MODELO TI-TO

TOTAL INMOVILIZADO PROMEDIO -
TOTAL DE ÓRDENES A EMITIR

OPTIMIZACIÓN DE TI SUJETO A TO

$$\begin{cases} TI = \sum_i \frac{1}{2} \cdot q_i \cdot b_i \quad \Rightarrow \quad Min \\ TO = \sum_i \frac{D_i}{q_i} \end{cases}$$

$$L = \sum_i \frac{1}{2} \cdot q_i \cdot b_i + \lambda \cdot \left[\sum_i \frac{D_i}{q_i} - TO \right]$$

$$\frac{\partial L}{\partial q_i} = \frac{1}{2} \cdot b_i - \frac{\lambda \cdot D_i}{q_i^2} = 0 \quad \Rightarrow \quad q_{oi} = \sqrt{\frac{2 \cdot \lambda \cdot D_i}{b_i}}$$

$$\frac{\partial L}{\partial \lambda} = \sum_i \frac{D_i}{q_i} - TO = 0 \quad \Rightarrow \quad TO = \sum_i \frac{D_i}{q_{oi}}$$

$$TO = \sum_i \frac{D_i}{\sqrt{\frac{2 \cdot \lambda \cdot D_i}{b_i}}} = \sum_i \sqrt{\frac{D_i \cdot b_i}{2 \cdot \lambda}} \quad \Rightarrow \quad \sqrt{2 \cdot \lambda} = \frac{\sum_i \sqrt{D_i \cdot b_i}}{TO}$$

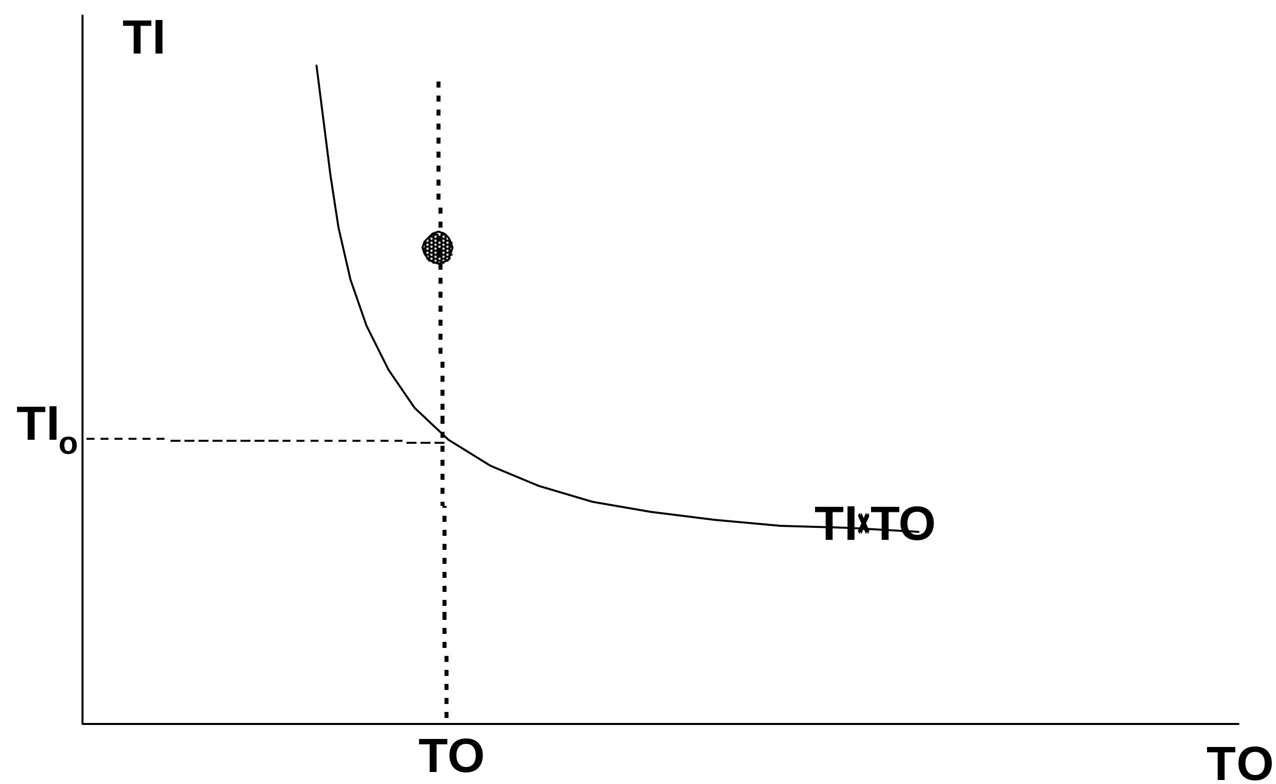
$$\lambda = \frac{\left[\sum_i \sqrt{D_i \cdot b_i} \right]^2}{2 \cdot TO^2}$$

