P/P/M/N

- Arribos Po
- Servicio Po
- Régimen permanente
- M canales
- 1 cola
- FIFO
- Capacidad finita (N clientes)
- Sin impaciencia
- Población infinita

Parámetros

- λ

$$T_a = \frac{1}{\lambda}$$

- μ

$$T_s = \frac{1}{\mu}$$

- M

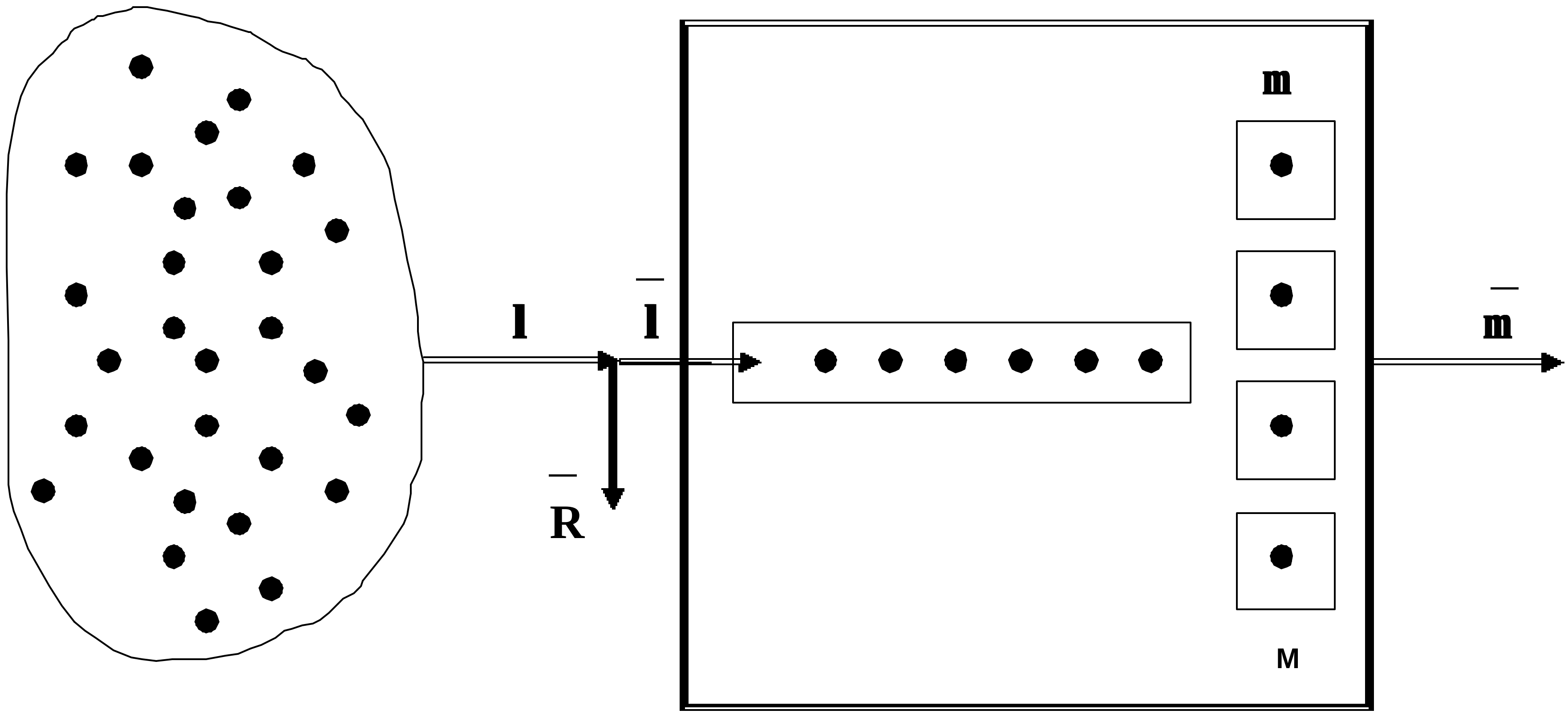
- N

Variables

- p(n)
- L_c
- L
- H
- W_c
- W

$\bar{\lambda}$

$\bar{\mu}$


l
l̄
R̄
m
m̄
M

$$\lambda_n = \begin{cases} \lambda & \text{para } n < N \\ 0 & \text{para } n = N \end{cases}$$

$$\mu_n = \begin{cases} 0 & \text{para } n = 0 \\ n \cdot \mu & \text{para } 1 \leq n < M \\ M \cdot \mu & \text{para } M \leq n \leq N \end{cases}$$

$$p(n) = \frac{\lambda_{n-1}}{\mu_n} \cdot p(n-1)$$

Para n = 1:

$$p(1) = \frac{\lambda_0}{\mu_1} \cdot p(0) = \frac{\lambda}{\mu} \cdot p(0) \quad \Rightarrow \quad p(1) = \rho \cdot p(0)$$

