

MODELOS DE DECISIÓN

- NO RESTRINGIDOS

MAX: $f(X)$

o

MIN: $f(X)$

Ejemplo: MIN : $3 \cdot x_1 + \frac{2}{x_1} + \ln x_2 + 4 \cdot x_1 \cdot x_2$

- RESTRINGIDOS

(PROGRAMAS MATEMÁTICOS)

PROGRAMACIÓN MATEMÁTICA

MAX: $Z = f(x)$

Sujeto a:

$$\left\{ \begin{array}{l} g_1(x) \leq b_1 \\ g_2(x) \leq b_2 \\ \dots\dots\dots \\ g_m(x) \leq b_m \end{array} \right.$$

PROGRAMACIÓN LINEAL

Maximizar

$$\sum c_j \cdot x_j$$

sujeto a un conjunto
de restricciones

$$\sum a_{ij} \cdot x_j \leq b_i$$

siendo

$$x_j \geq 0$$

FUNCIÓN OBJETIVO

Maximizar

$$Z = \sum c_j \odot x_j$$



FUNCIONAL

Ejemplo: $Z = 6 \odot x_1 + 8 \odot x_2 + 3 \odot x_3$



Coeficientes del funcional

FUNCIÓN OBJETIVO

Maximizar

$$Z = \underbrace{\sum c_j \cdot x_j}_{\text{FUNCIONAL}}$$

FUNCIONAL

Ejemplo: $Z = 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$

↓ ↓ ↓
Variables de decisión

RESTRICCIONES

Conjunto de inecuaciones o ecuaciones

**CONDICIONES
DE VÍNCULO**

$$\left\{ \begin{array}{l} \sum a_{ij} \cdot x_j \leq b_i \\ \sum a_{ij} \cdot x_j \geq b_i \\ \sum a_{ij} \cdot x_j = b_i \end{array} \right.$$

CONDICIONES DE VÍNCULO

Ejemplo: $12 \odot x_1 + 9 \odot x_2 + 4 \odot x_3 \leq 500$

↓ ↓ ↓ ↓

COEFICIENTES TECNOLÓGICOS

↓

RHS

CONDICIONES DE LAS VARIABLES

X_j

- NO NEGATIVIDAD
- CONTINUIDAD

CONDICIONES DE LOS TÉRMINOS INDEPENDIENTES ("RHS")

b_i

- NO NEGATIVIDAD
(en su forma estándar)

$$6 \odot x_1 + 9 \odot x_2 \leq 4 \odot x_3$$

INCORRECTO

$$6 \odot x_1 + 9 \odot x_2 - 4 \odot x_3 \leq 0$$

$$\frac{x_2}{4} + \frac{x_5}{6} \geq 10$$


INCORRECTO

$$0,25 \cdot x_2 + 0,1667 \cdot x_5 \geq 10$$

$$\sum \mathbf{a}_{ij} \cdot \mathbf{x}_j \leq \mathbf{b}_i$$

$$\sum \mathbf{a}_{ij} \cdot \mathbf{x}_j \geq \mathbf{b}_i$$

$$\sum \mathbf{a}_{ij} \cdot \mathbf{x}_j = \mathbf{b}_i$$

INECUACIÓN  ECUACIÓN

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

VARIABLES “SLACKS”

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 + x_4 = 500$$



DE HOLGURA


INECUACIÓN  ECUACIÓN

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100$$

VARIABLES “SLACKS”

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 - x_4 = 100$$



SUPERFLUA

FORMA NATURAL

MAX: $6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$

Sujeto a:

$$\left\{ \begin{array}{l} 12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500 \\ 3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100 \\ 7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200 \end{array} \right.$$

siendo: $x_j \geq 0$

FORMAS DE FORMULACIÓN DE UN MODELO DE PL

- **FORMA NATURAL:** Restricciones de “ \leq ”, “ \geq ” y “ $=$ ”
- **FORMA CANÓNICA**
 - De MAX: Todas las restricciones de “ \leq ”
 - De MIN: Todas las restricciones de “ \geq ”
- **FORMA ESTÁNDAR**
 - Todas las restricciones de “ $=$ ”

FORMA ESTÁNDAR

MAX: $6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$

Sujeto a:

$$\left\{ \begin{array}{l} 12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 + x_4 = 500 \\ 3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 + x_5 = 700 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 - x_6 = 100 \\ 7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200 \end{array} \right.$$

siendo: $x_j \geq 0$

FORMA CANÓNICA DE MAX

NATURAL

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200$$

$$x_j \geq 0$$

CANÓNICA

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700$$

$$-2 \cdot x_1 - 2 \cdot x_2 - 3 \cdot x_3 \leq -100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 \leq 200$$

$$-7 \cdot x_1 - 4 \cdot x_2 - 3 \cdot x_3 \leq -200$$

$$x_j \geq 0$$

FORMA CANÓNICA DE MIN

NATURAL

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200$$

$$x_j \geq 0$$

CANÓNICA

$$\text{MIN: } -6 \cdot x_1 - 8 \cdot x_2 - 3 \cdot x_3$$

$$-12 \cdot x_1 - 9 \cdot x_2 - 4 \cdot x_3 \geq -500$$

$$-3 \cdot x_1 - 15 \cdot x_2 - 6 \cdot x_3 \geq -700$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \geq 100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 \geq 200$$

$$-7 \cdot x_1 - 4 \cdot x_2 - 3 \cdot x_3 \geq -200$$

$$x_j \geq 0$$

FORMA CANÓNICA

MAX: $c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_k x_k$

Sujeto a:

$$\left\{ \begin{array}{l} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1k} x_k \leq b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \dots + a_{2k} x_k \leq b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + \dots + a_{3k} x_k \leq b_3 \\ \dots \\ a_{m1} x_1 + a_{m2} x_2 + a_{m3} x_3 + \dots + a_{mk} x_k \leq b_m \end{array} \right.$$

siendo: $x_j \geq 0$

FORMA ESTÁNDAR


MAX: $c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_k x_k$

Sujeto a

$$\left\{ \begin{array}{l} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1k} x_k + x_{k+1} = b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \dots + a_{2k} x_k + x_{k+2} = b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + \dots + a_{3k} x_k + x_{k+3} = b_3 \\ \dots\dots\dots \\ a_{m1} x_1 + a_{m2} x_2 + a_{m3} x_3 + \dots + a_{mk} x_k + x_n = b_m \end{array} \right.$$

siendo $x_j \geq 0$

FORMA MATRICIAL EXPANDIDA

X1	X2	X3		RHS
6	8	3		MAX
12	9	4	\leq	500
3	15	6	\leq	700
2	2	3	\geq	100
7	4	3	$=$	200

En un taller metalúrgico se fabrican dos tipos de piezas A y B, que deben seguir los siguientes procesos:

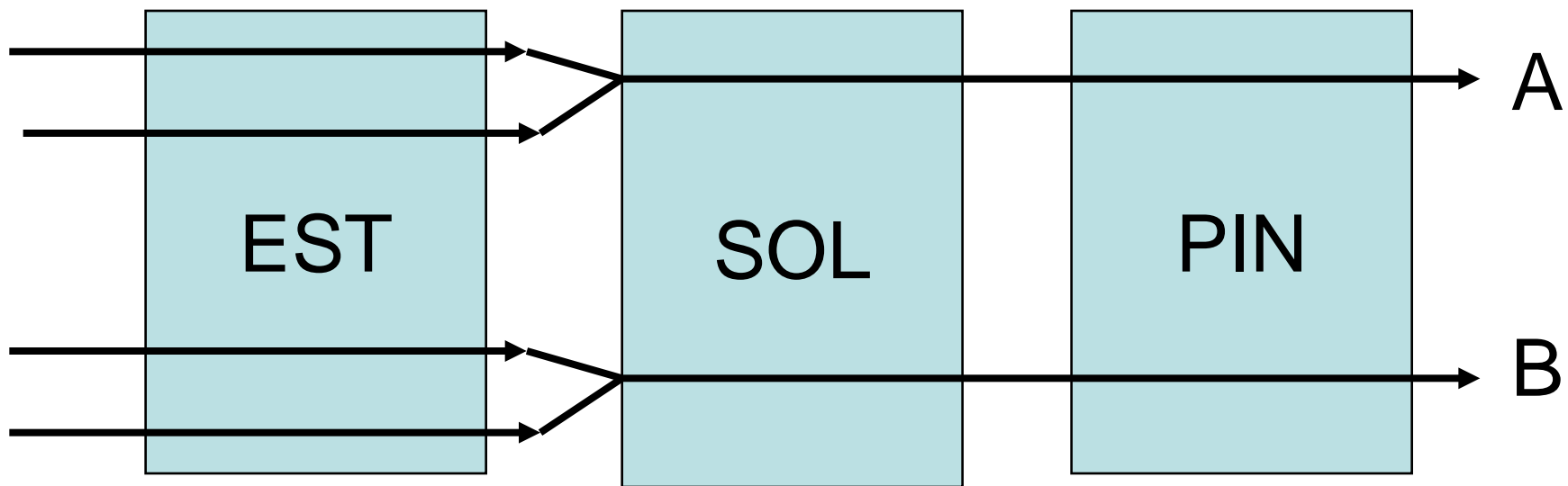
1. Estampado en hojas metálicas
2. Soldado
3. Pintado

La operación de estampado consiste en preparar partes idénticas que luego serán soldadas de a pares, formando la pieza A. El mismo proceso se realiza para la pieza B.

La utilidad unitaria es de \$ 4 para la pieza A y \$ 3 para la pieza B. Se desea establecer el programa semanal de producción que maximice la utilidad del taller con respecto a las piezas consideradas.

Los insumos de equipos son los siguientes, para la realización de cada una de las operaciones (expresados en segundos por pieza):

Operación	A	B	Tiempo disponible (seg./semana)
Estampado de c/parte	3	8	48.000
Soldado	12	6	42.000
Pintado	9	9	36.000



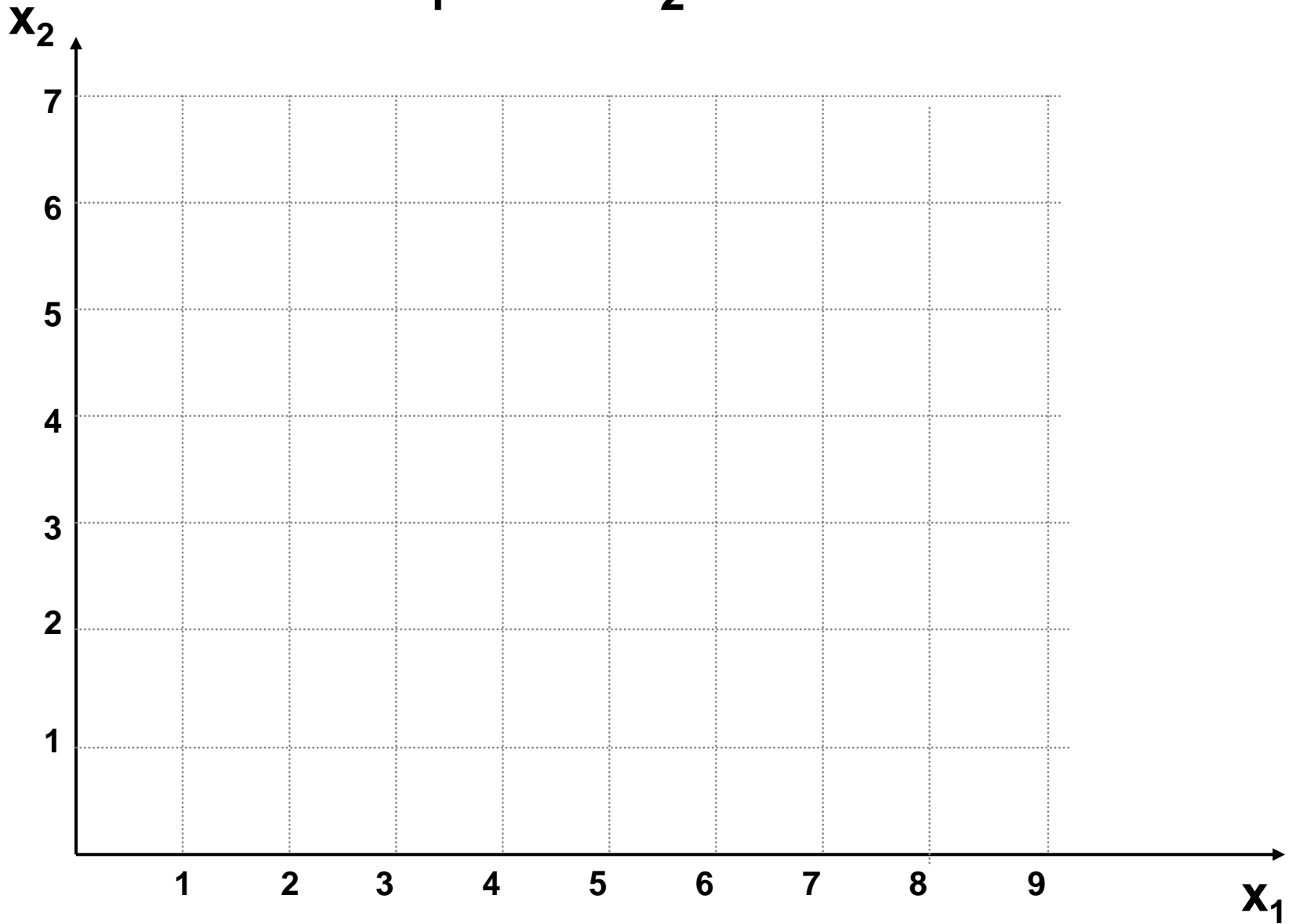
$$\text{MAX: } \mathbf{Z = 4 x_1 + 3 x_2}$$

Sujeto a:

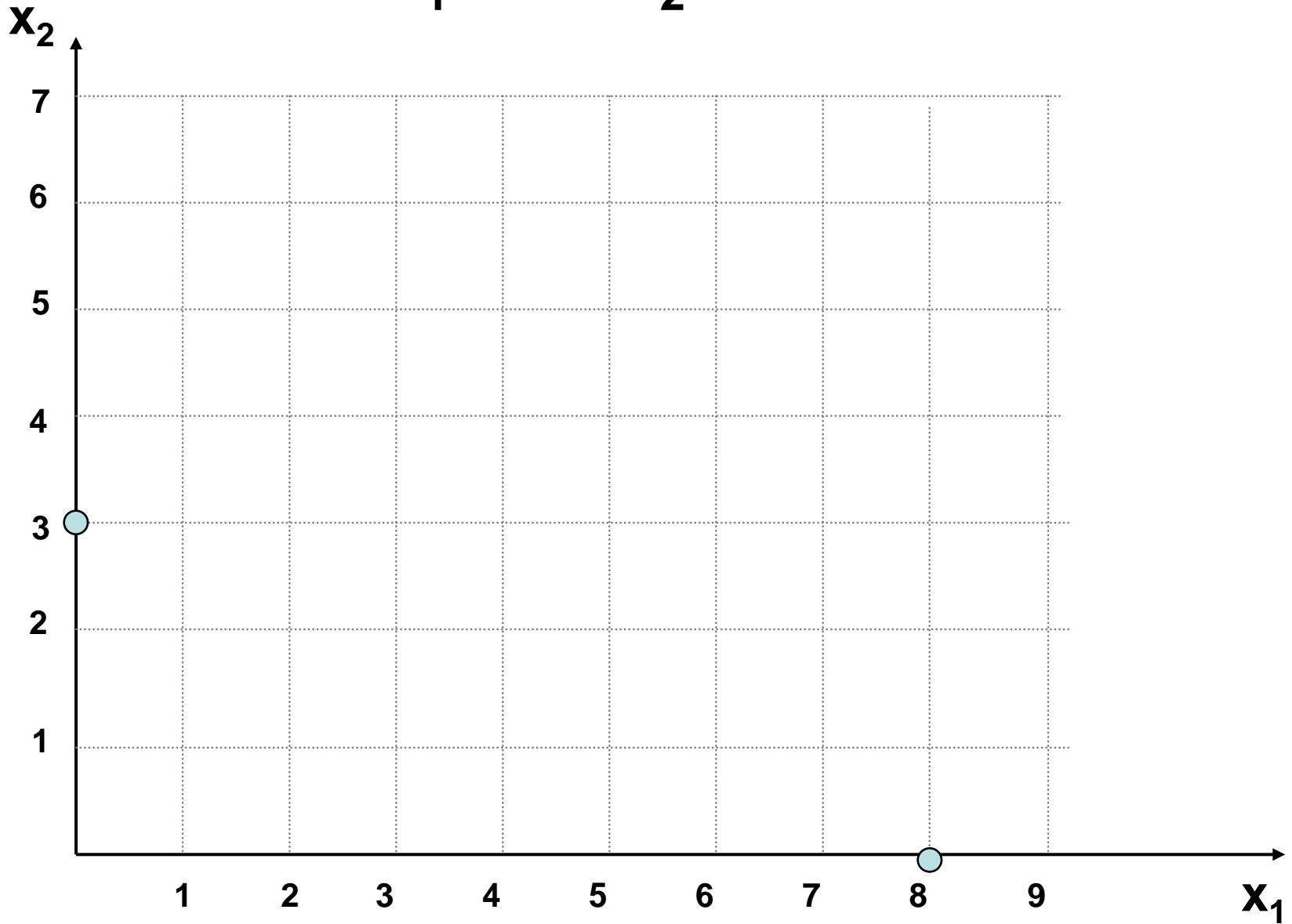
$$\left\{ \begin{array}{l} \mathbf{6 x_1 + 16 x_2 \leq 48000} \\ \mathbf{12 x_1 + 6 x_2 \leq 42000} \\ \mathbf{9 x_1 + 9 x_2 \leq 36000} \end{array} \right.$$

