P/P/M

- Arribos Po
- Servicio Po
- Régimen permanente
- M canales
- 1 cola
- FIFO
- Capacidad infinita
- Sin impaciencia
- Población infinita

Parámetros

- λ

$$T_a = \frac{1}{\lambda}$$

- μ

$$T_s = \frac{1}{\mu}$$

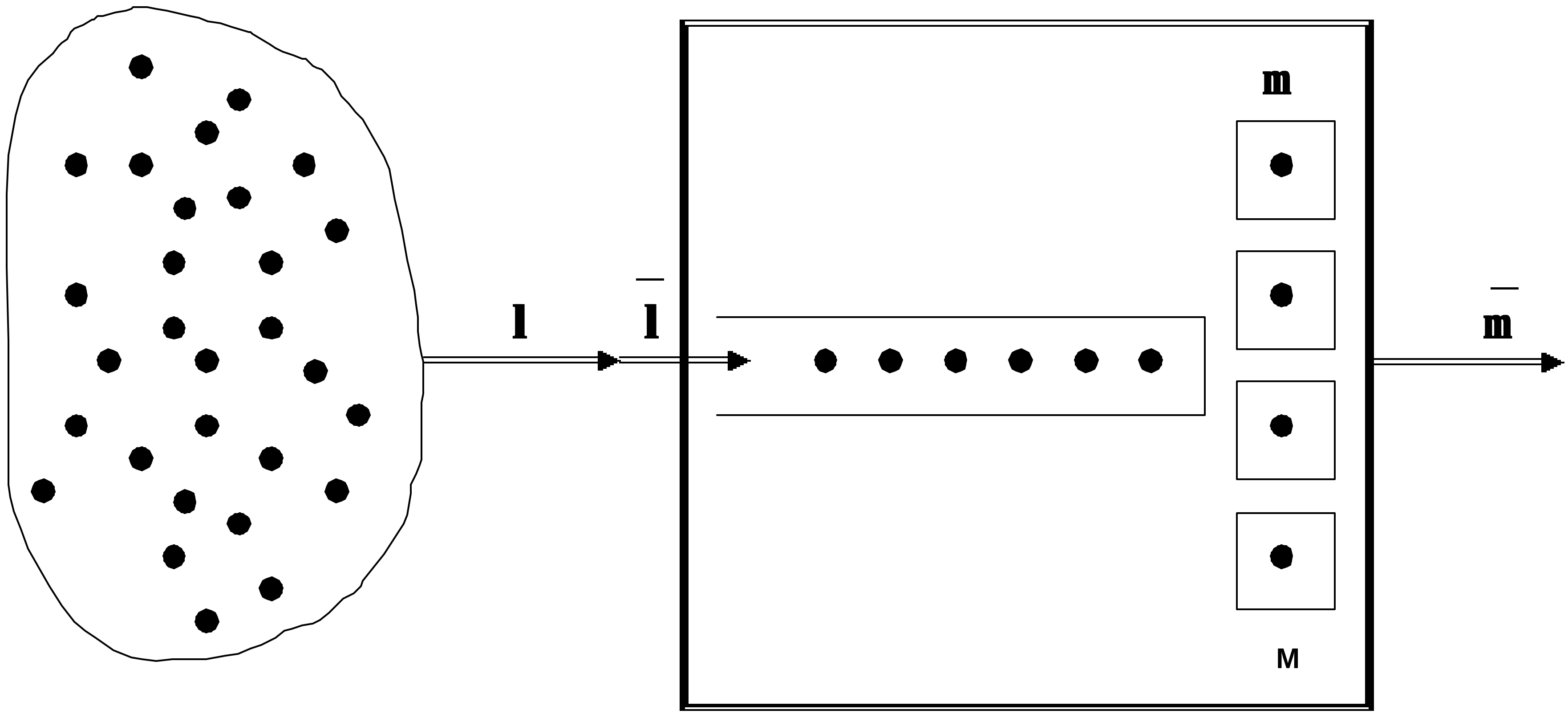
- M

Variables

- p(n)
- L_c
- L
- H
- W_c
- W
- $\varphi(W_c)$
- $\varphi(W)$

$\bar{\lambda}$

$\bar{\mu}$



l
l̄
m
m̄
M

$$\lambda_n = \lambda \qquad \text{para todo n.}$$

$$\lambda_n = \lambda \cdot p(i/n)$$

$$\mu_n = \begin{cases} 0 & \text{para } n = 0 \\ n \cdot \mu & \text{para } 1 \le n < M \\ M \cdot \mu & \text{para } n \ge M \end{cases}$$

$$\boxed{p(n) = \frac{\lambda_{n-1}}{\mu_n} \cdot p(n-1)}$$

$$p(1) = \frac{\lambda_0}{\mu_1} \cdot p(0) = \frac{\lambda}{\mu} \cdot p(0) \quad \Rightarrow \quad p(1) = \rho \cdot p(0)$$

$$p(2) = \frac{\lambda_1}{\mu_2} \cdot p(1) = \frac{\lambda}{2\mu} \cdot p(0) \cdot \frac{\lambda}{\mu} \quad \Rightarrow \quad p(2) = \frac{\rho^2}{2} \cdot p(0)$$

