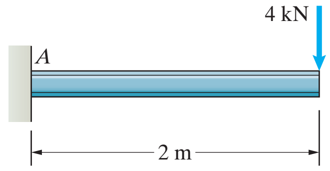
Problem 5.1 In Active Example 5.1, suppose that the beam is subjected to a 6kN-m counterclockwise couple at the right end in addition to the 4-kN downward force. Draw a sketch of the beam showing its new loading. Draw the free-body diagram of the beam and apply the equilibrium equations to determine the reactions at A.



Solution: The equilibrium equations are

$\Sigma F_x : A_x = 0$

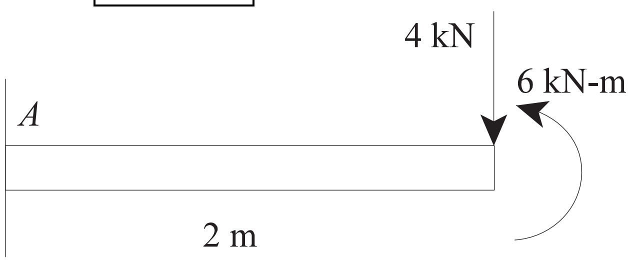
$\Sigma F_y : A_y - 4\text{ kN} = 0$

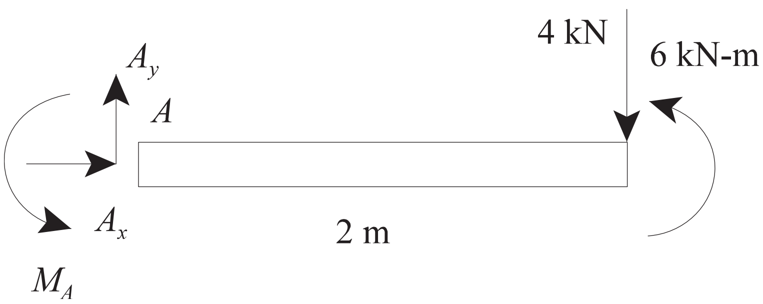
$\Sigma M_A : M_A - (4\text{ kN})(2\text{ m})$

$+ (6\text{ kN-m}) = 0$

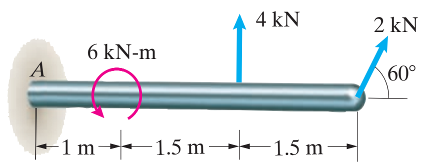
Solving yields

$$\boxed{\begin{aligned} A_x &= 0 \\ A_y &= 4\text{ kN} \\ M_A &= 2\text{ kN-m} \end{aligned}}$$

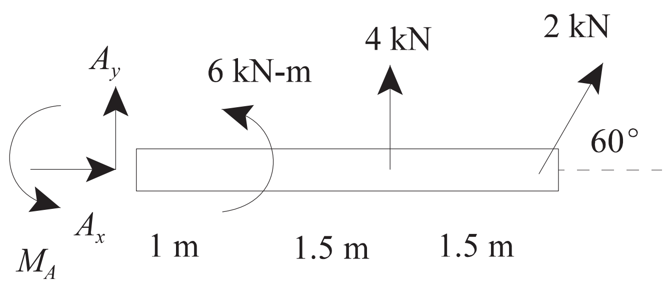




Problem 5.2 The beam has a fixed support at A and is loaded by two forces and a couple. Draw the free-body diagram of the beam and apply equilibrium to determine the reactions at A.



Solution: The free-body diagram is drawn.



The equilibrium equations are

$\Sigma F_x : A_x + (2\text{ kN})\cos 60^\circ = 0$

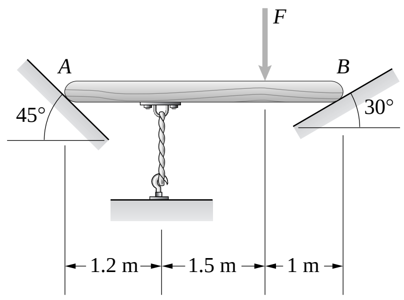
$\Sigma F_y : A_y + (4\text{ kN}) + (2\text{ kN})\sin 60^\circ = 0$

$\Sigma M_A : M_A + (6\text{ kN-m}) + (4\text{ kN})(2.5\text{ m}) + (2\text{ kN})\sin 60^\circ(4\text{ m} = 0)$

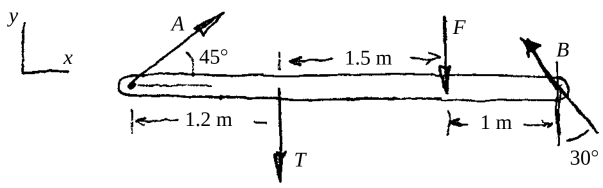
We obtain: $\boxed{A_x = -1\text{ kN}, A_y = -5.73\text{ kN}, M_A = -22.9\text{ kN-m}}$

Problem 5.3 The beam is subjected to a load $F = 400$ N and is supported by the rope and the smooth surfaces at A and B.

(a) Draw the free-body diagram of the beam.
(b) What are the magnitudes of the reactions at A and B?



Solution:



$$\sum F_X = 0: \quad A\cos 45^\circ - B\sin 30^\circ = 0$$

$$\sum F_Y = 0: \quad A\sin 45^\circ + B\cos 30^\circ - T - 400\text{ N} = 0$$

$$\circlearrowleft + \sum M_A = 0: \quad -1.2T - 2.7(400) + 3.7B\cos 30^\circ = 0$$

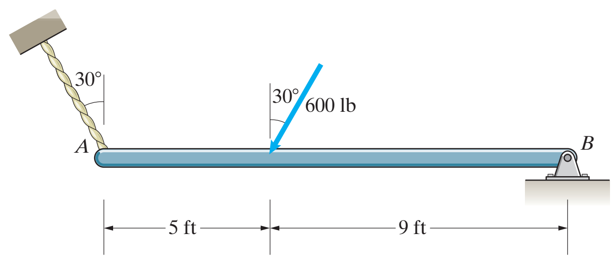
Solving, we get

$A = 271$ N

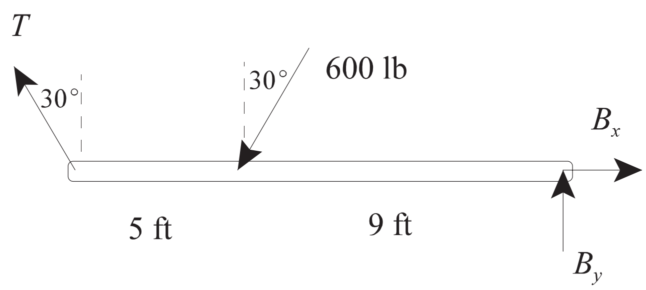
$B = 383$ N

$T = 124$ N

Problem 5.4 (a) Draw the free-body diagram of the beam. (b) Determine the tension in the rope and the reactions at B.



Solution: Let T be the tension in the rope.



The equilibrium equations are:

$\Sigma F_x : -T\sin 30^\circ - (600\text{ lb})\sin 30^\circ + B_x = 0$

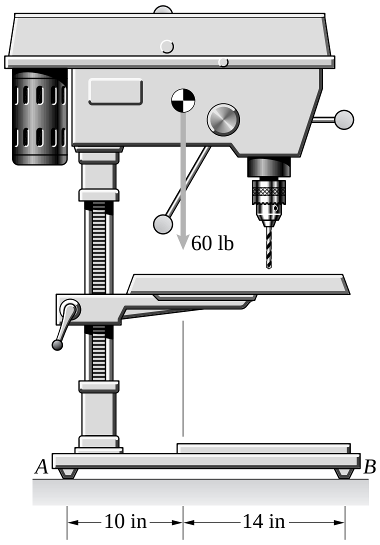
$\Sigma F_y : T\cos 30^\circ - (600\text{ lb})\cos 30^\circ + B_y = 0$

$\Sigma M_B : (600\text{ lb})\cos 30^\circ(9\text{ ft}) - T\cos 30^\circ(14\text{ ft}) = 0$

Solving yields $\boxed{T = 368\text{ lb}, B_x = 493\text{ lb}, B_y = 186\text{ lb}}$

Problem 5.5 (a) Draw the free-body diagram of the 60-lb drill press, assuming that the surfaces at A and B are smooth.

(b) Determine the reactions at A and B.



Solution: The system is in equilibrium.

(a) The free body diagram is shown.
(b) The sum of the forces:

$$\sum F_X = 0, \quad \sum F_Y = F_A + F_B - 60 = 0$$

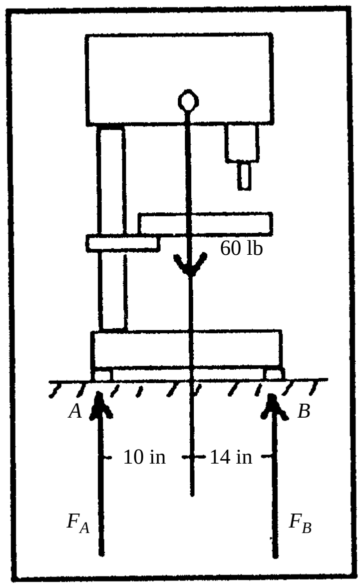
The sum of the moments about point A:

$$\sum M_A = -10(60) + 24(F_B) = 0,$$

from which $F_B = \frac{600}{24} = 25$ lb

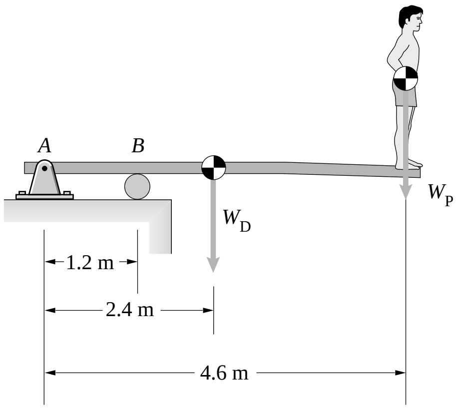
Substitute into the force balance equation:

$F_A = 60 - F_B = 35$ lb



Problem 5.6 The masses of the person and the diving board are 54 kg and 36 kg, respectively. Assume that they are in equilibrium.

(a) Draw the free-body diagram of the diving board.
(b) Determine the reactions at the supports A and B.



Solution:

(a)

(b) $\sum F_X = 0$: $A_X = 0$

$\sum F_Y = 0$: $A_Y + B_Y - (54)(9.81) - 36(9.81) = 0$

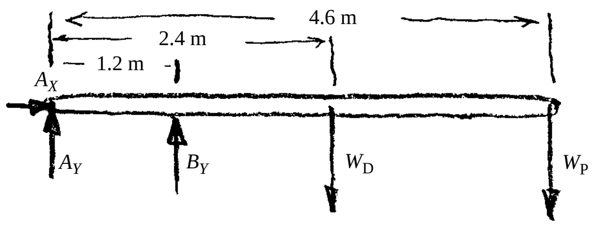
$\sum M_A = 0$: $1.2B_Y - (2.4)(36)(9.81)$

$- (4.6)(54)(9.81) = 0$

Solving: $A_X = 0$ N

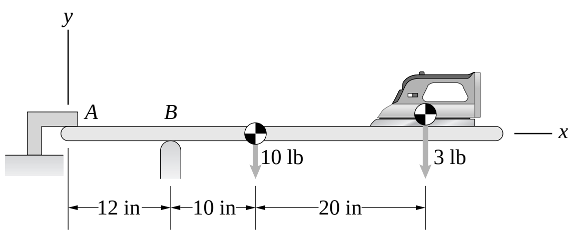
$A_Y = -1.85$ kN

$B_Y = 2.74$ kN



Problem 5.7 The ironing board has supports at A and B that can be modeled as roller supports.

(a) Draw the free-body diagram of the ironing board.
(b) Determine the reactions at A and B.



Solution: The system is in equilibrium.

(a) The free-body diagram is shown.
(b) The sums of the forces are:

$$\sum F_X = 0,$$

$$\sum F_Y = -F_A + F_B - 10 - 3 = 0.$$

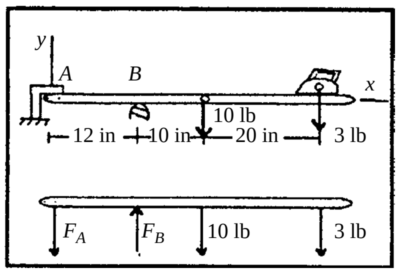
The sum of the moments about A is

$$\sum M_A = 12F_B - 22(10) - 42(3) = 0,$$

from which $F_B = \frac{346}{12} = 28.833$ in.

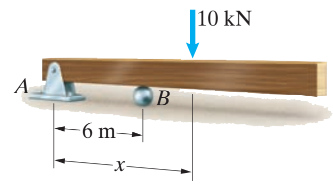
Substitute into the force balance equation:

$F_A = 13 - F_B = +15.833$ lb



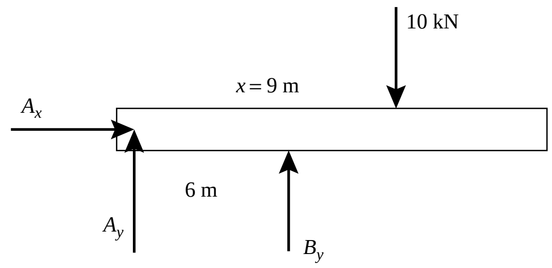
Problem 5.8 The distance $x = 9$ m.

(a) Draw the free-body diagram of the beam.
(b) Determine the reactions at the supports.



Solution:

(a) The FBD
(b) The equilibrium equations



$$\sum F_x : A_x = 0$$

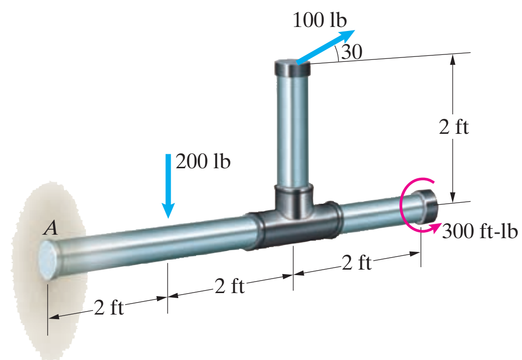
$$\sum F_y : A_y + B_y - 10 \text{ kN} = 0$$

$$\sum M_A : B_y(6 \text{ m}) - (10 \text{ kN})(9 \text{ m}) = 0$$

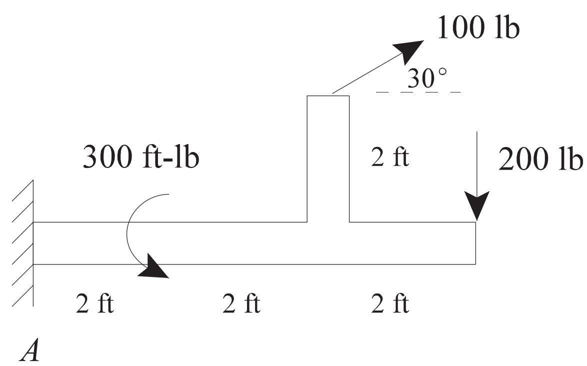
Solving we find

$$\boxed{A_x = 0, \ A_y = -5 \text{ kN}, \ B_y = 15 \text{ kN}}$$

Problem 5.9 In Example 5.2, suppose that the 200-lb downward force and the 300 ft-lb counterclockwise couple change places; the 200-lb downward force acts at the right end of the horizontal bar, and the 300 ft-lb counterclockwise couple acts on the horizontal bar 2 ft to the right of the support A. Draw a sketch of the object showing the new loading. Draw the free-body diagram of the object and apply the equilibrium equations to determine the reactions at A.



Solution: The sketch and free-body diagram are shown.