$$TI_o = \sum_i \frac{1}{2} \cdot q_{io} \cdot b_i = \sum_i \frac{1}{2} \cdot \sqrt{\frac{2 \cdot \lambda \cdot D_i}{b_i}} \cdot b_i = \sqrt{2 \cdot \lambda} \cdot \sum_i \frac{\sqrt{D_i \cdot b_i}}{2}$$

$$TI_o = \frac{\sum_i \sqrt{D_i \cdot b_i}}{TO} \cdot \sum_i \frac{\sqrt{D_i \cdot b_i}}{2} \quad \Rightarrow \quad TI_o = \frac{1}{2} \cdot \frac{\left[\sum_i \sqrt{D_i \cdot b_i} \right]^2}{TO}$$

$$TI_o = \lambda \cdot TO \quad \Rightarrow \quad \lambda = \frac{TI_o}{TO}$$

$$TI_o \cdot TO = \frac{1}{2} \cdot \left[\sum_i \sqrt{D_i \cdot b_i} \right]^2$$



OPTIMIZACIÓN DE TO SUJETO A TI

$$\begin{cases} TO = \sum_i \frac{D_i}{q_i}{}_i \quad \Rightarrow \quad Min \\ TI = \sum_i \frac{1}{2} \cdot q_i \cdot b_i \end{cases}$$

$$L = \sum_i \frac{D_i}{q_i} + \mu \cdot \left[\sum_i \frac{1}{2} q_i \cdot b_i - TI \right]$$

$$\frac{\partial L}{\partial q_i} = -\frac{D_i}{q_i^2} + \frac{\mu \cdot b_i}{2} = 0 \quad \Rightarrow \quad q_{oi} = \sqrt{\frac{2 \cdot D_i}{\mu \cdot b_i}}$$

$$\frac{\partial L}{\partial \mu} = \sum_i \frac{1}{2} \cdot q_i \cdot b_i - TI = 0 \quad \Rightarrow \quad TI = \sum_i \frac{1}{2} \cdot q_i \cdot b_i$$

$$TI = \sum_i \frac{1}{2} \cdot \sqrt{\frac{2 \cdot D_i}{\mu \cdot b_i}} \cdot b_i \quad \Rightarrow \quad \sqrt{2 \cdot \mu} = \frac{\sum_i \sqrt{D_i \cdot b_i}}{TI}$$

