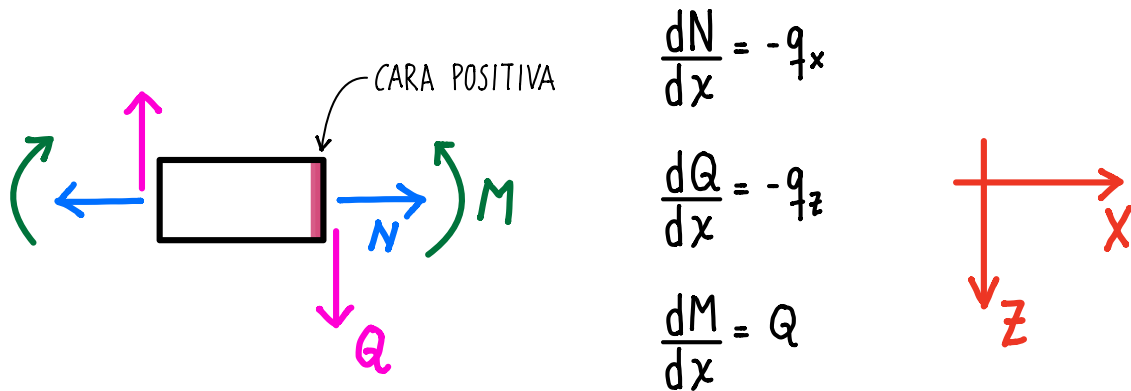
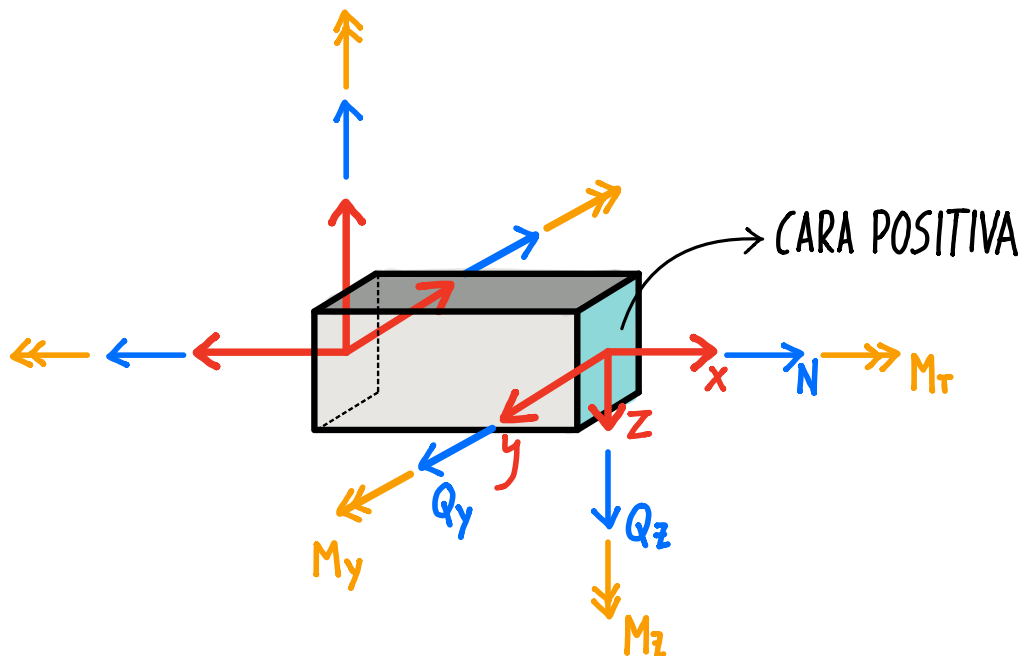


## DIAGRAMAS DE CARACTERÍSTICAS 3D

YA SABÍAMOS :



AHORA EN 3D APARECEN :