The equilibrium equations are

$\Sigma F_x : A_x + (100 \text{ lb}) \cos 30^\circ = 0$

$\Sigma F_y : A_y + (100 \text{ lb}) \sin 30^\circ - 200 \text{ lb} = 0$

$\Sigma M_A : M_A + (300 \text{ ft-lb})$

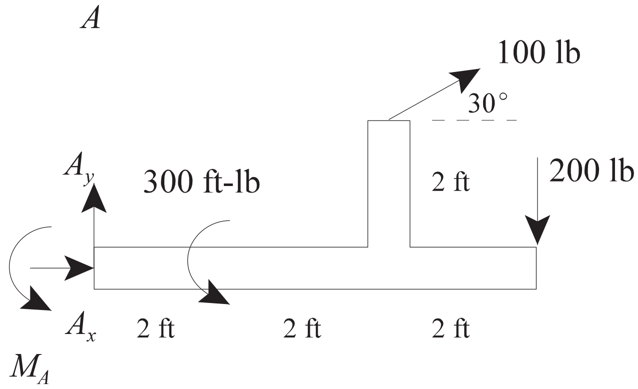
$+ (100 \text{ lb}) \sin 30^\circ (4 \text{ ft})$

$- (100 \text{ lb}) \cos 30^\circ (2 \text{ ft})$

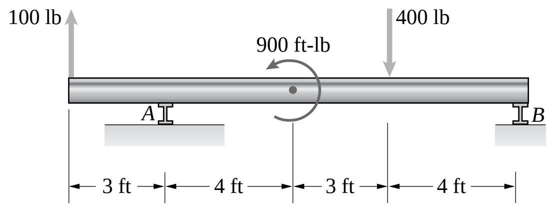
$- (200 \text{ lb})(6 \text{ ft}) = 0$

We obtain

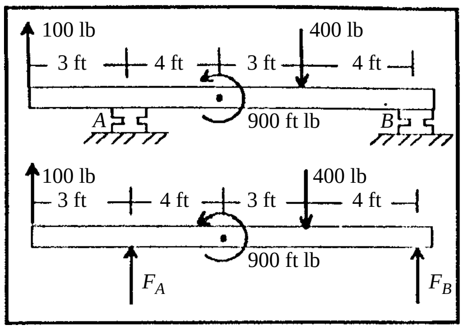
$$\boxed{\begin{aligned} A_x &= -86.6 \text{ lb} \\ A_y &= 150 \text{ lb} \\ M_A &= 873 \text{ ft-lb} \end{aligned}}$$



Problem 5.10 (a) Draw the free-body diagram of the beam.
(b) Determine the reactions at the supports.



Solution: (a) Both supports are roller supports. The free body diagram is shown. (b) The sum of the forces:



$$\sum F_X = 0,$$

and $\sum F_Y = F_A + F_B + 100 - 400 = 0.$

The sum of the moments about A is

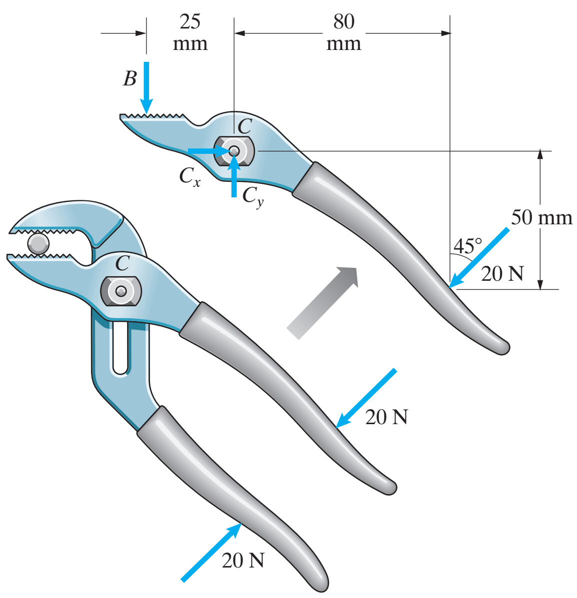
$$\sum M_A = -3(100) + 900 - 7(400) + 11F_B = 0.$$

From which $F_B = \dfrac{2200}{11} = 200$ lb

Substitute into the force balance equation to obtain

$F_A = 300 - F_B = 100$ lb

Problem 5.11 The person exerts 20-N forces on the pliers. The free-body diagram of one part of the pliers is shown. Notice that the pin at C connecting the two parts of the pliers behaves like a pin support. Determine the reactions at C and the force B exerted on the pliers by the bolt.



Solution: The equilibrium equations

$$\sum M_C : B(25 \text{ mm}) - 20 \text{ N} \cos 45^\circ (80 \text{ mm})$$

$$- 20 \text{ N} \sin 45^\circ (50 \text{ mm}) = 0$$

$$\sum F_x : C_x - 20 \text{ N} \sin 45^\circ = 0$$

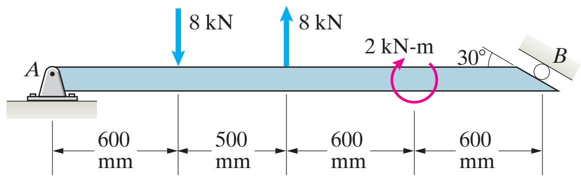
$$\sum F_y : C_y - B - 20 \text{ N} \cos 45^\circ = 0$$

Solving:

$$\boxed{B = 73.5 \text{ N}, \; C_x = 14.14 \text{ N}, \; C_y = 87.7 \text{ N}}$$

Problem 5.12 (a) Draw the free-body diagram of the beam.

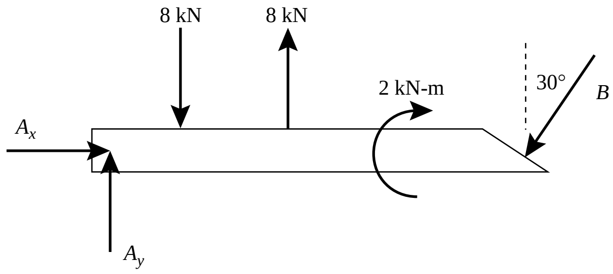
(b) Determine the reactions at the pin support A.



Solution:

(a) The FBD

(b) The equilibrium equations



$$\sum M_A : -(8\text{ kN})(0.6\text{ m}) + (8\text{ kN})(1.1\text{ m}) - 2\text{ kNm}$$

$$- B\cos 30^\circ (2.3\text{ m}) = 0$$

$$\sum F_x : A_x - B\sin 30^\circ = 0$$

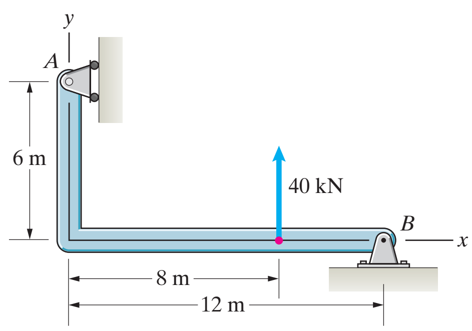
$$\sum F_y : A_y - 8\text{ kN} + 8\text{ kN} - B\cos 30^\circ = 0$$

Solving

$$\boxed{A_x = 0.502\text{ kN},\ A_y = 0.870\text{ kN},\ B = 1.004\text{ kN}}$$

Problem 5.13 (a) Draw the free-body diagram of the beam.

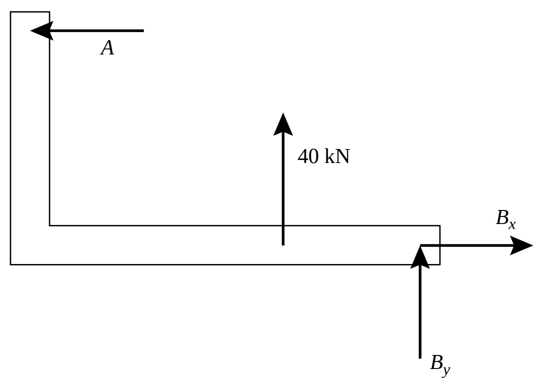
(b) Determine the reactions at the supports.



Solution:

(a) The FBD

(b) The equilibrium equations



$$\sum M_B : -(40\text{ kN})(4\text{ m}) + A(6\text{ m}) = 0$$

$$\sum F_x : -A + B_x = 0$$

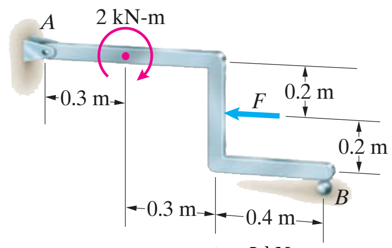
$$\sum F_y : 40\text{ kN} + B_y = 0$$

Solving we find

$$\boxed{A = B_x = 26.7\text{ kN},\ B_y = -40\text{ kN}}$$

Problem 5.14 (a) Draw the free-body diagram of the beam.

(b) If $F = 4$ kN, what are the reactions at A and B?



Solution:

(a) The free-body diagram

(b) The equilibrium equations

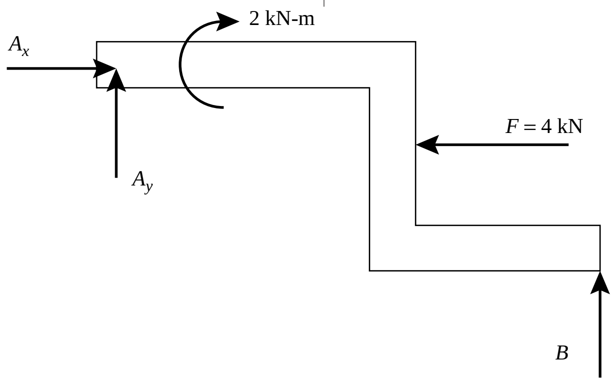
$$\sum M_A : -2 \text{ kN-m} - 4 \text{ kN}(0.2 \text{ m}) + B(1.0 \text{ m}) = 0$$

$$\sum F_x : A_x - 4 \text{ kN} = 0$$

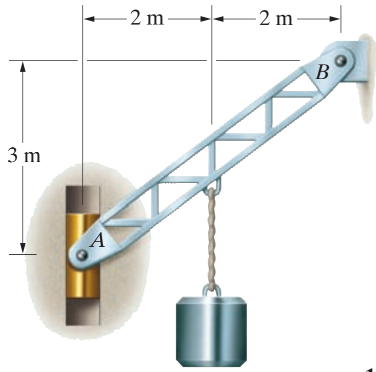
$$\sum F_y : A_y + B = 0$$

Solving:

$$\boxed{A_x = 4 \text{ kN}, \ A_y = -2.8 \text{ kN}, \ B = 2.8 \text{ kN}}$$



Problem 5.15 In Example 5.3, suppose that the attachment point for the suspended mass is moved toward point B such that the horizontal distance from A to the attachment point increases from 2 m to 3 m. Draw a sketch of the beam AB showing the new geometry. Draw the free-body diagram of the beam and apply the equilibrium equations to determine the reactions at A to B.



Solution: From Example 5.3, we know that the mass of the suspended object is 2-Mg. The sketch and free-body diagram are shown.

The equilibrium equations are

$\Sigma F_x : A_x + B_x = 0$

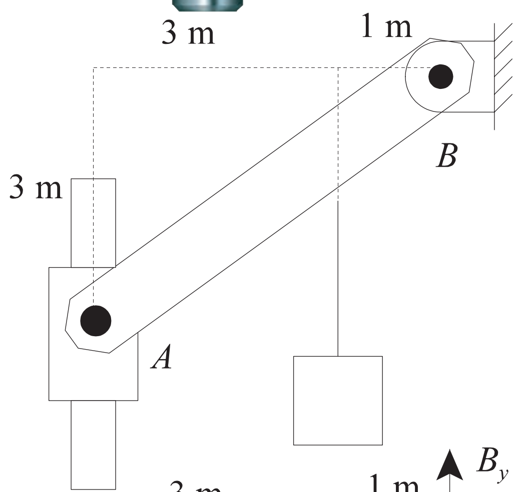
$\Sigma F_y : B_y - (2000\text{ kg})(9.81\text{ m/s}^2) = 0$

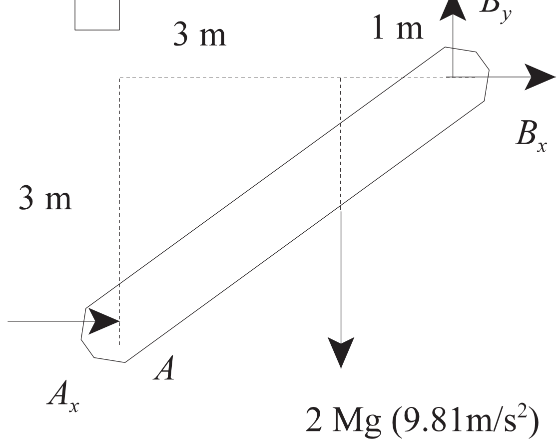
$\Sigma M_B : A_x(3\text{ m})$

$+ (2000\text{ kg})(9.81\text{ m/s}^2)(1\text{ m}) = 0$

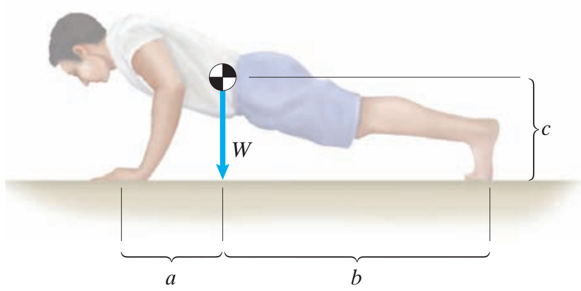
We obtain

$$A_x = -6.54\text{ kN}, \quad B_x = 6.54\text{ kN}, \quad B_y = 19.6\text{ kN}$$





Problem 5.16 A person doing push-ups pauses in the position shown. His 180-lb weight W acts at the point shown. The dimensions a = 15 in, b = 42 in, and c = 16 in. Determine the normal force exerted by the floor on each of his hands and on each of his feet.



Solution: The free-body diagram is shown. The equilibrium equations are

$\Sigma F_y : 2H + 2F - 180 \text{ lb} = 0$

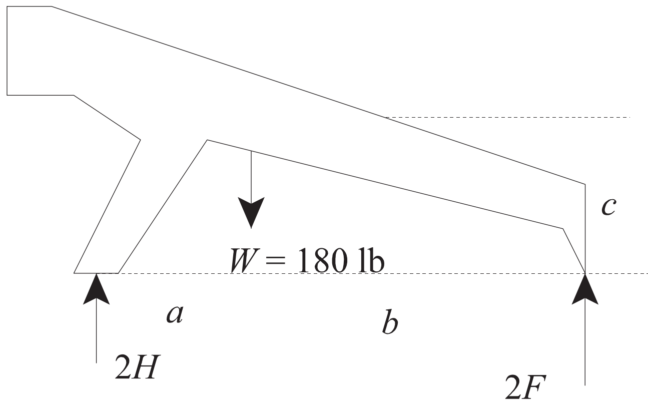
$\Sigma M_H : -Wa + 2F(a + b)\ 0$

We find that

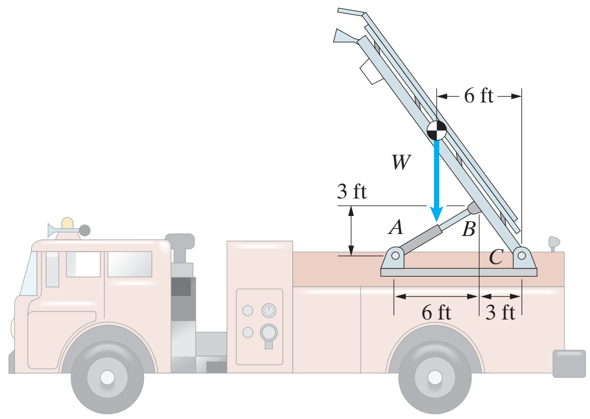
$$H = 66.3 \text{ lb}, F = 23.7 \text{ lb}$$

Thus

66.3 lb on each hand 23.7 lb on each foot



Problem 5.17 The hydraulic piston AB exerts a 400-lb force on the ladder at B in the direction parallel to the piston. Determine the weight of the ladder and the reactions at C.