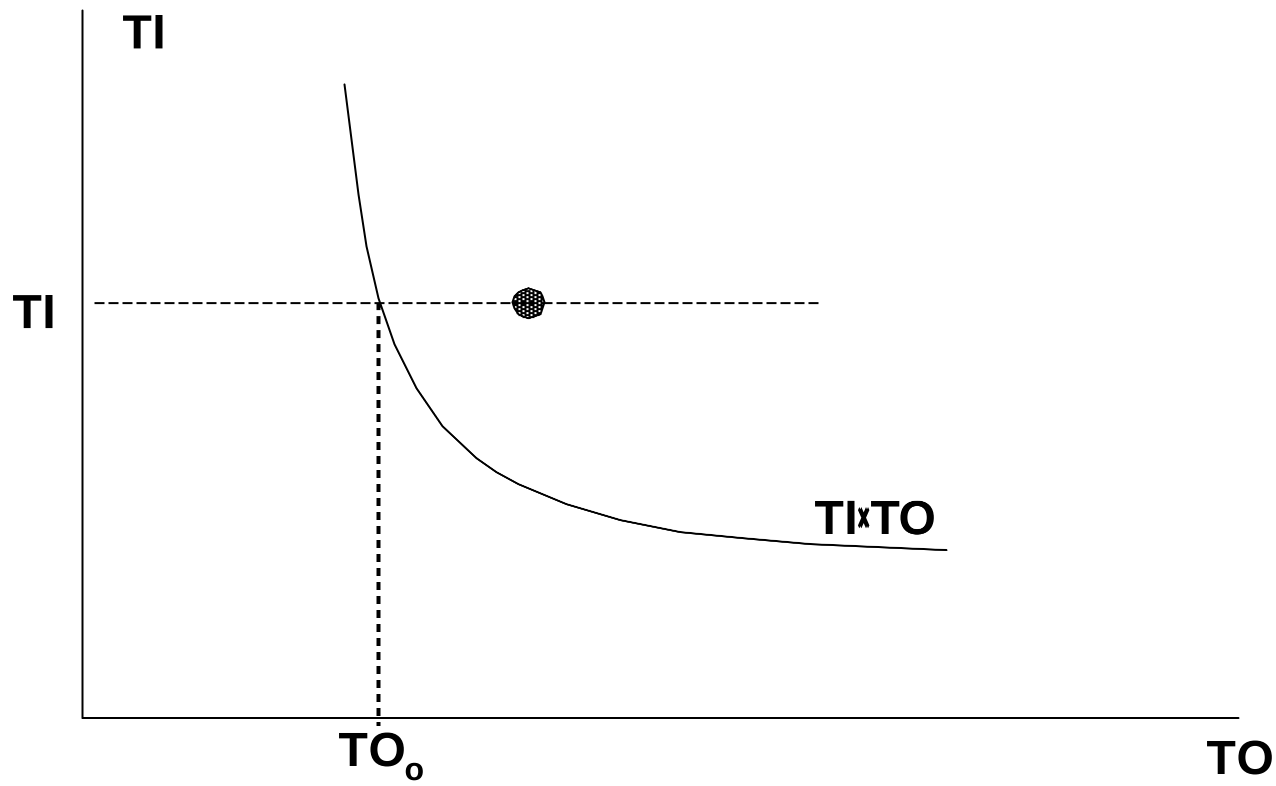
$$\mu = \frac{\left[\sum_i \sqrt{D_i \cdot b_i}\right]^2}{2 \cdot TI^2}$$

$$TO_o = \sum_i \frac{D_i}{q_{oi}} = \sum_i \frac{D_i}{\sqrt{\frac{2 \cdot D_i}{\mu \cdot b_i}}} = \frac{\sqrt{2 \cdot \mu} \cdot \sum_i \sqrt{D_i \cdot b_i}}{2}$$

