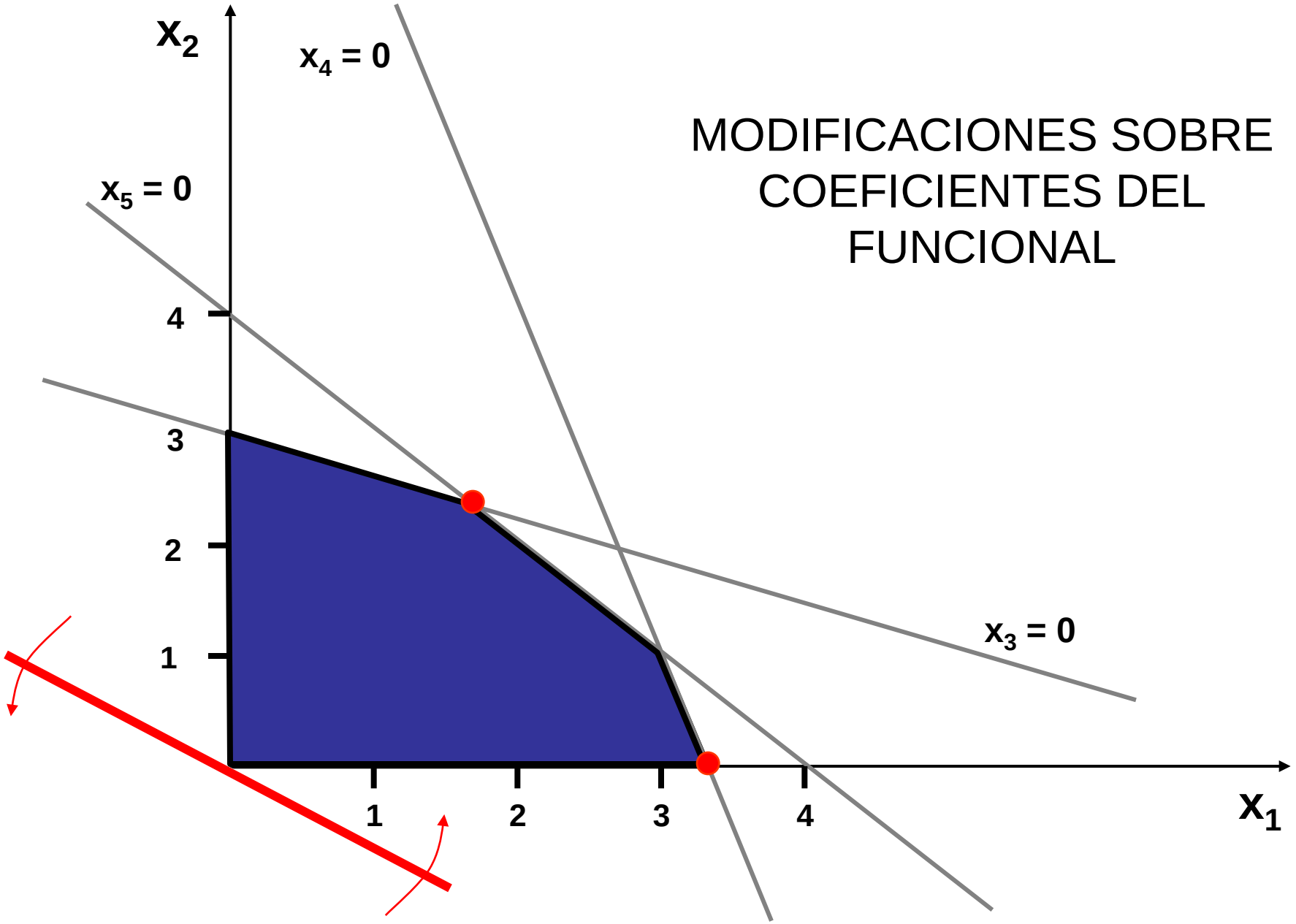
MODIFICACIONES SOBRE COEFICIENTES DEL FUNCIONAL



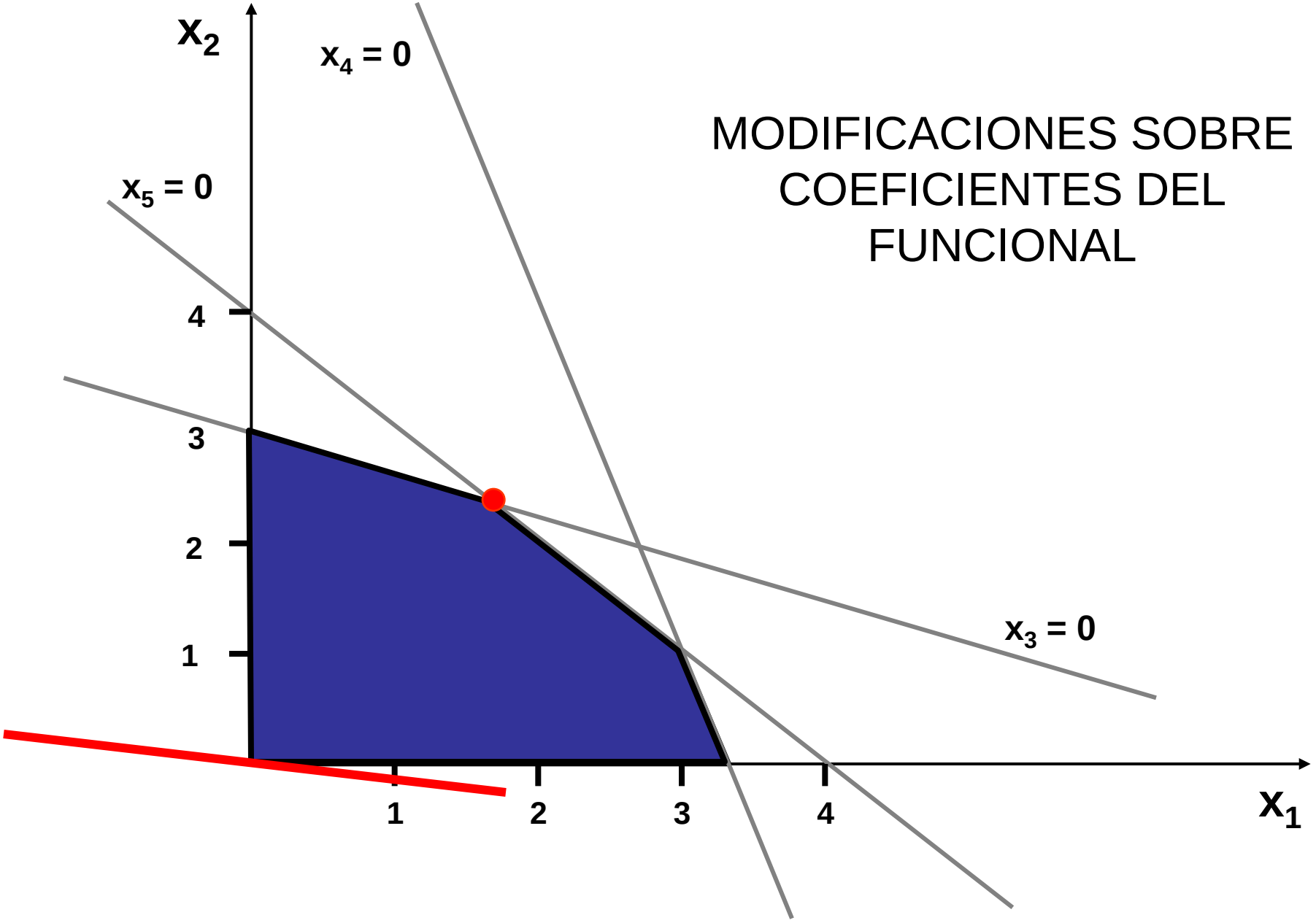
MODIFICACIONES SOBRE COEFICIENTES DEL FUNCIONAL



MODIFICACIONES SOBRE COEFICIENTES DEL FUNCIONAL



MODIFICACIONES SOBRE COEFICIENTES DEL FUNCIONAL



MAX: $Z = 4x_1 + 6x_2$

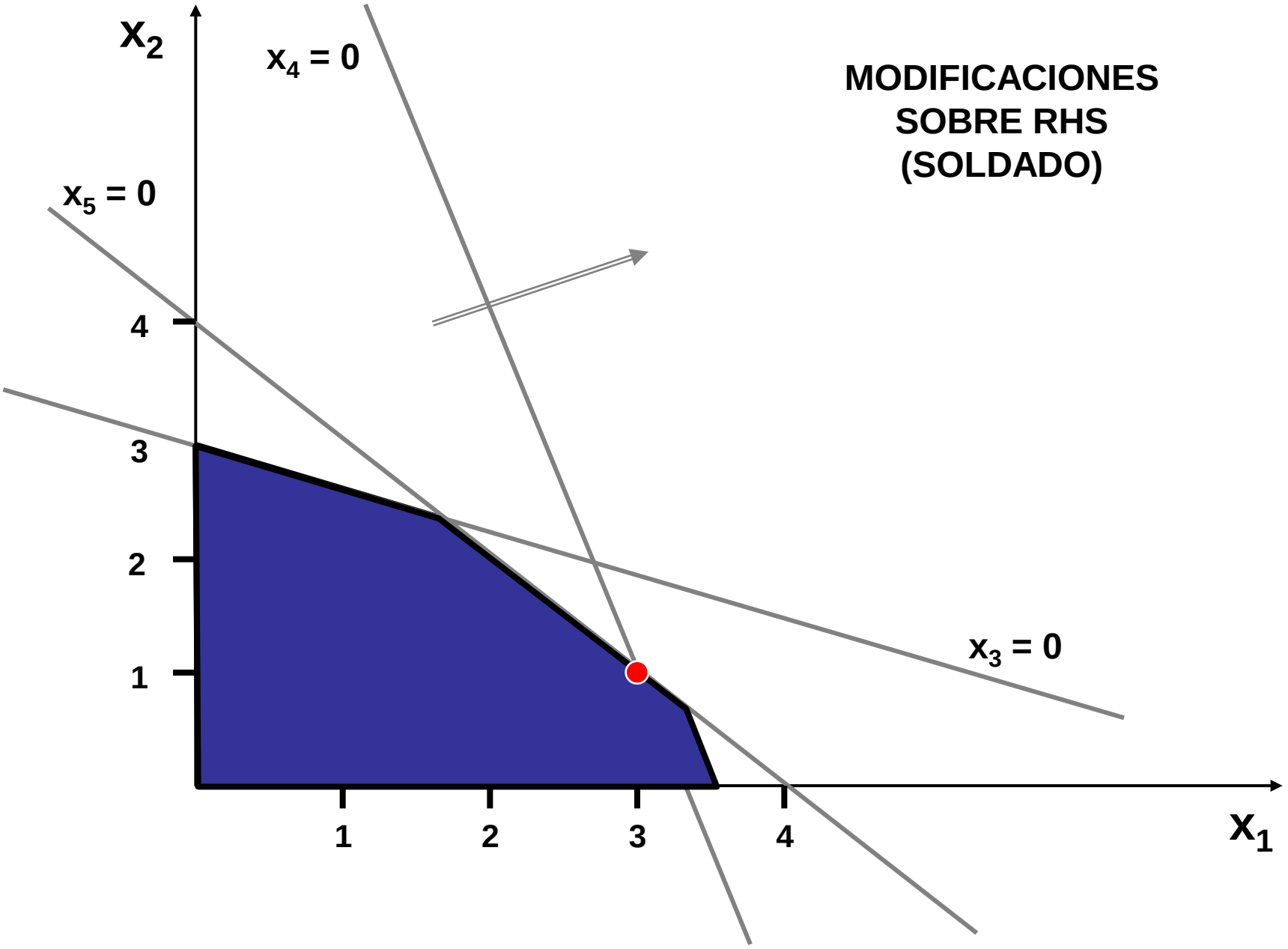
$$6x_1 + 16x_2 \leq 48000$$
$$12x_1 + 6x_2 \leq 42000$$
$$9x_1 + 9x_2 \leq 36000$$