$$p(3) = \frac{\lambda_2}{\mu_3} \cdot p(2) = \frac{\lambda}{3\mu} \cdot p(0) \cdot \frac{\lambda^2}{2\mu^2} \quad \Rightarrow \quad p(3) = \frac{\rho^3}{6} \cdot p(0)$$

$$p(4) = \frac{\lambda_3}{\mu_4} \cdot p(3) = \frac{\lambda}{4\mu} \cdot p(0) \cdot \frac{\lambda^3}{6\mu^3} \quad \Rightarrow \quad p(4) = \frac{\rho^4}{24} \cdot p(0)$$

$$\boxed{p(n) = \frac{\rho^n}{n!} \cdot p(0)} \qquad \text{para } n < M$$

$$\lambda_n = \lambda \qquad \text{para todo n.}$$

$$\mu_n = \begin{cases} 0 & \text{para } n = 0 \\ n \cdot \mu & \text{para } 1 \le n < M \\ M \cdot \mu & \text{para } n \ge M \end{cases}$$

$$\boxed{p(n) = \frac{\lambda_{n-1}}{\mu_n} \cdot p(n-1)}$$

$$p(M) = \frac{\rho^M}{M!} \cdot p(0)$$

$$p(M+1) = \frac{\rho^{M+1}}{M! \cdot M} \cdot p(0)$$

$$p(M+2) = \frac{\rho^{M+2}}{M! \cdot M^2} \cdot p(0)$$

$$p(M+3) = \frac{\rho^{M+3}}{M! \cdot M^3} \cdot p(0)$$

$$p(n) = \frac{\rho^n}{M! \cdot M^{(n-M)}} \cdot p(0) \qquad \boxed{p(n) = \left(\frac{\rho}{M}\right)^n \frac{M^M}{M!} \cdot p(0)} \qquad \text{para } n \ge M$$

$$\sum_{0}^{\infty} p(n) = 1$$

$$\sum_{0}^{M-1} p(n) + \sum_{M}^{\infty} p(n) = 1$$

$$\sum_{0}^{M-1} \frac{\rho^n}{n!} \cdot p(0) + \sum_{M}^{\infty} \left(\frac{\rho}{M}\right)^n \cdot \frac{M^M}{M!} \cdot p(0) = 1$$

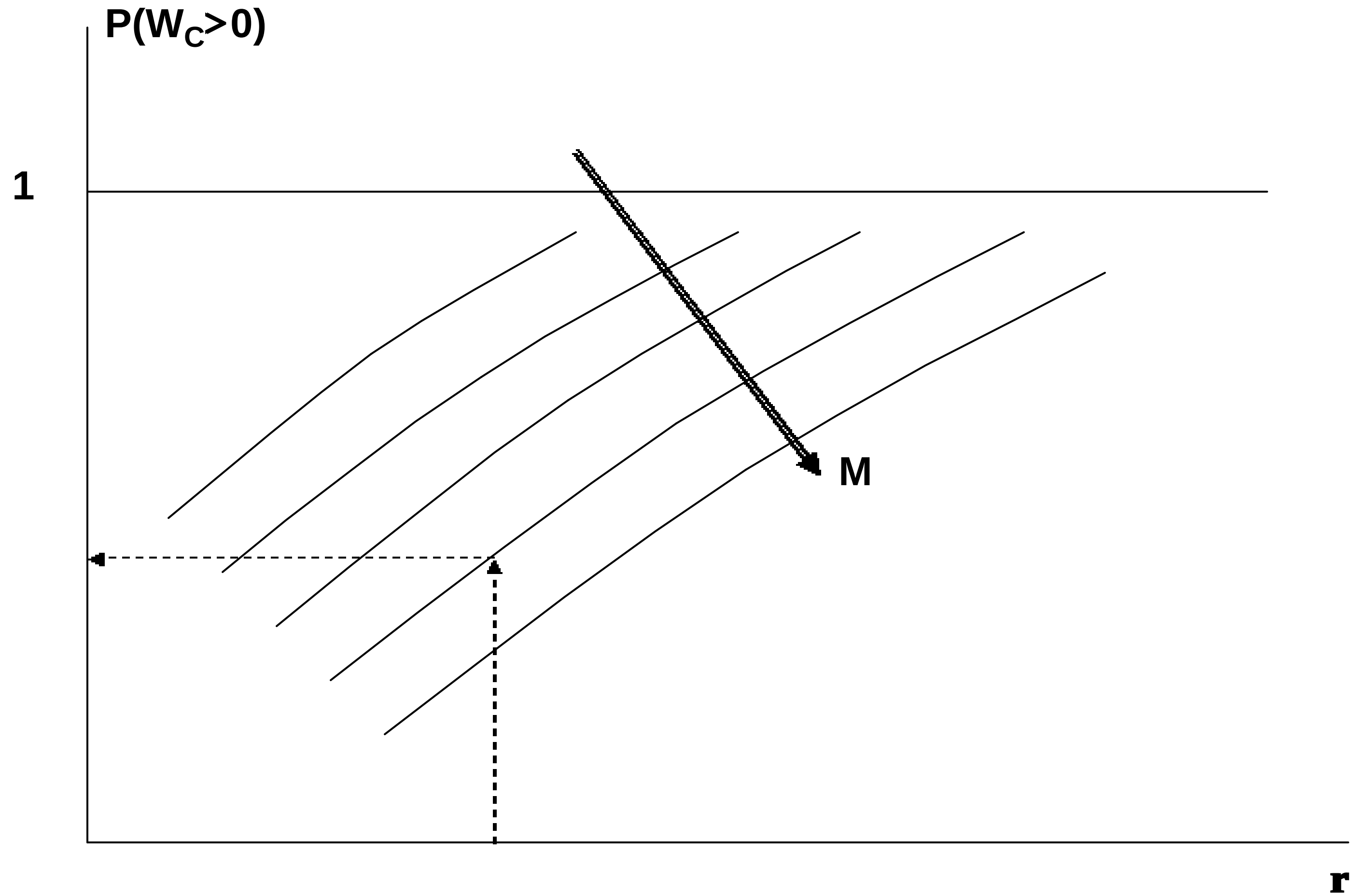
$$p(0) = \frac{1}{\sum_{0}^{M-1} \frac{\rho^n}{n!} + \frac{M^M}{M!} \cdot \sum_{M}^{\infty} \left(\frac{\rho}{M}\right)^n}$$