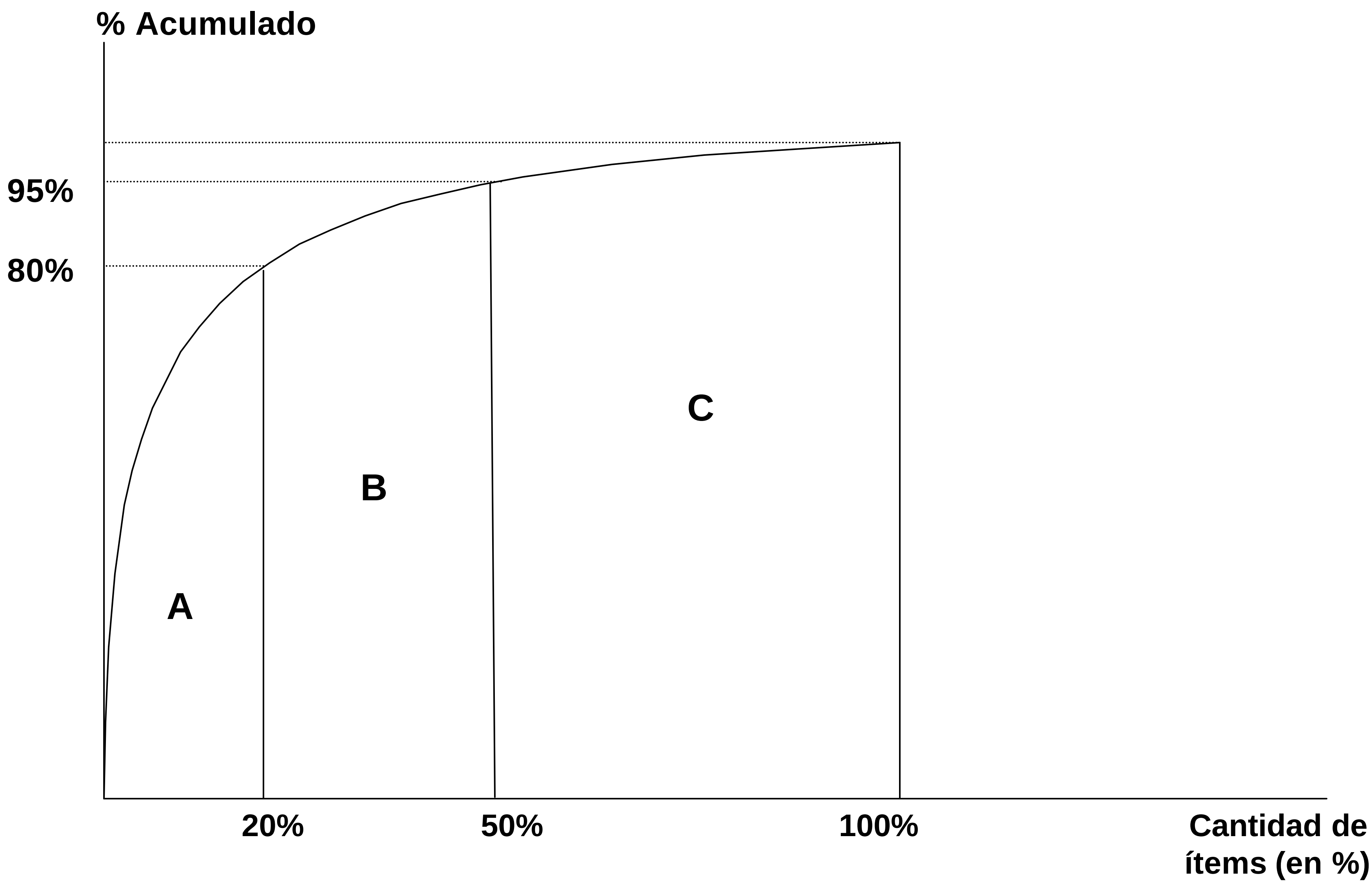
$$TO_o = \frac{\sum_i \sqrt{D_i \cdot b_i}}{TI} \cdot \frac{\sum_i \sqrt{D_i \cdot b_i}}{2} \quad \Rightarrow \quad TO_o = \frac{1}{2} \cdot \frac{\left[\sum_i \sqrt{D_i \cdot b_i}\right]^2}{TI}$$

$$TO_o = \mu \cdot TI \quad \Rightarrow \quad \mu = \frac{TO_o}{TI}$$

$$TI \cdot TO_o = \frac{1}{2} \cdot \left[\sum_i \sqrt{D_i \cdot b_i} \right]^2$$