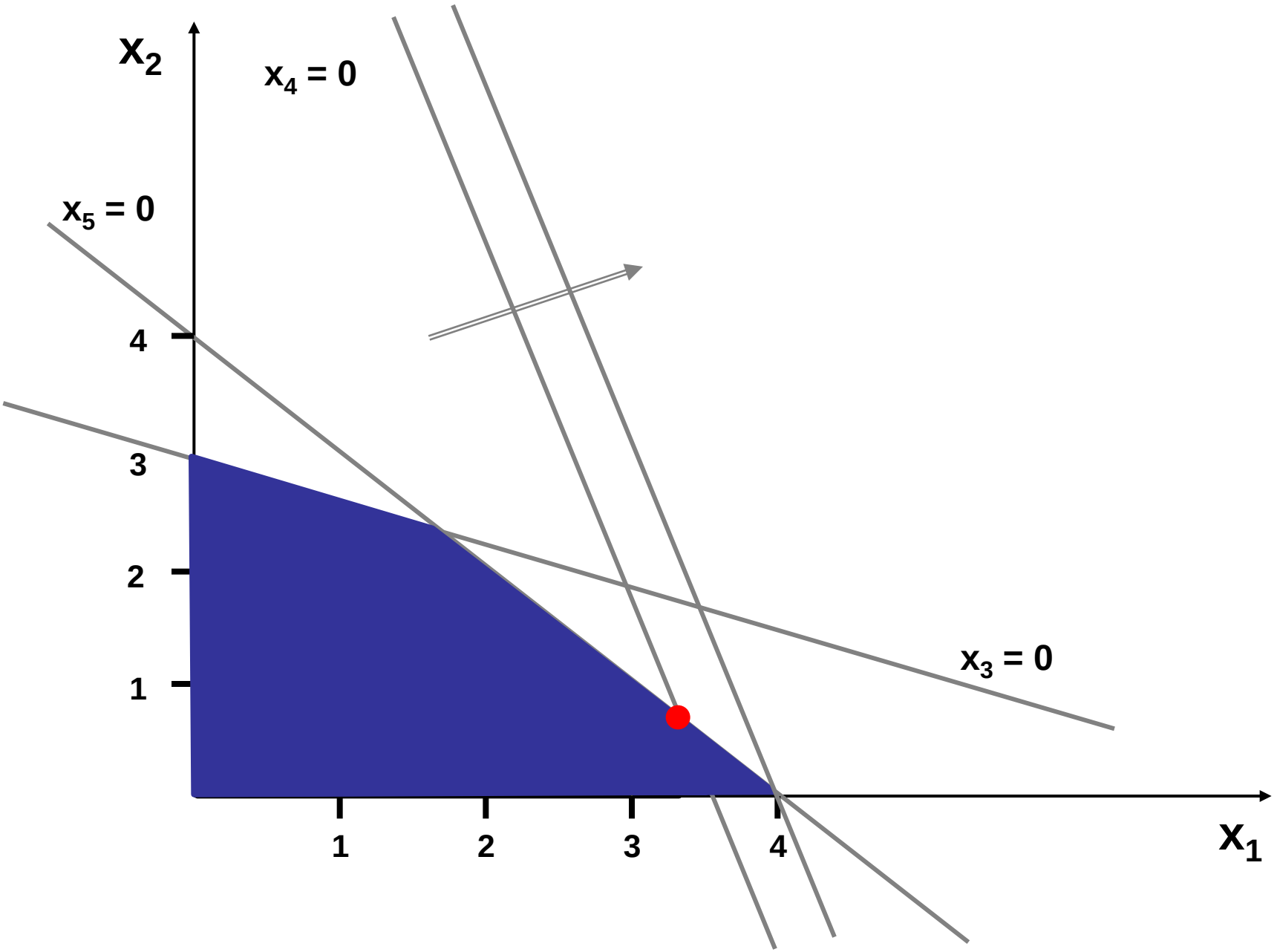
		c_j	4	6	0	0	0
c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
4	x_1	3.000	1			1/6	-1/9
6	x_2	1.000		1		-1/6	2/9
Z = 15.000			0	0	0	1/6	2/9
Z = 18.000			0	0	0	-1/3	8/9

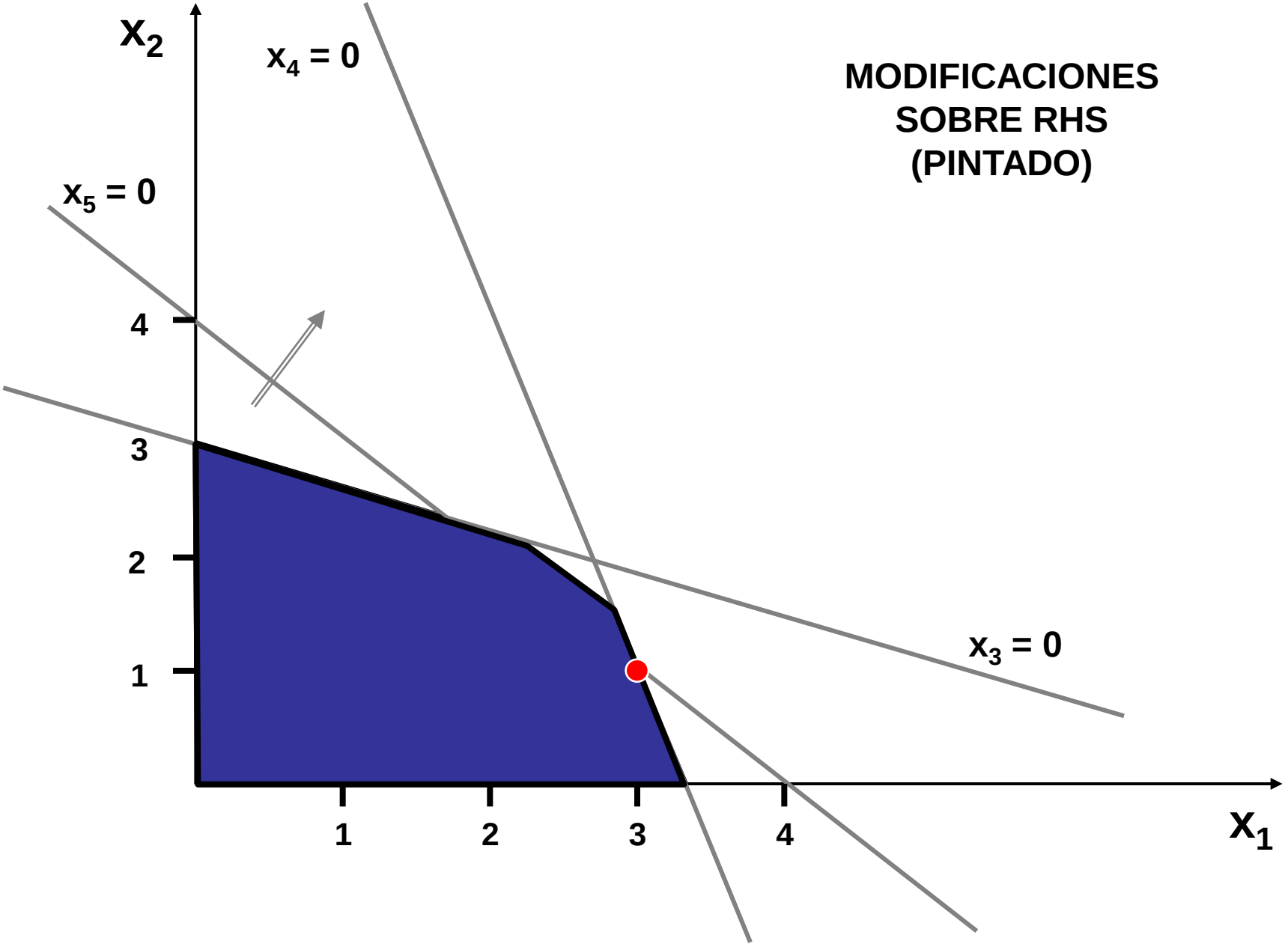
MAX: $Z = 4\,x_1 + \cancel{3}\,6\,x_2$

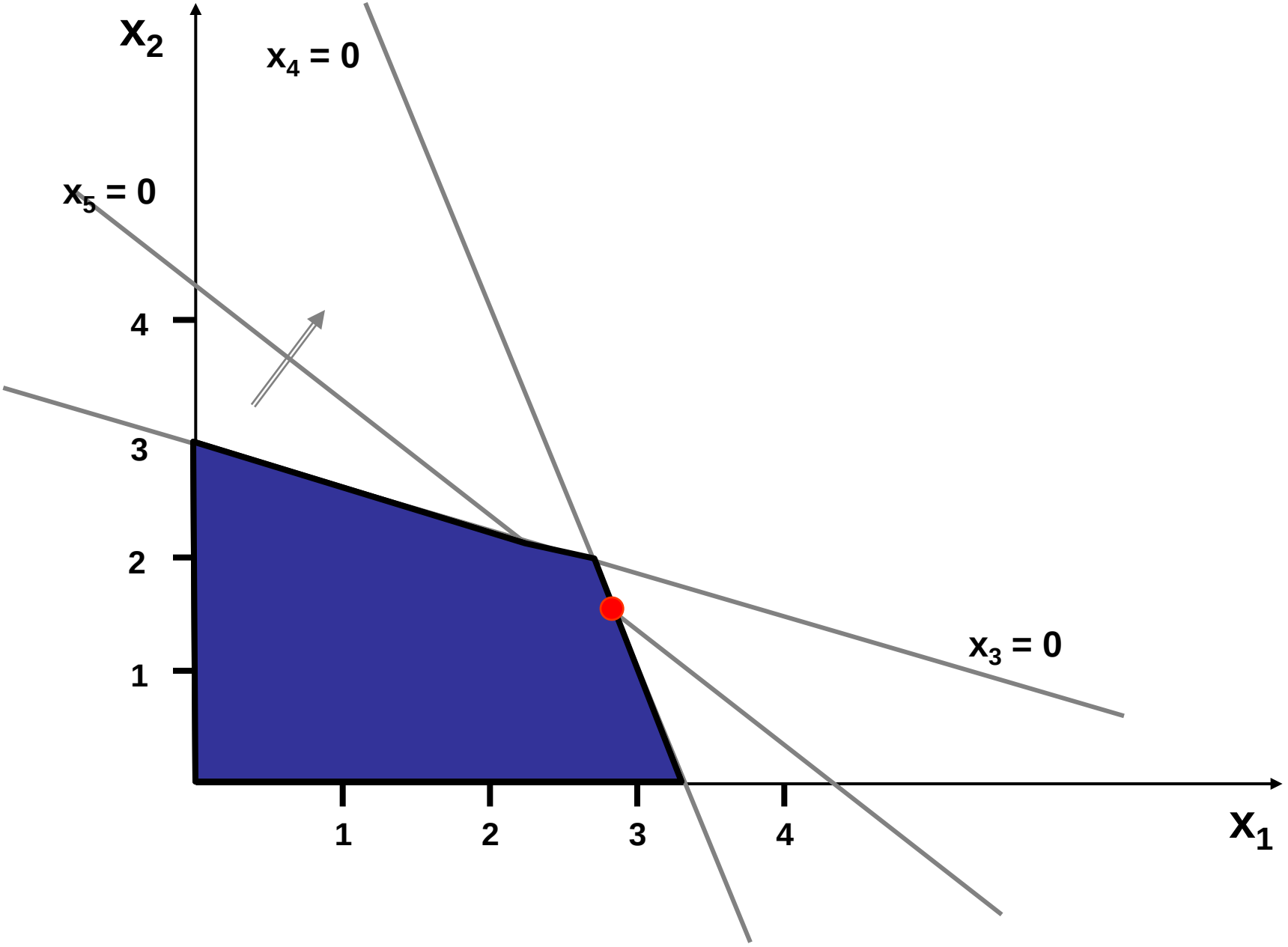
$$6\,x_1 + 16\,x_2 \leq 48000$$
$$12\,x_1 + 6\,x_2 \leq 42000$$
$$9\,x_1 + 9\,x_2 \leq 36000$$

