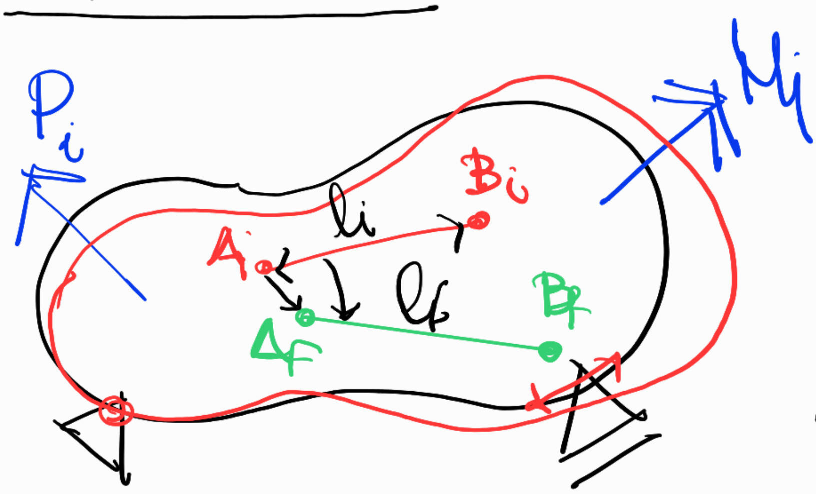
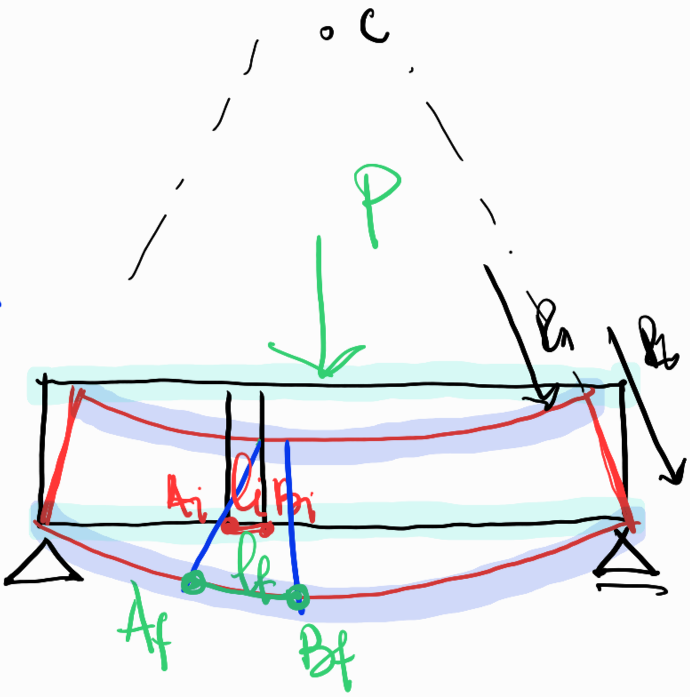
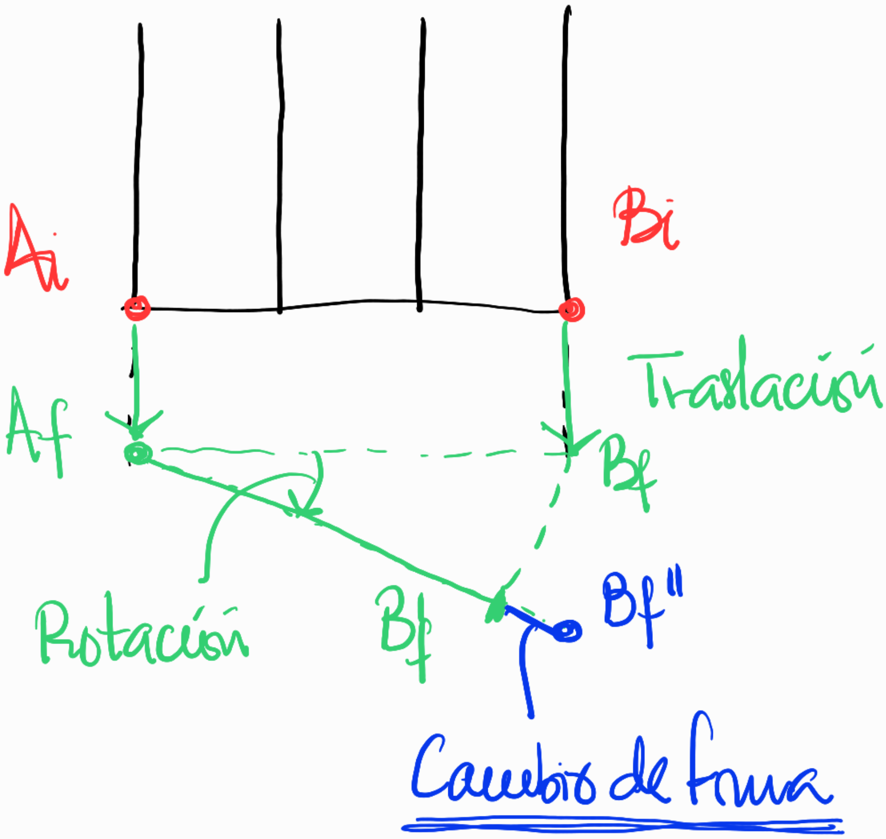
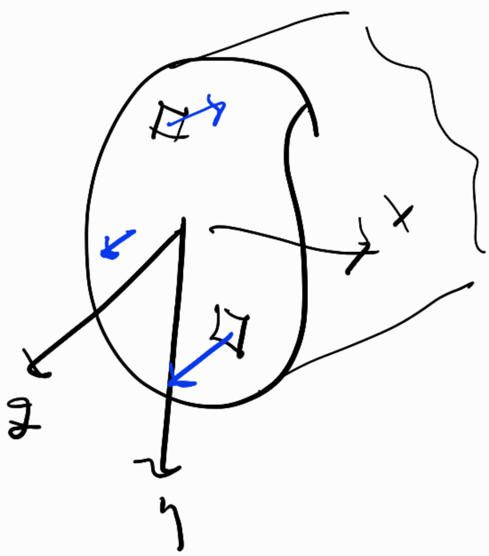
# Deformaciones