RELACIONES DIFERENCIALES ENTRE  
CARGAS EXTERIORES Y  
ESFUERZOS INTERNOS

$$\frac{dN}{dx} = -q_x$$

$$\frac{dQ_z}{dx} = -q_z$$

$$\frac{dQ_y}{dx} = -q_y$$

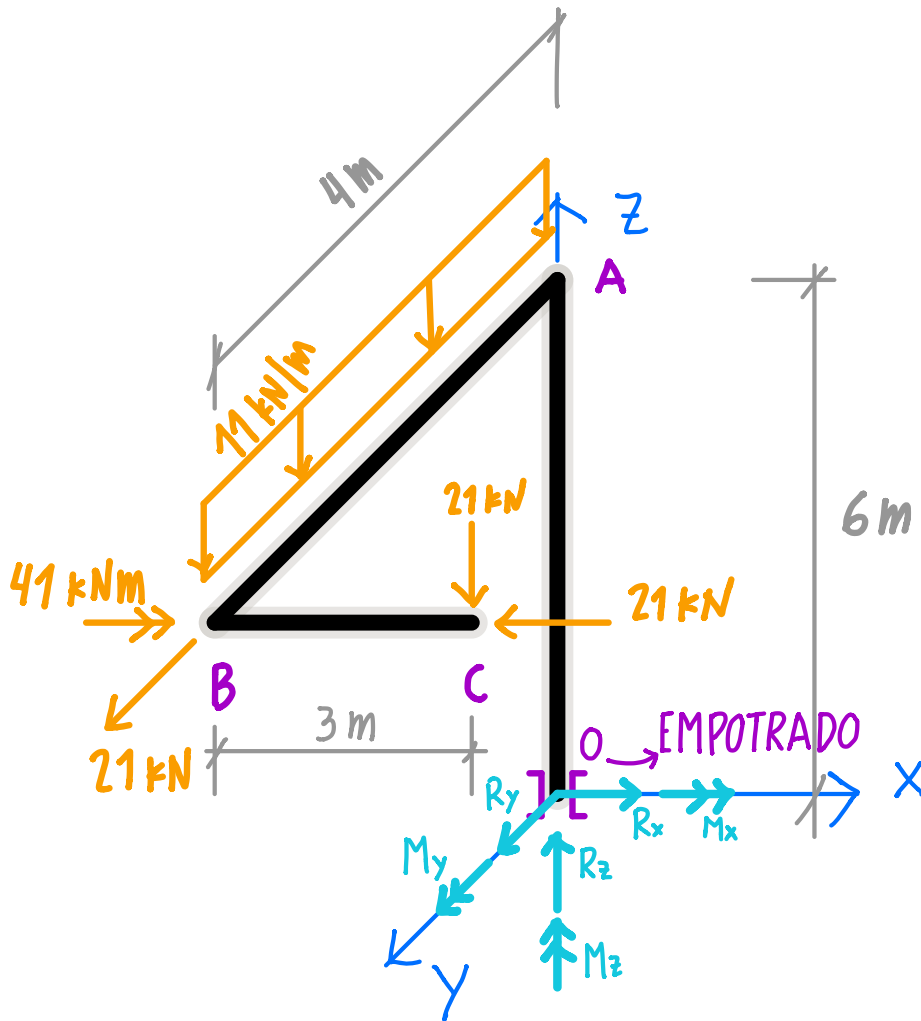
$$\frac{dM_t}{dx} = m_t$$

$$\frac{dM_y}{dx} = Q_z$$

$$\frac{dM_z}{dx} = -Q_y$$

# EJERCICIOS

## 1.1 TRAZAR LOS DIAG. DE ESF. CARACTERÍSTICOS



### CHECK LIST

- 1- R.V
- 2- TERNAS LOCALES
- 3- ANÁLISIS EXPLORATORIO (E.D Y PINTA)
- 4- DIAGRAMAS ( $N, Q_y, Q_z, M_x, M_y, M_z$ )
- 5- CHECK NUDOS

