Variables negativas con cota mínima

$x_j \geq -a$ $\quad x_j - x'_j = -a \quad \therefore \quad x_j = x'_j - a$

MAX: $-3x_1 + 6x_2 - 4x_3$

Sujeto a:

$$4x_1 - x_2 - x_3 \geq 100$$

$$2x_1 + 3x_2 - 2x_3 \leq 100$$

$$6x_1 - x_2 + 4x_3 \leq 150$$

siendo: x_1, x_2 no negativas

y: $x_3 \geq -15$

Variables negativas con cota mínima

$x_3 \geq -15$ $x_3 - x'_3 = -15$ $\therefore$ $x_3 = x'_3 - 15$

MAX: $-3x_1 + 6x_2 - 4x_3$

Sujeto a:

$$4x_1 - x_2 - x_3 \geq 100$$
$$2x_1 + 3x_2 - 2x_3 \leq 100$$
$$6x_1 - x_2 + 4x_3 \leq 150$$

siendo: x_1, x_2 no negativas

y: $x_3 \geq -15$

Variables negativas con cota mínima

$x_3 = x'_3 - 15$

MAX: $-3x_1 + 6x_2 - 4(x'_3 - 15)$

Sujeto a:

$$4x_1 - x_2 - (x'_3 - 15) \geq 100$$

$$2x_1 + 3x_2 - 2(x'_3 - 15) \leq 100$$

$$6x_1 - x_2 + 4(x'_3 - 15) \leq 150$$

siendo: x_1, x_2, x'_3 no negativas

Variables negativas con cota mínima

$x_3 = x'_3 - 15$

MAX: $-3x_1 + 6x_2 - 4x'_3 + 60$

Sujeto a:

$$4x_1 - x_2 - x'_3 + 15 \geq 100$$

$$2x_1 + 3x_2 - 2x'_3 + 30 \leq 100$$

$$6x_1 - x_2 + 4x'_3 - 60 \leq 150$$

siendo: x_1, x_2, x'_3 no negativas

Variables negativas con cota mínima

$x_3 = x'_3 - 15$

MAX: $-3x_1 + 6x_2 - 4x'_3 + x_4$

Sujeto a:

$$
\begin{aligned}
4x_1 - x_2 - x'_3 &\geq 85 \\
2x_1 + 3x_2 - 2x'_3 &\leq 70 \\
6x_1 - x_2 + 4x'_3 &\leq 210 \\
x_4 &= 60
\end{aligned}
$$

siendo: x_1, x_2, x'_3 no negativas

Variables negativas con cota mínima

$x_3 = x'_3 - 15$

Z	37.50
x_1	23.21
x_2	7.86
x'_3	0
x_4	60

→ $x_3 = -15$

Variables irrestrictas

$$x_i = x_i^+ - x_i^-$$

MIN $\quad 2x_1 + x_2 - 2x_3$

$$\begin{cases} x_1 + x_2 + 2x_3 \leq 10 \\ -4x_1 - x_2 + x_3 \leq 8 \\ x_2 \geq 2 \\ x_1 - x_2 \leq 5 \end{cases}$$

con: $\quad x_2, x_3 \geq 0$

Variables irrestrictas

$$x_1 = x_1^+ - x_1^-$$

MIN $\quad 2x_1 + x_2 - 2x_3$

$$\begin{cases} x_1 + x_2 + 2x_3 \leq 10 \\ -4x_1 - x_2 + x_3 \leq 8 \\ x_2 \geq 2 \\ x_1 - x_2 \leq 5 \end{cases}$$

con: $\quad x_2, x_3 \geq 0$

Variables irrestrictas

$$x_1 = x_1^+ - x_1^-$$

$$\text{MIN} \quad 2(x_1^+ - x_1^-) + x_2 - 2x_3$$

$$\begin{cases} (x_1^+ - x_1^-) + x_2 + 2x_3 \leq 10 \\ -4(x_1^+ - x_1^-) - x_2 + x_3 \leq 8 \\ x_2 \geq 2 \\ (x_1^+ - x_1^-) - x_2 \leq 5 \end{cases}$$

$$\text{con} \quad x_1^+, x_1^-, x_2, x_3 \geq 0$$

Variables irrestrictas

$$x_1 = x_1^+ - x_1^-$$

MIN $\quad 2x_1^+ - 2x_1^- + x_2 - 2x_3$

$$\begin{cases} x_1^+ - x_1^- + x_2 + 2x_3 \leq 10 \\ -4x_1^+ + 4x_1^- - x_2 + x_3 \leq 8 \\ x_2 \geq 2 \\ x_1^+ - x_1^- - x_2 \leq 5 \end{cases}$$

con $\quad x^+_1, x_1^-, x_2, x_3 \geq 0$

Variables irrestrictas

$$x_1 = x_1^+ - x_1^-$$

Z	- 10
x_1^+	0
x_1^-	1.33
x_2	2
x_3	4.67

Formulación con LINDO

```
MIN   2 x1 + x2  - 2 x3
ST
x1 + x2  + 2 x3  <=  10
- 4 x1 - x2  + x3  <=  8
x2  >=  2
x1 - x2 <=  5

END

FREE X1
```

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) -10.00000

VARIABLE	VALUE	REDUCED COST
X1	-1.333333	0.000000
X2	2.000000	0.000000
X3	4.666667	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.666667
3)	0.000000	0.666667
4)	0.000000	-1.000000
5)	8.333333	0.000000

NO. ITERATIONS= 3

x_1^+

PROGRAMACIÓN DE METAS

RESTRICCIÓN ESTRUCTURAL

$$9\ x_1 + 9\ x_2 \leq 36.000$$

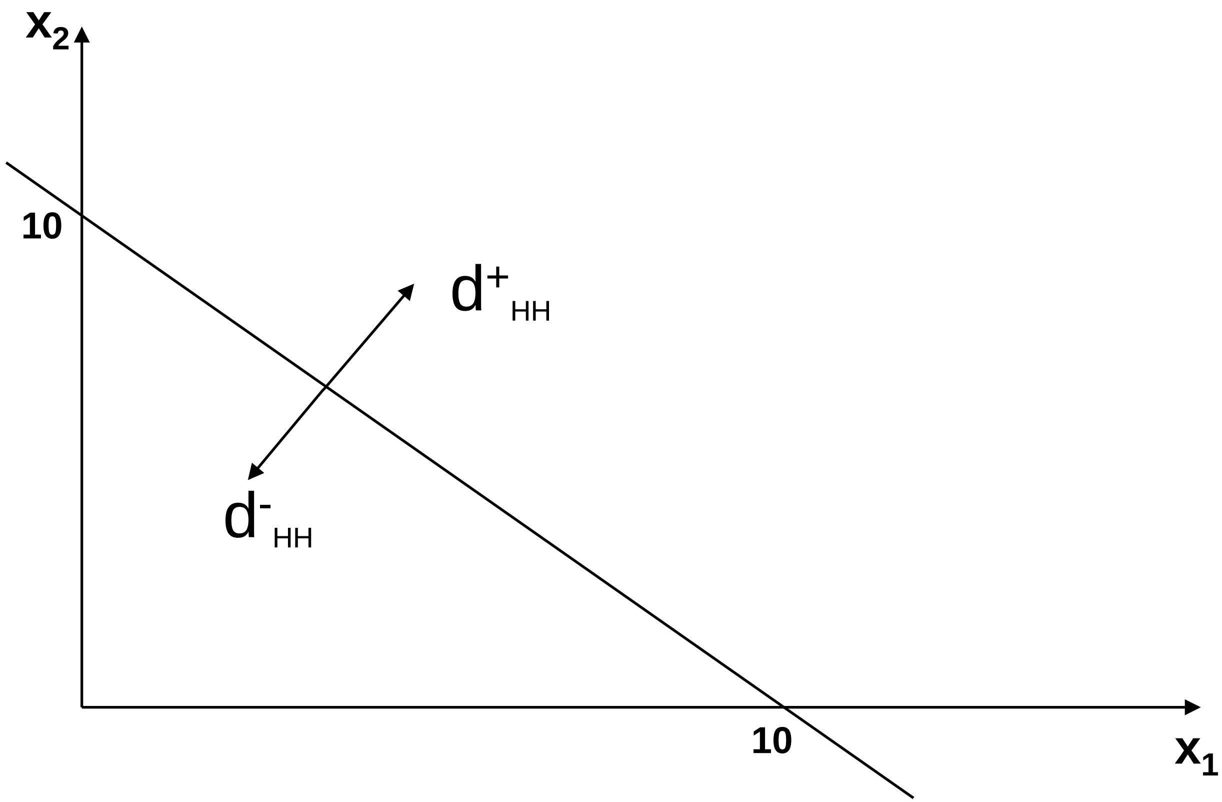
RESTRICCIÓN DE META

$$9\ x_1 + 9\ x_2 + d^-_P - d^+_P = 36.000$$

Ejemplo:

- Cada pieza A insume 10 hh
- Caba pieza B insume 10 hh
- Se dispone de 100 hh regulares
- Se desea
  - Utilizar a pleno la capacidad de MO
  - Mantener en un mínimo las hh extras

$$10\ x_1 + 10\ x_2 + d^-_{HH} - d^+_{HH} = 100$$

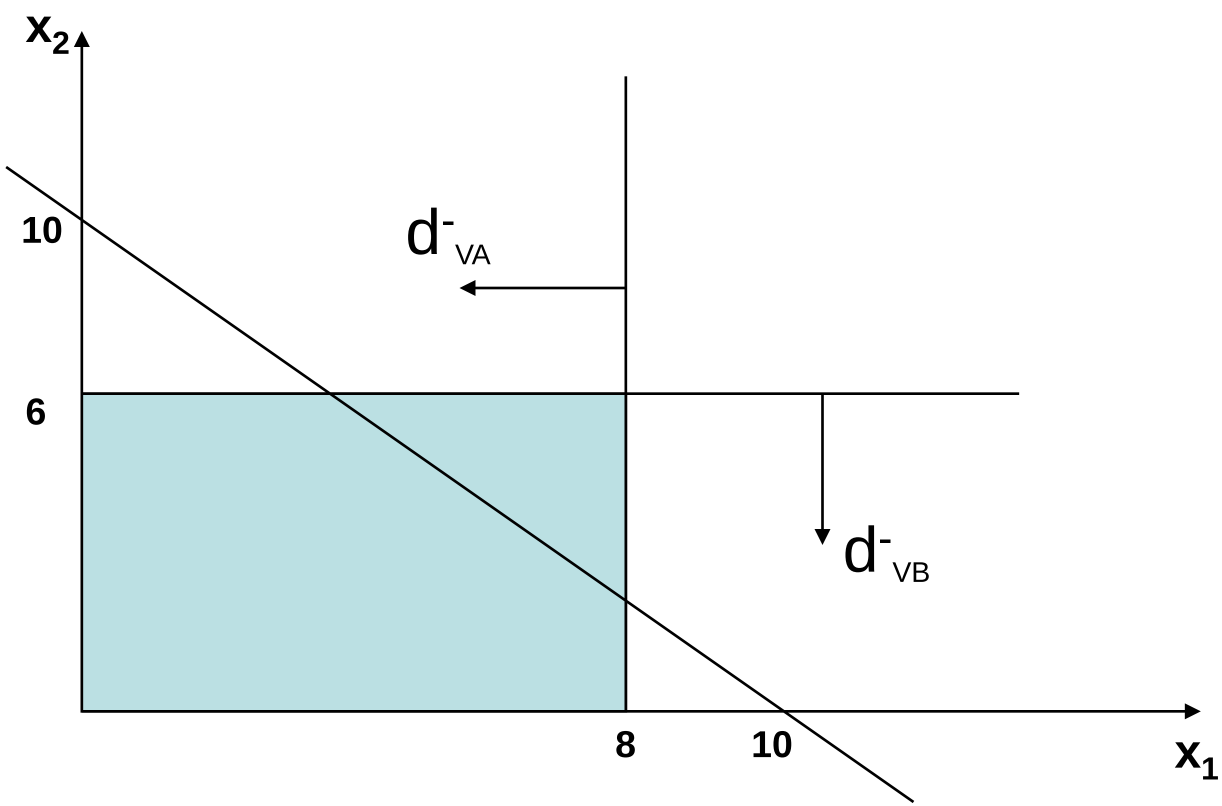


$$10\,x_1 + 10\,x_2 + d^-_{HH} - d^+_{HH} = 100$$

- Se pueden fabricar como máximo 8 piezas de A y 6 de B.
- Se desea vender toda la fabricación.
- Se sabe que la contribución marginal de A es el doble de la de B

$$x_1 + d^-_{VA} = 8$$

$$x_2 + d^-_{VB} = 6$$


x2
10
6
d-VA
d-VB
8
10
x1

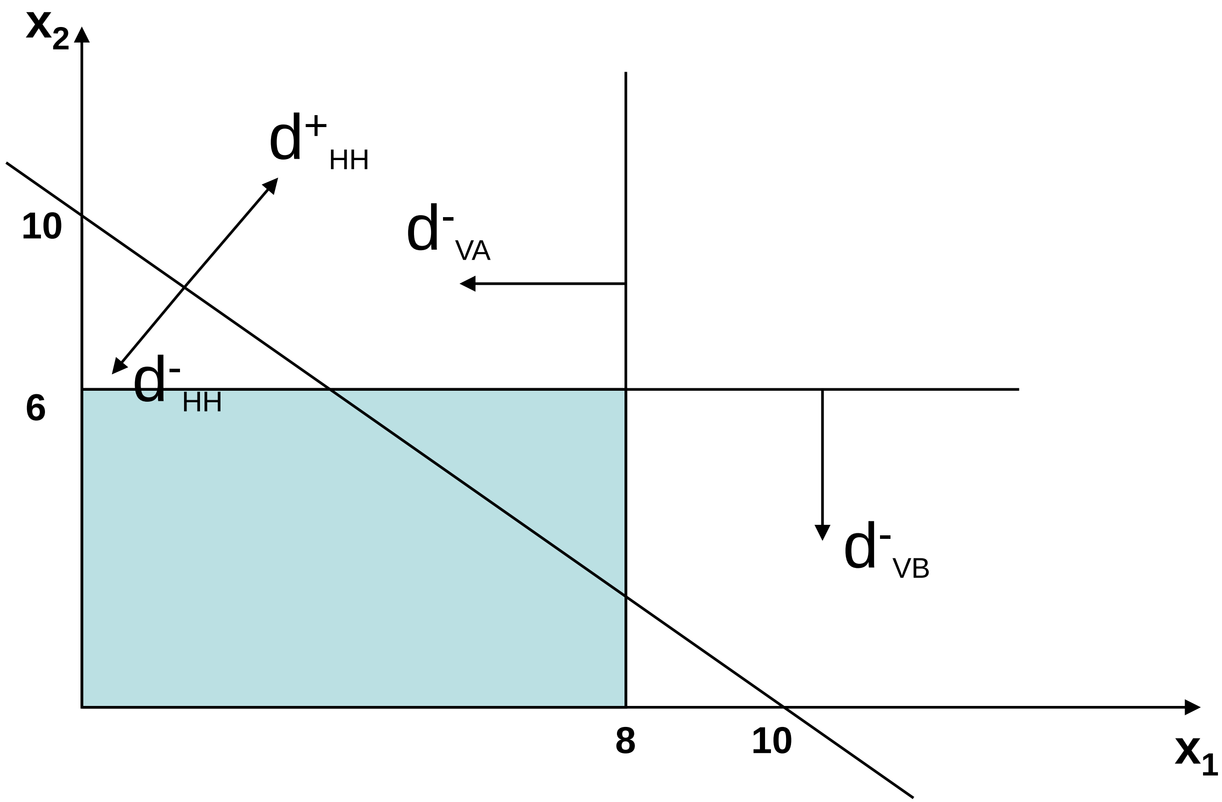
- Las prioridades son:

1. Mantener en un mínimo las horas extras
2. Vender toda la producción
3. Utilizar a pleno las horas regulares disponibles

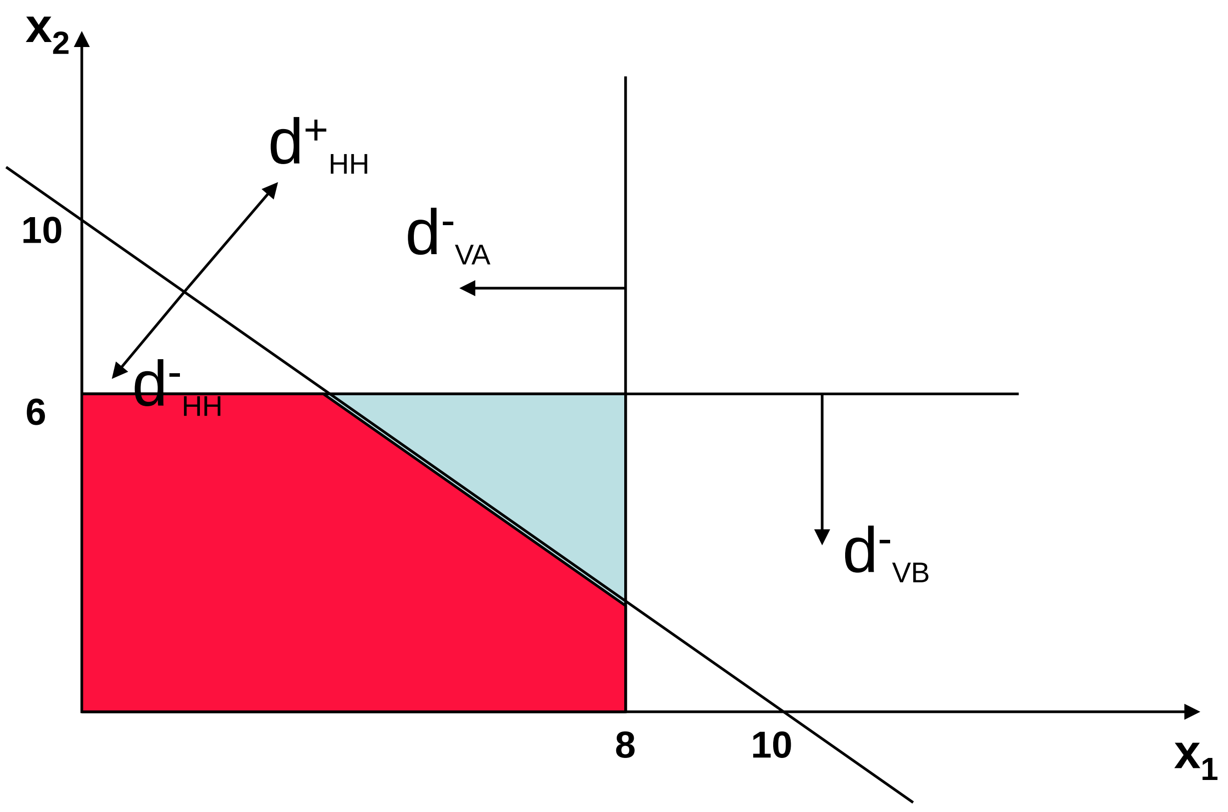
$$\begin{cases} 10\,x_1 + 10\,x_2 + d^-_{HH} - d^+_{HH} = 100 \\ x_1 + d^-_{VA} = 8 \\ x_2 + d^-_{VB} = 6 \end{cases}$$

$$Z = P_1\, d^+_{HH} + P_2\,(2\,d^-_{VA} + d^-_{VB}) + P_3\, d^-_{HH} \Rightarrow \text{Mín}$$

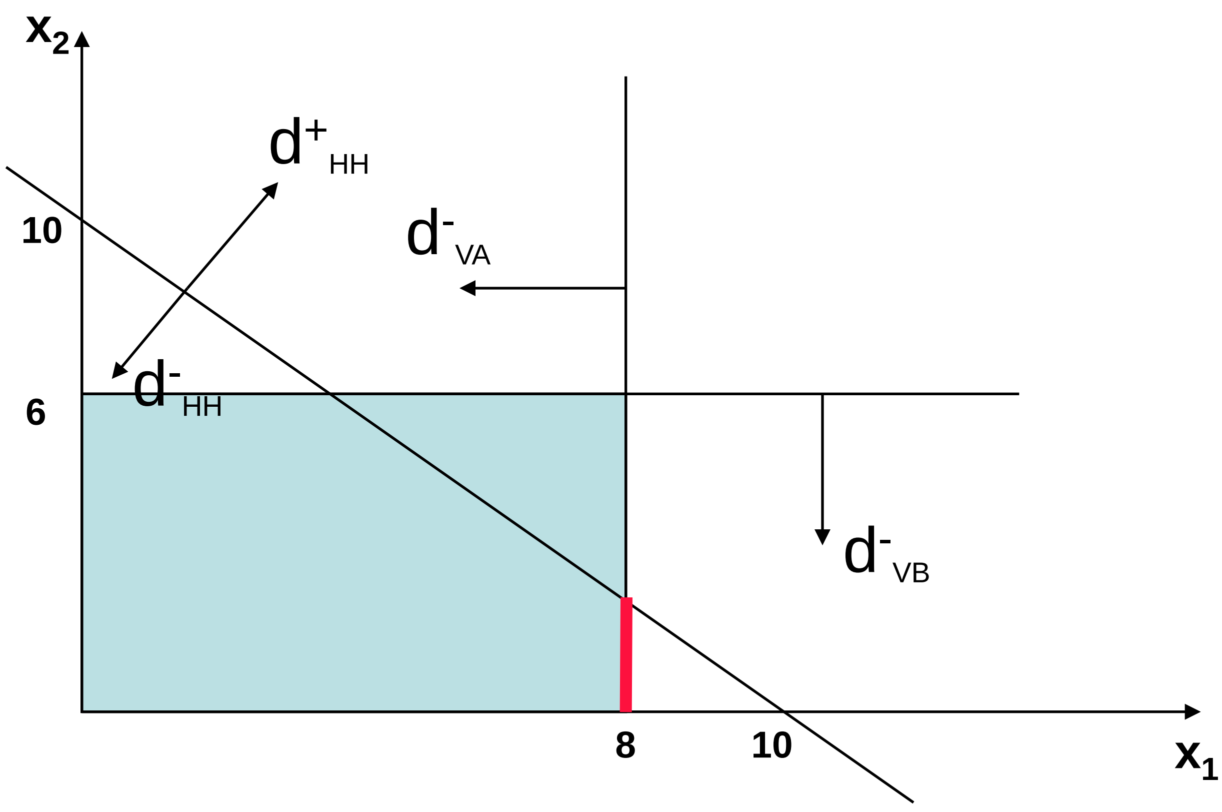
$$P_1 >> P_2 >> P_3$$



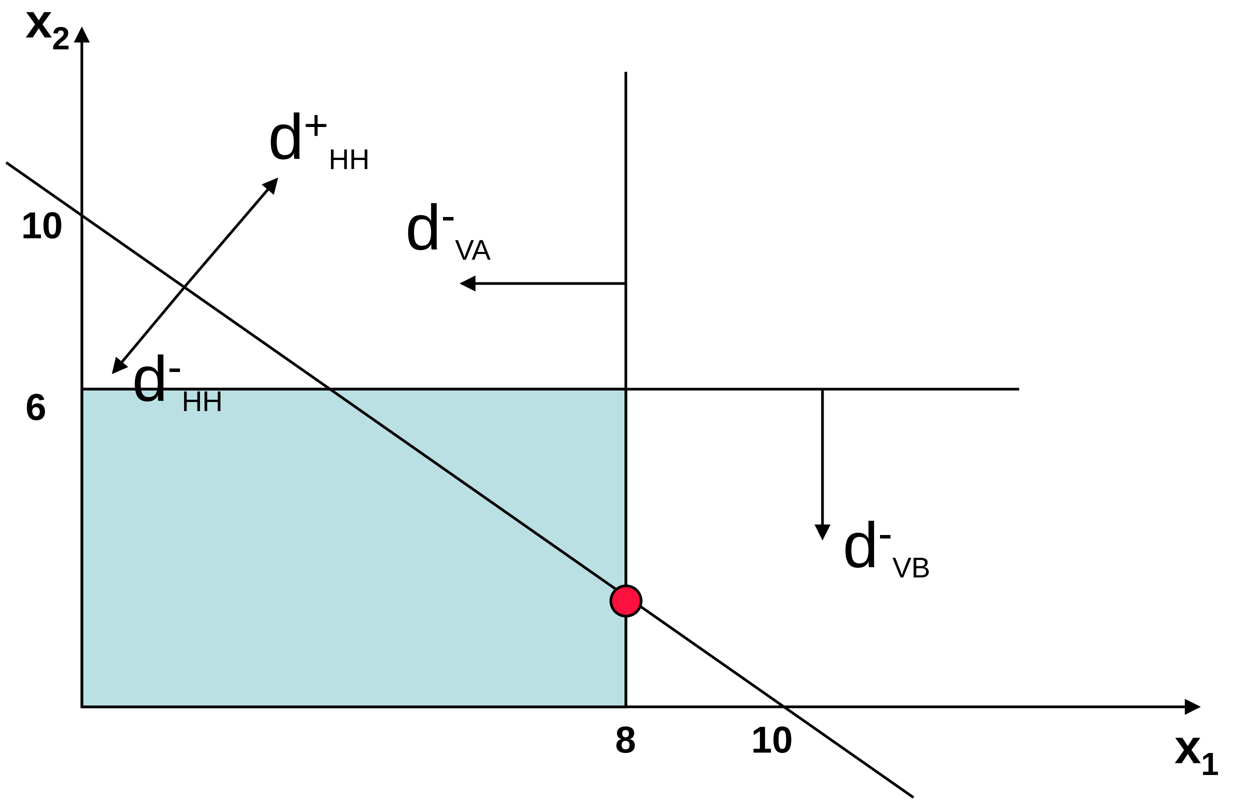
x2
10
6
d+HH
d-HH
d-VA
d-VB
8
10
x1



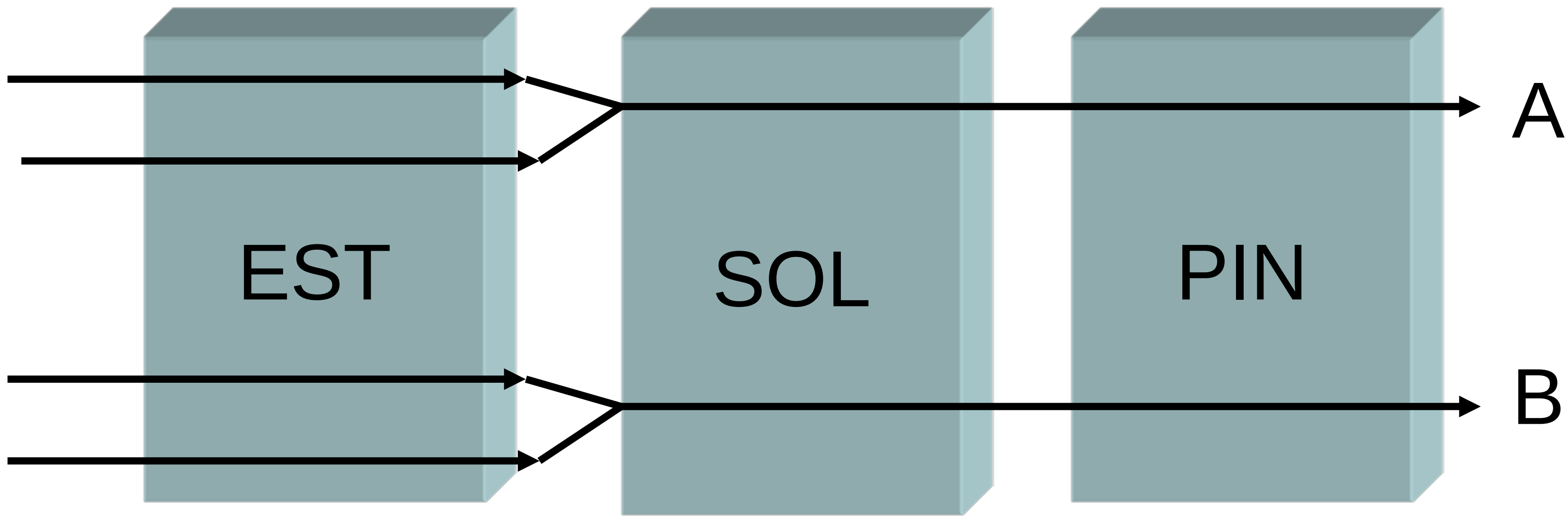
$$Z = P_1\, d^{+}_{HH} + P_2\,(2\, d^{-}_{VA} + d^{-}_{VB}) + P_3\, d^{-}_{HH} \Rightarrow \text{Mín}$$



$$Z = P_1\, d^+_{HH} + P_2\, (2\, d^-_{VA} + d^-_{VB}) + P_3\, d^-_{HH} \Rightarrow \text{Mín}$$



$$Z = P_1\, d^+_{HH} + P_2\,(2\, d^-_{VA} + d^-_{VB}) + P_3\, d^-_{HH} \Rightarrow \text{Mín}$$



