P/P/1/N

- Arribos Po
- Servicio Po
- Régimen permanente
- 1 canal
- 1 cola
- FIFO
- Capacidad finita igual a N
- Sin impaciencia
- Población infinita

Parámetros

- λ

$$T_a = \frac{1}{\lambda}$$

- μ

$$T_s = \frac{1}{\mu}$$

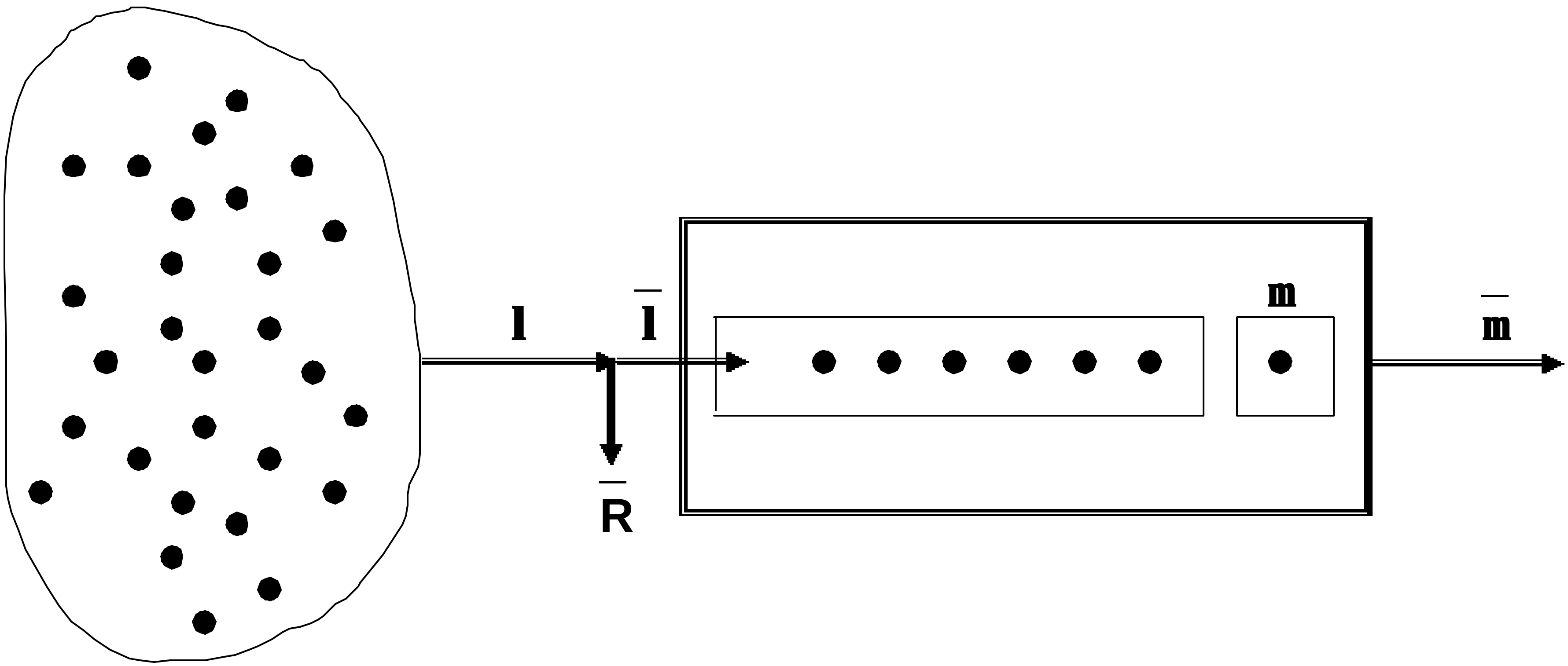
- M = 1
- N

Variables

- $p(n)$
- L_c
- L
- H
- W_c
- W

$\bar{\lambda}$

$\bar{\mu}$



l
$\bar{l}$
$\bar{R}$
m
$\bar{m}$

$$\lambda_n = \begin{cases} \lambda & n = 0, 1, 2, 3, \ldots. \\ 0 & n = N \end{cases}$$

$$\lambda_n = \lambda \cdot p(i/n)$$

$$\mu_n = \begin{cases} 0 & n = 0 \\ \mu & n = 1, 2, 3, \ldots., N \end{cases}$$

$$\boxed{p(n) = \frac{\lambda_{n-1}}{\mu_n} \cdot p(n-1)}$$

$$p(1) = \frac{\lambda_0}{\mu_1} \cdot p(0) = \frac{\lambda}{\mu} \cdot p(0) \quad \Rightarrow \quad p(1) = \rho \cdot p(0)$$