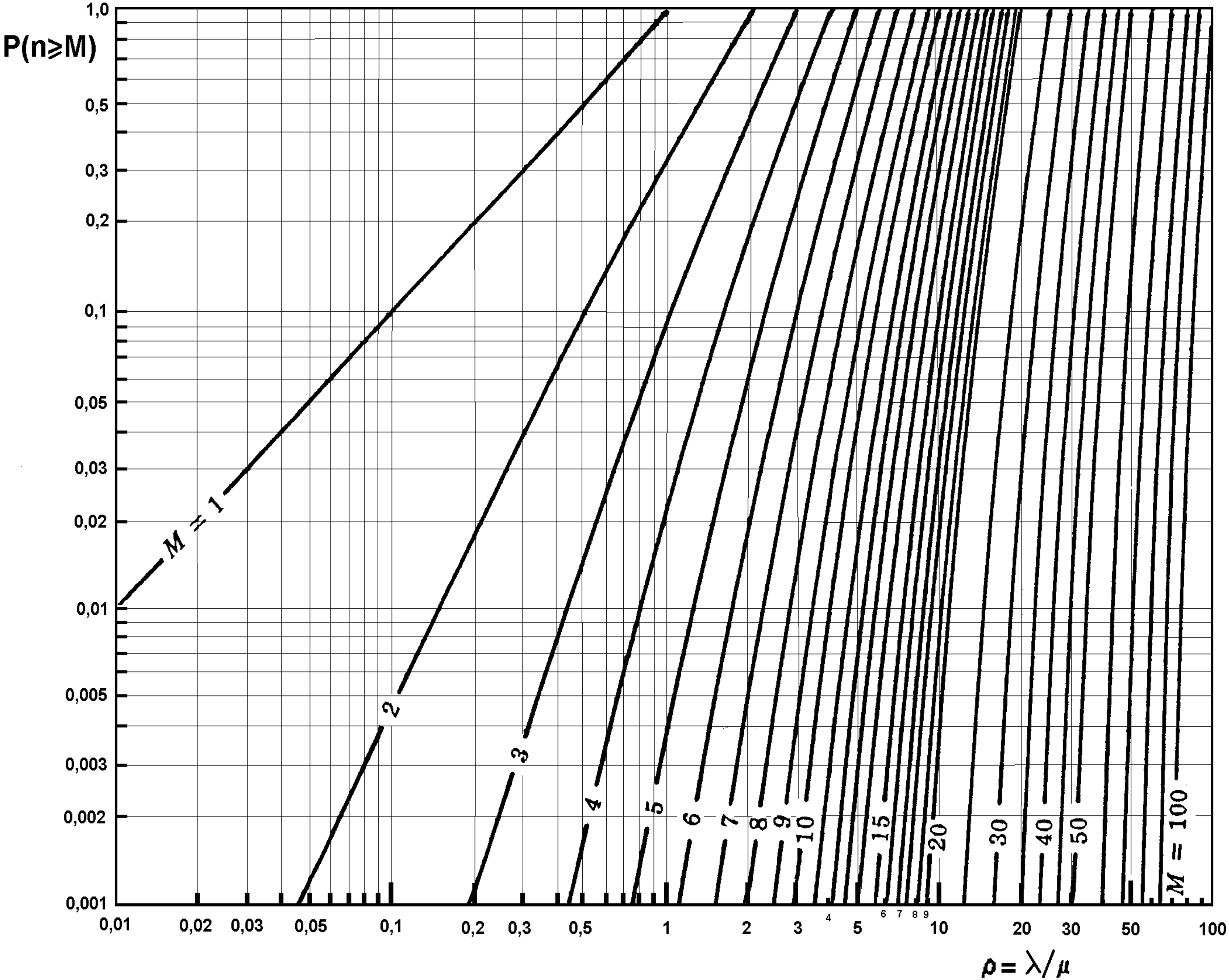
$$\frac{\rho}{M} < 1$$

$$\sum_{M}^{\infty}\left(\frac{\rho}{M}\right)^{n} = \frac{\left(\frac{\rho}{M}\right)^{M}\cdot\left[\left(\frac{\rho}{M}\right)^{\infty}-1\right]}{\frac{\rho}{M}-1} = \frac{-\frac{\rho}{M^{M}}^{M}}{-\left(1-\frac{\rho}{M}\right)} = \frac{\rho^{M}}{M^{M}\cdot\left(1-\frac{\rho}{M}\right)} =$$

$$= \frac{\rho^{M}}{M^{M}\frac{(M-\rho)}{M}} = \frac{\rho^{M}}{M^{M-1}\cdot(M-\rho)}$$

$$p(0) = \frac{1}{\sum_{0}^{M-1}\frac{\rho^{n}}{n!} + \frac{\rho^{M}}{(M-1)!\cdot(M-\rho)}}$$