EST
SOL
PIN
A
B

Operación	A	B	Tiempo disponible (seg./semana)
Estampado de c/parte	3	8	48.000
Soldado	12	6	42.000
Pintado	9	9	36.000

MAX: $Z = 4\,x_1 + 3\,x_2$

Sujeto a:

$$\begin{cases} 6\,x_1 + 16\,x_2 \leq 48000 \\ 12\,x_1 + 6\,x_2 \leq 42000 \\ 9\,x_1 + 9\,x_2 \leq 36000 \end{cases}$$

siendo: $x_1, x_2 \geq 0$ **y continuas**

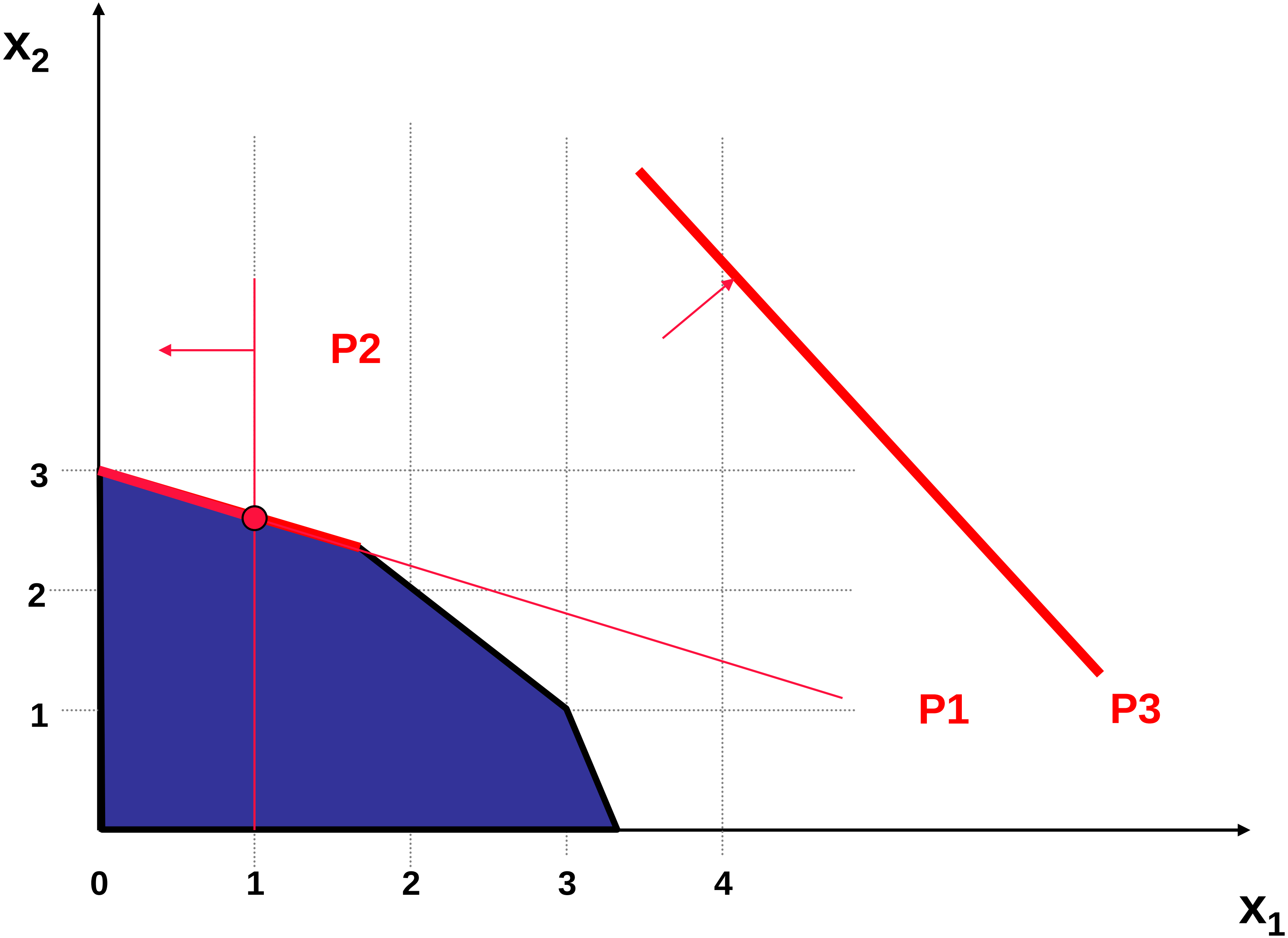
Ejemplo:E-S-P

- PRIORIDAD 1: Se desea utilizar a pleno la capacidad instalada de estampado.
- PRIORIDAD 2: Se desea fabricar no más de 1000 piezas de A.
- PRIORIDAD 3: Se quiere maximizar las utilidades

$$
\begin{cases}
6\,x_1 + 16\,x_2 + d^-_E = 48.000 \\
x_1 + d^-_A - d^+_A = 1.000 \\
4\,x_1 + 3\,x_2 + d^-_Z - d^+_Z = 100.000 \\
12\,x_1 + 6\,x_2 \leq 42000 \\
9\,x_1 + 9\,x_2 \leq 36000
\end{cases}
$$

$$Z = P_1\, d^-_E + P_2\, d^+_A + P_3\, d^-_Z \Rightarrow \text{Mín}$$

$$P_1 >> P_2 >> P_3$$



x2
3
2
1
0
1
2
3
4
x1
P2
P1
P3

PROGRAMACIÓN NO LINEAL

en el entorno de la PL

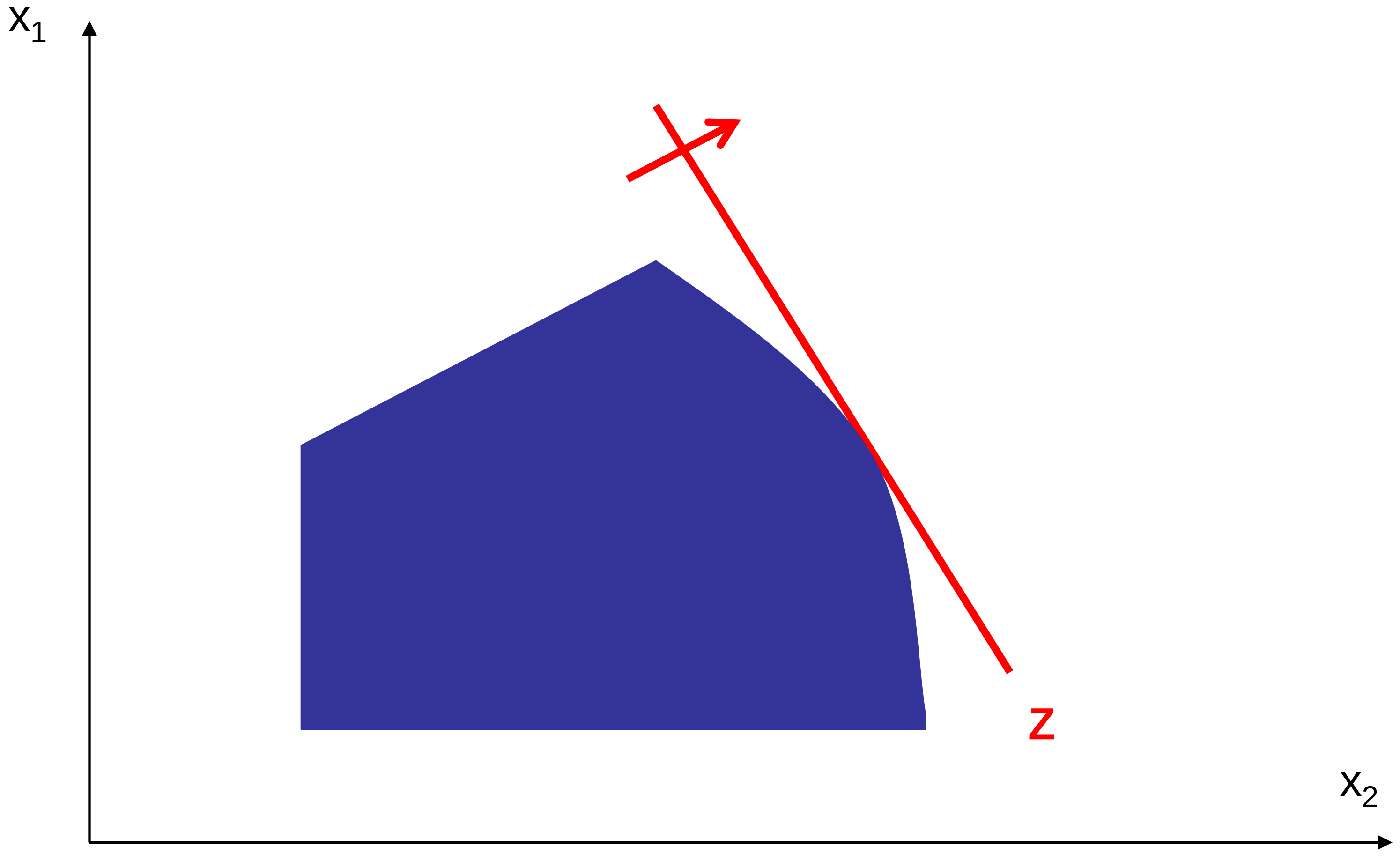
PROGRAMACIÓN MATEMÁTICA

MAX: $Z = f(x)$

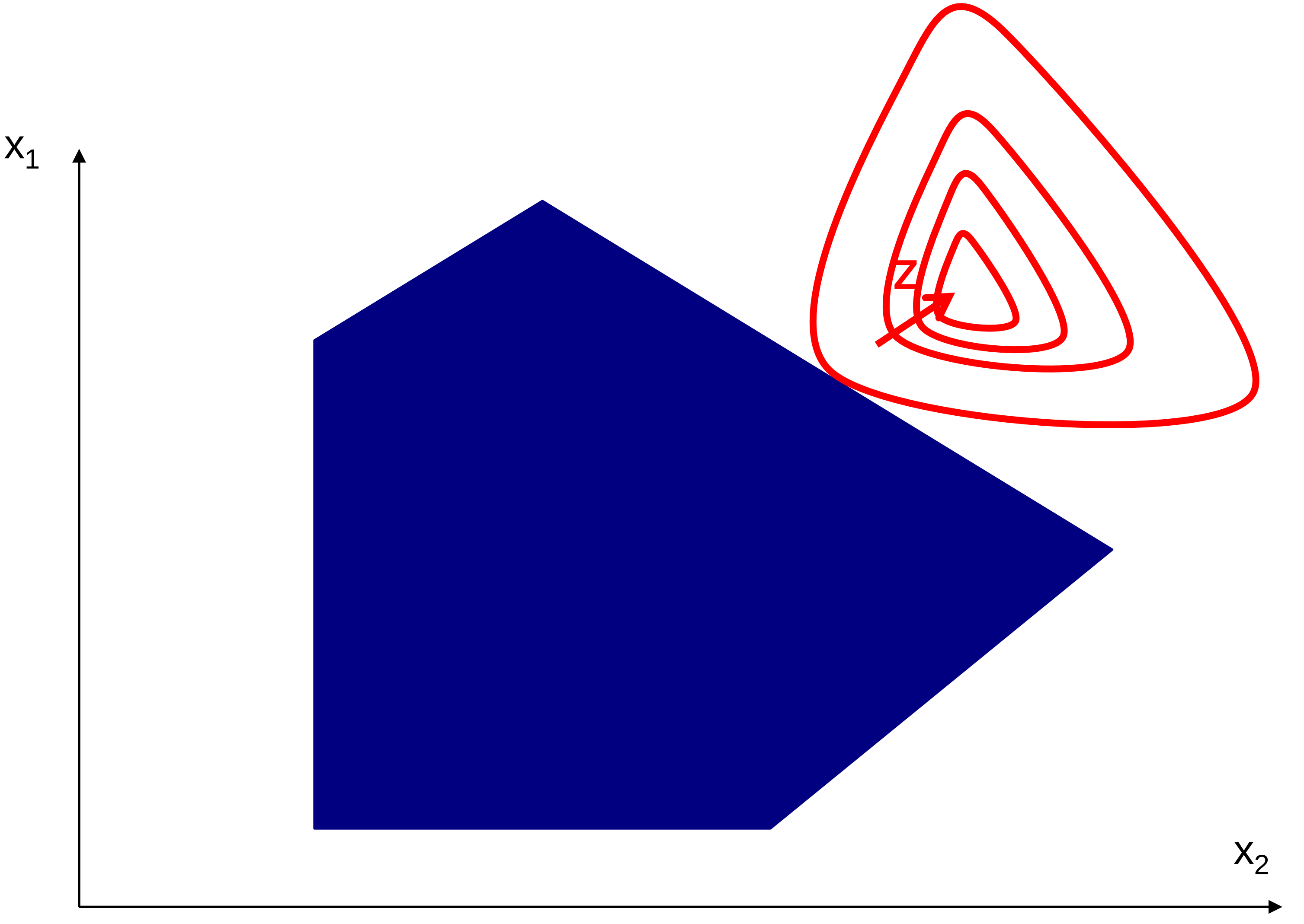
Sujeto a:

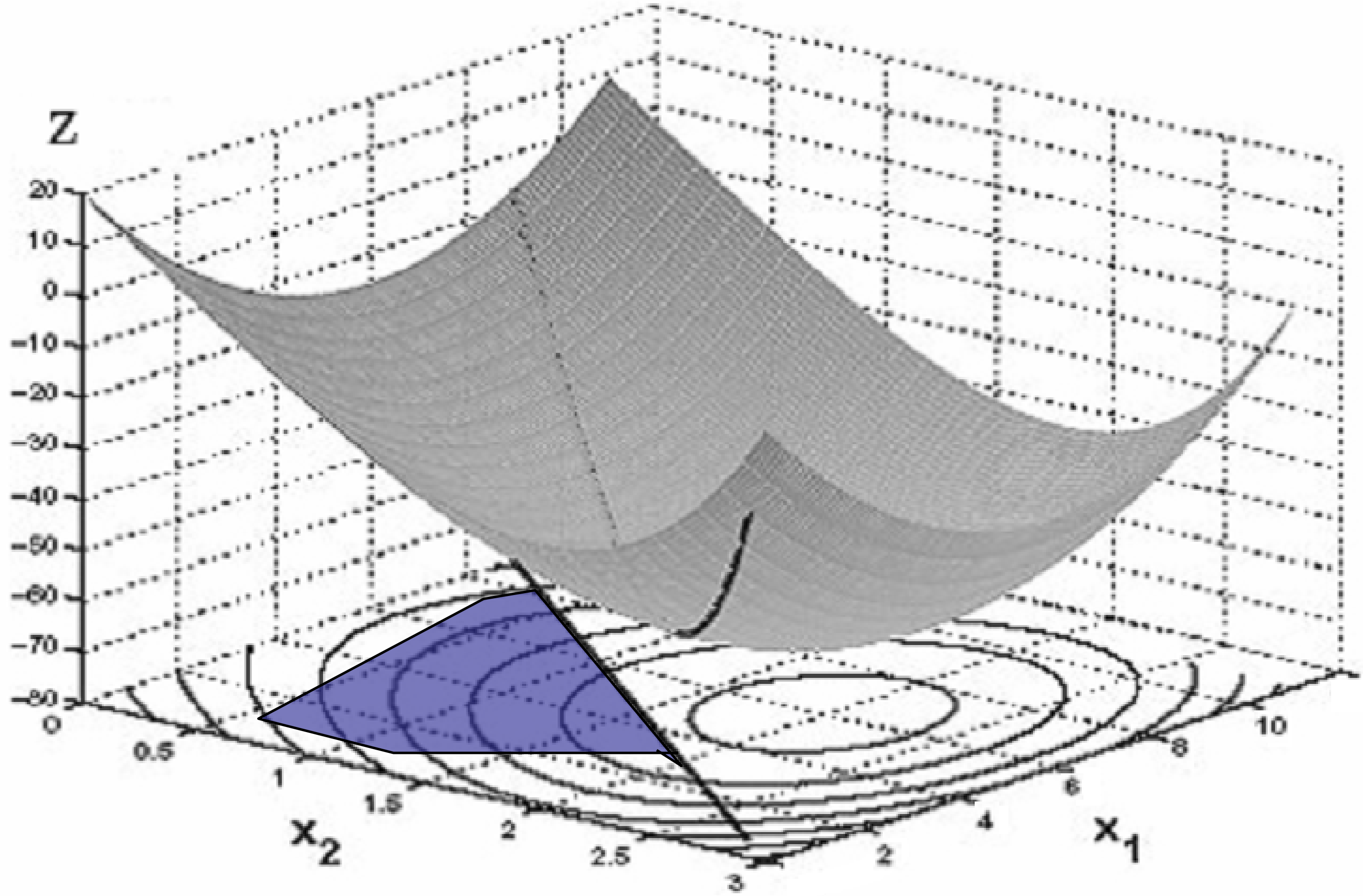
$$\begin{cases} g_1(x) \leq b_1 \\ g_2(x) \leq b_2 \\ \ldots\ldots\ldots \\ g_m(x) \leq b_m \end{cases}$$

RECINTO NO LINEAL, FUNCIONAL LINEAL

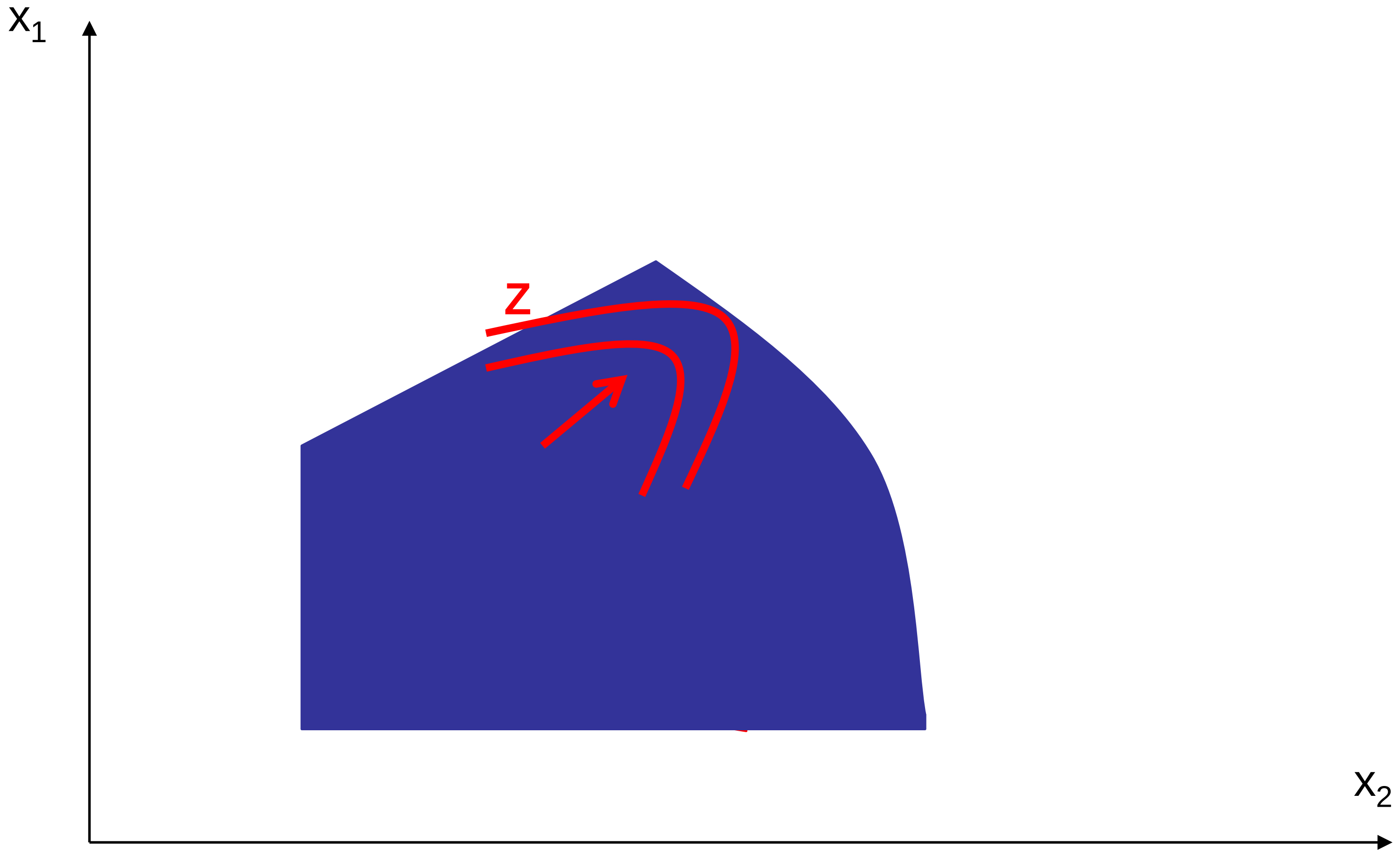


RECINTO LINEAL, FUNCIONAL NO LINEAL

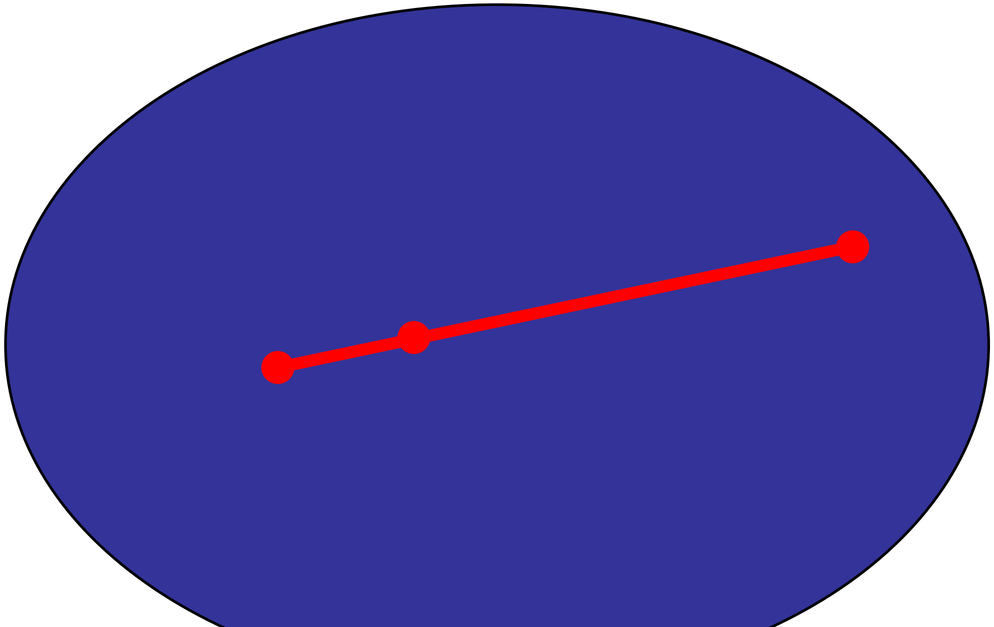



Z
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
0
0.5
1
1.5
2
2.5
3
x2
2
4
6
8
10
x1