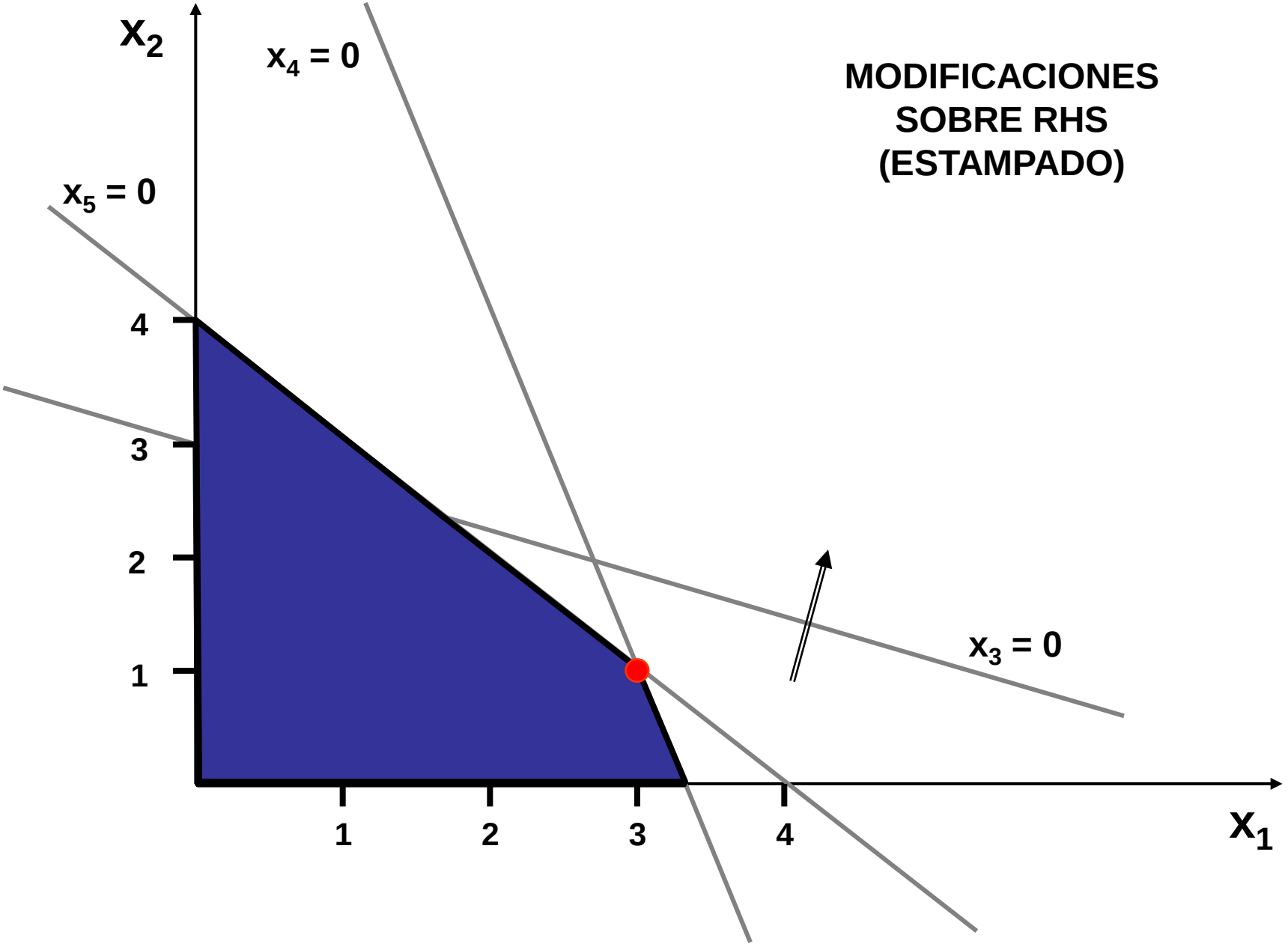
		c_j	4	6	0	0	0
c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_4	8.400			3/5	1	-26/15
4	x_1	1.600	1		-1/10		8/45
6	x_2	2.400		1	1/10		-1/15
Z = 18.400			0	0	1/5	0	14/15

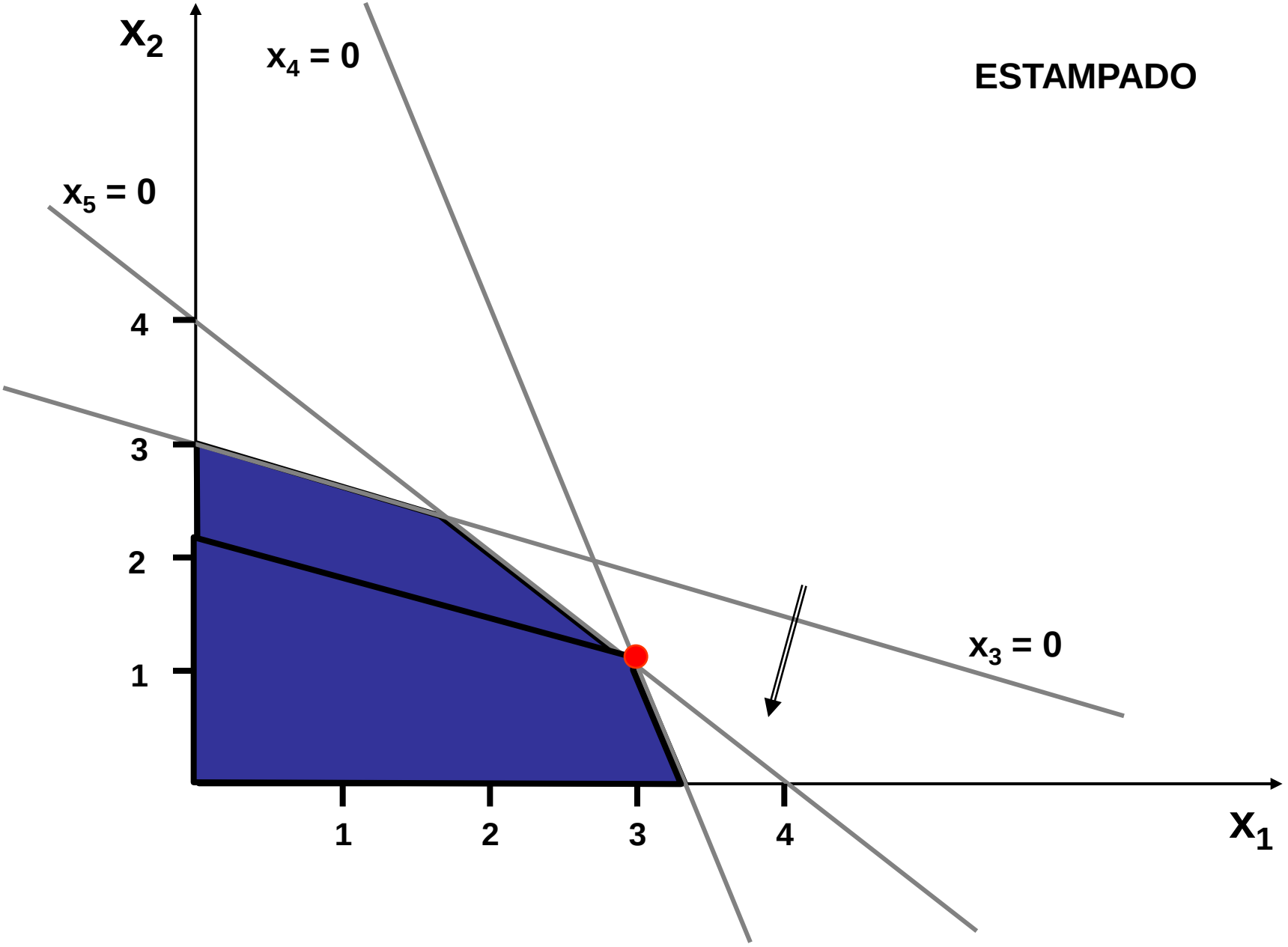

MODIFICACIONES
SOBRE RHS
(SOLDADO)
x_2
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
x_1
4
3
2
1
1
2
3
4

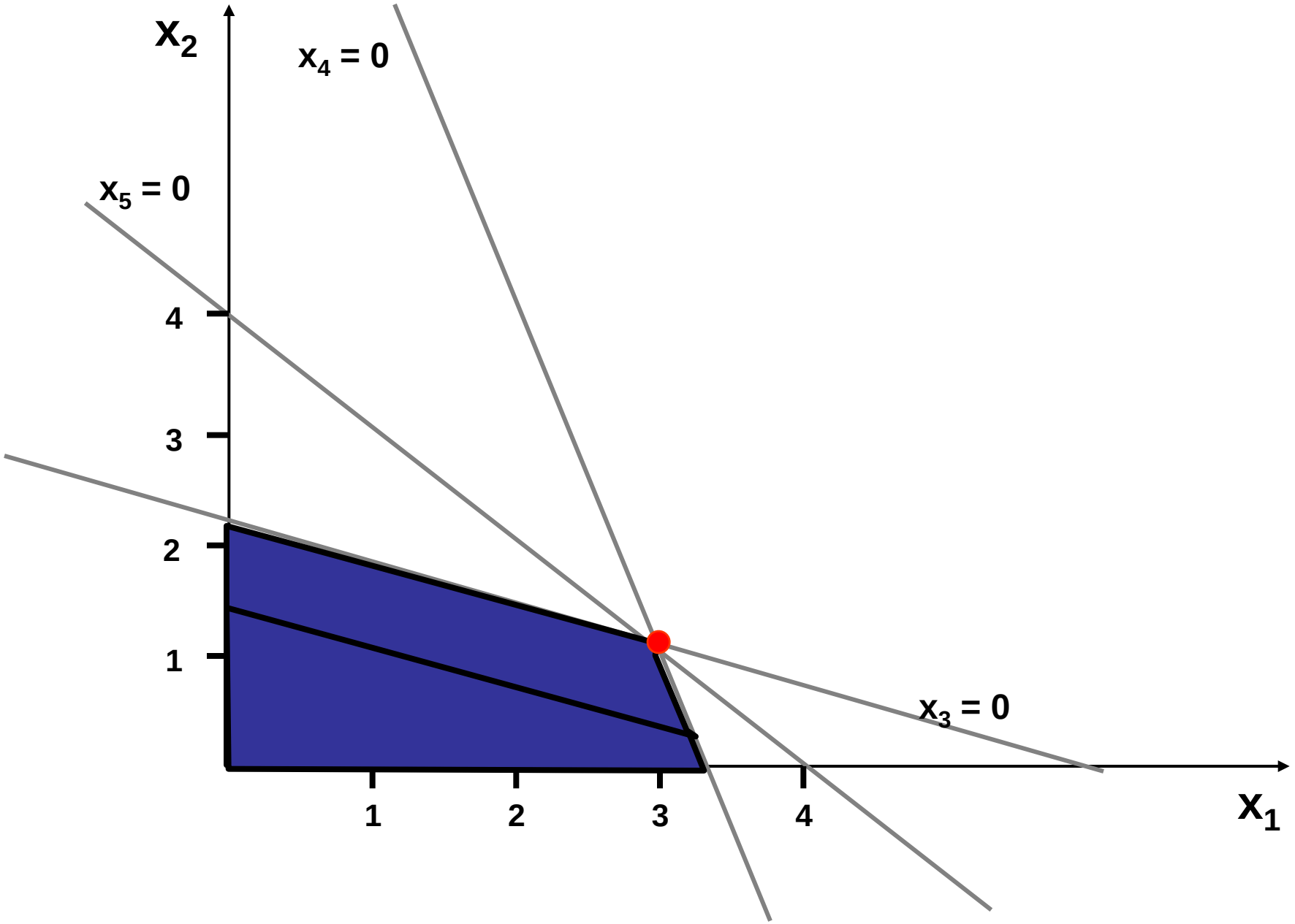

x2
x4 = 0
x5 = 0
4
3
2
1
x3 = 0
1
2
3
4
x1


MODIFICACIONES
SOBRE RHS
(PINTADO)
x2
x4 = 0
x5 = 0
x3 = 0
x1
1
2
3
4
1
2
3
4


x_2
$x_4 = 0$
$x_5 = 0$
4
3
2
1
$x_3 = 0$
1
2
3
4
x_1


MODIFICACIONES
SOBRE RHS
(ESTAMPADO)
x2
x1
x4 = 0
x5 = 0
x3 = 0
1
2
3
4
1
2
3
4


ESTAMPADO
x_2
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
x_1
4
3
2
1
1
2
3
4


x2
x4 = 0
x5 = 0
4
3
2
1
x3 = 0
1
2
3
4
x1

MAX: $Z = 4\,x_1 + 3\,x_2$

$6\,x_1 + 16\,x_2 \le 48.000$

$12\,x_1 + 6\,x_2 \le 42.000$ 54000

$9\,x_1 + 9\,x_2 \le 36.000$

MIN: $Z = 48.000\,y_1 + 42.000\,y_2 + 36.000\,y_3$

54.000

$6\,y_1 + 12\,y_2 + 9\,y_3 \ge 4$

$16\,y_1 + 6\,y_2 + 9\,y_3 \ge 3$

54.000

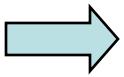
		c_j	48.000	42.000	36.000	0	0
c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5
54.000 ← 42.000	y_2	1/6	-5/3	1		-1/6	1/6
36.000	y_3	2/9	26/9		1	1/9	-2/9
Z = 15.000			-14.000	0	0	-3.000	-1.000
Z = 17.000			-34.000	0	0	-5.000	1.000