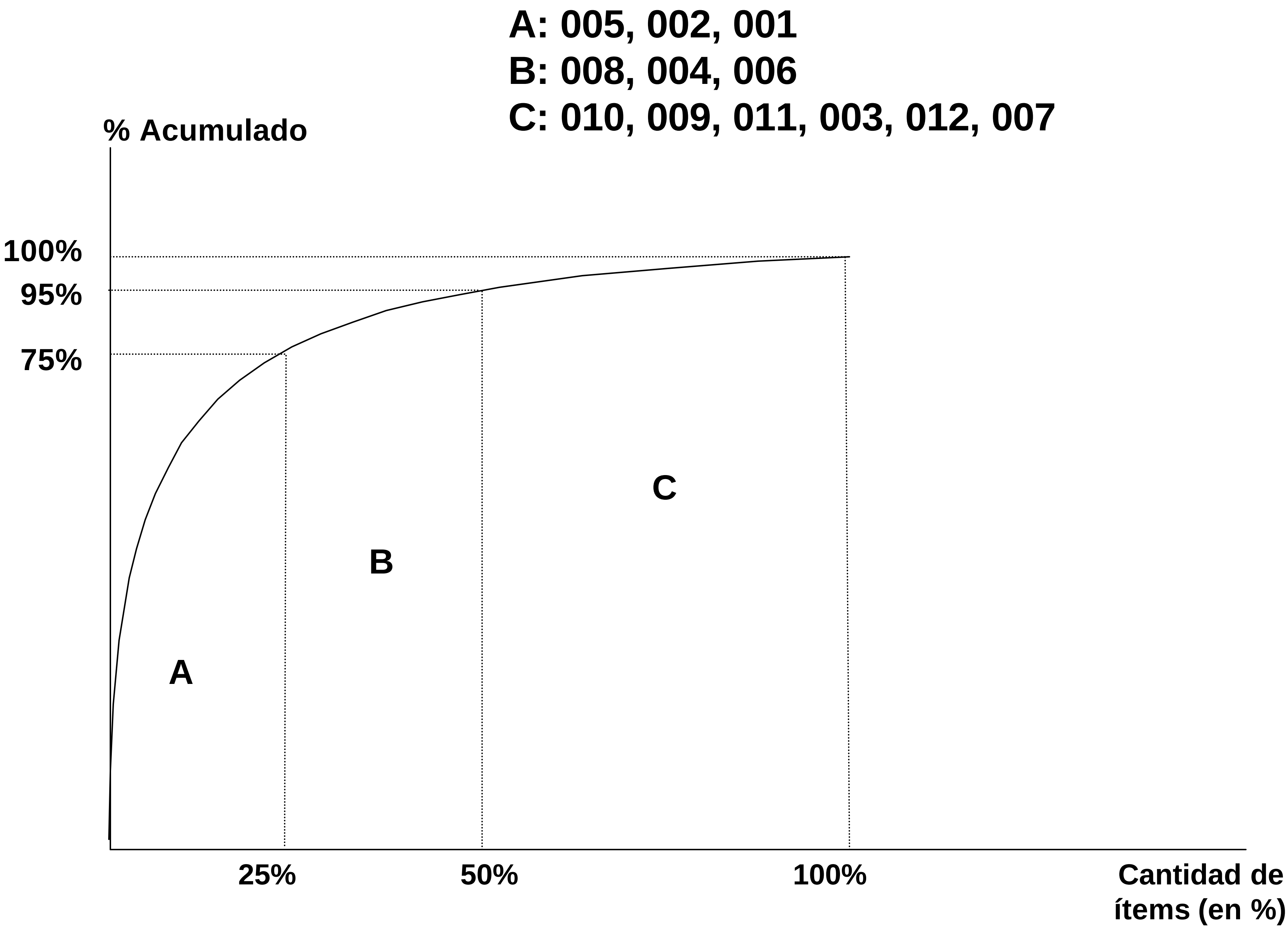


CURVA A – B C



Item	Precio b	Demanda D	Demanda Valorizada b*D	% sobre Demanda Valorizada
001	4	340	1.360	10,77
002	12	204	2.448	19,38
003	1	98	98	0,78
004	7	115	805	6,37
005	9	639	5.751	45,54
006	2	222	444	3,52
007	3	21	63	0,50
008	12	89	1.068	8,46
009	5	34	170	1,35
010	10	25	250	1,98
011	50	2	100	0,79
012	9	8	72	0,57
TOTAL			**12.629**	**100,00**

Item	Precio b	Demanda D	Demanda Valorizada b*D	% sobre Demanda Valorizada	% Demanda Valorizada Acumulado	% de Cantidad de Ítems Acumulado
005	9	639	5.751	45,54	45,54	8,33
002	12	204	2.448	19,38	64,92	16,67
001	4	340	1.360	10,77	75,69	25,00
008	12	89	1.068	8,46	84,15	33,33
004	7	115	805	6,37	90,52	41,67
006	2	222	444	3,52	94,04	50,00
010	10	25	250	1,98	96,02	58,33
009	5	34	170	1,35	97,36	66,67
011	50	2	100	0,79	98,16	75,00
003	1	98	98	0,78	98,93	83,33
012	9	8	72	0,57	99,50	91,67
007	3	21	63	0,50	100,00	100,00


A: 005, 002, 001
B: 008, 004, 006
C: 010, 009, 011, 003, 012, 007
% Acumulado
100%
95%
75%
A
B
C
25%
50%
100%
Cantidad de
ítems (en %)

Considerando los siguientes datos:

	A	***B***
D (u/año)	***1.000***	***900***
c_1 ($/(u.año)	***10***	***12***
k ($/lote)	***200***	***150***