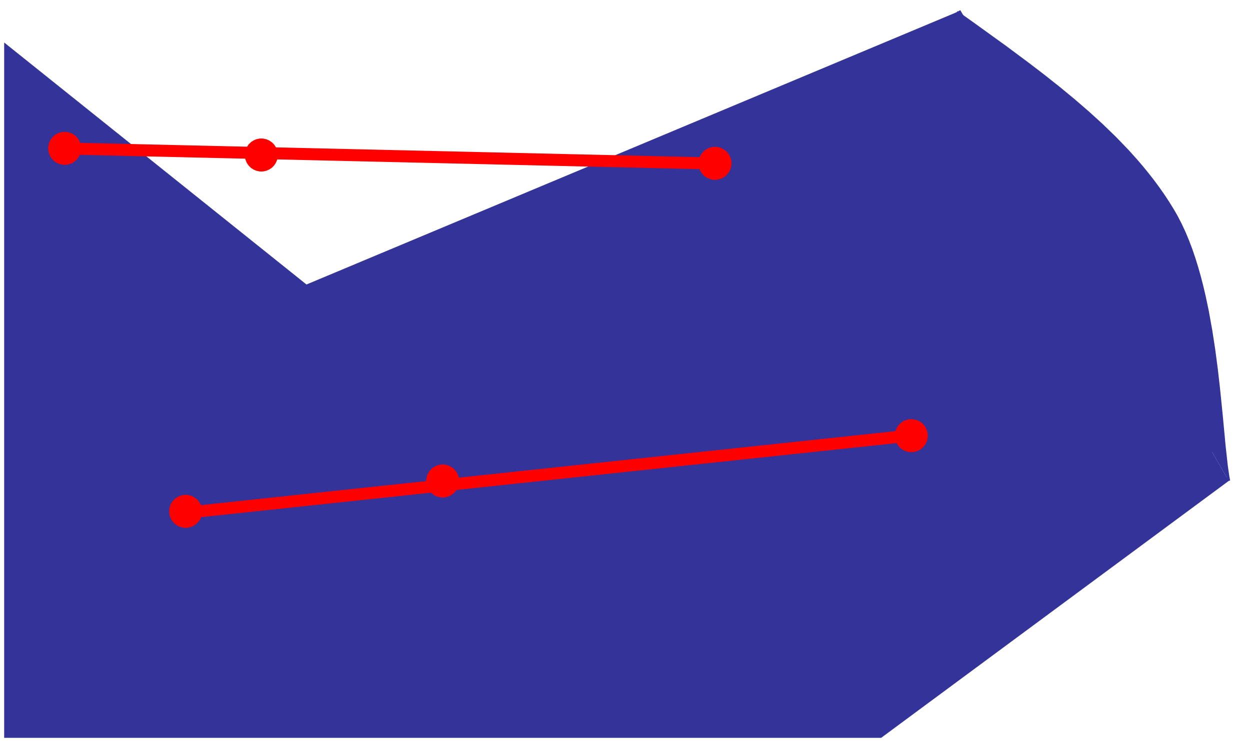
RECINTO NO LINEAL, FUNCIONAL NO LINEAL



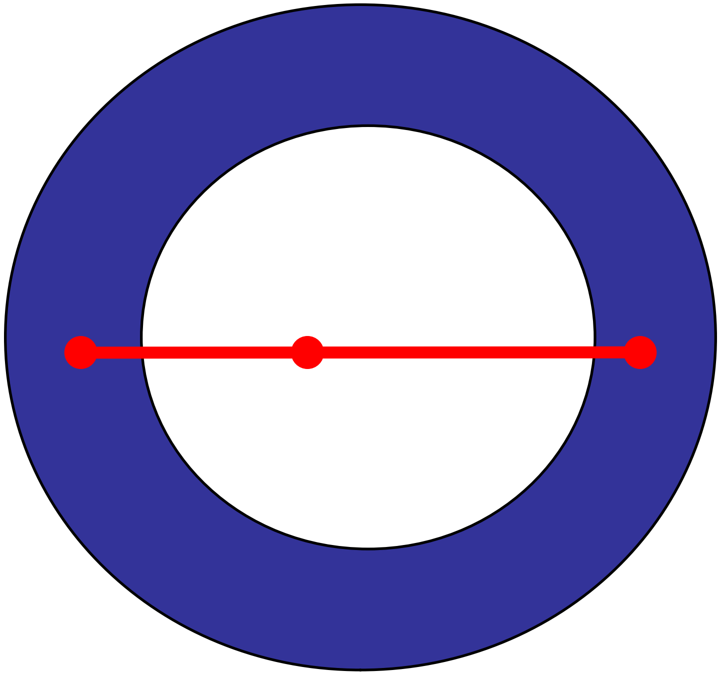
CONJUNTO CONVEXO



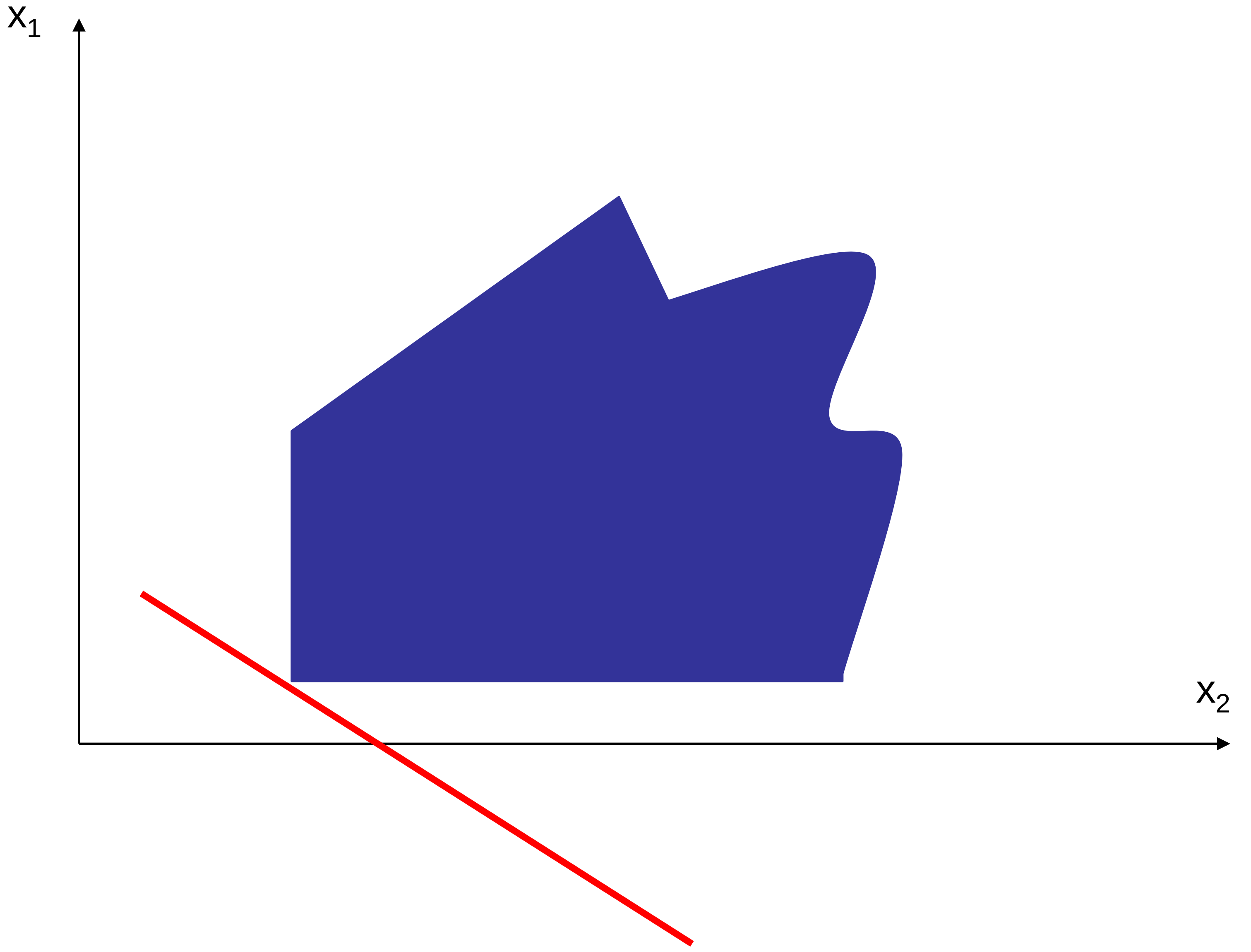
CONJUNTO NO CONVEXO



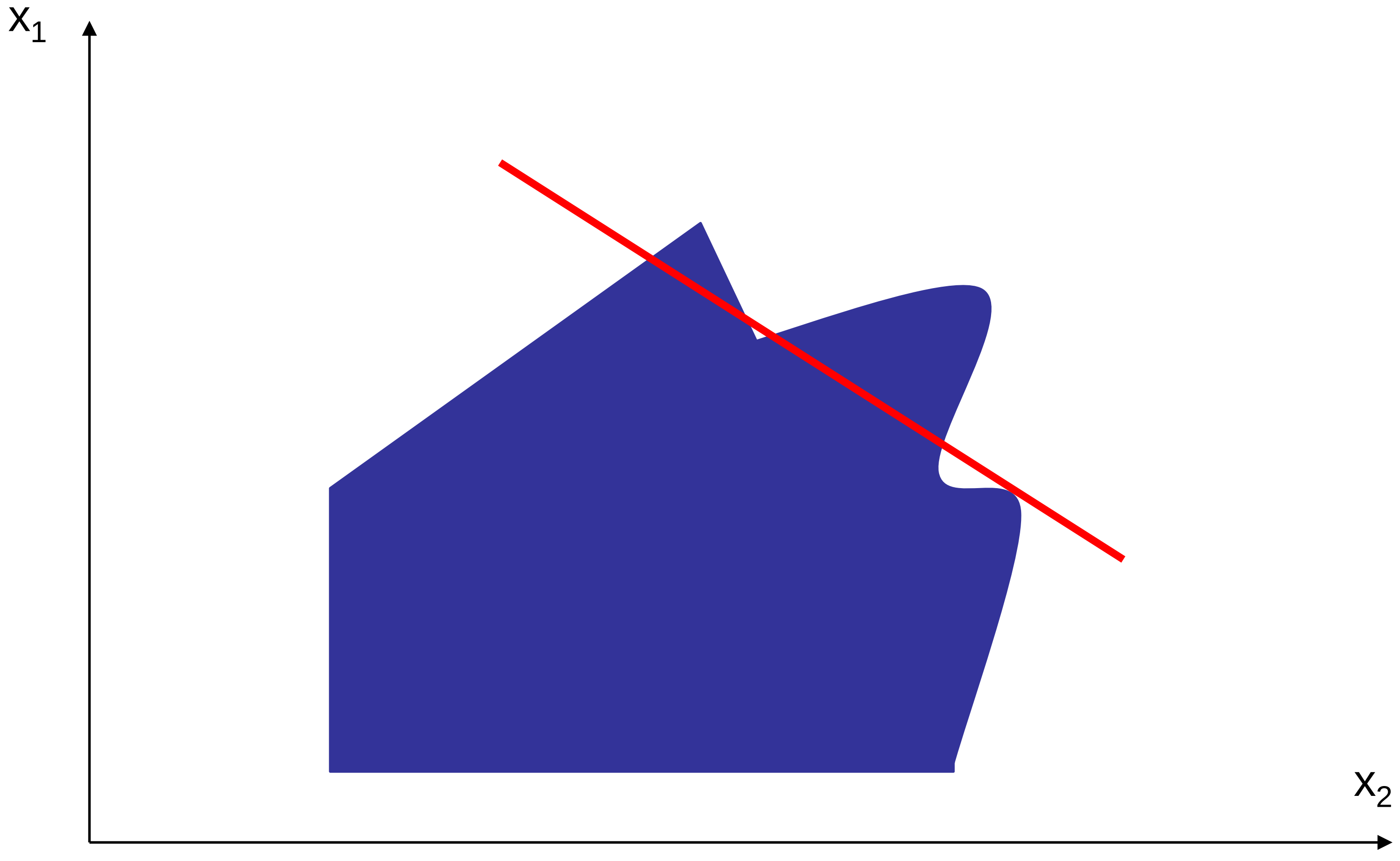
CONJUNTO NO CONVEXO



OPTIMOS LOCALES Y ÓPTIMO GLOBAL



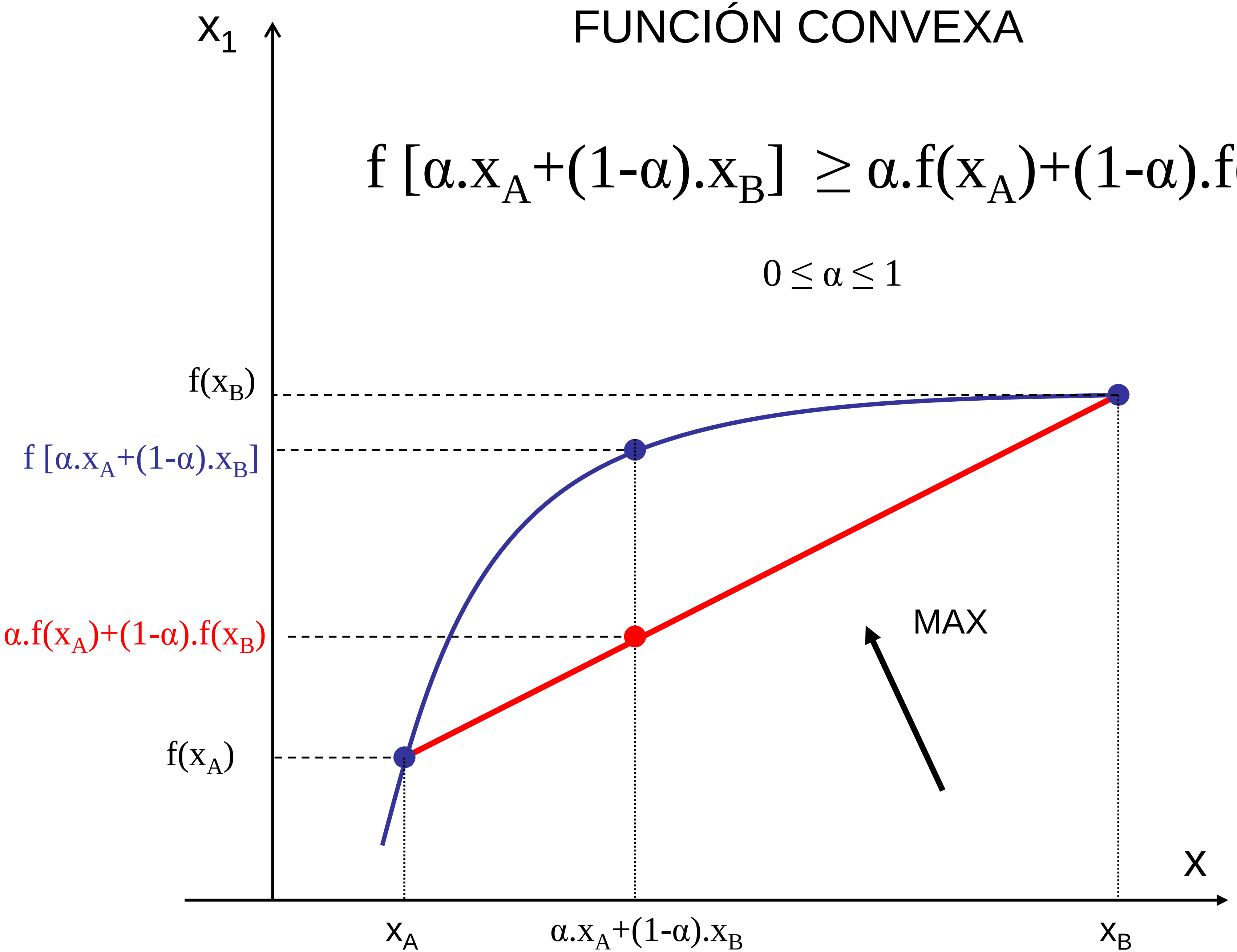
OPTIMOS LOCALES Y ÓPTIMO GLOBAL



FUNCIÓN CONVEXA

$$f\,[\alpha . x_A + (1-\alpha) . x_B] \geq \alpha . f(x_A) + (1-\alpha) . f(x_B)$$

