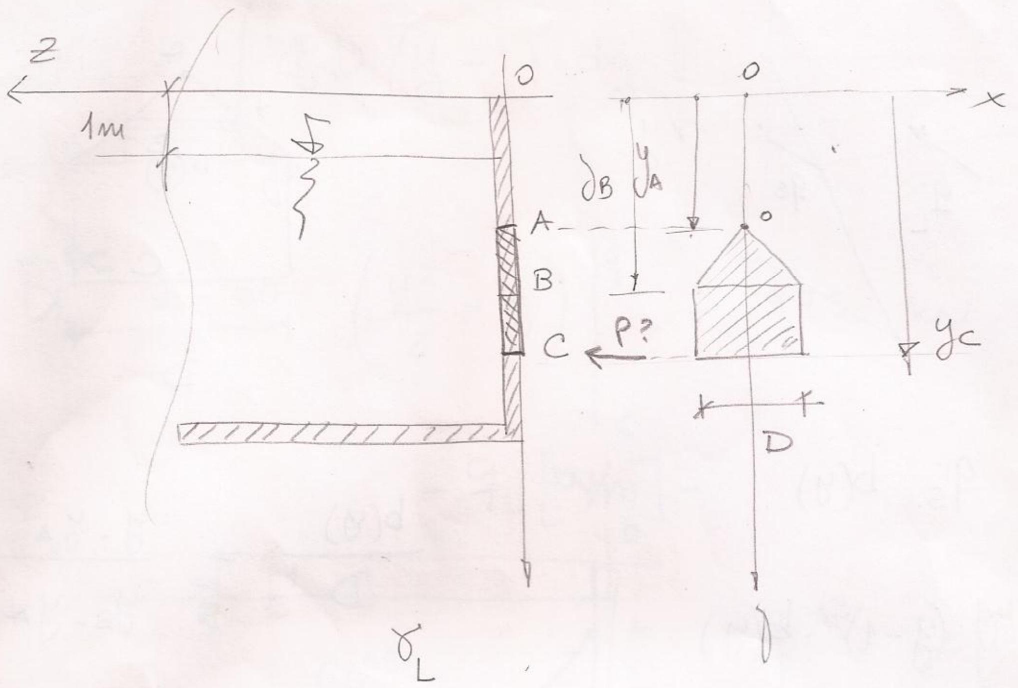
FUERZAS DISTRIBUIDAS

EJERCICIO (1)



DATOS

$D = 5m$

$y_A = 2m$

$y_B = 6m$

$y_C = 9m$

$\gamma_L = 10\ \frac{KN}{m^3}$

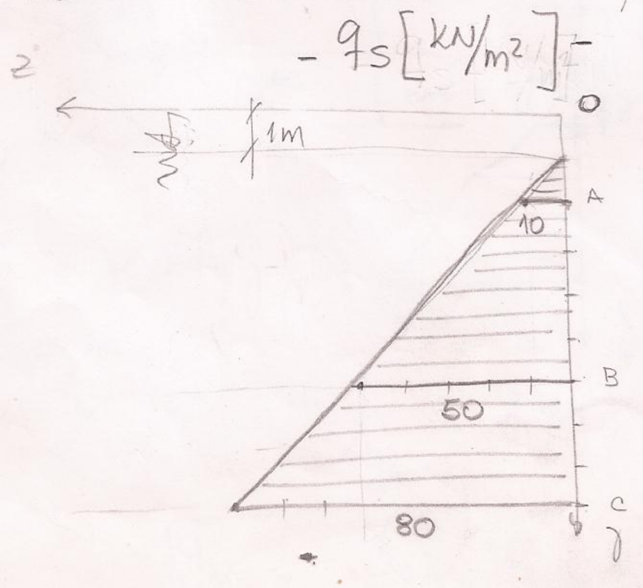
a) Dibujar el diagrama de carga específica superficial sobre la pared del depósito: $q_S = \gamma \cdot y = \left[KN/m^2\right]$

$q_S = \gamma\,(y-1)$ (En este caso el nivel del líquido está a 1m por debajo del eje z)

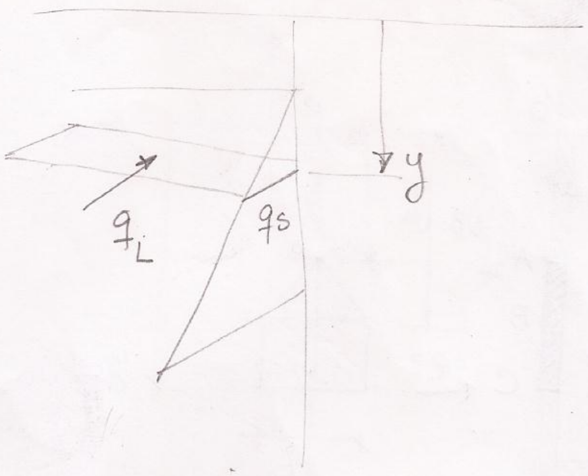
$q_S^A = \gamma\,(y_A - 1) = 10\ KN/m^2$