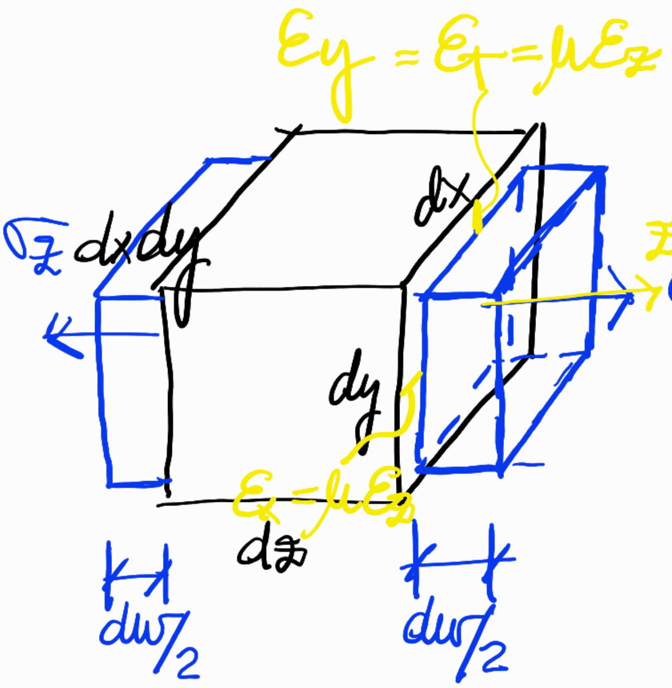

$$N = \int_A \sigma_{z(x;y)} \, dA$$

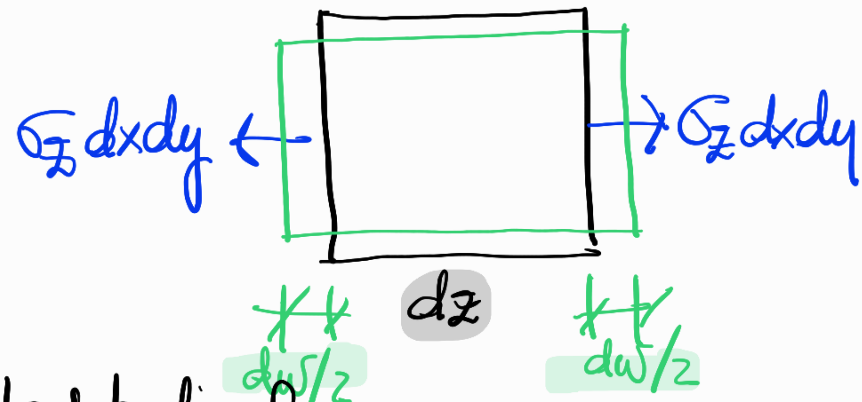


# Deformaciones longitudinales

funciones corrimiento

$x \to u \; ; y \to v \; ; z \to w$



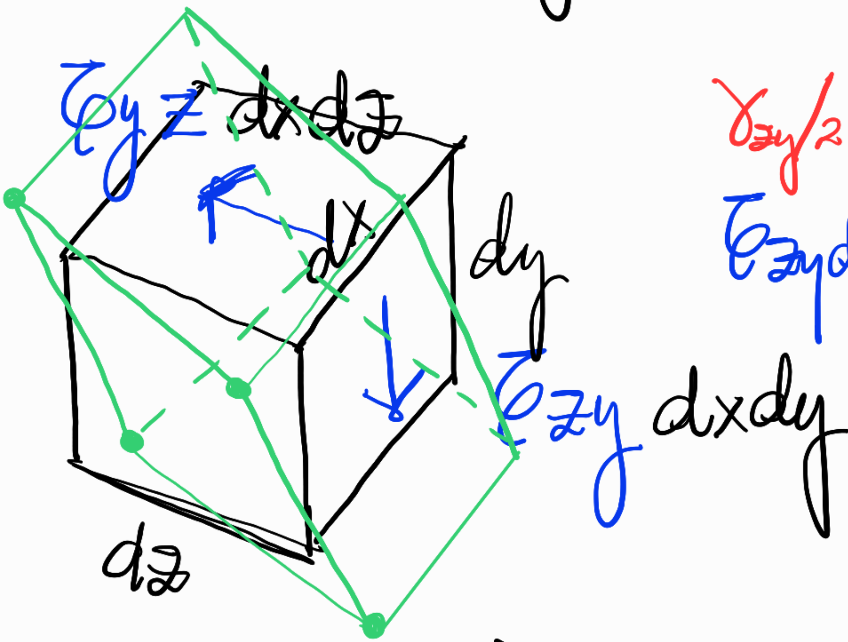



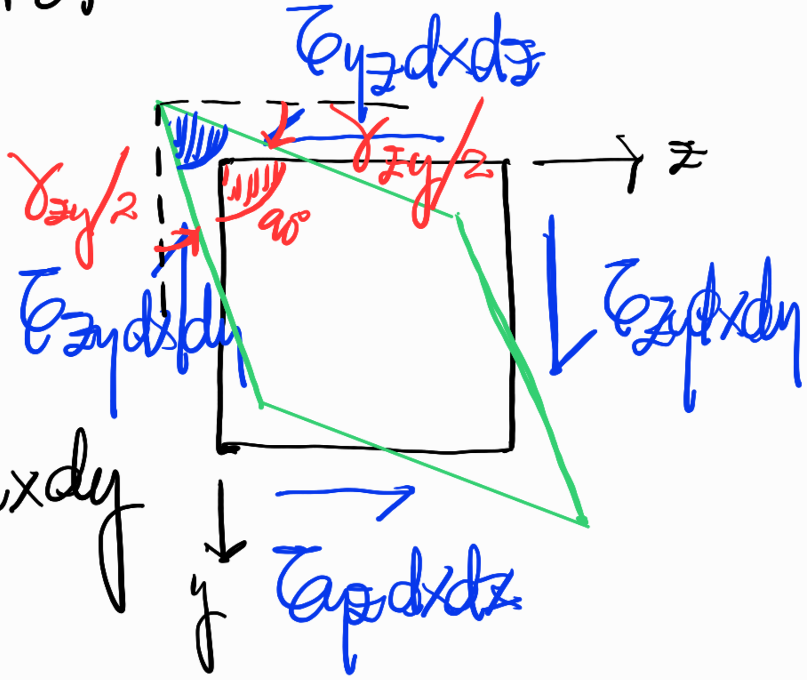

$\varepsilon$ = Deformación específica longitudinal

$$\varepsilon = \frac{l_f - l_i}{l_i} = \frac{(\cancel{dz} + dw) - \cancel{dz}}{dz} = \frac{dw}{dz}$$

$$\varepsilon_z = \frac{dw}{dz} \; ; \; \varepsilon_x = \frac{du}{dx} \; ; \; \varepsilon_y = \frac{dv}{dy}$$

# Deformaciones Angulares




$\gamma \rightarrow$ Distorsión angular

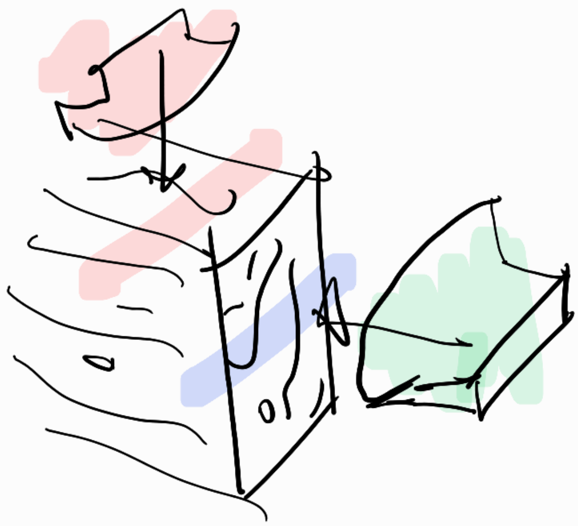
$\gamma_{zy}$ ; $\gamma_{xy}$ ; $\gamma_{xz}$