siendo: $x_1, x_2 \geq 0$ y continuas

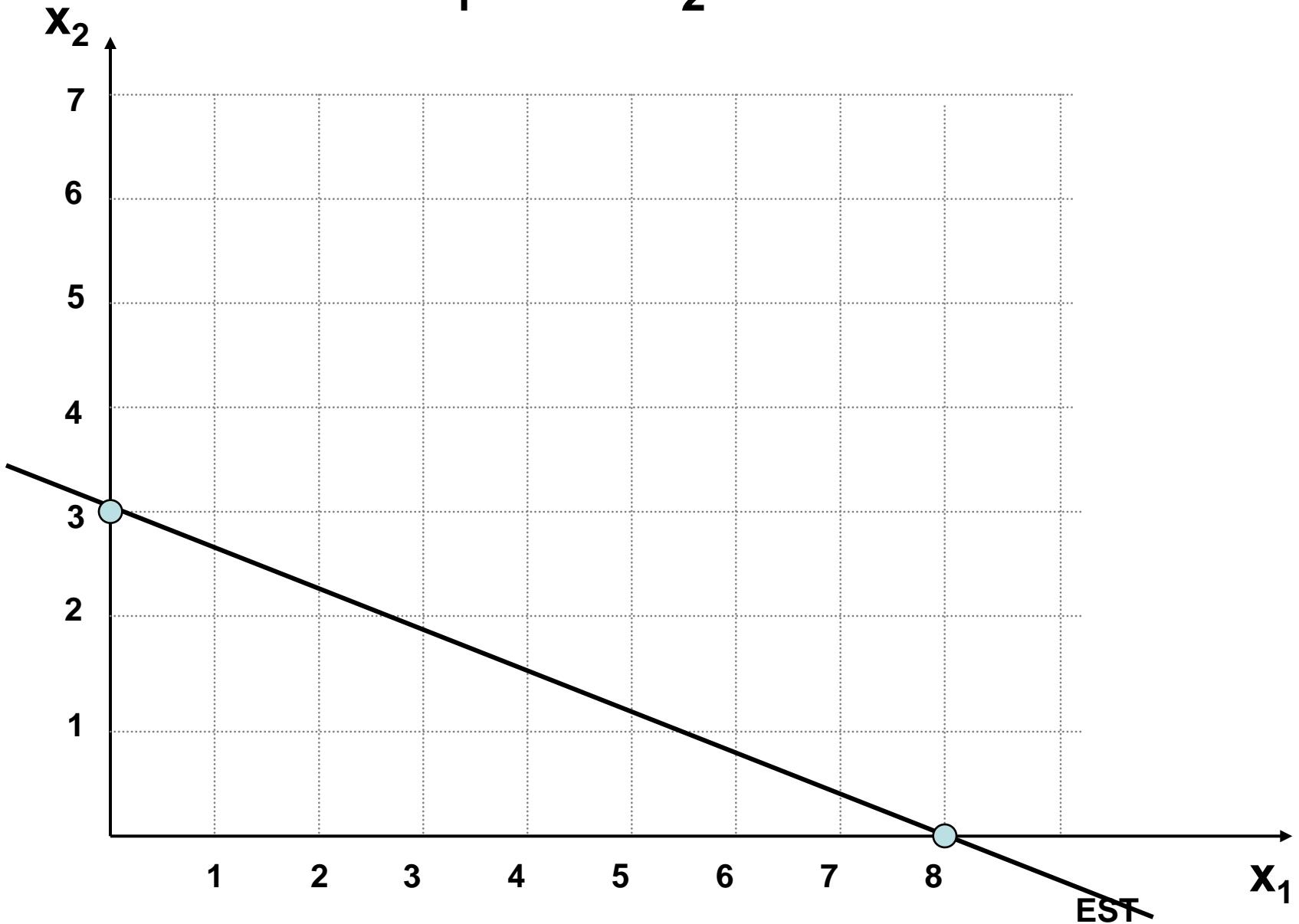
$$6x_1 + 16x_2 \leq 48000$$



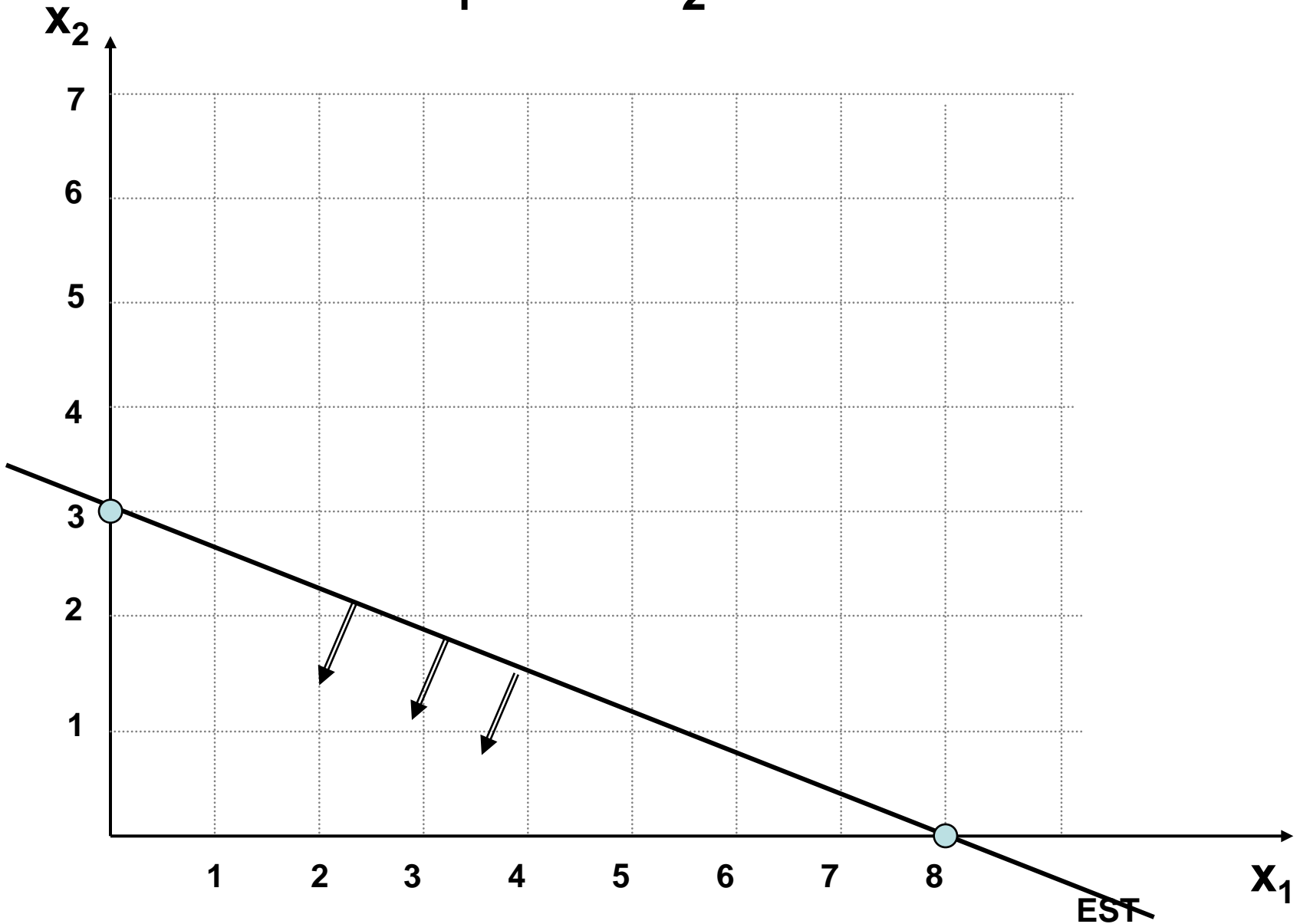
$$6x_1 + 16x_2 = 48000$$



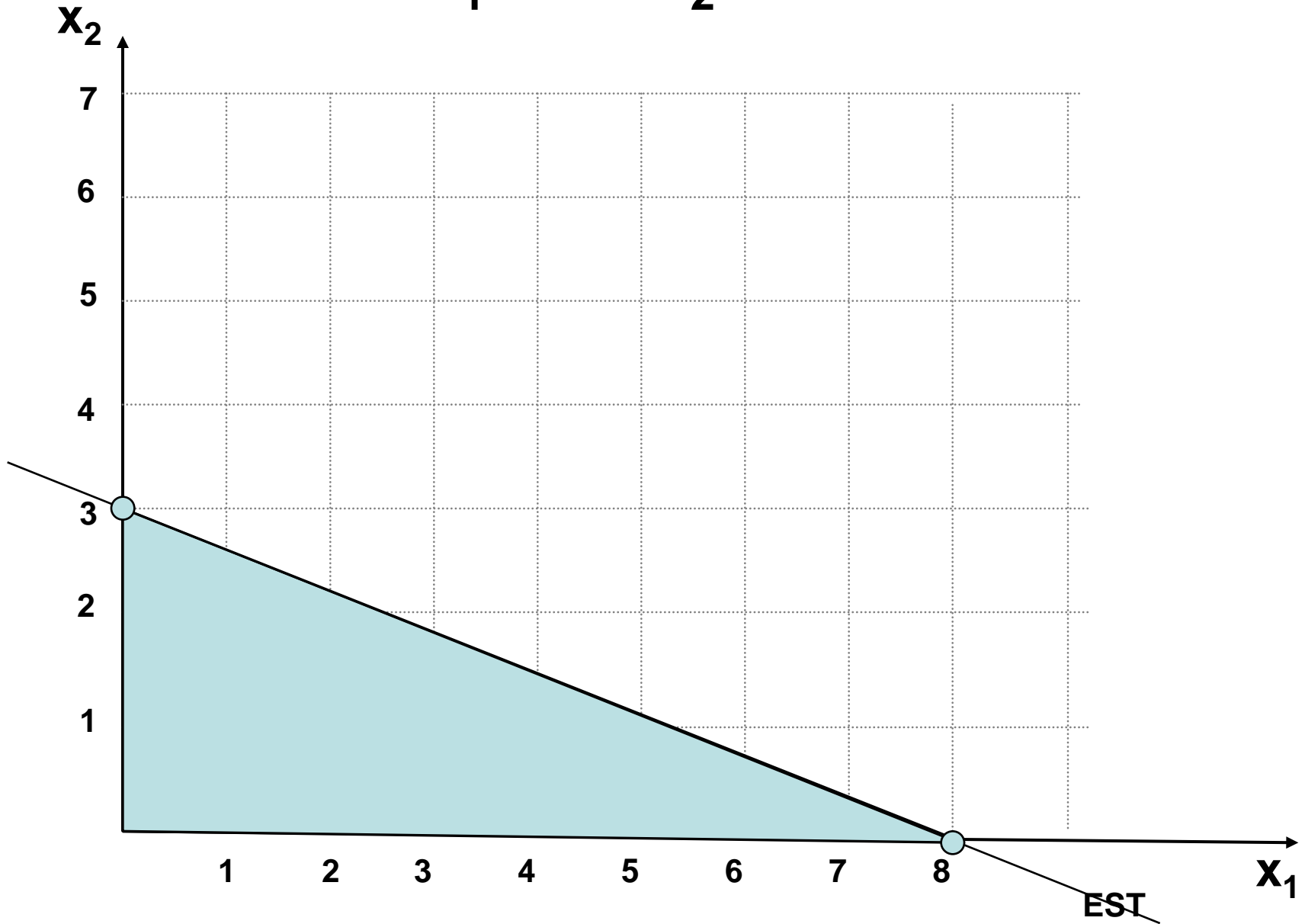
$$6x_1 + 16x_2 = 48000$$



$$6x_1 + 16x_2 \leq 48000$$

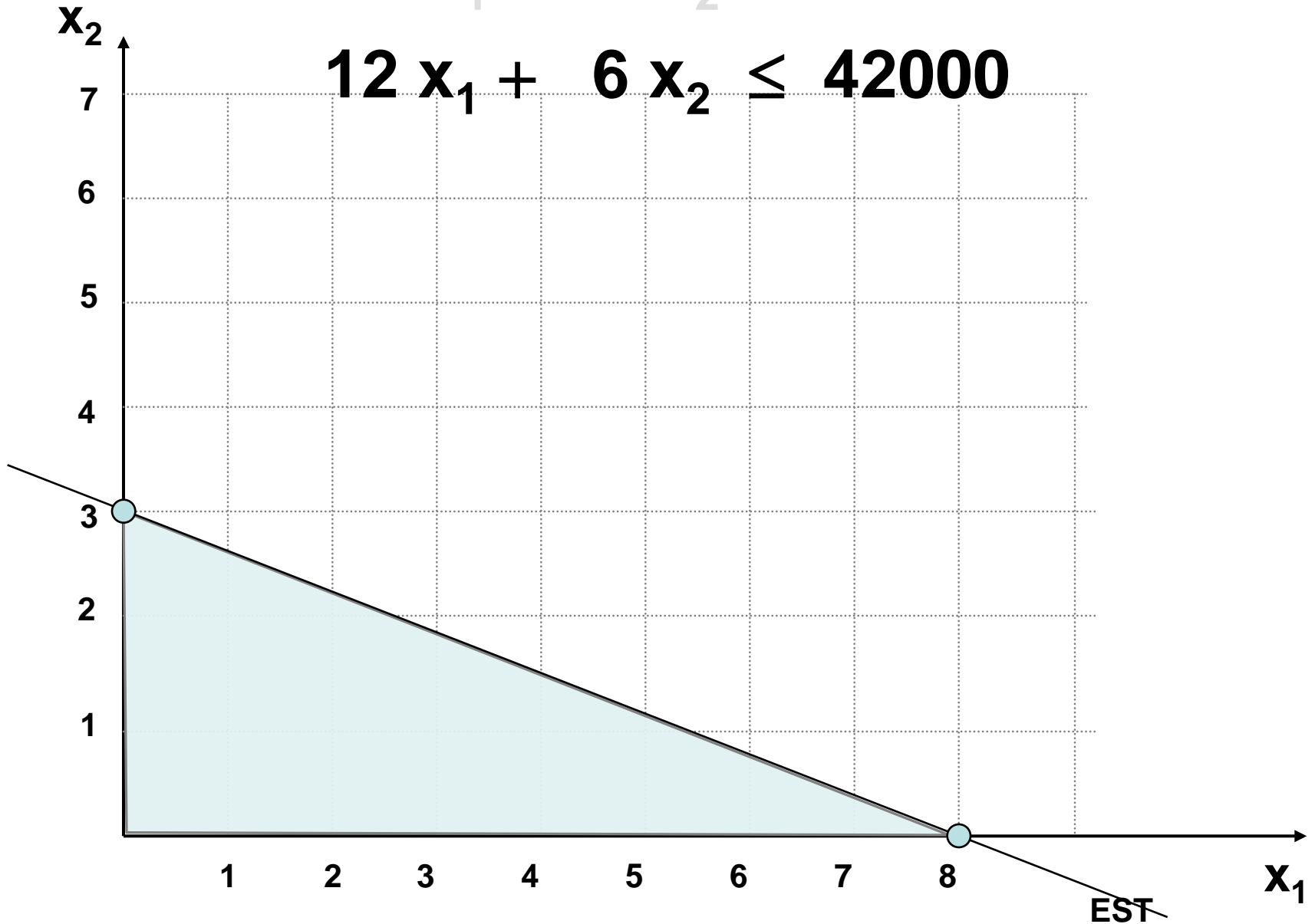


$$6x_1 + 16x_2 \leq 48000$$



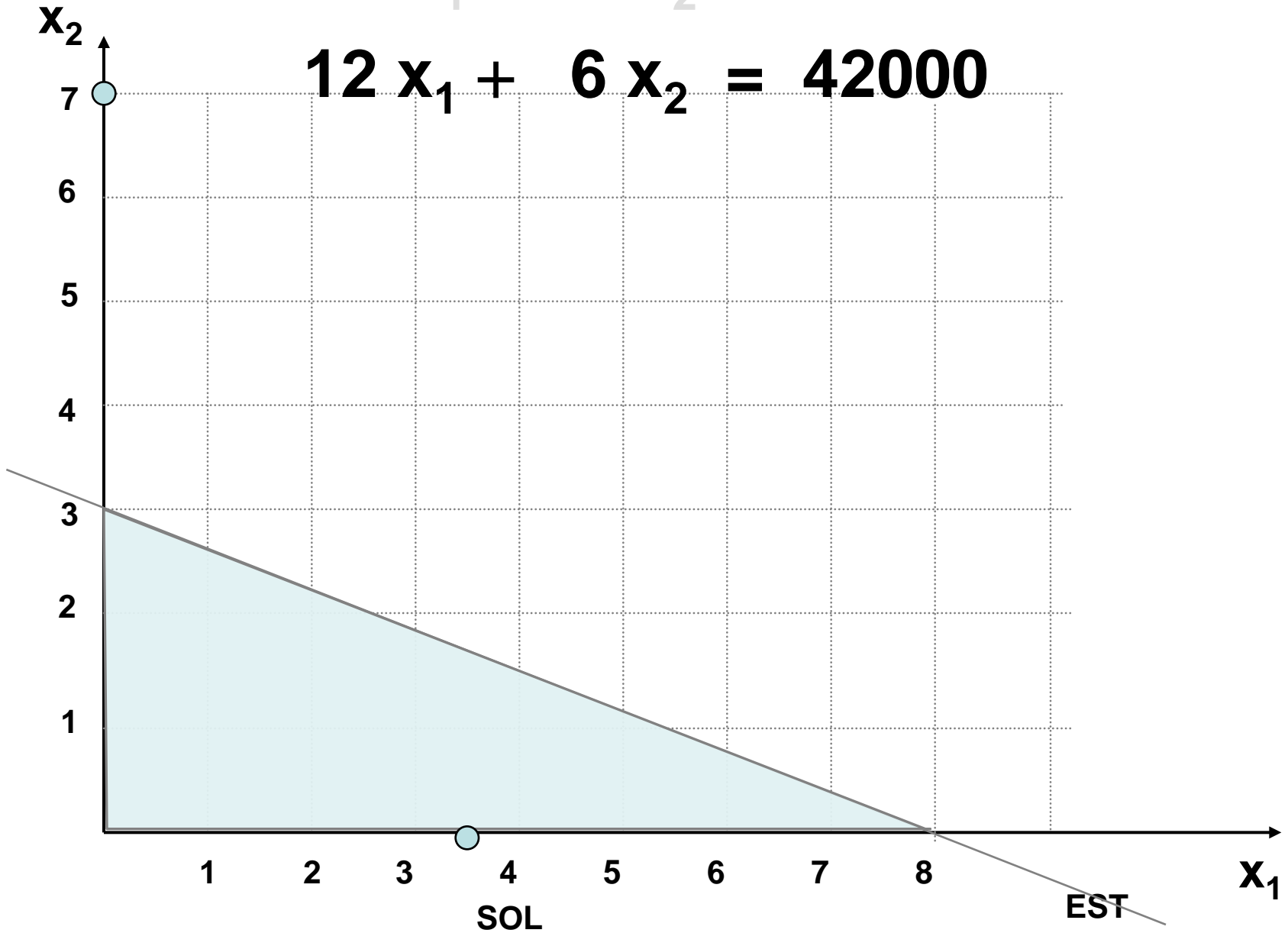
$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$



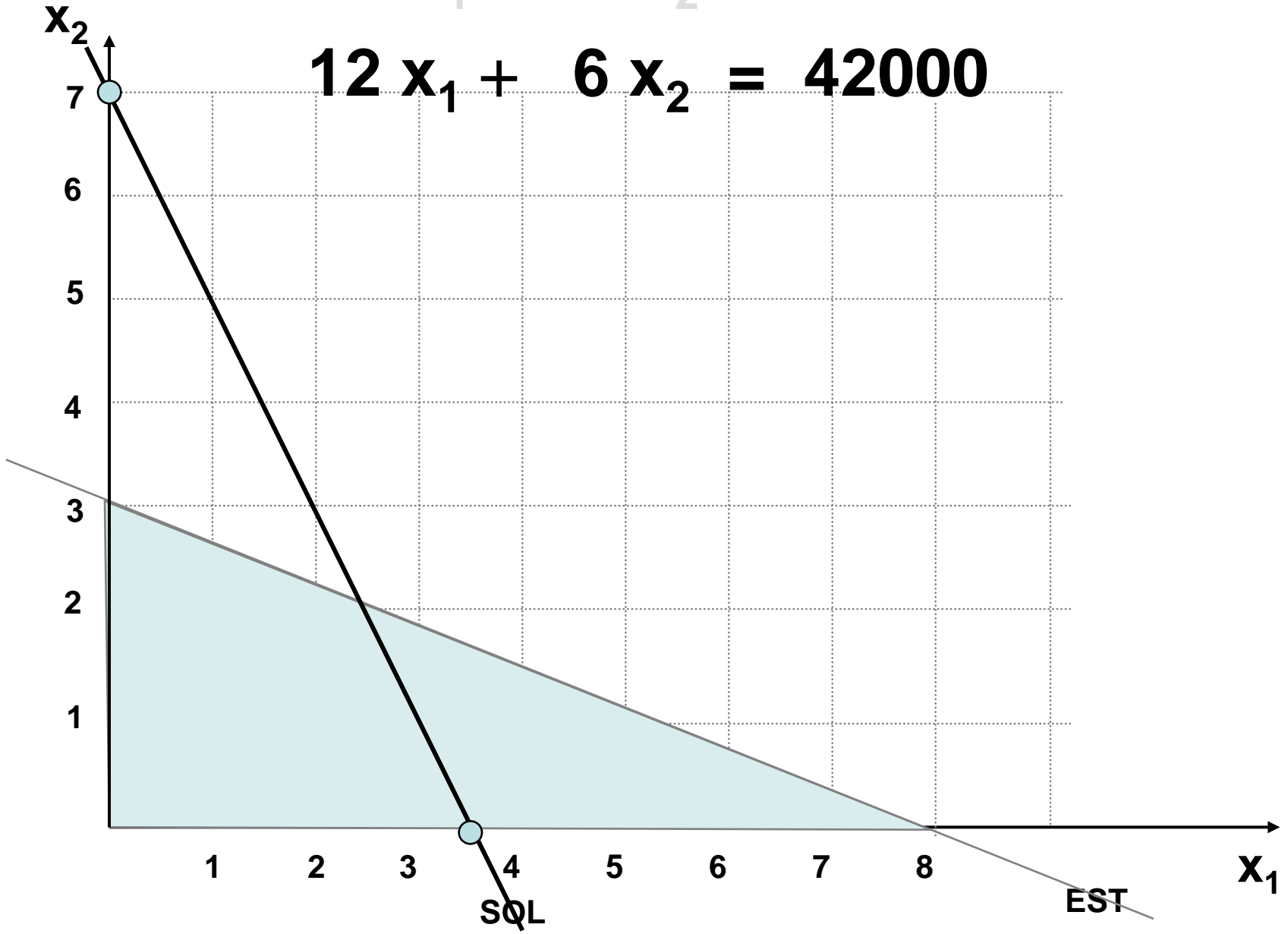
$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 = 42000$$



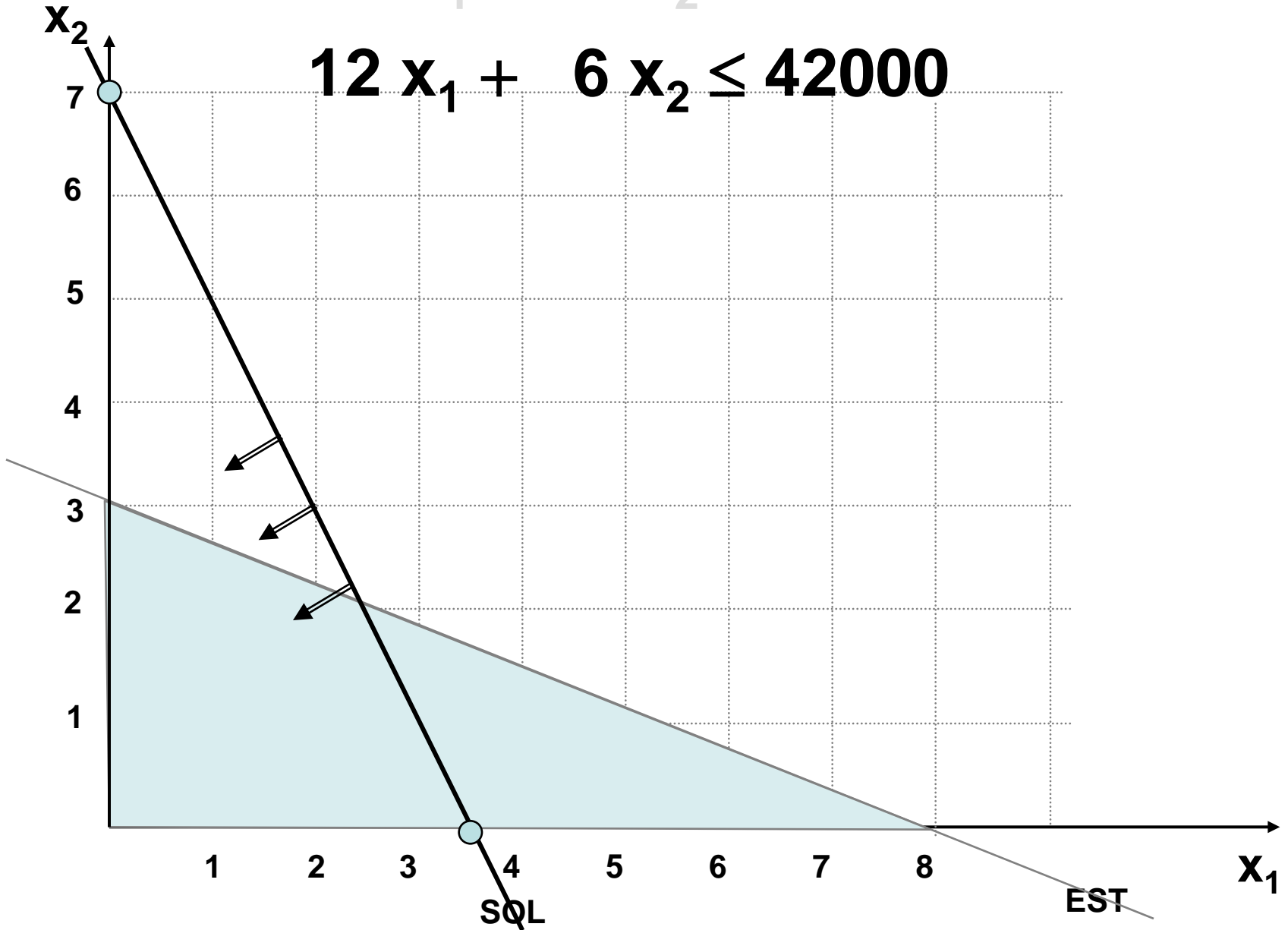
$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 = 42000$$



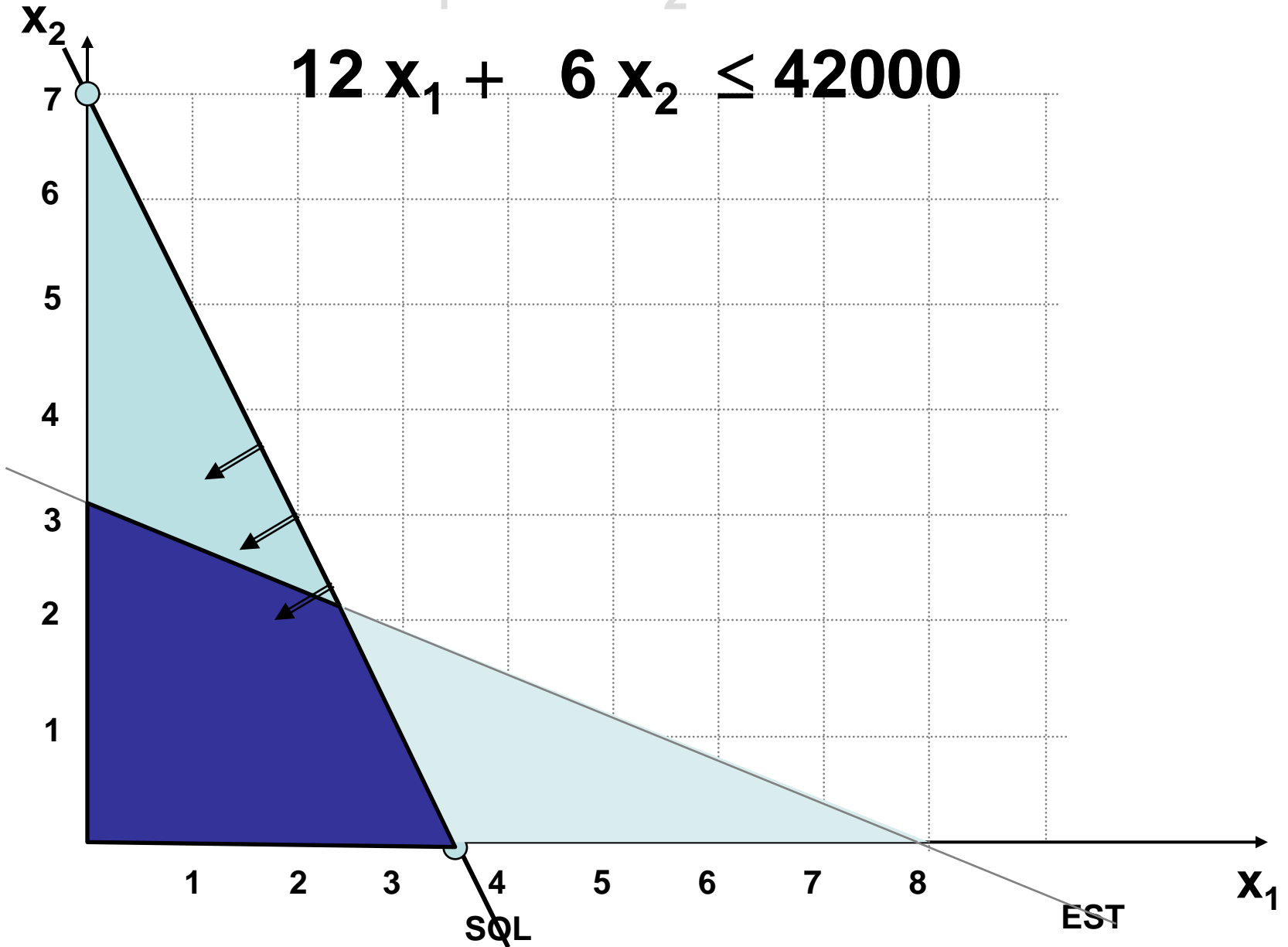
$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$



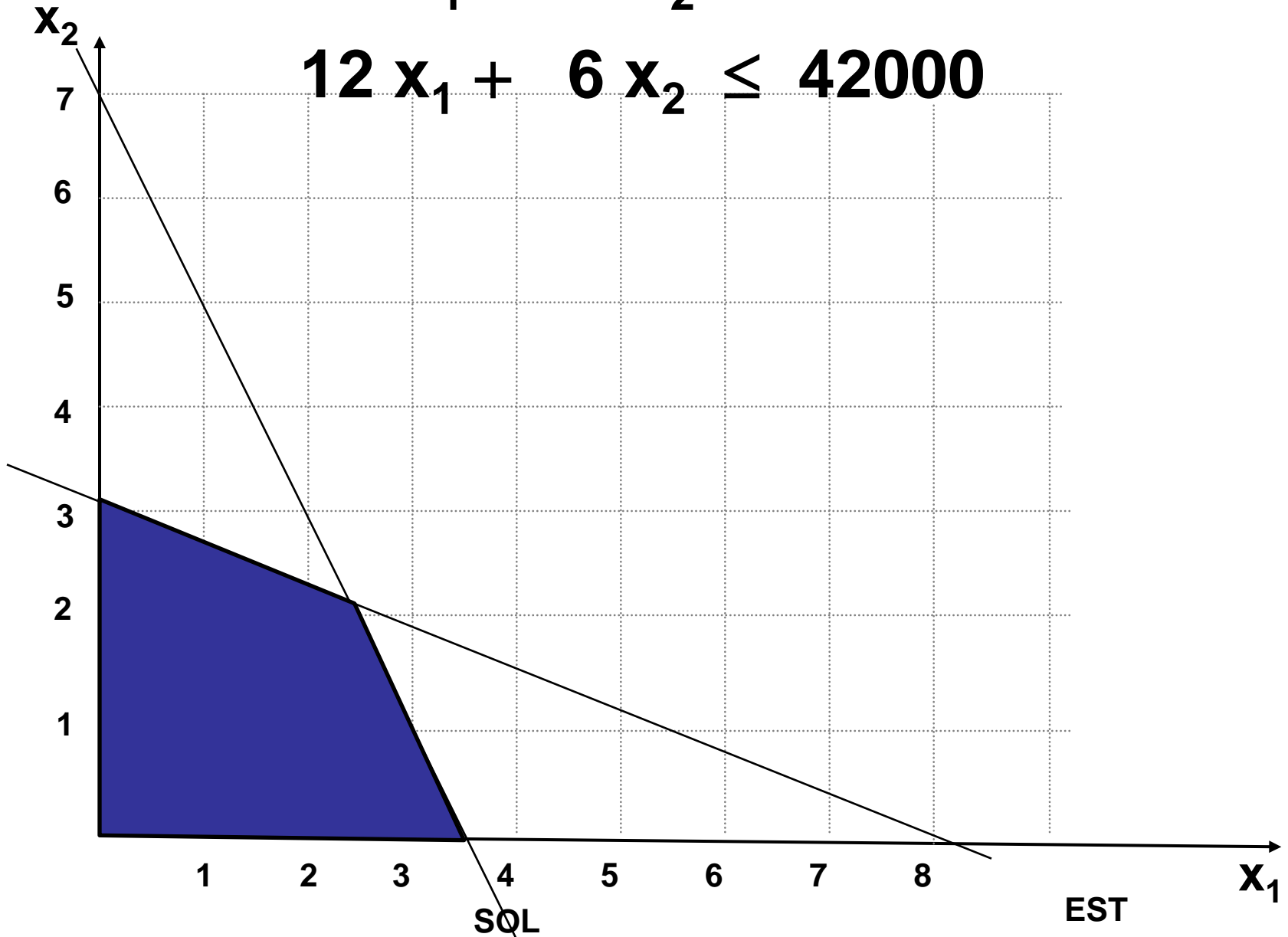
$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$