$$0 \leq \alpha \leq 1$$



Función separable

$$f(x_i) = f_1(x_1) + f_2(x_2) + f_3(x_3) + \ldots.. + f_N(x_N)$$

$$x_1^2 + 2 \cdot x_2 + e^{x_3}$$

$$x_1 + x_2^2 + 3 \cdot \log x_3$$

Función no separable

$$x_1 - \frac{x_2}{1 + x_3} + x_4 + x_4 \cdot e^{-x_3}$$

$$x_1 \odot x_2 + x_3 \odot \ln$$
$$x_4$$

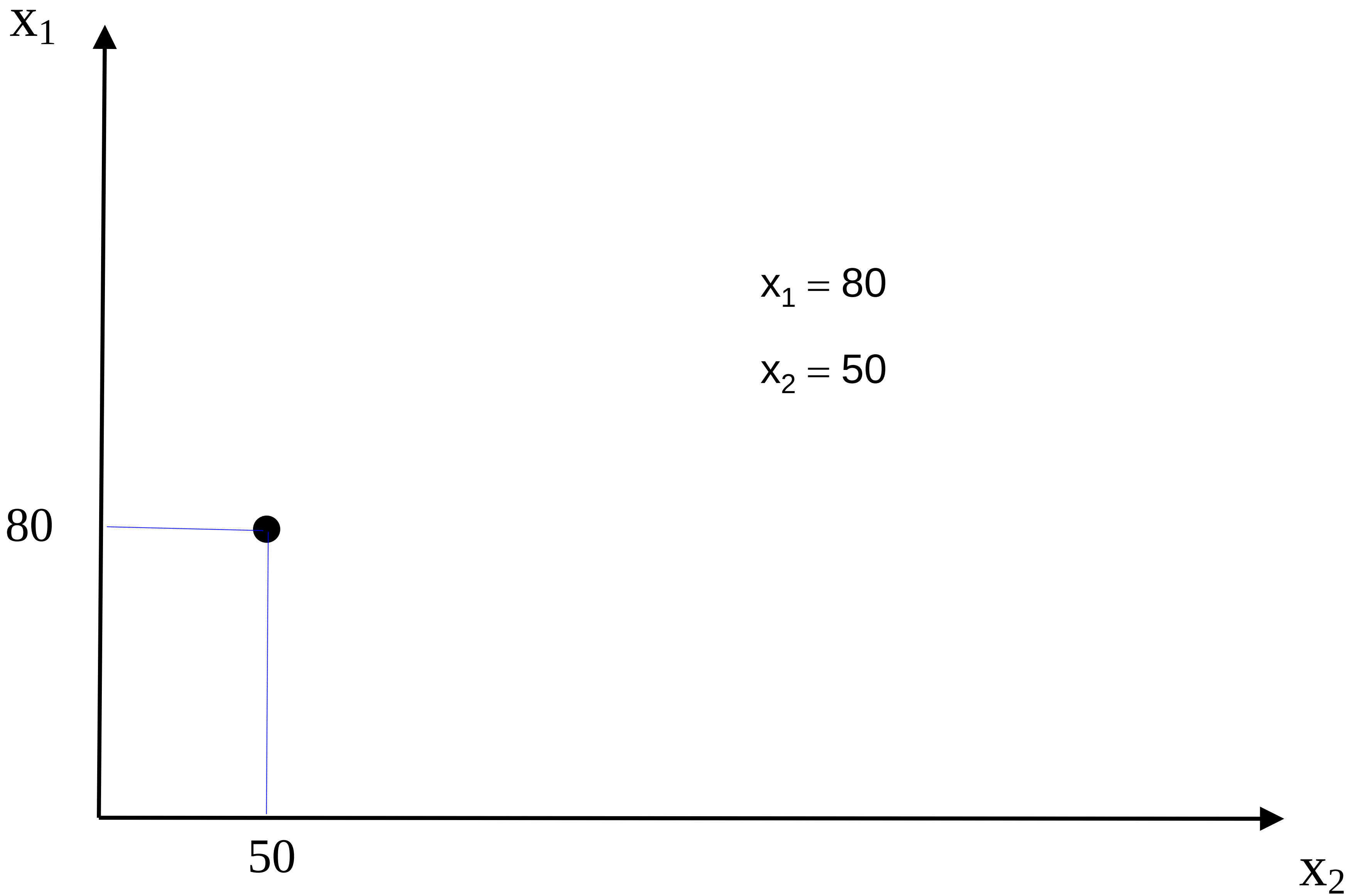
$x_1 \odot x_2$

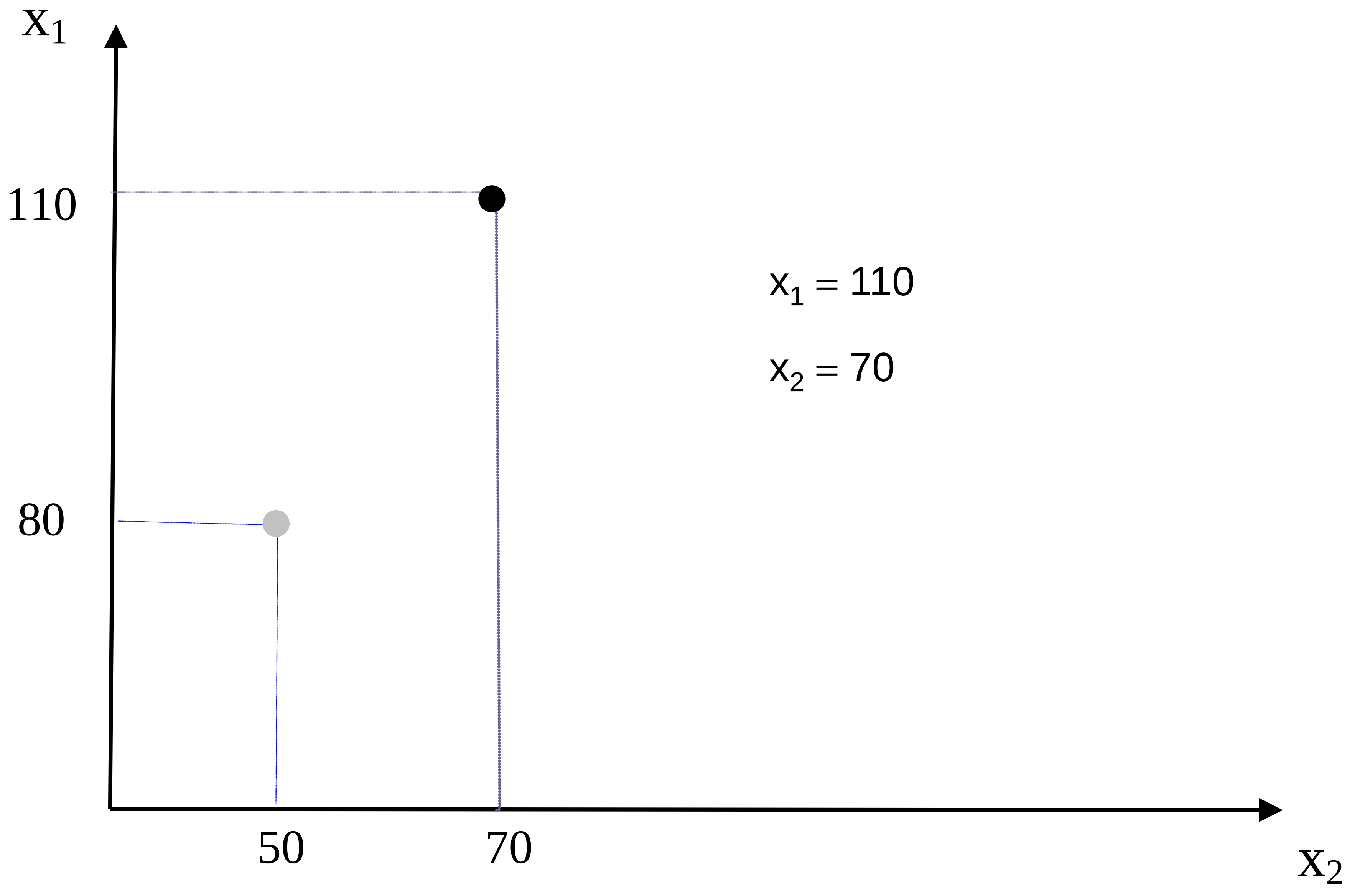
$x_1 = p + q$

$x_2 = p - q$

$x_1 \odot x_2 = (p + q) \odot (p - q) = p^2 - q^2$

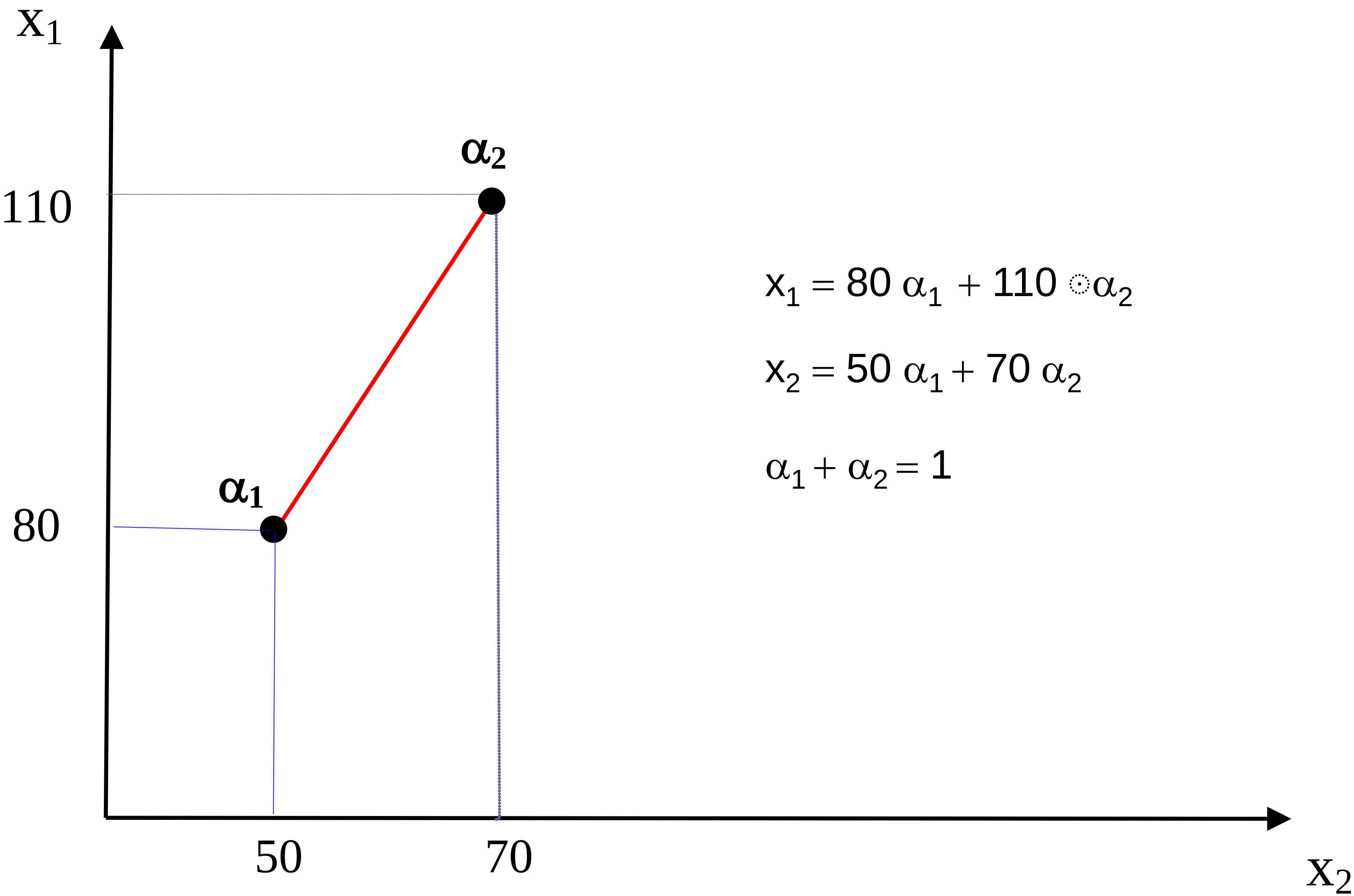
PROGRAMACIÓN SEPARABLE


x_1
$x_1 = 80$
$x_2 = 50$
80
50
x_2



x1
110
80
50
70
x2
x1 = 110
x2 = 70

Función lineal

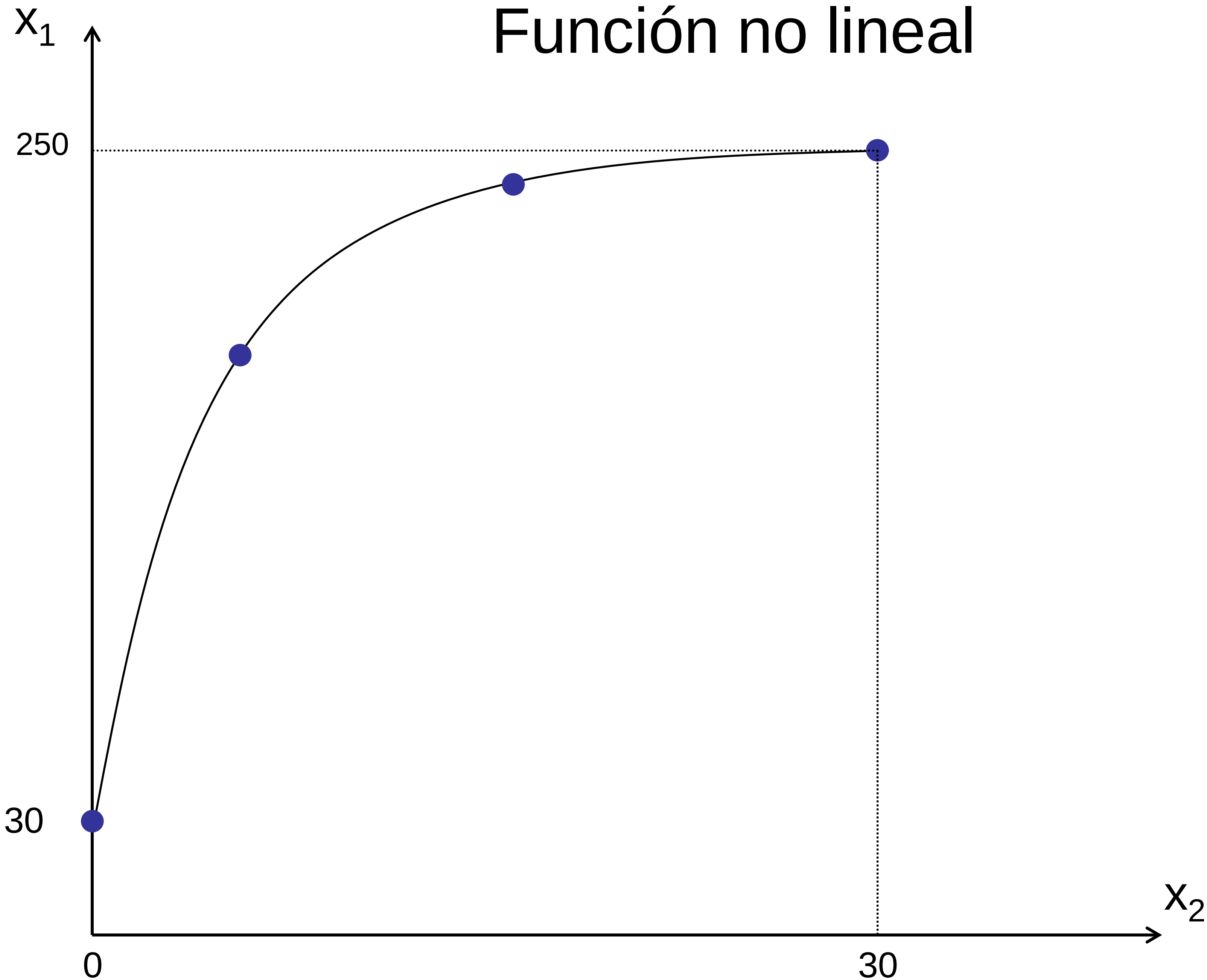


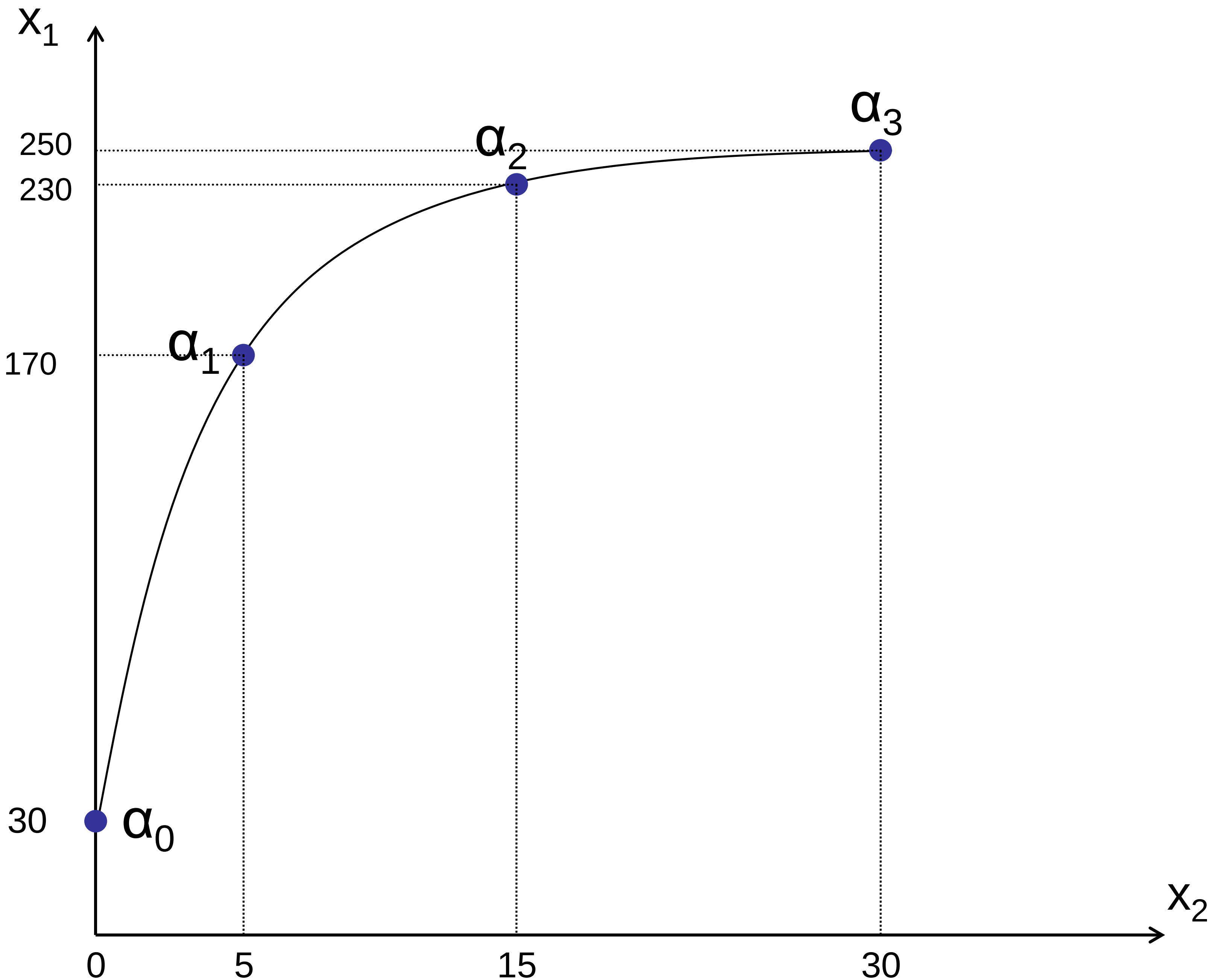
$x_1 = 80\,\alpha_1 + 110 \cdot \alpha_2$

$x_2 = 50\,\alpha_1 + 70\,\alpha_2$

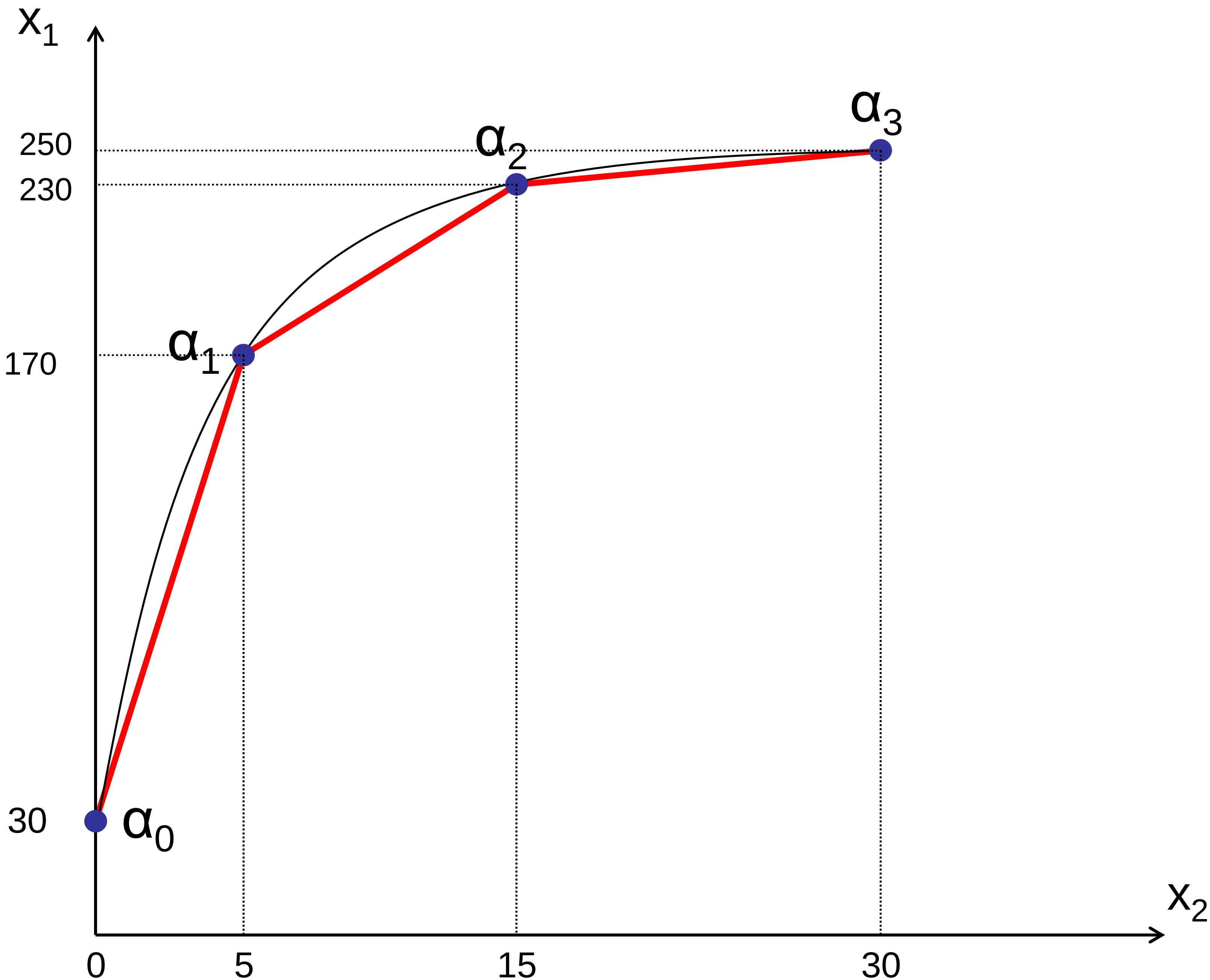
$\alpha_1 + \alpha_2 = 1$

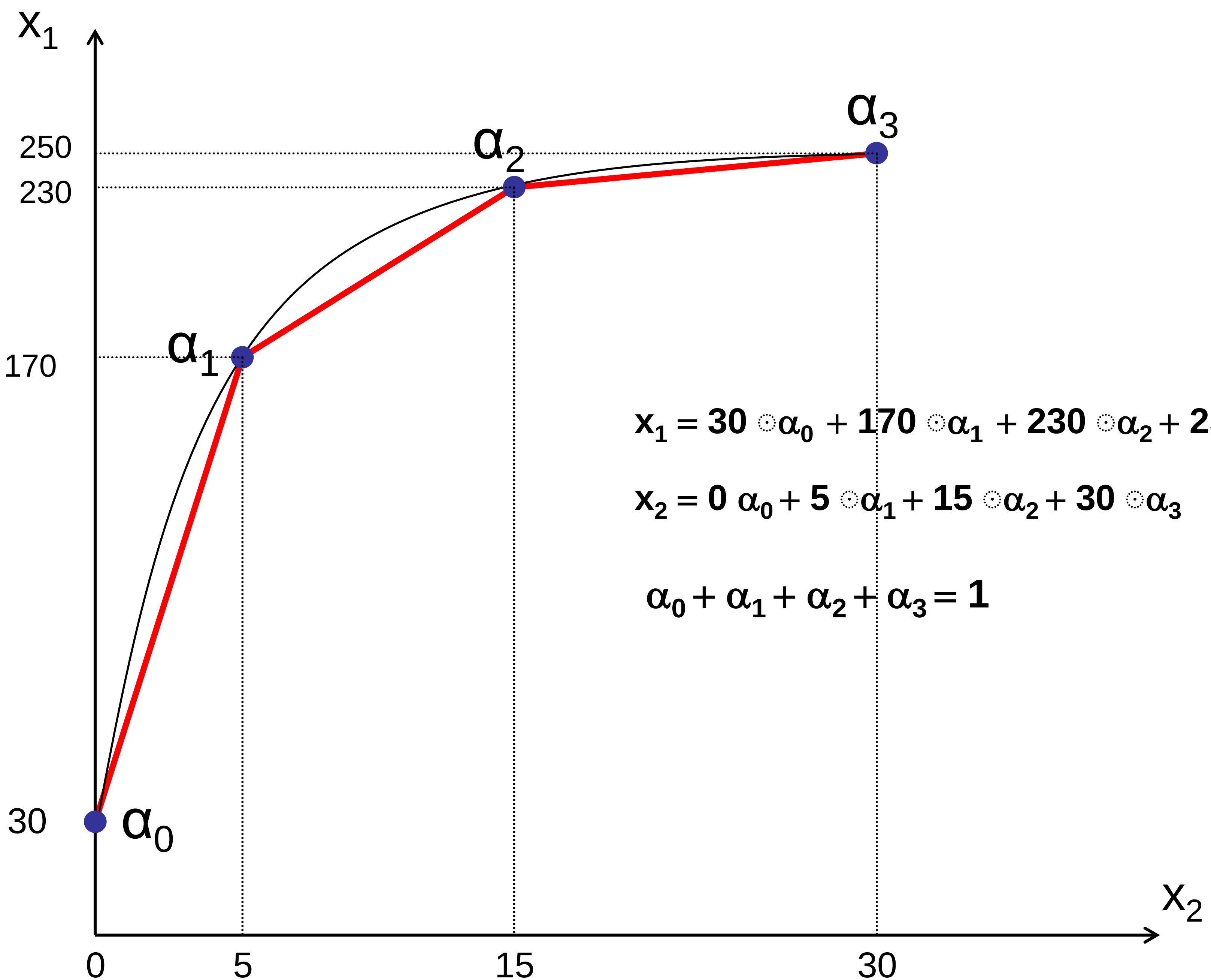
Función no lineal





x1
x2
250
230
170
30
0
5
15
30
α0
α1
α2
α3


x1
x2
250
230
170
30
0
5
15
30
α0
α1
α2
α3



$$x_1 = 30 \cdot \alpha_0 + 170 \cdot \alpha_1 + 230 \cdot \alpha_2 + 250 \cdot \alpha_3$$

$$x_2 = 0\ \alpha_0 + 5 \cdot \alpha_1 + 15 \cdot \alpha_2 + 30 \cdot \alpha_3$$

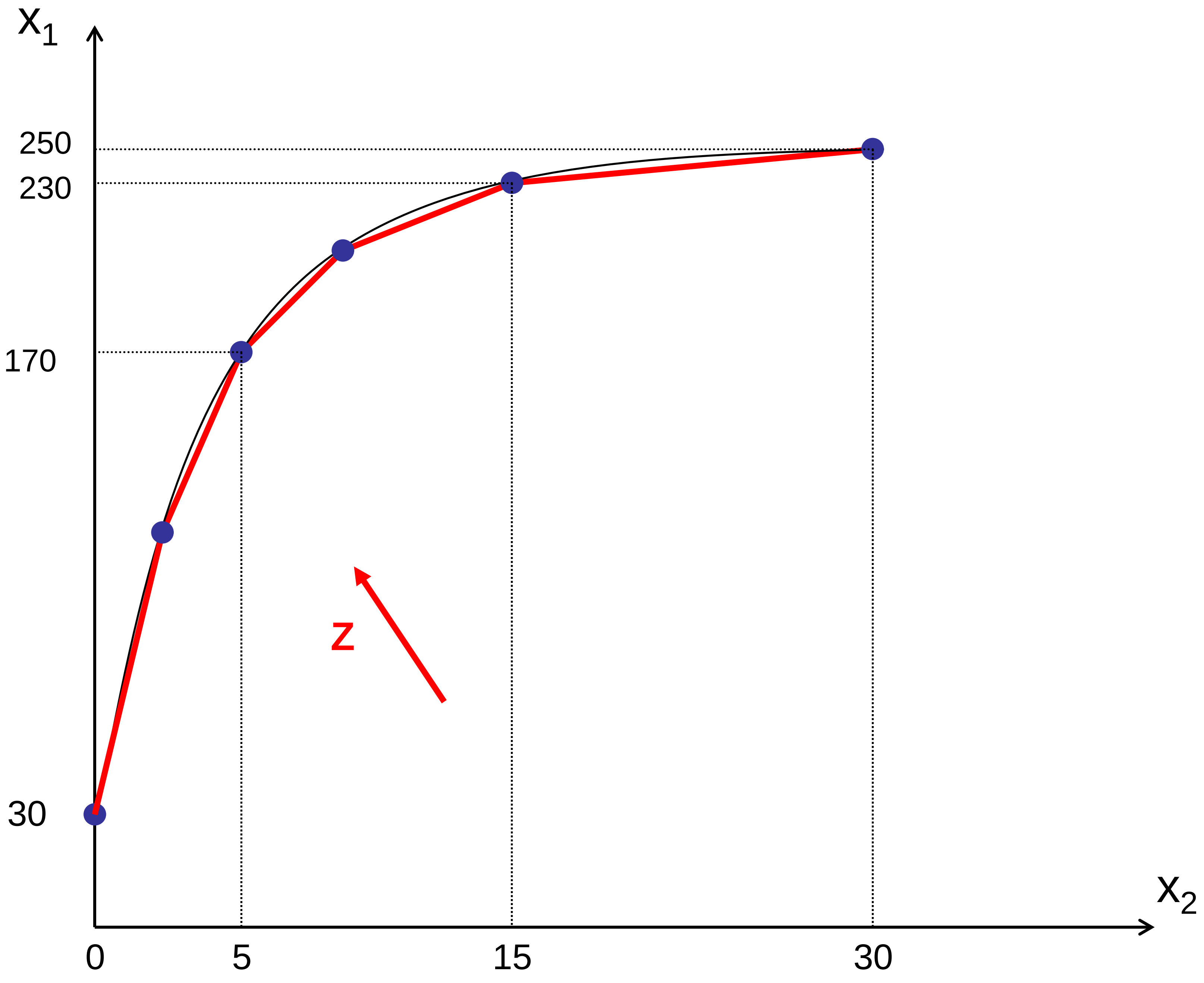
$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1$$

Vectores “mesh”

- La aproximación es tanto más precisa cuanto más vectores α_i se introduzcan

- El procedimiento es aplicable sólo a funciones convexas

- En la solución, debe estar activo una sola variable mesh o a lo sumo dos adyacentes.


x1
250
230
170
30
z
x2
0
5
15
30

Ejemplo:

$$\text{MIN} \quad 3\, x_1^2 + 4\, x_1 + 15\, x_2$$

$$x_1 + x_2 \geq 4$$

$$2\, x_1 + 0.5\, x_2 \leq 5$$

$$-3\, x_1 + 2.8\, x_2 \geq 2$$

Ejemplo:

$y = x_1^2$

x_1	y	Vector
0	0	α_0
1	1	α_1
2	4	α_2
2.5	6.25	α_3

$$\text{MIN } 3x_1^2 + 4x_1 + 15x_2$$

$$x_1 + x_2 \geq 4$$

$$2x_1 + 0.5x_2 \leq 5$$

$$-3x_1 + 2.8x_2 \geq 2$$

Ejemplo:

$y = x_1^2$

x_1	y	Vector
0	0	α_0
1	1	α_1
2	4	α_2
2.5	6.25	α_3

$$\text{MIN } 3y + 4x_1 + 15x_2$$

$$x_1 + x_2 \geq 4$$

$$2x_1 + 0.5x_2 \leq 5$$

$$-3x_1 + 2.8x_2 \geq 2$$

$$-x_1 + \alpha_1 + 2\alpha_2 + 2.5\alpha_3 = 0$$

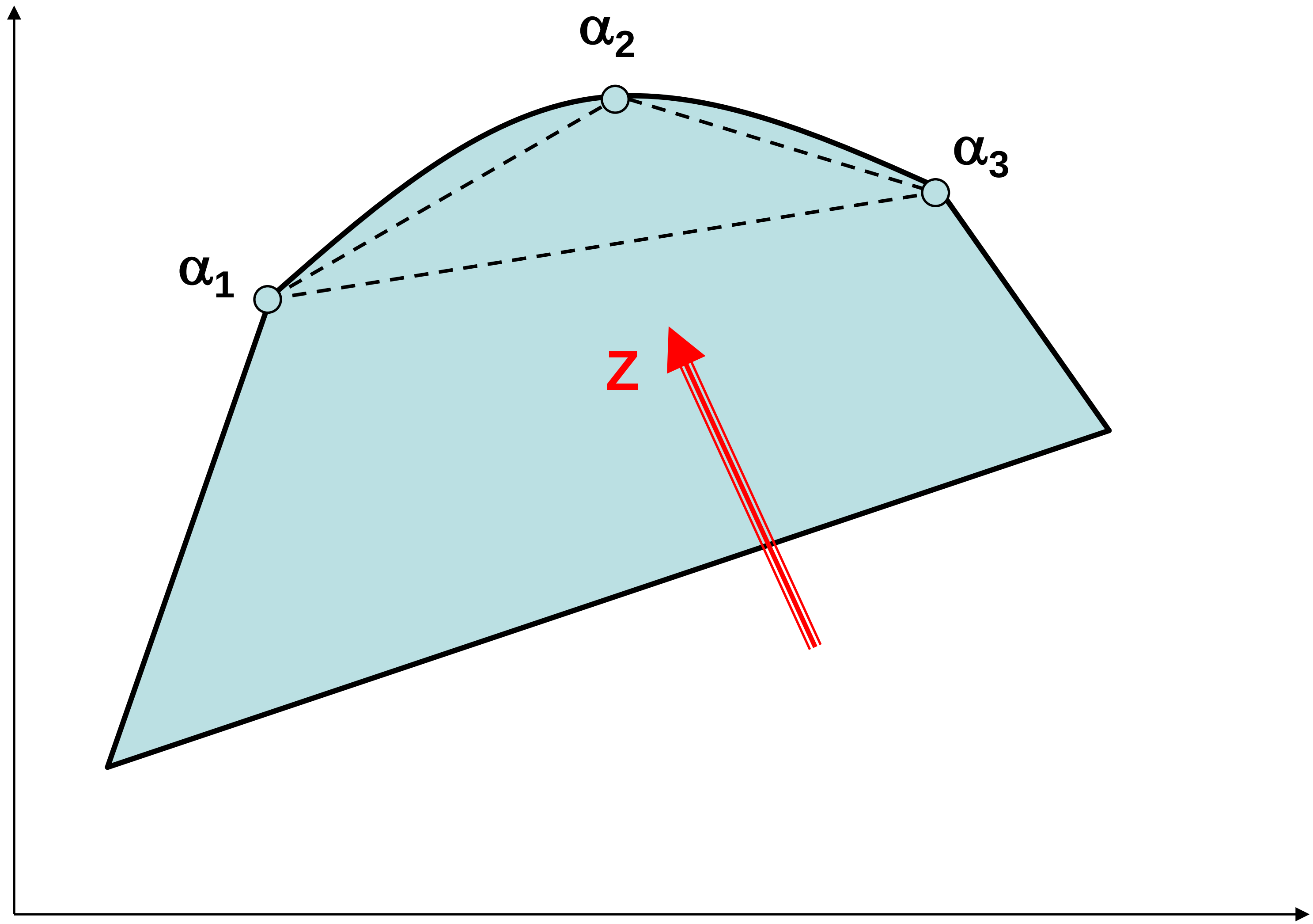
$$-y + \alpha_1 + 4\alpha_2 + 6.25\alpha_3 = 0$$