P(W_C>0)
1
M
r



P(n⩾M)
1,0
0,5
0,3
0,2
0,1
0,05
0,03
0,02
0,01
0,005
0,003
0,002
0,001
M = 1
2
3
4
5
6
7
8
9
10
15
20
30
40
50
M = 100
0,01
0,02
0,03
0,05
0,1
0,2
0,3
0,5
1
2
3
4
5
6
7
8
9
10
20
30
50
100
ρ = λ/μ

r	M												
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0.10	0,1000												
0,15	0,1500	0,0104											
0,20	0,2000	0,0181											
0,25	0,2500	0,0277											
0,30	0,3000	0,0391											
0,35	0,3500	0,0521											
0,40	0,4000	0,0666											
0,45	0,4500	0,0826	0,0113										
0,50	0,5000	0,1000	0,0151										
0,55	0,5500	0,1186	0,0195										
0,60	0,6000	0,1384	0,0246										
0,65	0,6500	0,1594	0,0304										
0,70	0,7000	0,1814	0,0369										
0,75	0,7500	0,2045	0,0441										
0,80	0,8000	0,2285	0,0520										
0,85	0,8500	0,2535	0,0606	0,0117									
0,90	0,9000	0,2793	0,0700	0,0143									
0,95	0,9500	0,3059	0,0801	0,0171									
1,00		0,3333	0,0909	0,0204									
1,20		0,4499	0,1411	0,0370									
1,40		0,5764	0,2033	0,0603	0,0153								
1,60		0,7111	0,2737	0,0906	0,0258								
1,80		0,8526	0,3547	0,1285	0,0404	0,0111							
2,00			0,4444	0,1739	0,0597	0,0180							
2,20			0,5421	0,2267	0,0839	0,0274							
2,40			0,6471	0,2870	0,1350	0,0399	0,0125						
2,60			0,7588	0,3544	0,1486	0,0558	0,0187						

$$L = \sum_{0}^{\infty} n \, . \, p(n) = \sum_{0}^{M-1} n \cdot \frac{\rho^n}{n!} \cdot p(0) + \sum_{M}^{\infty} n \cdot \frac{M^M}{M!} \cdot \left(\frac{\rho}{M} \right)^n \cdot p(0)$$

$$L = \frac{\lambda \cdot \mu \cdot \rho^M}{(M-1)! \cdot (M\mu - \lambda)^2} \cdot p(0) + \rho$$

$$H = \sum_{0}^{M} n \cdot p(n) + \sum_{M+1}^{\infty} M \cdot p(n)$$

$$\boxed{H = \rho}$$

$$\bar{\lambda} = \sum_{0}^{\infty} \lambda_n \cdot p(n) \qquad \bar{\lambda} = \lambda$$

$$\boxed{\lambda = \mu \cdot H}$$

$$\bar{\mu} = \sum_{0}^{\infty} \mu_n \cdot p(n) \qquad \bar{\mu} = \sum_{1}^{M-1} n \cdot \mu \cdot p(n) + \sum_{M}^{\infty} M \cdot \mu \cdot p(n) =$$

$$= \mu \cdot \left[\sum_{1}^{M-1} n \cdot p(n) + \sum_{M}^{\infty} M \cdot p(n) \right] = \mu \cdot H$$