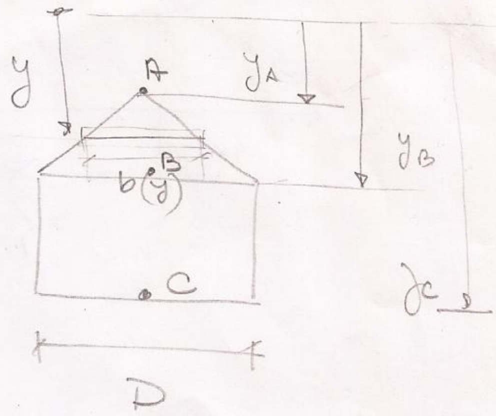
$q_S^B = \gamma\,(y_B - 1) = 50\ KN/m^2$

$q_S^C = \gamma\,(y_C - 1) = 80\ KN/m^2$



b) Dibujar el diagrama de carga específica lineal sobre el eje de simetría de la placa: $q_L \left[\frac{KN}{m}\right]$





$$q_L = q_s \cdot b(y)$$

$$\frac{b(y)}{D} = \frac{y - y_A}{y_B - y_A}$$

$$q_L = \gamma\,(y-1) \cdot b(y)$$

$$b(y) = D \cdot \left(\frac{y - y_A}{y_B - y_A} \right)$$

$$q_L^{AB} = \gamma (y-1) \cdot D \left(\frac{y - y_A}{y_B - y_A} \right)$$

$$q_L^{AB} = \frac{\gamma \cdot D}{y_B - y_A} \left(y^2 - y \cdot y_A - y + y_A \right)$$

$$q_L^{AB} = \frac{\gamma \cdot D}{y_B - y_A} \left[y^2 - y \left(y_A + 1 \right) + y_A \right]$$

$$Q_L^{AB} = \int_{y_A}^{y_B} q_L^{AB}\, dy.$$

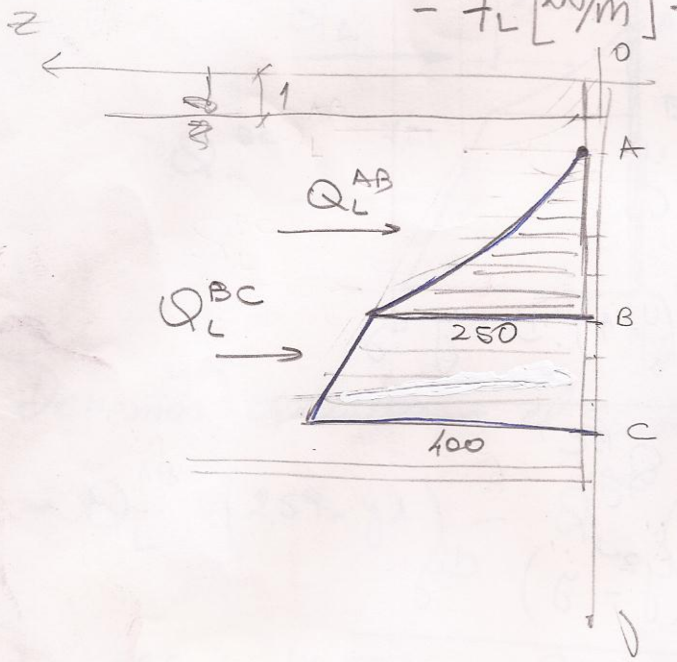
$$Q_L^{AB} = 366,6 \text{ KN}$$

$$q_L^{BC} = q_s \cdot D = \gamma \cdot (y-1) \cdot D$$

$$Q_L^{BC} = \int_{y_B}^{y_C} \gamma \cdot D \, (y-1) \, dy$$

$$Q_L^{BC} = 50 \cdot \left(\frac{y^2}{2} - y \right) \Big|_6^9 = 975 \text{ kN}$$

- q_L [kN/m] -



$$q_L^{AB} = \frac{25}{2} \left(y^2 - 3y + 2 \right)$$

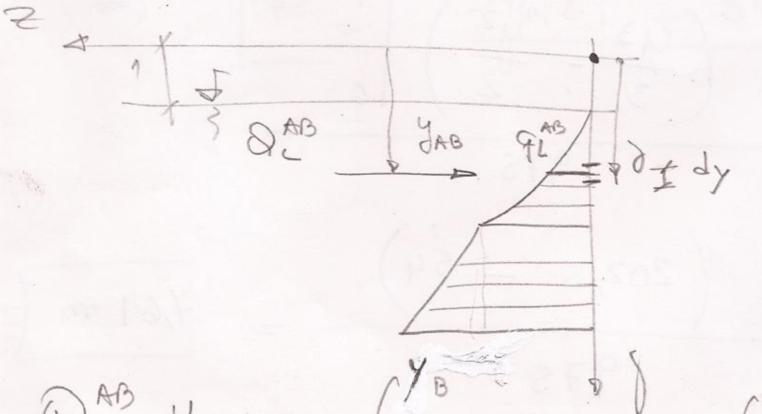
$$Q^A = \frac{25}{2} (4 - 6 + 2) = 0$$

$$Q^B = \frac{25}{2} (36 - 18 + 2)$$

$$q_L^{BC} = 50 (y-1)$$

$$Q^C = 400$$

c) Hallar la resultante del empuje del líquido.