$$p(2) = \frac{\lambda_1}{\mu_2} \cdot p(1) = \frac{\lambda}{\mu} \cdot p(0) \cdot \frac{\lambda}{\mu} \quad \Rightarrow \quad p(2) = \rho^2 \cdot p(0)$$

$$p(3) = \frac{\lambda_2}{\mu_3} \cdot p(2) = \frac{\lambda}{\mu} \cdot p(0) \cdot \frac{\lambda^2}{\mu^2} \quad \Rightarrow \quad p(3) = \rho^3 \cdot p(0)$$

$$\boxed{p(n) = \rho^n \cdot p(0)}$$

$$\sum_{0}^{N} p(n) = 1$$

$$\sum_{0}^{N} \rho^{n} \cdot p(0) = 1 \qquad \therefore \qquad p(0) \cdot \sum_{0}^{N} \rho^{n} = 1$$

$$p(0) = \frac{1}{\sum_{0}^{N} \rho^{n}} \qquad \sum_{0}^{N} \rho^{n} = \frac{\rho^{0} \cdot \left(\rho^{N+1} - 1\right)}{\rho - 1} = \frac{1 - \rho^{N+1}}{1 - \rho}$$

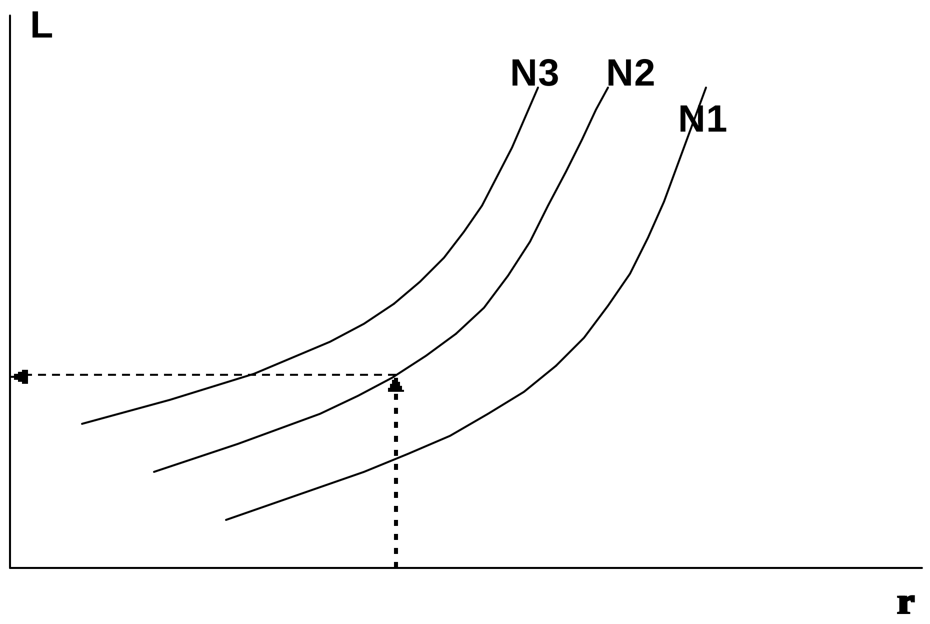
$$p(0) = \frac{1 - \rho}{1 - \rho^{N+1}} \qquad \text{Para } \rho = 1 \quad \Rightarrow \quad p(0) = \lim_{\rho \to 1} \frac{1 - \rho}{1 - \rho^{N+1}} = \frac{1}{N + 1}$$

$$p(n) = \rho^{n} \cdot \frac{(1 - \rho)}{(1 - \rho)^{N+1}}$$

$$L = \sum_{0}^{N} n \cdot p(n)$$

$$L = \sum_{0}^{N} n \cdot \rho^{n} \cdot \frac{(1-\rho)}{(1-\rho)^{N+1}} = \frac{(1-\rho)}{(1-\rho)^{N+1}} \cdot \sum_{0}^{N} n.\rho^{n}$$

$$\boxed{L = \rho \cdot \frac{1-(N+1)\cdot\rho^{N} + N\cdot\rho^{N+1}}{(1-\rho)\cdot(1-\rho)^{N+1}}}$$