MAX: $Z = 4\,x_1 + 3\,x_2$

$6\,x_1 + 16\,x_2 \leq 48.000$

$12\,x_1 + 6\,x_2 \leq 42.000$ 54000

$9\,x_1 + 9\,x_2 \leq 36.000$



MIN: $Z = 4\,8.000\,y_1 + 42.000\,y_2 + 36.000\,y_3$

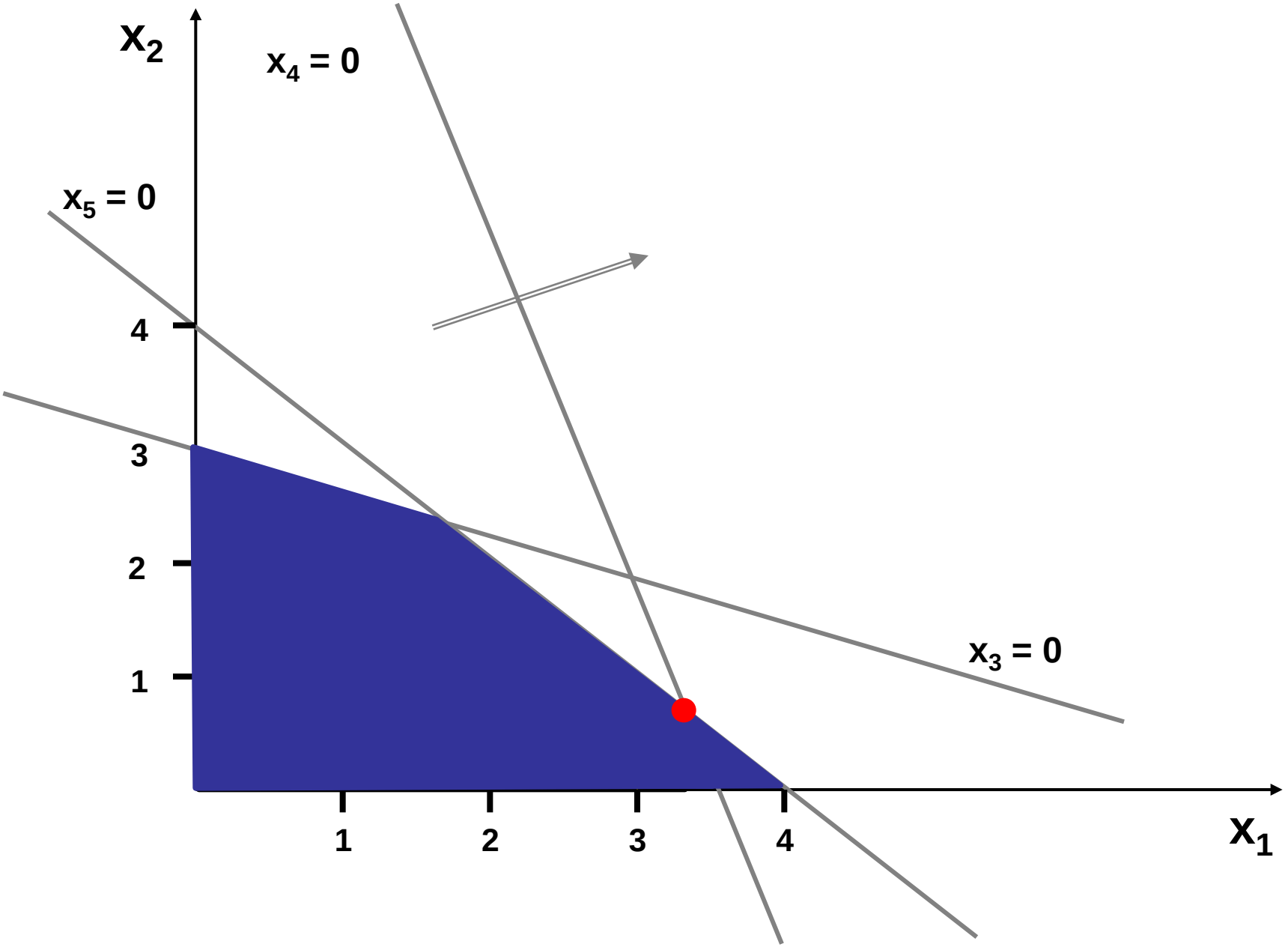
54.000

$6\,y_1 + 12\,y_2 + 9\,y_3 \geq 4$

$16\,y_1 + 6\,y_2 + 9\,y_3 \geq 3$

54.000

		c_j	48.000	42.000	36.000	0	0
c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5
0	y_5	1	-10	6		-1	1
36.000	y_3	4/9	2/3	4/3	1	-1/9	
Z = 16.000			-24.000	-6.000	0	-4.000	0


x2
x4 = 0
x5 = 0
4
3
2
1
x3 = 0
1
2
3
4
x1

AGREGADO DE UNA NUEVA ACTIVIDAD

$$\textbf{MAX:} \quad Z = 4\,x_1 + 3\,x_2 + 7{,}5\,x_6$$

$$\begin{cases} 6\,x_1 + 16\,x_2 + 23\,x_6 \leq 48.000 \\ 12\,x_1 + 6\,x_2 + 18\,x_6 \leq 42.000 \\ 9\,x_1 + 9\,x_2 + 18\,x_6 \leq 36.000 \end{cases}$$

SOLUCIÓN ÓPTIMA ANTERIOR: $y_1 = 0 \quad y_2 = 1/6 \quad y_3 = 2/9$

$23 \cdot y_1 + 18 \cdot y_2 + 18 \cdot y_3 = 23 \cdot 0 + 18 \cdot 1/6 + 18 \cdot 2/9 = 7 < 7{,}5$

		c_j	4	3	0	0	0	7,5
c_k	x_k	**B**	A_1	A_2	A_3	A_4	A_5	A_6
0	x_3	48.000	6	16	1			**23**
0	x_4	42.000	12	6		1		**18**
0	x_5	36.000	9	9			1	**18**
Z = 0			- 4	- 3	0	0	0	

0	x_3	14.000			1	5/3	-26/9	
4	x_1	3.000	1			1/6	-1/9	
3	x_2	1.000		1		-1/6	2/9	
Z = 15.000			0	0	0	1/6	2/9	

		c_j	4	3	0	0	0	7,5
c_k	x_k	B	A_1	A_2	A_3	A_4	A_5	A_6
0	x_3	48.000	6	16	1			23
0	x_4	42.000	12	6		1		18
0	x_5	36.000	9	9			1	18
Z = 0			- 4	- 3	0	0	0	

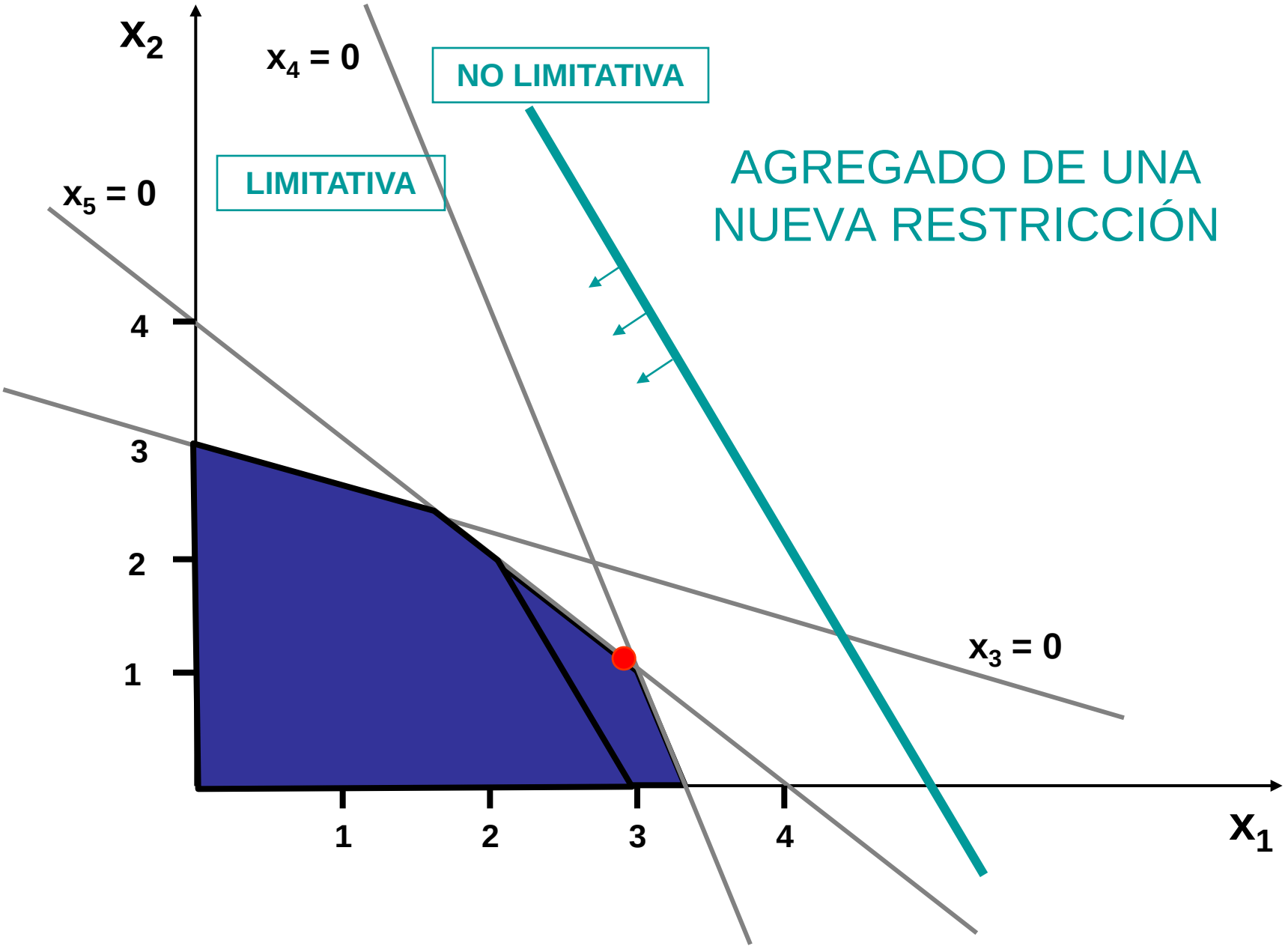
0	x_3	14.000			1	5/3	-26/9	
4	x_1	3.000	1			1/6	-1/9	
3	x_2	1.000		1		-1/6	2/9	
Z = 15.000			0	0	0	1/6	2/9	

$$\begin{vmatrix} 1 & 5/3 & -26/9 \\ & 1/6 & -1/9 \\ & -1/6 & 2/9 \end{vmatrix} . \begin{vmatrix} 23 \\ 18 \\ 18 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

		c_j	4	3	0	0	0	7,5
c_k	x_k	**B**	A_1	A_2	A_3	A_4	A_5	A_6
0	x_3	48.000	6	16	1			23
0	x_4	42.000	12	6		1		18
0	x_5	36.000	9	9			1	18
Z = 0			- 4	- 3	0	0	0	