e imponiendo la condición de que se deben emitir anualmente 9 órdenes entre "A" y "B", hallar los lotes óptimos de cada artículo.

$$q_{oA} = \sqrt{\frac{2 \cdot k_A \cdot D_A}{T \cdot c_{1A}}} = \sqrt{\frac{2 \cdot 200 \cdot 1.000}{1 \cdot 10}} = 200$$

$$q_{oB} = \sqrt{\frac{2 \cdot k_B \cdot D_B}{T \cdot c_{1B}}} = \sqrt{\frac{2 \cdot 150 \cdot 900}{1 \cdot 12}} = 150$$

$$n = \frac{D_A}{q_A} + \frac{D_B}{q_B} = \frac{1000}{200} + \frac{900}{150} = 11 \text{ órdenes}$$

No cumple

$$q_A = \sqrt{\frac{2 \cdot k_A \cdot D_A + 2 \cdot \lambda \cdot D_A}{T \cdot c_{1A}}}$$

$$q_B = \sqrt{\frac{2 \cdot k_B \cdot D_B + 2 \cdot \lambda \cdot D_B}{T \cdot c_{1B}}}$$

$$\frac{D_A}{q_A} + \frac{D_B}{q_B} = TO$$

$$q_A^* = 238{,}32$$

$$q_B^* = 187{,}34$$

$$\lambda = 83{,}99$$

$$n = \frac{1000}{238{,}32} + \frac{900}{187{,}34} = 9 \text{ órdenes}$$

$$CTE = \frac{1}{2} \cdot q_A \cdot c_{1A} \cdot T + k_A \cdot \frac{D_A}{q_A} + \frac{1}{2} \cdot q_B \cdot c_{1B} \cdot T + k_B \cdot \frac{D_B}{q_B} = 3.875{,}47 \frac{\$}{\text{año}}$$

```
MIN = CTE;
CTE = CTEa + CTEb;
CTEa = 0.5*qa*c1a+ ka*Da/qa;
na = Da / qa;
CTEb = 0.5*qb*c1b+ kb*Db/qb;
nb * qb = Db;
Da = 1000;
c1a = 10;
ka = 200;
Db = 900;
c1b = 12;
kb = 150;
na + nb = n;
n = 9 ;
END
```