Vamos a tener que estudiar el material

Los materiales q' vamos a estudiar van a ser $\rightarrow$ Continuos

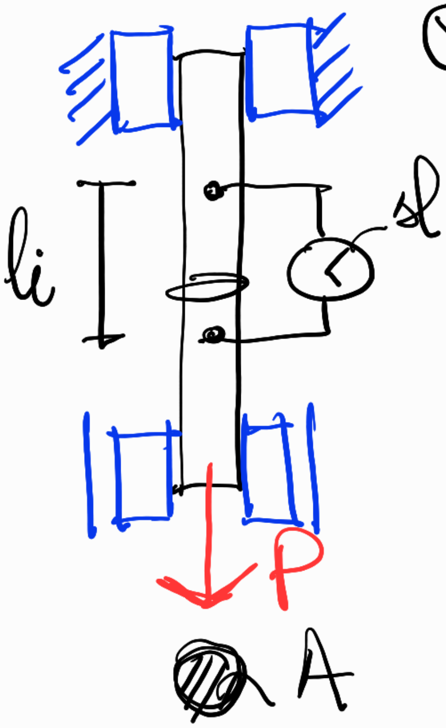
↳ Homogeneos

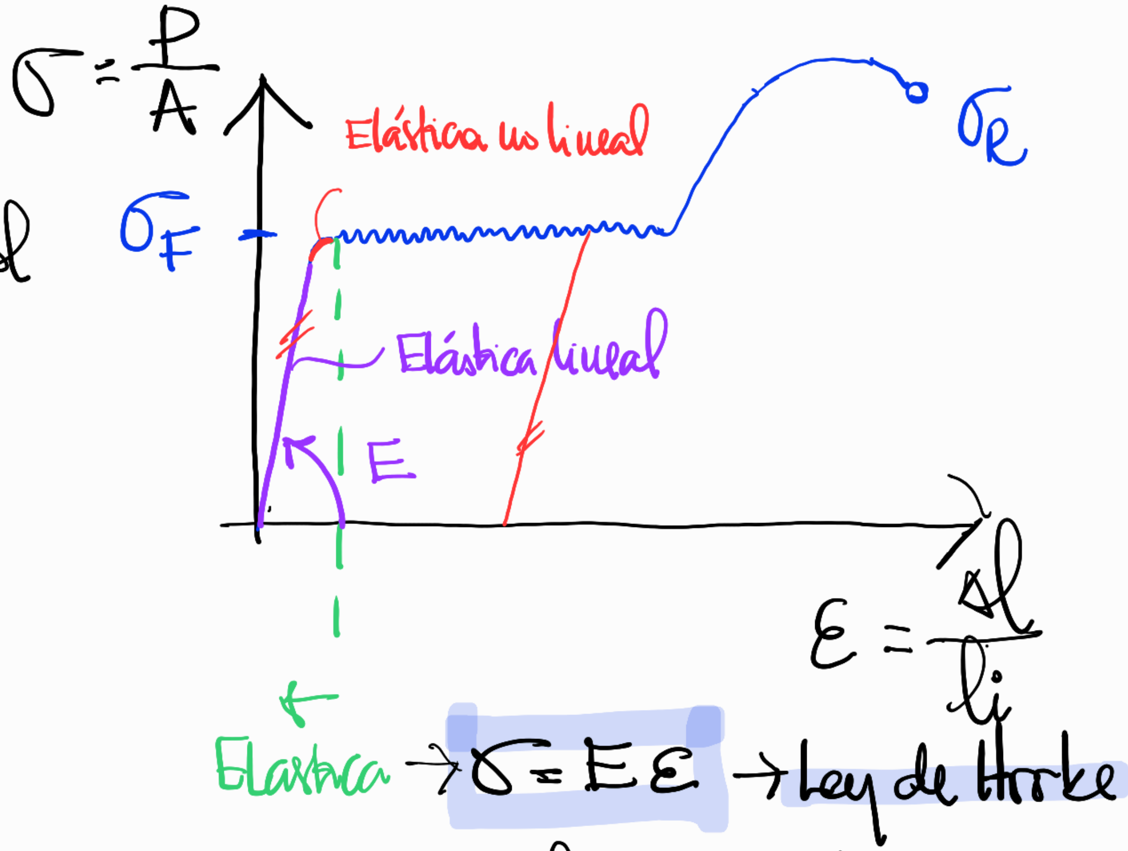
↳ Isotropos



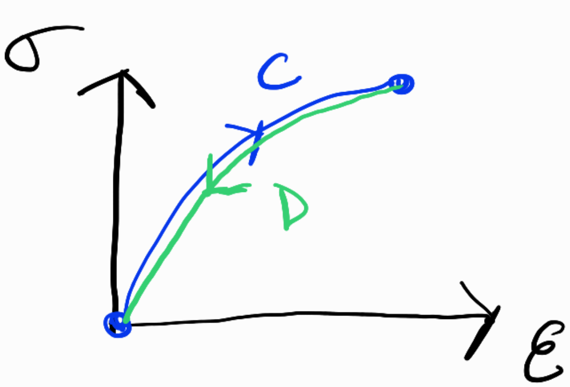
# Material

## Ensayo simple de tracción

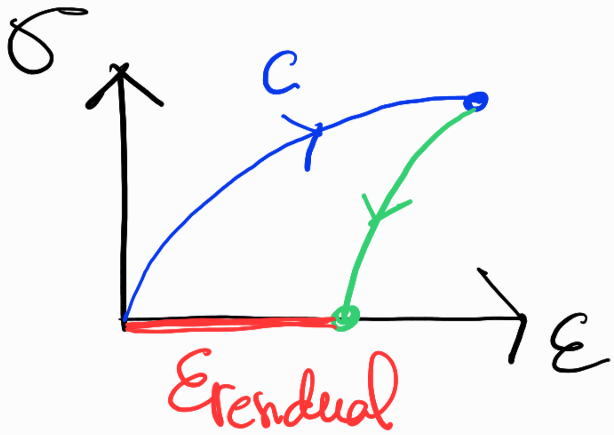


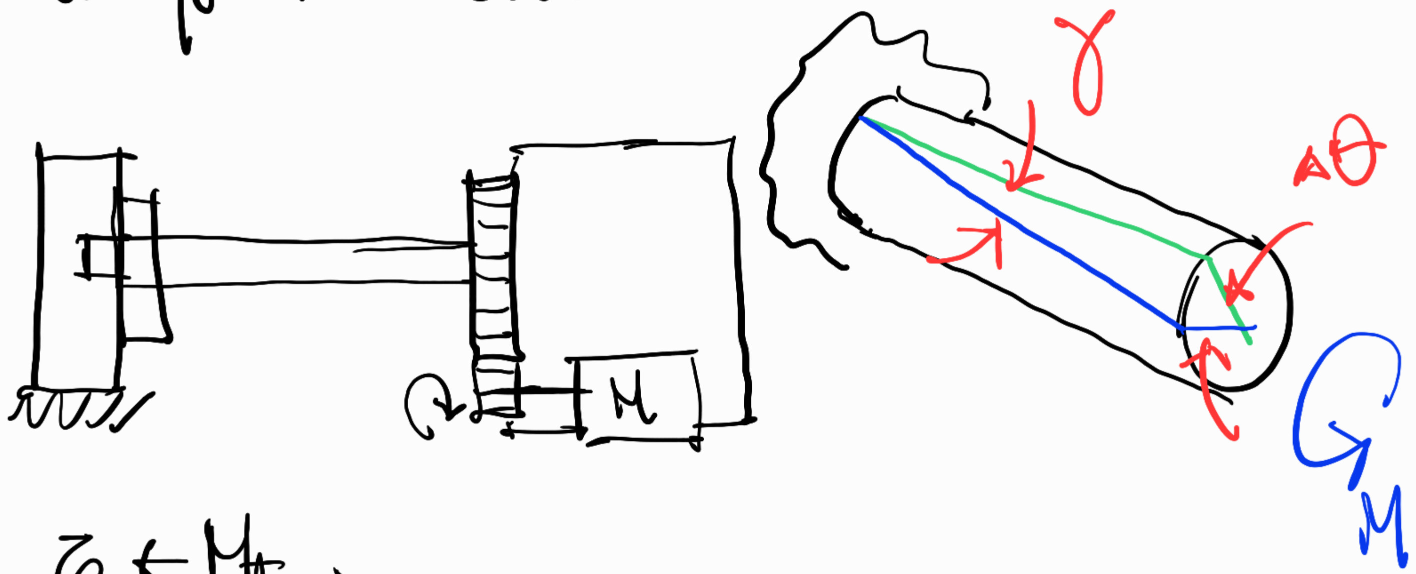

### Material Elástico

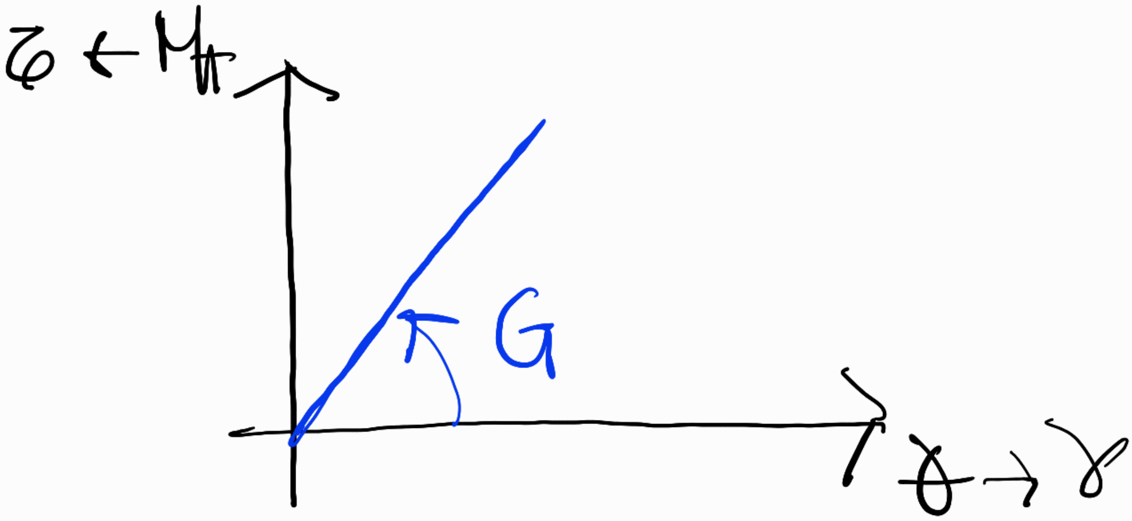



### Material Elasto-Plástico



# Ensayo en a Torsión




$$\boxed{\tau = G\gamma}$$ → Ley de Hooke

En términos mecánicos vamos a usar 3 parámetros del material

→ E = Mód. Elasticidad longitudinal

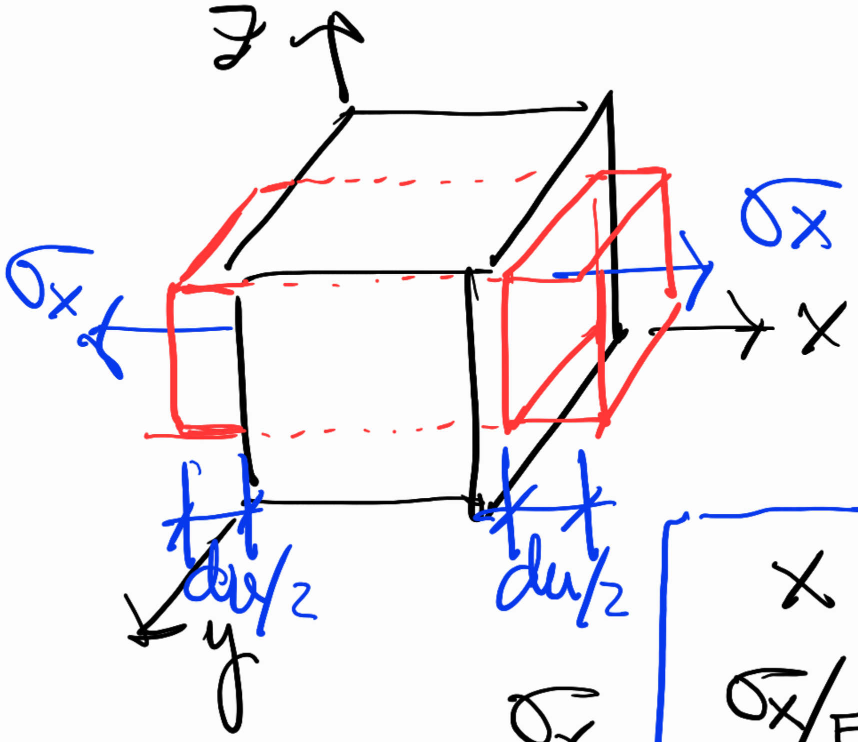
→ G = Mód. Elasticidad Transversal