$$6x_1 + 16x_2 \leq 48000$$

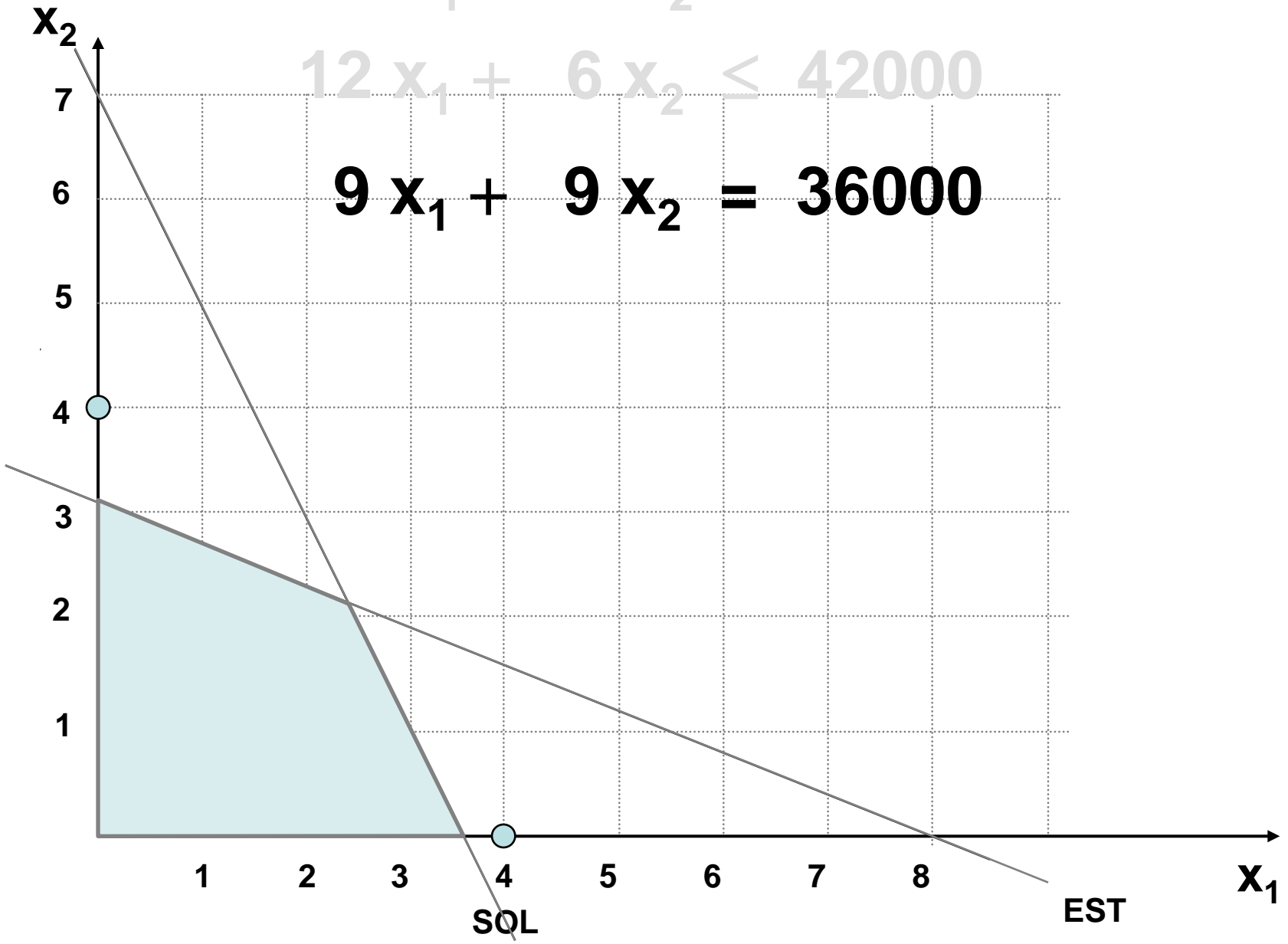
$$12x_1 + 6x_2 \leq 42000$$



$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$

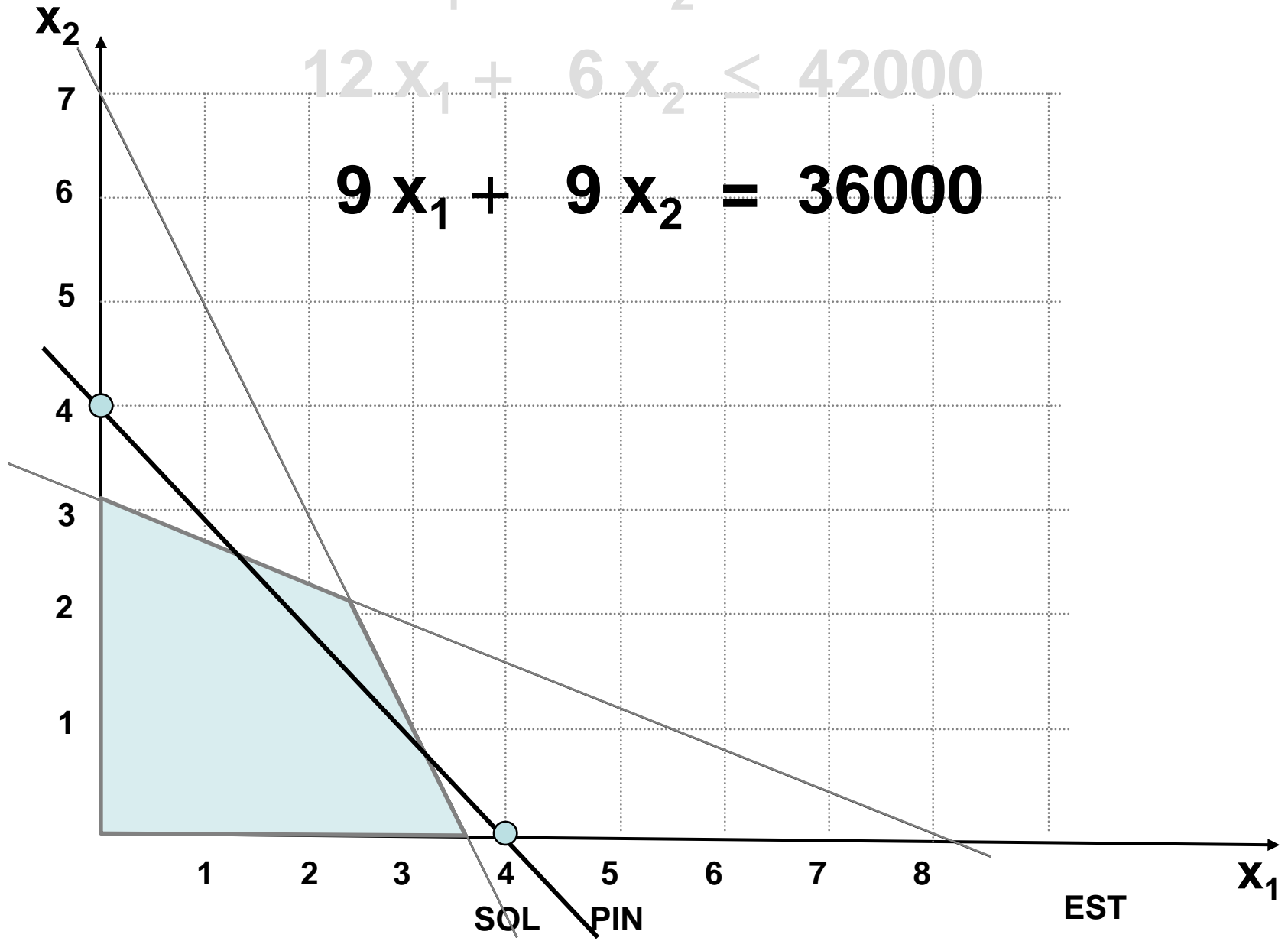
$$9x_1 + 9x_2 = 36000$$



$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$

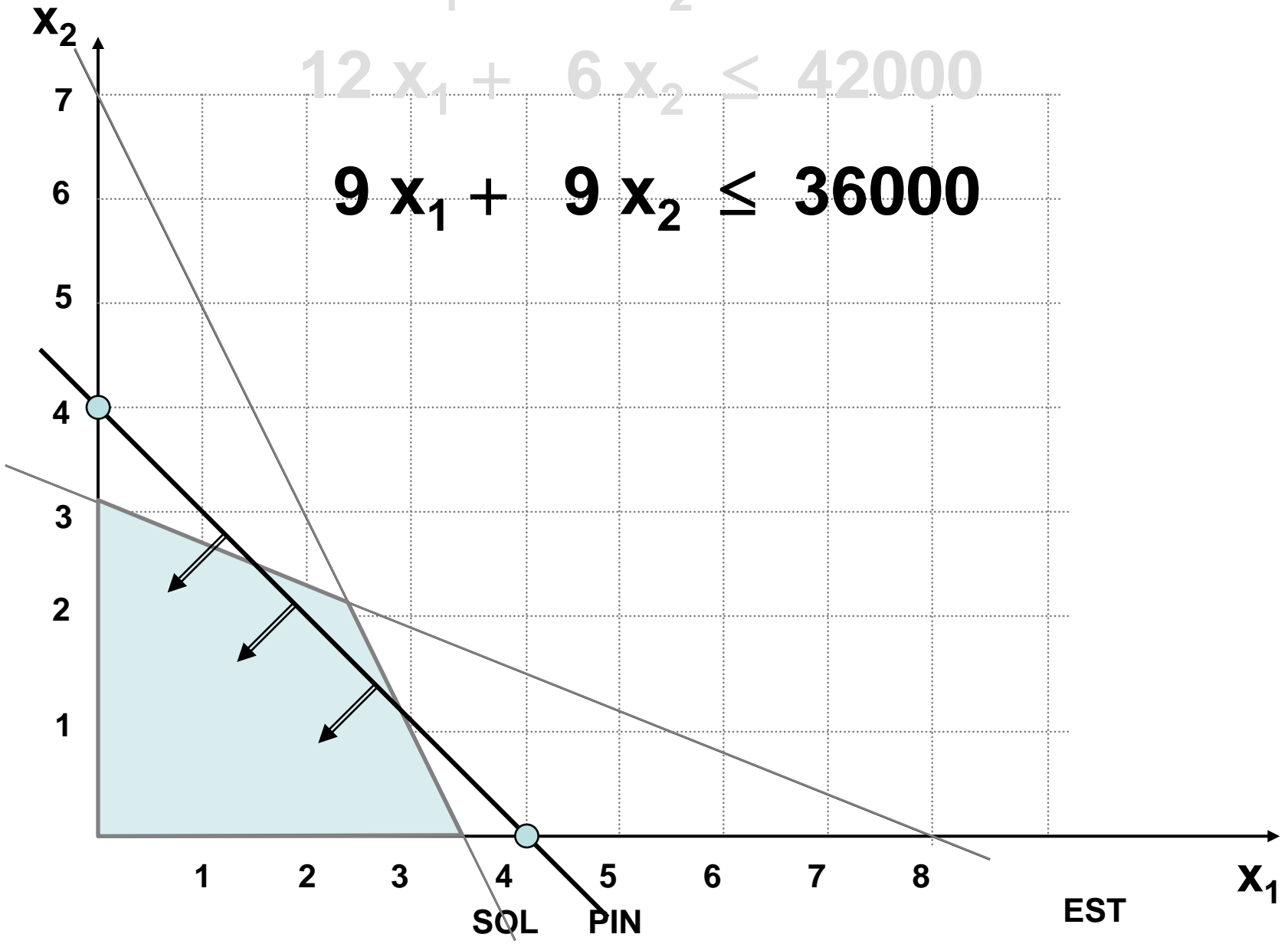
$$9x_1 + 9x_2 = 36000$$



$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$

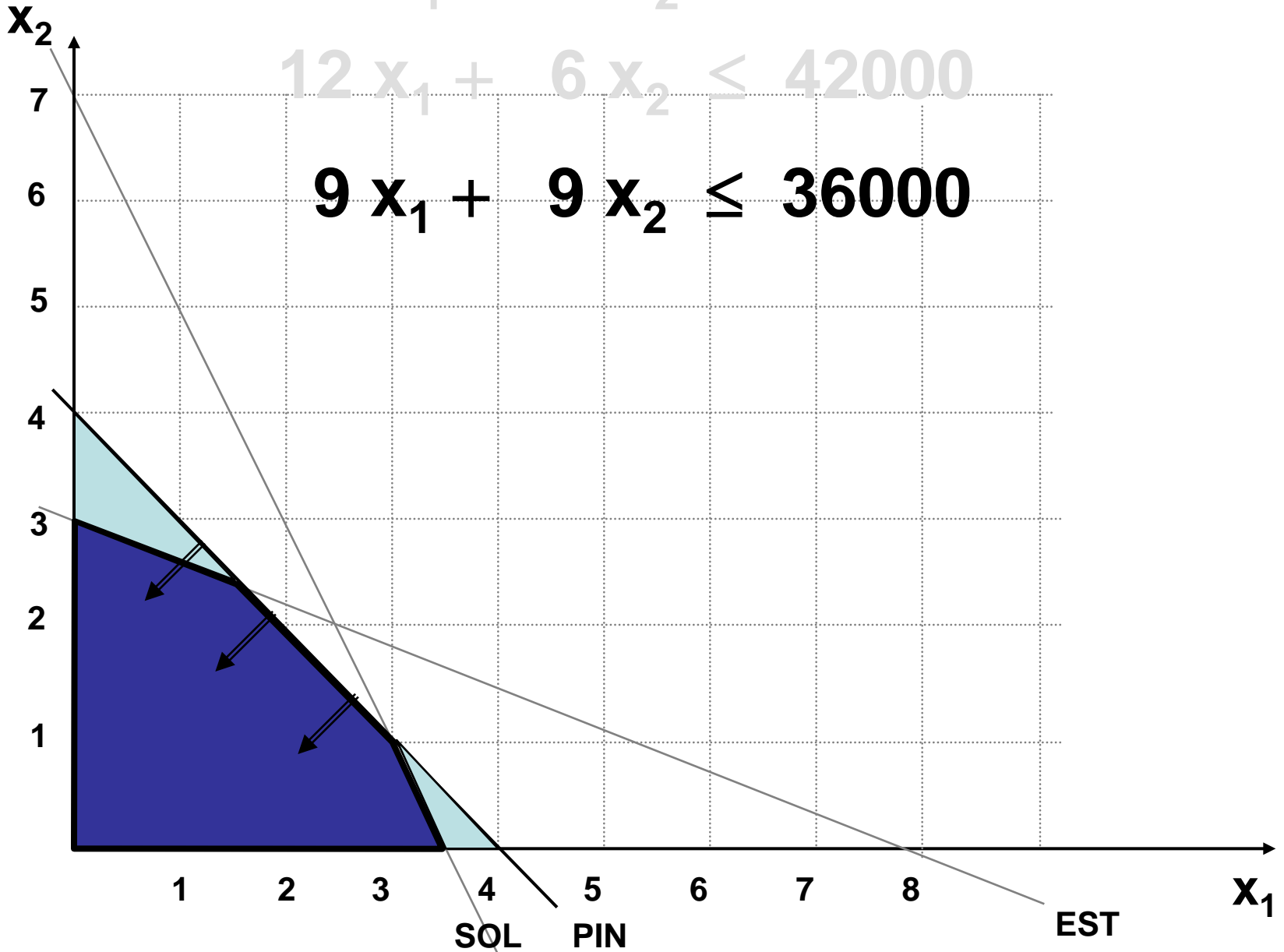
$$9x_1 + 9x_2 \leq 36000$$



$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$

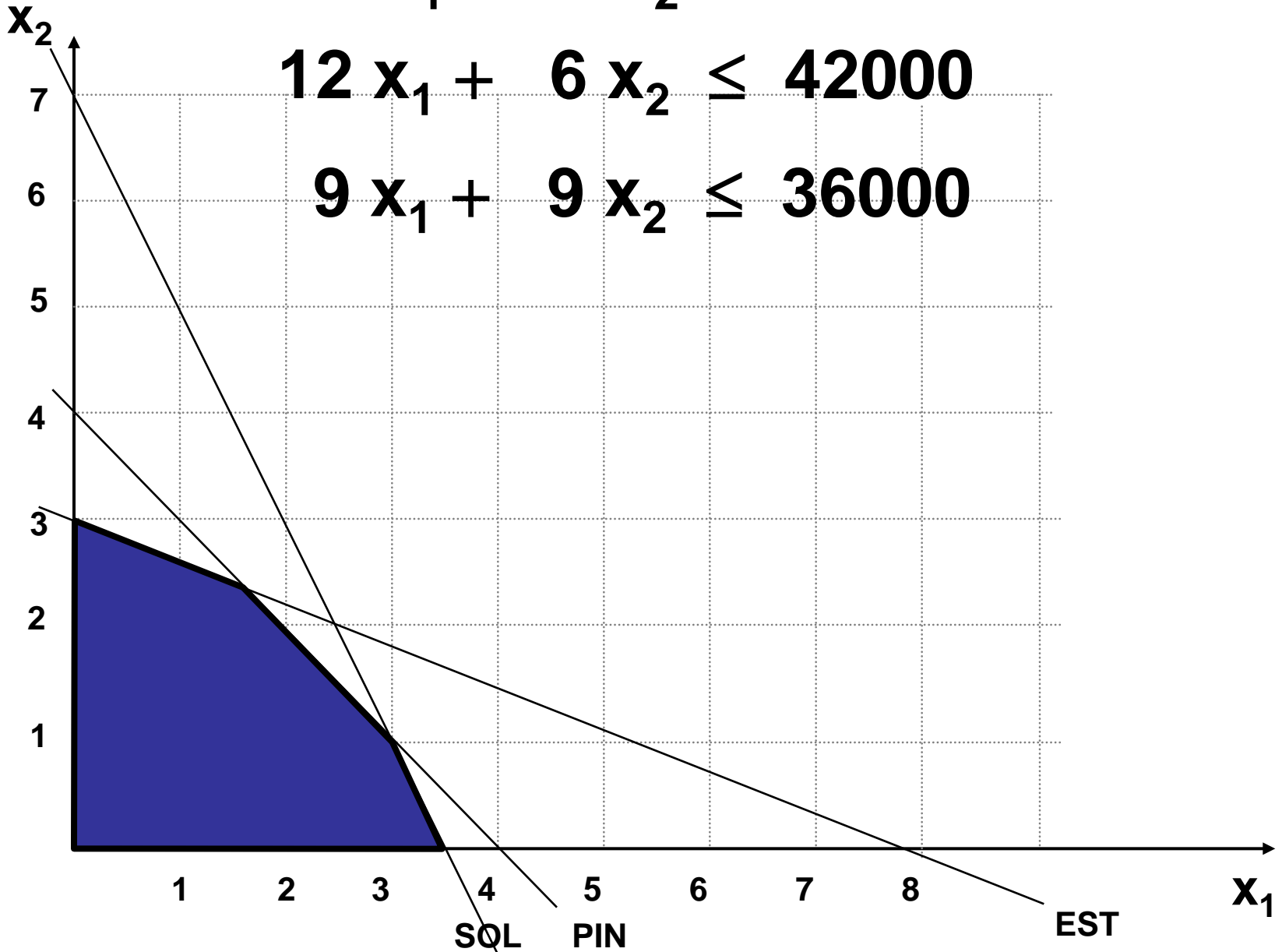
$$9x_1 + 9x_2 \leq 36000$$

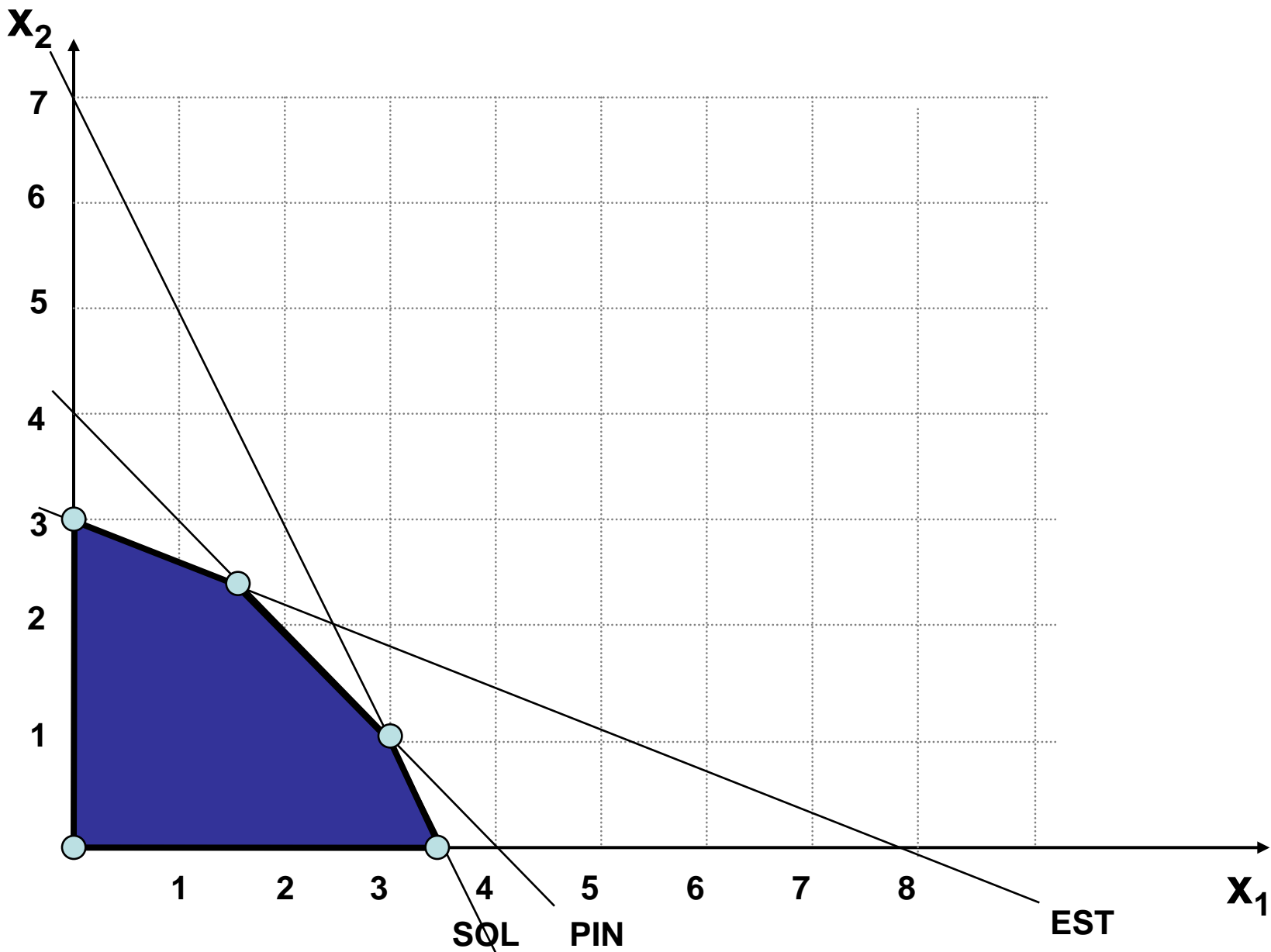


$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$