Local optimal solution found at step: 5

Objective value: 3875.474

Variable	Value	Reduced Cost
CTE	3875.474	0.0000000
CTEA	2030.810	0.0000000
CTEB	1844.664	0.0000000
BA	0.0000000	0.0000000
DA	1000.000	0.0000000
QA	238.3216	-0.1562145E-07
C1A	10.00000	0.0000000
KA	200.0000	0.0000000
NA	4.196011	0.0000000
BB	0.0000000	0.0000000
DB	900.0000	0.0000000
QB	187.3443	0.0000000
C1B	12.00000	0.0000000
KB	150.0000	0.0000000
NB	4.803989	0.0000000
N	9.000000	0.0000000

Row	Slack or Surplus	Dual Price
1	3875.474	-1.000000
2	0.0000000	-1.000000
3	0.0000000	-1.000000
4	0.0000000	-83.98594
5	0.0000000	0.0000000
6	0.0000000	-1.000000
7	0.0000000	-0.4482973
8	0.0000000	0.0000000
9	0.0000000	-1000.000
10	0.0000000	-1.191608
11	0.0000000	-119.1608
12	0.0000000	-4.196011
13	0.0000000	-900.0000
14	0.0000000	-1.248962
15	0.0000000	-93.67215
16	0.0000000	-4.803989
17	0.0000000	83.98594
18	0.0000000	83.98594

Una pequeña compañía adquiere tres tipos de subcomponentes. La administración estableció que la inversión máxima en estos ítems no debe exceder $ 15.000.

Suponiendo que la tasa de inmovilización es del 20%, calcular los lotes óptimos de compra.

ITEM	***1***	***2***	***3***
Demandas anuales	*1.000*	*1.000*	*2.000*
Costo de orden de fabricación ($)	*50*	*20*	*80*
Costo unitarios de compra	*50*	*40*	*50*

$$CTE = b_1 \cdot D_1 + \frac{1}{2} \cdot c_{11} \cdot q_1 \cdot T + k_1 \cdot \frac{D_1}{q_1} + b_2 \cdot D_2 + \frac{1}{2} \cdot c_{12} \cdot q_2 \cdot T +$$

$$k_2 \cdot \frac{D_2}{q_2} + b_3 \cdot D_3 + \frac{1}{2} \cdot c_{13} \cdot q_3 \cdot T + k_3 \cdot \frac{D_3}{q_3} \quad \Rightarrow \quad \text{Min}$$

Sujeto a :

$$b_1 \cdot q_1 + b_2 \cdot q_2 + b_3 \cdot q_3 \leq 15.000$$

$$L = b_1 \cdot D_1 + \frac{1}{2} \cdot c_{11} \cdot q_1 \cdot T + k_1 \cdot \frac{D_1}{q_1} + b_2 \cdot D_2 + \frac{1}{2} \cdot c_{12} \cdot q_2 \cdot T + k_2 \cdot \frac{D_2}{q_2} + b_3 \cdot D_3 + \frac{1}{2} \cdot c_{13} \cdot q_3 \cdot T$$

$$+ k_3 \cdot \frac{D_3}{q_3} + \lambda \cdot \left(b_1 \cdot q_1 + b_2 \cdot q_2 + b_3 \cdot q_3 - 15.000 \right) \Rightarrow \quad \text{Min}$$