0	x_3	14.000			1	5/3	-26/9	1
4	x_1	3.000	1			1/6	-1/9	1
3	x_2	1.000		1		-1/6	2/9	1
Z = 15.000			0	0	0	1/6	2/9	-0,5

		c_j	4	3	0	0	0	7,5
$\mathbf{c_k}$	$\mathbf{x_k}$	**B**	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$	$\mathbf{A_5}$	$\mathbf{A_6}$
0	$\mathbf{x_3}$	13.000		-1	1	11/6	-28/4	
4	$\mathbf{x_1}$	2.000	1	-1		1/3	-1/3	
7,5	$\mathbf{x_6}$	1.000		1		-1/6	2/9	1
Z = 15.500			0	1/2	0	1/12	1/3	0


x_2
$x_4 = 0$
NO LIMITATIVA
LIMITATIVA
$x_5 = 0$
AGREGADO DE UNA NUEVA RESTRICCIÓN
4
3
2
1
$x_3 = 0$
1
2
3
4
x_1

AGREGADO DE UNA NUEVA RESTRICCIÓN

$$\textbf{MAX:} \quad Z = 4x_1 + 3x_2$$

$$\begin{cases} 6x_1 + 16x_2 \leq 48.000 \\ 12x_1 + 6x_2 \leq 42.000 \\ 9x_1 + 9x_2 \leq 36.000 \\ 2x_1 + x_2 \leq 6.000 \end{cases}$$

SOLUCIÓN ÓPTIMA ANTERIOR: $x_1 = 3.000 \quad x_2 = 1.000$

$2 \cdot x_1 + x_2 = 2 \cdot 3.000 + 1.000 = 7.000 \not\leq 6.000$

		c_j	48.000	42.000	36.000	0	0	6.000
c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5	A'_6
0	y_4	4	6	12	9	-1		2
0	y_5	3	16	6	9		-1	1

c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5	A'_6
42.000	y_2	1/6	-5/3	1		-1/6	1/6	
36.000	y_3	2/9	26/9		1	1/9	-2/9	
Z = 15.000			-14.000	0	0	-3.000	-1.000	

$$\begin{vmatrix} 1/6 & -1/6 \\ -1/9 & 2/9 \end{vmatrix} . \begin{vmatrix} 2 \\ 1 \end{vmatrix} = \begin{vmatrix} 1/6 \\ 0 \end{vmatrix}$$

		c_j	48.000	42.000	36.000	0	0	6.000
c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5	A'_6
0	y_4	4	6	12	9	-1		2
0	y_5	3	16	6	9		-1	1

c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5	A'_6
42.000	y_2	1/6	-5/3	1		-1/6	1/6	1/6
36.000	y_3	2/9	26/9		1	1/9	-2/9	0
Z = 15.000			-14.000	0	0	-3.000	-1.000	1.000

		c_j	**48.000**	**42.000**	**36.000**	**0**	**0**	**6.000**
$\mathbf{c'_k}$	$\mathbf{y_k}$	**B'**	$\mathbf{A'_1}$	$\mathbf{A'_2}$	$\mathbf{A'_3}$	$\mathbf{A'_4}$	$\mathbf{A'_5}$	$\mathbf{A'_6}$
6.000	$\mathbf{y_6}$	1	-10	6		-1	1	1
36.000	$\mathbf{y_3}$	2/9	25/9		1	1/9	-2/9	
Z = 14.000			**-4.000**	**-6.000**	**0**	**-2.000**	**-2000**	**0**

VARIABLES BÁSICAS

c_k	x_k	**B**	A_j
c_i	x_i	b_i	a_{ij}
$Z \rightarrow$			$z_j - c_j$

VARIABLES BÁSICAS

c_k	x_k	B	A_j
c_i	x_i	b_i	a_{ij}
$Z \rightarrow$ Máx			$z_j - c_j$

$$\Delta c_+ \cdot a_{ij-} = -(z_j - c_j)_+$$

LÍMITE SUPERIOR: $\Delta c = c_{sup} - c_j$

VARIABLES BÁSICAS

a_{ij}	MAX	MIN
SUP.	-	
INF.		

VARIABLES BÁSICAS

c_k	x_k	B	A_j
c_i	x_i	b_i	a_{ij}
$Z \rightarrow$ Máx			$z_j - c_j$

$$\Delta c_{-} \cdot a_{ij+} = -(z_j - c_j)_{+}$$

LÍMITE INFERIOR: $\Delta c = c_{INF} - c_j$

VARIABLES BÁSICAS

a_{ij}	MAX	MIN
SUP.	-	
INF.	+	

VARIABLES BÁSICAS

c_k	x_k	B	A_j
c_i	x_i	b_i	a_{ij}
$Z \rightarrow$ Mín			$z_j - c_j$

$$\Delta c_+ \cdot a_{ij+} = -(z_j - c_j)_-$$

LÍMITE SUPERIOR: $\Delta c = c_{sup} - c_j$

VARIABLES BÁSICAS

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	

VARIABLES BÁSICAS

c_k	x_k	B	A_j
c_i	x_i	b_i	a_{ij}
$Z \rightarrow$ Mín			$z_j - c_j$

$$\Delta c_{-} \cdot a_{ij} = - (z_j - c_j)_{-}$$

LÍMITE INFERIOR: $\Delta c = c_{INF} - c_j$

VARIABLES BÁSICAS

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

$$\Delta c \cdot a_{ij} = - (z_j - c_j)$$

$$(c_{j\,LIM} - c_j) \cdot a_{ij} = - (z_j - c_j)$$

$$c_{j\,LIM} = c_j \pm \frac{z_j - c_j}{a_{ij}}$$

$$c_{j\,LIM} = c_j \pm \left| \frac{z_j - cj}{a_{ij}} \right|_{MIN}$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

VARIABLES NO BÁSICAS

		c_j	c_j
c_k	x_k	**B**	A_j
$Z \rightarrow$ Máx			z_j-c_j

LÍMITE SUPERIOR

$$(z_j\text{-}c_j)_{+} - \Delta c_{+} = 0$$

VARIABLES NO BÁSICAS

		c_j	c_j
c_k	x_k	**B**	A_j
Z → Máx			z_j-c_j

LÍMITE SUPERIOR

$$c_{j\ sup} = (z_j - c_j)_+ + c_j$$

VARIABLES NO BÁSICAS

		c_j	c_j
c_k	x_k	B	A_j
			a_{ij}
Z → Máx			z_j-c_j

$$(z_j\text{-}c_j)_{+} - \Delta c_{-} = 0$$

LÍMITE INFERIOR **NO EXISTE**

VARIABLES NO BÁSICAS

		c_j	c_j
c_k	x_k	**B**	A_j
			a_{ij}
$Z \rightarrow$ Mín			$z_j - c_j$

LÍMITE SUPERIOR

NO EXISTE

$$(z_j - c_j)_{-} - \Delta c_{+} = 0$$

VARIABLES NO BÁSICAS

		c_j	c_j
c_k	x_k	**B**	A_j
			a_{ij}
$Z \rightarrow$ Mín			z_j-c_j

$$(z_j\text{-}c_j)^{-} - \Delta c^{-} = 0$$

LÍMITE INFERIOR

VARIABLES NO BÁSICAS

		c_j	c_j
c_k	x_k	B	A_j
			a_{ij}
Z → Mín			z_j-c_j

$$c_{j\ INF} = -(z_j - c_j) + c_j$$

LÍMITE INFERIOR

VARIABLES NO BÁSICAS

c_j LIM	MAX	MIN
SUP.	$c_j + (z_j - c_j)$	$\nexists$
INF.	$\nexists$	$c_j - (z_j - c_j)$

RANGO DE c_1

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
4	x_1	3.000	1			1/6	-1/9
3	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9

$$c_{1SUP} = c_j + \frac{z_j - c_j}{a_{ij}}$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

RANGO DE c_1

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
~~4~~ 6	x_1	3.000	1			1/6	-1/9
3	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9
Z = 16.000 (Máx)			0	0	0	1/2	0*

$$c_{1SUP} = 4 + \frac{2/9}{1/9} = 6$$

RANGO DE c_1

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
4	x_1	3.000	1			1/6	-1/9
3	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9

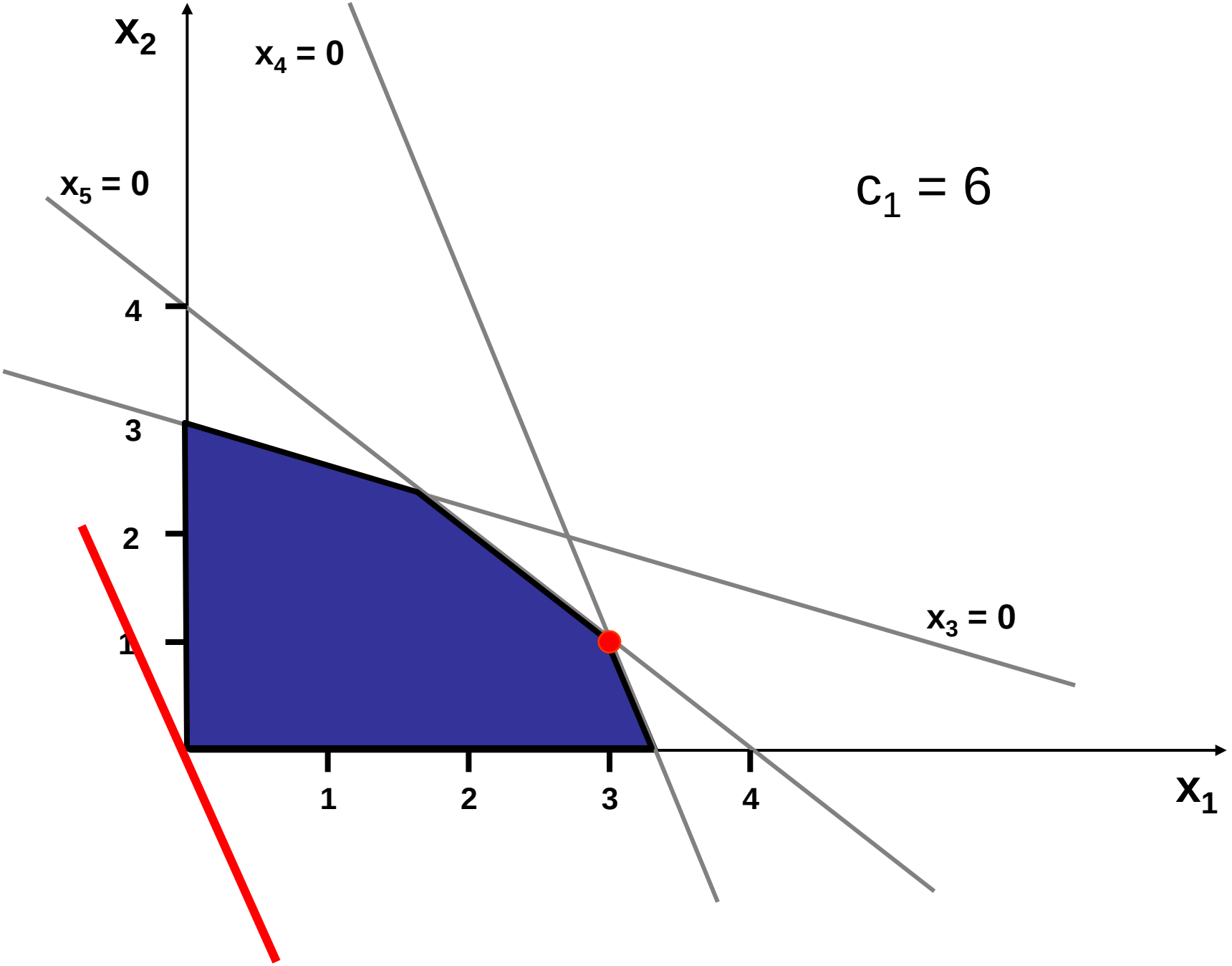
$$c_{1\,INF} = c_j - \frac{z_j - c_j}{a_{ij}}$$