Solution: The free-body diagram of the ladder is shown. The angle between the piston AB and the horizontal is

$$\alpha = \tan^{-1}(3/6) = 26.6^\circ$$

The equilibrium equations are

$\Sigma F_x : C_x + (400 \text{ lb}) \cos\alpha = 0$

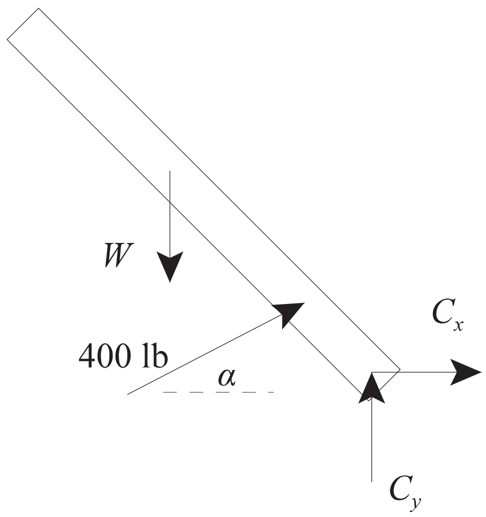
$\Sigma F_y : C_y + (400 \text{ lb}) \sin\alpha - W = 0$

$\Sigma M_C : W(6 \text{ ft}) - (400 \text{ lb}) \cos\alpha (3 \text{ ft})$

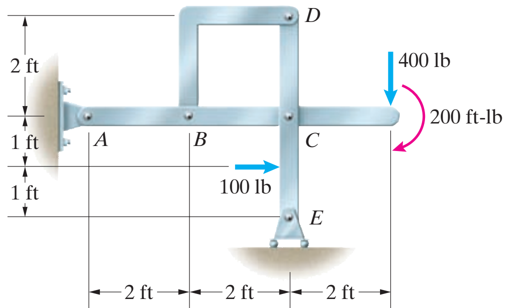
$\quad - (400 \text{ lb}) \sin\alpha (3 \text{ ft}) = 0$

Solving yields

$C_x = -358$ lb, $C_y = 89.4$ lb, $W = 268$ lb



Problem 5.18 Draw the free-body diagram of the structure by isolating it from its supports at A and E. Determine the reactions at A and E.



Solution: The free-body diagram is shown.

The equilibrium equations are

$\Sigma F_x : A_x + (100 \text{ lb}) = 0$

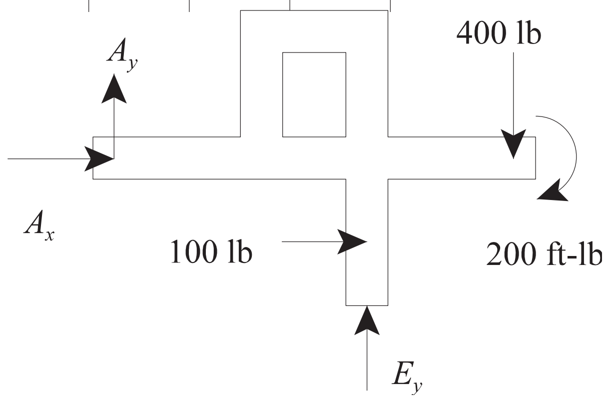
$\Sigma F_y : A_y - (400 \text{ lb}) + E_y = 0$

$\Sigma M_A : (100 \text{ lb})(1 \text{ ft}) - (400 \text{ lb})(6 \text{ ft})$

$\quad - (200 \text{ ft-lb}) + E_y(4 \text{ ft}) = 0$

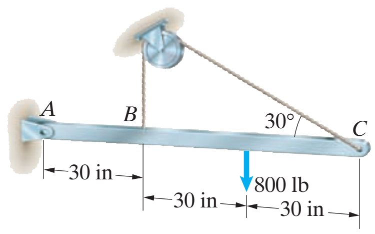
Solving yields

$$\boxed{\begin{aligned} A_x &= -100 \text{ lb} \\ A_y &= -225 \text{ lb} \\ E_y &= 625 \text{ lb} \end{aligned}}$$



Problem 5.19 (a) Draw the free-body diagram of the beam.

(b) Determine the tension in the cable and the reactions at A.



Solution:

(a) The FBD

(b) The equilibrium equations

$\sum M_A : -(800 \text{ lb})(60 \text{ in}) + T(30 \text{ in})$

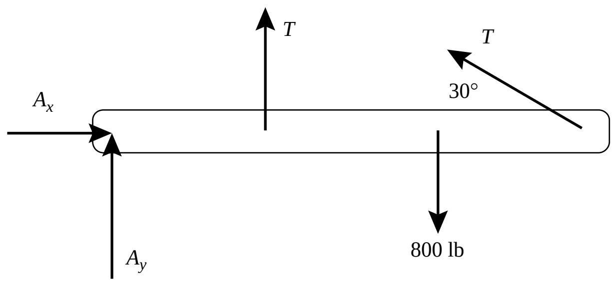
$\quad + T \sin 30^\circ (90 \text{ in}) = 0$

$\sum F_x : A_x - T\cos 30^\circ = 0$

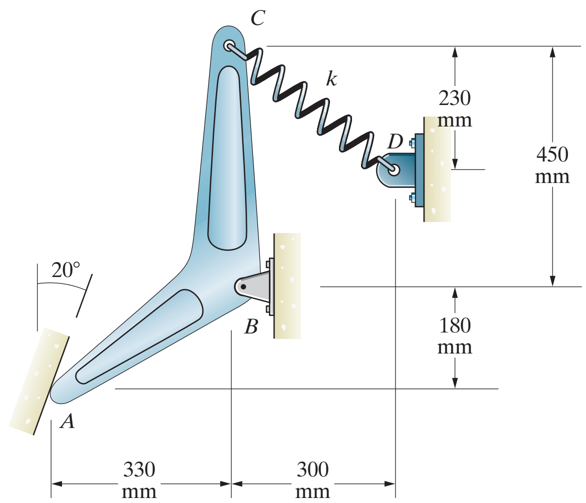
$\sum F_y : A_y + T + T\sin 30^\circ - 800 \text{ lb} = 0$

Solving:

$$\boxed{A_x = 554 \text{ lb}, \; A_y = -160 \text{ lb}, \; T = 640 \text{ lb}}$$



Problem 5.20 The unstretched length of the spring *CD* is 350 mm. Suppose that you want the lever *ABC* to exert a 120-N normal force on the smooth surface at *A*. Determine the necessary value of the spring constant *k* and the resulting reactions at *B*.



Solution: We have

$$F = k(\sqrt{(0.23\text{ m})^2 + (0.3\text{ m}^2)} - 0.35\text{ m})$$

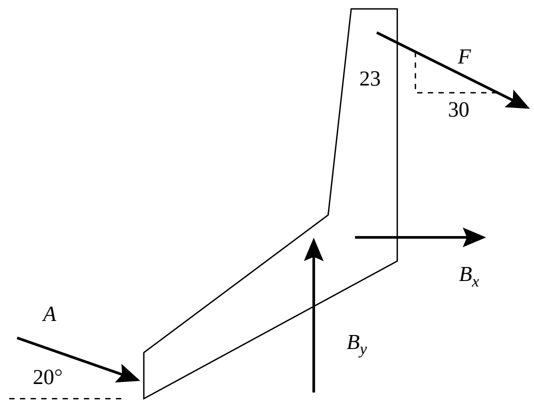
$$A = 120\text{ N}$$

$$\sum M_B : -\frac{30}{\sqrt{1429}}F(0.45\text{ m}) + A\cos 20^\circ(0.18\text{ m})$$

$$+ A\sin 20^\circ(0.33\text{ m}) = 0$$

$$\sum F_x : A\cos 20^\circ + B_x + \frac{30}{\sqrt{1429}}F = 0$$

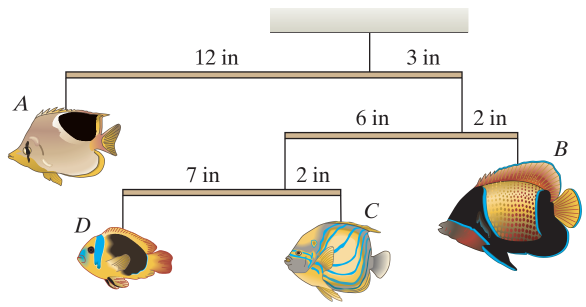
$$\sum F_y : -A\sin 20^\circ + B_y - \frac{23}{\sqrt{1429}}F = 0$$



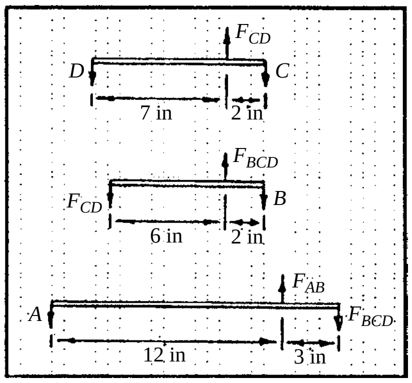
Solving we find:

$$\boxed{k = 3380\text{ N/m},\ B_x = -188\text{ N},\ B_y = 98.7\text{ N}}$$

Problem 5.21 The mobile is in equilibrium. The fish B weighs 27 oz. Determine the weights of the fish A, C, and D. (The weights of the crossbars are negligible.)



Solution: Denote the reactions at the supports by F_{AB}, F_{CD}, and F_{BCD} as shown. Start with the crossbar supporting the weights C and D. The sum of the forces is



$$\sum F_Y = -C - D + F_{CD} = 0,$$

from which $F_{CD} = C + D$.

For the cross bar supporting the weight B, the sum of the forces is

$$\sum F_Y = -B + F_{BCD} - F_{CD} = 0,$$

from which, substituting, $F_{BCD} = B + C + D$.

For the crossbar supporting C and D, the sum of the moments about the support is

$$\sum M_{CD} = 7D + 2C = 0,$$

from which $D = \frac{2C}{7}$.

For the crossbar supporting B, the sum of the moments is

$$\sum M_{BCD} = 6F_{CD} - 2B = 0,$$

from which, substituting from above

$$F_{CD} = \frac{2B}{6} = C + D = C + \frac{2C}{7} = \frac{9C}{7},$$

or $C = 7B/27 = 7$ oz,

and $D = 2C/7 = 2$ oz.

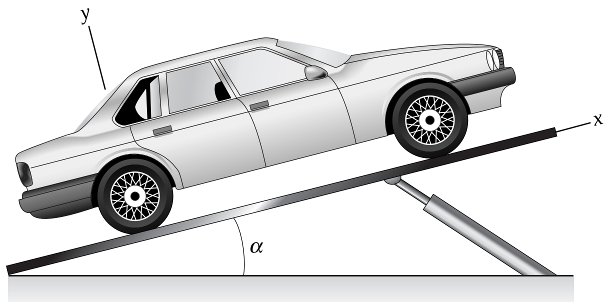
The sum of the moments about the crossbar supporting A is

$$\sum M_{AB} = 12A - 3F_{BCD} = 0,$$

from which, substituting from above,

$$A = \frac{3(B + C + D)}{12} = \frac{27 + 7 + 2}{4} = 9 \text{ oz}$$

Problem 5.22 The car's wheelbase (the distance between the wheels) is 2.82 m. The mass of the car is 1760 kg and its weight acts at the point $x = 2.00$ m, $y = 0.68$ m. If the angle $\alpha = 15°$, what is the total normal force exerted on the two rear tires by the sloped ramp?



Solution: Split W into components:

$W \cos\alpha$ acts $\perp$ to the incline

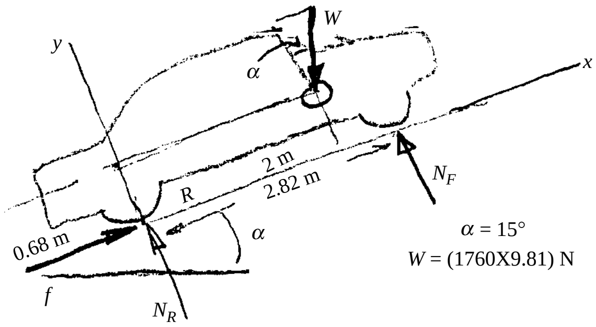
$W \sin\alpha$ acts parallel to the incline

$\sum F_X$: $f - W \sin\alpha = 0$

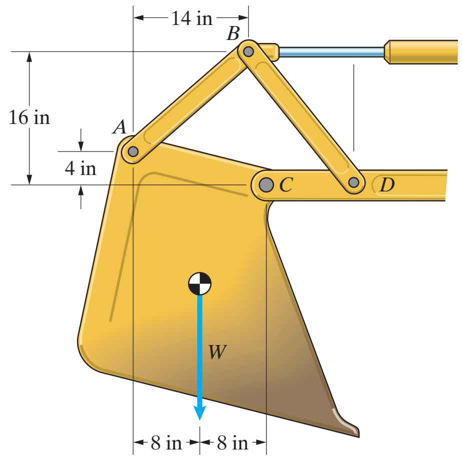
$\sum F_Y$: $N_R + N_F - W \cos\alpha = 0$

$\sum M_R$: $(-2)(W \cos\alpha) + (0.68)W \sin\alpha + 2.82 N_F = 0$

Solving: $N_R = 5930$ N, $N_F = 10750$ N



Problem 5.23 The link AB exerts a force on the bucket of the excavator at A that is parallel to the link. The weight $W = 1500$ lb. Draw the free-body diagram of the bucket and determine the reactions at C. (The connection at C is equivalent to a pin support of the bucket.)



Solution: The free-body diagram is shown.
The angle between the link AB and the horizontal is

$$\alpha = \tan^{-1}(12/14) = 40.6°$$

The equilibrium equations are

$\Sigma F_x : C_x + T_{AB} \cos\alpha = 0$

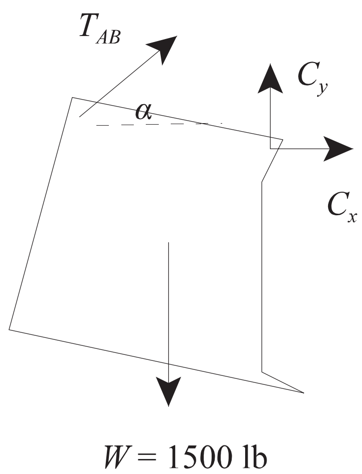
$\Sigma F_y : C_y + T_{AB} \sin\alpha - (1500 \text{ lb}) = 0$

$\Sigma M_C : (1500 \text{ lb})(8 \text{ in}) - T_{AB} \cos\alpha(4 \text{ in})$

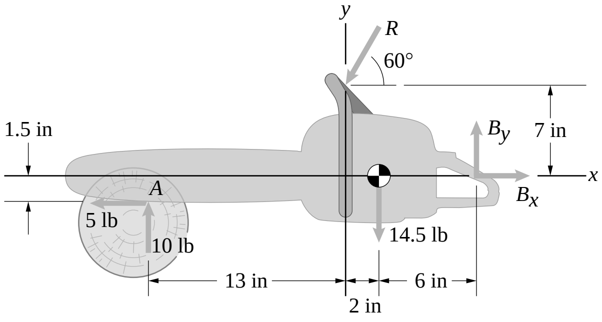
$- T_{AB} \sin\alpha(16 \text{ in}) = 0$

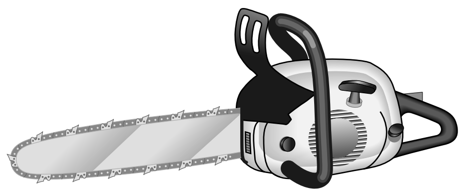
Solving yields

$$\boxed{T_{AB} = 892 \text{ lb}, C_x = -677 \text{ lb}, C_y = 919 \text{ lb}}$$



Problem 5.24 The 14.5-lb chain saw is subjected to the loads at A by the log it cuts. Determine the reactions R, B_x, and B_y that must be applied by the person using the saw to hold it in equilibrium.





Solution: The sum of the forces are

$$\sum F_X = -5 + B_X - R\cos 60^\circ = 0.$$

$$\sum F_Y = 10 - 14.5 + B_Y - R\sin 60^\circ = 0.$$

The sum of the moments about the origin is

$$\sum M_O = 7R\cos 60^\circ + 8B_Y - 2(14.5) - 13(10) - 5(1.5) = 0.$$

From which $7R\cos 60^\circ + 8B_Y - 166.5 = 0$. Collecting equations and reducing to 3 equations in 3 unknowns:

$$B_X + 0B_Y - 0.5R = 5$$

$$0B_X + B_Y - 0.866R = 4.5$$

$$0B_X + 8B_Y + 3.5R = 166.5.$$

Solving:

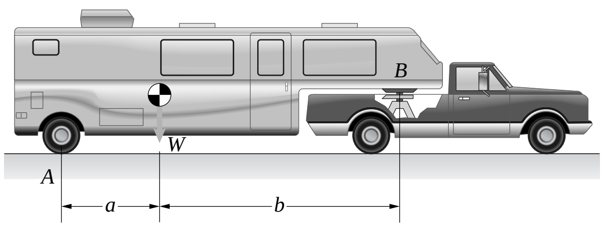
$$B_X = 11.257 \text{ lb},$$

$$B_Y = 15.337 \text{ lb},$$

and $R = 12.514$ lb

Problem 5.25 The mass of the trailer is 2.2 Mg (megagrams). The distances $a = 2.5$ m and $b = 5.5$ m. The truck is stationary, and the wheels of the trailer can turn freely, which means that the road exerts no horizontal force on them. The hitch at B can be modeled as a pin support.