### 1- REACCIONES DE VÍNCULO → 3D: 6 INCÓGNITAS

$$\sum F_x = -21 \text{ kN} + R_x = 0 \rightarrow R_x = 21 \text{ kN}$$

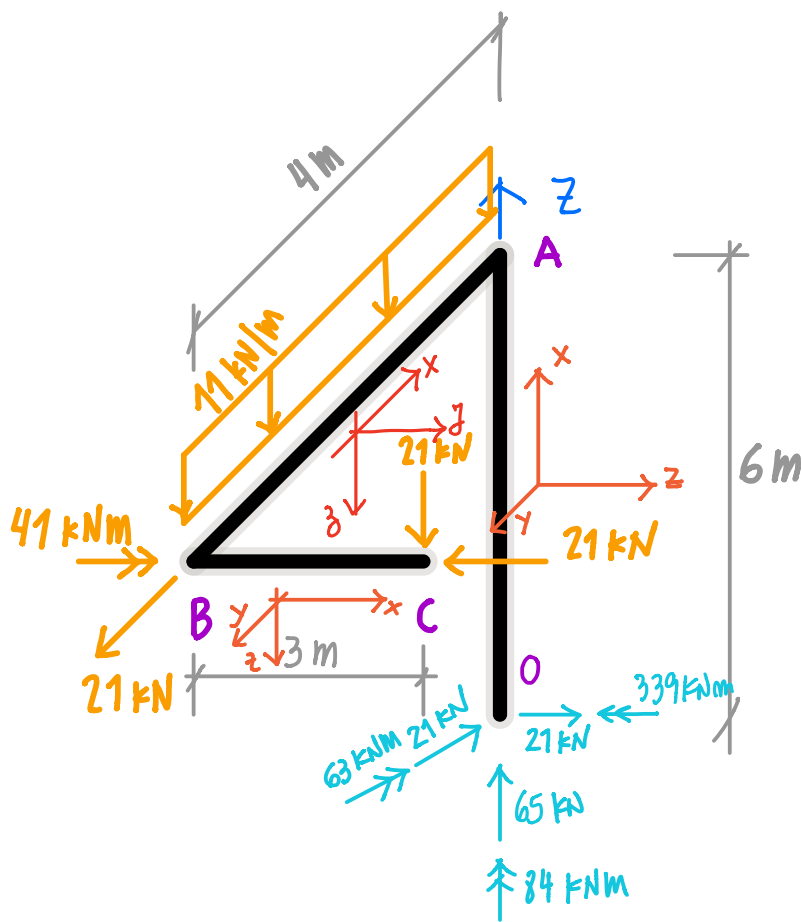
$$\sum F_y = 21 \text{ kN} + R_y = 0 \rightarrow R_y = -21 \text{ kN}$$

$$\sum F_z = -(11 \text{ kN/m} \cdot 4 \text{ m}) - 21 \text{ kN} + R_z = 0 \rightarrow R_z = 65 \text{ kN}$$

$$\sum M_x^0 = M_x + 4 \text{ m} \cdot 21 \text{ kN} + (11 \text{ kN/m} \cdot 4 \text{ m}) \cdot \frac{4 \text{ m}}{2} + 41 \text{ kNm} + 6 \text{ m} \cdot 21 \text{ kN} = 0 \rightarrow M_x = -339 \text{ kNm}$$

$$\sum M_y^0 = M_y + 6 \text{ m} \cdot 21 \text{ kN} - 3 \text{ m} \cdot 21 \text{ kN} = 0 \rightarrow M_y = -63 \text{ kNm}$$

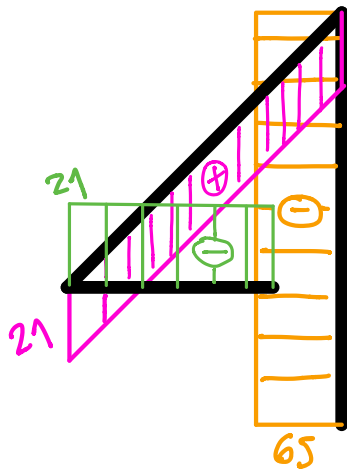
$$\sum M_z^0 = M_z - 4 \text{ m} \cdot 21 \text{ kN} = 0 \rightarrow M_z = 84 \text{ kNm}$$



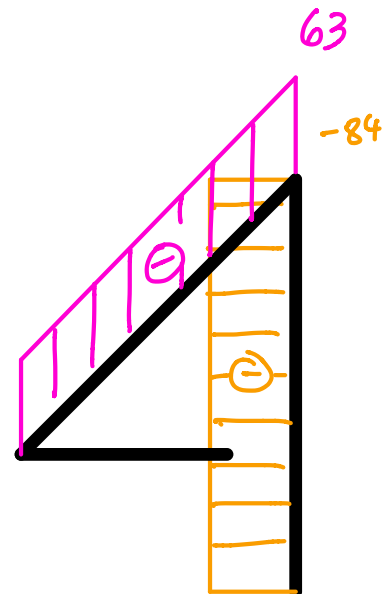
Ahora los diagramas se dibujan en 3D

- \*  $N$  y  $M_T$  en el Plano  $x-z$
- \*  $Q_y$  en el Plano  $x-y$
- \*  $M_z$  en el Plano  $y-z$
- \*  $Q_z$  en el Plano  $x-z$
- \*  $M_y$  en el Plano  $x-y$