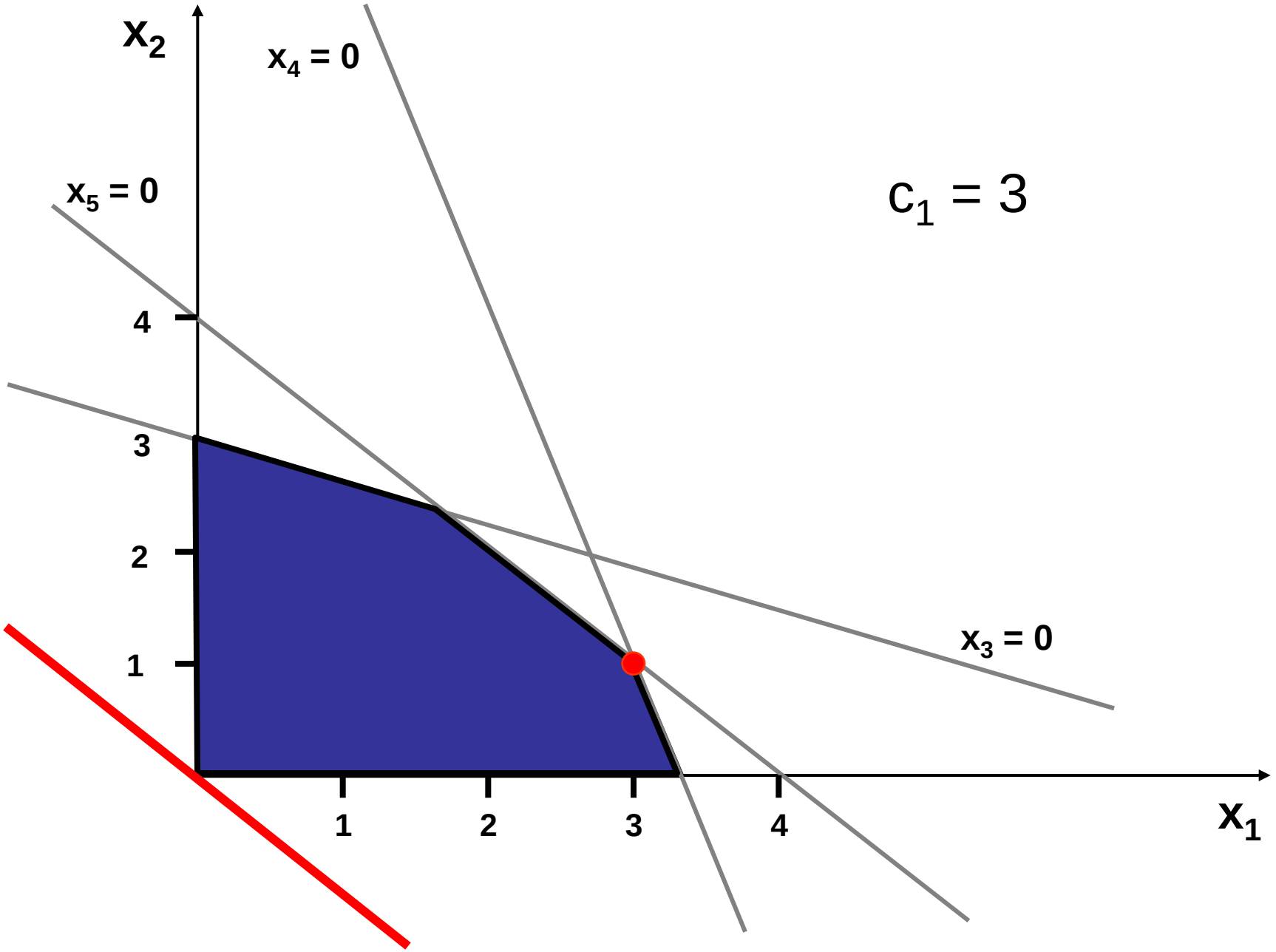
a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

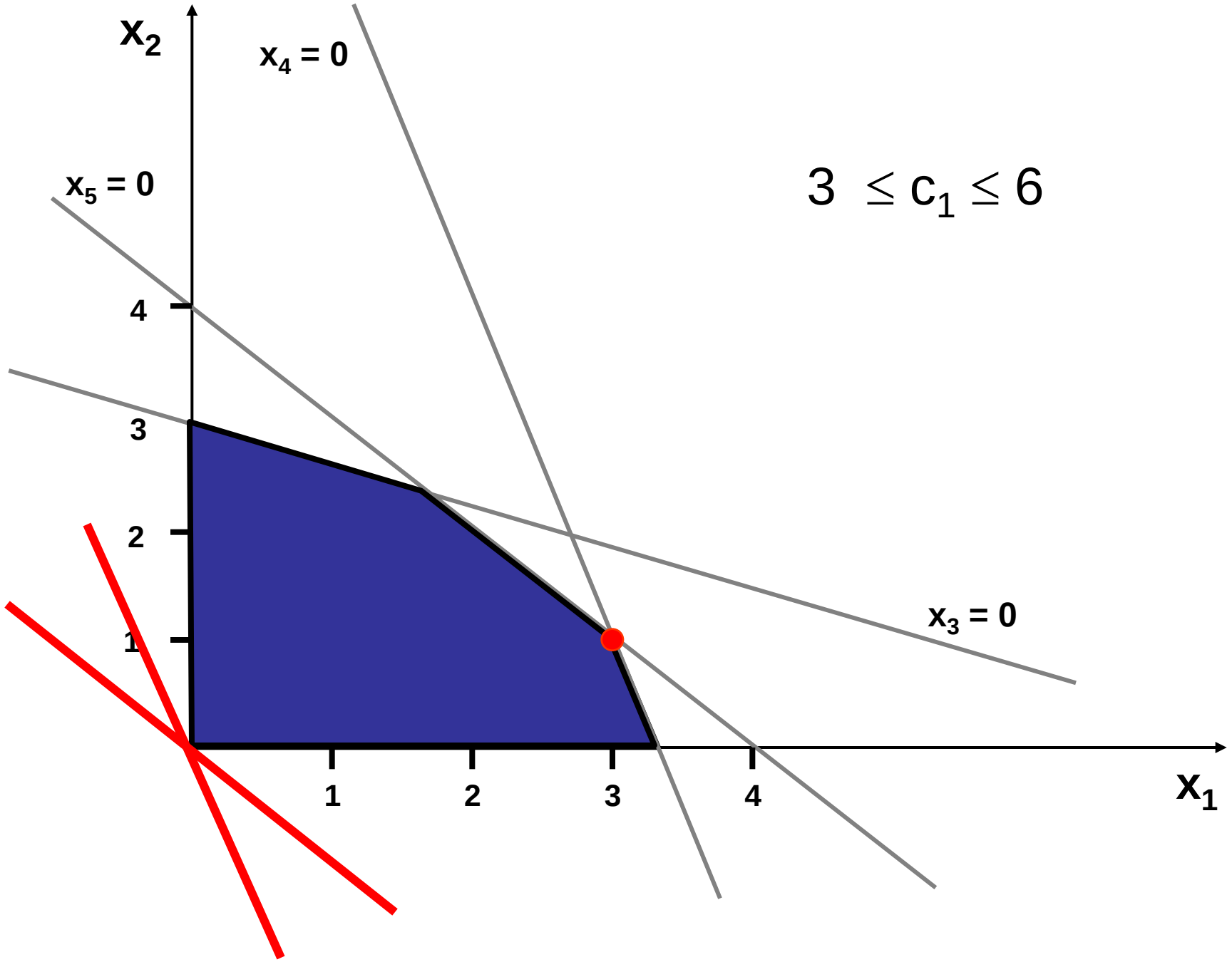
RANGO DE c_1

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
~~4~~ 3	x_1	3.000	1			1/6	-1/9
3	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9
Z = 12.000 (Máx)			0	0	0	0*	1/3

$$c_{1\,\mathrm{INF}} = 4 - \frac{1/6}{1/6} = 3$$


x2
x4 = 0
x5 = 0
c1 = 6
4
3
2
1
x3 = 0
1
2
3
4
x1


x2
x4 = 0
x5 = 0
c1 = 3
4
3
2
1
x3 = 0
1
2
3
4
x1



$3 \leq c_1 \leq 6$

RANGO DE c_2

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
4	x_1	3.000	1			1/6	-1/9
3	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9

$$c_{2\,SUP} = c_j + \frac{z_j - c_j}{a_{ij}}$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

RANGO DE c_2

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
4	x_1	3.000	1			1/6	-1/9
~~3~~ 4	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9
Z = 16.000 (Máx)			0	0	0	0*	4/9

$$c_{2\,SUP} = 3 + \frac{1/6}{1/6} = 4$$

RANGO DE c_2

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
4	x_1	3.000	1			1/6	-1/9
3	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9

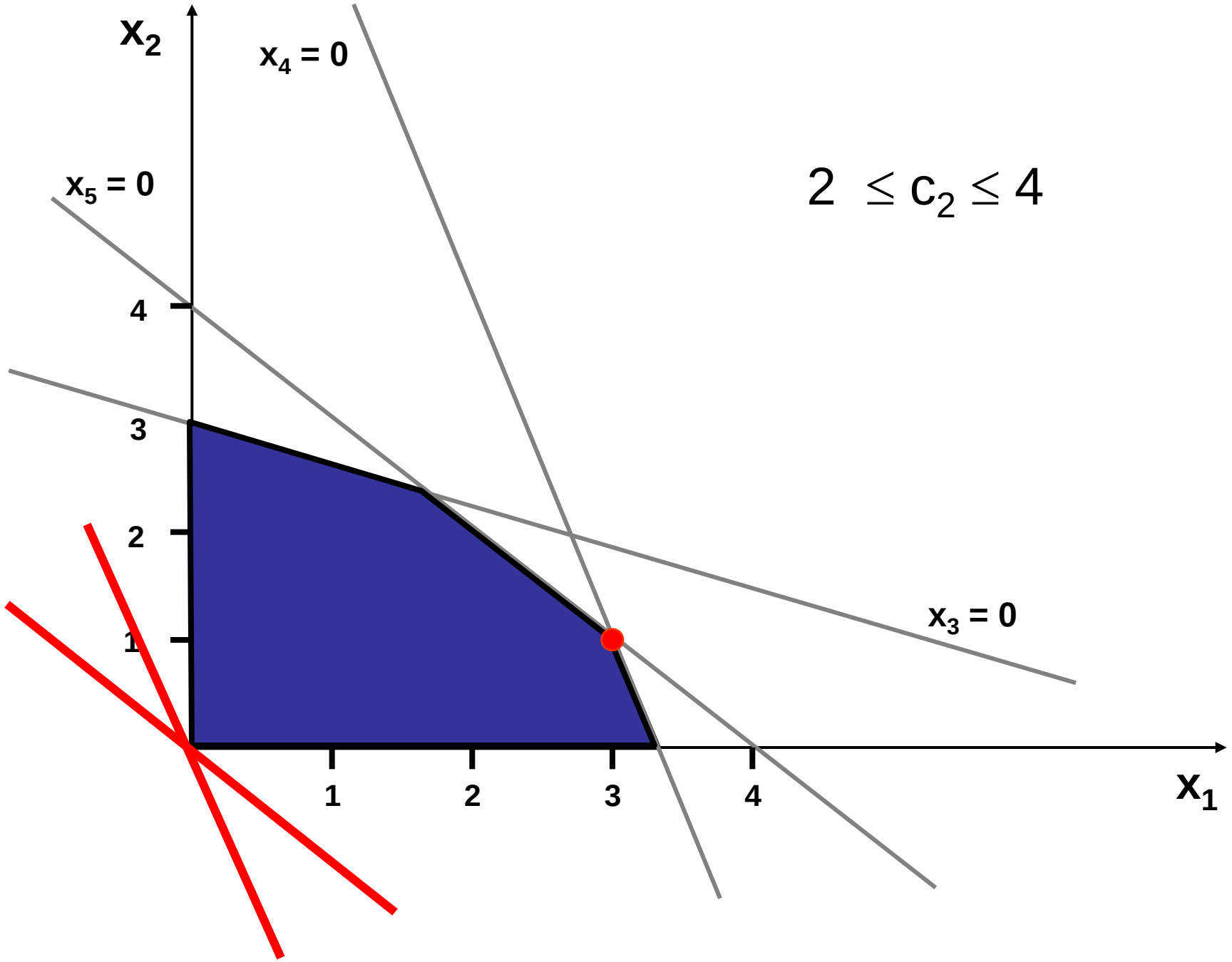
$$c_{2\,INF} = c_j - \frac{z_j - c_j}{a_{ij}}$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