(a) Draw the free-body diagram of the trailer.
(b) Determine the total normal force exerted on the rear tires at A and the reactions exerted on the trailer at the pin support B.



Solution:

(a) The free body diagram is shown.
(b) The sum of forces:

$$\sum F_X = B_X = 0.$$

$$\sum F_Y = F_A - W + F_B = 0.$$

The sum of the moments about A:

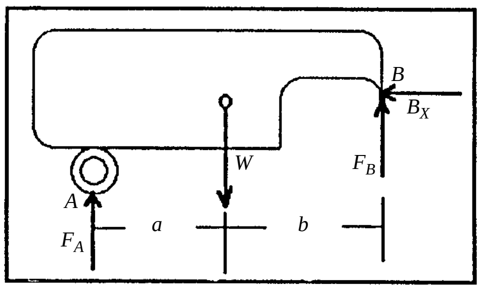
$$\sum M_A = -aW + (a + b)F_B = 0,$$

from which

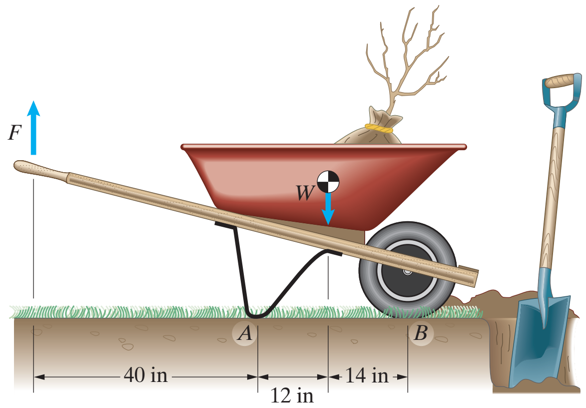
$$F_B = \frac{aW}{a+b} = \frac{2.5(2.2 \times 10^3)(9.81)}{(2.5 + 5.5)} = 6.744 \text{ kN}$$

Substitute into the force equation:

$$F_A = W - F_B = 14.838 \text{ kN}$$



Problem 5.26 The total weight of the wheelbarrow and its load is $W = 100$ lb. (a) What is the magnitude of the upward force F necessary to lift the support at A off the ground? (b) What is the magnitude of the downward force necessary to raise the wheel off the ground?



Solution: The free-body diagram is shown. The equilibrium equations are

$$\Sigma F_y : A + B + F - W = 0$$

$$\Sigma M_A : B(26 \text{ in}) - W(12 \text{ in})$$

$$- F(40 \text{ in}) = 0$$

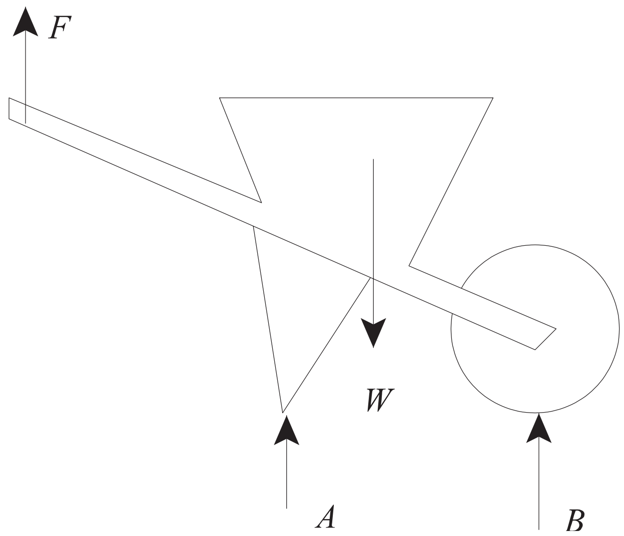
(a) Set $A = 0$ and solve. We find that

$$F = 21.2 \text{ lb}$$

(b) Set $B = 0$ and solve. We find that

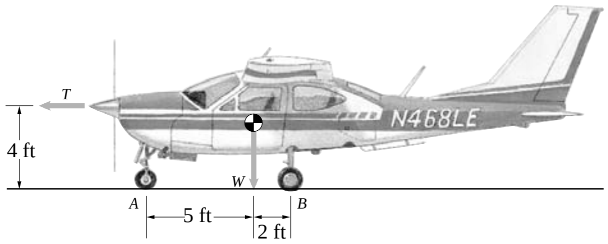
$$F = -30 \text{ lb}$$

So we have $\boxed{(a)\ 21.2 \text{ lb},\ (b)\ 30 \text{ lb.}}$



Problem 5.27 The airplane's weight is $W = 2400$ lb. Its brakes keep the rear wheels locked. The front (nose) wheel can turn freely, and so the ground exerts no horizontal force on it. The force T exerted by the airplane's propeller is horizontal.

(a) Draw the free-body diagram of the airplane. Determine the reaction exerted on the nose wheel and the total normal reaction on the rear wheels
(b) when $T = 0$,
(c) when $T = 250$ lb.



Solution: (a) The free body diagram is shown. (b) The sum of the forces:

$$\sum F_X = B_X = 0$$

$$\sum F_Y = A_Y - W + B_Y = 0.$$

The sum of the moments about A is

$$\sum M_A = -5W + 7B_Y = 0,$$

from which $B_Y = \frac{5W}{7} = 1714.3$ lb

Substitute from the force balance equation:

$$A_Y = W - B_Y = 685.7 \text{ lb}$$

(c) The sum of the forces:

$$\sum F_X = -250 + B_X = 0,$$

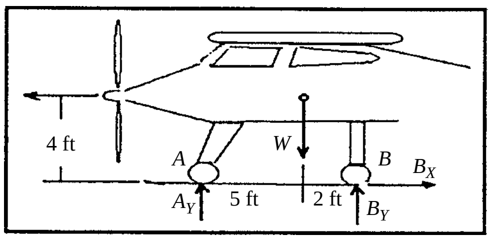
from which $B_X = 250$ lb

$$\sum F_Y = A_Y - W + B_Y = 0.$$

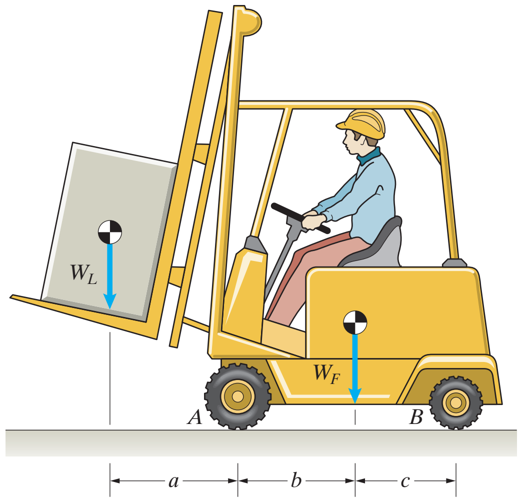
The sum of the moments about A:

$$\sum M_A = (250)(4) - 5W + 7B_Y = 0,$$

from which $B_Y = 1571.4$ lb. Substitute into the force balance equation to obtain: $A_Y = 828.6$ lb



Problem 5.28 A safety engineer establishing limits on the load that can be carried by a forklift analyzes the situation shown. The dimensions are $a = 32$ in, $b = 30$ in, and $c = 26$ in. The combined weight of the forklift and operator is $W_F = 1200$ lb. As the weight W_L supported by the forklift increases, the normal force exerted on the floor by the rear wheels at B decreases. The forklift is on the verge of tipping forward when the normal force at B is zero. Determine the value of W_L that will cause this condition.



Solution: The equilibrium equations and the special condition for this problem are

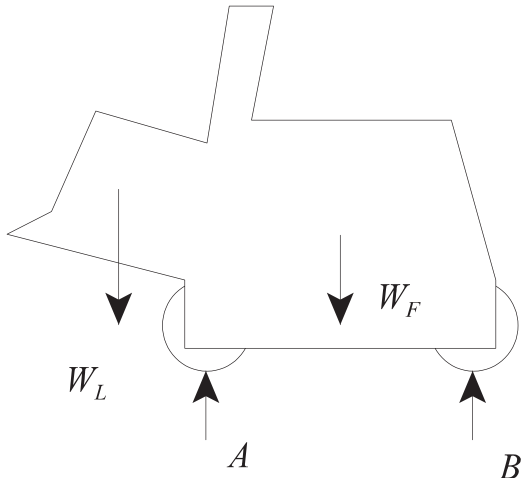
$\Sigma F_y : A + B - W_L - (1200 \text{ lb}) = 0$

$\Sigma M_A : W_L a - W_F b + Bc = 0$

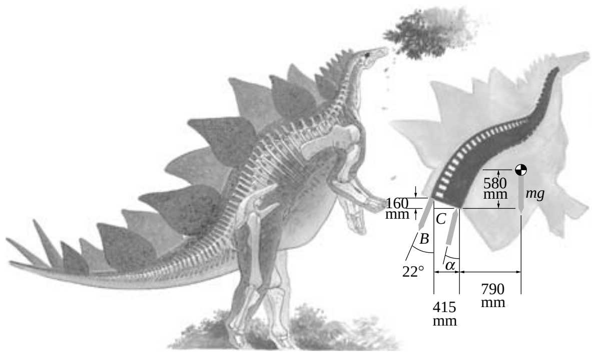
$B = 0$

We obtain

$\boxed{W_L = 1125 \text{ lb}}$



Problem 5.29 Paleontologists speculate that the stegosaur could stand on its hind limbs for short periods to feed. Based on the free-body diagram shown and assuming that $m = 2000$ kg, determine the magnitudes of the forces B and C exerted by the ligament–muscle brace and vertebral column, and determine the angle α.



Solution: Take the origin to be at the point of application of the force C. The position vectors of the points of application of the forces B and W are:

$$\mathbf{r}_B = -415\mathbf{i} + 160\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_W = 790\mathbf{i} + 580\mathbf{j} \text{ (mm)}.$$

The forces are

$$\mathbf{C} = C(\mathbf{i}\cos(90^\circ - \alpha) + \mathbf{j}\sin(90^\circ - \alpha))$$

$$= C(\mathbf{i}\sin\alpha + \mathbf{j}\cos\alpha).$$

$$\mathbf{B} = B(\mathbf{i}\cos(270^\circ - 22^\circ) + \mathbf{j}\sin(270^\circ - 22^\circ))$$

$$= B(-0.3746\mathbf{i} - 0.9272\mathbf{j}).$$

$$\mathbf{W} = -2(9.81)\mathbf{j} = -19.62\mathbf{j} \text{ (kN)}.$$

The moments about C,

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -415 & 160 & 0 \\ -0.3746B & -0.9272B & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 790 & 580 & 0 \\ 0 & -19.62 & 0 \end{vmatrix} = 0$$

$$= 444.72B - 15499.8 = 0,$$

from which

$$B = \frac{15499.8}{444.72} = 34.85 \text{ kN}.$$

The sums of the forces:

$$\sum \mathbf{F}_X = (C\sin\alpha - 0.3746B)\mathbf{i} = 0,$$

from which $C\sin\alpha = 13.06$ kN.

$$\sum \mathbf{F}_Y = (C\cos\alpha - 0.9272B - 19.62)\mathbf{j} = 0,$$

from which $C\cos\alpha = 51.93$ kN.

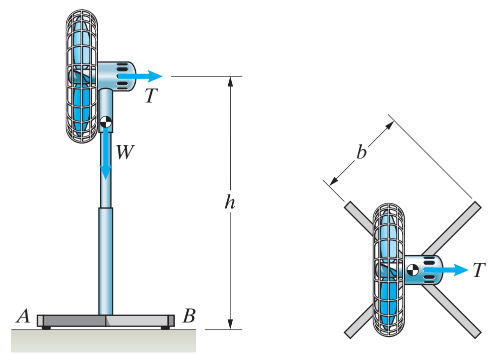
The angle α is

$$\alpha = \tan^{-1}\left(\frac{13.06}{51.93}\right) = 14.1^\circ.$$

The magnitude of $\mathbf{C}$,

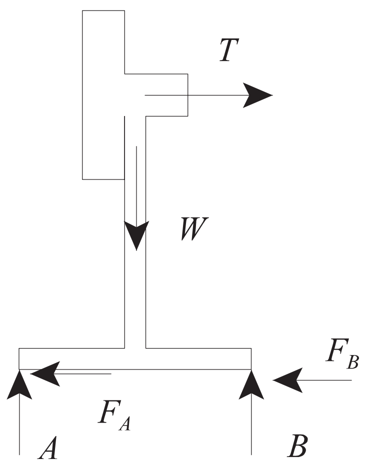
$$C = \sqrt{13.06^2 + 51.93^2} = 53.55 \text{ kN}$$

Problem 5.30 The weight of the fan is $W = 20$ lb. Its base has four equally spaced legs of length $b = 12$ in. Each leg has a pad near the end that contacts the floor and supports the fan. The height $h = 32$ in. If the fan's blade exerts a thrust $T = 2$ lb, what total normal force is exerted on the two legs at A?



Side View Top View

Solution: The free-body diagram is shown.



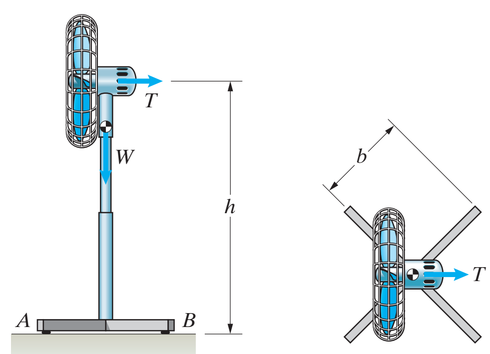
The equilibrium equations are

$$\Sigma F_y : A + B - W = 0$$

$$\Sigma M_B : W\frac{b}{\sqrt{2}} - A\frac{2b}{\sqrt{2}} - Th = 0$$

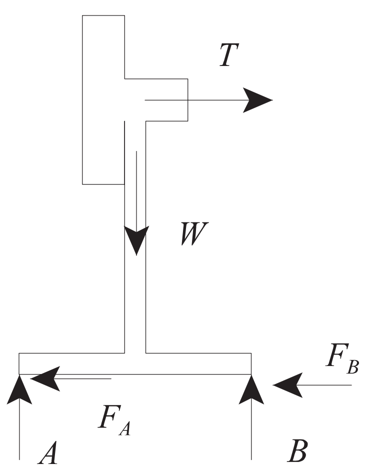
We obtain $\boxed{A = 6.23 \text{ lb}}$

Problem 5.31 The weight of the fan is $W = 20$ lb. Its base has four equally spaced legs of length $b = 12$ in. Each leg has a pad near the end that contacts the floor and supports the fan. The height $h = 32$ in. As the thrust T of the fan increases, the normal force supported by the two legs at A decreases. When the normal force at A is zero, the fan is on the verge of tipping over. Determine the value of T that will cause this condition.



Side View Top View

Solution: The free-body diagram is shown.



The equilibrium equations are

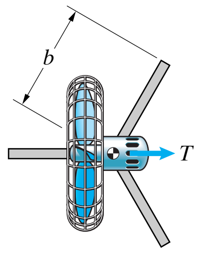
$$\Sigma F_y : A + B - W = 0$$

$$\Sigma M_B : W\frac{b}{\sqrt{2}} - A\frac{2b}{\sqrt{2}} - Th = 0$$

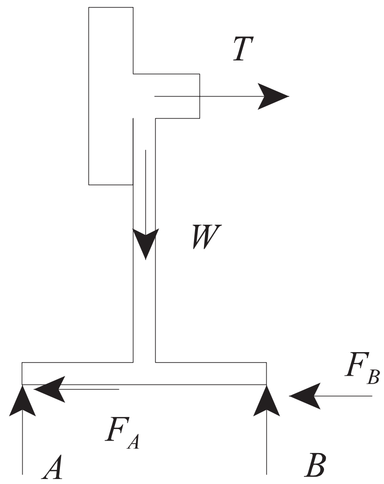
We set $A = 0$ and solve to obtain

$$\boxed{T = 5.30 \text{ lb}}$$

Problem 5.32 In a measure to decrease costs, the manufacturer of the fan described in Problem 5.31 proposes to support the fan with three equally spaced legs instead of four. An engineer is assigned to analyze the safety implications of the change. The weight of the fan decreases to $W = 19.6$ lb. The dimensions b and h are unchanged. What thrust T will cause the fan to be on the verge of tipping over in this case? Compare your answer to the answer to Problem 5.31.