$$L_C = \sum_{1}^{N} (n-1) \cdot p(n) = \sum_{1}^{N} n \cdot p(n) - \sum_{1}^{N} p(n) = \sum_{0}^{N} n \cdot p(n) - \left[1 - p(0)\right] = L - \left[1 - p(0)\right]$$

$$L_C = \rho^2 \cdot \frac{1 - N \cdot \rho^{N-1} + N \cdot \rho^{N}}{(1-\rho) \cdot (1-\rho)^{N+1}}$$

$$H = \sum_{1}^{N} p(n) = 1 - p(0)$$

$$H = 1 - \frac{1-\rho}{1-\rho^{N+1}}$$

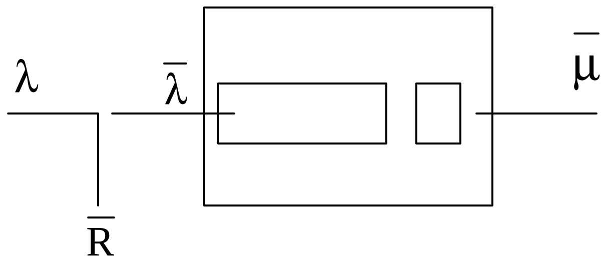
$$L = L_C + H$$

$$PA = \frac{H}{M}$$

$$\bar{\lambda} = \sum_{0}^{N} \lambda_n \cdot p(n) = \sum_{0}^{N-1} \lambda \cdot p(n) = \lambda \cdot \sum_{0}^{N-1} p(n) = \lambda \cdot [1 - p(N)]$$

$$\bar{\mu} = \sum_{0}^{N} \mu_n \cdot p(n) = \sum_{1}^{\infty} \mu \cdot p(n) = \mu \cdot [1 - p(0)] = \mu \cdot H$$

$$\bar{\lambda} = \bar{\mu}$$



$$\lambda \cdot [1 - p(N)] = \mu \cdot [1 - p(0)] \quad \Longrightarrow \quad \rho = \frac{1 - p(0)}{1 - p(N)}$$

$$\bar{R} = \lambda \cdot p(N) \quad \Longrightarrow \quad \bar{R} = \lambda \cdot \rho^N \frac{1 - \rho}{1 - \rho^{N+1}}$$

$$\lambda = \bar{\lambda} + \bar{R} \qquad PR = \frac{\bar{R}}{\lambda}$$

$$W_C = \frac{L_C}{\bar{\bar{\mu}}} = \frac{L_C}{\mu \cdot [1 - p(0)]} = \frac{L_C}{\mu \cdot \left[1 - \frac{1-\rho}{1-\rho^{N+1}}\right]}$$