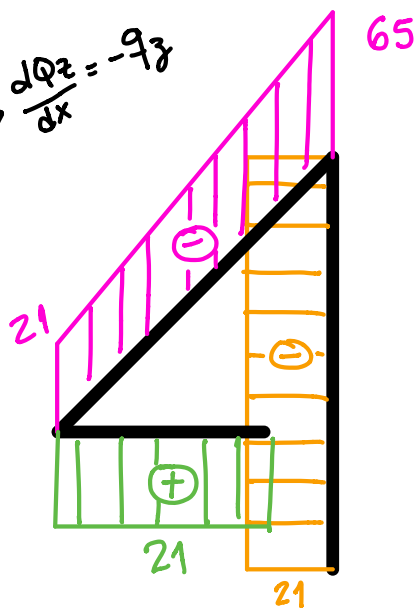
$N$



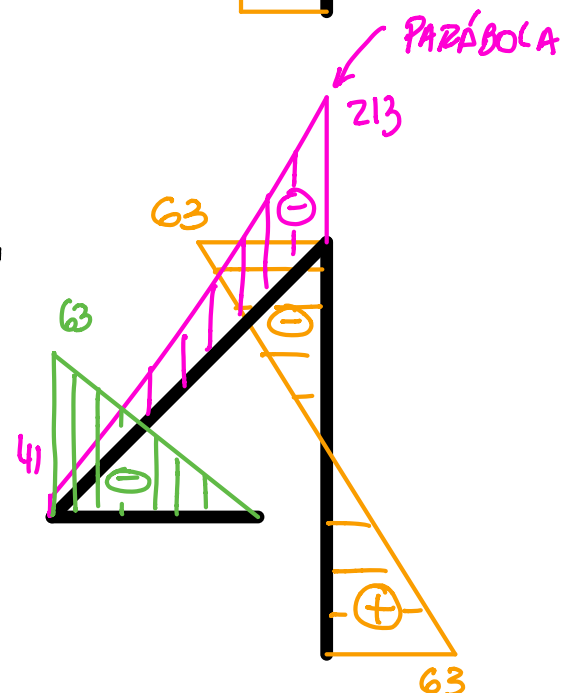
$M_T$



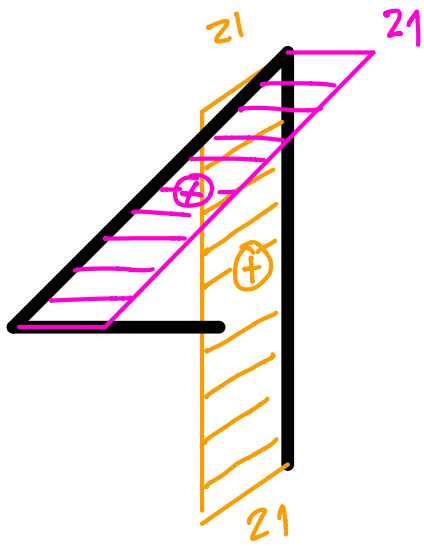
$Q_z \rightarrow \frac{dQ_z}{dx} = -9z$



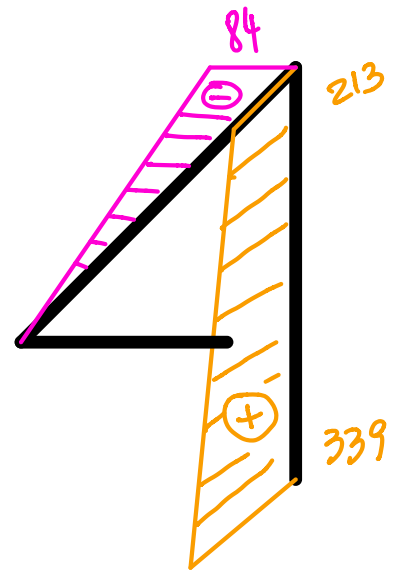
$M_y$



$Q_y$

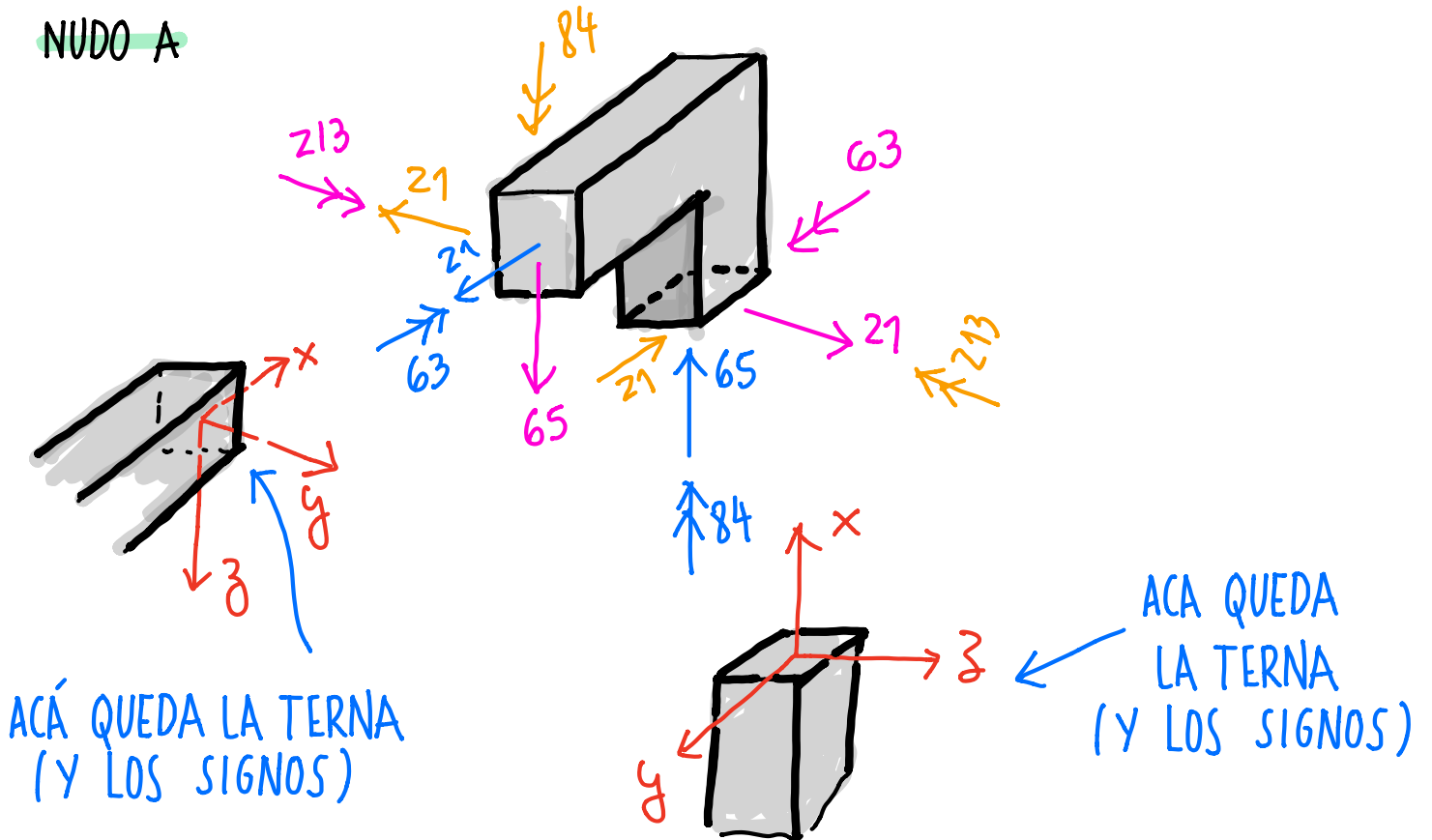


$M_z$

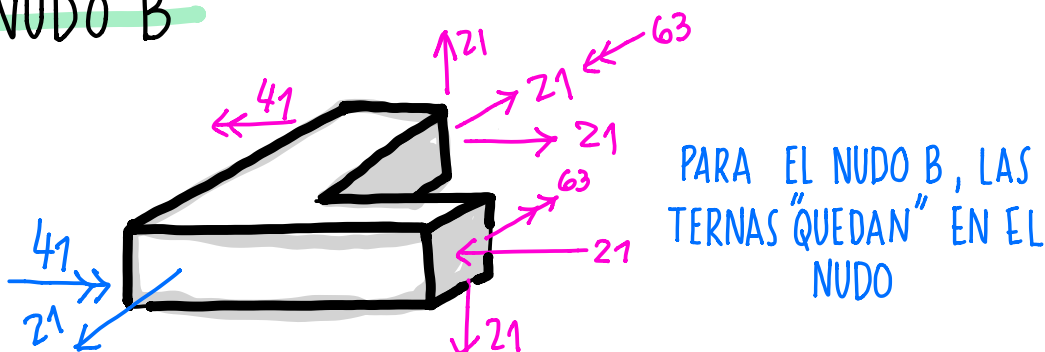