Solution: The free-body diagram is shown.



The equilibrium equations are

$$\Sigma F_y : A + B - W = 0$$

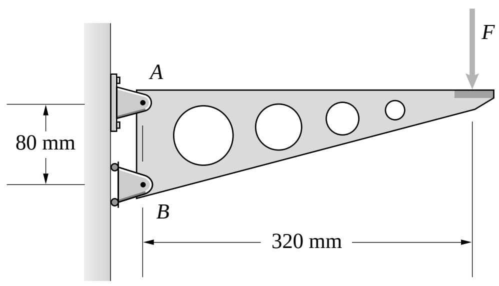
$$\Sigma M_B : Wb\cos 60^\circ - A(b + b\cos 60^\circ) - Th = 0$$

We set $A = 0$ and solve to obtain

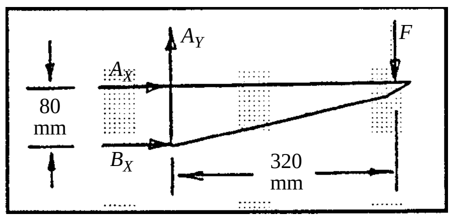
$$\boxed{T = 3.68 \text{ lb}}$$

This configuration is less stable than the one in Problem 5.31 using four legs.

Problem 5.33 A force $F = 400$ N acts on the bracket. What are the reactions at A and B?



Solution: The joint A is a pinned joint; B is a roller joint. The pinned joint has two reaction forces A_X, A_Y. The roller joint has one reaction force B_X. The sum of the forces is



$$\sum F_X = A_X + B_X = 0,$$

$$\sum F_Y = A_Y - F = 0,$$

from which

$$A_Y = F = 400 \text{ N}.$$

The sum of the moments about A is

$$\sum M_A = 0.08B_X - 0.320F = 0,$$

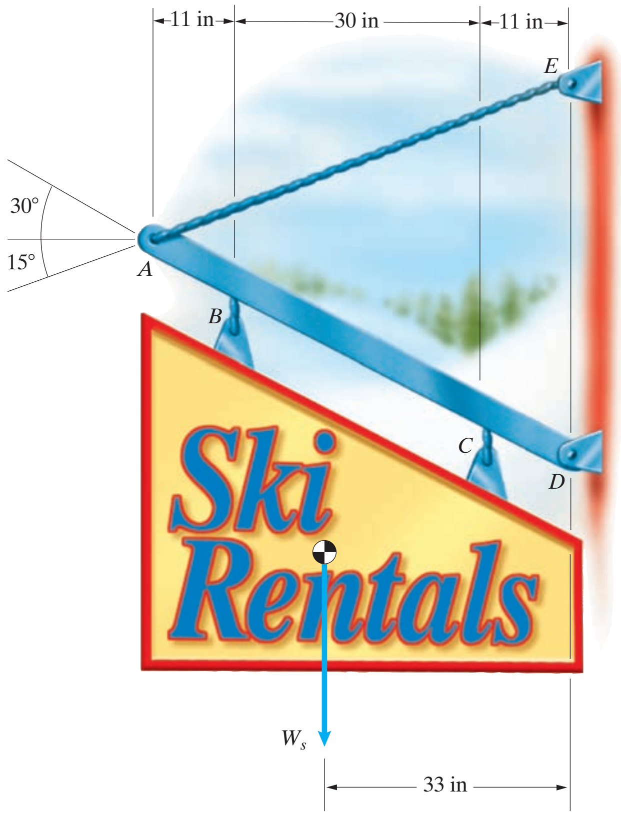
from which

$$B_X = \frac{0.320(400)}{0.08} = 1600 \text{ N}.$$

Substitute into the sum of forces equation to obtain:

$$A_X = -B_X = -1600 \text{ N}$$

Problem 5.34 The sign's weight $W_S = 32$ lb acts at the point shown. The 10-lb weight of bar AD acts at the midpoint of the bar. Determine the tension in the cable AE and the reactions at D.



Solution: Treat the bar AD and sign as one single object. Let T_{AE} be the tension in the cable. The equilibrium equations are

$$\Sigma F_x : T_{AE}\cos 15^\circ + D_x = 0$$

$$\Sigma F_y : T_{AE}\sin 15^\circ + D_y - W_s - (10 \text{ lb}) = 0$$

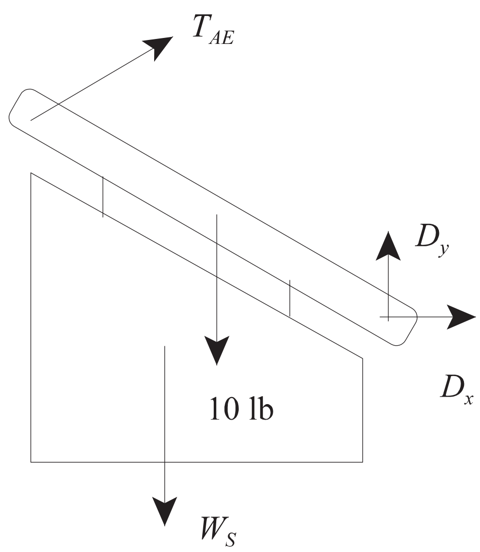
$$\Sigma M_D : -T_{AE}\cos 15^\circ(52 \text{ in})\tan 30^\circ$$

$$- T_{AE}\sin 15^\circ(52 \text{ in})$$

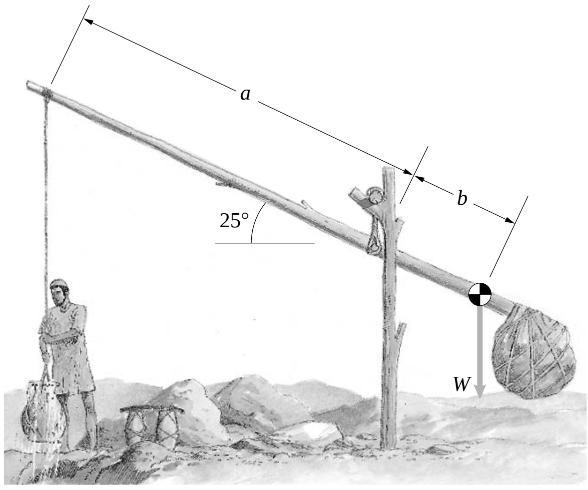
$$+ (32 \text{ lb})(33 \text{ in}) + (10 \text{ lb})(26 \text{ in}) = 0$$

Solving yields

$$T_{AE} = 31.0 \text{ lb}$$
$$D_x = -29.9 \text{ lb}$$
$$D_y = 34.0 \text{ lb}$$



Problem 5.35 The device shown, called a *swape* or *shadoof*, helps a person lift a heavy load. (Devices of this kind were used in Egypt at least as early as 1550 B.C. and are still in use in various parts of the world.) The dimensions $a = 3.6$ m and $b = 1.2$ m. The mass of the bar and counterweight is 90 kg, and their weight W acts at the point shown. The mass of the load being lifted is 45 kg. Determine the vertical force the person must exert to support the stationary load (a) when the load is just above the ground (the position shown); (b) when the load is 1 m above the ground. Assume that the rope remains vertical.



Solution:

$$\sum M_O : (441\text{ N} - F)(3.6\text{ m } \cos\theta)$$

$$-(883\text{ N})(1.2\text{ m } \cos\theta) = 0$$

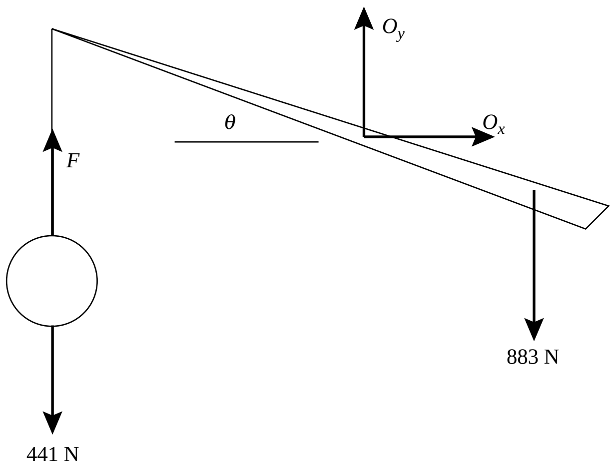
Solving we find

$$F = 147.2\text{ N}$$

Notice that the angle θ is not a part of this answer therefore

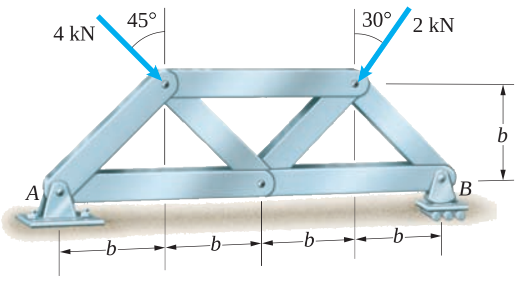
(a) $\boxed{F = 147.2\text{ N}}$

(b) $\boxed{F = 147.2\text{ N}}$



Problem 5.36 This structure, called a *truss*, has a pin support at A and a roller support at B and is loaded by two forces. Determine the reactions at the supports.

Strategy: Draw a free-body diagram, treating the entire truss as a single object.



Solution:

$$\sum M_A : -(4\text{ kN})\sqrt{2}b - (2\text{ kN } \cos 30^\circ)3\text{ b}$$

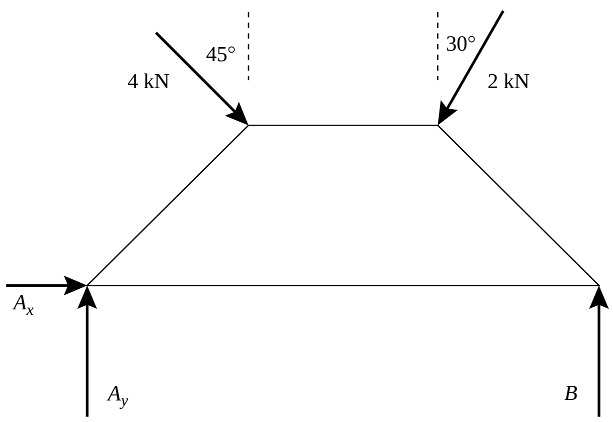
$$+ (2\text{ kN } \sin 30^\circ)b + B(4\text{ b}) = 0$$

$$\sum F_x : A_x + 4\text{ kN } \sin 45^\circ - 2\text{ kN } \sin 30^\circ = 0$$

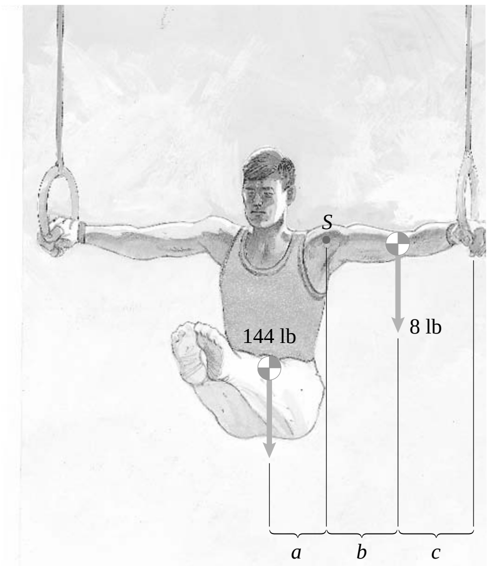
$$\sum F_y : A_y - 4\text{ kN } \cos 45^\circ - 2\text{ kN } \cos 30^\circ + B = 0$$

Solving:

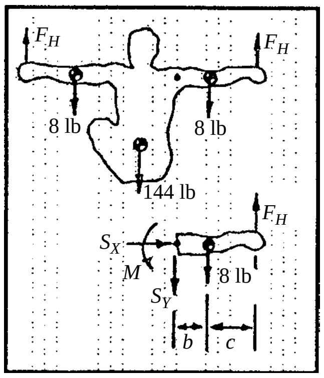
$$\boxed{A_x = -1.828\text{ kN},\ A_y = 2.10\text{ kN},\ B = 2.46\text{ kN}}$$



Problem 5.37 An Olympic gymnast is stationary in the “iron cross” position. The weight of his left arm and the weight of his body *not including his arms* are shown. The distances are $a = b = 9$ in and $c = 13$ in. Treat his shoulder S as a fixed support, and determine the magnitudes of the reactions at his shoulder. That is, determine the force and couple his shoulder must support.



Solution: The shoulder as a built-in joint has two-force and couple reactions. The left hand must support the weight of the left arm and half the weight of the body:



$$F_H = \frac{144}{2} + 8 = 80 \text{ lb}.$$

The sum of the forces on the left arm is the weight of his left arm and the vertical reaction at the shoulder and hand:

$$\sum F_X = S_X = 0.$$

$$\sum F_Y = F_H - S_Y - 8 = 0,$$

from which $S_Y = F_H - 8 = 72$ lb. The sum of the moments about the shoulder is

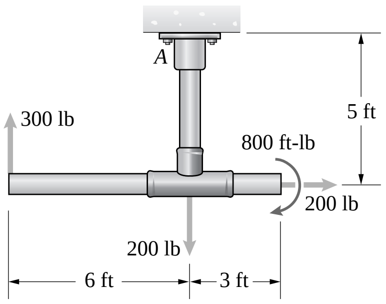
$$\sum M_S = M + (b + c)F_H - b8 = 0,$$

where M is the couple reaction at the shoulder. Thus

$$M = b8 - (b + c)F_H = -1688 \text{ in lb} = 1688 \text{ (in lb)} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= 140.67 \text{ ft lb}$$

Problem 5.38 Determine the reactions at A.



Solution: The built-in support at A is a two-force and couple reaction support. The sum of the forces for the system is

$$\sum F_X = A_X + 200 = 0,$$

from which

$$A_X = -200 \text{ lb}$$

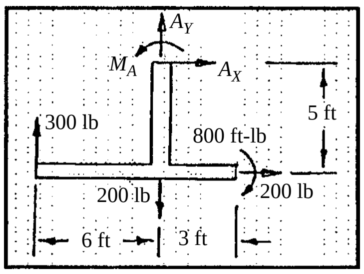
$$\sum F_Y = A_Y + 300 - 200 = 0,$$

from which $A_Y = -100$ lb

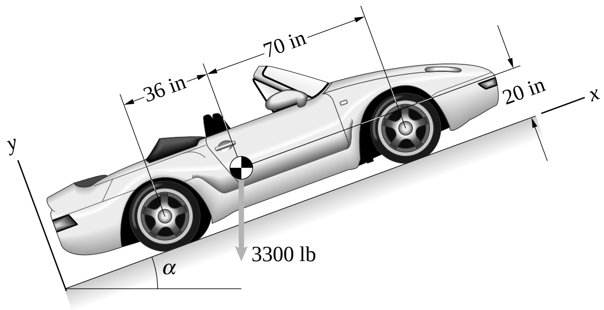
The sum of the moments about A:

$$\sum M = -6(300) + 5(200) - 800 + M_A = 0,$$

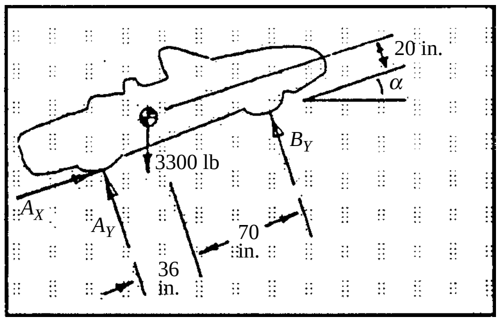
from which $M_A = 1600$ ft lb which is the couple at A.



Problem 5.39 The car's brakes keep the rear wheels locked, and the front wheels are free to turn. Determine the forces exerted on the front and rear wheels by the road when the car is parked (a) on an up slope with $\alpha = 15^\circ$; (b) on a down slope with $\alpha = -15^\circ$.