RANGO DE c_2

c_k	x_k	B	A_1	A_2	A_3	A_4	A_5
0	x_3	14.000			1	5/3	-26/9
4	x_1	3.000	1			1/6	-1/9
~~2~~ 3	x_2	1.000		1		-1/6	2/9
Z = 15.000 (Máx)			0	0	0	1/6	2/9
Z = 14.000 (Máx)			0	0	0	1/3	0*

$$c_{2\,\mathrm{INF}} = 3 - \frac{2/9}{2/9} = 2$$



$$2 \leq c_2 \leq 4$$

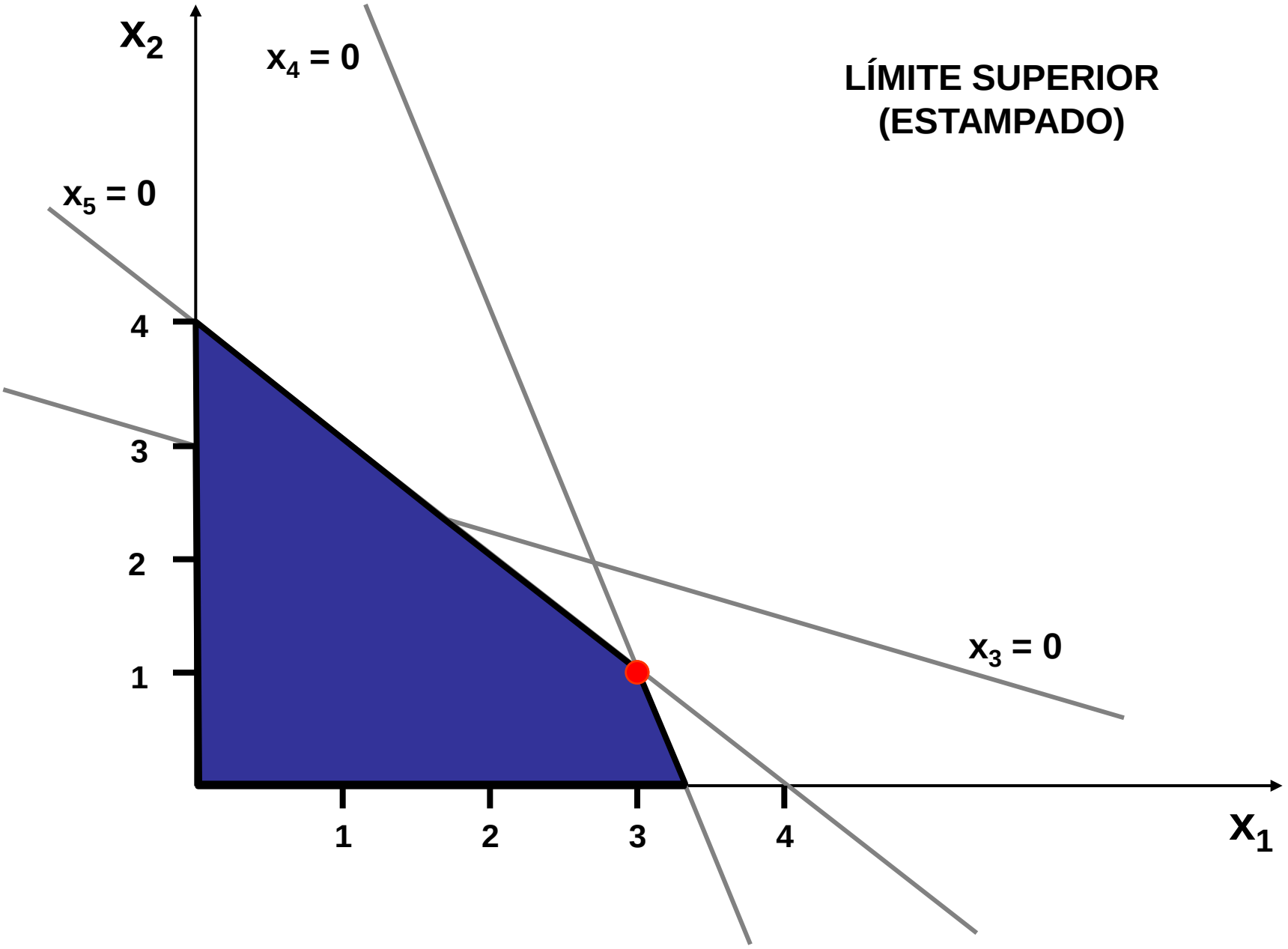
RANGO DE b_1 (EST)

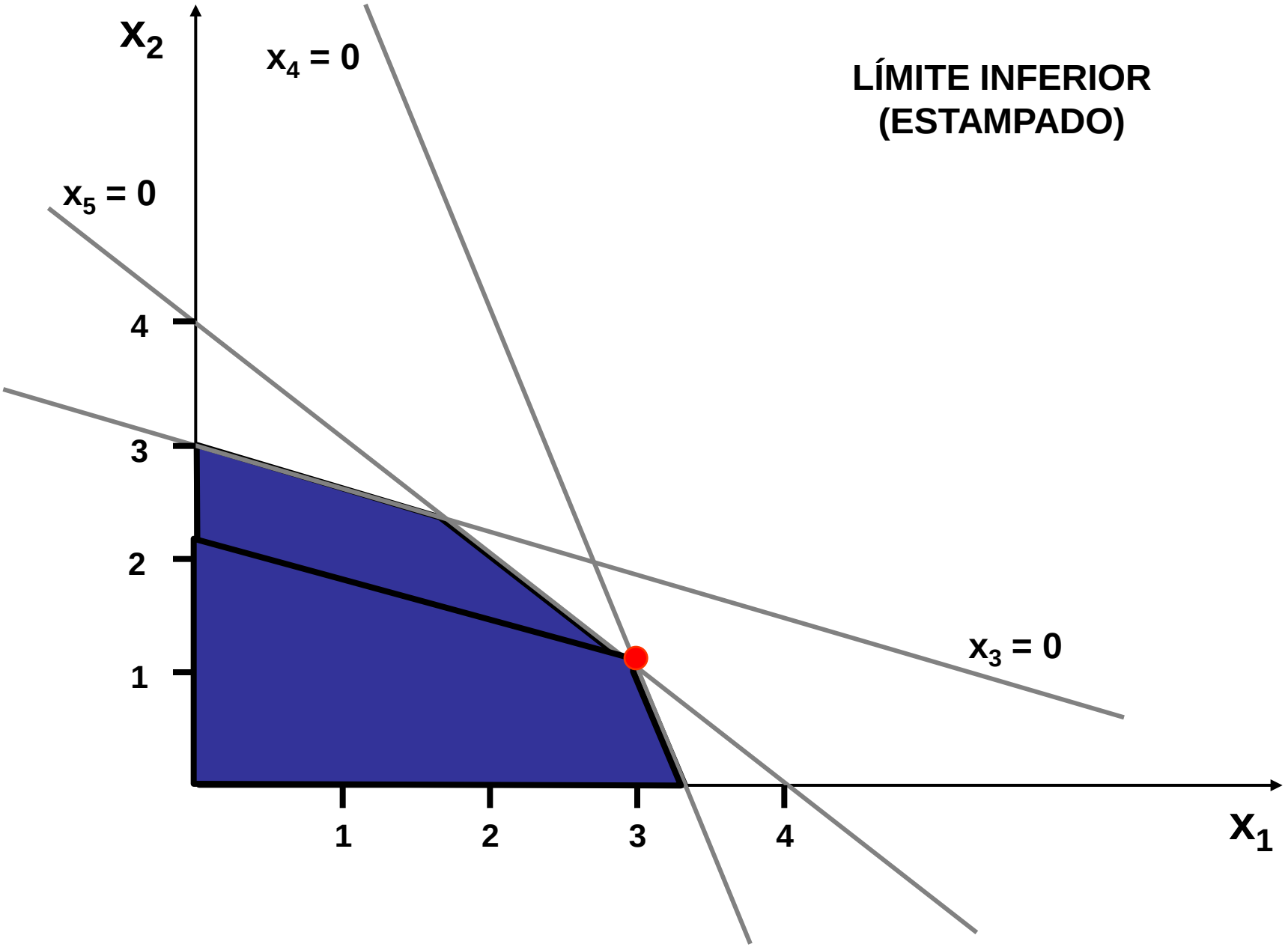
		c_j	34.000 ~~48.000~~	42.000	36.000	0	0
c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5
42.000	y_2	1/6	-5/3	1		-1/6	1/6
36.000	y_3	2/9	26/9		1	1/9	-2/9
Z = 15.000 (Mín)			-14.000	0	0	-3.000	-1.000
Z = 15.000 (Mín)			0*	0	0	-3.000	-1.000

$b_{1\ SUP} = \infty$

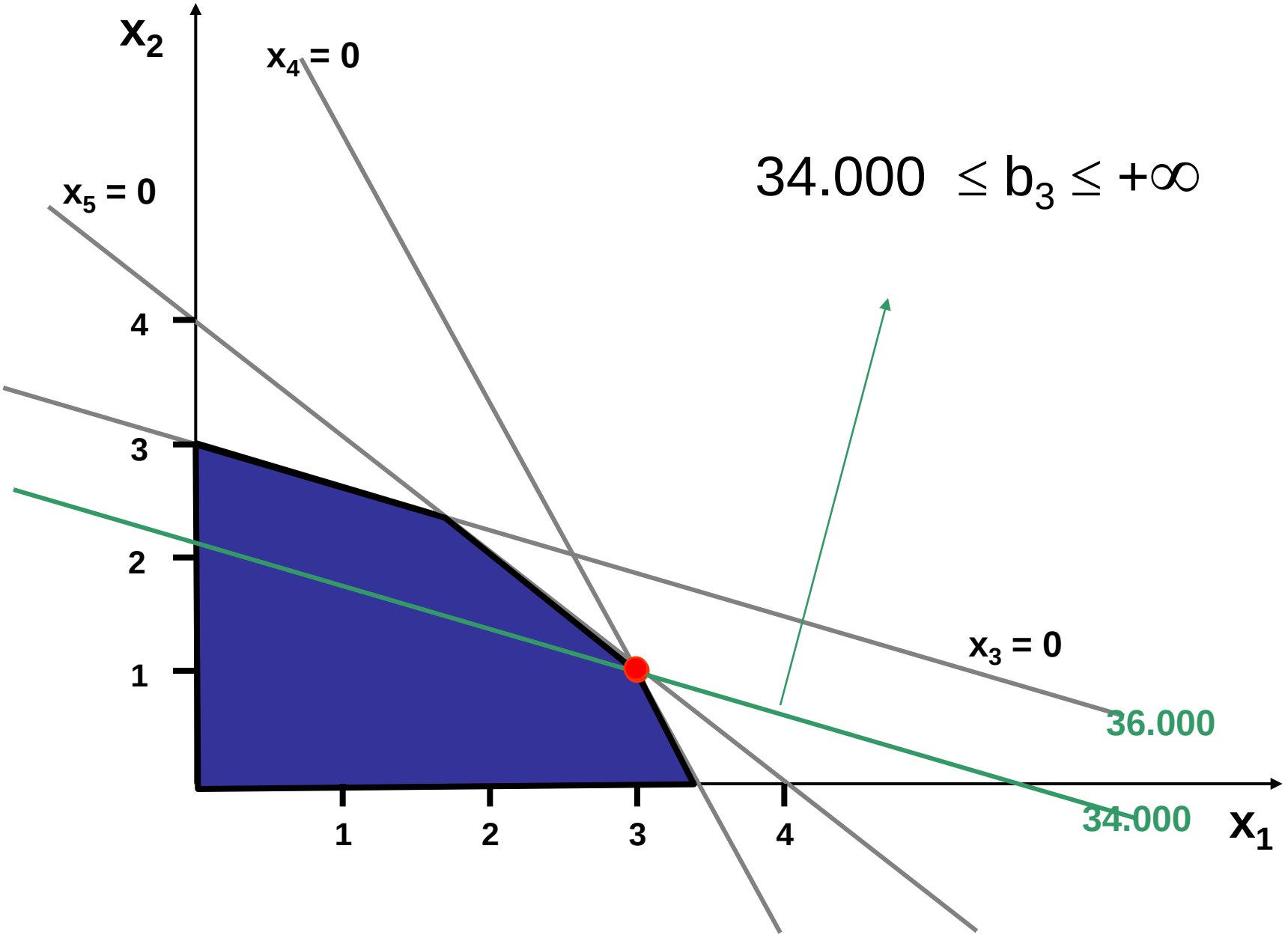
$b_{1\ INF} = 48.000 - 14.000 = 34.000$

$c_{j\ LIM}$	MAX	MIN
SUP.	$c_j + (z_j - c_j)$	∞
INF.	$-\infty$	$c_j - (z_j - c_j)$


x2
x4 = 0
x5 = 0
LÍMITE SUPERIOR
(ESTAMPADO)
4
3
2
1
x3 = 0
1
2
3
4
x1



LÍMITE INFERIOR (ESTAMPADO)


x_2
$x_4 = 0$
$x_5 = 0$
$34.000 \leq b_3 \leq +\infty$
4
3
2
1
$x_3 = 0$
36.000
34.000
1
2
3
4
x_1