→ $\mu$ = coef. de Poisson

$$\mu = -\frac{\varepsilon_T}{\varepsilon_L} \qquad 0 < \mu < 0{,}5$$

$$G = \frac{E}{2(1+\mu)}$$

Relaciones entre tensiones y deformaciones




$$\frac{du}{dx} = \varepsilon_x = \frac{\sigma_x}{E}$$

| | X | Y | Z |
|---|---|---|---|
| $\sigma_x$ | $\sigma_x/E$ | $-\mu\,\sigma_x/E$ | $-\mu\,\sigma_x/E$ |
| $\sigma_y$ | $-\mu\,\sigma_y/E$ | $\sigma_y/E$ | $-\mu\,\sigma_y/E$ |
| $\sigma_z$ | $-\mu\,\sigma_z/E$ | $-\mu\,\sigma_z/E$ | $\sigma_z/E$ |

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\mu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

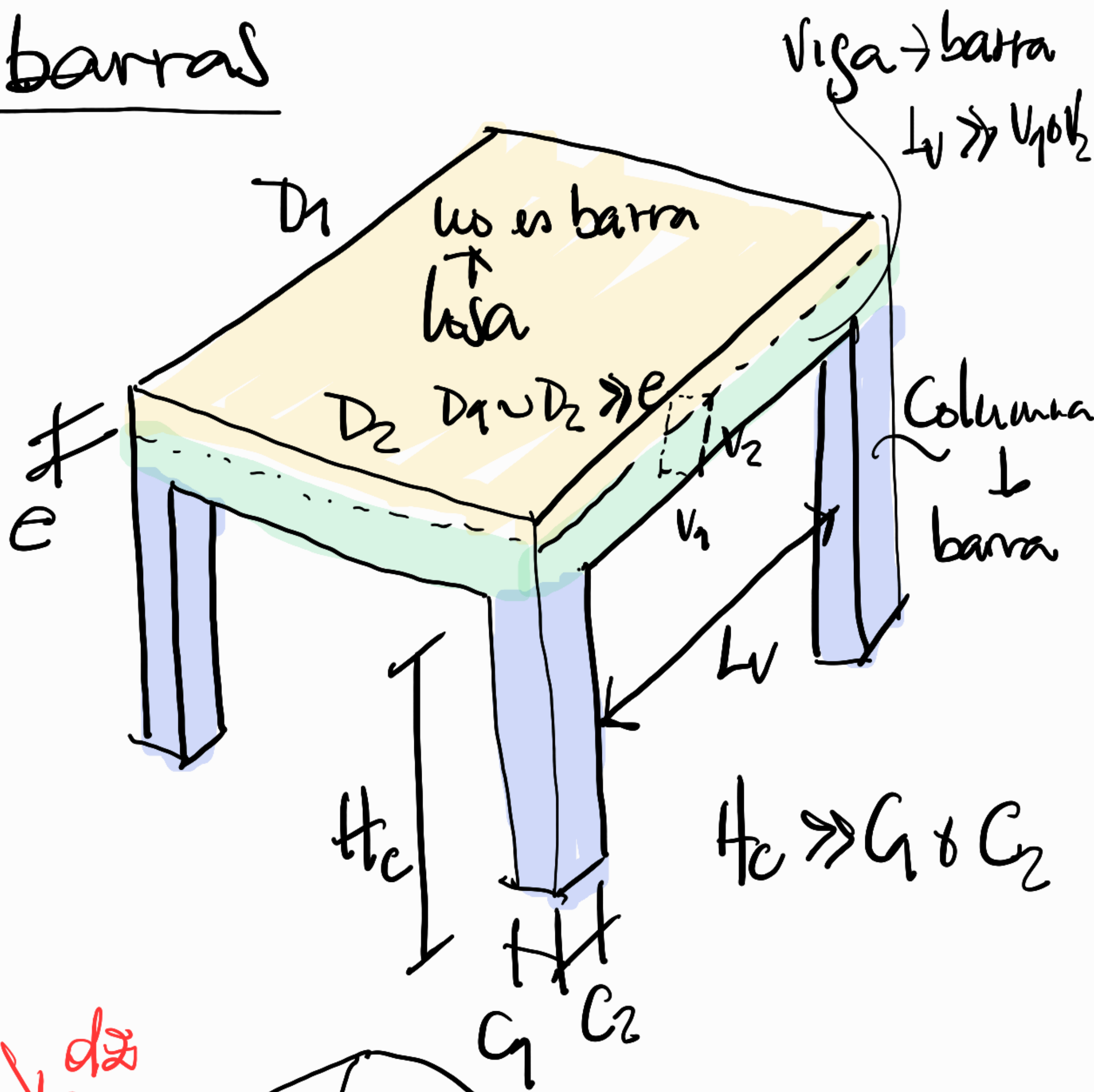
$$\varepsilon_z = -\mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