Solution: The rear wheels are two force reaction support, and the front wheels are a one force reaction support. Denote the rear wheels by A and the front wheels by B, and define the reactions as being parallel to and normal to the road. The sum of forces:



$$\sum F_X = A_X - 3300 \sin 15^\circ = 0,$$

from which

$$A_X = 854.1 \text{ lb}.$$

$$\sum F_Y = A_Y - 3300 \cos 15^\circ + B_Y = 0.$$

Since the mass center of the vehicle is displaced above the point A, a component of the weight ($20W \sin \alpha$) produces a positive moment about A, whereas the other component ($36W \cos \alpha$) produces a negative moment about A. The sum of the moments about A:

$$\sum M_A = -36(3300 \cos 15^\circ) + 20(3300 \sin 15^\circ) + B_Y(106) = 0,$$

from which

$$B_Y = \frac{+97669}{106} = 921.4 \text{ lb}.$$

Substitute into the sum of forces equation to obtain $A_Y = 2266.1$ lb

(b) For the car parked down-slope the sum of the forces is

$$\sum F_X = A_X + 3300 \sin 15^\circ = 0,$$

from which $A_X = -854$ lb

$$\sum F_Y = A_Y - 3300 \cos 15^\circ + B_Y = 0.$$

The component ($20W \sin \alpha$) now produces a negative moment about A. The sum of the moments about A is

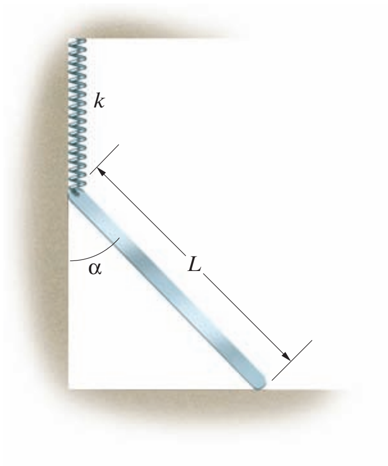
$$\sum M_A = -3300(36) \cos 15^\circ - 3300(20) \sin 15^\circ + 106 B_Y = 0,$$

from which

$$B_Y = \frac{131834}{106} = 1243.7 \text{ lb}.$$

Substitute into the sum of forces equation to obtain $A_Y = 1943.8$ lb

Problem 5.40 The length of the bar is $L = 4$ ft. Its weight $W = 6$ lb acts at the midpoint of the bar. The floor and wall are smooth. The spring is unstretched when the angle $\alpha = 0$. If the bar is in equilibrium when $\alpha = 40^\circ$, what is the spring constant k?



Solution: The free-body diagram is shown.

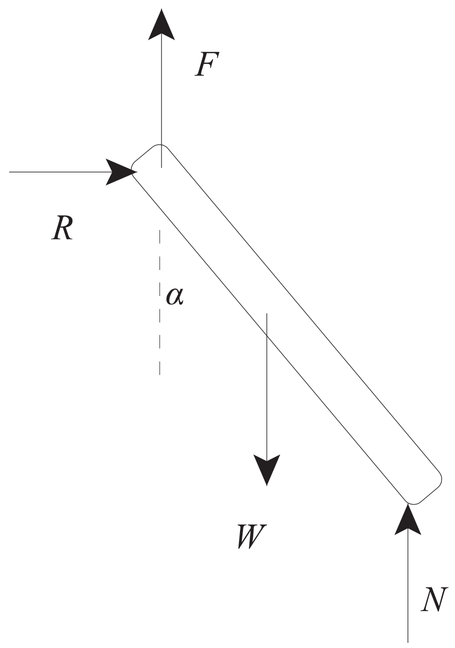
The stretch in the spring is $L - L\cos\alpha$, so the upward force exerted on the bar by the spring is $F = kL\,(1 - \cos\alpha)$. Let N and R be the normal forces exerted by the floor and the wall, respectively. The equilibrium equations for the bar are

$\Sigma F_x : R = 0$

$\Sigma F_y : F + N - W = 0$

$\Sigma M_{\text{bottom}} : W\dfrac{L}{2}\sin\alpha - RL\cos\alpha$

$- FL\sin\alpha = 0$

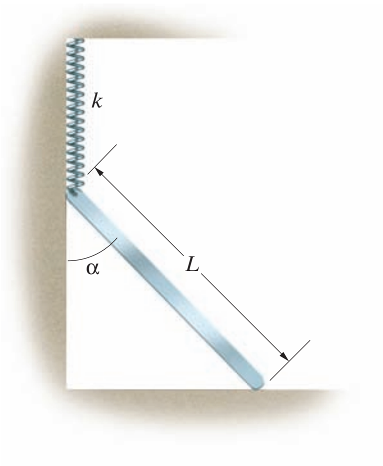


Because $R = 0$, the moment equation can be solved for the force exerted by the spring.

$$F = 0.5W = 3 \text{ lb} = kL\,(1 - \cos\alpha)$$

Solving yields $\boxed{k = 3.21 \text{ lb/ft}}$

Problem 5.41 The weight W of the bar acts at its midpoint. The floor and wall are smooth. The spring is unstretched when the angle $\alpha = 0$. Determine the angle α at which the bar is in equilibrium in terms of W, k, and L.



Solution: The free-body diagram is shown.
The stretch in the spring is $L - L\cos\alpha$, so the upward force exerted on the bar by the spring is $F = kL(1 - \cos\alpha)$. Let N and R be the normal forces exerted by the floor and the wall, respectively. The equilibrium equations for the bar are

$$\Sigma F_x : R = 0$$

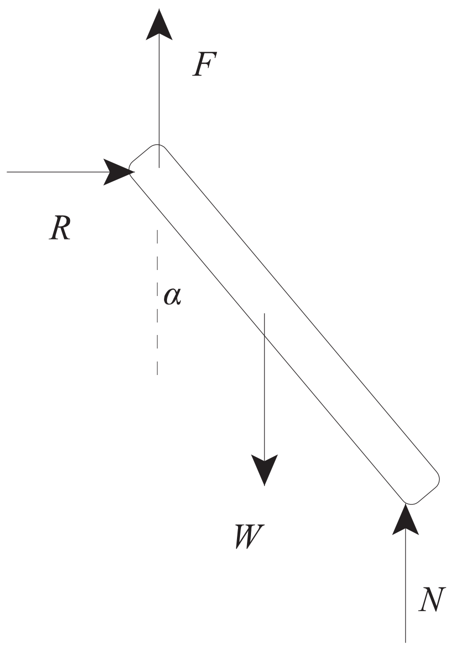
$$\Sigma F_y : F + N - W = 0$$

$$\Sigma M_{\text{bottom}} : W\frac{L}{2}\sin\alpha - RL\cos\alpha - FL\sin\alpha = 0$$

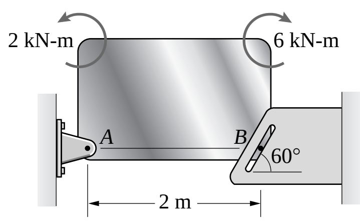
Because $R = 0$, the moment equation can be solved for the force exerted by the spring.

$$F = \frac{W}{2} = kL(1 - \cos\alpha)$$

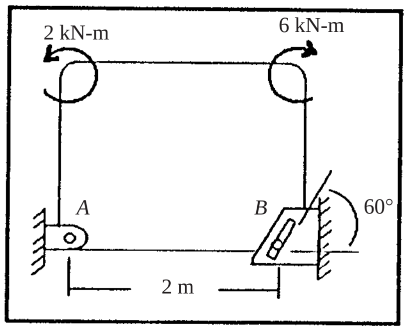
Solving yields $\boxed{\alpha = \cos^{-1}\left(1 - \frac{W}{2L}\right)}$

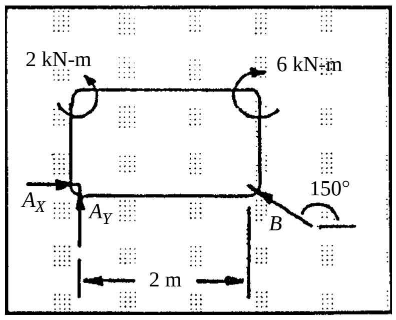


Problem 5.42 The plate is supported by a pin in a smooth slot at B. What are the reactions at the supports?



Solution: The pinned support is a two force reaction support. The smooth pin is a roller support, with a one force reaction. The reaction at B forms an angle of $90^\circ + 60^\circ = 150^\circ$ with the positive x axis. The sum of the forces:





$$\sum F_X = A_X + B\cos 150^\circ = 0$$

$$\sum F_Y = A_Y + B\sin 150^\circ = 0$$

The sum of the moments about B is

$$\sum M_B = -2A_Y + 2 - 6 = 0,$$

from which

$$A_Y = -\frac{4}{2} = -2 \text{ kN}.$$

Substitute into the force equations to obtain

$$B = \frac{A_Y}{\sin 150^\circ} = 4 \text{ kN},$$

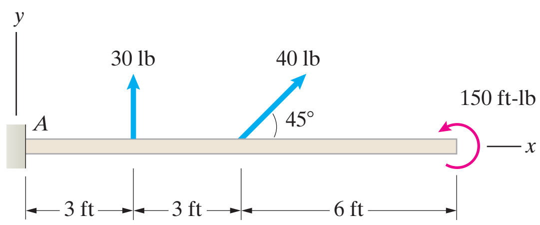
and $A_X = -B\cos 150^\circ = 3.464$ kN.

The horizontal and vertical reactions at B are

$$B_X = 4\cos 150^\circ = -3.464 \text{ kN},$$

and $B_Y = 4\sin 150^\circ = 2$ kN.

Problem 5.43 Determine the reactions at the fixed support A.



Solution: The free-body diagram is shown.
The equilibrium equations are

$$\Sigma F_x : A_x + (40 \text{ lb}) \cos 45^\circ = 0$$

$$\Sigma F_y : A_y + (30 \text{ lb})$$

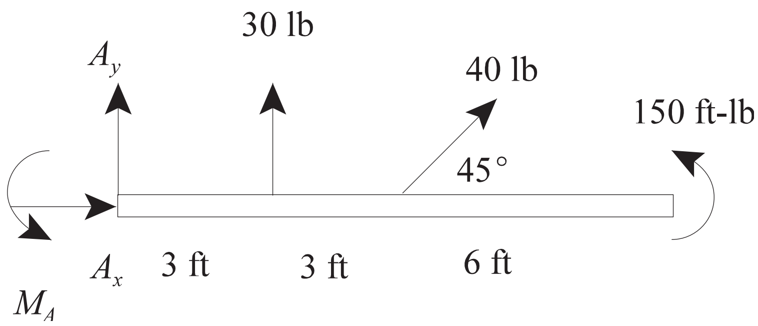
$$+ (40 \text{ lb}) \sin 45^\circ = 0$$

$$\Sigma M_A : M_A + (30 \text{ lb})(3 \text{ ft})$$

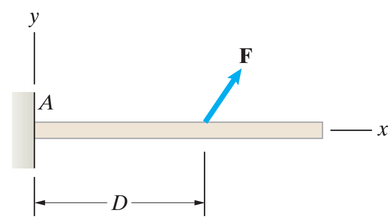
$$+ (40 \text{ lb}) \sin 45^\circ (6 \text{ ft})$$

$$+ (150 \text{ ft-lb}) = 0$$

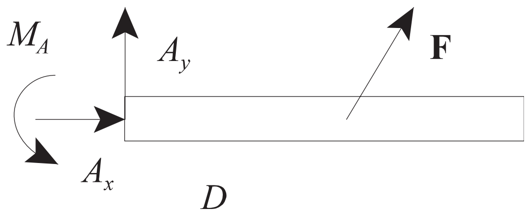
Solving yields $\boxed{A_x = -28.3 \text{ lb}, A_y = -58.3 \text{ lb}, M_A = -410 \text{ ft-lb}.}$



Problem 5.44 Suppose the you want to represent the two forces and couple acting on the beam in Problem 5.43 by an equivalent force **F** as shown. (a) Determine **F** and the distance D at which its line of action crosses the x axis. (b) Assume that **F** is the only load acting on the beam and determine the reactions at the fixed support A. Compare your answers to answers to Problem 5.43.



Solution: The free-body diagram is shown.



(a) To be equivalnet, **F** must equal the sum of the two forces:

$$\mathbf{F} = (30 \text{ lb})\mathbf{j} + (40 \text{ lb})(\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j})$$

$$\boxed{\mathbf{F} = (28.3\mathbf{i} + 58.3\mathbf{j}) \text{ lb}}$$

The force **F** must be placed so that the moment about a point due to **F** is equal to the moment about the same point due to the two forces and couple. Evaluating the moments about the origin,

$$(58.3 \text{ lb})D = (30 \text{ lb})(3 \text{ ft}) + (40 \text{ lb}) \sin 45^\circ (6 \text{ ft}) + (150 \text{ ft-lb})$$

The distance $\boxed{D = 7.03 \text{ ft}}$

(b) The equilibrium equations are

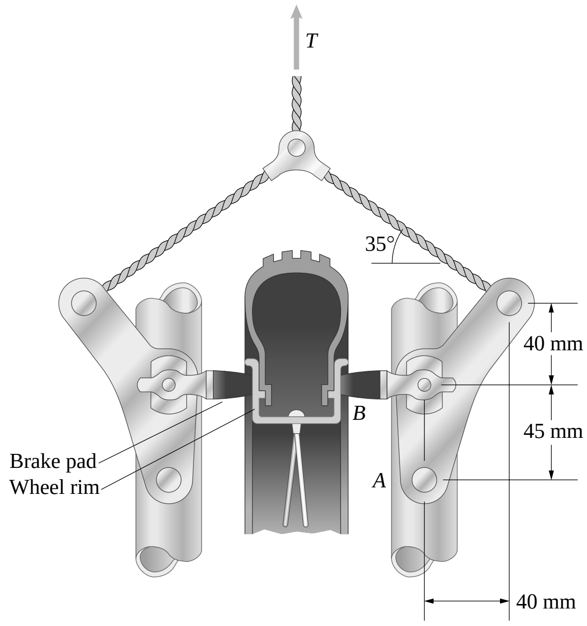
$$\Sigma F_x : A_x + (28.3 \text{ lb}) = 0$$

$$\Sigma F_y : A_y + (58.3 \text{ lb}) = 0$$

$$\Sigma M_A : M_A + (58.3 \text{ lb})(7.03 \text{ ft}) = 0$$

Solving yields $\boxed{A_x = -28.3 \text{ lb}, A_y = -58.3 \text{ lb}, M_A = -410 \text{ ft-lb}.}$

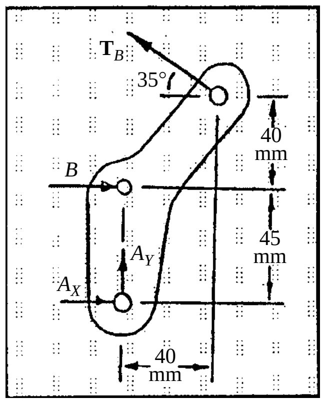
Problem 5.45 The bicycle brake on the right is pinned to the bicycle's frame at A. Determine the force exerted by the brake pad on the wheel rim at B in terms of the cable tension T.



Solution: From the force balance equation for the cables: the force on the brake mechanism T_B in terms of the cable tension T is

$$T - 2T_B \sin 35^\circ = 0,$$

from which $T_B = \dfrac{T}{2\sin 35^\circ} = 0.8717T$.



Take the origin of the system to be at A. The position vector of the point of attachment of B is $\mathbf{r}_B = 45\mathbf{j}$ (mm). The position vector of the point of attachment of the cable is $\mathbf{r}_C = 40\mathbf{i} + 85\mathbf{j}$ (mm).

The force exerted by the brake pad is $\mathbf{B} = -B\mathbf{i}$. The force vector due the cable tension is

$$\mathbf{T}_B = T_B(\mathbf{i}\cos 145^\circ + \mathbf{j}\sin 145^\circ) = T_B(-0.8192\mathbf{i} + 0.5736\mathbf{j}).$$

The moment about A is

$$\mathbf{M}_A = \mathbf{r}_B \times \mathbf{B} + \mathbf{r}_C \times \mathbf{T}_B = 0$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 45 & 45 \\ -B & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 40 & 85 & 85 \\ -0.8192 & 0.5736 & 0 \end{vmatrix} T_B = 0$$

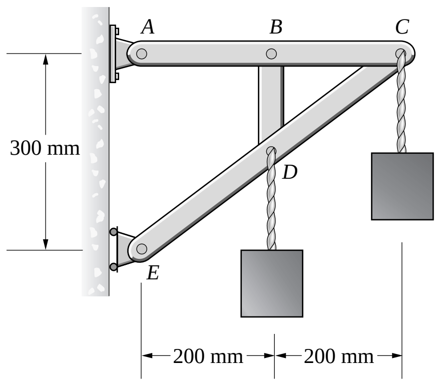
$$\mathbf{M}_A = (45B + 92.576T_B)\mathbf{k} = 0,$$

from which $B = \dfrac{92.576T_B}{45} = 2.057T_B$.