$$9x_1 + 9x_2 \leq 36000$$





x_2

$$Z = 4x_1 + 3x_2$$

3

2

1

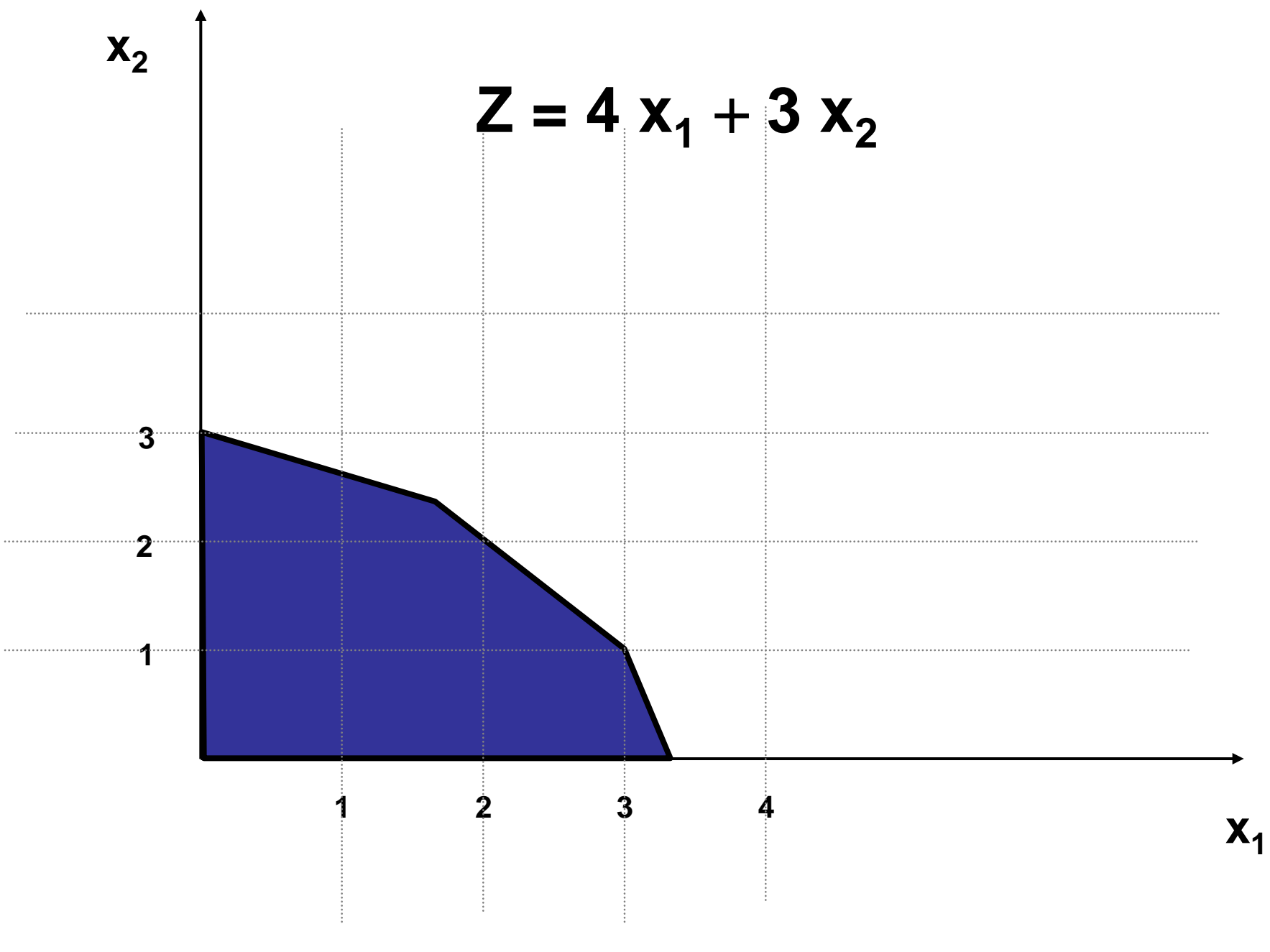
1

2

3

4

x_1



x_2

$$0 = 4x_1 + 3x_2$$

3

2

1

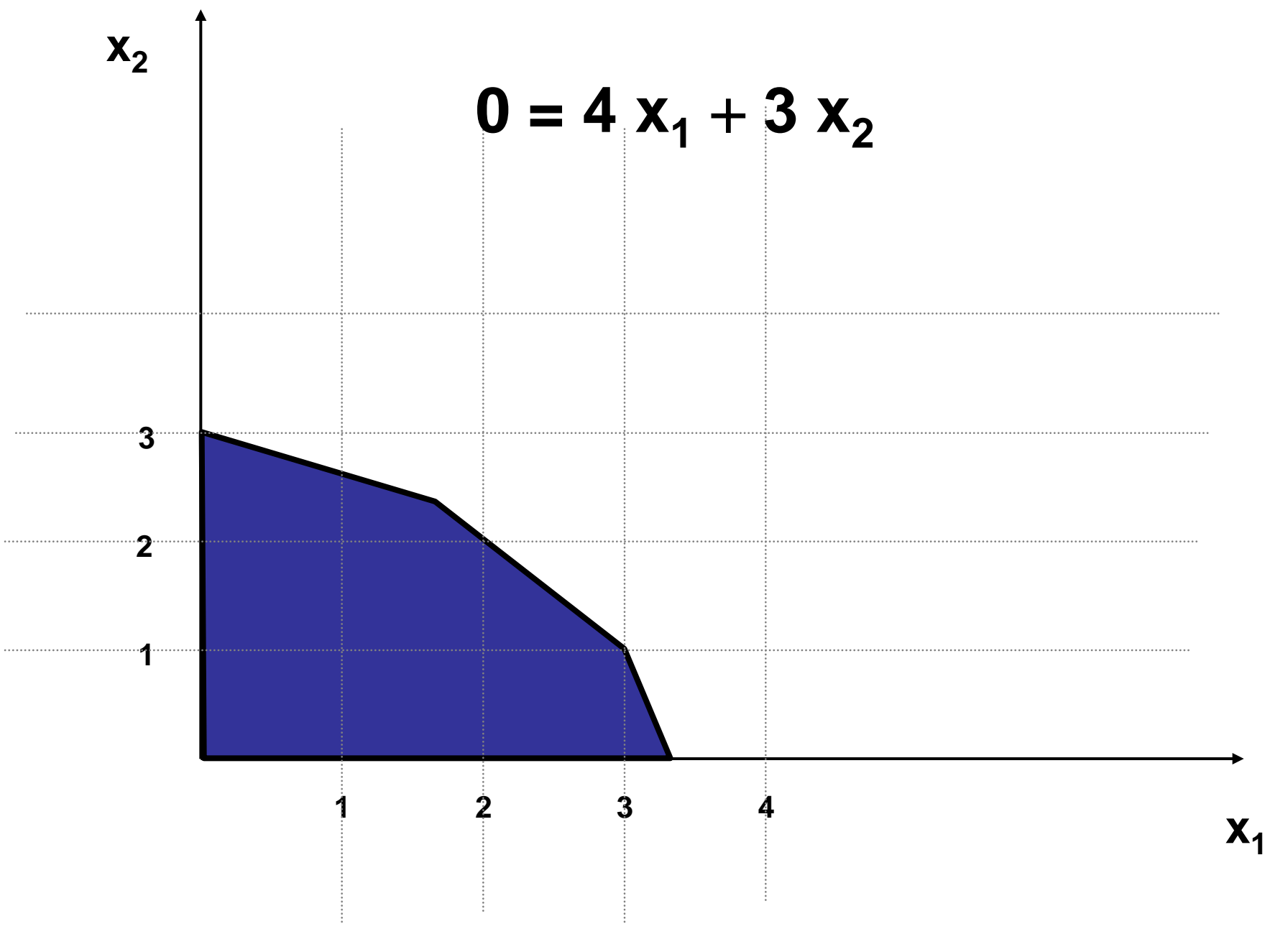
1

2

3

4

x_1



x_2

$$0 = 4x_1 + 3x_2$$

$$x_1 = -\frac{3}{4}x_2$$

3

2

1

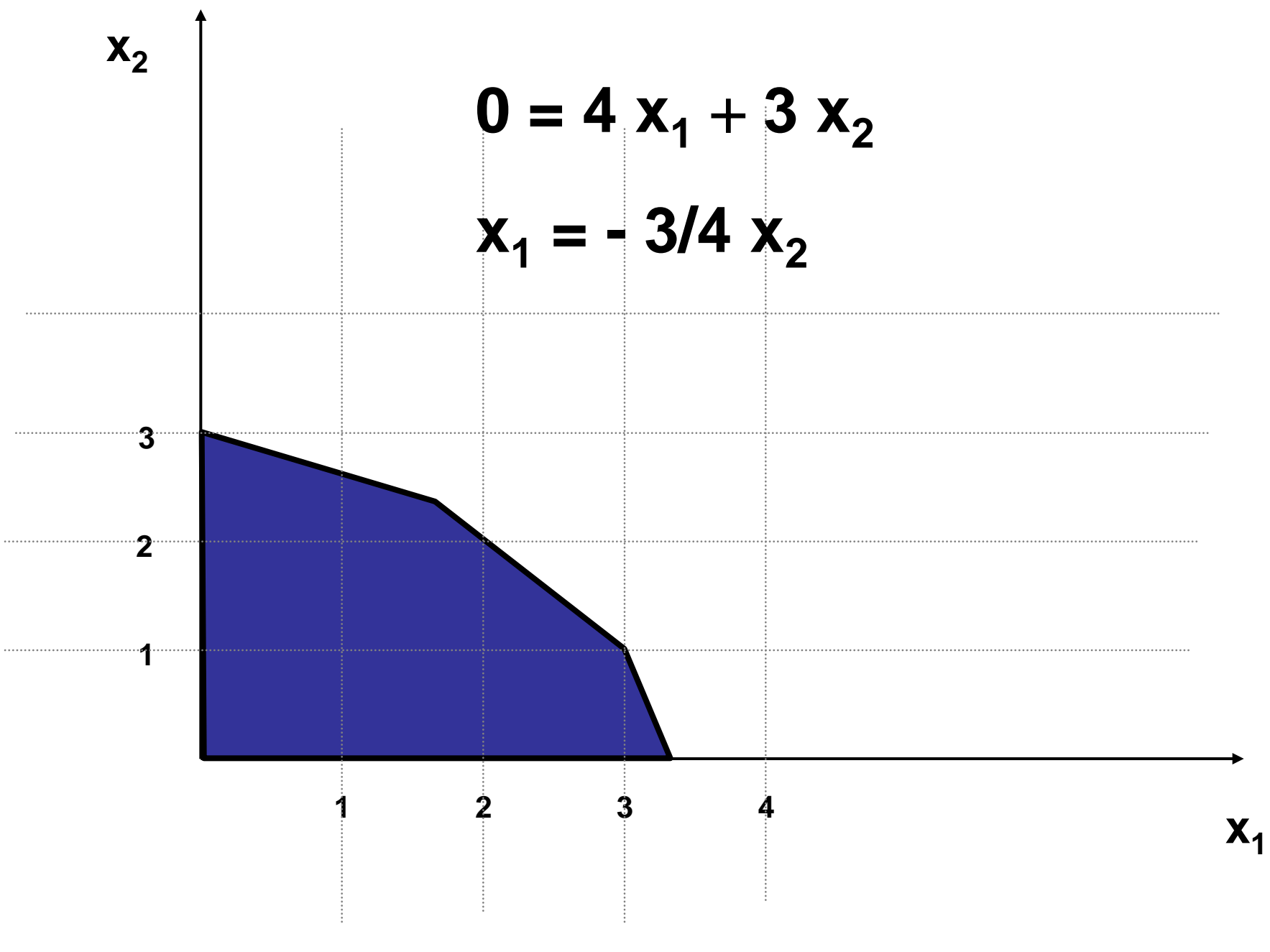
1

2

3

4

x_1



x_2

$$0 = 4x_1 + 3x_2$$

$$x_1 = -\frac{3}{4}x_2$$

Para $x_2 = 0 \rightarrow x_1 = 0$

3

2

1

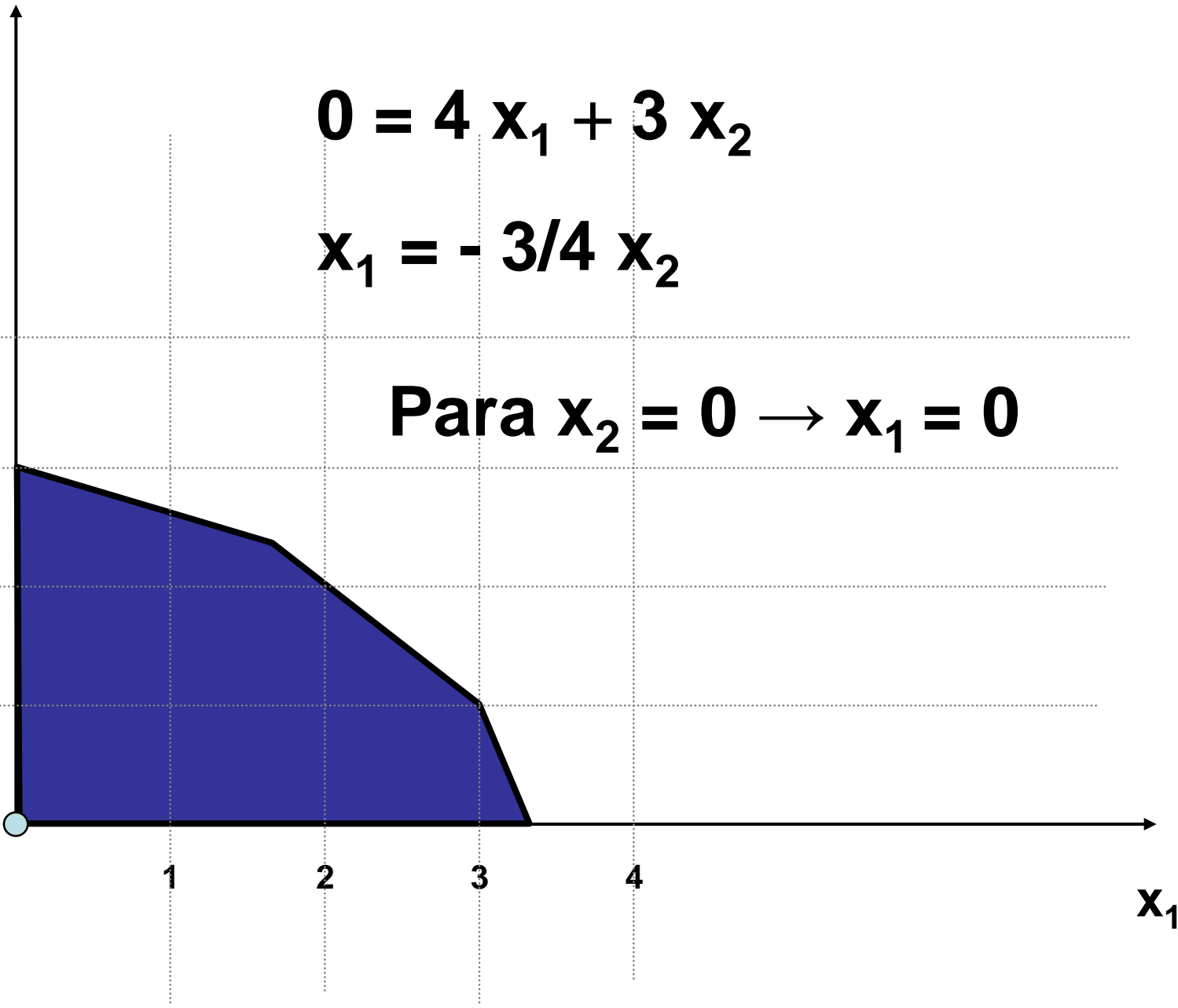
1

2

3

4

x_1

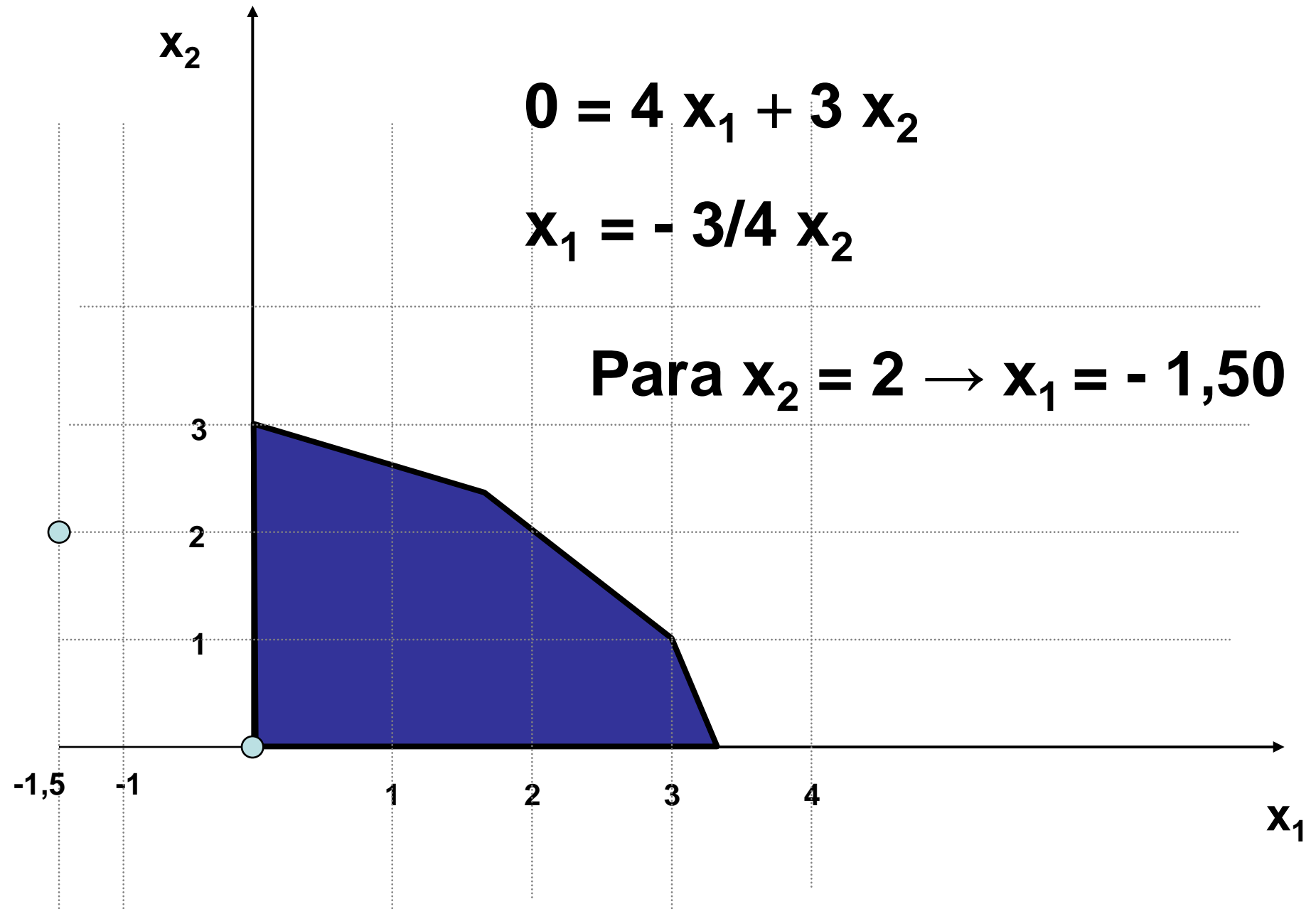


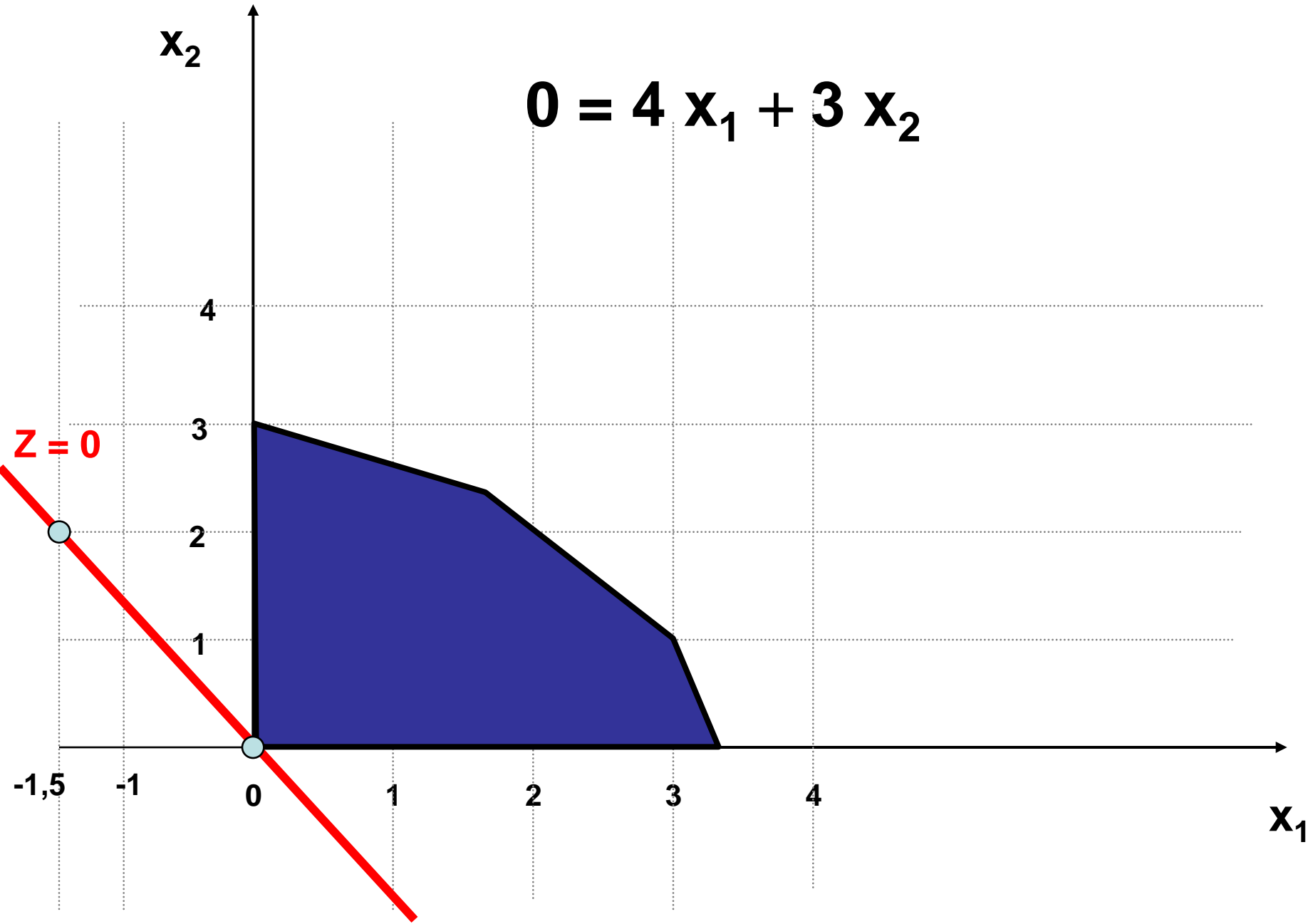
x_2

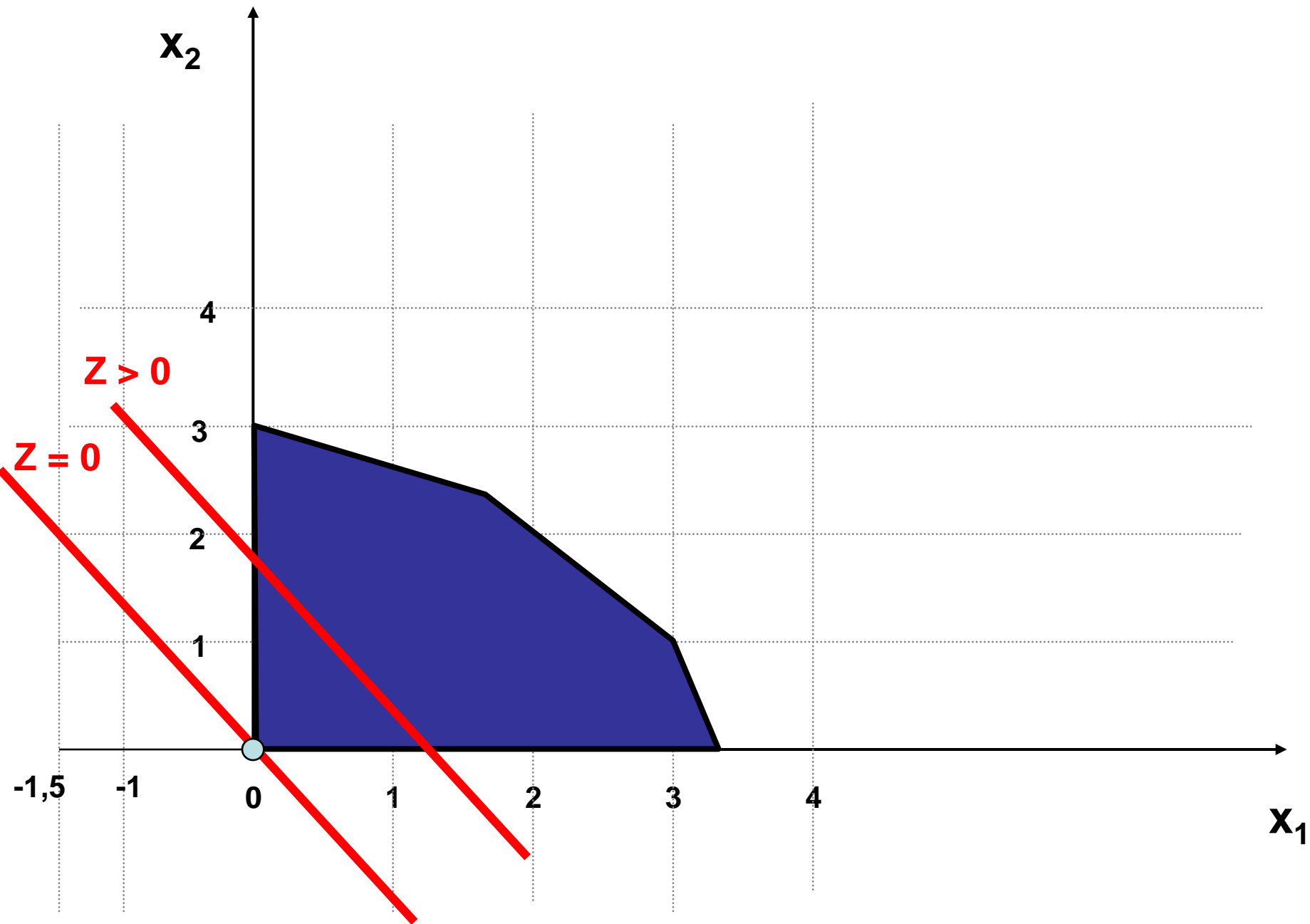
$$0 = 4x_1 + 3x_2$$

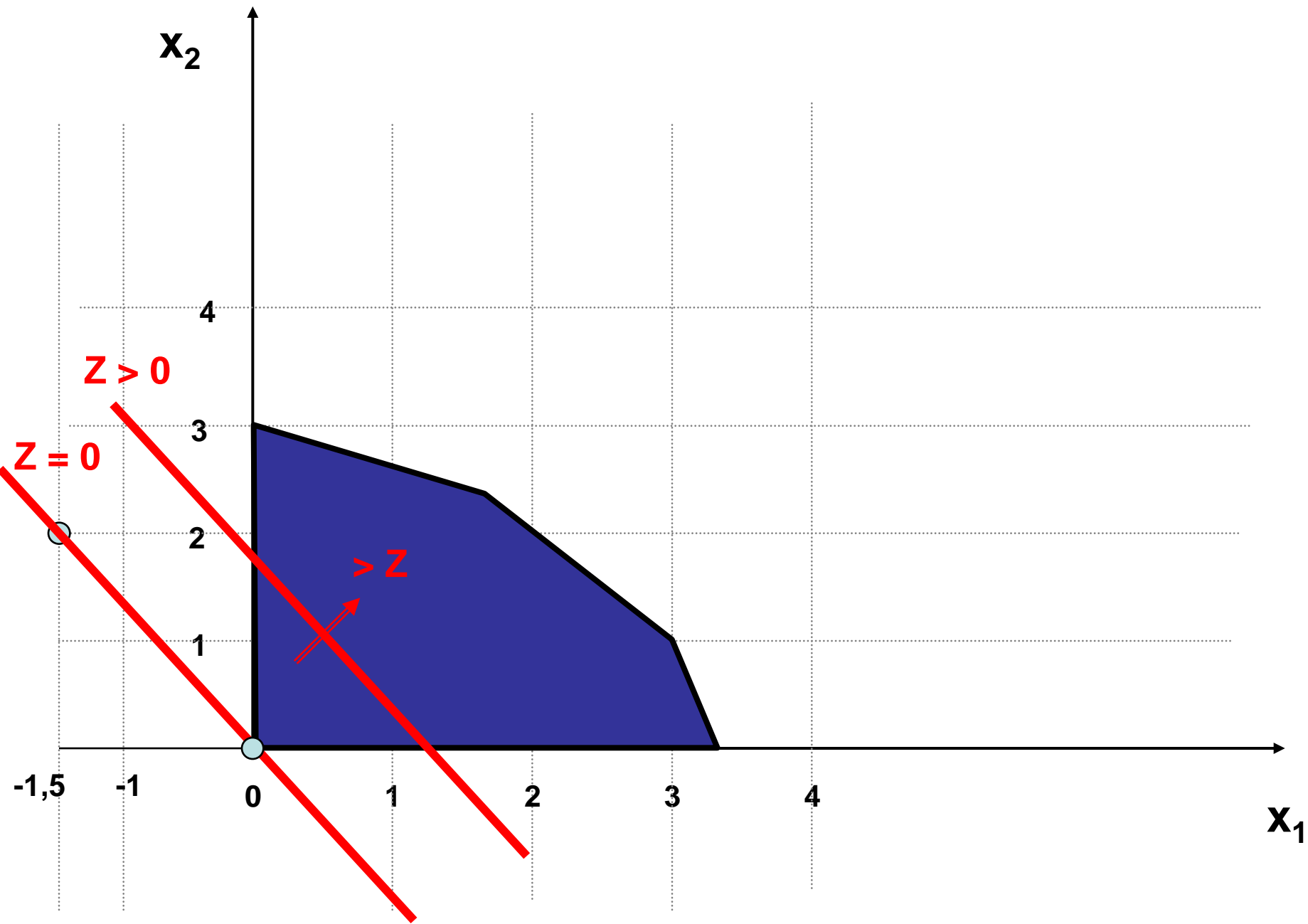
$$x_1 = -\frac{3}{4}x_2$$