$$L_C = \sum_{M}^{\infty} (n - M) \cdot p(n)$$

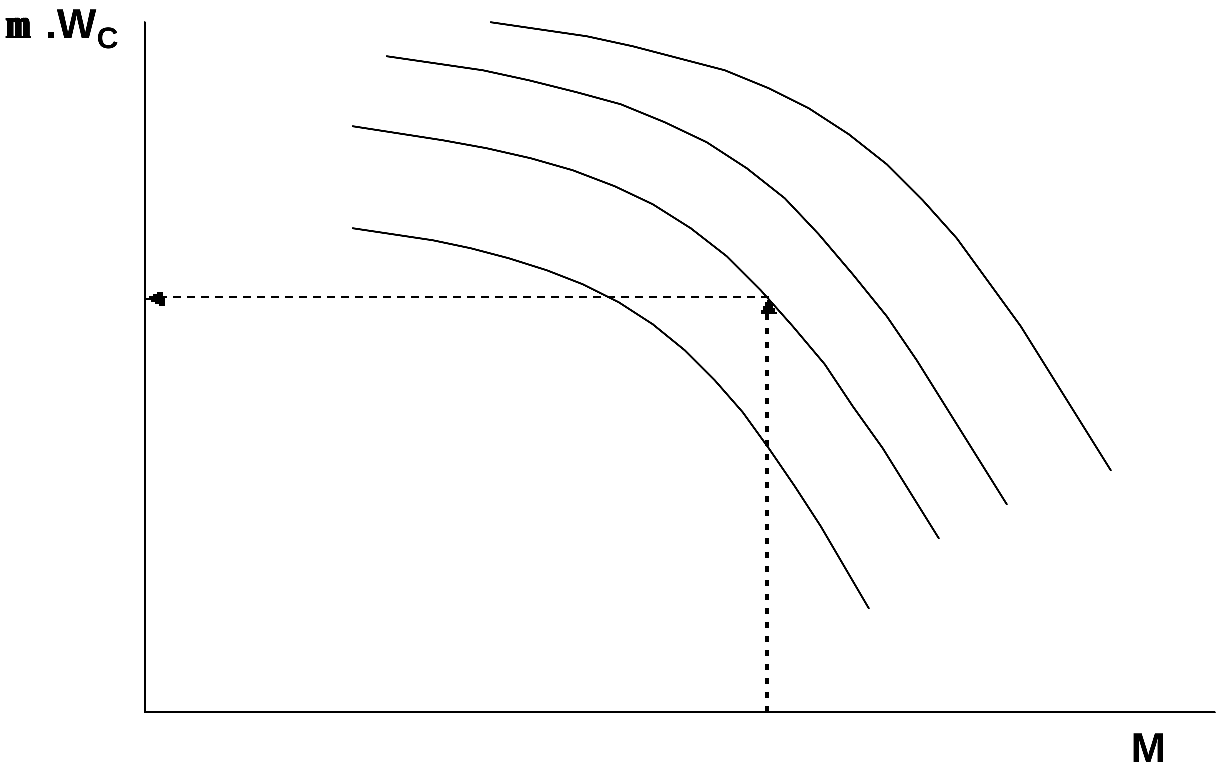
$$L_C = \frac{\lambda \cdot \mu \cdot \rho^M}{(M-1)! \cdot (M\mu - \lambda)^2} \cdot p(0)$$