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1$$

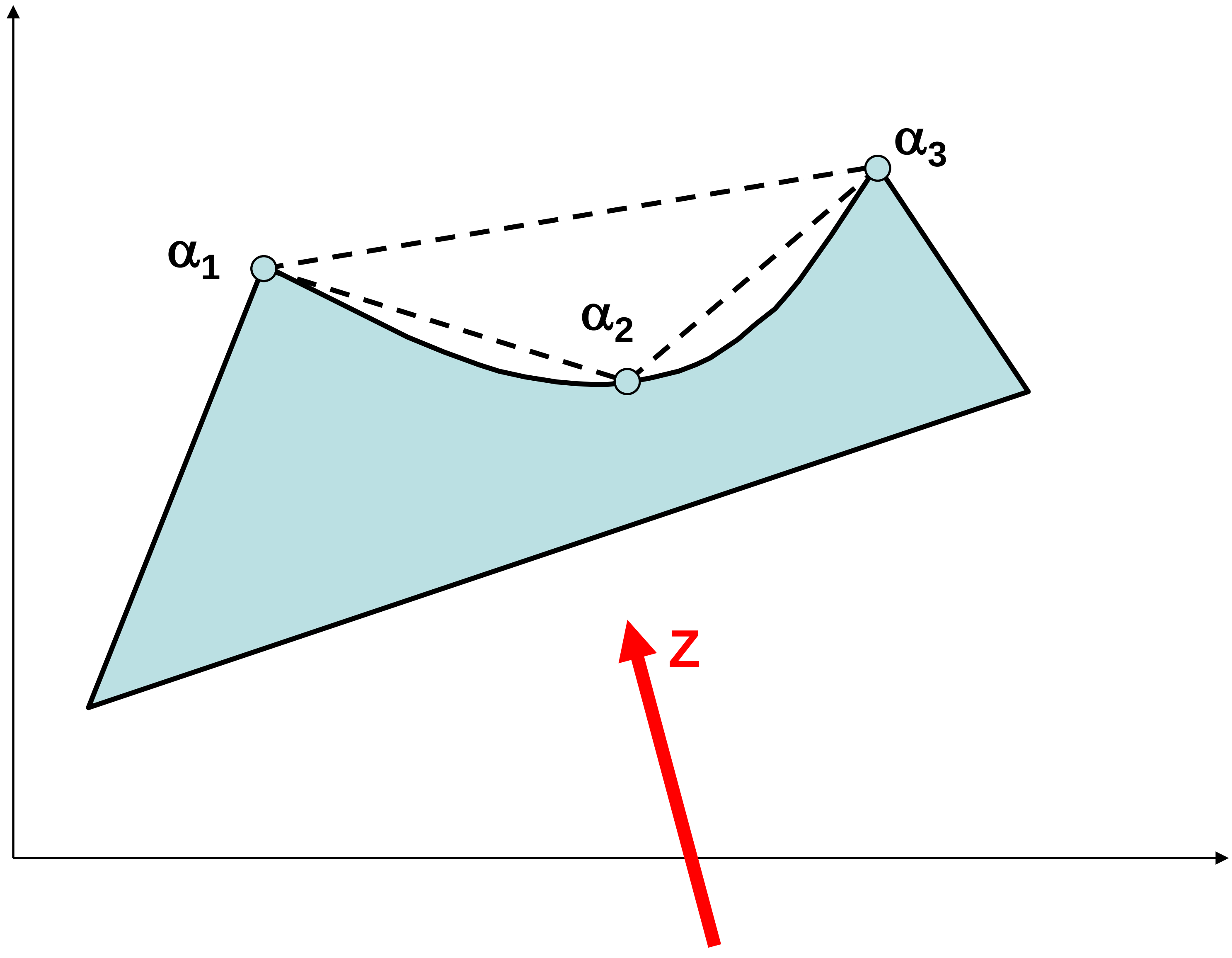
OBJECTIVE FUNCTION VALUE

Z) 50.82759

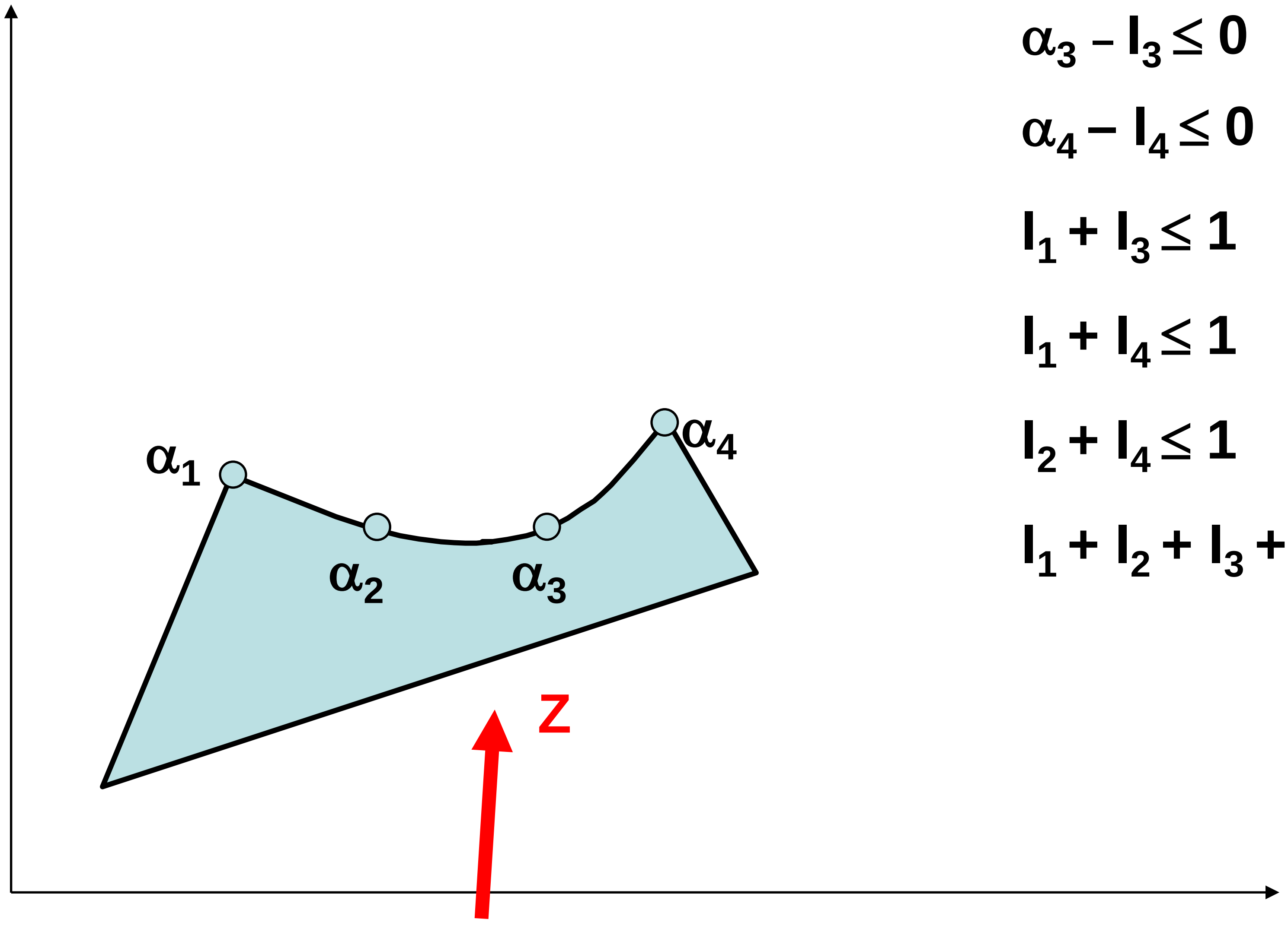
VARIABLE	VALUE
x_1	1.586207
x_2	2.413793
y	2.758621
α_1	0.413793
α_2	0.586207
α_3	0.000000
α_0	0.000000



α2
α3
α1
z



α1
α2
α3
Z



$\alpha_1 - I_1 \leq 0$

$\alpha_2 - I_2 \leq 0$

$\alpha_3 - I_3 \leq 0$

$\alpha_4 - I_4 \leq 0$

$I_1 + I_3 \leq 1$

$I_1 + I_4 \leq 1$

$I_2 + I_4 \leq 1$

$I_1 + I_2 + I_3 + I_4 \leq 2$

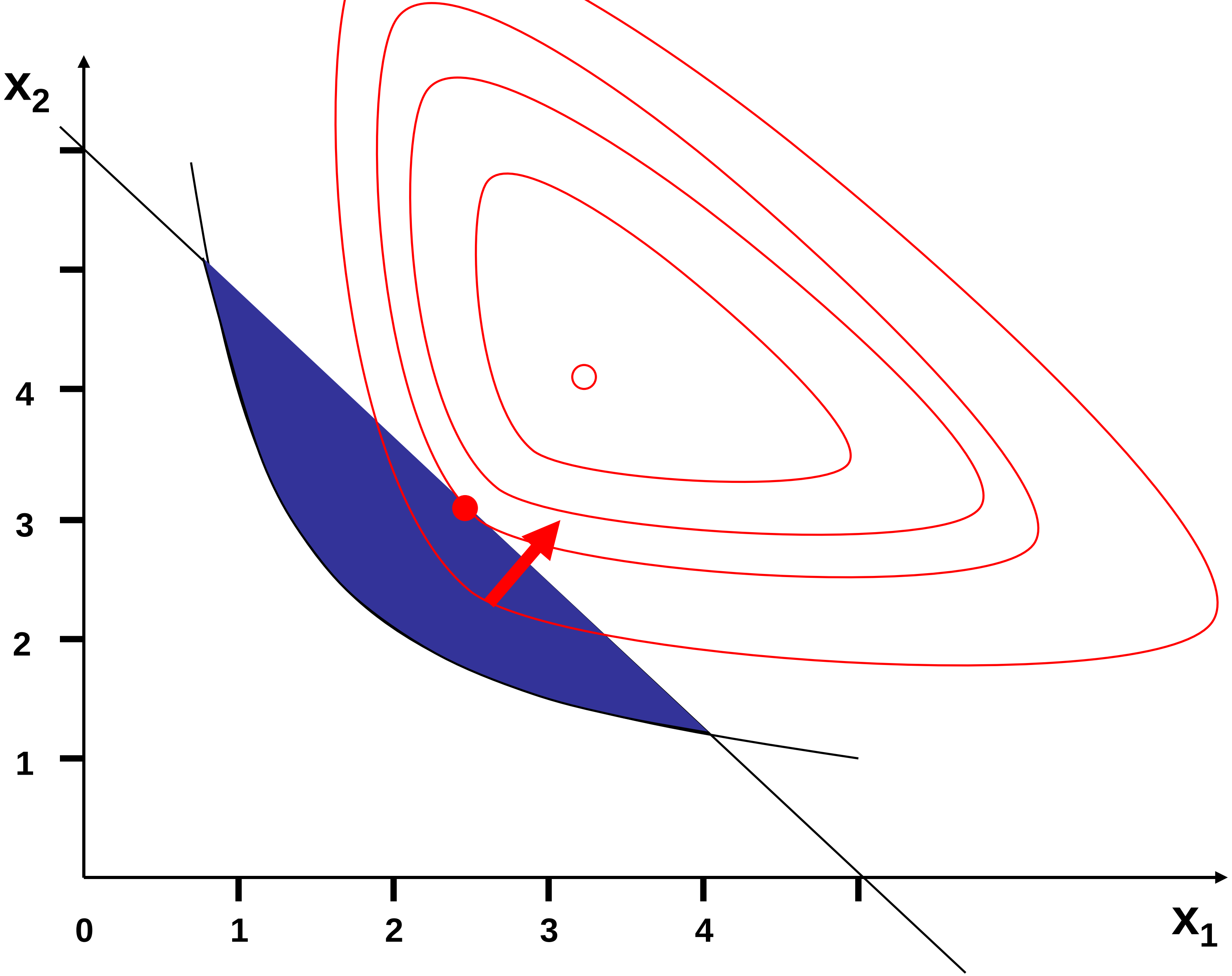
EJEMPLO:

$$\text{MIN}: \quad 150 \cdot x_1 + \frac{125}{x_1} + 200 \cdot x_2 + \frac{160}{x_2}$$

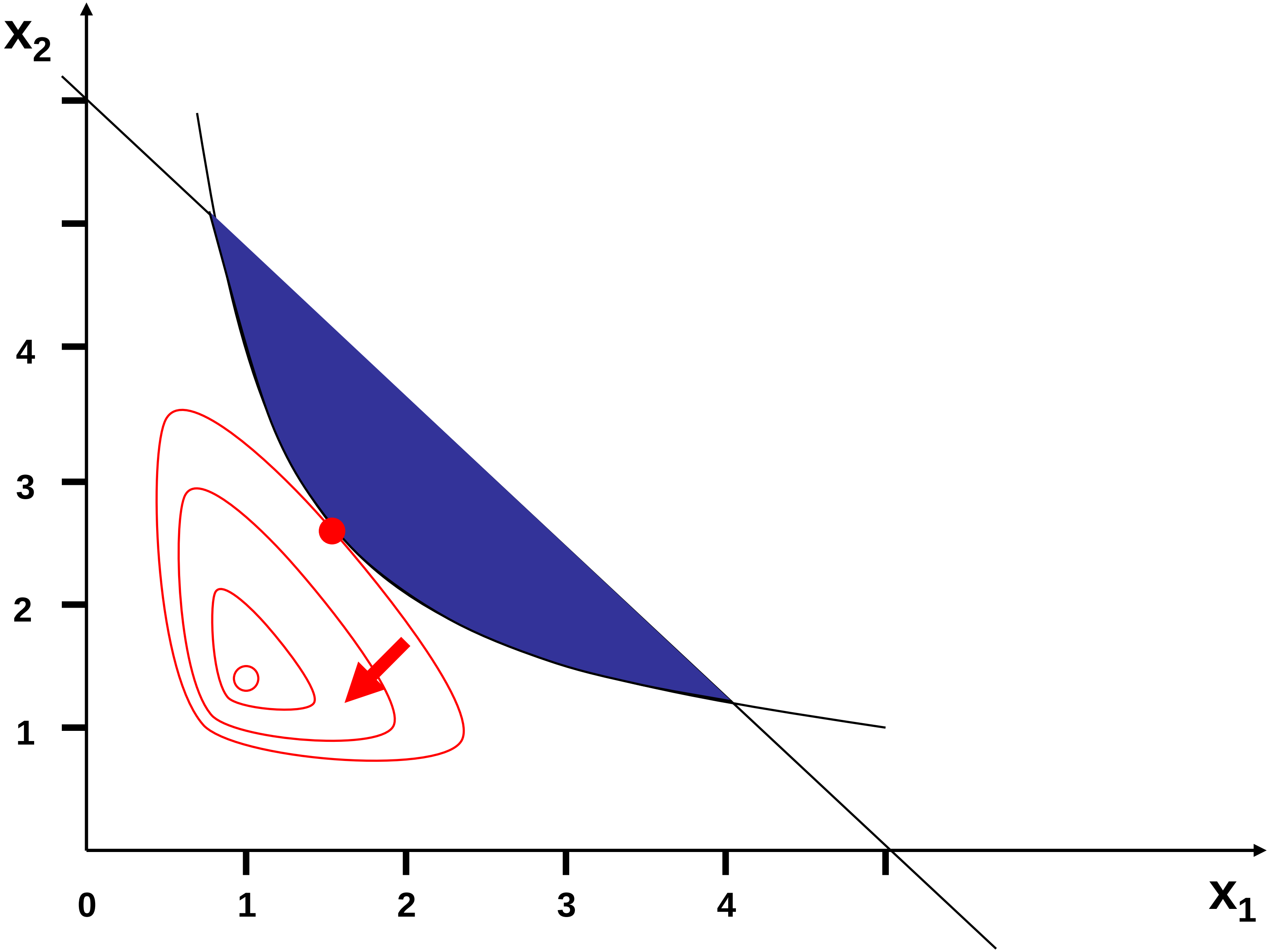
ST

$$\begin{cases} 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 \leq 3 \\ \dfrac{4}{x_1} + \dfrac{6}{x_2} \leq 5 \end{cases}$$

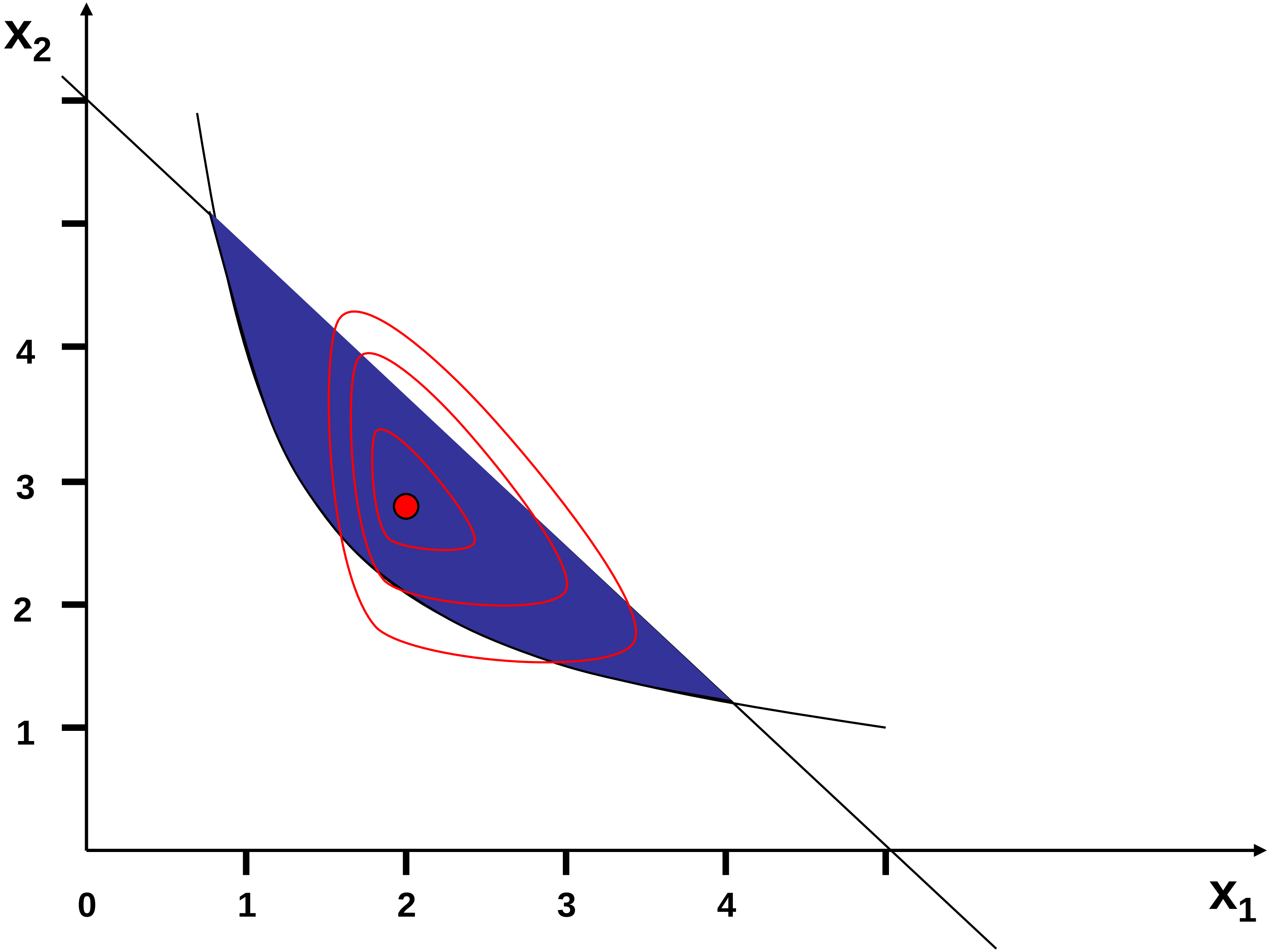
$$x_i \geq 0$$



x_2
4
3
2
1
0
1
2
3
4
x_1



x2
4
3
2
1
0
1
2
3
4
x1



x2
4
3
2
1
0
1
2
3
4
x1

$$\text{MIN}: \quad 150 \cdot x_1 + \frac{125}{x_1} + 200 \cdot x_2 + \frac{160}{x_2}$$

ST

$$\begin{cases} 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 \le 3 \\ \dfrac{4}{x_1} + \dfrac{6}{x_2} \le 5 \end{cases}$$

$$y_1 = \frac{1}{x_1}$$

$$y_2 = \frac{1}{x_2}$$

$$\text{MIN}: \quad 150 \cdot x_1 + \frac{125}{x_1} + 200 \cdot x_2 + \frac{160}{x_2}$$

ST

$$\begin{cases} 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 \le 3 \\ \dfrac{4}{x_1} + \dfrac{6}{x_2} \le 5 \end{cases}$$

$$y_1 = \frac{1}{x_1}$$

$$y_2 = \frac{1}{x_2}$$

$$\text{MIN}: \quad 150 \cdot x_1 + 125 \cdot y_1 + 200 \cdot x_2 + 160 \cdot y_2$$

ST

$$\begin{cases} 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 \le 3 \\ 4 \cdot y_1 + 6 \cdot y_2 \le 5 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

$$y_1 = \frac{1}{x_1}$$

x_1	y_1	
1	1	A_1
2	0.5	A_2
3	0.33	A_3
4	0.25	A_4
5	0.20	A_5

$$\begin{cases} -x_1 + A_1 + 2\cdot A_2 + 3\cdot A_3 + 4\cdot A_4 + 5\cdot A_5 = 0 \\ -y_1 + A_1 + 0.5\cdot A_2 + 0.33\cdot A_3 + 0.25\cdot A_4 + 0.2\cdot A_5 = 0 \\ A_1 + A_2 + A_3 + A_4 + A_5 = 1 \end{cases}$$

$$y_2 = \frac{1}{x_2}$$

x_2	y_2	
1	1	B_1
2	0.5	B_2
3	0.33	B_3
4	0.25	B_4
5	0.20	B_5
6	0.17	B_6

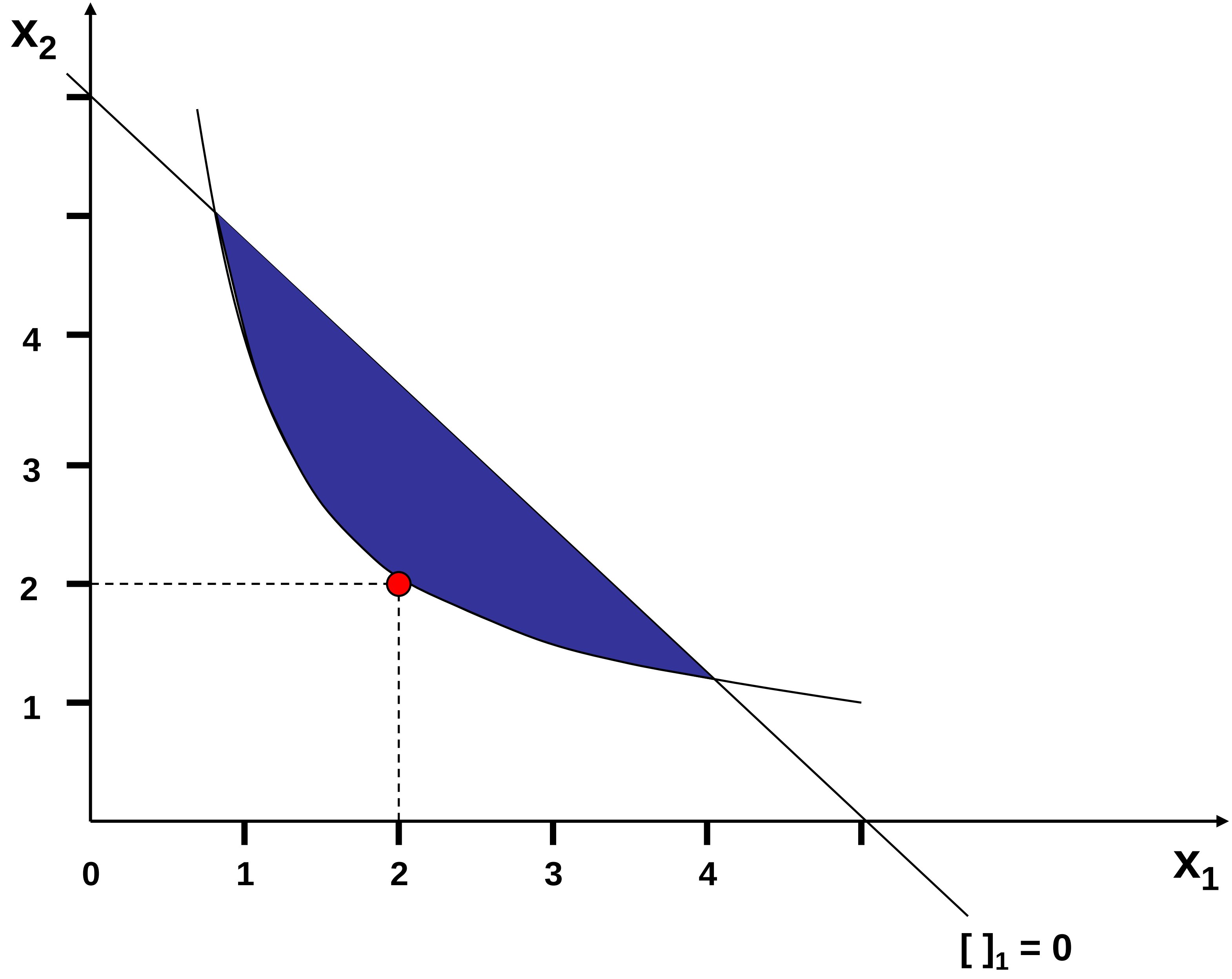
$$\begin{cases} -x_2 + B_1 + 2 \cdot B_2 + 3 \cdot B_3 + 4 \cdot B_4 + 5 \cdot B_5 + 6 \cdot B_6 = 0 \\ -y_2 + B_1 + 0.5 \cdot B_2 + 0.33 \cdot B_3 + 0.25 \cdot B_4 + 0.2 \cdot B_5 + 0.1677 \cdot B_6 = 0 \\ B_1 + B_2 + B_3 + B_4 + B_5 + B_6 = 1 \end{cases}$$

$$\text{MIN}: \quad 150 \cdot x_1 + 125 \cdot y_1 + 200 \cdot x_2 + 160 \cdot y_2$$