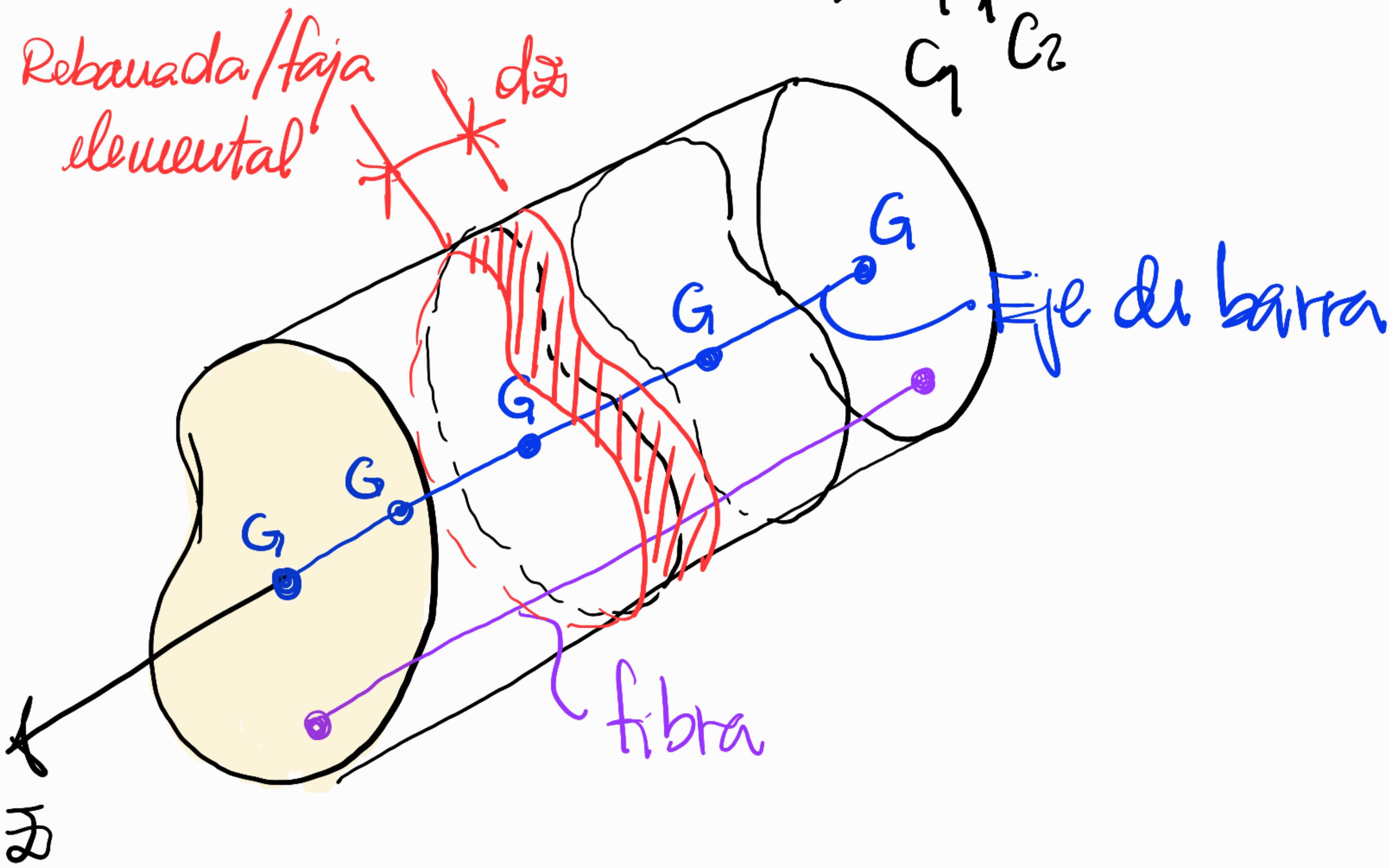
↳ Ley Generalizada de Hooke

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} 1/E & -\mu/E & -\mu/E & & & \\ -\mu/E & 1/E & -\mu/E & & 0 & \\ -\mu/E & -\mu/E & 1/E & & & \\ \hline & & & 1/G & 0 & 0 \\ & 0 & & 0 & 1/G & 0 \\ & & & 0 & 0 & 1/G \end{array}\right] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

# Teoria de barras

barra
↓
la longitud del elemento predomina s/ las otras dos dimensiones

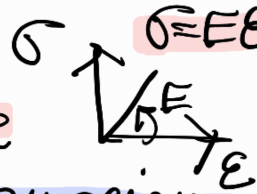


# Hipótesis

- MATERIAL CONTINUO
- MATERIAL Homogéneo
- MATERIAL Isótropo
- LINEALIDAD MECÁNICA → Ley de Hooke

$\sigma = E\varepsilon$



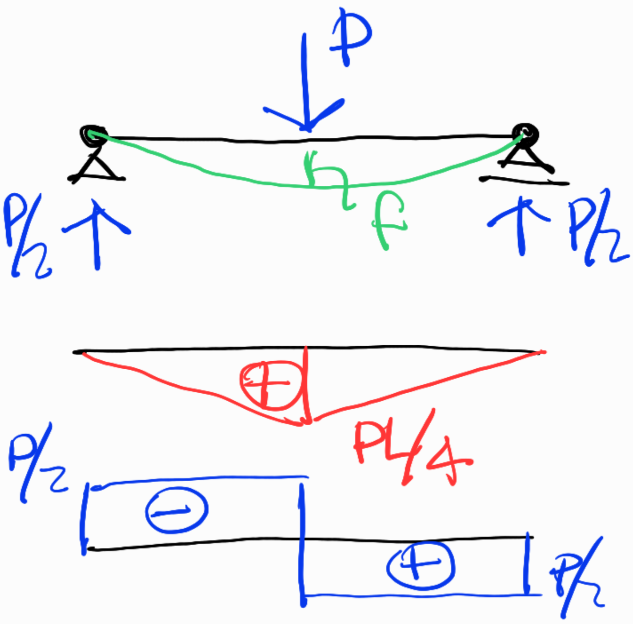

- Linealidad Cinemática → Pequeñas deformaciones

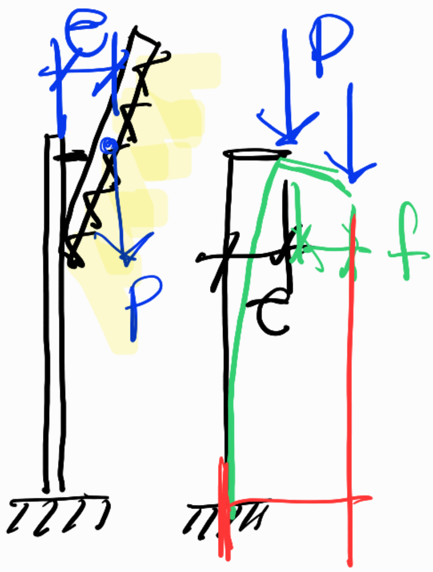
$$\downarrow$$

$$\theta \cong \operatorname{sen}\theta \cong \operatorname{tg}\theta$$

$$\cos\theta \cong 1$$

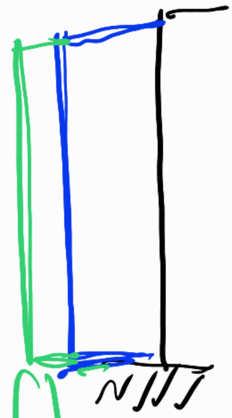
- Linealidad estática → El equilibrio se alcanza con independencia de las deformaciones




∃ H.L.E

1º orden

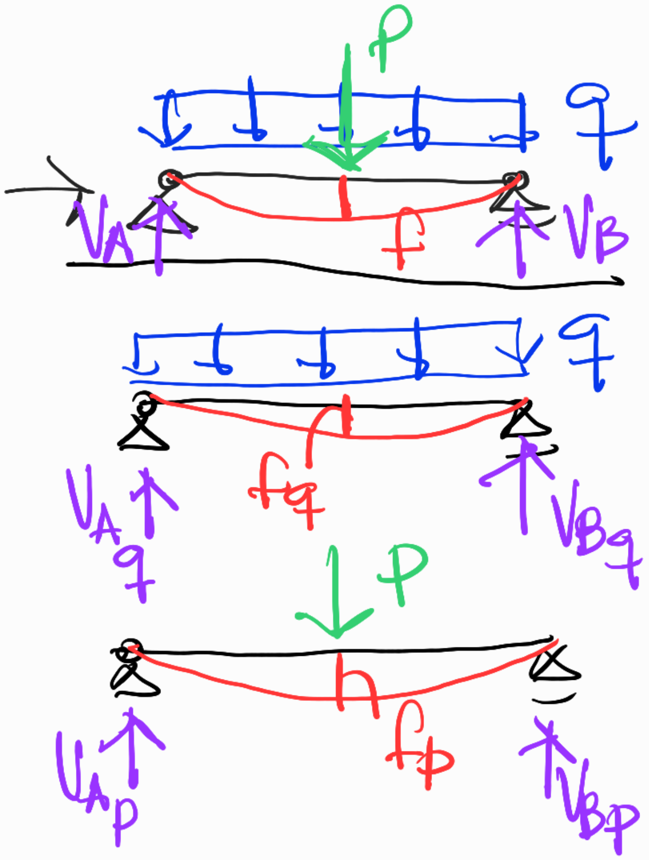


P(e + f)

∄ HLE

2º orden

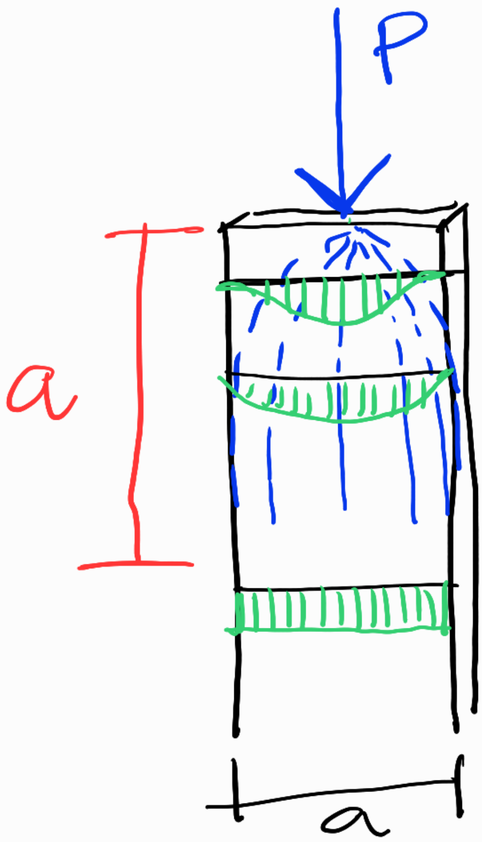
$\left.\begin{array}{l}\exists LM \\ \exists LC \\ \exists LE\end{array}\right\}$ PSEfecto →