Para n = 2:

$$p(2) = \frac{\lambda_1}{\mu_2} \cdot p(1) = \frac{\lambda}{2\mu} \cdot p(0) \cdot \frac{\lambda}{\mu} \quad \Rightarrow \quad p(2) = \frac{\rho^2}{2} \cdot p(0)$$

Para n = 3:

$$p(3) = \frac{\lambda_2}{\mu_3} \cdot p(2) = \frac{\lambda}{3\mu} \cdot p(0) \cdot \frac{\lambda^2}{2\mu^2} \quad \Rightarrow \quad p(3) = \frac{\rho^3}{6} \cdot p(0)$$

Para n = 4:

$$p(4) = \frac{\lambda_3}{\mu_4} \cdot p(3) = \frac{\lambda}{4\mu} \cdot p(0) \cdot \frac{\lambda^3}{6\mu^3} \quad \Rightarrow \quad p(4) = \frac{\rho^4}{24} \cdot p(0)$$

………..

Para n < M

$$p(n) = \frac{\rho^n}{n!} \cdot p(0)$$

Para n = M:

$$p(M) = \frac{\rho^M}{M!} \cdot p(0)$$

Para n = M+1:

$$p(M+1) = \frac{\rho^{M+1}}{M! \cdot M} \cdot p(0)$$

Para n = M+2:

$$p(M+2) = \frac{\rho^{M+2}}{M! \cdot M^2} \cdot p(0)$$

Para n = M+3:

$$p(M+3) = \frac{\rho^{M+3}}{M! \cdot M^3} \cdot p(0)$$

……….

Para M n N

$$p(n) = \frac{\rho^n}{M! \cdot M^{(n-M)}} \cdot p(0)$$

$$p(n) = \left(\frac{\rho}{M}\right)^n \cdot \frac{M^M}{M!} \cdot p(0)$$

$$\sum_{0}^{N} p(n) = 1$$

$$\sum_{0}^{M-1} p(n) + \sum_{M}^{N} p(n) = 1$$

$$\sum_{0}^{M-1} \frac{\rho^{n}}{n!} \cdot p(0) + \sum_{M}^{N} \left(\frac{\rho}{M} \right)^{n} \cdot \frac{M^{M}}{M!} \cdot p(0) = 1$$

$$p(0) = \frac{1}{\sum_{0}^{M-1} \frac{\rho^{n}}{n!} + \frac{M^{M}}{M!} \cdot \sum_{M}^{N} \left(\frac{\rho}{M} \right)^{n}}$$

$$p(0)=\frac{1}{\sum_{0}^{M-1}\frac{\rho^{n}}{n!}+\frac{\rho^{M}}{(M-1)!}\cdot\left[1-\left(\frac{\rho}{M}\right)^{N-M+1}\right]\cdot\frac{1}{(M-\rho)}}$$

$$L=\sum_{0}^{N}n\cdot p(n)=\sum_{0}^{M-1}n\cdot\frac{\rho^{n}}{n!}\cdot p(0)+\sum_{M}^{N}n\cdot\frac{M^{M}}{M!}\cdot\left(\frac{\rho}{M}\right)^{n}\cdot p(0)$$

$$L=\frac{\frac{\rho}{M}\cdot\rho^{M}\cdot p(0)}{M!\cdot\left(1-\frac{\rho}{M}\right)^{2}}\cdot\left[1-\left(\frac{\rho}{M}\right)^{N-M}-(N-M)\cdot\left(\frac{\rho}{M}\right)^{N-M}\cdot\left(1-\frac{\rho}{M}\right)\right]+\rho\cdot\left[1-\left(\frac{\rho}{M}\right)^{N}\cdot\frac{M^{M}}{M!}\cdot p(0)\right]$$

$$H=\sum_{0}^{M-1}n\cdot p(n)+\sum_{M}^{N}M\cdot p(n)$$

$$H=\rho\cdot\left[1-\left(\frac{\rho}{M}\right)^{N}\cdot\frac{M^{M}}{M!}\cdot p(0)\right]$$

$$L_C = \sum_{M}^{N} (n - M) \cdot p(n)$$

$$L_C = \frac{\frac{\rho}{M} \cdot \rho^M}{M! \cdot \left(1 - \frac{\rho}{M}\right)^2} \cdot \left[1 - \left(\frac{\rho}{M}\right)^{N-M} - (N - M) \cdot \left(\frac{\rho}{M}\right)^{N-M} \cdot \left(1 - \frac{\rho}{M}\right)\right] \cdot p(0)$$