ST

$$\begin{cases} 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 \leq 3 \\ 4 \cdot y_1 + 6 \cdot y_2 \leq 5 \\ \\ -x_1 + A_1 + 2 \cdot A_2 + 3 \cdot A_3 + 4 \cdot A_4 + 5 \cdot A_5 = 0 \\ -y_1 + A_1 + 0.5 \cdot A_2 + 0.33 \cdot A_3 + 0.25 \cdot A_4 + 0.2 \cdot A_5 = 0 \\ A_1 + A_2 + A_3 + A_4 + A_5 = 1 \\ \\ -x_2 + B_1 + 2 \cdot B_2 + 3 \cdot B_3 + 4 \cdot B_4 + 5 \cdot B_5 + 6 \cdot B_6 = 0 \\ -y_2 + B_1 + 0.5 \cdot B_2 + 0.33 \cdot B_3 + 0.25 \cdot B_4 + 0.2 \cdot B_5 + 0.1677 \cdot B_6 = 0 \\ B_1 + B_2 + B_3 + B_4 + B_5 + B_6 = 1 \end{cases}$$

$$x_i, y_i, A_i, B_i \geq 0$$



x2
4
3
2
1
0
1
2
3
4
x1
[ ]1 = 0

LP OPTIMUM FOUND AT STEP 4

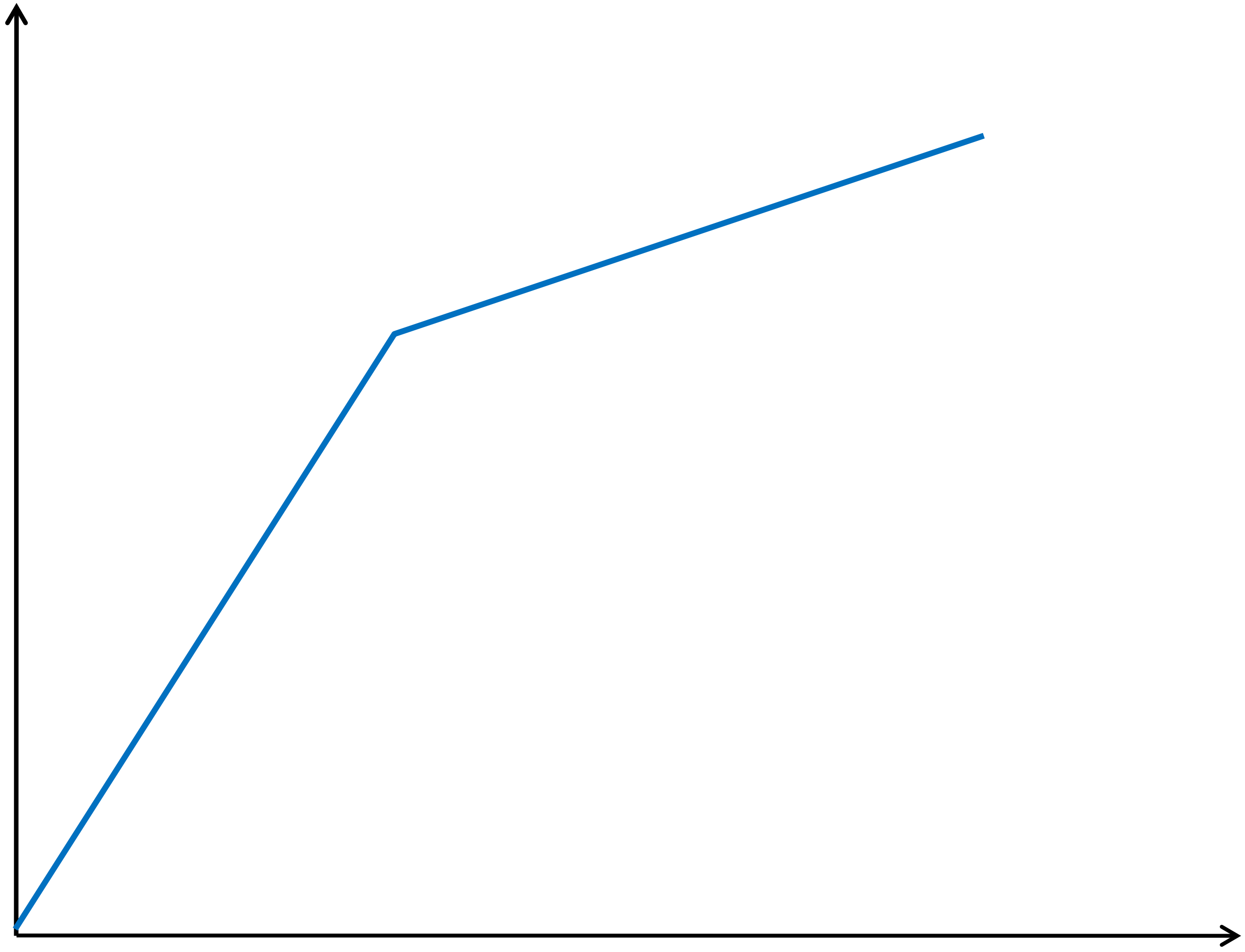
OBJECTIVE FUNCTION VALUE

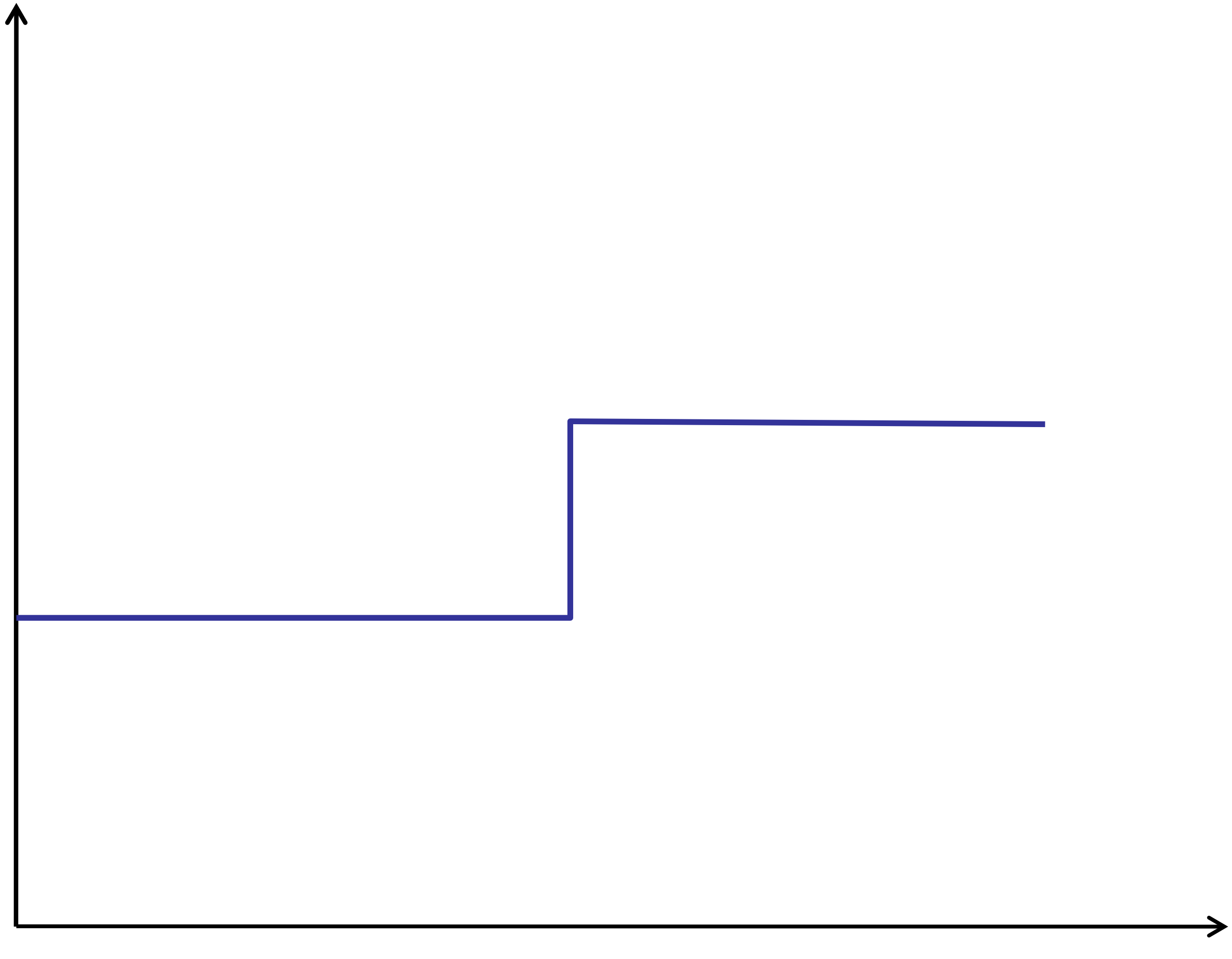
1) 842.5000 842.1916

VARIABLE	VALUE		REDUCED COST
X1	2.000000	1.95	0.000000
Y1	0.500000		0.000000
X2	2.000000	2.03478	0.000000
Y2	0.500000		0.000000
A1	0.000000		0.000000
A2	1.000000		0.000000
A3	0.000000		99.989998
A4	0.000000		225.000000
A5	0.000000		360.000000
B1	0.000000		11.250000
B2	1.000000		0.000000
B3	0.000000		129.569244
B4	0.000000		294.375000
B5	0.000000		473.250000
B6	0.000000		659.180725

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.800000	0.000000
3)	0.000000	43.750000

FORMULACIONES DE RESTRICCIONES U OBJETIVOS LINEALES DE A TRAMOS





MAX: $Z = 4\,x_1 + c_2\,x_2$

Sujeto a:

$$
\begin{cases}
6\,x_1 + 16\,x_2 \leq 48000 \\
12\,x_1 + 6\,x_2 \leq 42000 \\
9\,x_1 + 9\,x_2 \leq 36000
\end{cases}
$$

siendo: $x_1, x_2 \geq 0$ y continuas

$c_2 = 3$ para $x_2 \leq 500$

$c_2 = 5$ para $x_2 > 500$

$$6\,x_1 + 16\,x_2 \le 48000$$

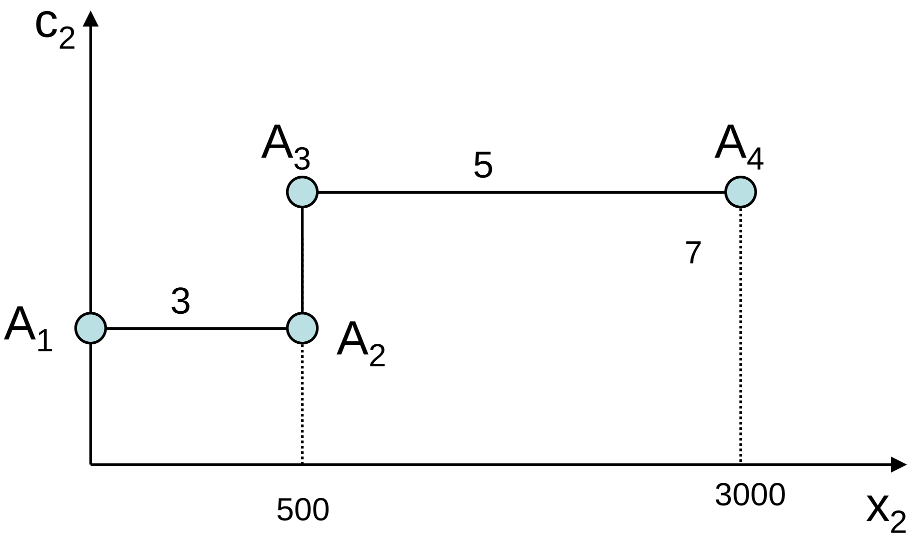
$$12\,x_1 + 6\,x_2 \le 42000$$

$$9\,x_1 + 9\,x_2 \le 36000$$

$$-x_2 + 0\,A_1 + 500\,A_2 + 500\,A_3 + 3000\,A_4 = 0$$

$$A_1 + A_2 + A_3 + A_4 = 1$$

MAX: $Z = 4\,x_1 + 1500\,A_2 + 2500\,A_3 + 15000\,A_4$



OBJECTIVE FUNCTION VALUE

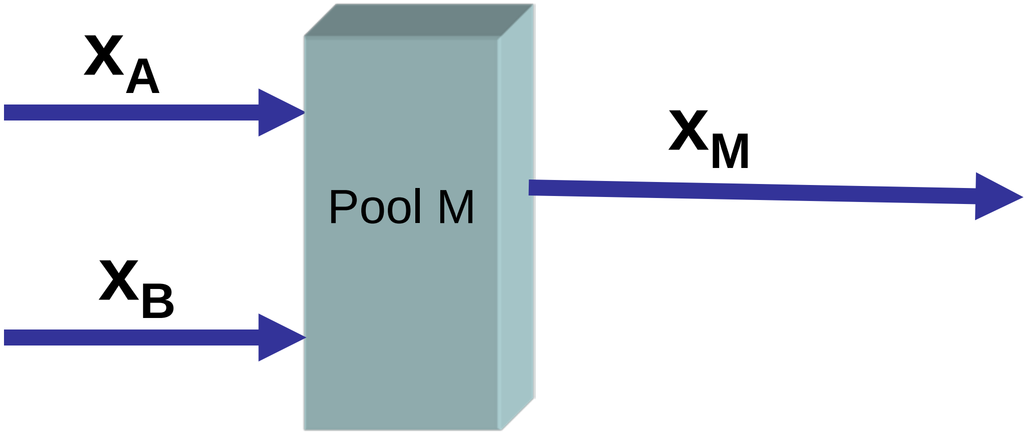
1)	18400.00

VARIABLE	VALUE	REDUCED COST
X1	1600.000000	0.000000
A2	0.000000	1000.000000
A3	0.240000	0.000000
A4	0.760000	0.000000
X2	2400.000000	0.000000
A1	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.100000
3)	8400.000000	0.000000
4)	0.000000	0.377778
5)	0.000000	-5.000000
6)	0.000000	0.000000

PROBLEMA DE MEZCLA

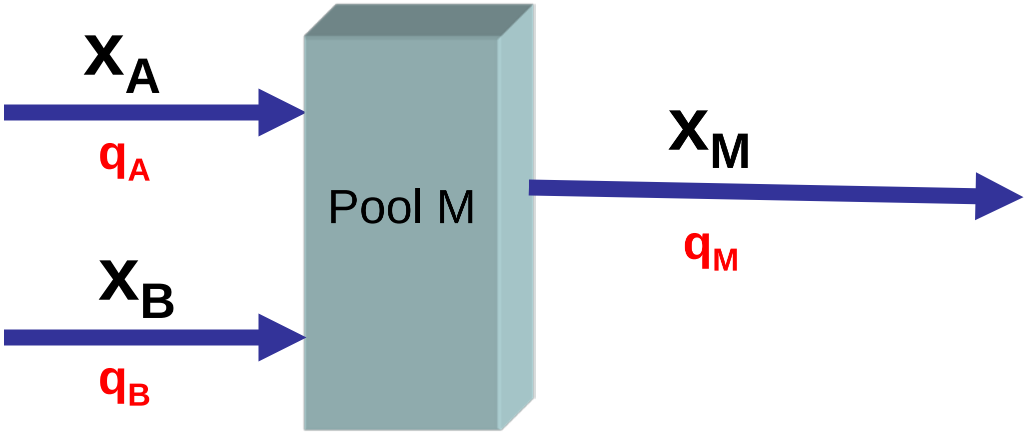
$$x_A + x_B - x_M = 0$$



PROBLEMA DE MEZCLA

$$x_A + x_B - x_M = 0$$

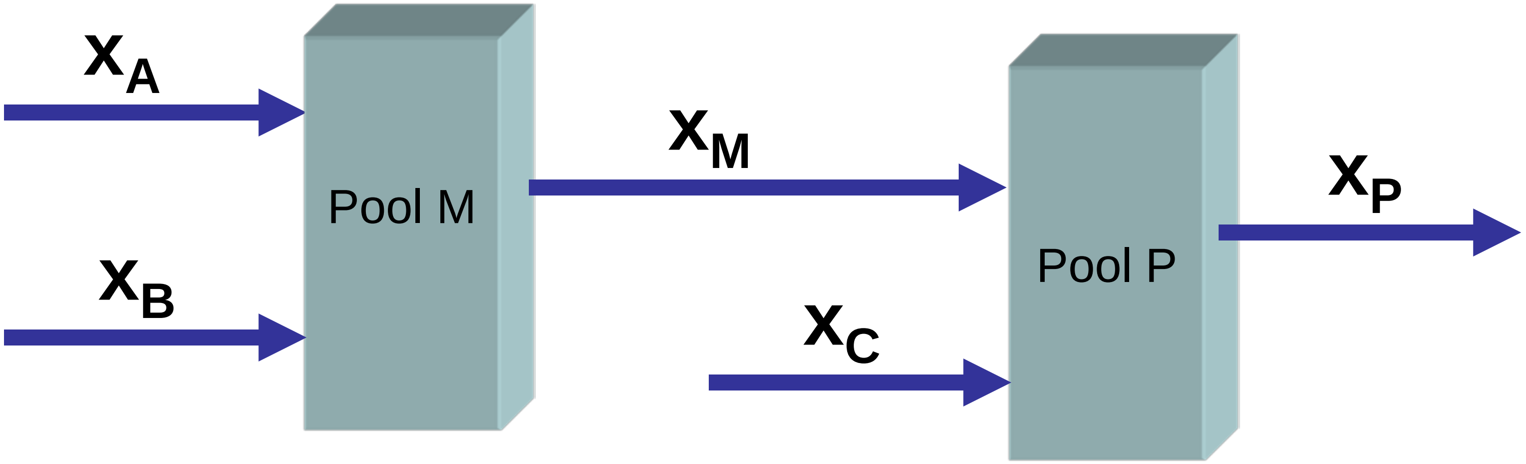
$$q_A \cdot x_A + q_B \cdot x_B - q_P \cdot x_P \leq 0$$



POOLING

$$x_A + x_B - x_M = 0$$

$$x_M + x_C - x_P = 0$$



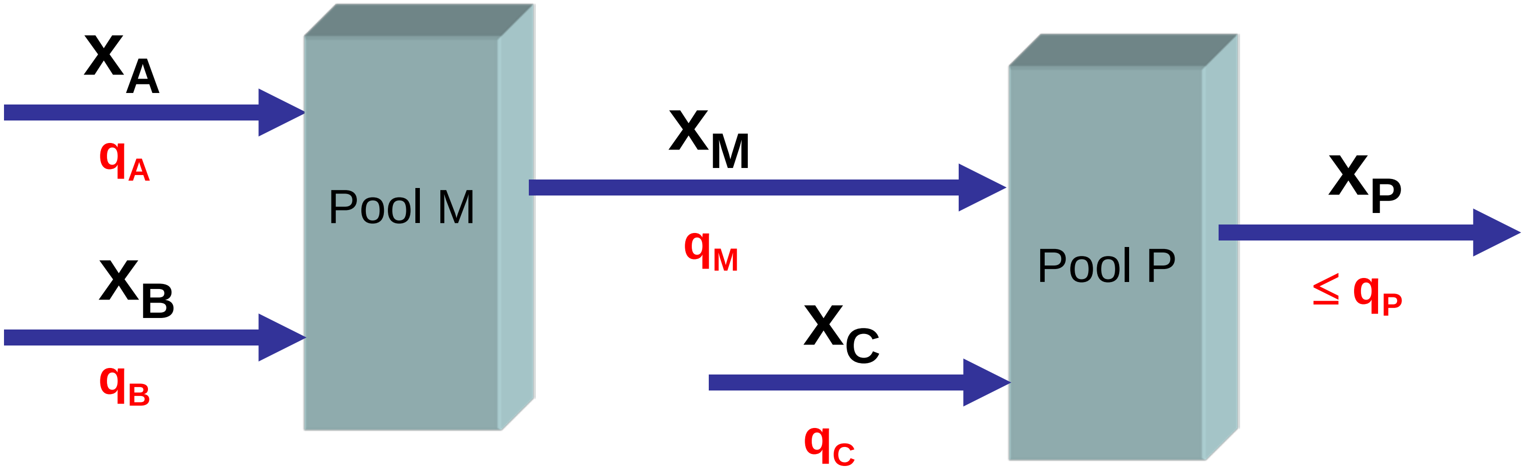
POOLING

$$x_A + x_B - x_M = 0$$

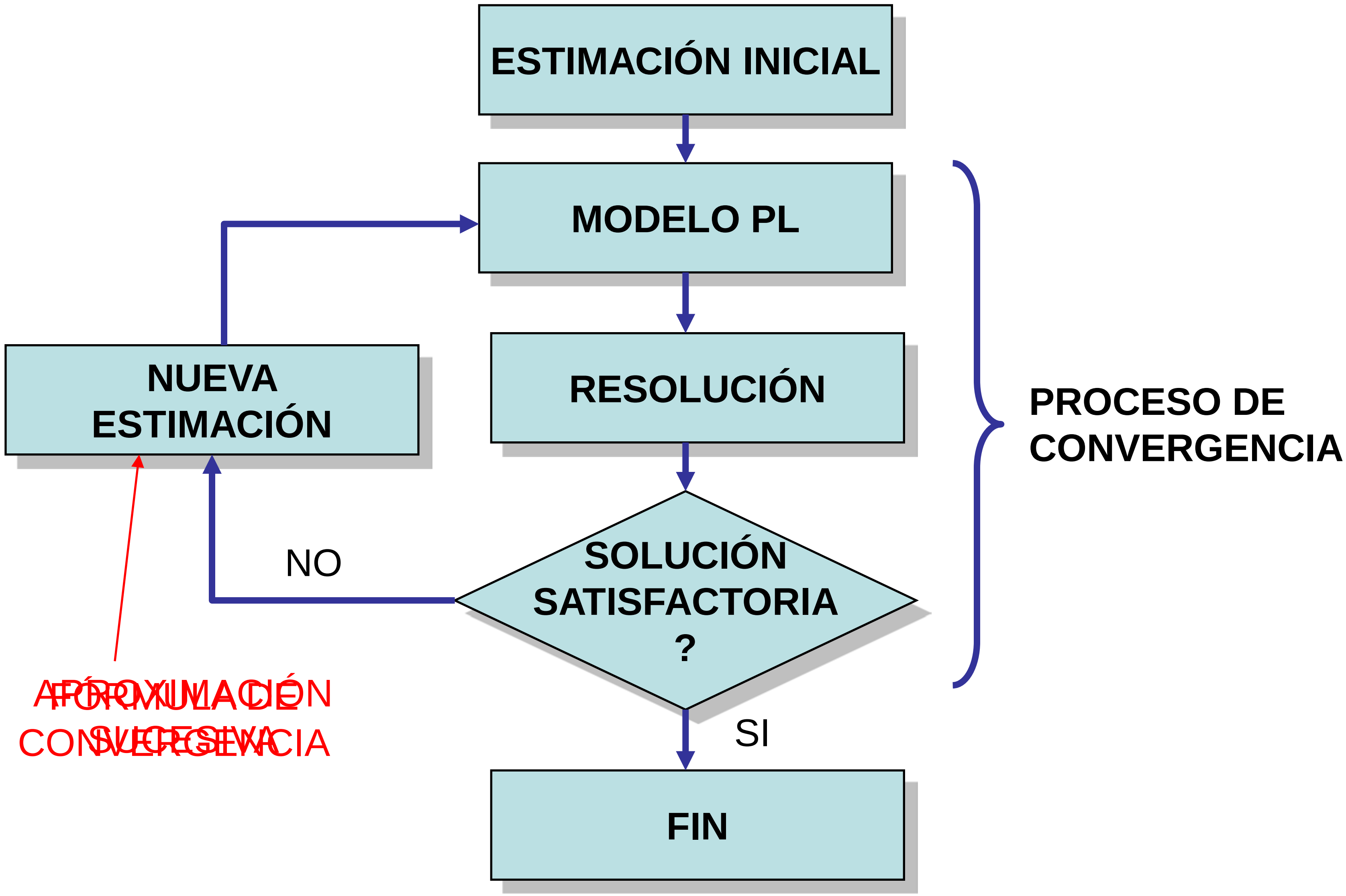
$$x_M + x_C - x_P = 0$$

$$q_A \cdot x_A + q_B \cdot x_B - q_M \cdot x_M = 0$$

$$q_M \cdot x_M + q_C \cdot x_C - q_P \cdot x_P \leq 0$$



PROGRAMACIÓN LINEAL SUCESIVA


ESTIMACIÓN INICIAL
MODELO PL
RESOLUCIÓN
SOLUCIÓN SATISFACTORIA ?
NO
SI
FIN
NUEVA ESTIMACIÓN
PROCESO DE CONVERGENCIA
APROXIMACIÓN SUCESIVA
FÓRMULA DE CONVERGENCIA

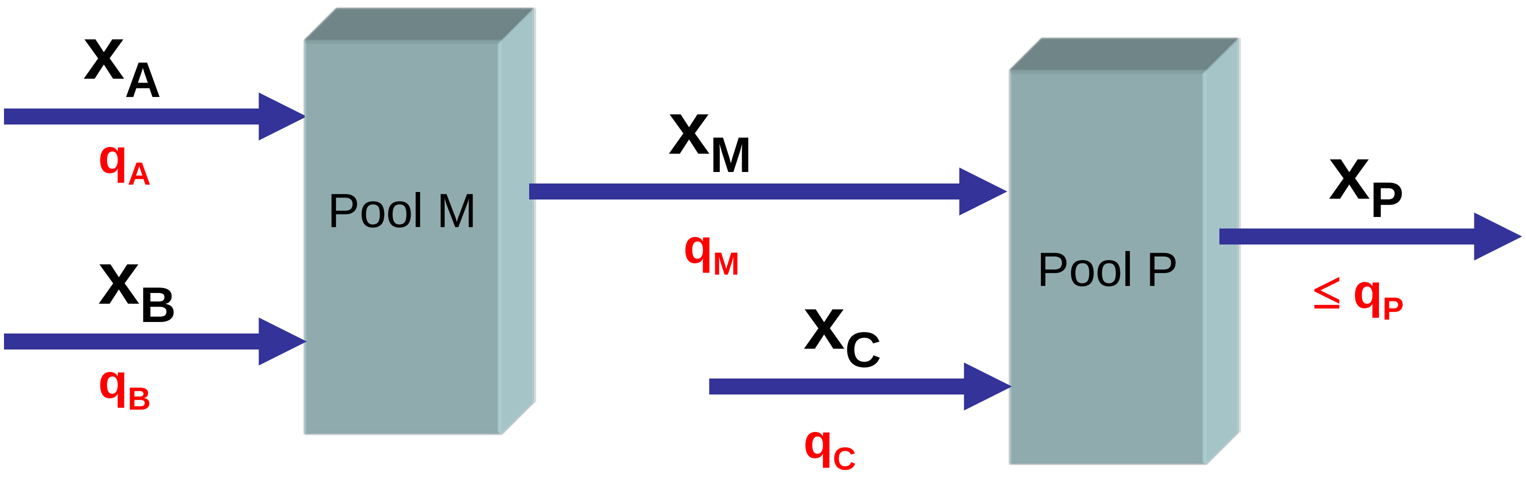
PLANTEO DE FÓRMULA DE RECURRENCIA

Ejemplo

	DISP	q_i
A	40	3.5
B	40	3.0
C	40	2.0

c_i
$10
$12
$15

REQ. P	$Q_{p\ MAX}$
100	2.8



$$x_A + x_B - x_M = 0$$

$$x_M + x_C - x_P = 0$$

$$q_M \cdot x_M + 2.0 \cdot x_C - 2.8 \cdot x_P \leq 0$$

$$3.5 \cdot x_A + 3.0 \cdot x_B - q_M \cdot x_M = 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\ X_A + 12\ x_B + 15\ x_C \Rightarrow \text{Min}$$

$$x_A + x_B - x_M = 0$$

$$x_M + x_C - x_P = 0$$

$$q_M \cdot x_M + 2.0 \cdot x_C - 2.8 \cdot x_P \leq 0$$

$$3.5 \cdot x_A + 3.0 \cdot x_B - q_M \cdot x_M = 0 \quad \Rightarrow \quad q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B}$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