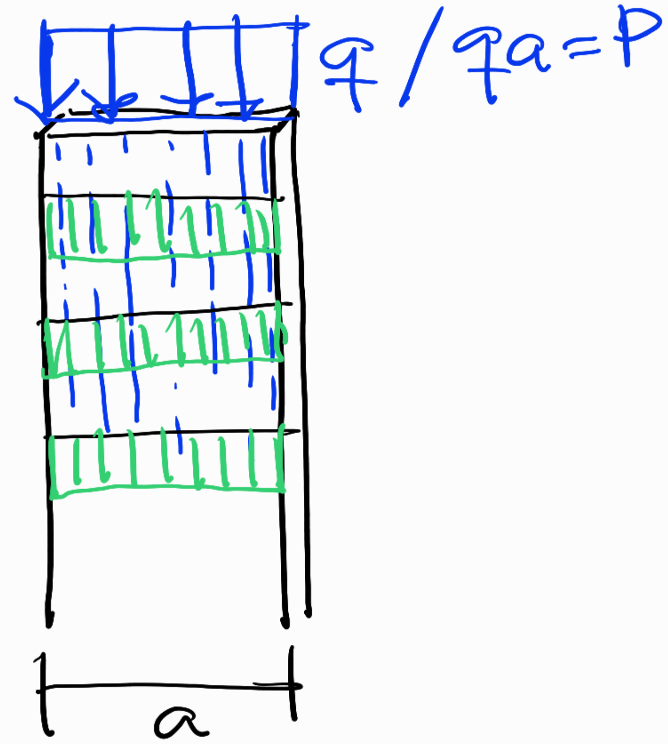



$f = f_q + f_p$

$V_A = V_{Aq} + V_{Ap}$

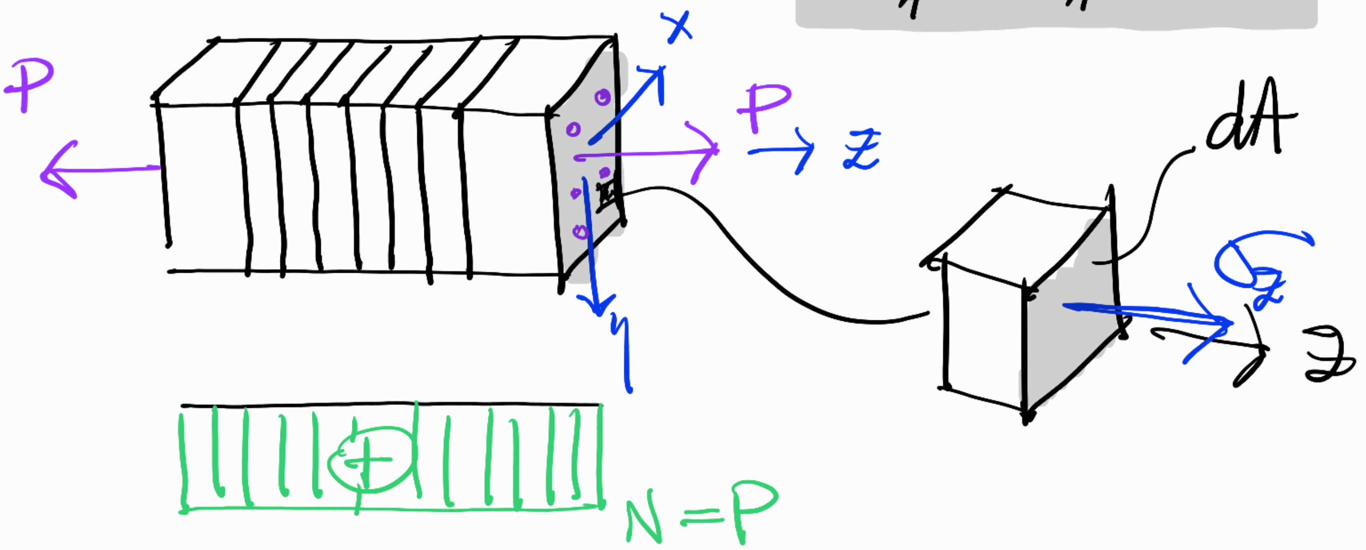
Ppio de Saint Venant → Vamos a estudiar las barras lejos de los puntos donde existan perturbaciones



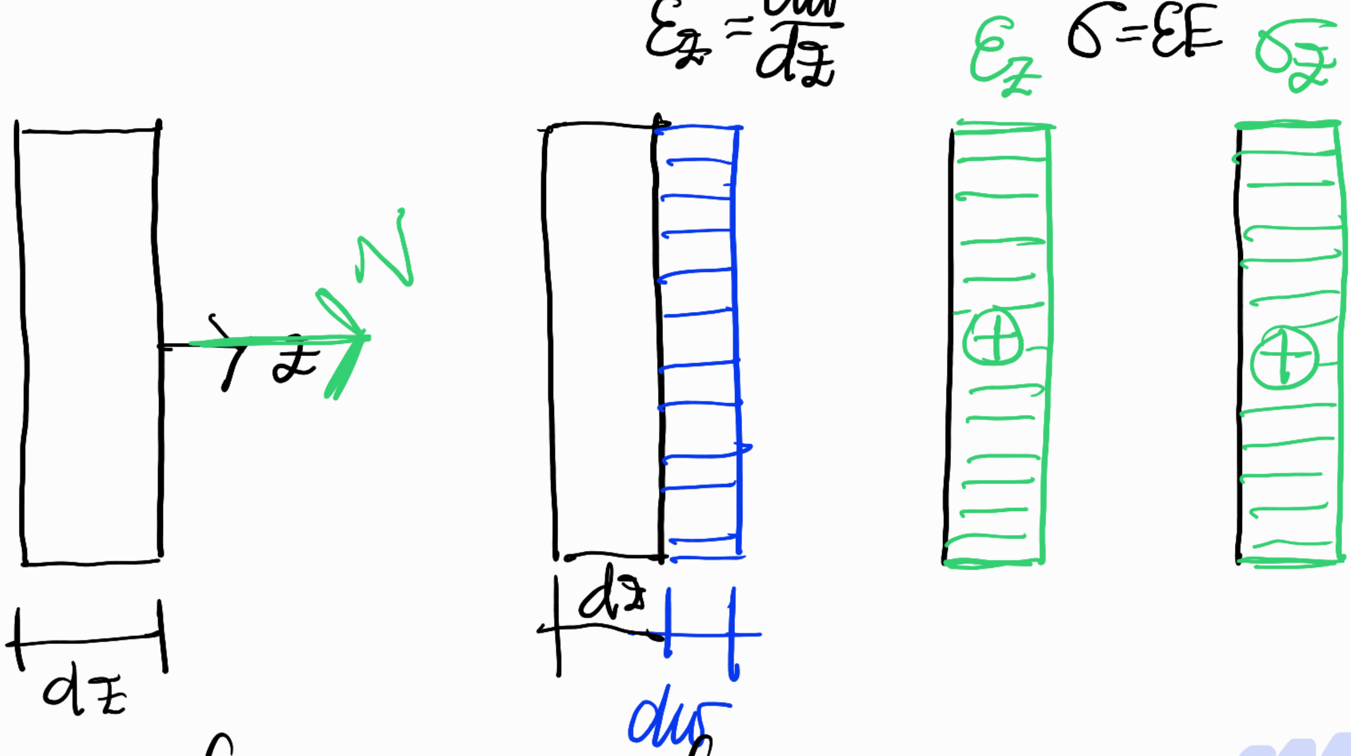
# Barras Solicitadas Axialmente en Régimen Elástico.

$$N = \int_A dN = \int_A \sigma_z \, dA$$



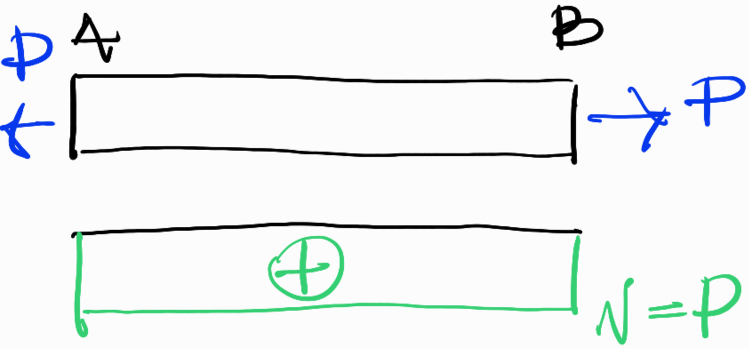

Hipótesis DE Secciones Planas y paralelas luego de la deformación.