## EQUILIBRIO DE NUDOS

### NUDO A

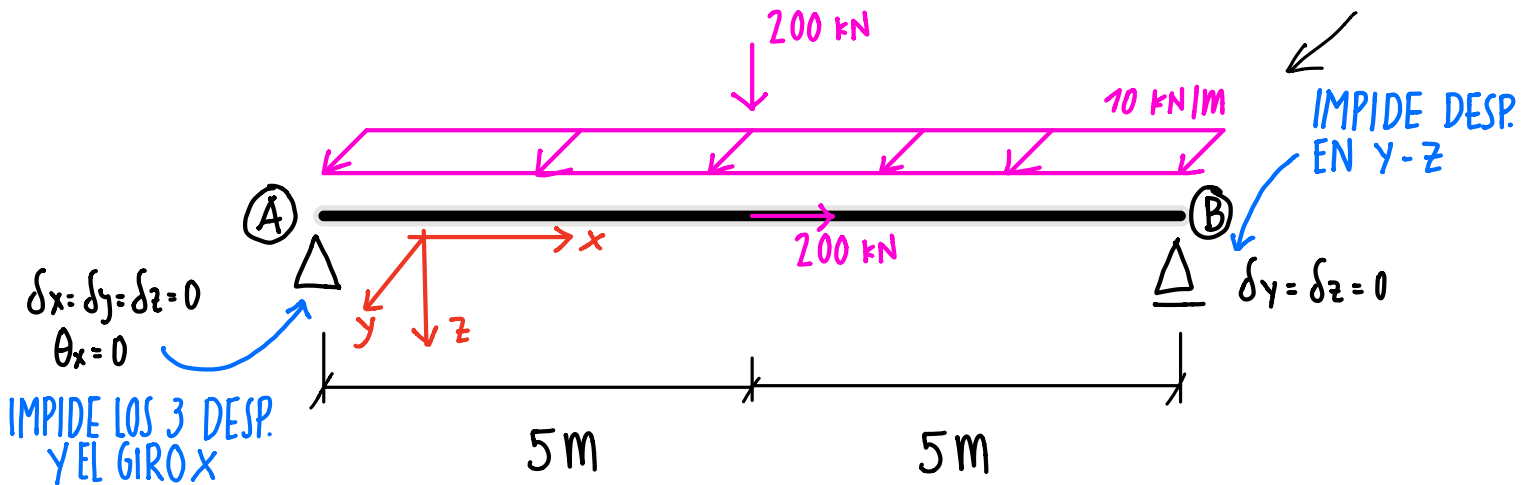


### NUDO B

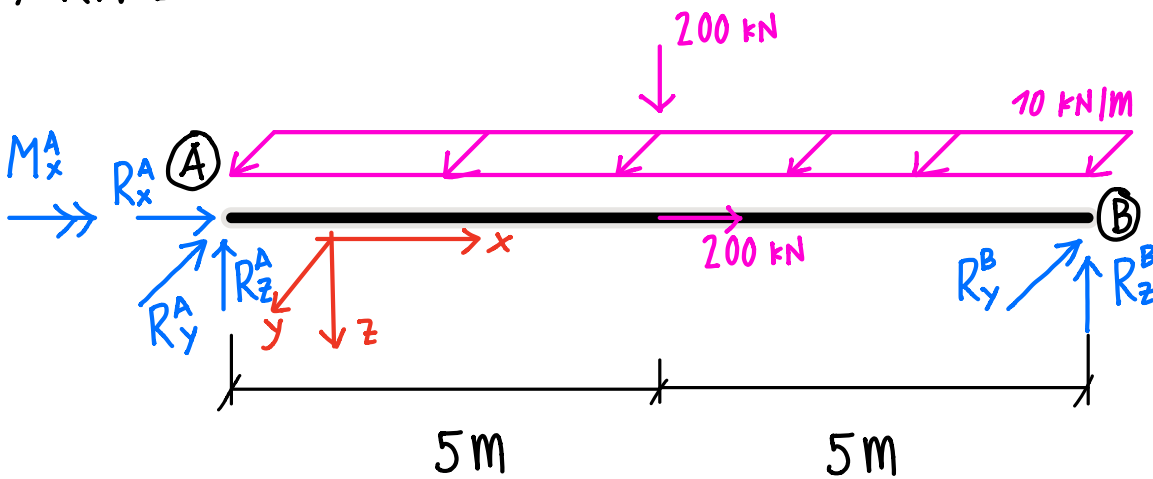


1.6

1.6 La viga de la figura soporta 3 cargas: una concentrada, vertical, dirección Z, una distribuida en plano horizontal, dirección Y, y una paralela al eje de la