$q_A = 3.5$
$q_B = 3.0$
$q_C = 2.0$
$q_P = 2.8$

$$x_A + x_B - x_M = 0$$

$$x_M + x_C - x_P = 0$$

$$q_M \cdot x_M + 2.0 \cdot x_C - 2.8 \cdot x_P \leq 0$$

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B}$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}25 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

$q_A = 3{,}5$
$q_B = 3$
$q_C = 2$
$q_P = 2{,}8$

X_A	40,00
X_B	24,00
X_C	36,00
X_M	64,00
X_P	100,00
Z	1228,00

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3125$$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}3125 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

$q_A = 3{,}5$
$q_B = 3$
$q_C = 2$
$q_P = 2{,}8$

X_A	**40,00**
X_B	**20,95**
X_C	**39,04**
X_M	**60,95**
X_P	**100,00**
Z	**1237,14**

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3281$$

$x_A + x_B - X_M = 0$

$x_M + x_C - X_P = 0$

$3{,}3281 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$

$x_P = 100$

$x_A \leq 40$

$x_B \leq 40$

$x_C \leq 40$

$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$

$q_A = 3{,}5$
$q_B = 3$
$q_C = 2$
$q_P = 2{,}8$

X_A	40,00
X_B	20,24
X_C	39,76
X_M	60,23
X_P	100,00
Z	1239,29

$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3320$$

$q_A = 3,5$
$q_B = 3$
$q_C = 2$
$q_P = 2,8$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3,3320 \cdot x_M + 2 \cdot x_C - 2,8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

X_A	40,00
X_B	20,06
X_C	39,94
X_M	60,06
X_P	100,00
Z	1239,82

$$q_M = \frac{3,5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3,3330$$

$$x_A + x_B - X_M = 0$$

$$x_M + x_C - X_P = 0$$

$$3{,}3330 \cdot x_M + 2 \cdot x_C - 2{,}8 \cdot X_P \leq 0$$

$$x_P = 100$$

$$x_A \leq 40$$

$$x_B \leq 40$$

$$x_C \leq 40$$

$$Z = 10\, X_A + 12\, x_B + 15\, x_C \Rightarrow \text{Min}$$

$q_A = 3{,}5$
$q_B = 3$
$q_C = 2$
$q_P = 2{,}8$

X_A	**40,00**
X_B	**20,06**
X_C	**39,94**
X_M	**60,06**
X_P	**100,00**
Z	**1239,82**

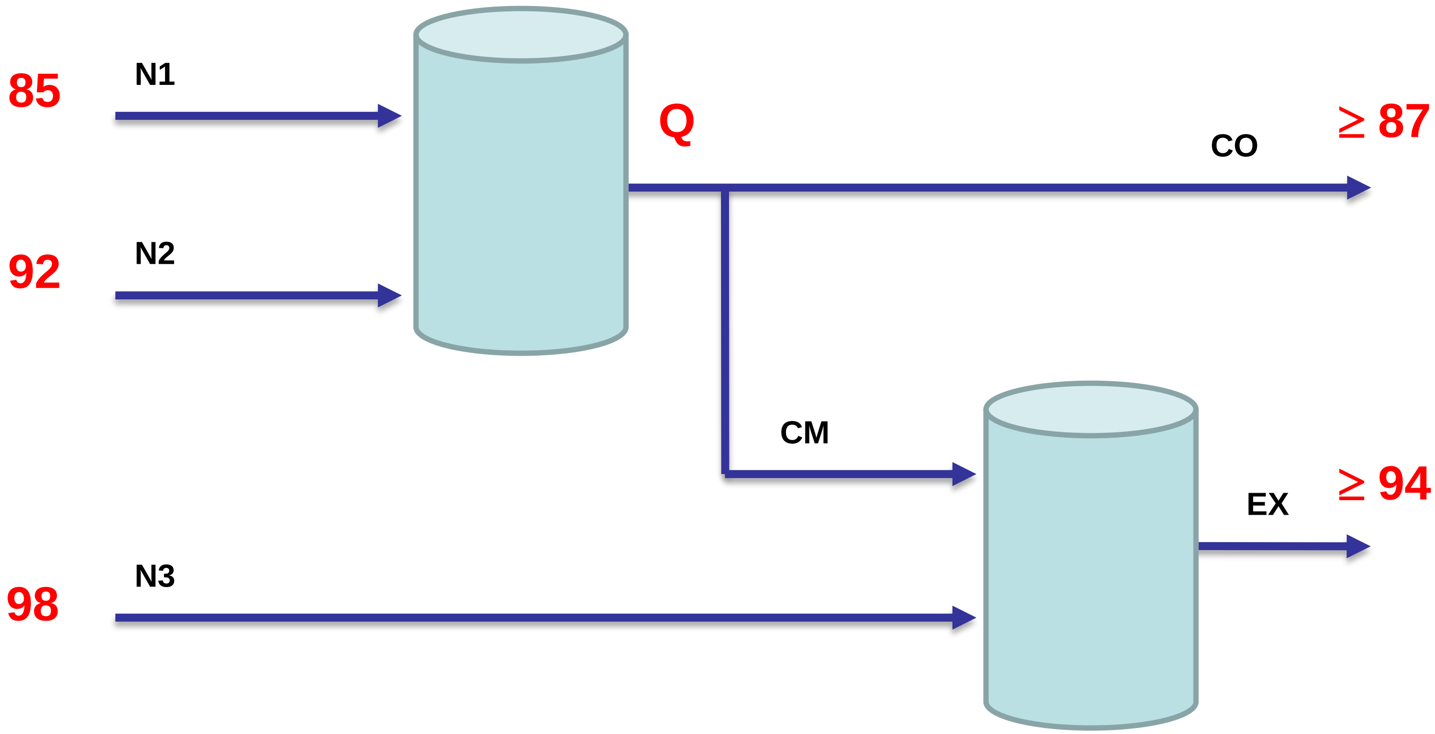
$$q_M = \frac{3{,}5 \cdot x_A + 3 \cdot x_B}{x_A + x_B} = 3{,}3330$$

```
Local optimal solution found.
Objective value:  1240.000
Extended solver steps: 0
Total solver iterations:  21


        Variable           Value        Reduced Cost
              XA        40.00000            0.000000
              XB        20.00000            0.000000
              XC        40.00000            0.000000
              XM        60.00000            0.000000
              XP        100.0000            0.000000
              QM        3.333333            0.000000

             Row    Slack or Surplus      Dual Price
               1        1240.000           -1.000000
               2        0.000000           -21.00000
               3        0.000000           -21.00000
               4       0.2290108E-08        3.000000
               5      -0.2290136E-08        3.000000
               6        0.000000           -12.59994
               7        0.000000            0.500000
               8        20.00000            0.000000
               9        0.000000            0.000000
```

APROXIMACIÓN SUCESIVA


85
N1
92
N2
98
N3
Q
CO
≥ 87
CM
EX
≥ 94

$$MAX \quad 1.2\,EX + 1\,CO - 0.8\,N1 - 0.85\,N2 - 1.25\,N3$$

2) $N1 \leq 5000$

3) $N2 \leq 3000$

4) $N3 \leq 2000$

5) $CO \geq 2500$

6) $CO \leq 3500$

7) $EX \leq 4000$

8) $N1 + N2 - CM - CO = 0$

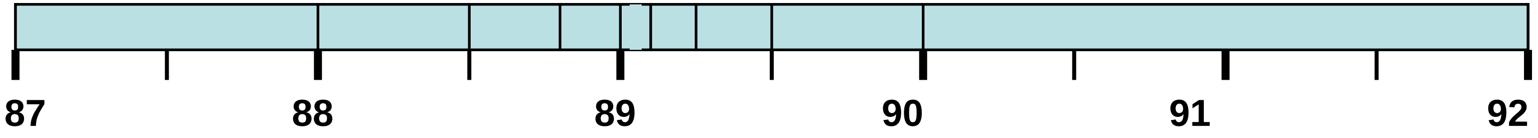
9) $CM + N3 - EX = 0$

10) $85\,N1 + 92\,N2 - \boxed{Q \,.\, CM} - \boxed{Q \,.\, CO} = 0$

11) $\boxed{Q \;\; CM} + 98\;\; N3 - 94\;\; EX \geq 0$

Iteración	Q	Z
1	87	990,82
2	88	1029,76
3	89	1094,29
4	90	945,00
5	88.5	1057,96
6	89.5	1038,89
7	88.75	1074,96
8	89,25	1075,08
9	88,875	1084,71
10	89,125	1.096,39
11	89,05	1098,46
12	89,09	1101,88

89.09324 1102.162



FORMULADO Y RESUELTO CON LINGO

```
MAX = 1.2 * EX + 1 * CO - 0.8 * N1 - 0.85 * N2 - 1.25 * N3;
    N1 < 5000 ;
    N2 < 3000 ;
    N3 < 2000 ;
    CO > 2500 ;
    CO < 3500 ;
    EX < 4000 ;
    N1 + N2 - CM - CO = 0;
    CM + N3 - EX = 0;
    85 * N1 + 92 * N2 - Q * CM - Q * CO = 0 ;
    Q *CM + 98 * N3 - 94 * EX > 0 ;
```

Local optimal solution found.

Objective value: 1102.162

Total solver iterations: 73

Variable	Value	Reduced Cost
EX	3630.405	0.000000
CO	3500.000	0.000000
N1	2130.405	0.000000
N2	3000.000	0.000000
N3	2000.000	0.000000
CM	1630.405	0.000000
Q	89.09324	0.000000

Row	Slack or Surplus	Dual Price
1	1102.162	1.000000
2	2869.595	0.000000
3	0.000000	0.9334451E-01
4	0.000000	0.2077503
5	1000.000	0.000000
6	0.000000	0.1161794
7	369.5949	0.000000
8	0.000000	0.9406118
9	0.000000	4.857131
10	-0.5839043E-05	-0.2047779E-01
11	0.5839043E-05	-0.6443757E-01

Otro ejemplo de recurrencia

$$Z = 5 \cdot x_1 + 4 \cdot x_2 + 2 \cdot x_3 \Rightarrow MAX$$

$$x_1 \cdot x_2 + x_3 \leq 40 \quad \longrightarrow \quad x_1 \leq \frac{40 - x_3}{x_2}$$

$$x_1 + 2 \cdot x_2 \leq 20$$

$$3 \cdot x_2 + x_3 \leq 30$$

$$4 \cdot x_1 + x_3 \leq 30$$

$$x_i \geq 0$$

Expresión de recurrencia: $x_1 = \dfrac{40 - x_3}{x_2}$

$Z = 5 \cdot x_1 + 4 \cdot x_2 + 2 \cdot x_3 \Rightarrow \text{MAX}$

$x_1 \leq$ Valor Máximo

$x_1 + 2 \cdot x_2 \leq 20$

$3 \cdot x_2 + x_3 \leq 30$

$4 \cdot x_1 + x_3 \leq 30$

$x_i \geq 0$

Valor máximo de x_1	RESOLUCIÓN DEL PL				$x_1 \leq \frac{40 - x_3}{x_2}$
	Z	x_1	x_2	x_3	
7.5	72.72727	5.454545	7.272727	8.181818	4.37000
4.37	70.19666	4.370000	5.826667	12.520000	4.71625
4.71625	71.00459	4.716250	6.288333	11.135000	4.5902
4.5902	70.71046	4.590200	6.120266	11.639200	4.6339
4.6339	70.81243	4.633900	6.178534	11.464399	4.6185
4.6185	70.77650	4.618500	6.158000	11.525999	4.6239
4.6239	70.78910	4.623900	6.165200	11.504400	4.6220
4.6220	70.78467	4.622000	6.162667	11.511999	4.6227
4.6227	70.78630	4.622700	6.163600	11.509199	4.6224
4.6224	70.78560	4.622400	6.163200	11.510401	4.6225
4.6225	70.78584	4.622500	6.163333	11.510000	4.6225

EL MISMO PROBLEMA RESUELTO POR PROG. SEPARABLE

$$Z = 5\cdot x_1 + 4\cdot x_2 + 2\cdot x_3 \Rightarrow \text{MAX}$$

$$x_1\cdot x_2 + x_3 \leq 40$$

$$x_1 + 2\cdot x_2 \leq 20$$

$$3\cdot x_2 + x_3 \leq 30$$

$$4\cdot x_1 + x_3 \leq 30$$

$$x_i \geq 0$$

$$x_1 \cdot x_2 + x_3 \leq 40$$

$$- x_1 + p - q = 0$$

$$- x_2 + p + q = 0$$

$$pc - qc + x_3 \leq 40$$

$$- p + 2 \cdot A1 + 4 \cdot A2 + 6 \cdot A3 + 8 \cdot A4 + 10 \cdot A5 = 0$$

$$- pc + 4 \cdot A1 + 16 \cdot A2 + 36 \cdot A3 + 64 \cdot A4 + 100 \cdot A5 = 0$$

$$A0 + A1 + A2 + A3 + A4 + A5 = 1$$

$$- q + 2 \cdot B1 + 4 \cdot B2 + 6 \cdot B3 + 8 \cdot B4 + 10 \cdot B5 = 0$$

$$- qc + 4 \cdot B1 + 16 \cdot B2 + 36 \cdot B3 + 64 \cdot B4 + 100 \cdot B5 = 0$$

$$B0 + B1 + B2 + B3 + B4 + B5 = 1$$

$$Z = 5\cdot x_1 + 4\cdot x_2 + 2\cdot x_3 \Rightarrow MAX$$

$$-x_1 + p - q = 0$$

$$-x_2 + p + q = 0$$

$$pc - qc + x_3 \le 40$$

$$x_1 + 2\cdot x_2 \le 20$$

$$3\cdot x_2 + x_3 \le 30$$

$$4\cdot x_1 + x_3 \le 30$$

$$-p + 2\cdot A1 + 4\cdot A2 + 6\cdot A3 + 8\cdot A4 + 10\cdot A5 = 0$$

$$-pc + 4\cdot A1 + 16\cdot A2 + 36\cdot A3 + 64\cdot A4 + 100\cdot A5 = 0$$

$$A0 + A1 + A2 + A3 + A4 + A5 = 1$$

$$-q + 2\cdot B1 + 4\cdot B2 + 6\cdot B3 + 8\cdot B4 + 10\cdot B5 = 0$$

$$-qc + 4\cdot B1 + 16\cdot B2 + 36\cdot B3 + 64\cdot B4 + 100\cdot B5 = 0$$

$$B0 + B1 + B2 + B3 + B4 + B5 = 1$$

$$x_i, p, pc, q, qc, Aj, Bj \ge 0$$

LINDO

```
MAX 5 x1 + 4 x2 + 2 x3
ST
- x1 + p - q = 0
- x2 + p + q = 0
pc - qc + x3 < 40
x1 + 2 x2 < 20
3 x2 + x3 < 30
4 x1 + x3 < 30
- p + 2 A1 + 4 A2 + 6 A3 + 8 A4 + 10 A5 = 0
- pc + 4  A1 + 16 A2 + 36 A3 + 64 A4 + 100 A5 = 0
A0 + A1 + A2 + A3 + A4 + A5 = 1
- q + 2 B1 + 4 B2 + 6 B3 + 8 B4 + 10 B5 = 0
- qc + 4  B1 + 16 B2 + 36 B3 + 64 B4 + 100 B5 = 0
B0 + B1 + B2 + B3 + B4 + B5 = 1
```

1) 72.68750

VARIABLE	VALUE	REDUCED COST
X1	5.437500	0.000000
X2	7.250000	0.000000
X3	8.250000	0.000000
P	6.343750	0.000000
Q	0.906250	0.000000
PC	40.812500	0.000000
QC	9.062500	0.000000
A1	0.000000	5.250000
A2	0.000000	1.750000
A3	0.828125	0.000000
A4	0.171875	0.000000
A5	0.000000	1.750000
A0	0.000000	10.500000
B1	0.000000	3.500000
B2	0.000000	5.250000
B3	0.000000	5.250000
B4	0.000000	3.500000
B5	0.090625	0.000000
B0	0.909375	0.000000

```
B0 - IB0 < 0
B1 - IB1 < 0
B2 - IB2 < 0
B3 - IB3 < 0
B4 - IB4 < 0
B5 - IB5 < 0

IB0 + IB2 < 1
IB0 + IB3 < 1
IB0 + IB4 < 1
IB0 + IB5 < 1

IB1 + IB3 < 1
IB1 + IB4 < 1
IB1 + IB5 < 1

IB2 + IB4 < 1
IB2 + IB5 < 1

IB3 + IB5 < 1

END

INT IB0
INT IB1
INT IB2
INT IB3
INT IB4
INT IB5
```

LP OPTIMUM FOUND AT STEP 23

OBJECTIVE VALUE = 72.6875000

OBJECTIVE FUNCTION VALUE

1) 70.81818

VARIABLE	VALUE	REDUCED COST
IB0	1.000000	0.000000
IB1	1.000000	0.000000
IB2	0.000000	-2.545455
IB3	0.000000	-7.636364
IB4	0.000000	-15.272727
IB5	0.000000	-25.454546
X1	4.636364	0.000000
X2	6.181818	0.000000
X3	11.454545	0.000000
P	5.409091	0.000000
Q	0.772727	0.000000
PC	30.090910	0.000000
QC	1.545455	0.000000
A1	0.000000	2.545455
A2	0.295455	0.000000
A3	0.704545	0.000000
A4	0.000000	2.545455
A5	0.000000	7.636364
A0	0.000000	7.636364
B1	0.386364	0.000000
B2	0.000000	0.000000
B3	0.000000	0.000000
B4	0.000000	0.000000
B5	0.000000	0.000000
B0	0.613636	0.000000

Objective value: 70.78583

Variable	Value	Reduced Cost
X1	4.622499	0.0000000
X2	6.163332	0.0000000
X3	11.51000	0.0000000

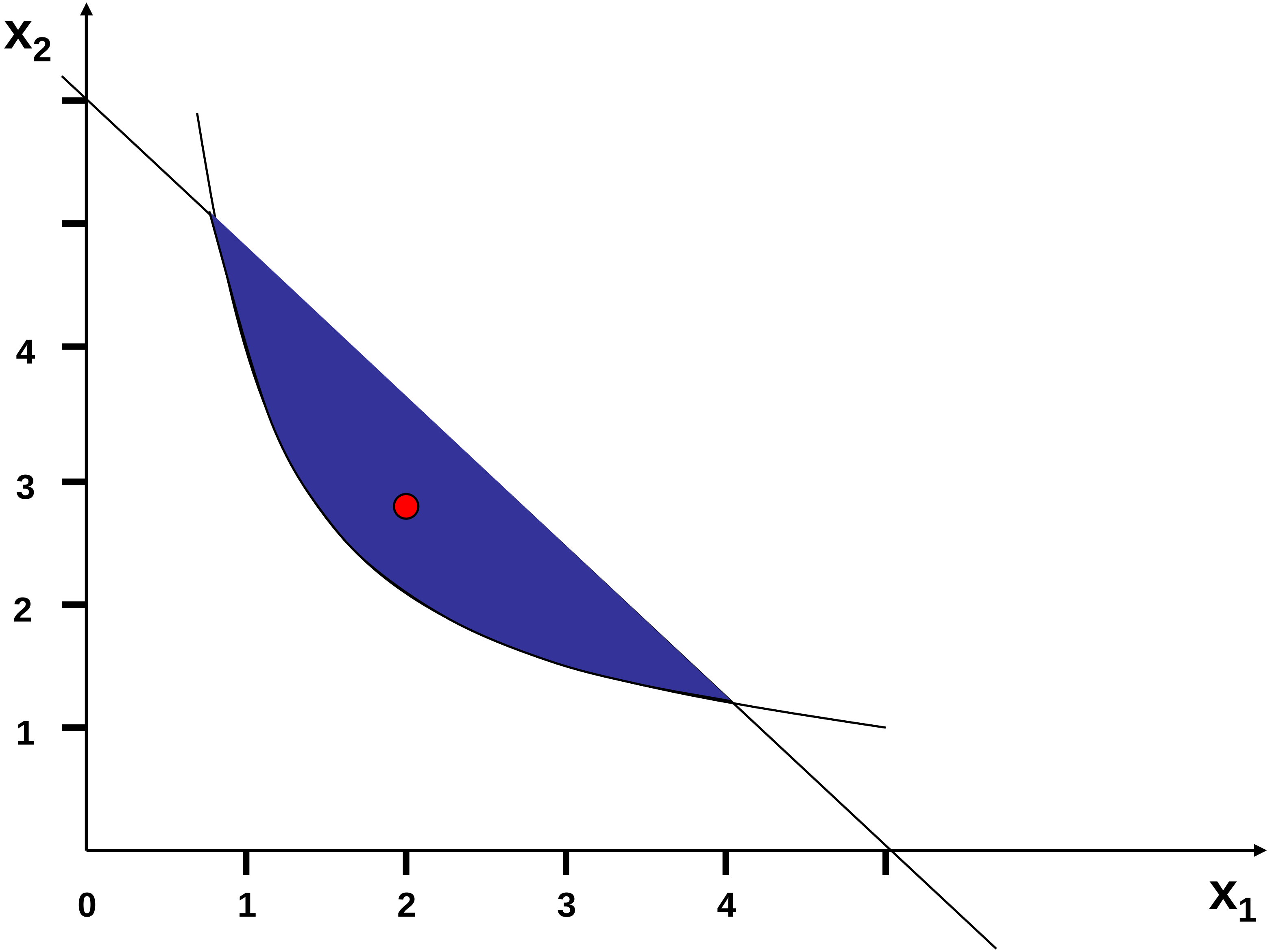
Row	Slack or Surplus	Dual Price
1	70.78583	1.000000
2	-0.8594789E-08	0.2802243
3	3.050837	0.0000000
4	0.0000000	0.9015545
5	0.0000000	0.8182212

Limitaciones

- PROGRAMACIÓN SEPARABLE
  - Aplicable a:
    - Funciones separables
    - Funciones convexas sobre conjuntos convexos
      (de otra forma requiere Prog Binaria)

    Se incrementa considerablemente cantidad de variables

- RECURRENCIA
  - Se deben correr varios programas
  - La solución optima debe estar restringida



x2
4
3
2
1
0
1
2
3
4
x1

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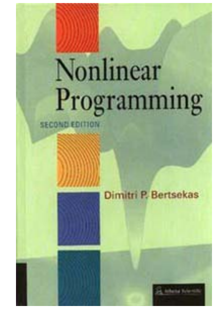
  - George Dantzig, Mukund Thapa: Linear Programming
  - Wayne L. Winston: Investigación de Operaciones. Aplicaciones y Algoritmos
- PROGRAMACIÓN DE METAS
  - Frederick S. Hillier, Gerald J. Lieberman, Frederick Hillier: Introducción a Investigación de Operaciones
- PROGRAMACIÓN SEPARABLE
  - Harvey Wagner: Principles of Operations Research
  - Paul Jensen & Jonathan Bard: Operations Research, Models and Methods
- PROGRAMACIÓN SUCESIVA
  - Internet

MÉTODOS NO BASADOS EN PL

- CADA PROBLEMA DE OPTIMIZACIÓN NO-LINEAL ES ÚNICO
- LOS ALGORITMOS DE RESOLUCIÓN SON COMPLEJOS
- ES DIFÍCIL PREDECIR LA PERFORMANCE DE UN ALGORITMO PARA UN PROBLEMA DADO
- EXISTEN LIMITACIONES PARA RESOLVER PROBLEMAS DE GRAN ESCALA

Bertsekas, Dimitri P. *Nonlinear Programming*. 2nd ed. Athena Scientific Press, 1999. ISBN: 1886529000.