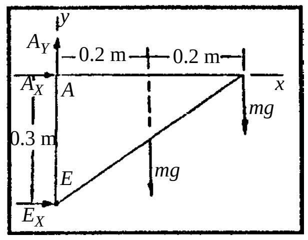
Substitute the expression for the cable tension:

$$B = (2.057)(0.8717)T = 1.793T$$

Problem 5.46 The mass of each of the suspended weights is 80 kg. Determine the reactions at the supports at A and E.



Solution: From the free body diagram, the equations of equilibrium for the rigid body are



$$\sum F_x = A_X + E_X = 0,$$

$$\sum F_y = A_Y - 2(80)(9.81) = 0,$$

and $\sum M_A = 0.3E_X - 0.2(80)(9.81) - 0.4(80)(9.81) = 0.$

We have three equations in the three components of the support reactions. Solving for the unknowns, we get the values

$A_X = -1570$ N,

$A_Y = 1570$ N,

and $E_X = 1570$ N.

Problem 5.47 The suspended weights in Problem 5.46 are each of mass m. The supports at A and E will each safely support a force of 6 kN magnitude. Based on this criterion, what is the largest safe value of m?

Solution: Written with the mass value of 80 kg replaced by the symbol m, the equations of equilibrium from Problem 5.46 are

$$\sum F_x = A_X + E_X = 0,$$

$$\sum F_y = A_Y - 2\ m(9.81) = 0,$$

and $\sum M_A = 0.3E_X - 0.2\ m(9.81) - 0.4\ m(9.81) = 0.$

We also need the relation

$|A| = \sqrt{A_X^2 + A_Y^2} = 6000$ N.

We have four equations in the three components of the support reactions plus the magnitude of A. This is four equations in four unknowns. Solving for the unknowns, we get the values

$A_X = -4243$ N,

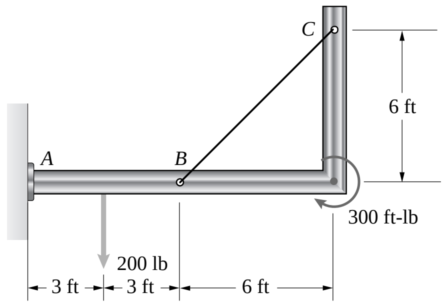
$A_Y = 4243$ N,

$E_X = 4243$ N,

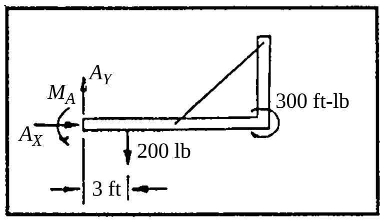
and $m = 216.5$ kg.

Note: We could have gotten this result by a linear scaling of all of the numbers in Problem 5.46.

Problem 5.48 The tension in cable BC is 100 lb. Determine the reactions at the built-in support.



Solution: The cable does not exert an external force on the system, and can be ignored in determining reactions. The built-in support is a two-force and couple reaction support. The sum of forces:



$$\sum F_X = A_X = 0.$$

$$\sum F_Y = A_Y - 200 = 0,$$

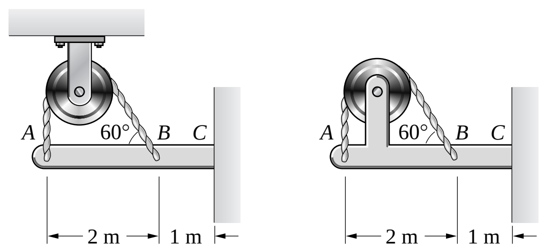
from which $A_Y = 200$ lb.

The sum of the moments about A is

$$\sum M = M_A - (3)(200) - 300 = 0,$$

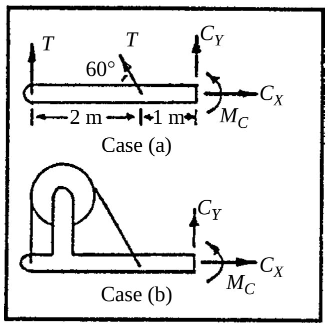
from which $M_A = 900$ ft lb

Problem 5.49 The tension in cable AB is 2 kN. What are the reactions at C in the two cases?



(a) (b)

Solution: *First Case:* The sum of the forces:



$$\sum F_X = C_X - T\cos 60^\circ = 0,$$

from which $C_X = 2(0.5) = 1$ kN

$$\sum F_Y = C_Y + T\sin 60^\circ + T = 0,$$

from which $C_Y = -1.866(2) = -3.732$ kN.

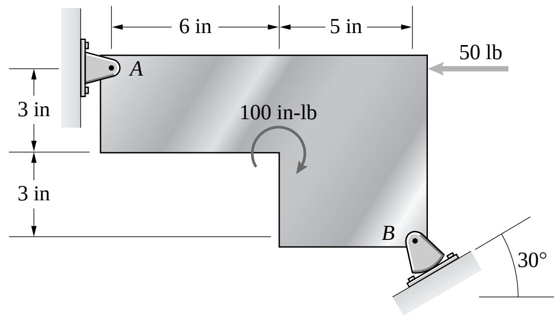
The sum of the moments about C is

$$\sum M = M_C - T\sin 60^\circ - 3T = 0,$$

from which $M_C = 3.866(2) = 7.732$ kN

Second Case: The weight of the beam is ignored, hence there are no external forces on the beam, and the reactions at C are zero.

Problem 5.50 Determine the reactions at the supports.



Solution: The reaction at A is a two-force reaction. The reaction at B is one-force, normal to the surface.

The sum of the forces:

$$\sum F_X = A_X - B\cos 60^\circ - 50 = 0.$$

$$\sum F_Y = A_Y + B\sin 60^\circ = 0.$$

The sum of the moments about A is

$$\sum M_A = -100 + 11B\sin 60^\circ - 6B\cos 60^\circ = 0,$$

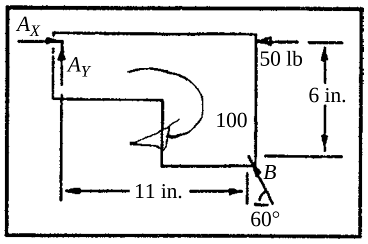
from which

$$B = \frac{100}{(11\sin 60^\circ - 6\cos 60^\circ)} = 15.3 \text{ lb}.$$

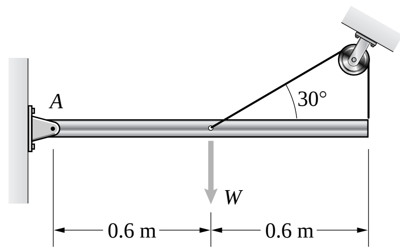
Substitute into the force equations to obtain

$$A_Y = -B\sin 60^\circ = -13.3 \text{ lb}$$

and $A_X = B\cos 60^\circ + 50 = 57.7$ lb



Problem 5.51 The weight $W = 2$ kN. Determine the tension in the cable and the reactions at A.



Solution: *Equilibrium Eqns*:

$$\sum F_X = 0: \quad A_X + T\cos 30^\circ = 0$$

$$\sum F_Y = 0: \quad A_Y + T + T\sin 30^\circ - W = 0$$

$$\circlearrowleft + \sum M_A = 0: \quad (-0,6)(W) + (0.6)(T\sin 30^\circ)$$

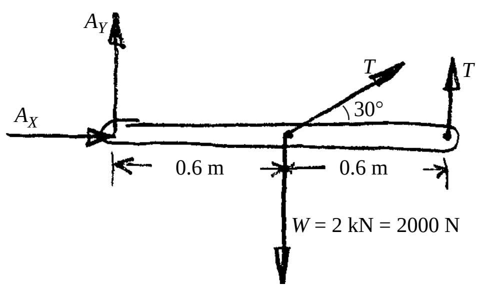
$$+ (1,2)(T) = 0$$

Solving, we get

$A_X = -693$ N,

$A_Y = 800$ N,

$T = 800$ N



Problem 5.52 The cable shown in Problem 5.51 will safely support a tension of 6 kN. Based on this criterion, what is the largest safe value of the weight W?

Solution: The equilibrium equations in the solution of problem are

$$\sum F_X = 0: \quad A_X + T\cos 30^\circ = 0$$

$$\sum F_Y = 0: \quad A_Y + T + T\sin 30^\circ - W = 0$$

$$\circlearrowleft + \sum M_A = 0: \quad (-0,6)(W) + (0,6)(T\sin 30^\circ)$$

$$+ (1,2)(T) = 0$$

We previously had 3 equations in the 3 unknowns A_X, A_Y and T (we knew W). In the current problem, we know T but don't know W. We again have three equations in three unknowns (A_X, A_Y, and W). Setting $T = 6$ kN, we solve to get

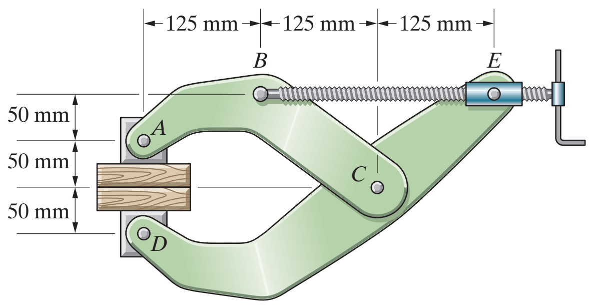
$A_X = -5.2$ kN

$A_Y = 6.0$ kN

$W = 15.0$ kN

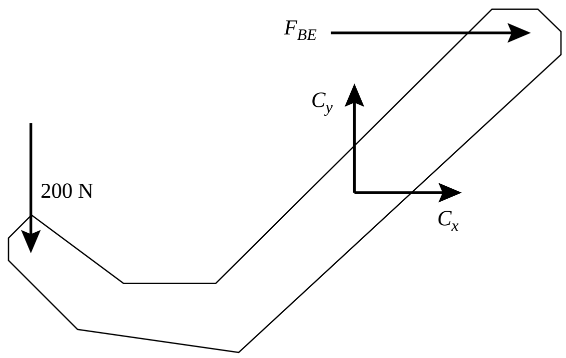
Problem 5.53 The blocks being compressed by the clamp exert a 200-N force on the pin at D that points from A toward D. The threaded shaft BE exerts a force on the pin at E that points from B toward E.

(a) Draw a free-body diagram of the arm DCE of the clamp, assuming that the pin at C behaves like a pin support.
(b) Determine the reactions at C.



Solution:

(a) The free-body diagram



(b) The equilibrium equations

$$\sum M_C : (200 \text{ N})(0.25 \text{ m}) - F_{BE}(0.1 \text{ m}) = 0$$

$$\sum F_x : C_x + F_{BE} = 0$$

$$\sum F_y : C_y - 200 \text{ N} = 0$$

Solving

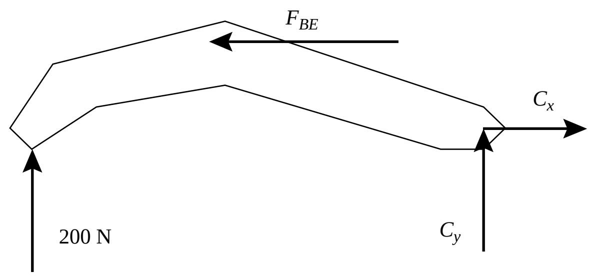
$$\boxed{C_x = -500 \text{ N},\ C_y = 200 \text{ N}}$$

Problem 5.54 Consider the clamp in Problem 5.53. The blocks being compressed by the clamp exert a 200-N force on the pin at A that points from D toward A. The threaded shaft BE exerts a force on the pin at B that points from E toward B.

(a) Draw a free-body diagram of the arm ABC of the clamp, assuming that the pin at C behaves like a pin support.
(b) Determine the reactions at C.

Solution:

(a) The free-body diagram
(b) The equilibrium equations



$$\sum M_C : -(200 \text{ N})(0.25 \text{ m}) + F_{BE}(0.1 \text{ m}) = 0$$

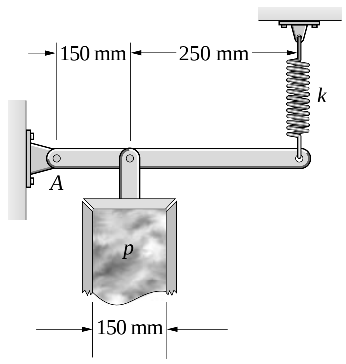
$$\sum F_x : -F_{BE} + C_x = 0$$

$$\sum F_y : 200 \text{ N} + C_y = 0$$

Solving we find

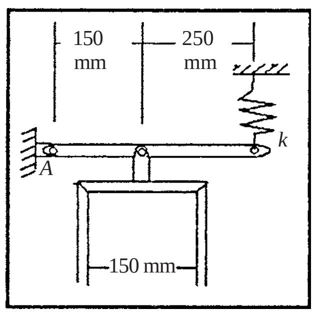
$$\boxed{C_x = 500 \text{ N}, \; C_y = -200 \text{ N}}$$

Problem 5.55 Suppose that you want to design the safety valve to open when the difference between the pressure p in the circular pipe (diameter = 150 mm) and the atmospheric pressure is 10 MPa (megapascals; a pascal is 1 N/m^2). The spring is compressed 20 mm when the valve is closed. What should the value of the spring constant be?



Solution: The area of the valve is

$$a = \pi \left(\frac{0.15}{2}\right)^2 = 17.671 \times 10^{-3} \text{ m}^2.$$

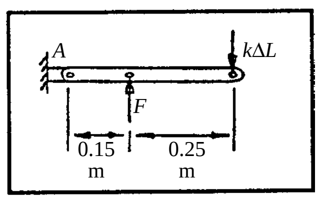


The force at opening is

$$F = 10a \times 10^6 = 1.7671 \times 10^5 \text{ N}.$$

The force on the spring is found from the sum of the moments about A,

$$\sum M_A = 0.15F - (0.4)k\Delta L = 0.$$

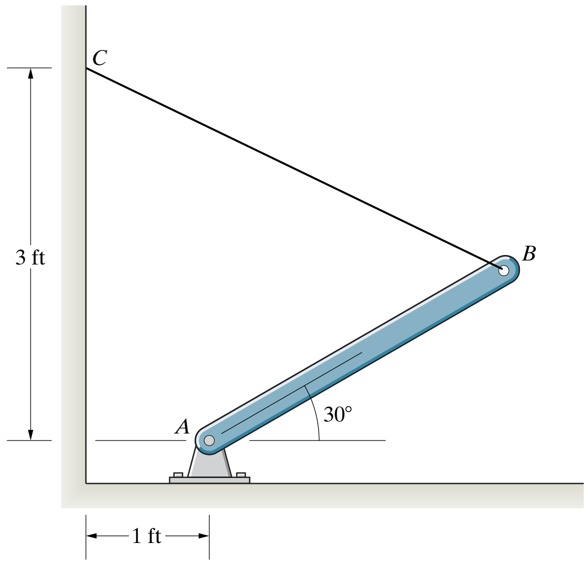


Solving,

$$k = \frac{0.15F}{(0.4)\Delta L} = \frac{0.15(1.7671 \times 10^5)}{(0.4)(0.02)}$$

$$= 3.313 \times 10^6 \ \frac{\text{N}}{\text{m}}$$

Problem 5.56 The 10-lb weight of the bar *AB* acts at the midpoint of the bar. The length of the bar is 3 ft. Determine the tension in the string *BC* and the reactions at *A*.