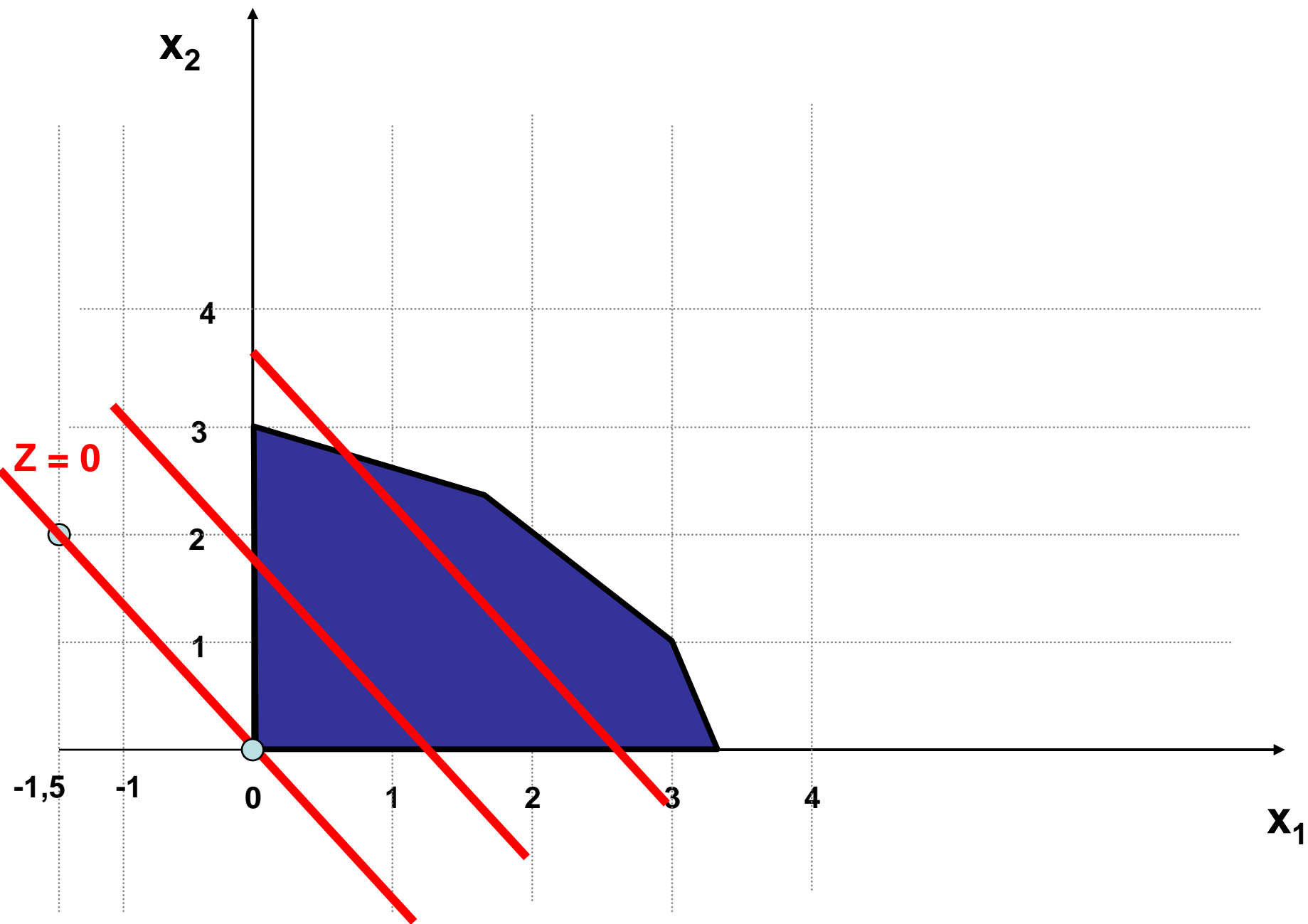
Para $x_2 = 2 \rightarrow x_1 = -1,50$

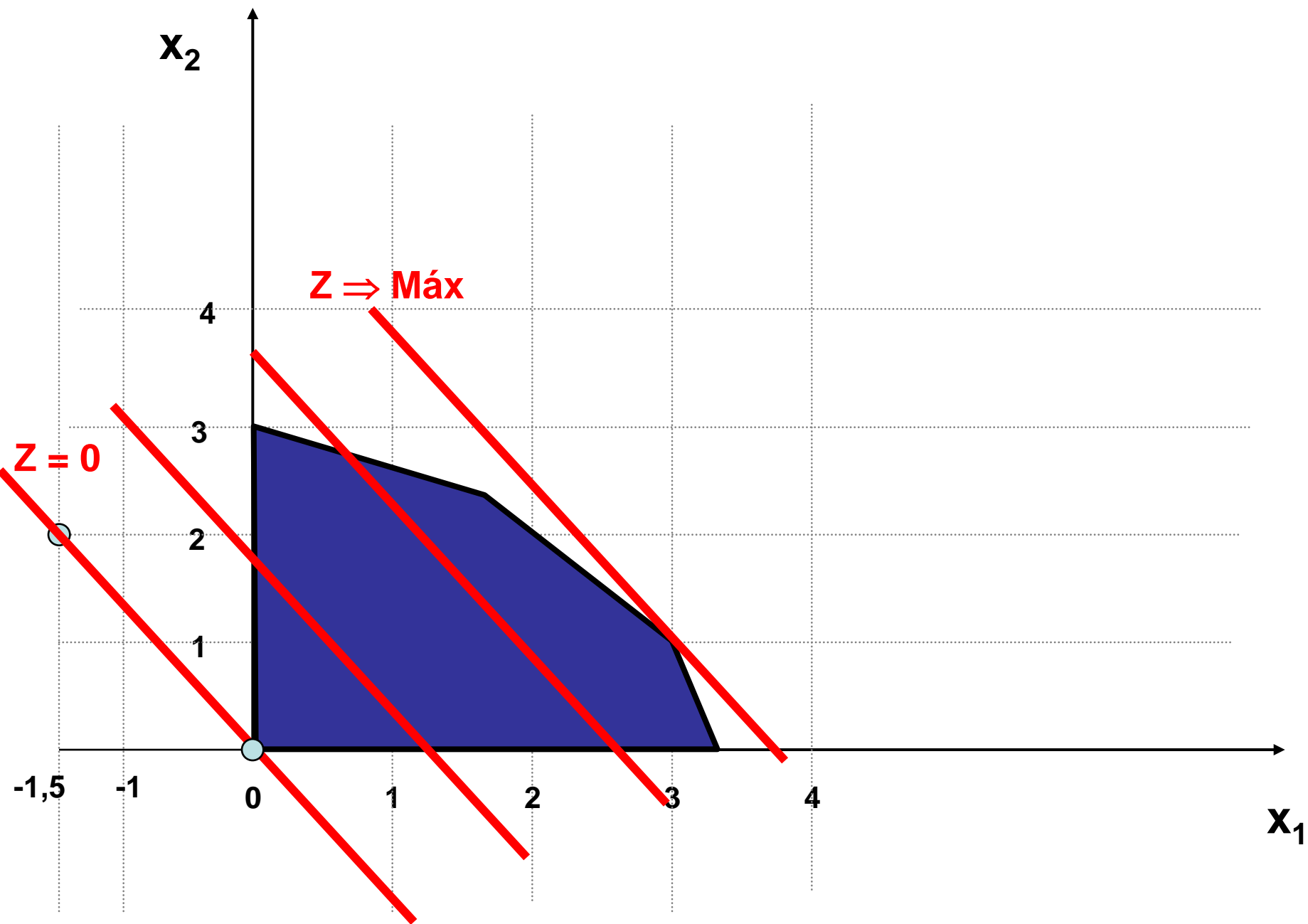


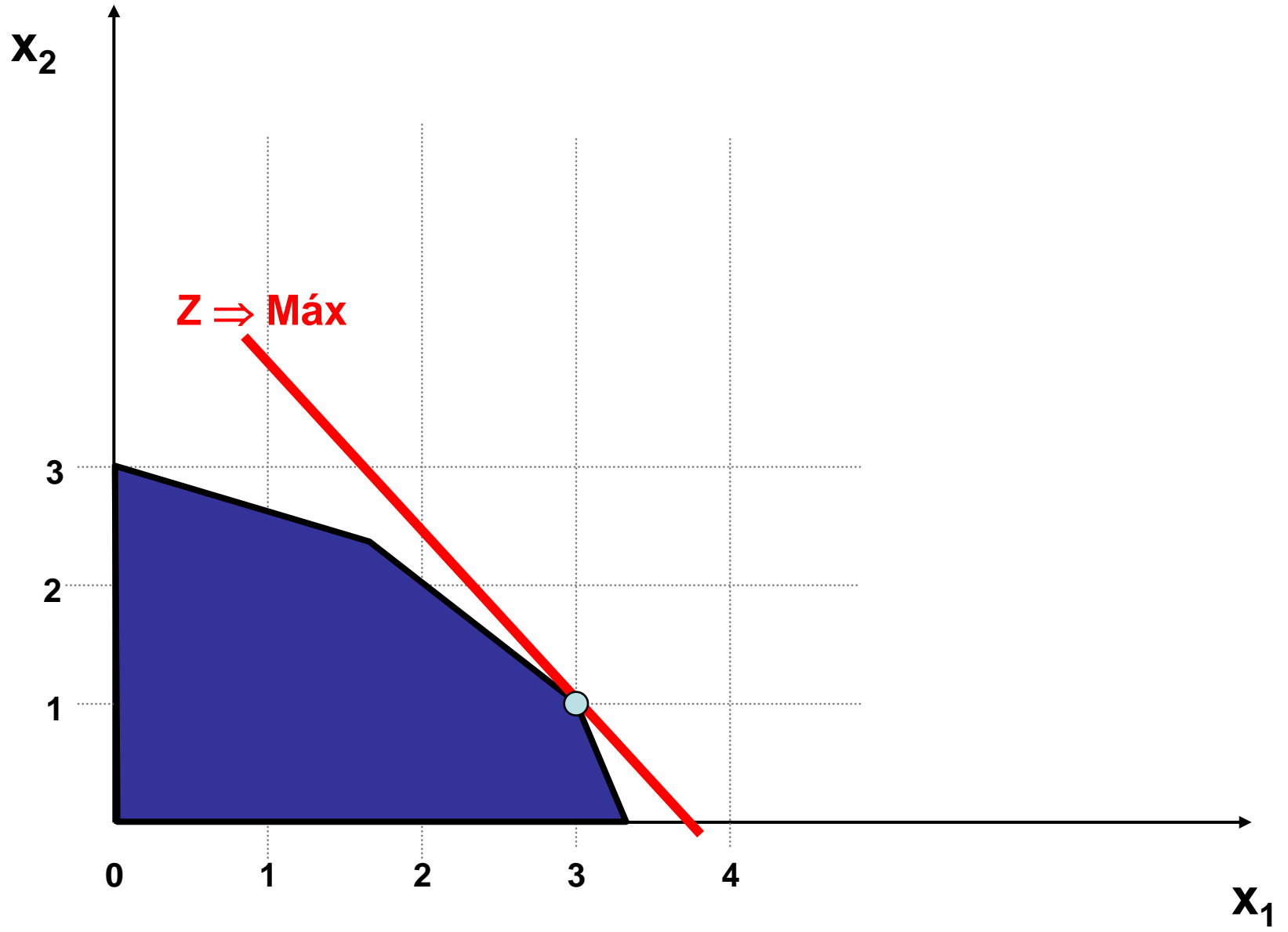


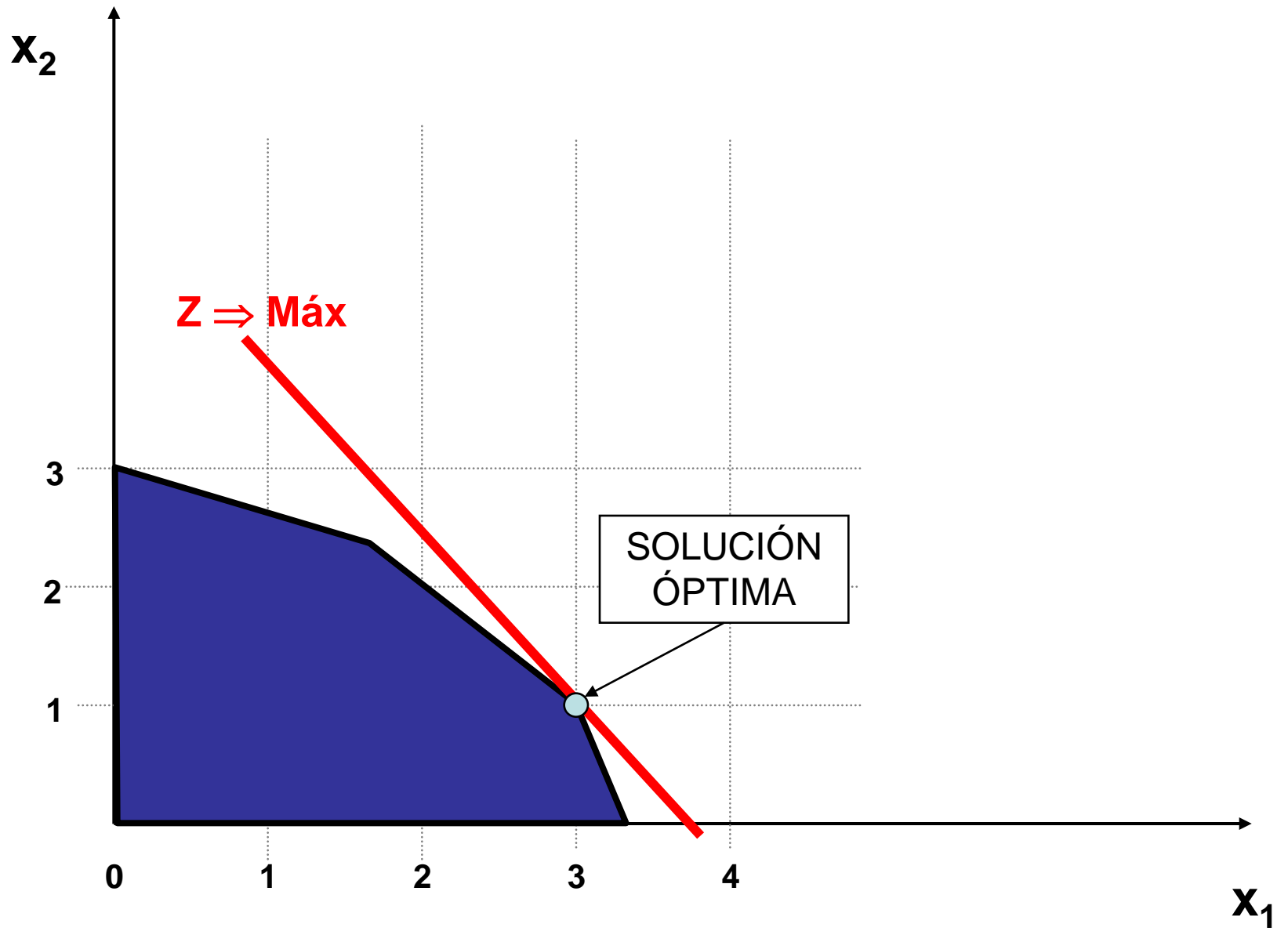


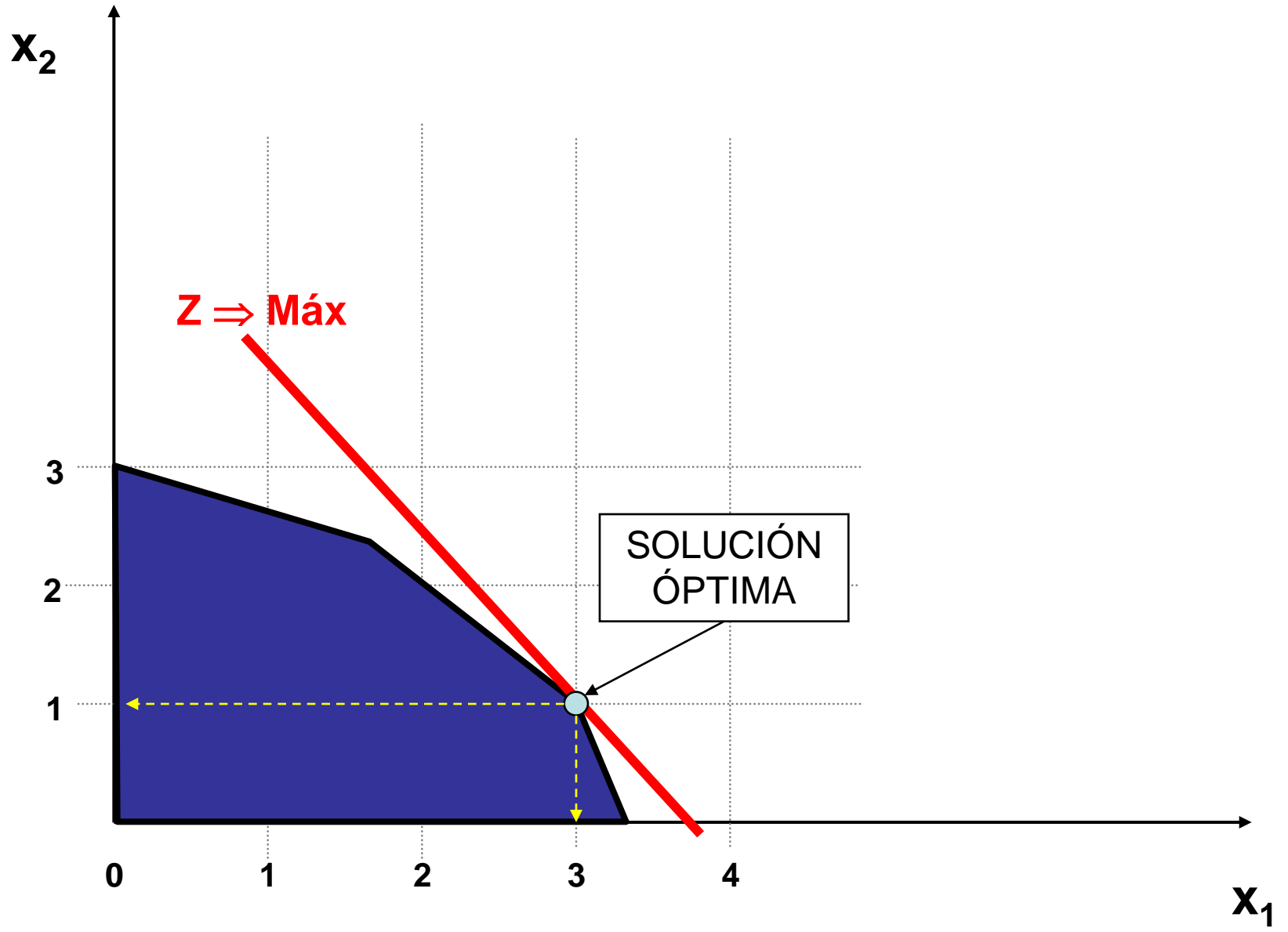


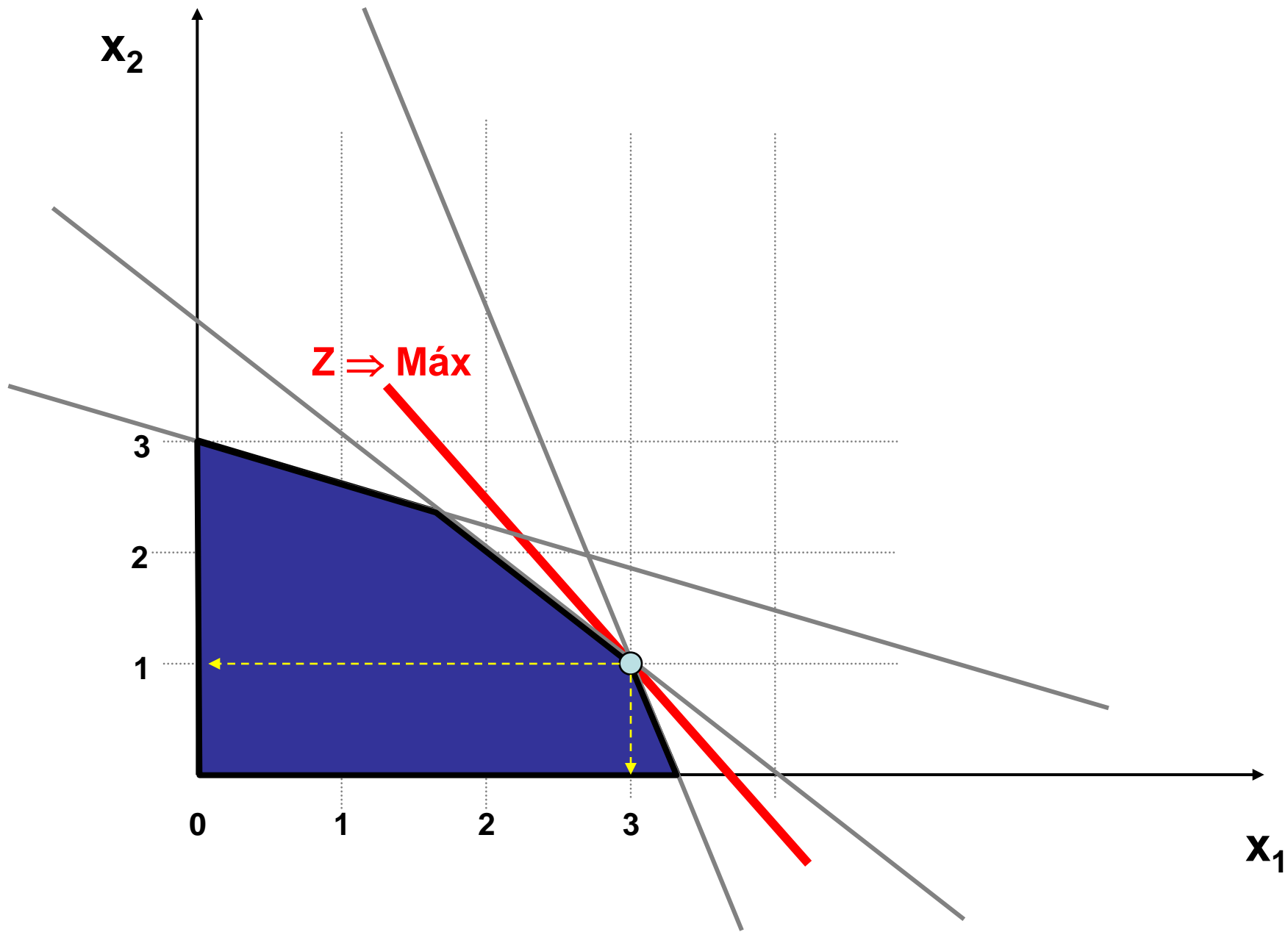


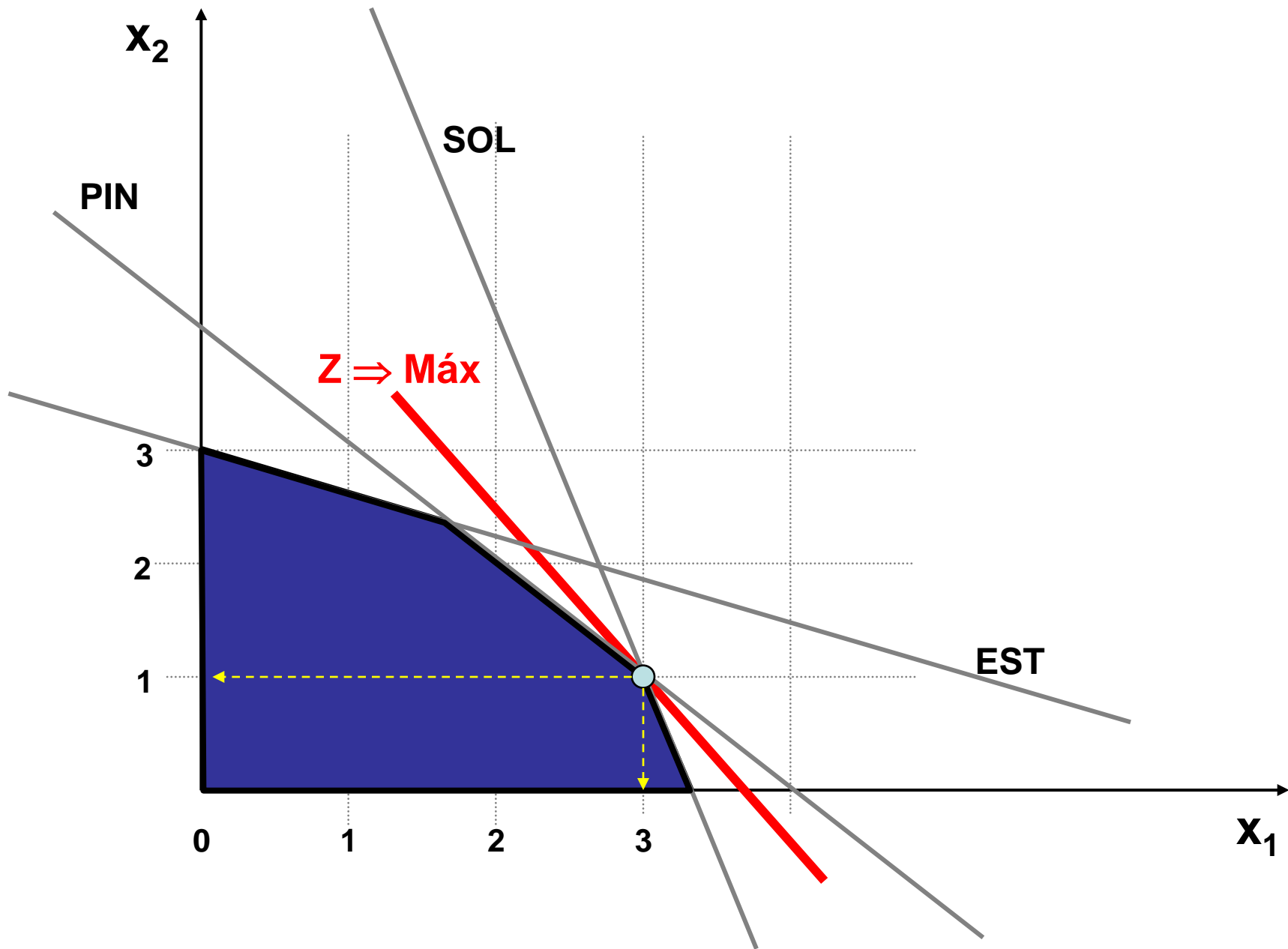


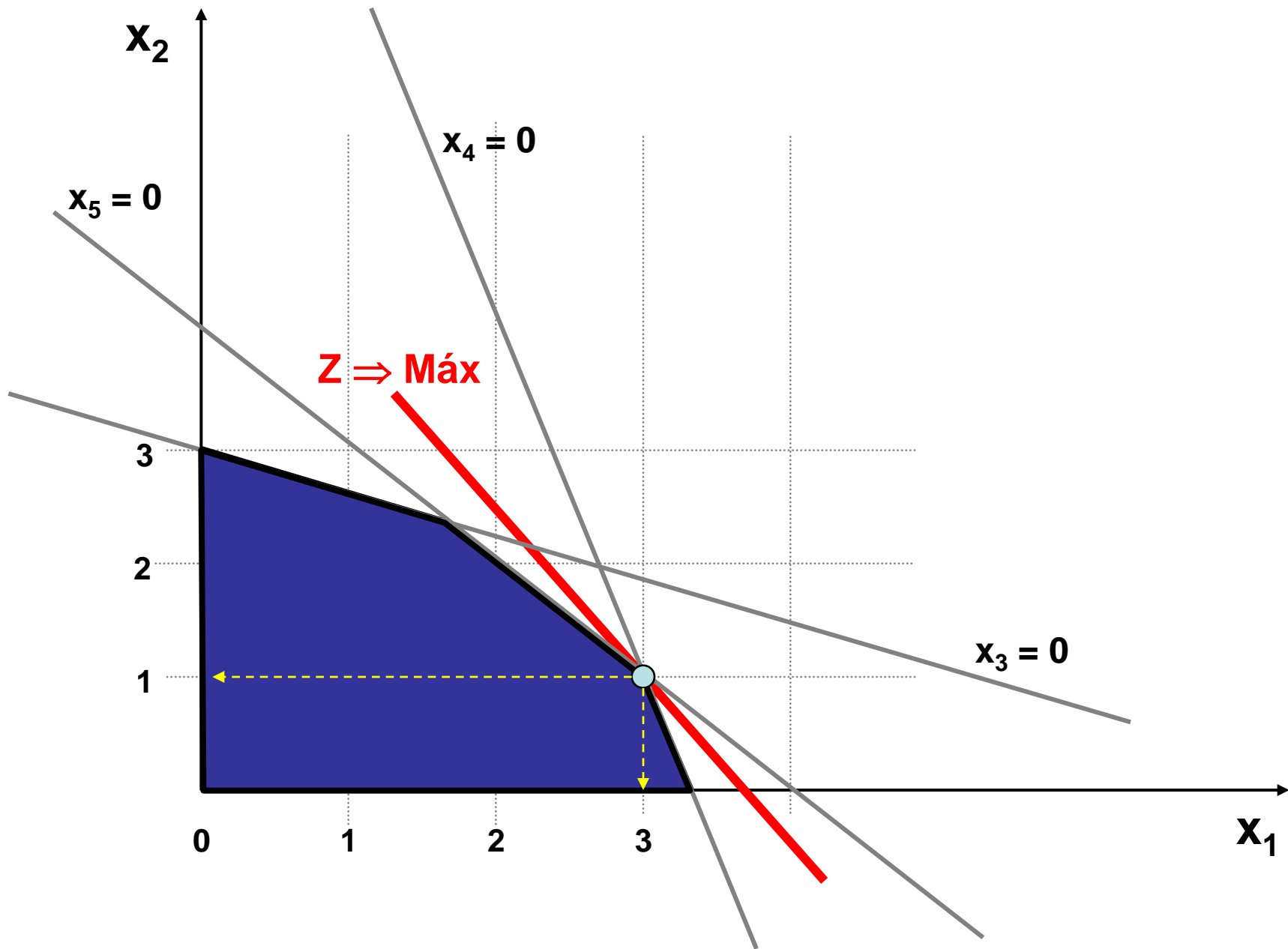


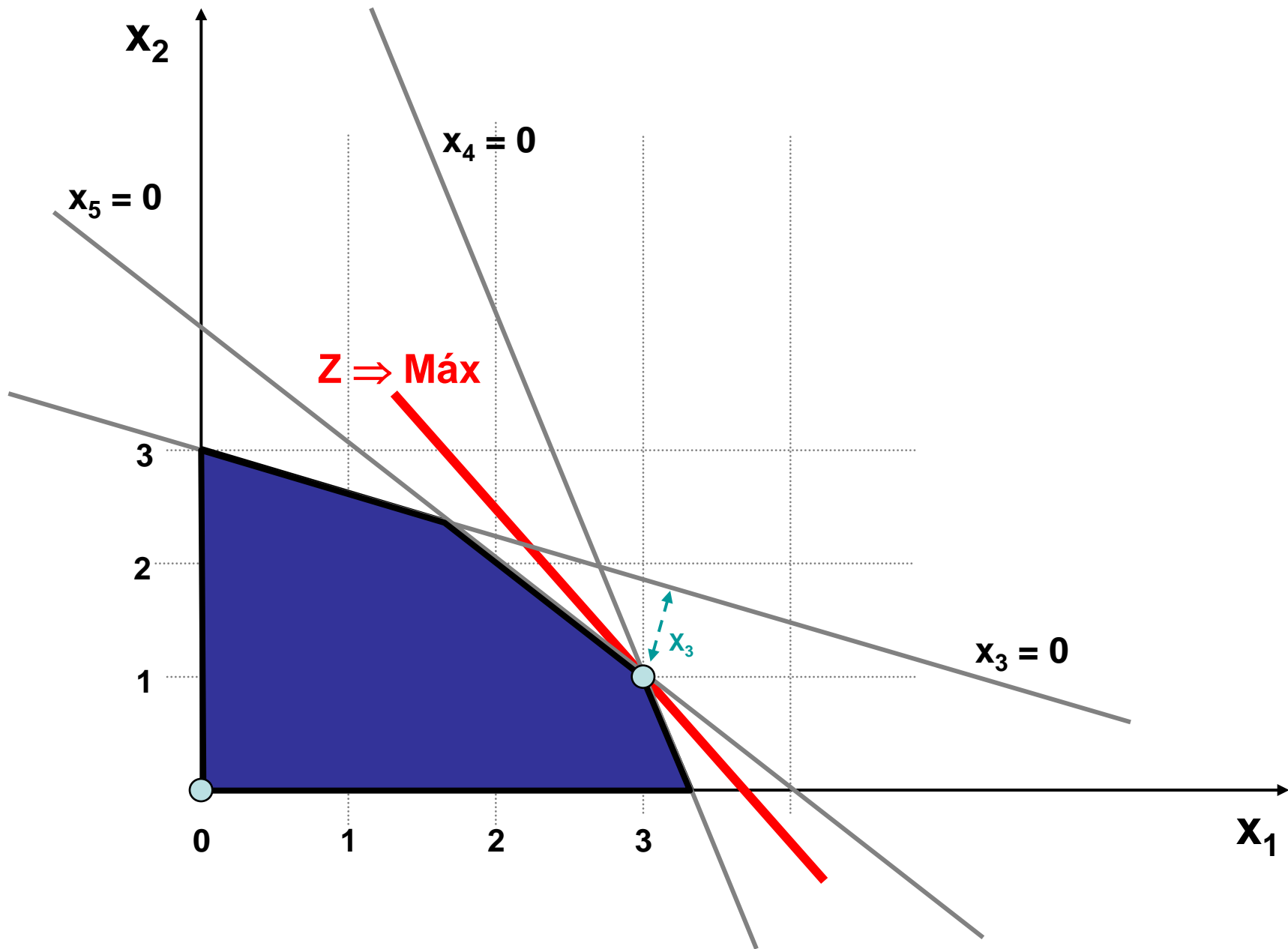


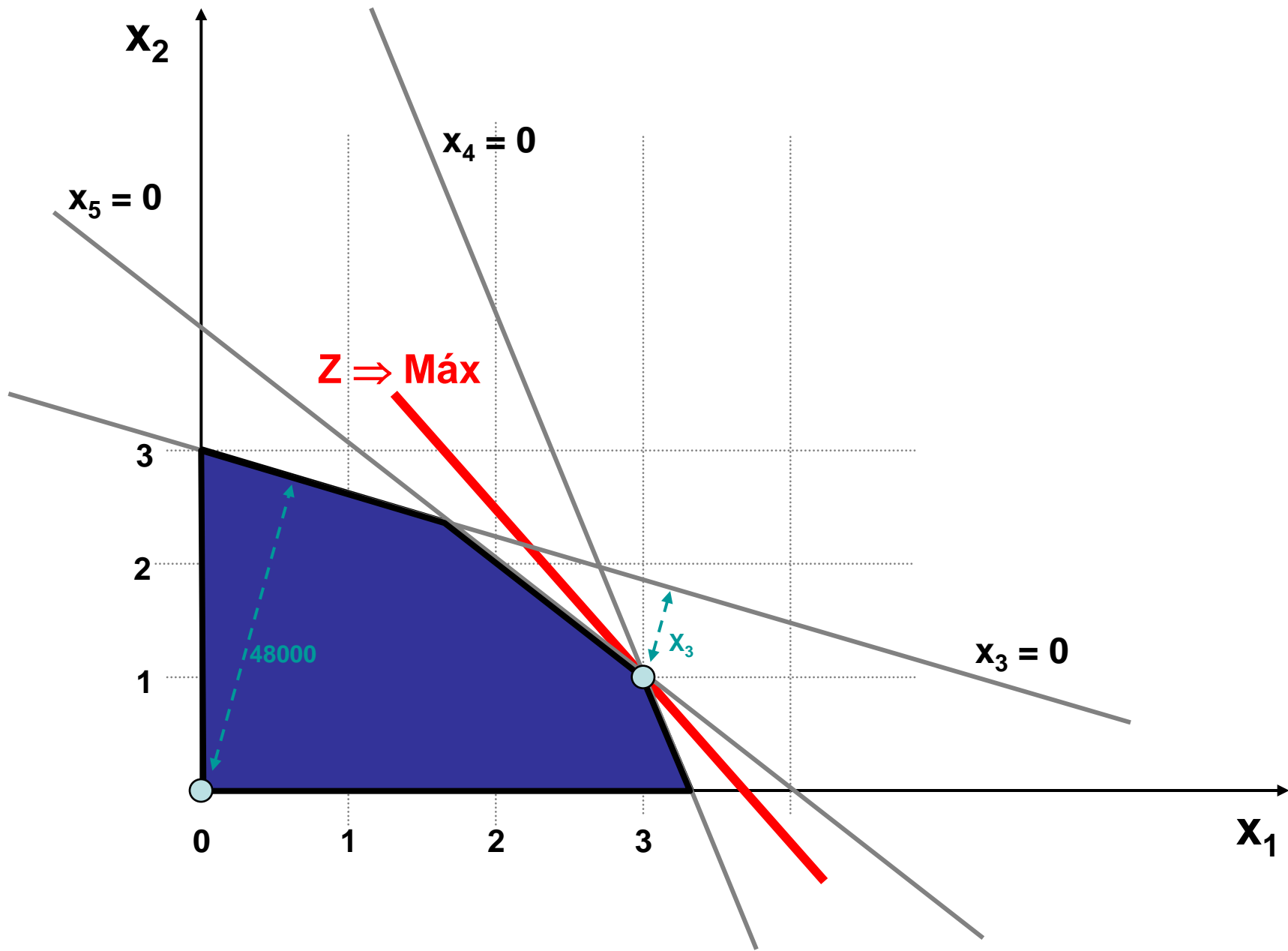












$$\text{MAX: } Z = 4 x_1 + 3 x_2$$

$$\left\{ \begin{array}{l} 6 x_1 + 16 x_2 \leq 48000 \\ 12 x_1 + 6 x_2 \leq 42000 \\ 9 x_1 + 9 x_2 \leq 36000 \end{array} \right.$$

$$x_1, x_2 \geq 0$$

FORMA MATRICIAL EXTENDIDA

	X_1	X_2	SIGNO	RHS