$$\varepsilon_z = \frac{dw}{dz} \qquad \sigma = \varepsilon E$$




$$N = \int_A \sigma_z \, dA = \sigma_z \int_A dA = \sigma_z A \rightarrow \sigma_z = \frac{N}{A}$$

$$\varepsilon_z = \frac{\sigma_z}{E} = \frac{N}{EA}$$




$$\Delta l_{AB} = \int_L dw = \int_L \varepsilon_z \, dz$$

$$\Delta l_{AB} = \int_L \frac{N}{EA} \, dz$$

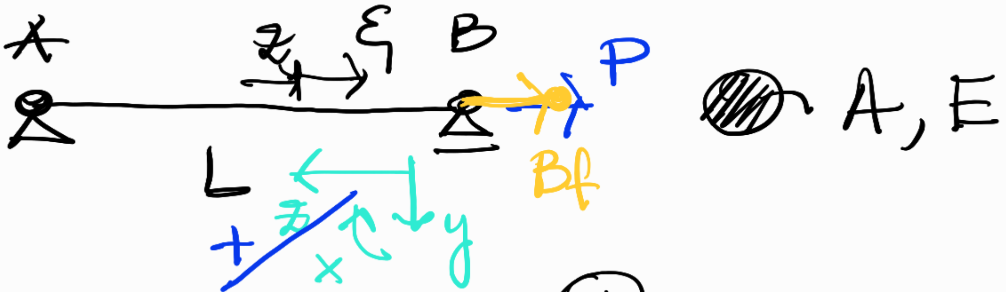
Si para $\underline{L}$ $\rightarrow$ $N = cte$

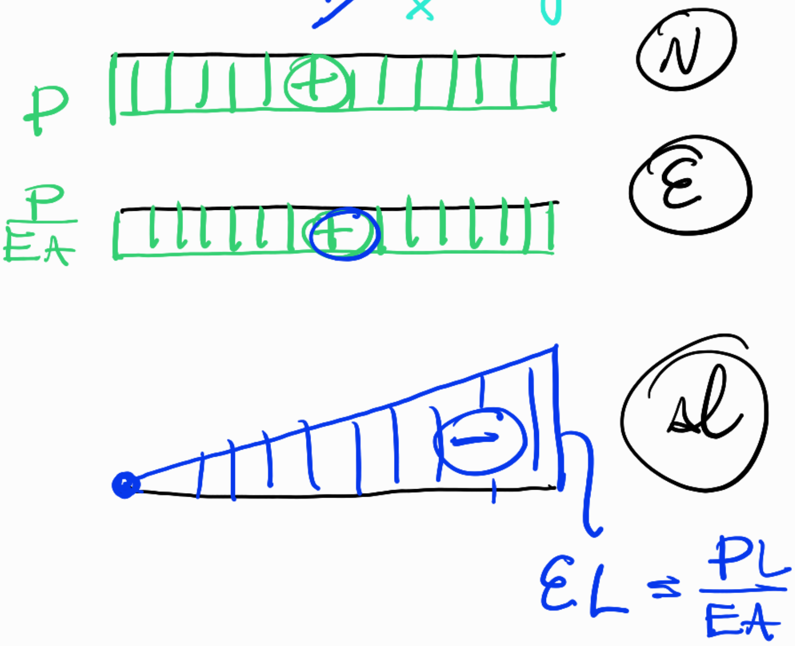
$EA = cte$

$$\Delta l_{AB} = \frac{N}{EA} \int_L dz = \frac{NL}{EA} \quad \varepsilon_z$$

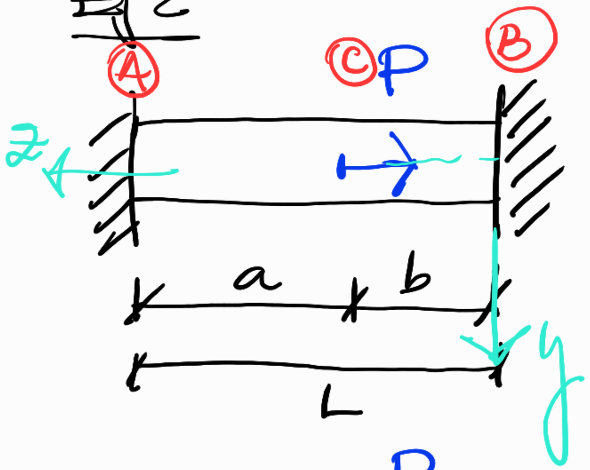
$$\Delta l_{AB} = \varepsilon_{AB} \, L$$

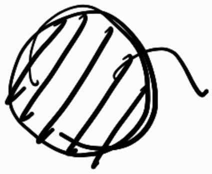
## Ej 1



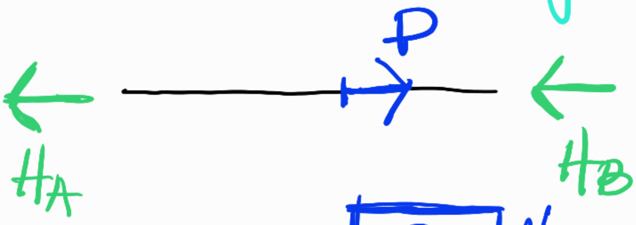



Ej 2

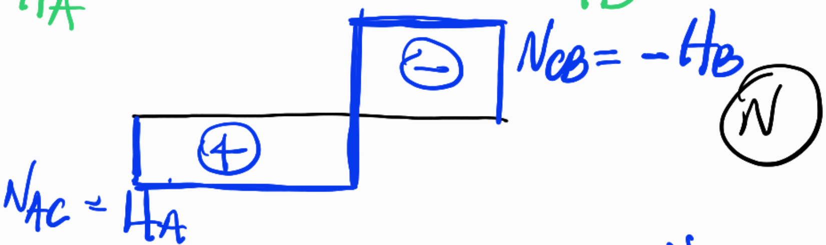


A; E



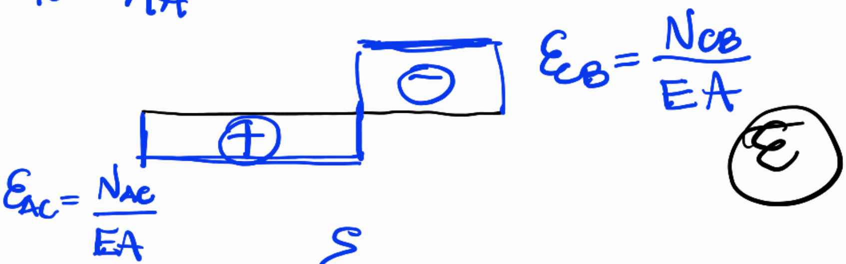

$$\sum F_z = 0 = H_A + H_B - P$$



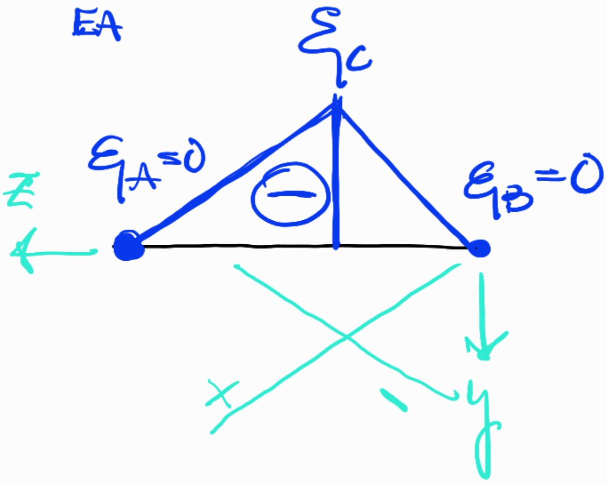

$$\delta_B = 0 = \overset{0}{\delta_A} + \Delta l_{AC} + \Delta l_{BC}$$

Ec. compatibilidad

$$0 = \Delta l_{AC} + \Delta l_{BC}$$




$$0 = \frac{N_{AC}\, a}{EA} + \frac{N_{CB}\, b}{EA}$$



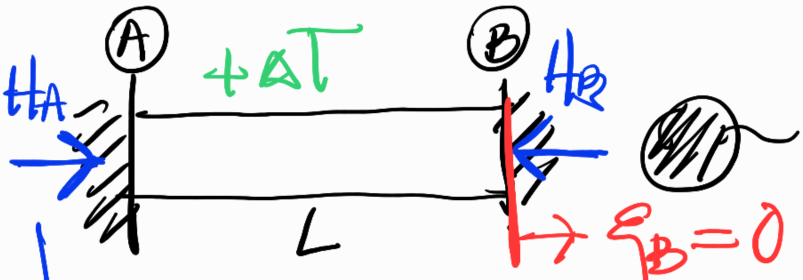

$$0 = \frac{H_A\, a}{\cancel{EA}} + \frac{(-H_B)\, b}{\cancel{EA}}$$

$$H_B\, b = H_A\, a \rightarrow H_B = H_A \frac{a}{b}$$

$$0 = H_A + H_B - P = H_A + H_A \frac{a}{b} - P$$

$$P = H_A \left(1 + \frac{a}{b}\right) = H_A \frac{a+b}{b} \rightarrow H_A = P\frac{b}{L}$$