viga, dirección X. Para dimensionar la viga es necesario conocer los 6 esfuerzos característicos en la sección media. Determinarlos, siendo  $L=10$  m. Indicar claramente dirección y sentido de los esfuerzos. Nota: el sistema de coordenadas globales coincide con el local.



1- R.V.E



$$\sum F_x = R_x^A + 200 \text{ kN} = 0 \rightarrow \boxed{R_x^A = -200 \text{ kN}}$$

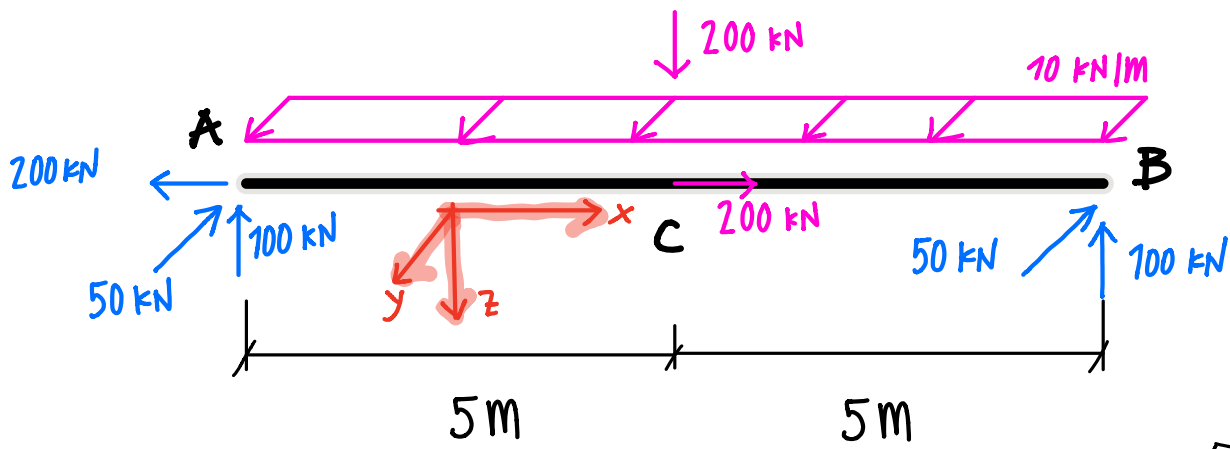
$$\sum M_y^A = -10 \text{ m} \cdot R_z^B + 5 \text{ m} \cdot 200 \text{ kN} = 0 \rightarrow \boxed{R_z^B = 100 \text{ kN}}$$

$$\sum F_z = -R_z^A - R_z^B + 200 \text{ kN} = 0 \rightarrow \boxed{R_z^A = 100 \text{ kN}}$$