Z)	4	3	→	MAX
EST)	6	16	≤	48000
SOL)	12	6	≤	42000
PIN)	9	9	≤	36000
Var.	NN	NN		

$$\text{MAX: } Z = 4 x_1 + 3 x_2$$

$$\left\{ \begin{array}{l} 6 x_1 + 16 x_2 + x_3 = 48000 \\ 12 x_1 + 6 x_2 + x_4 = 42000 \\ 9 x_1 + 9 x_2 + x_5 = 36000 \end{array} \right.$$

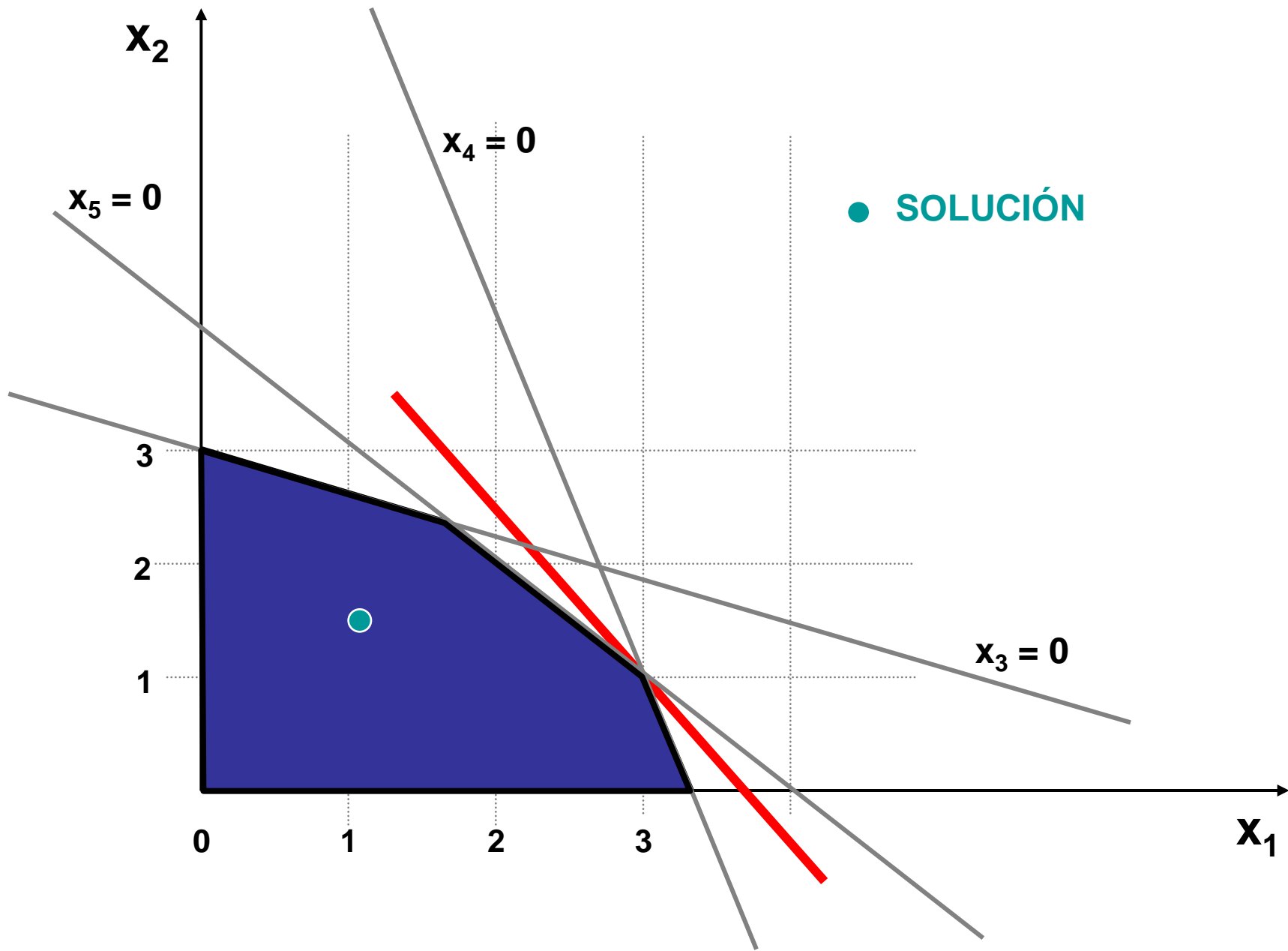
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

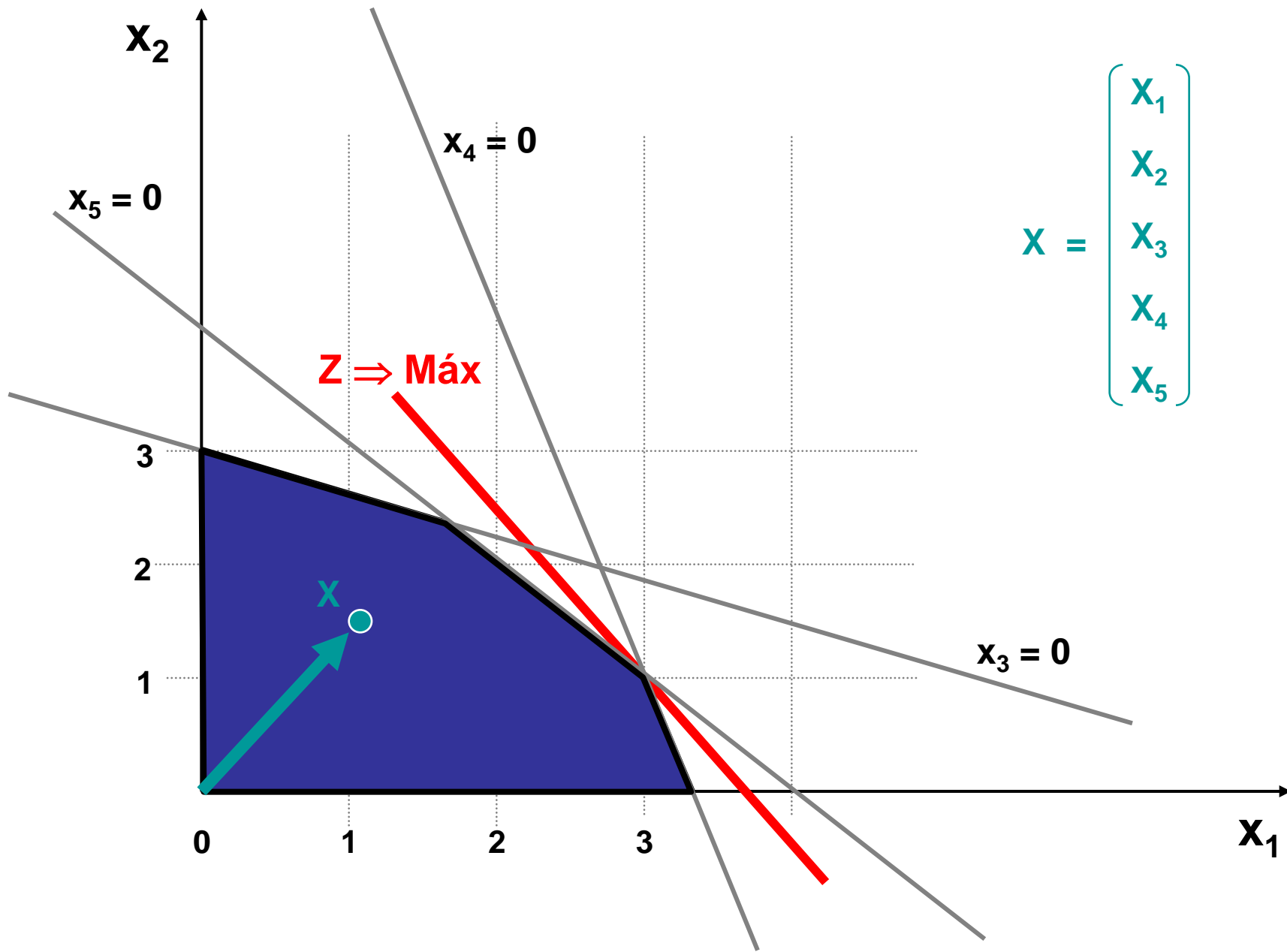
SISTEMA DE ECUACIONES LINEALES

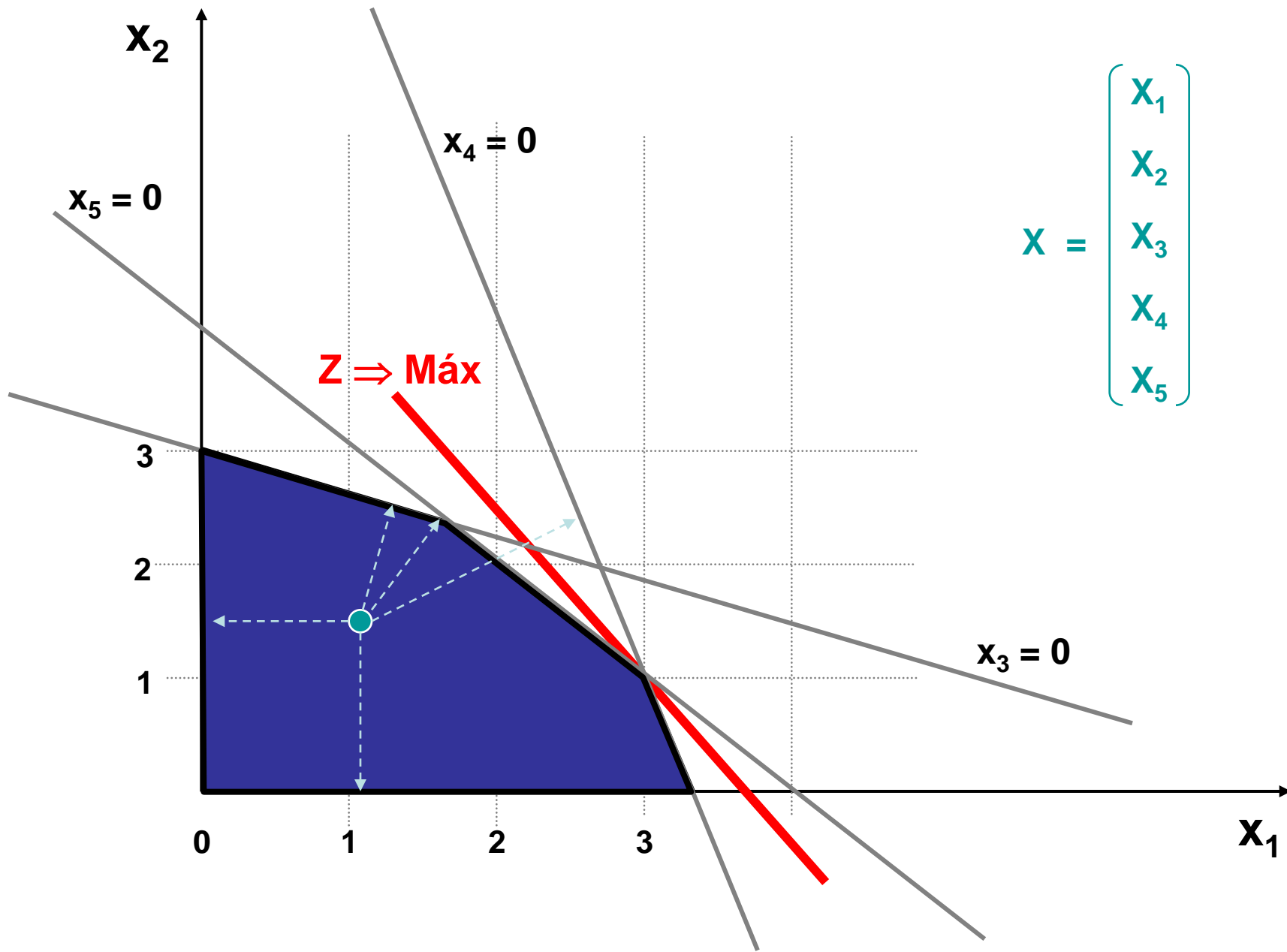
$$\left\{ \begin{array}{l} 6 x_1 + 16 x_2 + x_3 = 48000 \\ 12 x_1 + 6 x_2 + x_4 = 42000 \\ 9 x_1 + 9 x_2 + x_5 = 36000 \end{array} \right.$$

n = número de variables (5)

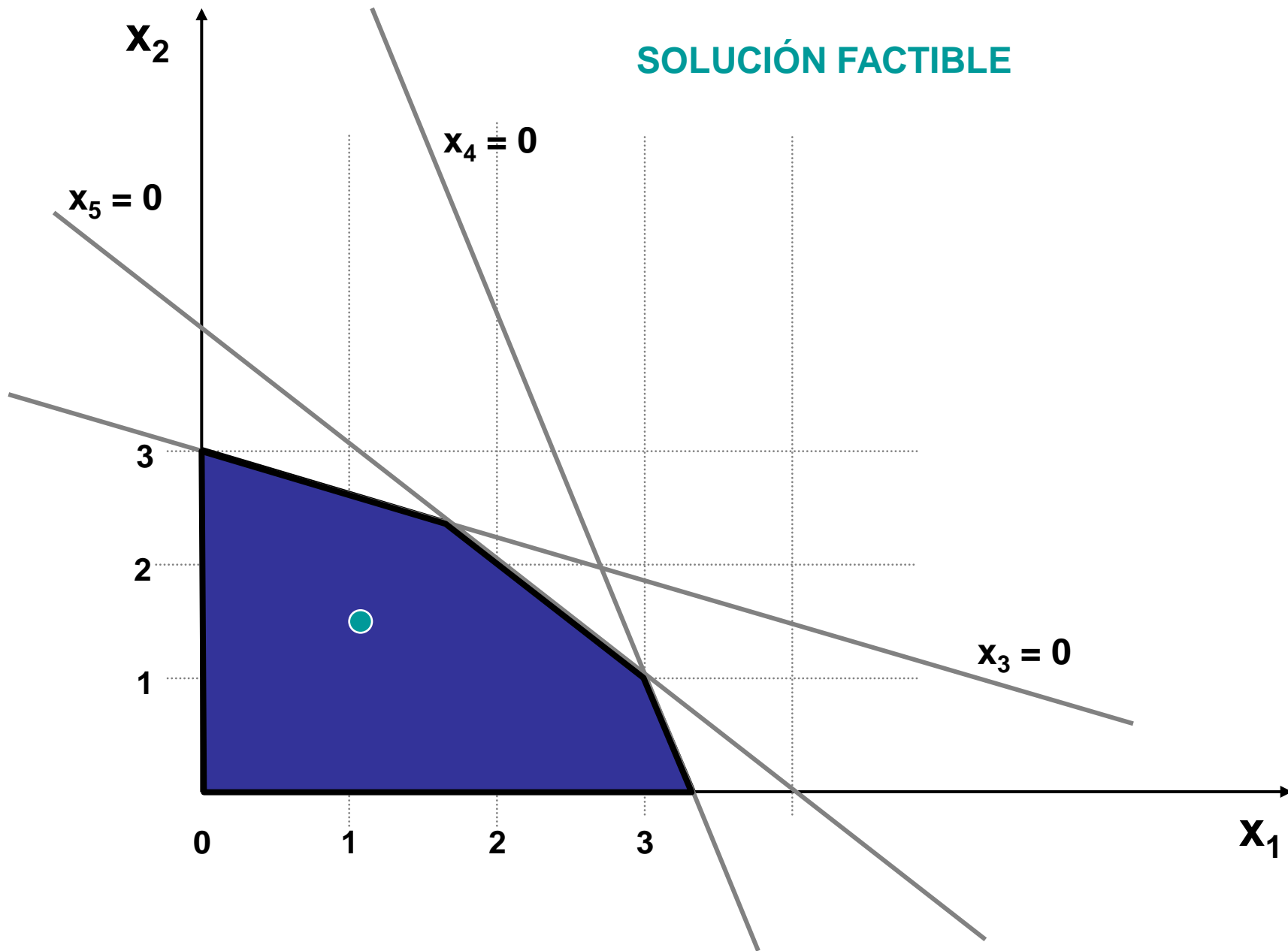
m = número de restricciones (3)