$$\overline{\lambda} = \sum_{0}^{N} \lambda_n \cdot p(n) \qquad \overline{\lambda} = \lambda \cdot [1 - p(N)]$$

$$\overline{\mu} = \sum_{0}^{N} \mu_n \cdot p(n) \qquad \overline{\mu} = \mu \cdot H$$

$$\overline{\lambda} = \overline{\mu}$$

$$\lambda \cdot [1 - p(N)] = \mu \cdot H$$

$$H = \rho \cdot [1 - p(N)]$$

$$PA = \frac{H}{M} = \frac{\rho \cdot (1 - p(n))}{M}$$

$$\overline{R} = \lambda - \overline{\lambda}$$

$$\overline{R} = \lambda \cdot p(N)$$

$$W_C = \frac{L_C}{\overline{\lambda}}$$

$$W = W_C + T_S$$

Ejemplos de funcionales

$$Z = c_e \cdot L + c_C \cdot M \quad \rightarrow \quad \text{Min}$$

$$\begin{cases} Z = c_L \cdot (N - M) + u \cdot \overline{R} \quad \rightarrow \quad \text{Min} \\ Z = u \cdot \overline{\mu} - c_L \cdot (N - M) \quad \rightarrow \quad \text{Max} \end{cases}$$

$$Z = c_s \cdot M \cdot \mu + u \cdot \overline{R} \quad \rightarrow \quad \text{Min}$$

A un sistema P/P/2/4, sin impaciencia, arriba un cliente cada 5 minutos, en promedio. La duración media del servicio es de 10 minutos. Determinar:

Probabilidad de que un cliente que arriba al sistema no tenga que esperar para recibir el servicio.

Porcentaje de ocupación del canal.

Probabilidad de que un cliente que arriba al sistema no pueda ingresar.

Tasa promedio de clientes que se retiran sin ser atendidos.

Número promedio de clientes esperando ser atendidos.

Número promedio de clientes en el sistema.

Ingreso de caja esperado, si cada servicio se cobra $50.

Tiempo promedio de permanencia de un cliente dentro del sistema.

Lucro cesante esperado.

$$\lambda = \frac{1}{5}\frac{\text{cl}}{\text{min}} \cdot 60\frac{\text{min}}{\text{h}} = 12\ \frac{\text{cl}}{\text{h}}$$

$$\mu = \frac{1}{10}\frac{\text{cl}}{\text{min}} \cdot 60\frac{\text{min}}{\text{h}} = 6\ \frac{\text{cl}}{\text{h}}$$

$$\rho = \frac{\lambda}{\mu} = \frac{12}{6} = 2$$

$\mathbf{l}_n = \mathbf{l}$ para n = 0, 1, 2 y 3

$\mathbf{l}_n = 0$ para n = 4

$\mathbf{m}_n = 0$ para n = 0

$\mathbf{m}_n = \mathbf{m}$ para n = 1

$\mathbf{m}_n = 2\,\mathbf{m}$ para n = 2, 3 y 4

$$p(1) = \frac{\lambda}{\mu} \cdot p(0) = \rho \cdot p(0)$$

p(1) = 0,2222

$$p(2) = \frac{\lambda}{2 \cdot \mu} \cdot p(1) = \frac{\rho}{2} \cdot \rho \cdot p(0) = \frac{\rho^2}{2} \cdot p(0)$$

p(2) = 0,2222

$$p(3) = \frac{\lambda}{2 \cdot \mu} \cdot p(2) = \frac{\rho}{2} \cdot \frac{\rho^2}{2} \cdot p(0) = \frac{\rho^3}{4} \cdot p(0)$$

p(3) = 0,2222

$$p(4) = \frac{\lambda}{2 \cdot \mu} \cdot p(3) = \frac{\rho}{2} \cdot \frac{\rho^3}{4} \cdot p(0) = \frac{\rho^4}{8} \cdot p(0)$$

p(4) = 0,2222

$$p(0) + \rho \cdot p(0) + \frac{\rho^2}{2} \cdot p(0) + \frac{\rho^3}{4} \cdot p(0) + \frac{\rho^4}{8} \cdot p(0) = 1$$

$$p(0) = \frac{1}{1 + \rho + \frac{\rho^2}{2} + \frac{\rho^3}{4} + \frac{\rho^4}{8}} = \frac{1}{1 + 2 + \frac{2^2}{2} + \frac{2^3}{4} + \frac{2^4}{8}} = \frac{1}{9} = 0{,}1111$$