$$W_C = \frac{L_C}{\bar{\lambda}}$$

$$W_C = \frac{L_C}{\lambda}$$

$$W = \frac{L}{\bar{\lambda}} = \frac{L}{\lambda}$$

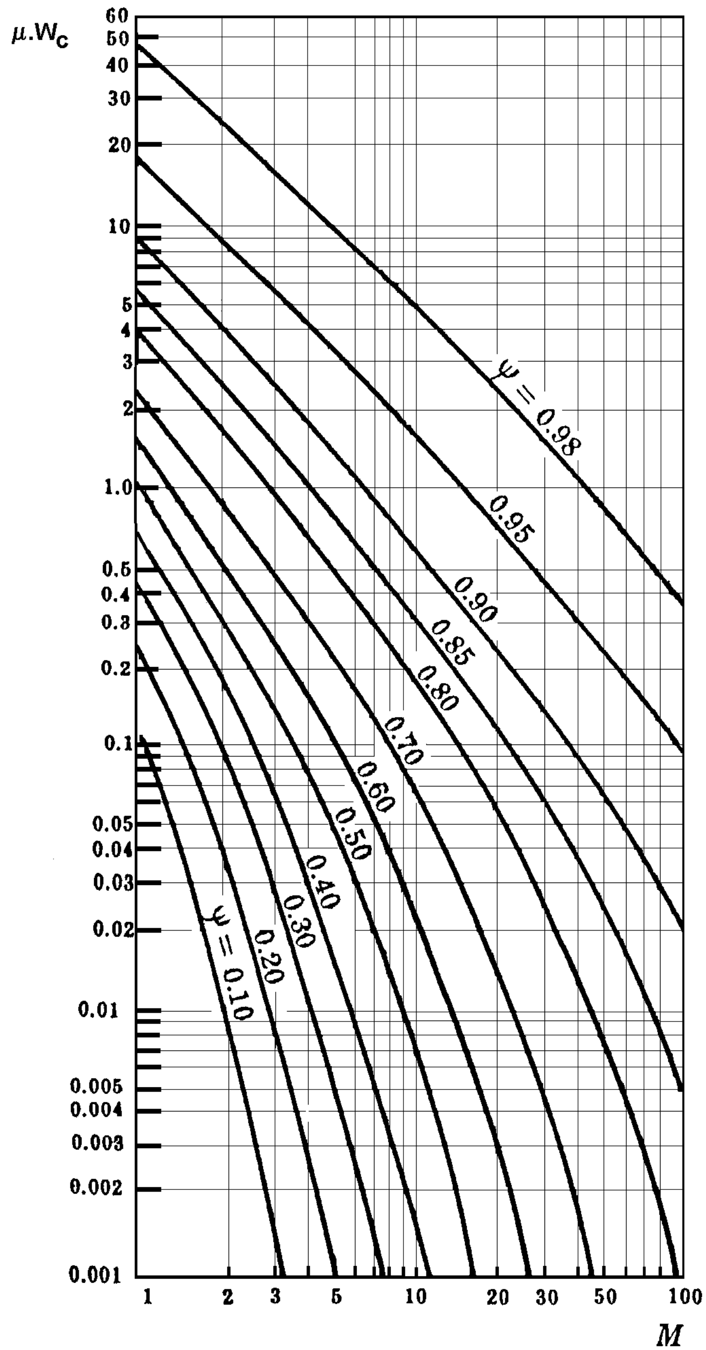
$$W = W_C + T_S$$



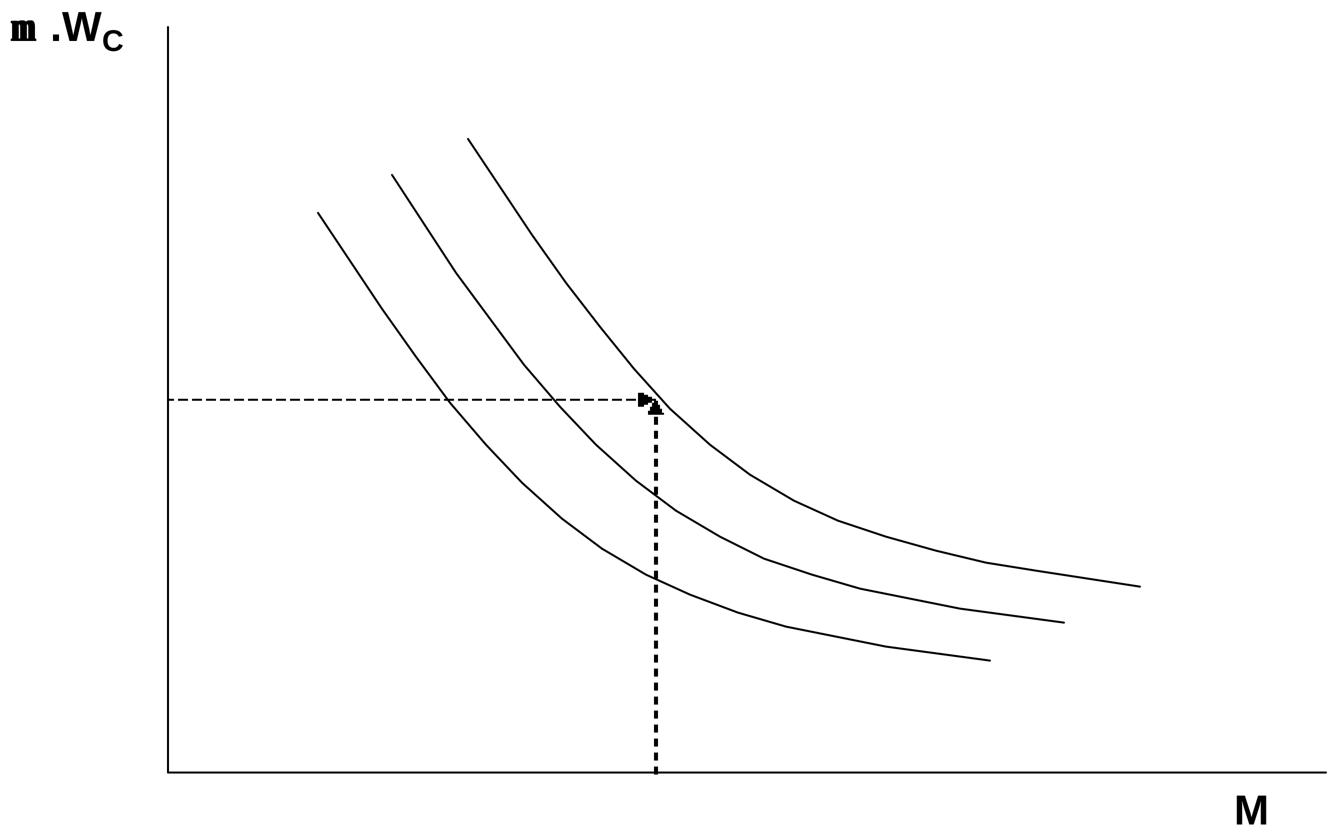
$$L_C = W_C \cdot \lambda$$

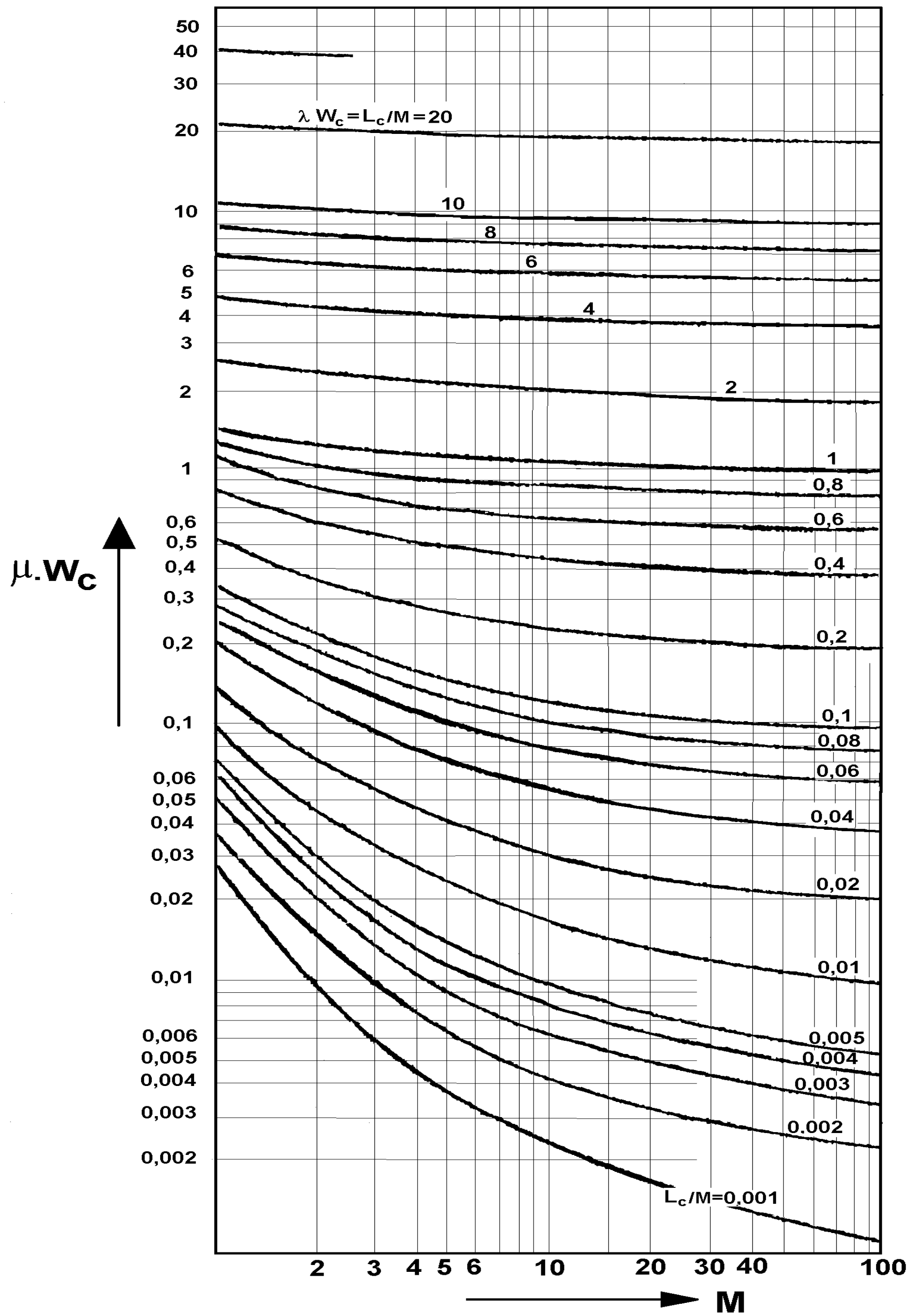
$$L = L_C + H = L_C + \rho$$

$$W = W_C + \frac{1}{\mu}$$

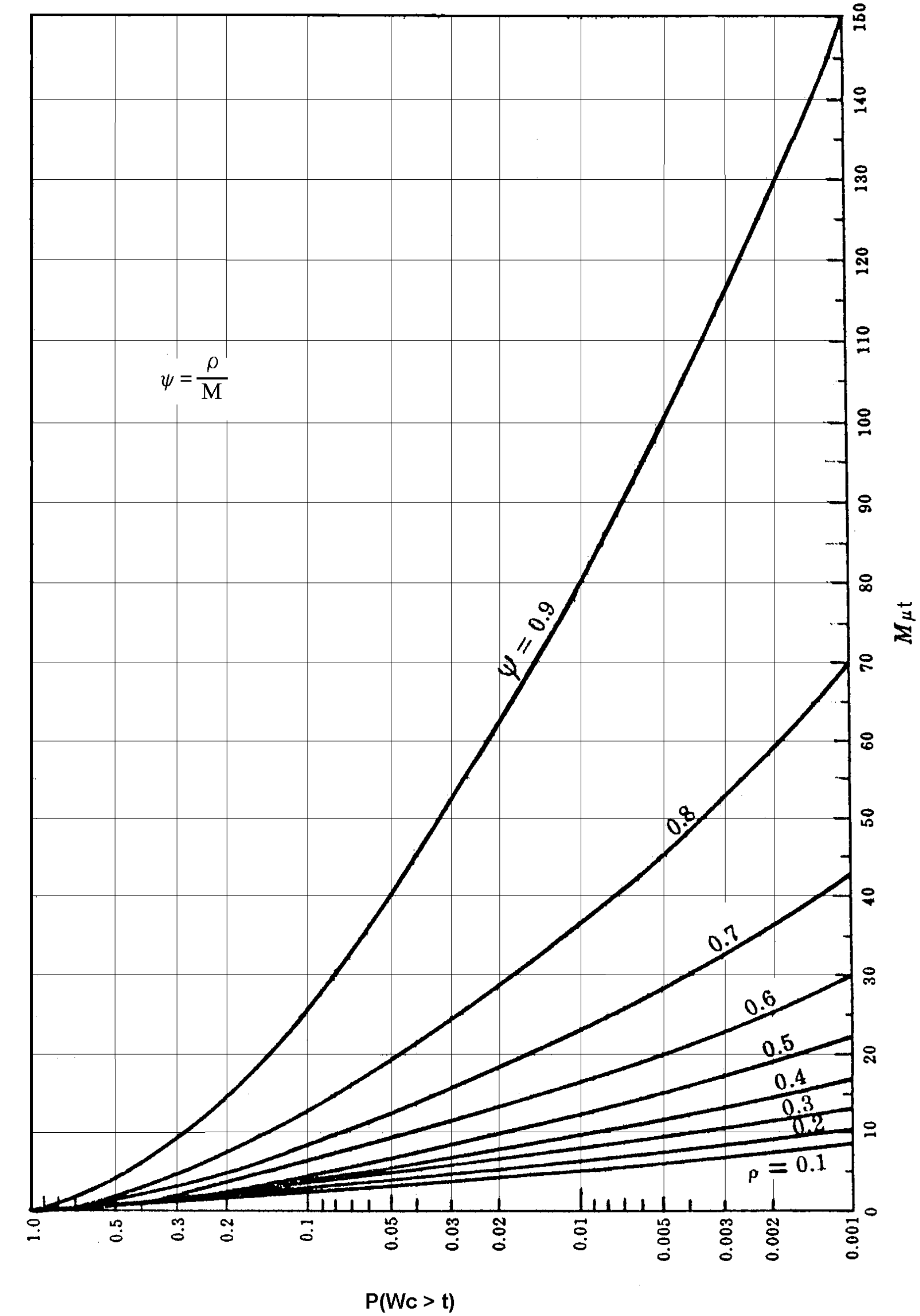


μ.W_C
60
50
40
30
20
10
5
4
3
2
1.0
0.5
0.4
0.3
0.2
0.1
0.05
0.04
0.03
0.02
0.01
0.005
0.004
0.003
0.002
0.001
1
2
3
5
10
20
30
50
100
M
ψ = 0.98
0.95
0.90
0.85