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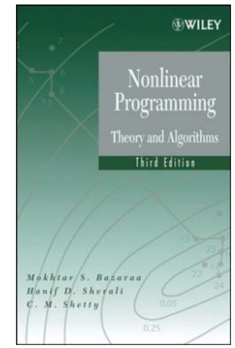

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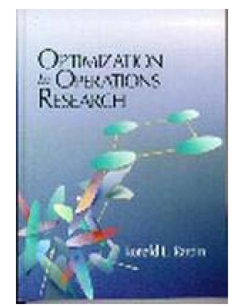

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http://www-unix.mcs.anl.gov/otc/Guide/faq/nonlinear-programming-faq.html

Nonlinear Programming
Frequently Asked Questions

Optimization Technology Center of
Northwestern University and Argonne National Laboratory

http://www.ee.ucla.edu/ee236b/

EE236B: Nonlinear Programming

UCLA Electrical Engineering Department
Prof. Lieven Vandenberghe

Winter Quarter 2007-2008

General Nonlinear Solver – Utiliza las técnicas siguientes:
Gradiente Reducido Generalizado (GRG).
Successive Linear Programming (SLP)

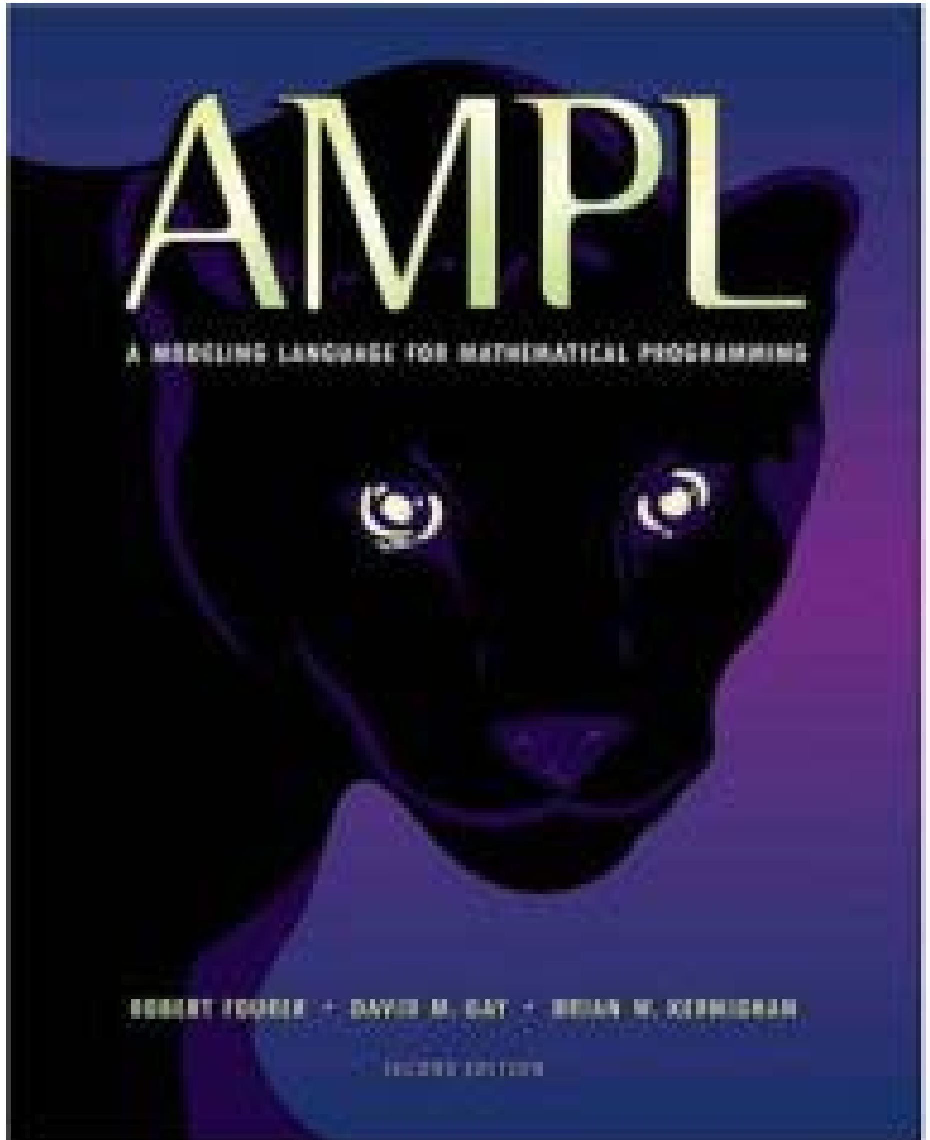
Global Solver – El optimizador no se detiene en soluciones locales.

Multistart Capability – Investiga y genera diferentes puntos de comienzo en el espacio de soluciones.

Quadratic Solver – Resuelve modelos con función objetivo y/o algunas restricciones que incluyan términos cuadráticos, incluso con restricciones binarias y enteras.

Linealización – Capacidad de linealizar funciones (usando técnicas tales como Programación Separable).

http://www.ziena.com


AMPL
A MODELING LANGUAGE FOR MATHEMATICAL PROGRAMMING
ROBERT FOURER • DAVID M. GAY • BRIAN W. KERNIGHAN

MULTIPLICADORES DE LAGRANGE

MAX = $f(x_1, x_2, \ldots\ldots, x_n)$

ST

$g_1 (x_1, x_2,\ldots, x_n) = b_1$

$g_2 (x_1, x_2,\ldots, x_n) = b_2$

……………………………

$g_m (x_1, x_2,\ldots, x_n) = b_m$

$$L = f(x_1, x_2, \ldots, x_N) + \sum_{1}^{m} \lambda_i \cdot \left[b_i - g_i(x_1, x_2, \ldots, x_n) \right]$$

Ejemplo

MAX: $Z = 0{,}003\ x_1^2 + 0{,}005\ x_2^2$

Sujeto a:

$$6\ x_1 + 16\ x_2 = 48000$$

siendo: $x_1, x_2 \geq 0$ **y continuas**

$$L = 0{,}003 \cdot x_1^2 + 0{,}005 \cdot x_2^2 + \lambda \cdot \left[6 \cdot x_1 + 16 \cdot x_2 - 48000\right]$$

$$\frac{\partial L}{\partial x_1} = 0{,}006 \cdot x_1 + 6 \cdot \lambda = 0 \quad \Rightarrow \quad x_1 = -\frac{6 \cdot \lambda}{0{,}006}$$

$$\frac{\partial L}{\partial x_2} = 0{,}01 \cdot x_1 + 16 \cdot \lambda = 0 \quad \Rightarrow \quad x_2 = -\frac{16 \cdot \lambda}{0{,}01}$$

$$\frac{\partial L}{\partial \lambda} = 6 \cdot x_1 + 16 \cdot x_2 = 48000$$

$x_1 = 1.265,8$

$x_2 = 2.025,28$

$\lambda = -1,2658$

$Z = 25.315,54$

CONDICIONES DE KARUSH-KUHN-TUCKER

MIN (o MAX) = $f(x_1, x_2, \ldots.., x_n)$

$g_1 (x_1, x_2,\ldots, x_n) \leq b_1$

$g_2 (x_1, x_2,\ldots, x_n) \leq b_2$

....

$g_m (x_1, x_2,\ldots, x_n) \leq b_m$

x_i no negativas

$$L = f(x_1, x_2, \ldots, x_N) \pm \sum_{1}^{m} \lambda_i \cdot \left[b_i - g_i (x_1, x_2, \ldots, x_n) \right]$$

$$\frac{\partial L}{\partial x_i} = 0$$

$$\lambda_i \cdot \left[b_i - g_i (x_1, x_2, \ldots, x_n) \right] = 0$$

$$\lambda_i \geq 0$$

EJEMPLO:

$$\text{MIN}: \quad 5 \cdot x_1 + \frac{125}{x_1} + 10 \cdot x_2 + \frac{160}{x_2}$$

ST

$$\begin{cases} 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 \leq 3 \\ \dfrac{4}{x_1} + \dfrac{6}{x_2} \leq 5 \end{cases}$$

$$x_i \geq 0$$

$$L = 5 \cdot x_1 + \frac{125}{x_1} + 10 \cdot x_2 + \frac{160}{x_2} + \lambda_1 \cdot \left[0,6 \cdot x_1 + 0,5 \cdot x_2 - 3\right]$$

$$+ \lambda_2 \cdot \left[\frac{4}{x_1} + \frac{6}{x_2} - 5\right] \rightarrow \text{Mín}$$

$$\frac{\partial L}{\partial x_1} = 5 - \frac{125}{x_1^2} + \lambda_1 \cdot 0,6 - \lambda_2 \cdot \frac{4}{x_1^2} = 0$$

$$x_1 = \sqrt{\frac{125 + 4 \cdot \lambda_2}{5 + 0,6 \cdot \lambda_1}}$$

$$L = 5 \cdot x_1 + \frac{125}{x_1} + 10 \cdot x_2 + \frac{160}{x_2} + \lambda_1 \cdot [0{,}6 \cdot x_1 + 0{,}5 \cdot x_2 - 3]$$

$$+ \lambda_2 \cdot \left[\frac{4}{x_1} + \frac{6}{x_2} - 5 \right] \rightarrow \text{Mín}$$

$$\frac{\partial L}{\partial x_2} = 10 - \frac{160}{x_2^2} + \lambda_1 \cdot 0{,}5 - \lambda_2 \cdot \frac{6}{x_1^2} = 0$$

$$x_2 = \sqrt{\frac{160 + 6 \cdot \lambda_2}{10 + 0{,}5 \cdot \lambda_1}}$$

CONDICIONES DE KARUSH-KUHN-TUCKER

(1) $$x_1 = \sqrt{\frac{125 + 4 \cdot \lambda_2}{5 + 0{,}6 \cdot \lambda_1}}$$

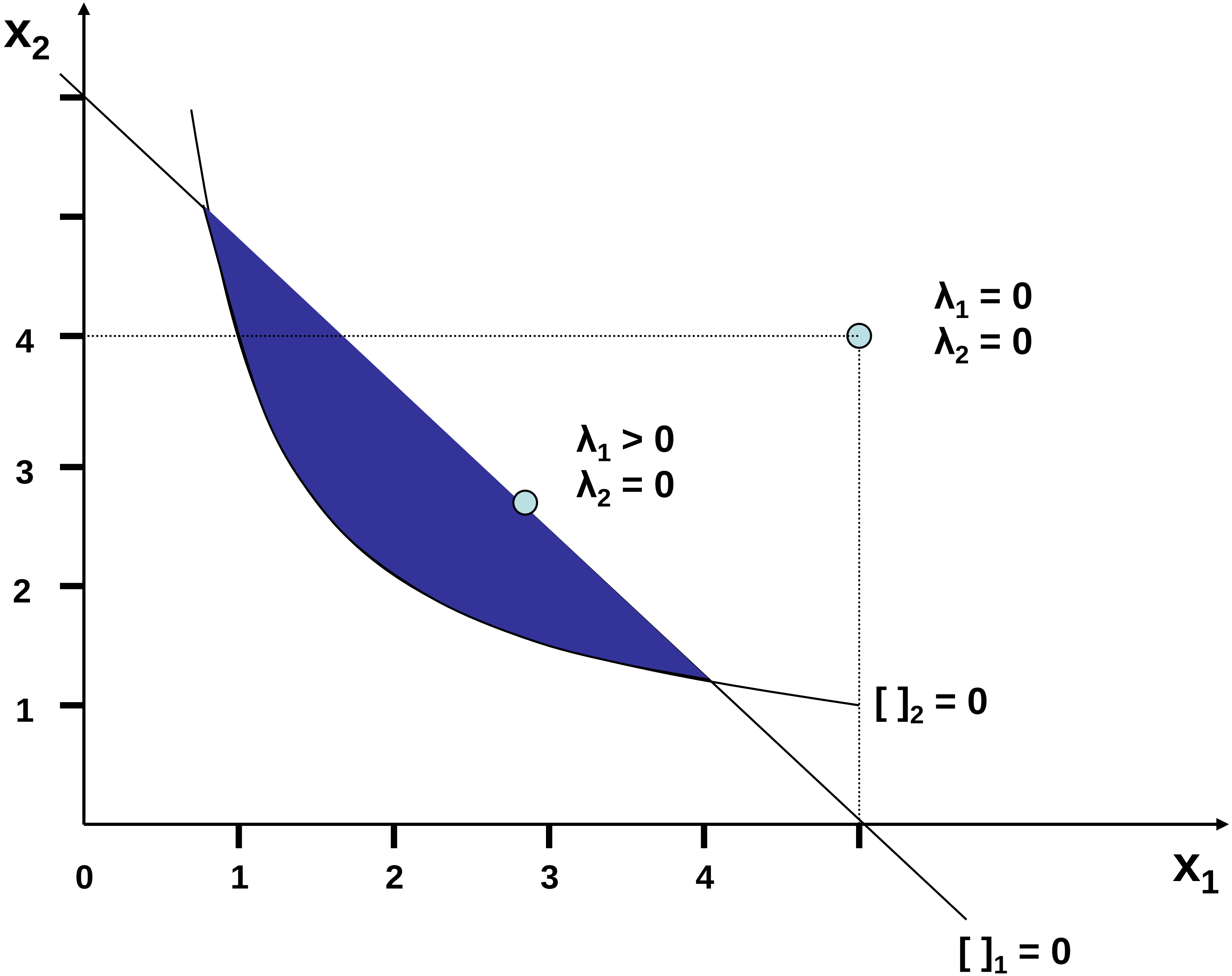
(2) $$x_2 = \sqrt{\frac{160 + 6 \cdot \lambda_2}{10 + 0{,}5 \cdot \lambda_1}}$$

(3) $$\lambda_1 \cdot \left[3 - 0{,}6 \cdot x_1 + 0{,}5 \cdot x_2\right] = 0$$

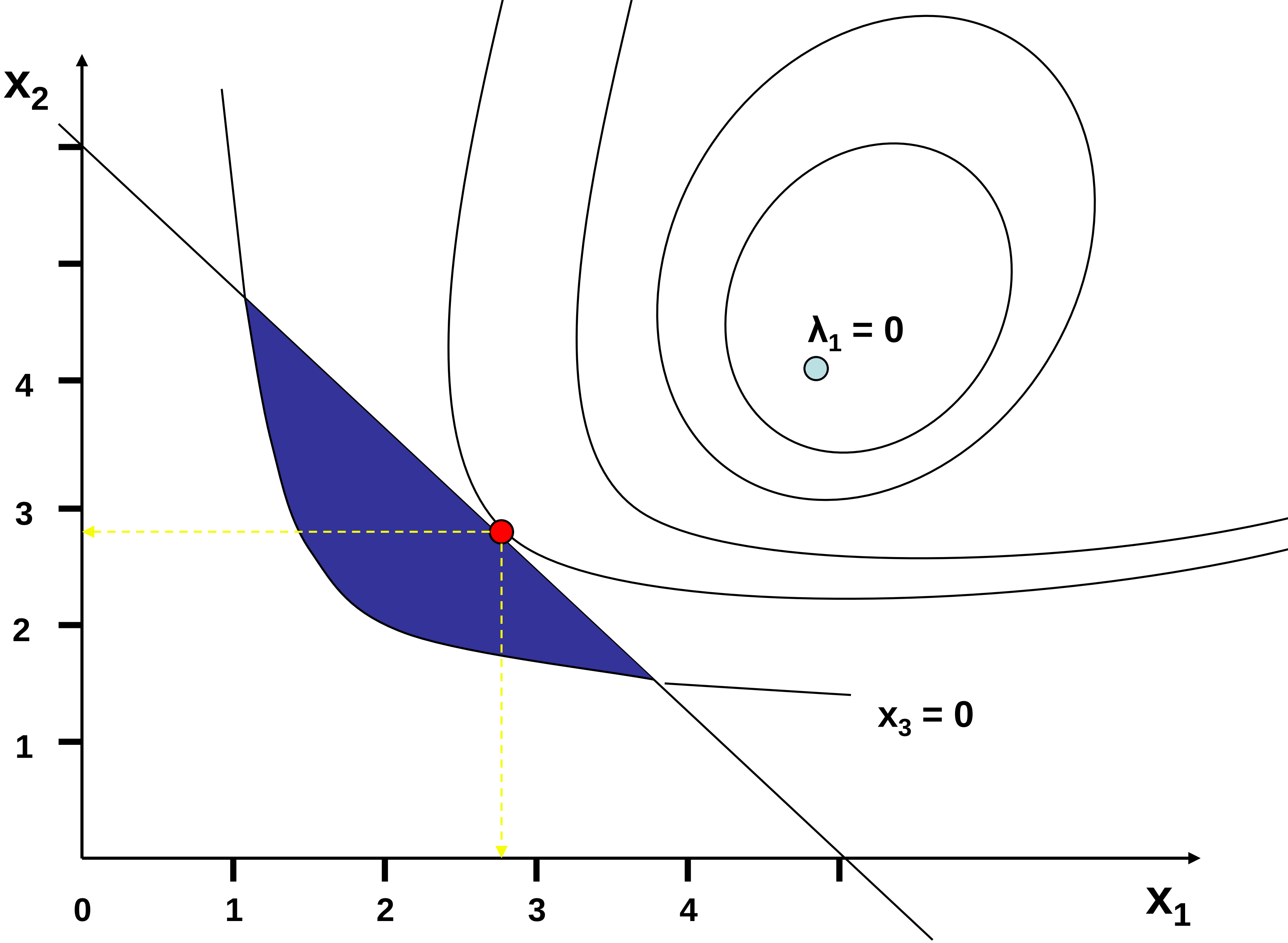
(4) $$\lambda_2 \cdot \left[5 - \left(\frac{4}{x_1} + \frac{6}{x_2}\right)\right] = 0$$

(5) $$\lambda_1 \geq 0$$

(6) $$\lambda_2 \geq 0$$



x2
4
3
2
1
0
1
2
3
4
x1
λ1 = 0
λ2 = 0
λ1 > 0
λ2 = 0
[ ]2 = 0
[ ]1 = 0



x_2
4
3
2
1
$\lambda_1 = 0$
$x_3 = 0$
0
1
2
3
4
x_1

λ_1	λ_2	x_1	x_2	
0	0	4	5	VULNERA 3
>0	**0**	**2,6693**	**2,7969**	**λ_1=20,9069**
0	>0			---
>0	>0			---

USANDO EL SISTEMA "LINGO"

```
MIN = 5 * x1 + 125 / x1 + 10 * x2 + 160 / x2;
0.6 * X1 + 0.5 * X2 < 3;
4 / X1 + 6 / X2 < 5;
```

```
 Local optimal solution found at step:              10
 Objective value:                             145.3511

           Variable           Value        Reduced Cost
                 X1        2.669250           0.0000000
                 X2        2.796900           0.3422487E-06

                Row    Slack or Surplus      Dual Price
                  1        145.3511            1.000000
                  2       0.0000000            20.90686
                  3        1.356219           0.0000000
```

MAX: $Z = 0{,}004\, x_1^2 - 0{,}006\, x_1 + 0{,}003\, x_2^2$

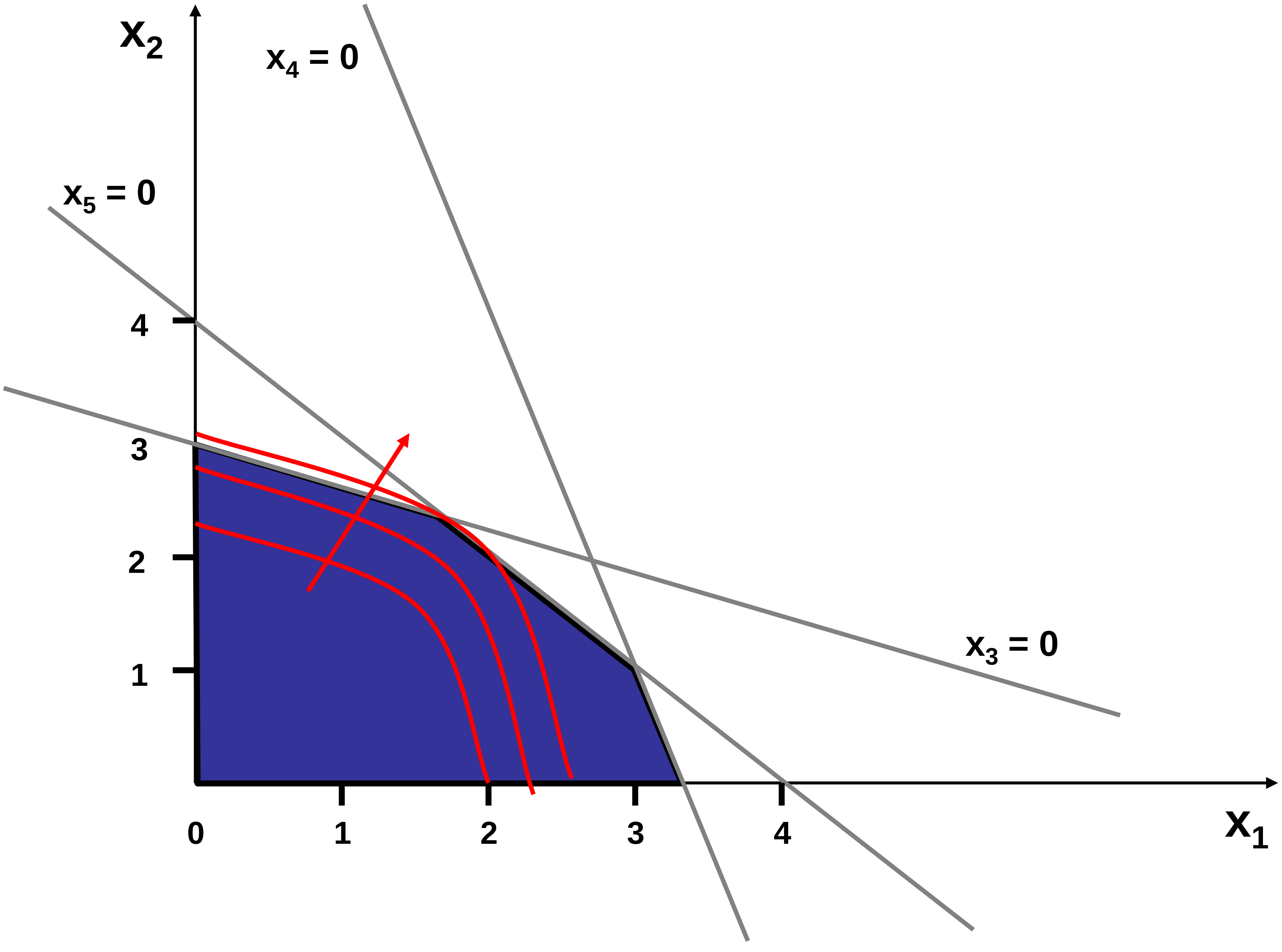
Sujeto a:

$$6\, x_1 + 16\, x_2 \leq 48000$$

$$12\, x_1 + 6\, x_2 \leq 42000$$

$$9\, x_1 + 9\, x_2 \leq 36000$$

siendo: $x_1, x_2 \geq 0$ **y continuas**



x_2
$x_4 = 0$
$x_5 = 0$
4
3
2
1
$x_3 = 0$
0
1
2
3
4
x_1

$$\mathbf{L} = 0{,}004\, x_1^2 - 0{,}006\, x_1 + 0{,}003\, x_2^2$$
$$- \lambda_1 (6\, x_1 + 16\, x_2 - 48000)$$
$$- \lambda_2 (12\, x_1 + 6\, x_2 - 42000)$$
$$- \lambda_3 (9\, x_1 + 9\, x_2 - 36000)$$

$$\frac{\partial L}{\partial x_1} = 0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 - 12 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$\mathbf{L = 0{,}004\, x_1^2 - 0{,}006\, x_1 + 0{,}003\, x_2^2}$$
$$\mathbf{- \lambda_1 (6\, x_1 + 16\, x_2 - 48000)}$$
$$\mathbf{- \lambda_2 (12\, x_1 + 6\, x_2 - 42000)}$$
$$\mathbf{- \lambda_3 (9\, x_1 + 9\, x_2 - 36000)}$$

$$\frac{\partial L}{\partial x_2} = 0{,}006 \cdot x_2 - 16 \cdot \lambda_1 - 6 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

CONDICIONES DE KKT

$$0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 - 12 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 16 \cdot \lambda_1 - 6 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$\lambda_1 (6\, x_1 + 16\, x_2 - 48000) = 0$$

$$\lambda_2 (12\, x_1 + 6\, x_2 - 42000) = 0$$

$$\lambda_3 (9\, x_1 + 9\, x_2 - 36000) = 0$$

$$\lambda_1 = \lambda_2 = \lambda_3 = 0 \Rightarrow \text{INCOMPATIBLE}$$

$$0{,}008 \cdot x_1 - 0{,}006 = 0$$

$$0{,}006 \cdot x_2 = 0$$

$$6\,x_1 + 16\,x_2 - 48000 = 0$$

$$12\,x_1 + 6\,x_2 - 42000 = 0$$

$$9\,x_1 + 9\,x_2 - 36000 = 0$$

$$\lambda_1 = \lambda_2 = 0 \Rightarrow \text{INCOMPATIBLE}$$

$$0{,}008 \cdot x_1 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 9 \cdot \lambda_3 = 0$$

$$6\,x_1 + 16\,x_2 - 48000 = 0$$

$$12\,x_1 + 6\,x_2 - 42000 = 0$$

$$\lambda_1 = \lambda_3 = 0 \Rightarrow \text{INCOMPATIBLE}$$

$$0{,}008 \cdot x_1 - 0{,}006 - 12 \cdot \lambda_2 = 0$$

$$0{,}006 \cdot x_2 - 6 \cdot \lambda_2 = 0$$

$$6\,x_1 + 16\,x_2 - 48000 = 0$$

$$9\,x_1 + 9\,x_2 - 36000 = 0$$

$\lambda_2 = \lambda_3 = 0 \Rightarrow$ **INCOMPATIBLE**

$$0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 = 0$$

$$0{,}006 \cdot x_2 - 16 \cdot \lambda_1 = 0$$

$$12\, x_1 + 6\, x_2 - 42000 = 0$$

$$9\, x_1 + 9\, x_2 - 36000 = 0$$

$\lambda_1 = 0 \Rightarrow$ INCOMPATIBLE

$$0{,}008 \cdot x_1 - 0{,}006 - 12 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 6 \cdot \lambda_2 - 9 \cdot \lambda_3 = 0$$

$$\mathbf{6\ x_1 + 16\ x_2 - 48000 = 0}$$

$\lambda_2 = 0$

$$0{,}008 \cdot x_1 - 0{,}006 - 6 \cdot \lambda_1 - 9 \cdot \lambda_3 = 0$$

$$0{,}006 \cdot x_2 - 16 \cdot \lambda_1 - 9 \cdot \lambda_3 = 0$$

$$6\, x_1 + 16\, x_2 - 48000 = 0$$

$$9\, x_1 + 9\, x_2 - 36000 = 0$$

$X_1 = 1600$

$X_2 = 2400$

$\lambda_1 = 0{,}1606$

$\lambda_2 = 1{,}3145$

FORMULACIÓN CON SISTEMA “LINGO”

```
MAX = 0.004 * x1^2 - 0.006 * X1 + 0.003*x2^2;

 6 * x1 + 16 * x2 < 48000;
12 * x1 +  6 * x2 < 42000;
 9 * x1 +  9 * X2 < 36000;
```

RESOLUCIÓN CON SISTEMA “LINGO”

Local optimal solution found at step: 5
Objective value: 27510.40

Variable	Value	Reduced Cost
X1	1600.000	0.0000000
X2	2400.000	0.0000000

Row	Slack or Surplus	Dual Price
1	27510.40	1.000000
2	0.0000000	0.1606000
3	8400.000	0.0000000
4	0.0000000	1.314489

MAX: $Z = 7\,x_1 + 3\,x_2$

Sujeto a:

$$6\,x_1 + 16\,x_2 \leq 48000$$

$$12\,x_1 + 6\,x_2 \leq 42000$$

$$0{,}00225\,x_1^2 + 0{,}002\,x_2^2 \leq 20250$$

siendo: $x_1, x_2 \geq 0$ y continuas