RANGO DE b_2 (SOL)

			c_j	48.000	48.000 ~~42.000~~	36.000	0	0
	c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5
48.000	~~42.000~~	y_2	1/6	-5/3	1		-1/6	1/6
	36.000	y_3	2/9	26/9		1	1/9	-2/9
Z = 15.000 (Mín)				-14.000	0	0	-3.000	-1.000
Z = 16.000 (Mín)				-24.000	0	0	-4.000	0*

$$b_{2\,SUP} = c_j + \frac{z_j - c_j}{a_{ij}}$$

$$b_{2\,SUP} = 42.000 + \frac{1.000}{1/6} = 48.000$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

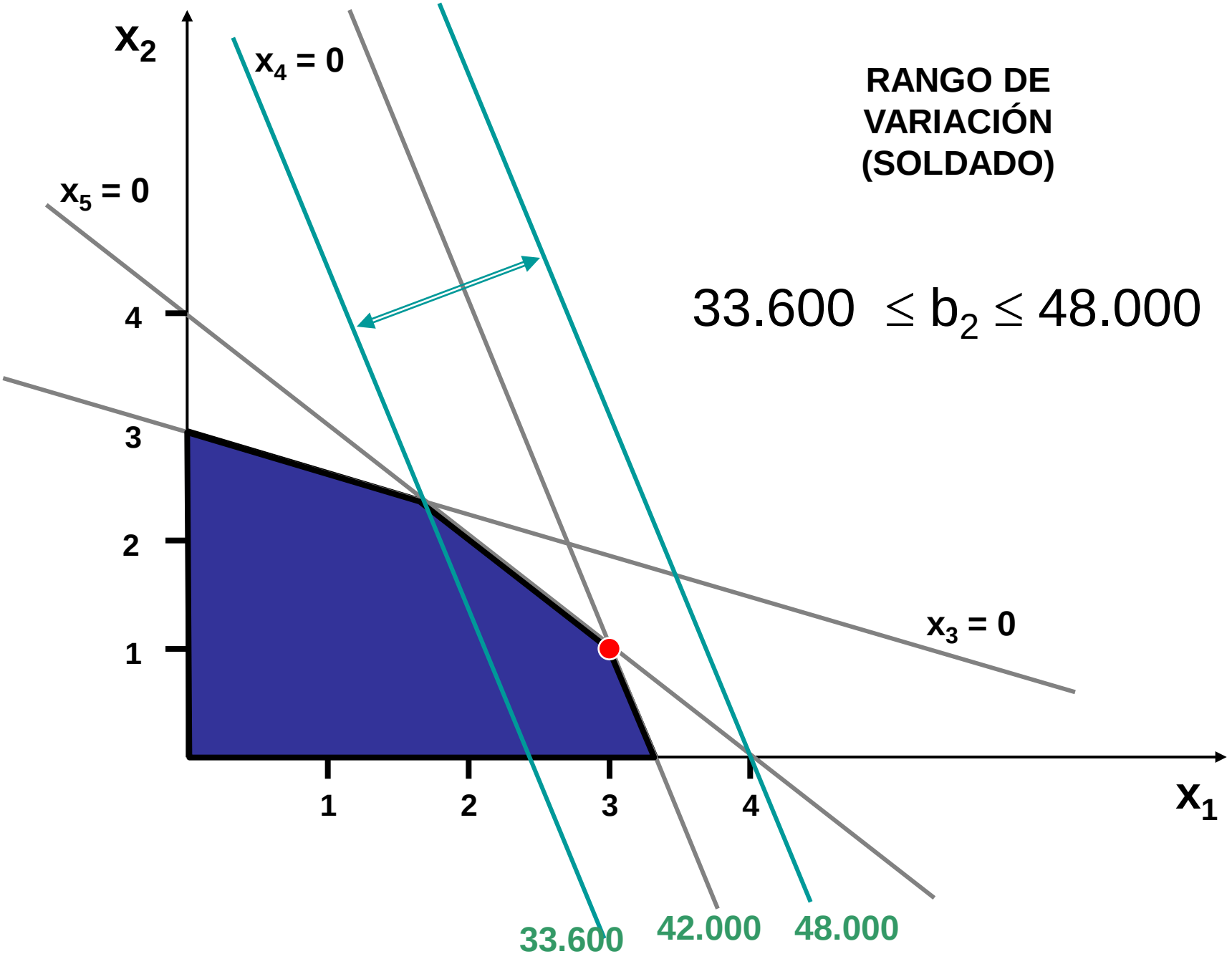
RANGO DE b_2 (SOL)

				33.600			
		c_j	48.000	~~42.000~~	36.000	0	0
c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5
33.600 ~~42.000~~	y_2	1/6	-5/3	1		-1/6	1/6
36.000	y_3	2/9	26/9		1	1/9	-2/9
Z = 15.000 (Mín)			-14.000	0	0	-3.000	-1.000
Z = 13.600 (Mín)			0*	0	0	-1.600	-2.400

$$b_{2\,INF} = c_j - \left| \frac{z_j - c_j}{a_{ij}} \right|_{MIN}$$

$$b_{2\,INF} = 42.000 - \frac{14.000}{5/3} = 33.600$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-


RANGO DE VARIACIÓN (SOLDADO)
33.600 ≤ b2 ≤ 48.000
x2
x1
x4 = 0
x5 = 0
x3 = 0
1
2
3
4
33.600
42.000
48.000

RANGO DE b_3 (PIN)

					40.846,15		
		c_j	48.000	42.000	~~36.000~~	0	0
c'_k	y_k	B'	A'_1	A'_2	A'_3	A'_4	A'_5
42.000	y_2	1/6	-5/3	1		-1/6	1/6
40.846,15 ~~36.000~~	y_3	2/9	26/9		1	1/9	-2/9
Z = 15.000 (Mín)			-14.000	0	0	-3.000	-1.000
Z = 16.077 (Mín)			0*	0	0	-2.461,5	-2.076,9

$$b_{3\,SUP} = c_j + \frac{z_j - c_j}{a_{ij}}$$

$$b_{3\,SUP} = 36.000 + \frac{14.000}{26/9} = 40.846{,}15$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-

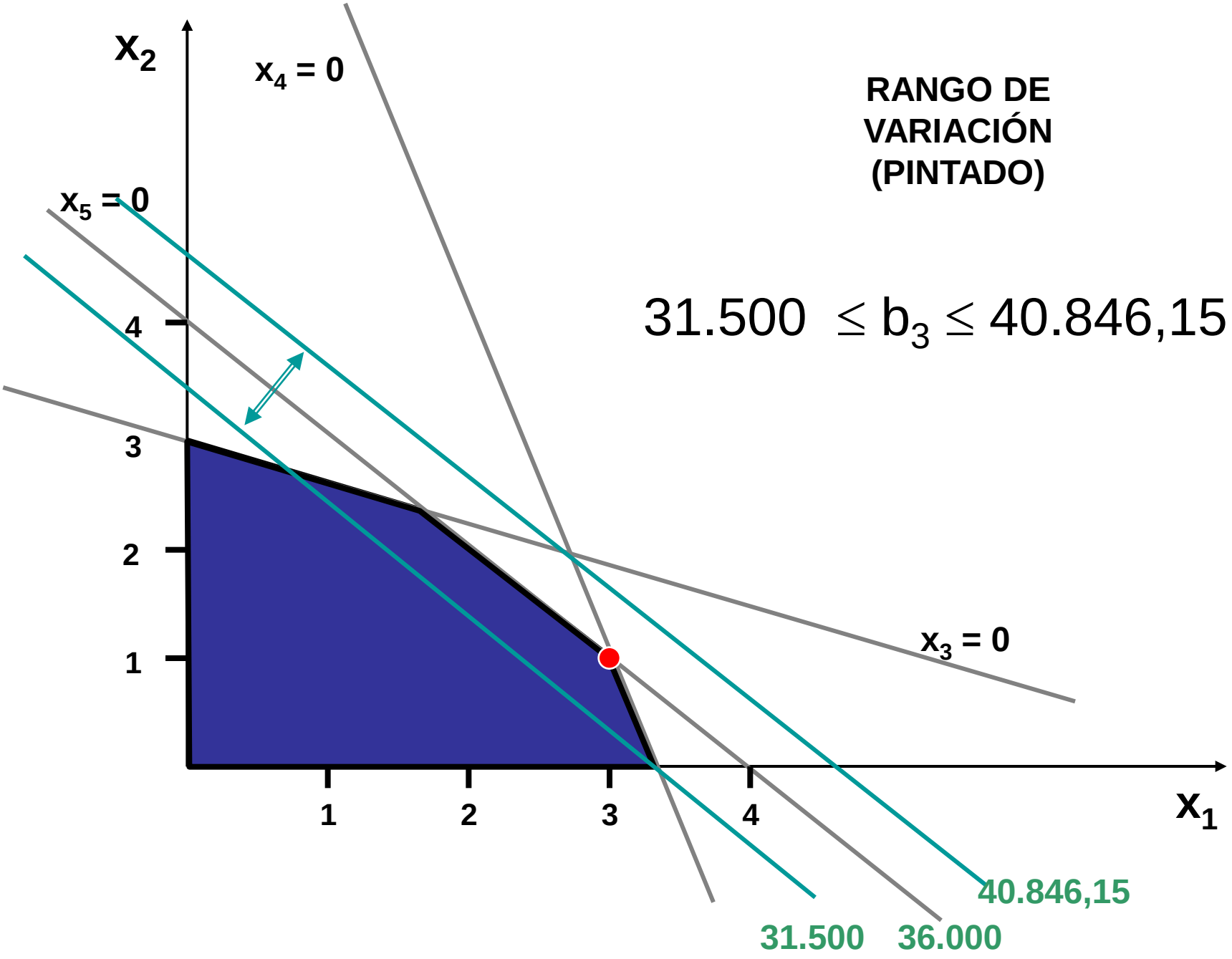
RANGO DE b_3 (PIN)

					31.500		
		c_j	**48.000**	**42.000**	**~~36.000~~**	**0**	**0**
c'_k	**y_k**	**B'**	**A'_1**	**A'_2**	**A'_3**	**A'_4**	**A'_5**
42.000	**y_2**	1/6	-5/3	1		-1/6	1/6
~~36.000~~ 31.500	**y_3**	2/9	26/9		1	1/9	-2/9
Z = 15.000 (Mín)			**-14.000**	**0**	**0**	**-3.000**	**-1.000**
Z = 14.000 (Mín)			**-27.000**	**0**	**0**	**-3.500**	**0***

$$b_{3\,INF} = c_j - \frac{z_j - c_j}{a_{ij}}$$

$$b_{3\,INF} = 36.000 - \frac{1.000}{2/9} = 31.500$$

a_{ij}	MAX	MIN
SUP.	-	+
INF.	+	-



RANGO DE VARIACIÓN (PINTADO)

$$31.500 \leq b_3 \leq 40.846{,}15$$