$$\boxed{M^0_{Q_L^{AB}} = \int_{y_A}^{y_B} q_L \, y \, dy}$$

$$Q_L^{AB} \cdot y_{AB} = \int_{y_A}^{y_B} q_L \cdot y \, dy = \int_{y_A}^{y_B} \gamma (y-1) \cdot b(y) \cdot y \, dy$$

$$\boxed{y_{AB}} = \frac{\int_{y_A}^{y_B} \frac{25}{2} \left(y^2 - 3y + 2 \right) y \, dy}{Q_L^{AB}}$$

$$y_{AB} = \frac{\int_2 \frac{25}{2} \left(y^3 - 3y^2 - 2y\right) dy}{Q_L^{AB}}$$

$$y_{AB} = \frac{\frac{25}{2} \left(\frac{y^4}{4} - y^3 - y^2\right)\Big|_2^6}{Q_L^{AB}}$$

$$\boxed{y_{AB} = \frac{950}{366{,}6} = 2{,}59 \text{ m}}$$

$$Q_L^{BC} \cdot y_{BC} = q_L^{BC} \cdot y$$

$$y_{BC} = \frac{\int_{y_B}^{y_C} \gamma \cdot (y-1) \cdot D \cdot y \, dy}{Q_L^{BC}}$$

$$y_{BC} = \frac{\int_6^9 \gamma \cdot D \left(y^2 - y\right) dy}{Q_L^{BC}}$$

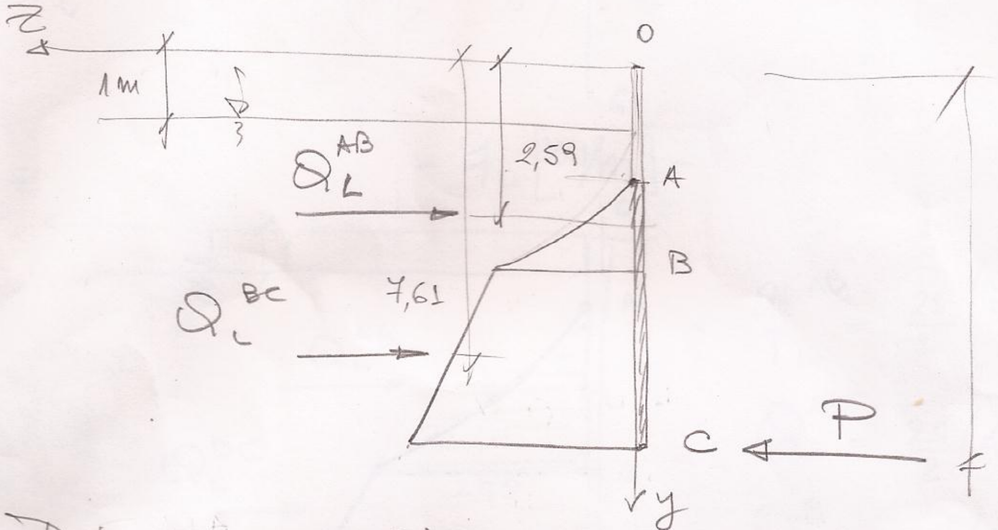
$$y_{BC} = \frac{50 \left(\frac{y^3}{3} - \frac{y^2}{2}\right)\Big|_6^9}{975}$$

$$\boxed{y_{BC} = \frac{50 \left(202{,}5 - 54\right)}{975} = 7{,}61 \text{ m}}$$

La Resultante del empuje del liquido es:

$$\boxed{Q_T = Q_L^{AB} + Q_L^{BC} = 366{,}6 + 975 = 1341{,}6 \text{ kN}}$$

d) Determinar la fuerza **P** requerida para cerrar la compuerta venciendo la presion del liquido.



Debemos equilibrar el sistema de los $\vec{F}$ de Empuje con P:

$$-Q_L^{AB}(2{,}59 - y_A) - Q_L^{BC}(7{,}61 - y_A) + P \cdot 9\text{m} = 0$$

$$-366{,}6 \cdot (0{,}59) - 975 \cdot 5{,}61 + P \cdot 9 = 0$$

$$\Rightarrow \boxed{P} = \frac{216{,}29 + 5469{,}75}{9} = \boxed{631{,}79 \text{ kN}}$$