0.80
0.70
0.60
0.50
0.40
0.30
0.20
ψ = 0.10


m .W$_C$
M



50
40
30
20
10
6
5
4
3
2
1
0,6
0,5
0,4
0,3
0,2
0,1
0,06
0,05
0,04
0,03
0,02
0,01
0,006
0,005
0,004
0,003
0,002
μ.Wc
λ Wc=Lc/M=20
10
8
6
4
2
1
0,8
0,6
0,4
0,2
0,1
0,08
0,06
0,04
0,02
0,01
0,005
0,004
0,003
0,002
Lc/M=0,001
2
3
4
5
6
10
20
30
40
100
M

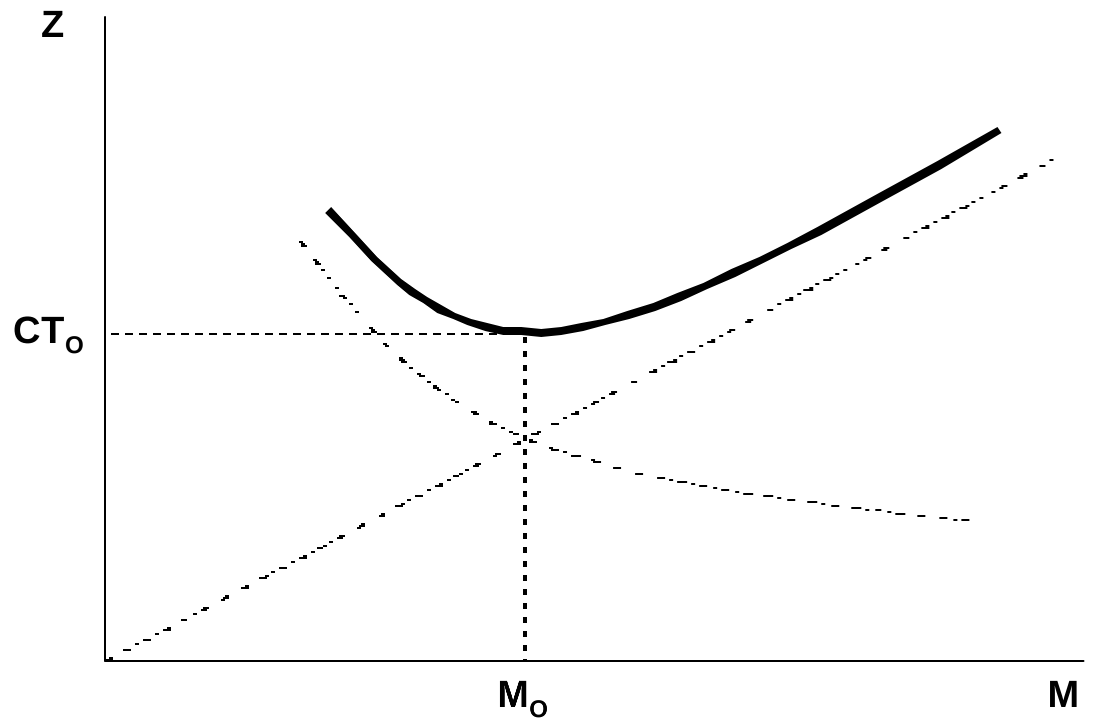


ψ = ρ/M
ψ = 0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
ρ = 0.1
Mμt
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
1.0
0.5
0.3
0.2
0.1
0.05
0.03
0.02
0.01
0.005
0.003
0.002
0.001
P(Wc > t)

ANÁLISIS ECONÓMICO

c_e: $\left(\frac{\$}{t \cdot cl}\right)$

c_C: $\left(\frac{\$}{canal \cdot t}\right)$



$$Z = c_e \cdot L + c_C \cdot M \quad \rightarrow \quad Min$$

OTROS EJEMPLOS DE FUNCIONAL

$$Z = c_e \cdot L_C + c_C \cdot M \quad \rightarrow \quad \text{Min}$$

$$Z = c_e \cdot L + c_D \cdot M \cdot \frac{H}{M} = c_e \cdot L + c_D \cdot H \quad \rightarrow \quad \text{Min}$$

$$Z = c_e \cdot L + c_C \cdot M + c_D \cdot H \quad \rightarrow \quad \text{Min}$$

$$Z = c_e \cdot L + c_V \cdot M \cdot \mu \quad \rightarrow \quad \text{Min}$$