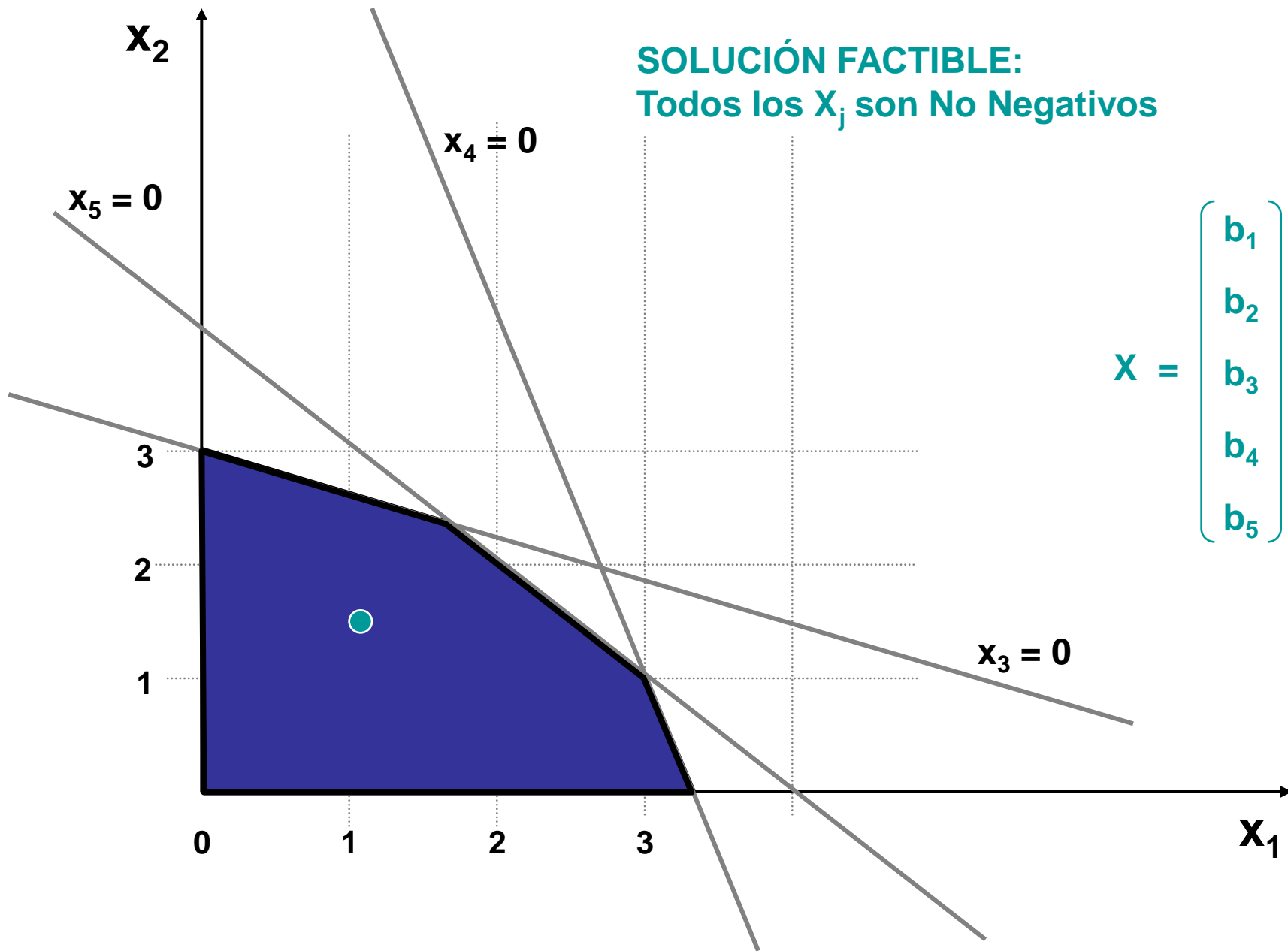




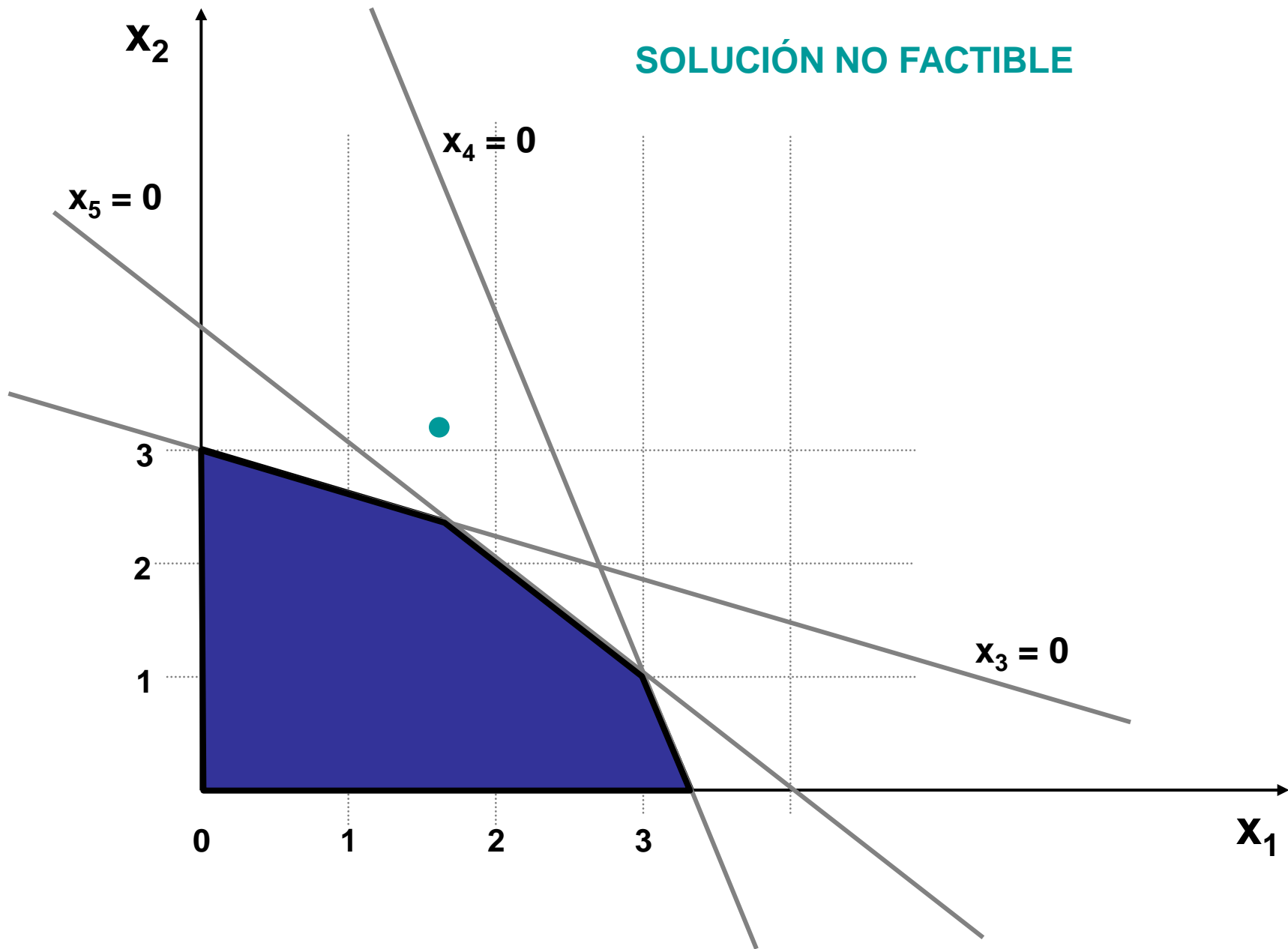
SOLUCIÓN FACTIBLE



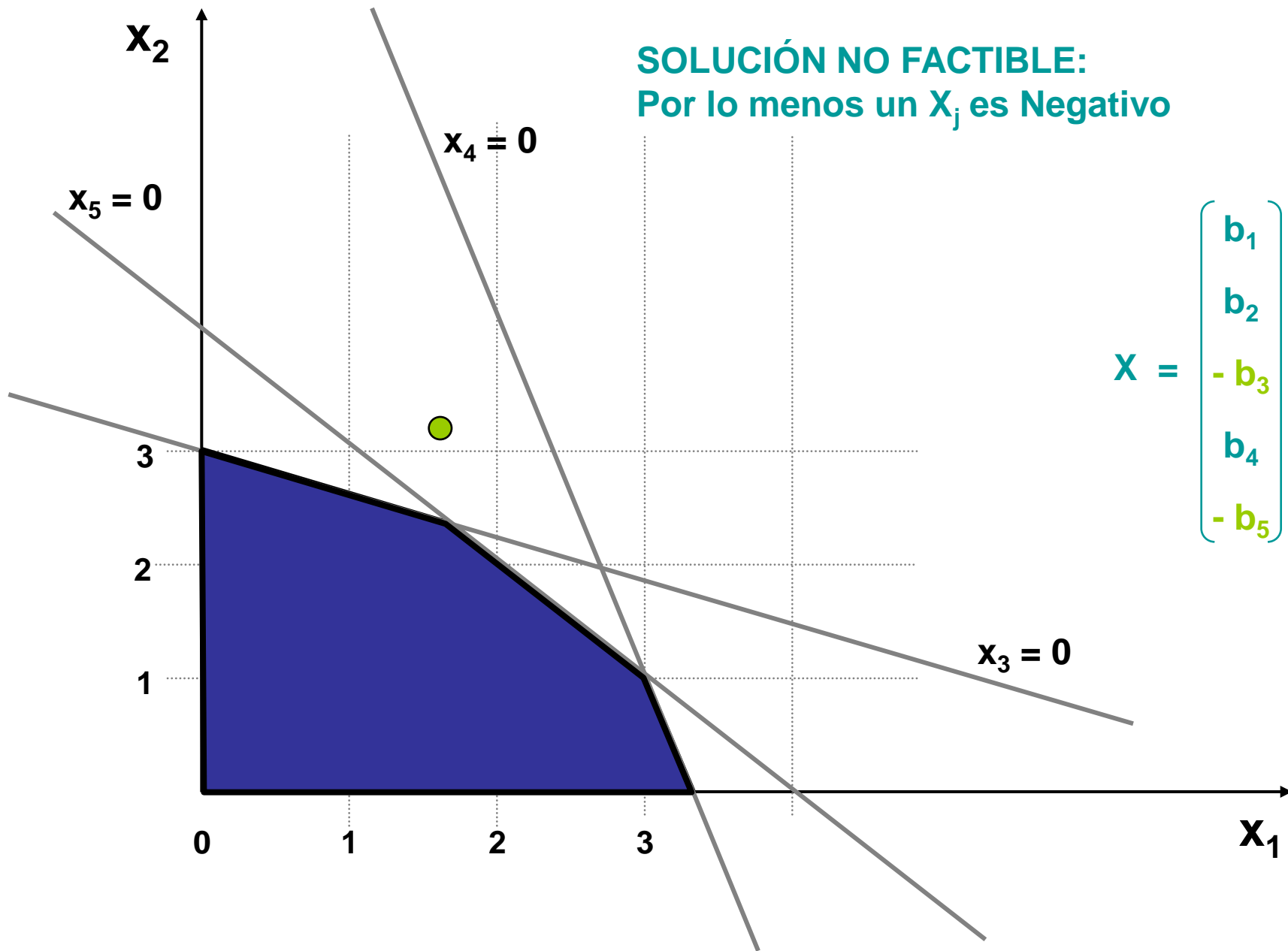
SOLUCIÓN FACTIBLE:
Todos los X_j son No Negativos



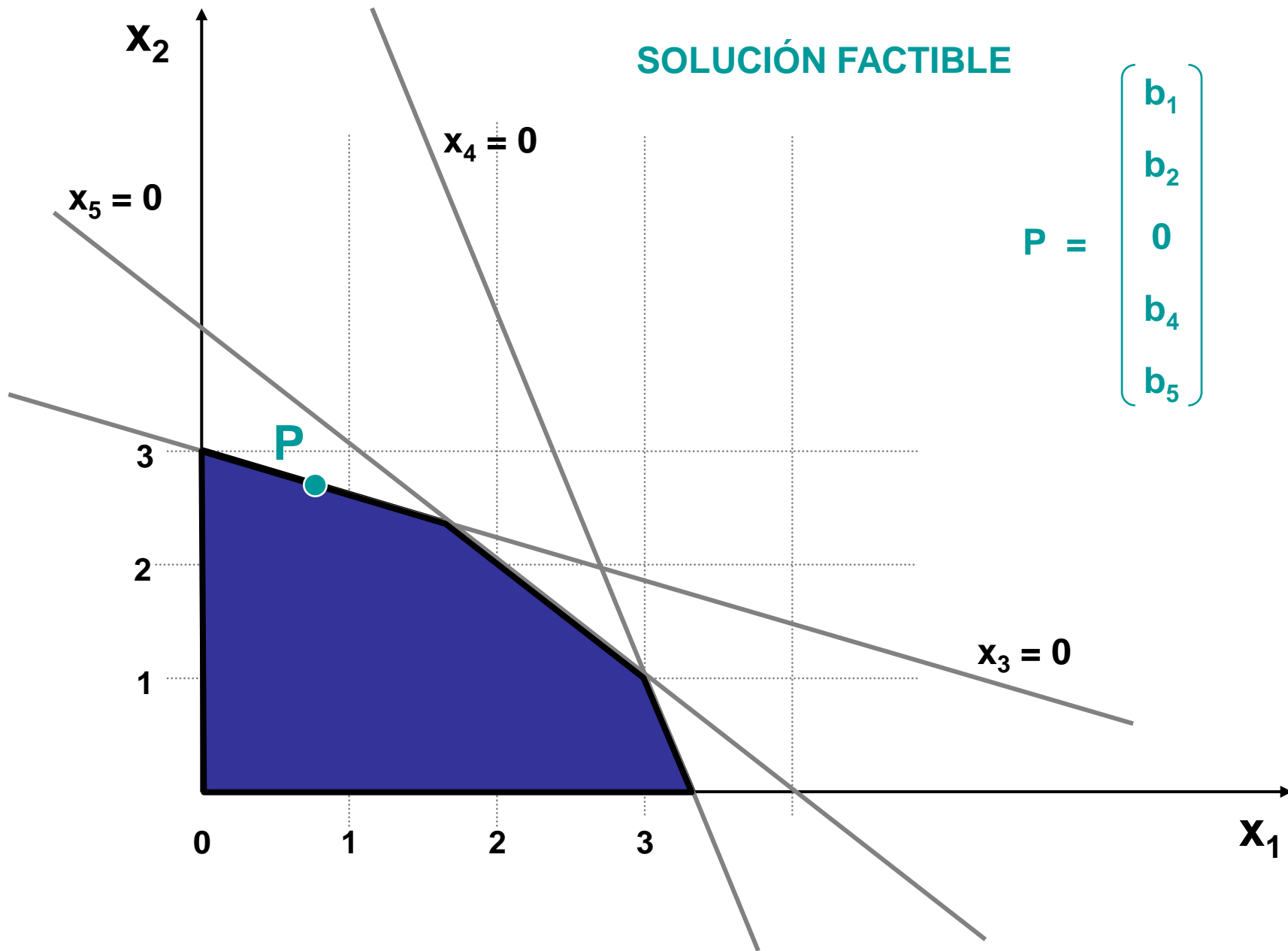
SOLUCIÓN NO FACTIBLE



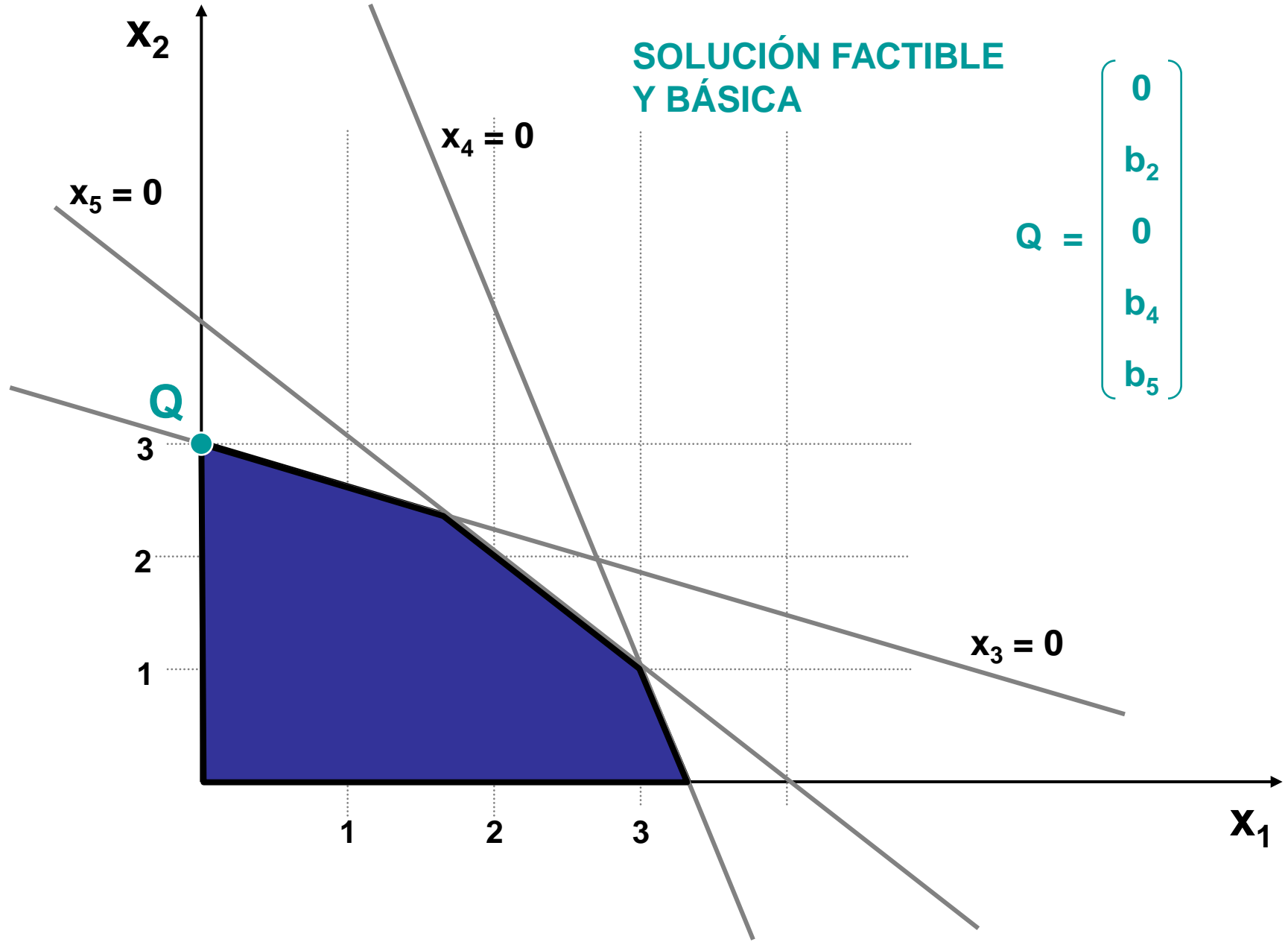
**SOLUCIÓN NO FACTIBLE:
Por lo menos un X_j es Negativo**



SOLUCIÓN FACTIBLE



SOLUCIÓN FACTIBLE Y BÁSICA



$$Q = \begin{pmatrix} 0 \\ b_2 \\ 0 \\ b_4 \\ b_5 \end{pmatrix}$$

x_2

$x_5 = 0$

$x_4 = 0$

Q

3

2

1

1

2

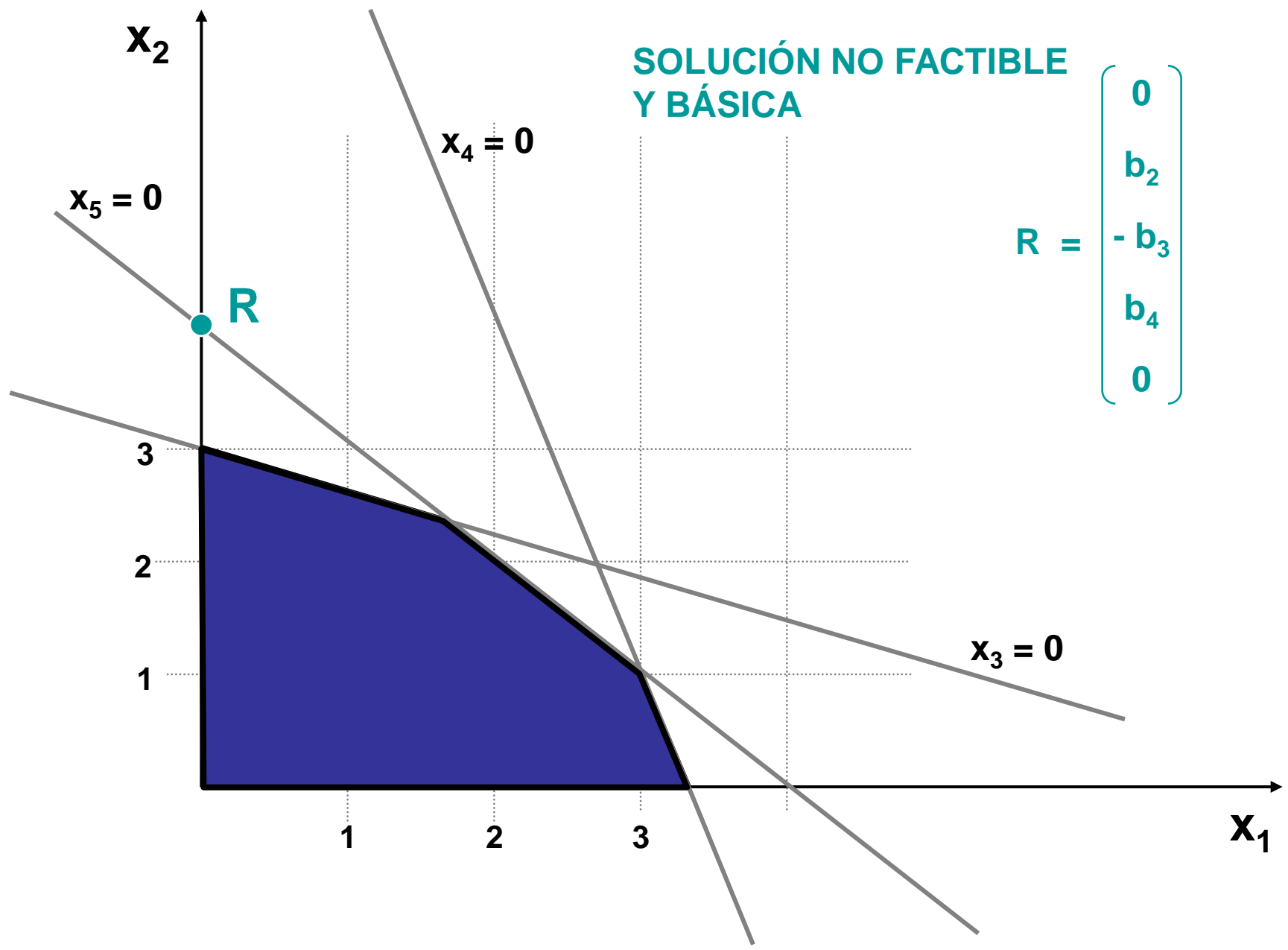
3

$x_3 = 0$

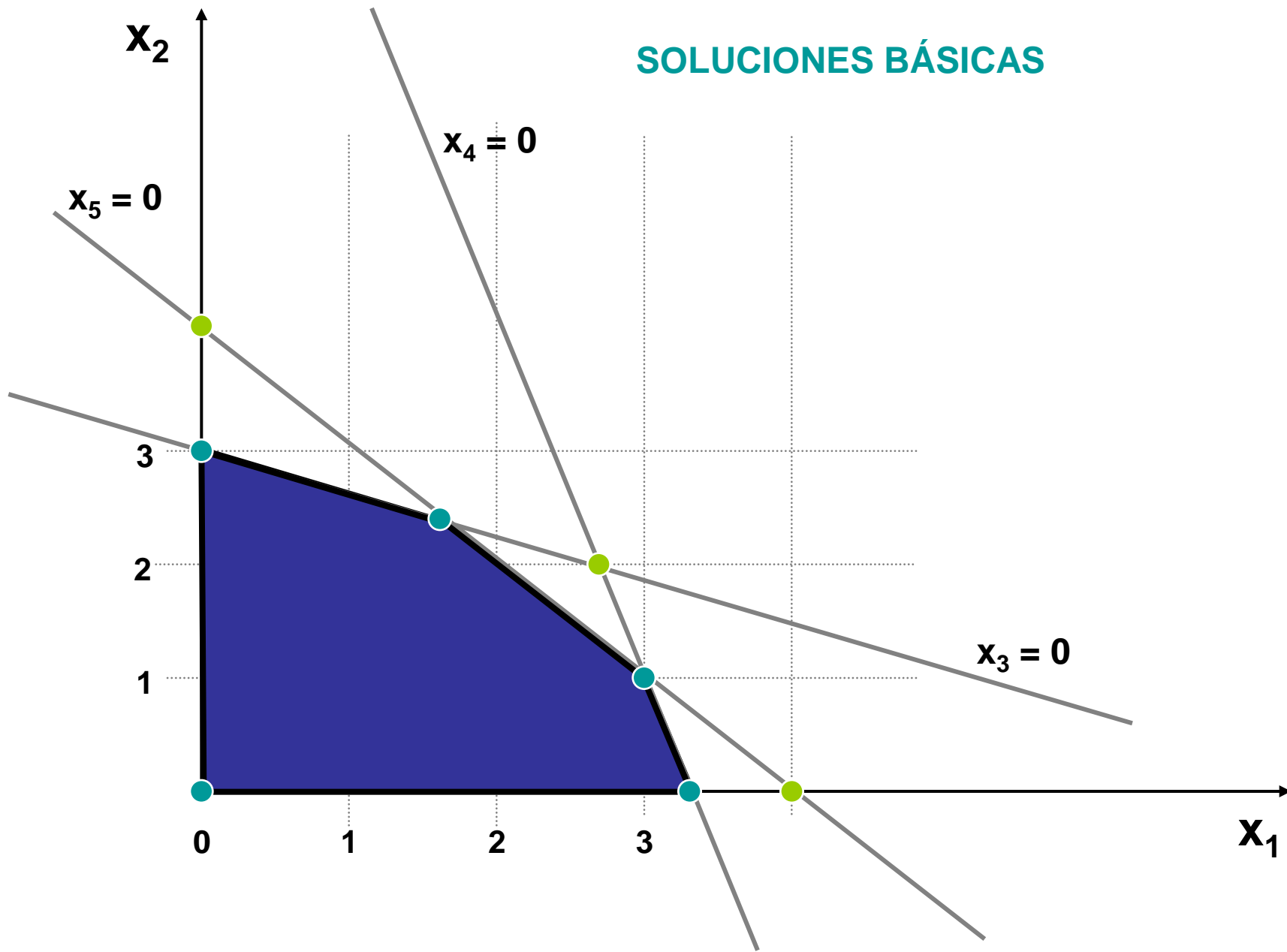
x_1

SOLUCIÓN NO FACTIBLE Y BÁSICA

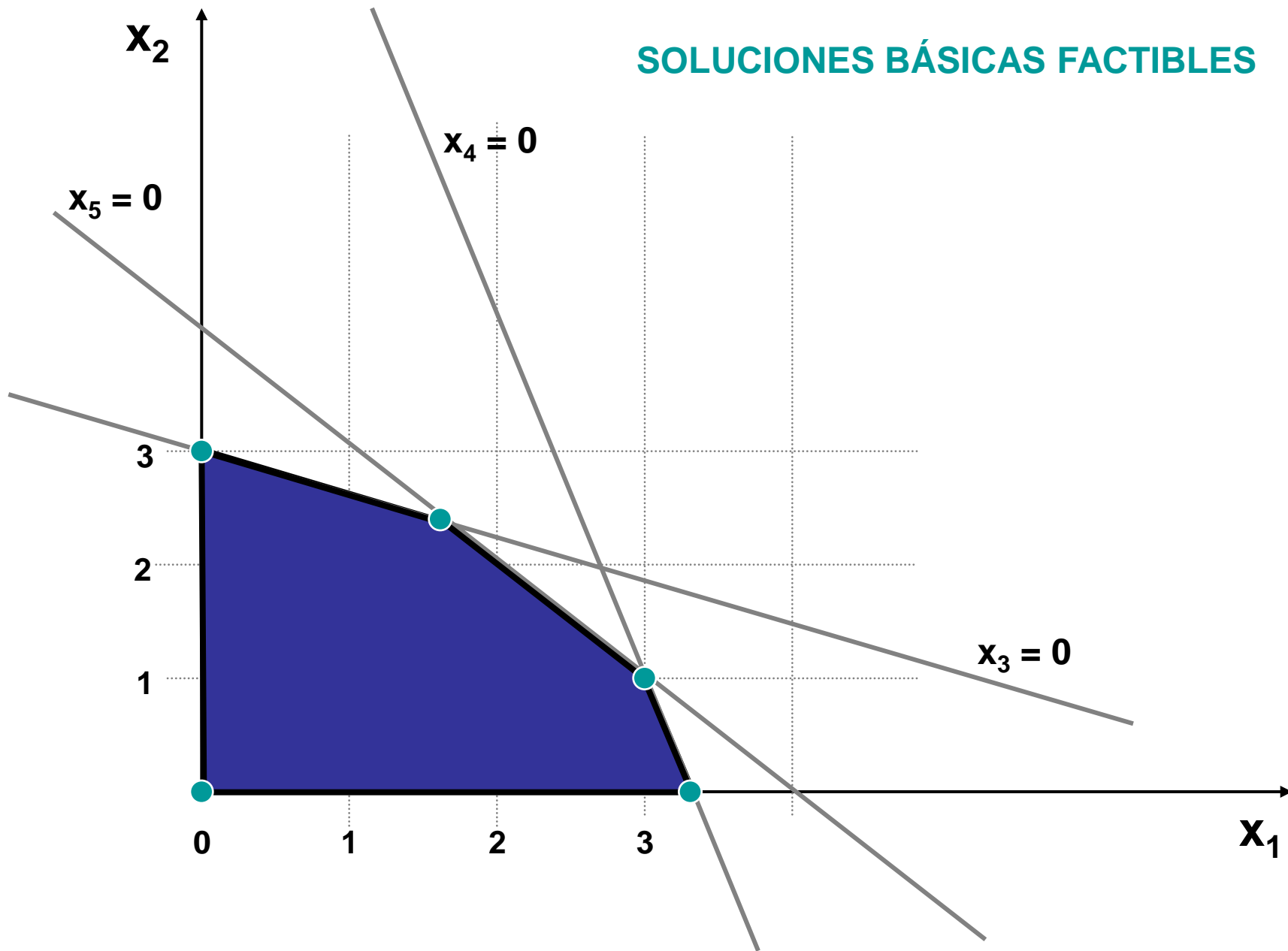
$$R = \begin{pmatrix} 0 \\ b_2 \\ -b_3 \\ b_4 \\ 0 \end{pmatrix}$$