$$W = W_C + T_S \qquad W = \frac{L}{\bar{\bar{\mu}}}$$

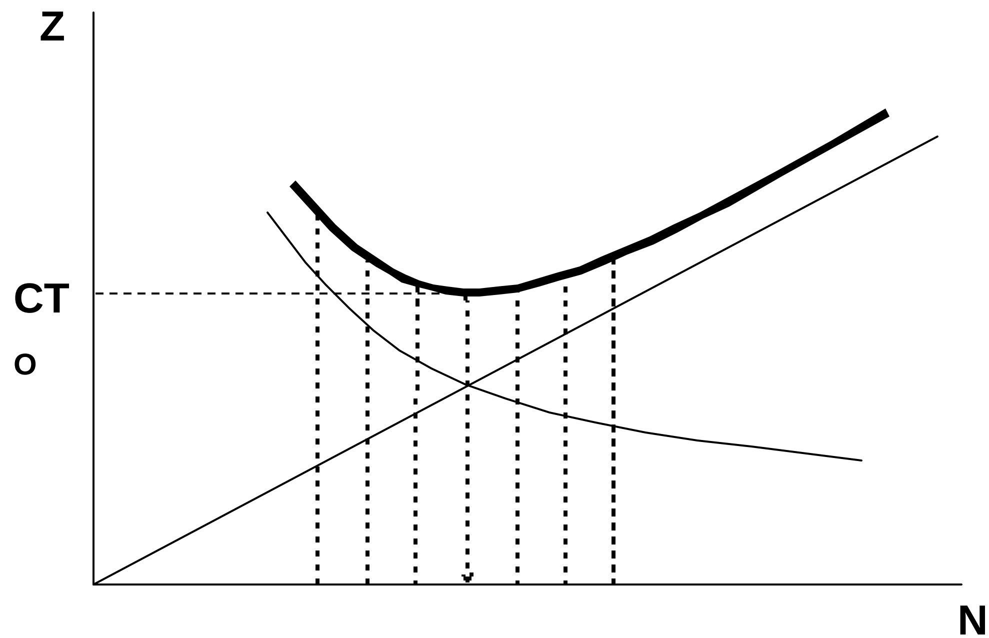
N = 10

ñ	0,00	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50	0,55	0,60	0,65	0,70	0,75	0,80	0,85	0,90	0,95
p(0)	1,0000	0,9000	0,8500	0,8000	0,7500	0,7000	0,6500	0,6000	0,5501	0,5002	0,4506	0,4015	0,3531	0,3061	0,2610	0,2188	0,1801	0,1457	0,1160
L	0,0000	0,1111	0,1765	0,2500	0,3333	0,4286	0,5384	0,6662	0,8165	0,9946	1,2069	1,4599	1,7600	2,1114	2,5149	2,9663	3,4559	3,9694	4,4898
ñ	1,00	1,05	1,10	1,15	1,20	1,25	1,30	1,35	1,40	1,45	1,50	1,55	1,60	1,65	1,70	1,75	1,80	1,85	1,90
p(0)	0,0909	0,0704	0,0540	0,0411	0,0311	0,0235	0,0177	0,0134	0,0101	0,0077	0,0058	0,0045	0,0034	0,0026	0,0020	0,0016	0,0012	0,0010	0,0008
L	5,0000	5,4856	5,9359	6,3451	6,7107	7,0337	7,3167	7,5636	7,7785	7,9656	8,1287	8,2712	8,3962	8,5063	8,6036	8,6900	8,7671	8,8362	8,8983
ñ	2,00	2,05	2,10	2,15	2,20	2,25	2,30	2,35	2,40	2,45	2,50	2,55	2,60	2,65	2,70	2,75	2,80	2,85	2,90
p(0)	0,0005	0,0004	0,0003	0,0003	0,0002	0,0002	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
L	9,0054	9,0517	9,0941	9,1329	9,1685	9,2015	9,2319	9,2602	9,2864	9,3109	9,3338	9,3552	9,3753	9,3942	9,4120	9,4287	9,4446	9,4596	9,4738

ANÁLISIS ECONÓMICO

$c_L \left(\frac{\$}{\text{lugar} \cdot t} \right)$

$u \left(\frac{\$}{cl} \right)$



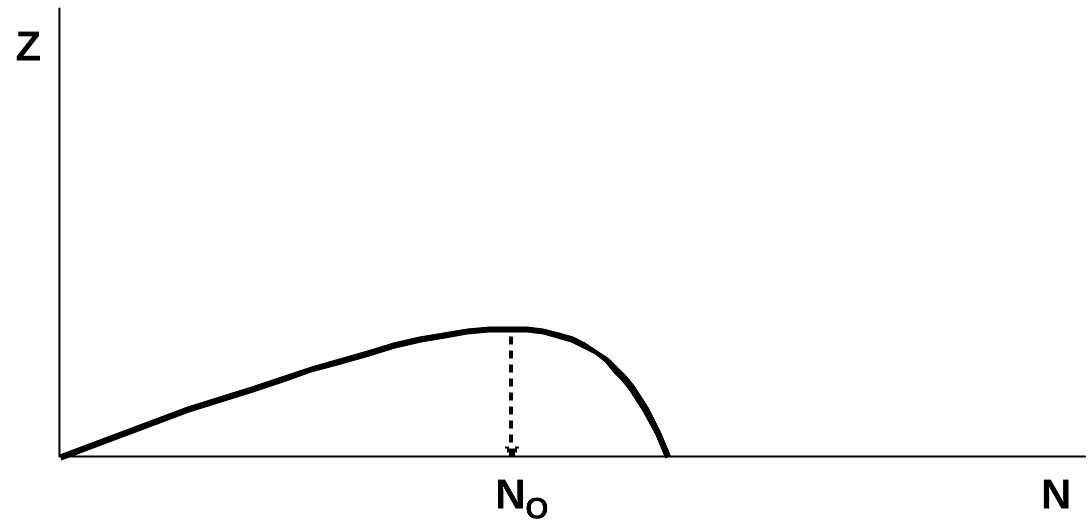
$$Z = c_L \cdot (N-1) + u \cdot \overline{R} \quad \Rightarrow \quad \text{Min}$$

$$Z = c_L \cdot (N-1) + u \cdot \lambda \cdot \rho^N \frac{1-\rho}{1-\rho^{N+1}} \Rightarrow \text{Min}$$