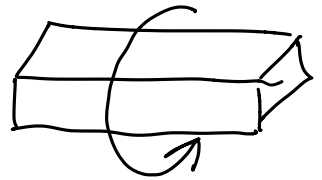
$$\sum M_z^A = -10 \text{ m} R_y^B + (10 \text{ kN/m} \cdot 10 \text{ m}) \cdot \frac{10 \text{ m}}{2} = 0 \rightarrow \boxed{R_y^B = 50 \text{ kN}}$$

$$\sum F_y = -R_y^A - R_y^B + (10 \text{ kN/m} \cdot 10 \text{ m}) = 0 \rightarrow \boxed{R_y^A = 50 \text{ kN}}$$

$$\sum M_x^A = \boxed{M_x^A = 0}$$



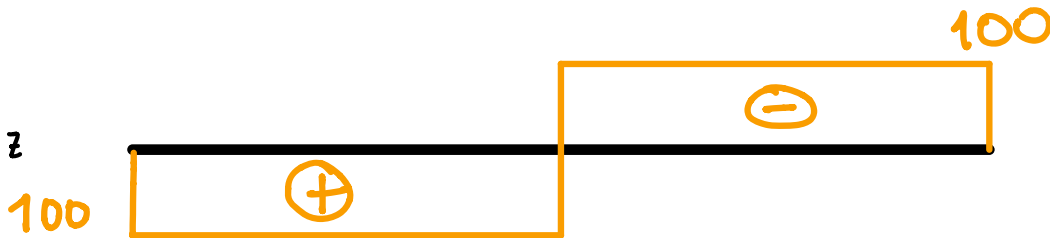
$M_T$



$N$

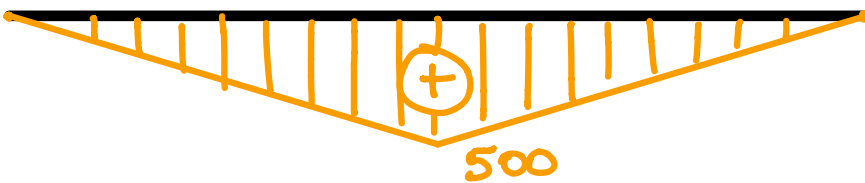


$Q_z$



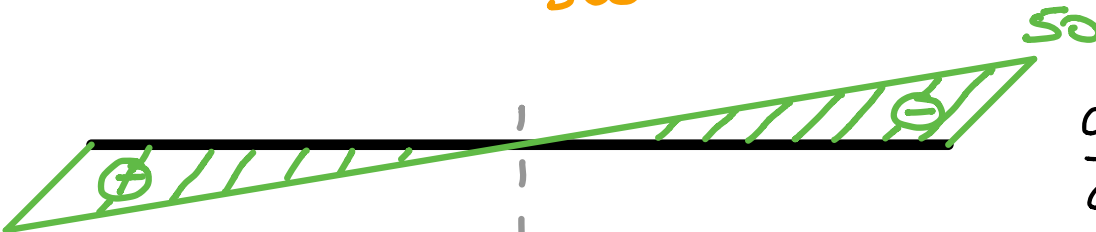
$$\frac{dQ_z}{dx} = -q_z$$

$M_y$



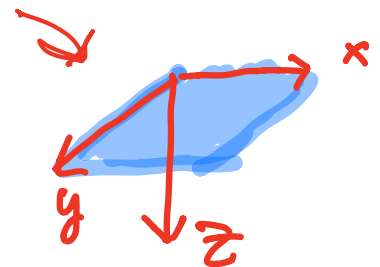
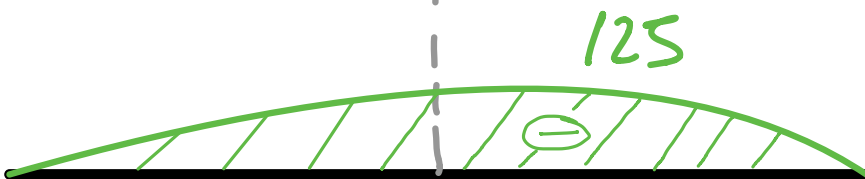
$$\frac{dM_y}{dx} = Q_z$$

$Q_y$



$$\frac{dQ_y}{dx} = -q_y$$

$M_z$



$$\frac{dM_z}{dx} = -Q_y$$