$$q^*_{o1} = \sqrt{\frac{2 \cdot k_1 \cdot D_1}{T \cdot b_1 \cdot i + 2 \cdot \lambda \cdot b_1}} = \sqrt{\frac{2 \cdot 50 \cdot 1.000}{1 \cdot 50 \cdot 0{,}20 + 2 \cdot \lambda \cdot 50}}$$

$$q^*_{o2} = \sqrt{\frac{2 \cdot k_2 \cdot D_2}{T \cdot b_2 \cdot i + 2 \cdot \lambda \cdot b_2}} = \sqrt{\frac{2 \cdot 20 \cdot 1.000}{1 \cdot 40 \cdot 0{,}20 + 2 \cdot \lambda \cdot 40}}$$

$$q^*_{o3} = \sqrt{\frac{2 \cdot k_3 \cdot D_3}{T \cdot b_3 \cdot i + 2 \cdot \lambda \cdot b_3}} = \sqrt{\frac{2 \cdot 80 \cdot 2.000}{1 \cdot 50 \cdot 0{,}20 + 2 \cdot \lambda \cdot 50}}$$

$q^*_{o1} = 89{,}43$ $\quad$ $q^*_{o2} = 63{,}24$ $\quad$ $q^*_{o3} = 159{,}98$ $\quad$ $\lambda = 0{,}025033$

```
MIN = CTE1 + CTE2  + CTE3  ;
CTE1 = b1 *D1 + 0.5*q1 *c11 + k1 *D1 /q1 ;
n1 = D1 / q1 ;
CTE2  = b2  *D2  + 0.5*q2  *c12  + k2  *D2  /q2  ;
n2  = D2  / q2  ;
CTE3  = b3  *D3  + 0.5*q3  *c13  + k3  *D3  /q3  ;
n3  = D3  / q3  ;
i = 0.20;
b1 = 50;
D1 = 1000;
c11 = i * b1;
k1 = 50;
b2  = 40;
D2  = 1000;
k2  = 20;
c12 = i * b2;
b3  = 50;
D3  = 2000;
k3  = 80;
c13 = i*b3;
IT = q1 * b1 + q2 * b2 + q3 * b3 ;
IT < 15000;
END
```

Local optimal solution found at step: 91
Objective value: 193375.5

Variable	Value	Reduced Cost
CTE1	51006.25	0.0000000
CTE2	40569.22	0.0000000
CTE3	101800.0	0.0000000
B1	50.00000	0.0000000
D1	1000.000	0.0000000
Q1	89.43104	0.0000000
C11	10.00000	0.0000000
K1	50.00000	0.0000000
N1	11.18180	0.0000000
B2	40.00000	0.0000000
D2	1000.000	0.0000000
Q2	63.23730	0.2117961E-06
C12	8.000000	0.0000000
K2	20.00000	0.0000000
N2	15.81345	0.0000000
B3	50.00000	0.0000000
D3	2000.000	0.0000000
Q3	159.9791	-0.1927393E-06
C13	10.00000	0.0000000
K3	80.00000	0.0000000
N3	12.50163	0.0000000
I	0.2000000	0.0000000
IT	15000.00	0.0000000

Row	Slack or Surplus	Dual Price
1	193375.5	-1.000000
2	0.0000000	-1.000000
3	0.0000000	0.0000000
4	0.0000000	-1.000000
5	0.0000000	0.0000000
6	0.0000000	-1.000000
7	0.0000000	0.0000000
8	0.0000000	-7500.000
9	0.0000000	-1011.182
10	0.0000000	-50.55909
11	0.0000000	-44.71552
12	0.0000000	-11.18180
13	0.0000000	-1007.907
14	0.0000000	-40.31627
15	0.0000000	-15.81345
16	0.0000000	-31.61865
17	0.0000000	-2020.003
18	0.0000000	-50.50007
19	0.0000000	-12.50163
20	0.0000000	-79.98956
21	0.0000000	-0.2503263E-01
22	0.0000000	0.2503263E-01