ANÁLISIS ECONÓMICO

$$c_L \left(\frac{\$}{\text{lugar} \cdot t} \right)$$

$$u \left(\frac{\$}{\text{cl}} \right)$$



$$Z = u \cdot \bar{\mu} - c_L \cdot (N-1) \quad \rightarrow \quad \text{Max}$$

A un sistema de colas, con capacidad limitada a un máximo de 4 clientes, arriban usuarios a una tasa media de 6 por hora, siendo la velocidad promedio de atención 8 clientes por hora.

Si cada servicio se abona $100, determinar el lucro cesante por el hecho de que los clientes se retiren sin ser atendidos.

Calcular también la longitud promedio del sistema y de la cola, como así también el tiempo promedio dentro del sistema que permanece un cliente.

$$\lambda = 6 \qquad \mu = 8$$

$$p(1) = \rho \cdot p(0) \quad = 0{,}245839$$

$$p(2) = \rho^2 \cdot p(0) \quad = 0{,}184379$$

$$p(3) = \rho^3 \cdot p(0) \quad = 0{,}138284$$

$$p(4) = \rho^4 \cdot p(0) \quad = 0{,}103713$$

$$p(0) + p(1) + p(2) + p(3) + p(4) = 1$$

$$p(0) + \rho \cdot p(0) + \rho^2 \cdot p(0) + \rho^3 \cdot p(0) + \rho^4 \cdot p(0) = 1$$

$$p(0) = \frac{1}{1 + \rho + \rho^2 + \rho^3 + \rho^4} = \frac{1}{1 + \frac{6}{8} + \left(\frac{6}{8}\right)^2 + \left(\frac{6}{8}\right)^3 + \left(\frac{6}{8}\right)^4} = 0{,}327785$$

$$\bar{\lambda} = \lambda \cdot [1 - p(4)] = 6 \cdot 0{,}896287 = 5{,}377721 \frac{cl}{h}$$

$$\bar{\mu} = \mu \cdot [1 - p(0)] = 8 \cdot 0{,}67221 = 5{,}377721 \frac{cl}{h}$$

$$\bar{R} = \lambda - \bar{\lambda}$$

$$\bar{R} = \lambda \cdot p(4) = 6 \cdot 0{,}103713 = 0{,}62227913 \frac{cl}{h}$$

$$LC = 100 \cdot \bar{R} = 62{,}227913 \frac{\$}{h}$$

$\lambda = 6 \qquad \mu = 8$

p(0) = 0,327785

p(1) = 0,245839

p(2) = 0,184379

p(3) = 0,138284

p(4) = 0,103713

$$L = 0.p(0) + 1 \cdot p(1) + 2 \cdot p(2) + 3 \cdot p(3) + 4 \cdot p(4)$$
$$= 0{,}245839 + 2 \cdot 0{,}184379 + 3 \cdot 0{,}138284 + 4 \cdot 0{,}103713 = 1{,}444302$$

$$L_C = 1 \cdot p(2) + 2 \cdot p(3) + 3 \cdot p(4) = 1 \cdot 0{,}184379 + 2 \cdot 0{,}138284 + 3 \cdot 0{,}103713 = 0{,}772087$$

$$W_C = \frac{L_C}{\bar{\lambda}} = \frac{0{,}772087}{5{,}377721} = 0{,}064876\text{h}$$

$$W = W_C + \frac{1}{\mu} = 0{,}064876 + \frac{1}{8} = 0{,}189876\,\text{h}$$

N = 4

ñ	0,00	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50	0,55	0,60	0,65	0,70	0,75	0,80	0,85	0,90	0,95
p(0)	1,0000	0,9000	0,8501	0,8003	0,7507	0,7017	0,6534	0,6062	0,5603	0,5161	0,4738	0,4337	0,3959	0,3606	0,3278	0,2975	0,2696	0,2442	0,2210
L	0,0000	0,1111	0,1761	0,2484	0,3284	0,4164	0,5121	0,6149	0,7242	0,8387	0,9572	1,0784	1,2008	1,3232	1,4443	1,5631	1,6786	1,7903	1,8975