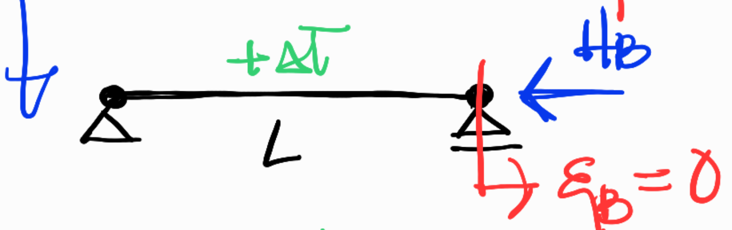
$$H_B = P\frac{a}{L}$$

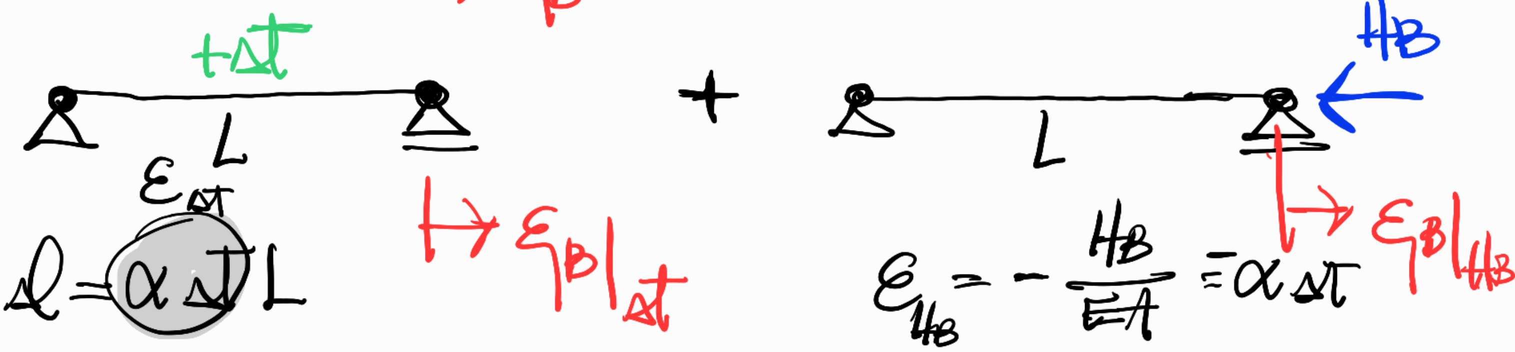



$A;\ E;\ \alpha$

↳ coef. de dilatación térmica

$(1/°C)$



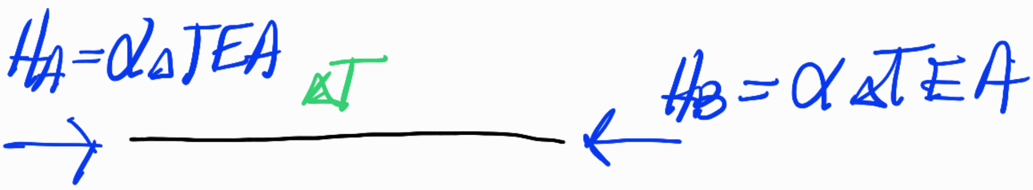

$$\ell = \underbrace{\alpha \Delta T}_{\varepsilon_{\Delta T}} L$$

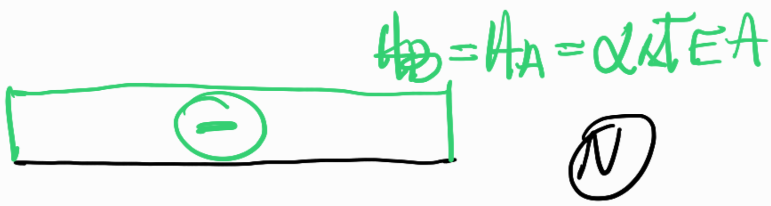
$$\varepsilon_{H_B} = -\frac{H_B}{EA} = \alpha \Delta T$$

$$0 = \varepsilon_B = \varepsilon_B|_{\Delta T} + \varepsilon_B|_{H_B}$$

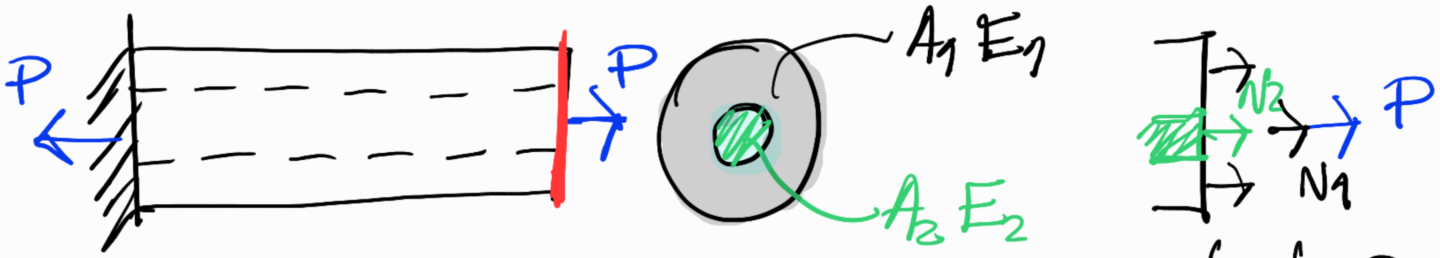
$$0 = (\alpha \Delta T \cancel{L}) + \left(-\frac{H_B \cancel{L}}{EA}\right)$$

$$H_B = \alpha \Delta T E A$$

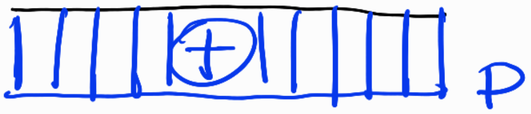




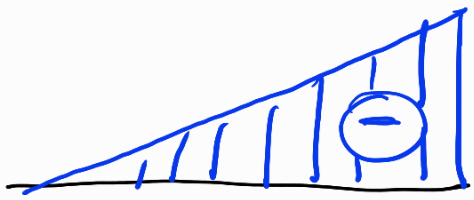



$$N_1 + N_2 = P$$





$$\varepsilon = \frac{P}{E_1A_1 + E_2A_2}$$




$$\Delta l_{AB} = \varepsilon_B = \varepsilon_{AB} L$$

$$= \frac{PL}{E_1A_1 + E_2A_2}$$

→ Rigidez total

$$\varepsilon_1 = \varepsilon_2 = \varepsilon$$

$$\varepsilon = \frac{N_1}{E_1A_1} = \frac{P\,\frac{E_1A_1}{E_1A_1+E_2A_2}}{\cancel{E_1A_1}}$$

$$\varepsilon_1$$

$$\varepsilon = P \Big/ (E_1A_1 + E_2A_2)$$

$$\Delta l_1 = \Delta l_2$$

$$\frac{N_1 \cancel{L}}{E_1A_1} = \frac{N_2 \cancel{L}}{E_2A_2}$$

$$N_1 = N_2 \frac{E_1A_1}{E_2A_2}$$

$$N_2 \left(\frac{E_1A_1}{E_2A_2} + 1\right) = P$$

$$N_2 \frac{E_1A_1 + E_2A_2}{E_2A_2} = P$$

$$N_2 = P \frac{E_2A_2}{E_1A_1 + E_2A_2}$$

$$N_1 = P \frac{E_1A_1}{E_1A_1 + E_2A_2}$$