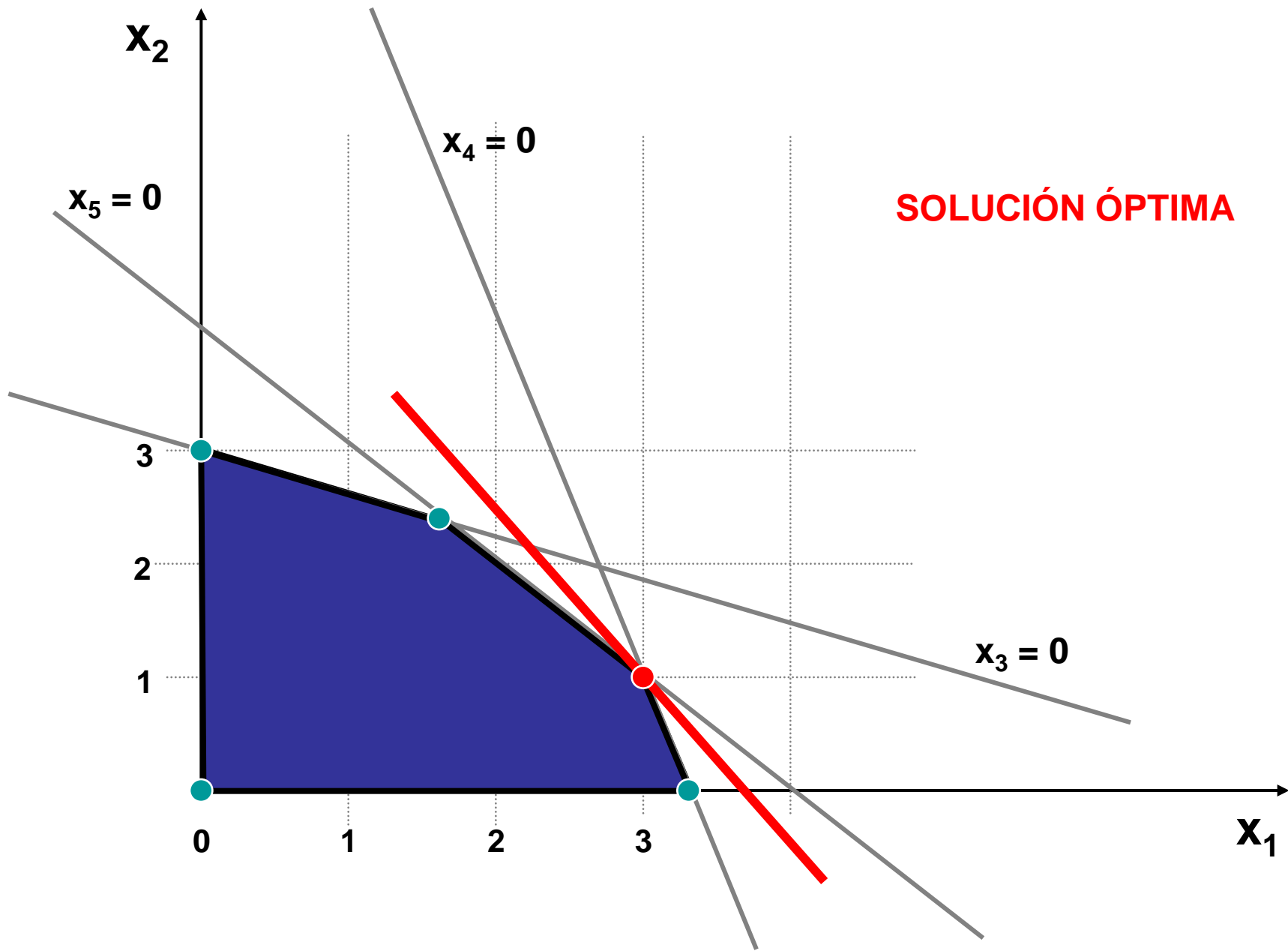


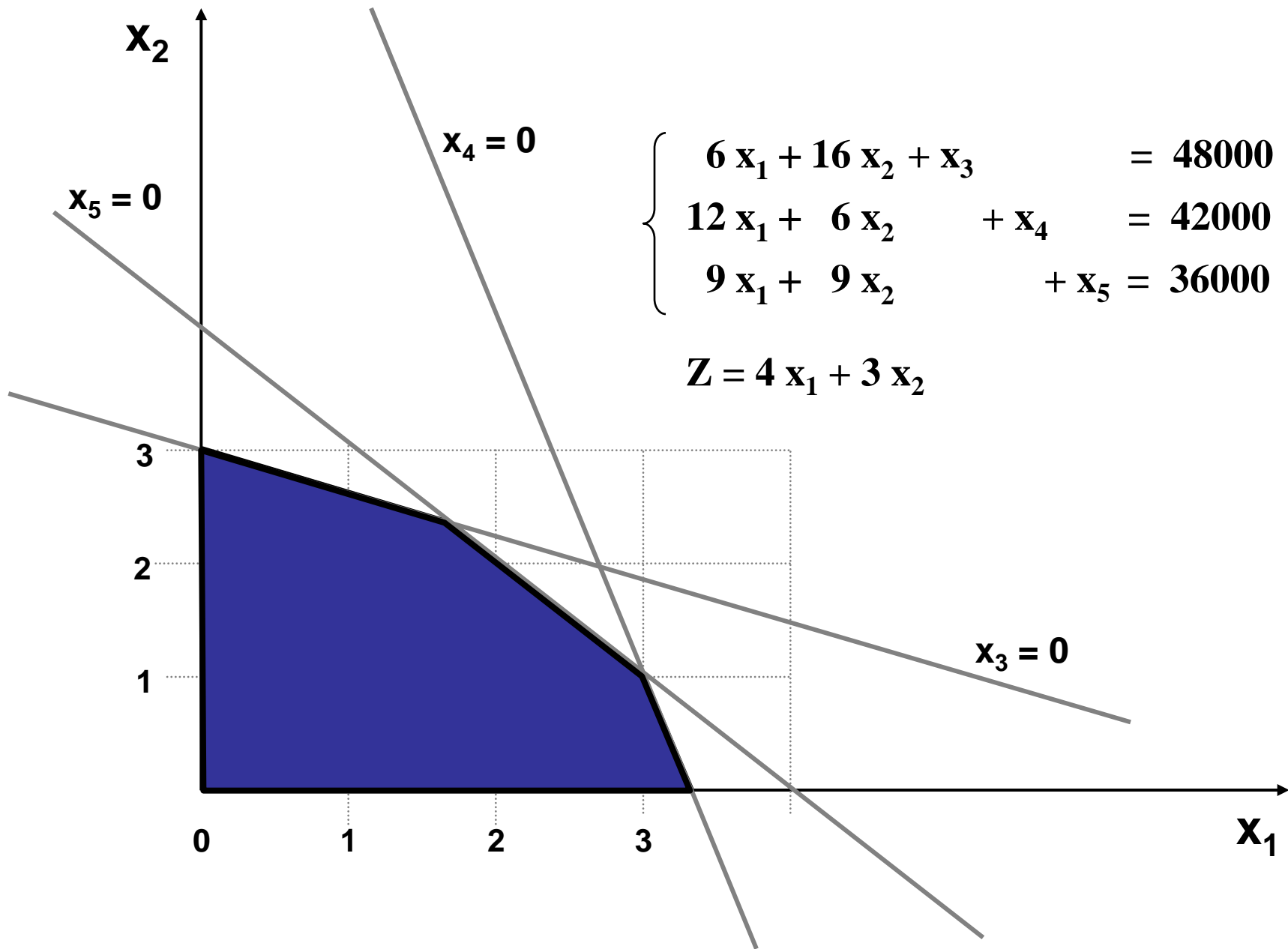
SOLUCIONES BÁSICAS



SOLUCIONES BÁSICAS FACTIBLES







$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48000 \\ 12x_1 + 6x_2 + x_4 = 42000 \\ 9x_1 + 9x_2 + x_5 = 36000 \end{cases}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \quad \mathbf{Z} = 4x_1 + 3x_2$$

{

$$\mathbf{x}_3 = 48000$$

$$\mathbf{x}_4 = 42000$$

$$\mathbf{x}_5 = 36000$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 48000 \\ 42000 \\ 36000 \end{pmatrix} \quad \mathbf{Z} = 0$$

{

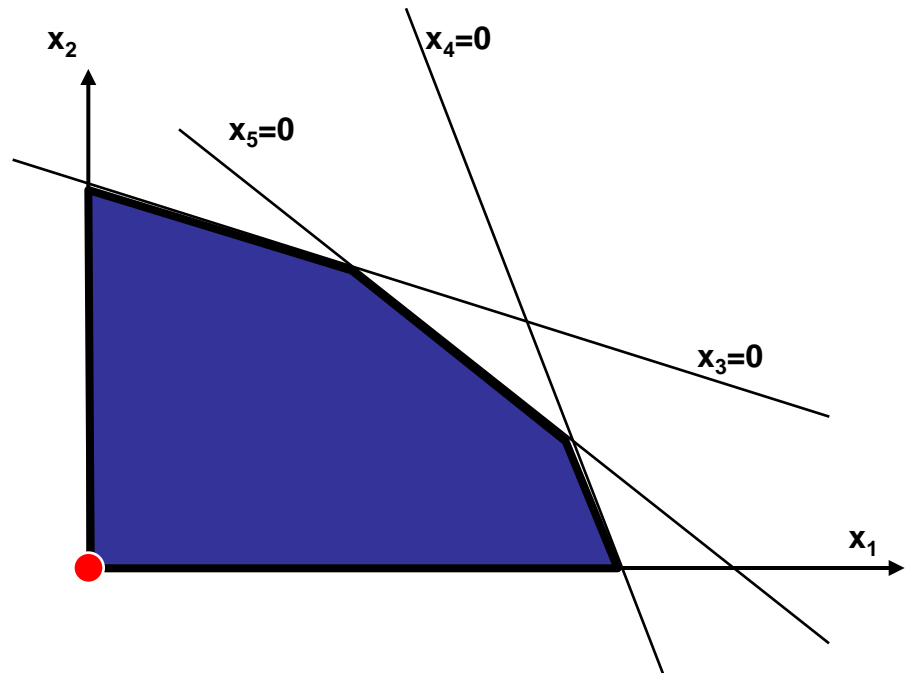
$$x_3 = 48000$$

$$x_4 = 42000$$

$$x_5 = 36000$$

$$X = \begin{pmatrix} 0 \\ 0 \\ 48000 \\ 42000 \\ 36000 \end{pmatrix}$$

$$Z = 0$$



$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48000 \\ 12x_1 + 6x_2 + x_4 = 42000 \\ 9x_1 + 9x_2 + x_5 = 36000 \end{cases}$$

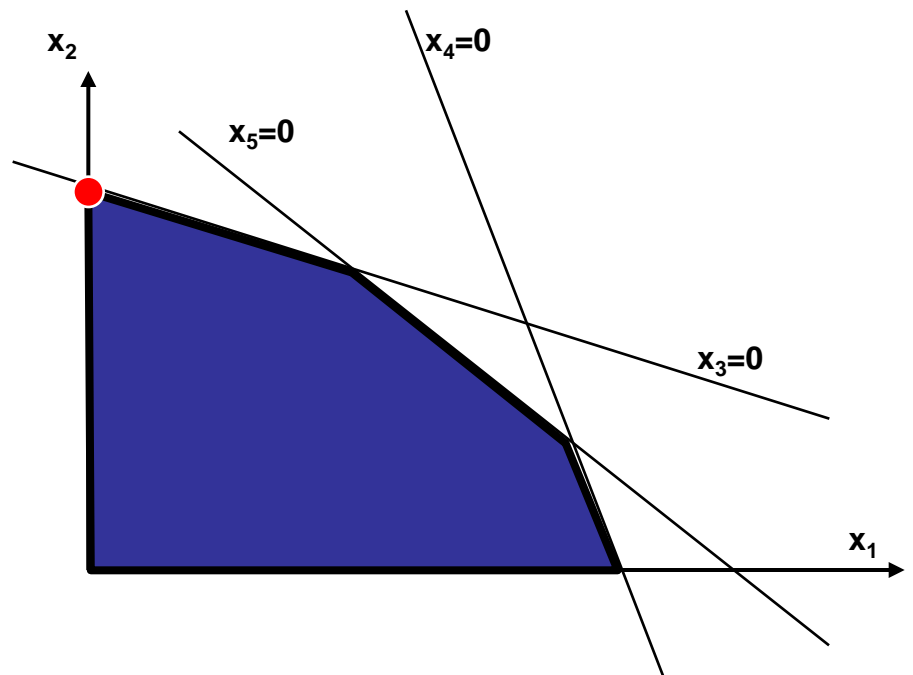
$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \quad \mathbf{Z} = 4x_1 + 3x_2$$

$$\left\{ \begin{array}{rcl} 16 x_2 & & = 48000 \\ 6 x_2 & + x_4 & = 42000 \\ 9 x_2 & & + x_5 = 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 3000 \\ 0 \\ 24000 \\ 9000 \end{pmatrix} \quad \mathbf{Z} = 9000$$

$$\left\{ \begin{array}{rcl} 16 x_2 & & = 48000 \\ 6 x_2 & + x_4 & = 42000 \\ 9 x_2 & & + x_5 = 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 3000 \\ 0 \\ 24000 \\ 9000 \end{pmatrix} \quad \mathbf{Z} = 9000$$



$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48000 \\ 12x_1 + 6x_2 + x_4 = 42000 \\ 9x_1 + 9x_2 + x_5 = 36000 \end{cases}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \quad \mathbf{Z} = 4x_1 + 3x_2$$

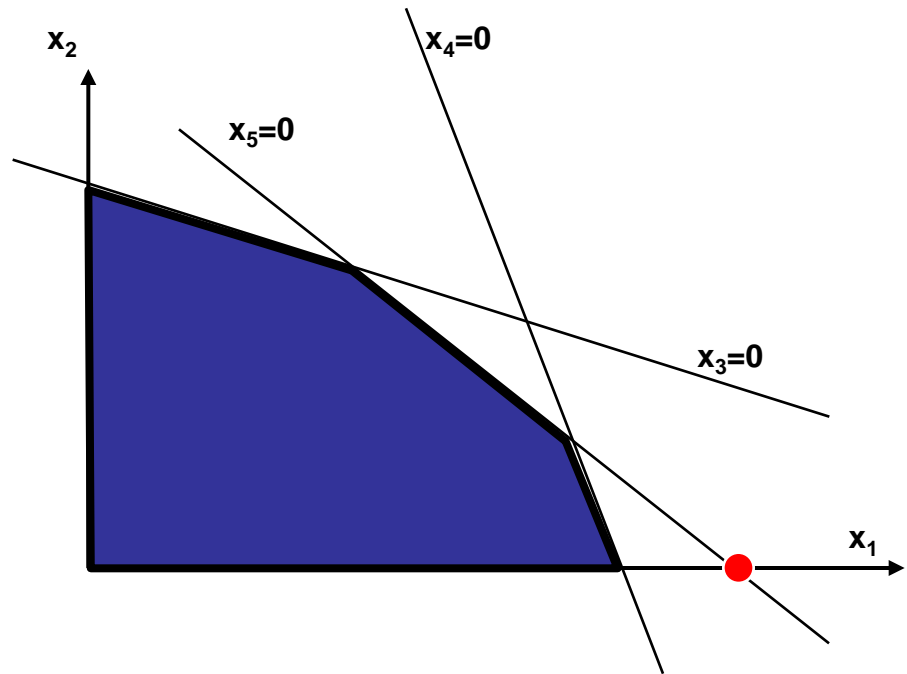
$$\begin{cases} 6x_1 & + x_3 & = 48000 \\ 12x_1 & + x_4 & = 42000 \\ 9x_1 & & = 36000 \end{cases}$$

$$\mathbf{x} = \begin{pmatrix} 4000 \\ 0 \\ 24000 \\ -6000 \\ 0 \end{pmatrix} \quad \text{NO FACTIBLE}$$

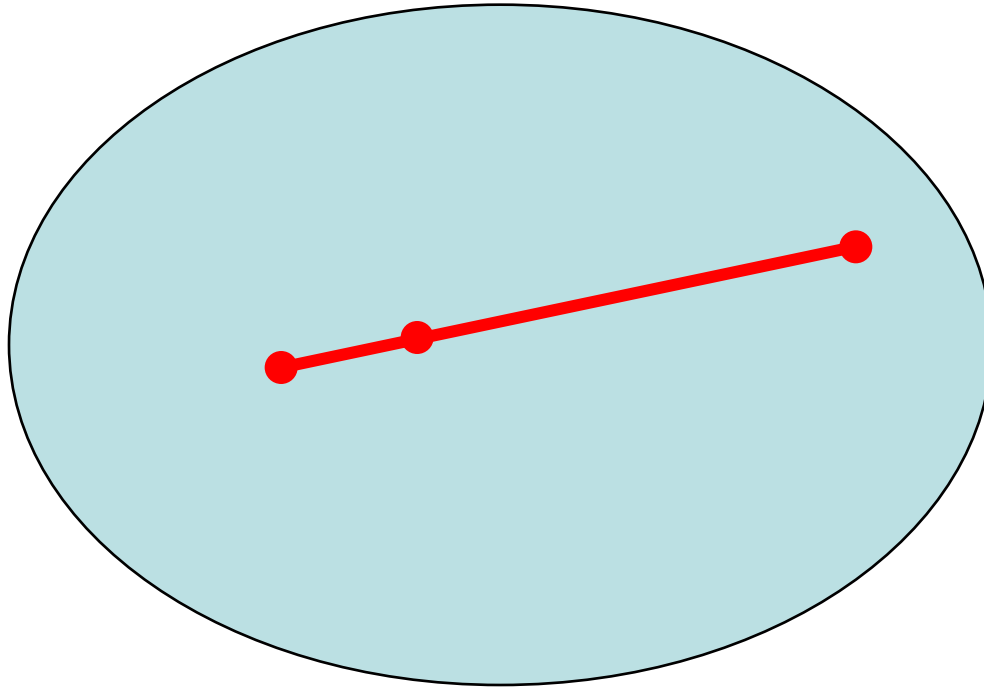
$$\begin{cases} 6x_1 & + x_3 & = 48000 \\ 12x_1 & + x_4 & = 42000 \\ 9x_1 & & = 36000 \end{cases}$$

$$X = \begin{pmatrix} 4000 \\ 0 \\ 24000 \\ -6000 \\ 0 \end{pmatrix}$$

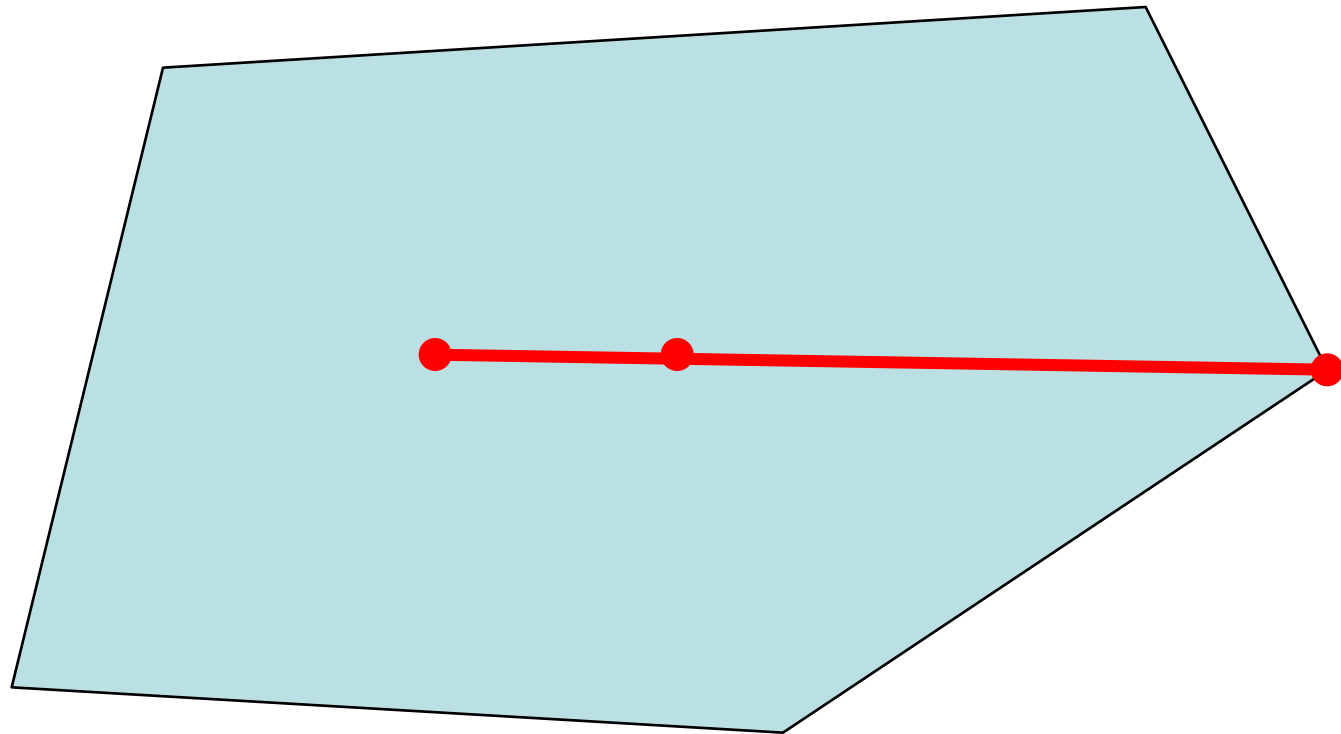
NO FACTIBLE



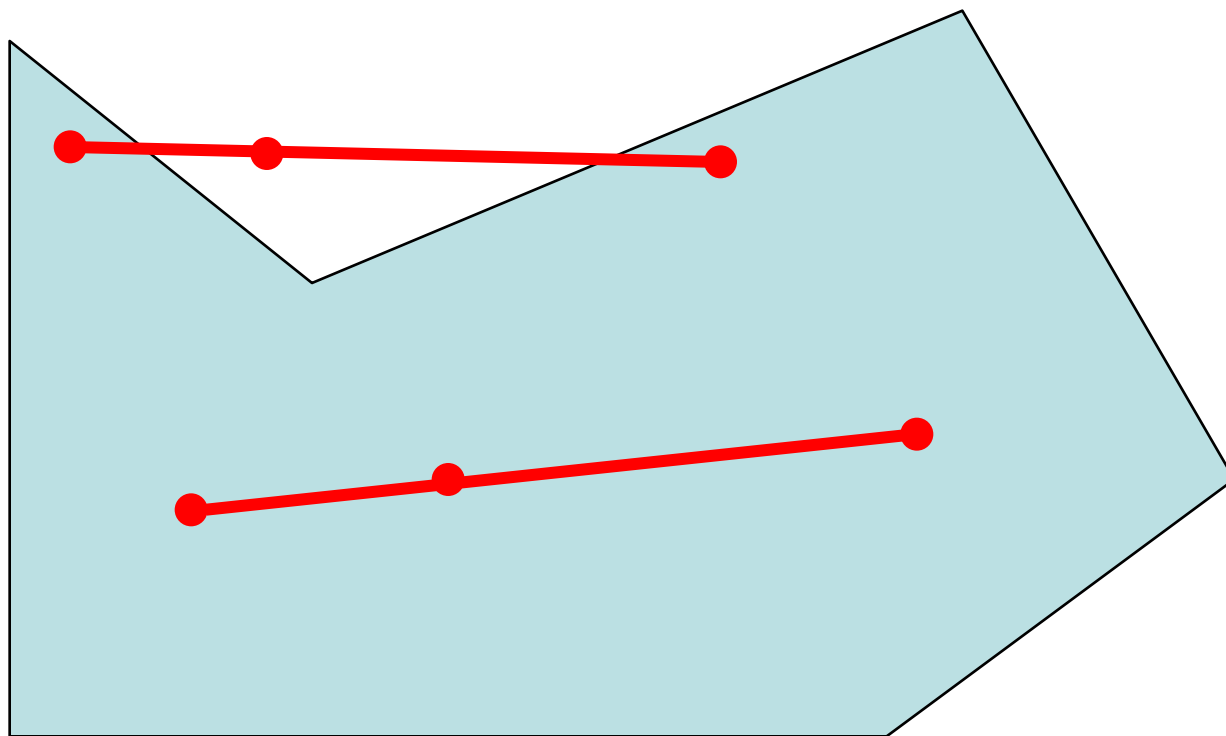
CONJUNTO CONVEXO



CONJUNTO CONVEXO



CONJUNTO NO CONVEXO



CONJUNTO NO CONVEXO