Solution: Geometry:

$$\tan\theta = \frac{3 \text{ ft} - 3 \text{ ft } \sin 30^\circ}{1 \text{ ft} + 3 \text{ ft } \cos 30^\circ} = 0.4169 \Rightarrow \theta = 22.63^\circ$$

The equilibrium equations

$$\sum M_A : T_{BC}\cos\theta(3 \text{ ft } \sin 30^\circ) + T_{BC}\sin\theta(3 \text{ ft } \cos 30^\circ)$$

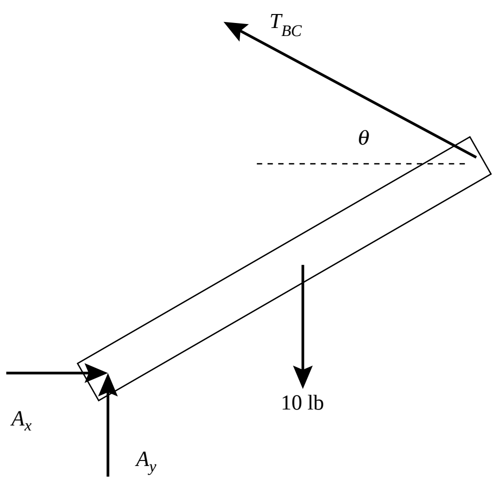
$$- (10 \text{ lb})(1.5 \text{ ft } \cos 30^\circ) = 0$$

$$\sum F_x : -T_{BC}\cos\theta + A_x = 0$$

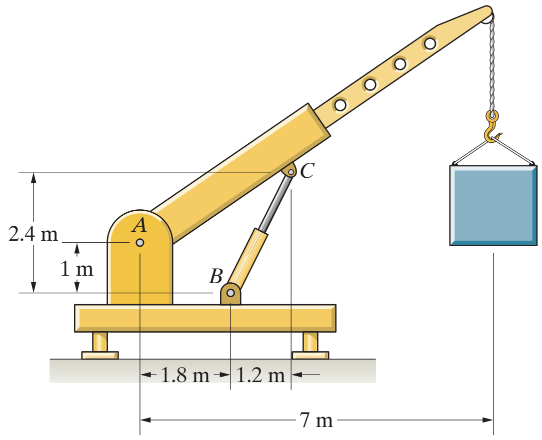
$$\sum F_y : T_{BC}\sin\theta - 10 \text{ lb} + A_y = 0$$

Solving:

$$\boxed{A_x = 5.03 \text{ lb}, \ A_y = 7.90 \text{ lb}, \ T = 5.45 \text{ lb}}$$



Problem 5.57 The crane's arm has a pin support at A. The hydraulic cylinder BC exerts a force on the arm at C in the direction parallel to BC. The crane's arm has a mass of 200 kg, and its weight can be assumed to act at a point 2 m to the right of A. If the mass of the suspended box is 800 kg and the system is in equilibrium, what is the magnitude of the force exerted by the hydraulic cylinder?



Solution: The free-body diagram of the arm is shown.

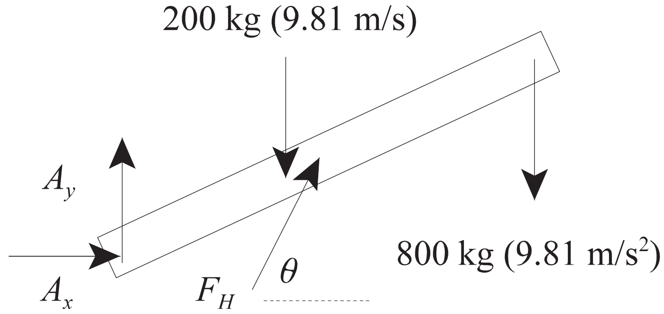
The angle $\theta = \tan^{-1}\left(\frac{2.4}{1.2}\right) = 63.4°$

The equilibrium equations are

$$\Sigma F_x : A_x + F_H \cos\theta = 0$$

$$\Sigma F_y : A_y + F_H \sin\theta - (200 \text{ kg})(9.81 \text{ m/s}^2) - (800 \text{ kg})(9.81 \text{ m/s}^2) = 0$$

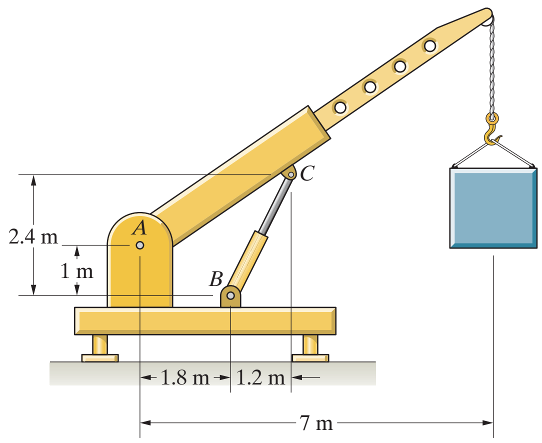
$$\Sigma M_A : F_H \sin\theta(3 \text{ m}) - F_H \cos\theta(1.4 \text{ m}) - (200 \text{ kg})(9.81 \text{ m/s}^2)(2 \text{ m}) - (800 \text{ kg})(9.81 \text{ m/s}^2)(7 \text{ m}) = 0$$



We obtain $A_x = -12.8$ kN, $A_y = -15.8$ kN, $F_H = 28.6$ kN

Thus $\boxed{F_H = 28.6 \text{ kN}}$

Problem 5.58 In Problem 5.57, what is the magnitude of the force exerted on the crane's arm by the pin support at A?



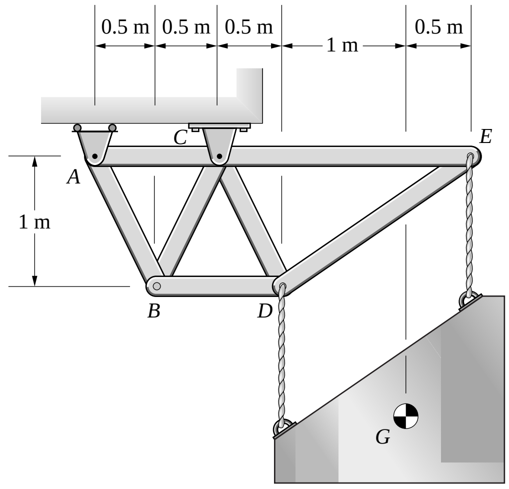
Solution: See the solution to Problem 5.57.

$A_x = -12.8$ kN, $A_y = -15.8$ kN, $F_H = 28.6$ kN

$$|\mathbf{A}| = \sqrt{(-12.8 \text{ kN})^2 + (-15.8 \text{ kN})^2} = 20.3 \text{ kN}$$

Thus $\boxed{|\mathbf{A}| = 20.3 \text{ kN}}$

Problem 5.59 A speaker system is suspended by the cables attached at D and E. The mass of the speaker system is 130 kg, and its weight acts at G. Determine the tensions in the cables and the reactions at A and C.



Solution: The weight of the speaker is $W = mg = 1275$ N. The equations of equilibrium for the entire assembly are

$$\sum F_x = C_X = 0,$$

$$\sum F_y = A_Y + C_Y - mg = 0$$

(where the mass $m = 130$ kg), and

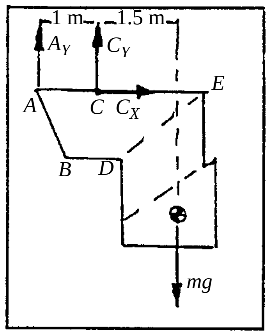
$$\sum M_C = -(1)A_Y - (1.5)mg = 0.$$

Solving these equations, we get

$C_X = 0,$

$C_Y = 3188$ N,

and $A_Y = -1913$ N.



From the free body diagram of the speaker alone, we get

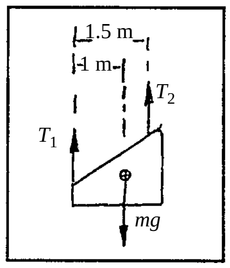
$$\sum F_y = T_1 + T_2 - mg = 0,$$

and $\sum M_{\text{left support}} = -(1)mg + (1.5)T_2 = 0.$

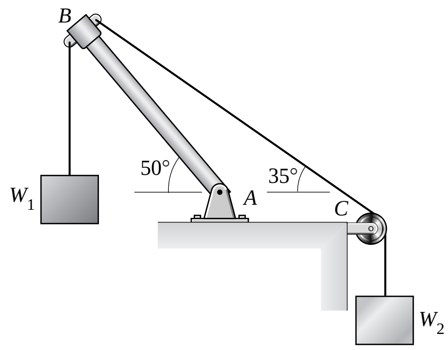
Solving these equations, we get

$T_1 = 425.$ N

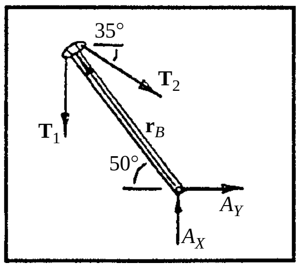
and $T_2 = 850$ N



Problem 5.60 The weight $W_1 = 1000$ lb. Neglect the weight of the bar AB. The cable goes over a pulley at C. Determine the weight W_2 and the reactions at the pin support A.



Solution: The strategy is to resolve the tensions at the end of bar AB into x- and y-components, and then set the moment about A to zero. The angle between the cable and the positive x axis is $-35°$. The tension vector in the cable is



$$\mathbf{T}_2 = W_2(\mathbf{i}\cos(-35°) + \mathbf{j}\sin(-35°)).$$

$$= W_2(0.8192\mathbf{i} - 0.5736\mathbf{j})(\text{lb}).$$

Assume a unit length for the bar. The angle between the bar and the positive x axis is $180° - 50° = 130°$. The position vector of the tip of the bar relative to A is

$$\mathbf{r}_B = \mathbf{i}\cos(130°) + \mathbf{j}\sin(130°), = -0.6428\mathbf{i} + 0.7660\mathbf{j}.$$

The tension exerted by W_1 is $\mathbf{T}_1 = -1000\mathbf{j}$. The sum of the moments about A is:

$$\sum \mathbf{M}_A = (\mathbf{r}_B \times \mathbf{T}_1) + (\mathbf{r}_B \times \mathbf{T}_2) = \mathbf{r}_B \times (\mathbf{T}_1 + \mathbf{T}_2)$$

$$= L \begin{vmatrix} \mathbf{i} & \mathbf{j} \\ -0.6428 & 0.7660 \\ 0.8191W_2 & -0.5736W_2 - 1000 \end{vmatrix}$$

$$\sum \mathbf{M}_A = (-0.2587W_2 + 642.8)\mathbf{k} = 0,$$

from which $W_2 = 2483.5$ lb

The sum of the forces:

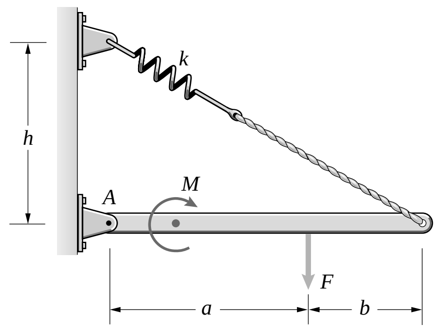
$$\sum \mathbf{F}_X = (A_X + W_2(0.8192))\mathbf{i} = 0,$$

from which $A_X = -2034.4$ lb

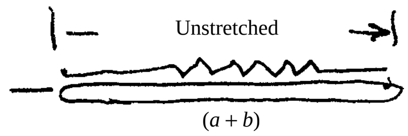
$$\sum \mathbf{F}_Y = (A_Y - W_2(0.5736) - 1000)\mathbf{j} = 0,$$

from which $A_Y = 2424.5$ lb

Problem 5.61 The dimensions $a = 2$ m and $b = 1$ m. The couple $M = 2400$ N-m. The spring constant is $k = 6000$ N/m, and the spring would be unstretched if $h = 0$. The system is in equilibrium when $h = 2$ m and the beam is horizontal. Determine the force F and the reactions at A.



Solution: We need to know the unstretched length of the spring, l_0



$l_0 = a + b = 3$ m

We also need the stretched length

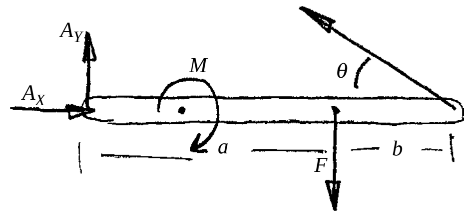
$$l^2 = h^2 + (a+b)^2$$

$$l = 3.61 \text{ m}$$

$$F_S = k(l - l_0)$$

$$\tan\theta = \frac{h}{(a+b)}$$

$$\theta = 33.69^\circ$$



Equilibrium eqns:

$$\sum F_X: \quad A_X - F_S\cos\theta = 0$$

$$\sum F_Y: \quad A_Y + F_S\sin\theta - F = 0$$

$$\circlearrowleft + \sum M_A: \quad -M - aF + (a+b)F_S\sin\theta = 0$$

$a = 2$ m, $b = 1$ m, $M = 2400$ N-m,

$h = 2$ m, $k = 6000$ N/m.

Substituting in and solving, we get

$F_S = 6000(l - l_0) = 3633$ N

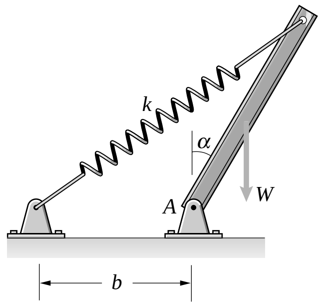
and the equilibrium equations yield

$A_X = 3023$ N

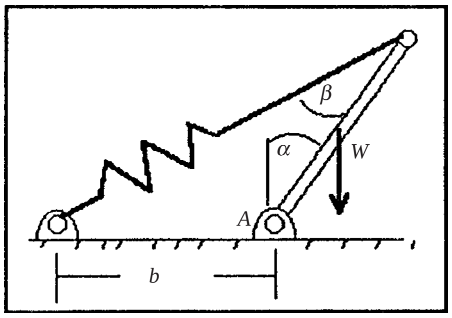
$A_Y = -192$ N

$F = 1823$ N

Problem 5.62 The bar is 1 m long, and its weight W acts at its midpoint. The distance $b = 0.75$ m, and the angle $\alpha = 30^\circ$. The spring constant is $k = 100$ N/m, and the spring is unstretched when the bar is vertical. Determine W and the reactions at A.



Solution: The unstretched length of the spring is $L = \sqrt{b^2 + 1^2} = 1.25$ m. The obtuse angle is $90 + \alpha$, so the stretched length can be determined from the cosine law:

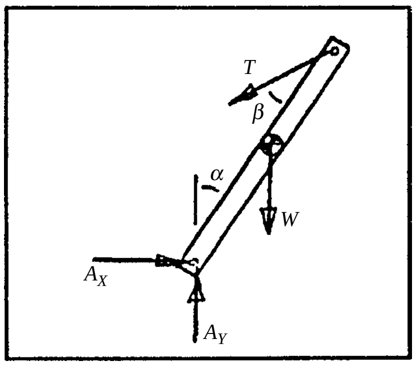


$$L_2^2 = 1^2 + 0.75^2 - 2(0.75)\cos(90 + \alpha) = 2.3125\ \text{m}^2$$

from which $L_2 = 1.5207$ m. The force exerted by the spring is

$$T = k\Delta L = 100(1.5207 - 1.25) = 27.1\ \text{N}.$$

The angle between the spring and the bar can be determined from the sine law:



$$\frac{b}{\sin\beta} = \frac{1.5207}{\sin(90 + \alpha)},$$

from which $\sin\beta = 0.4271$,

$$\beta = 25.28^\circ.$$

The angle the spring makes with the horizontal is $180 - 25.28 - 90 - \alpha = 34.72^\circ$. The sum of the forces:

$$\sum F_X = A_X - T\cos 34.72^\circ = 0,$$

from which $A_X = 22.25$ N.

$$\sum F_Y = A_Y - W - T\sin 34.72^\circ = 0.$$

The sum of the moments about A is

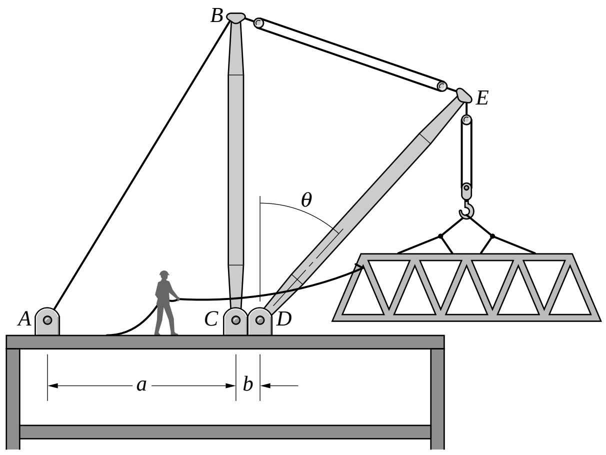
$$\sum M_A = T\sin 25.28^\circ - \left(\frac{W}{2}\right)\sin\alpha = 0,$$

from which

$$W = \frac{2T\sin 25.28^\circ}{\sin\alpha} = 46.25\ \text{N}.$$

Substitute into the force equation to obtain: $A_Y = W + T\sin 34.72^\circ = 61.66$ N

Problem 5.63 The boom derrick supports a suspended 15-kip load. The booms BC and DE are each 20 ft long. The distances are $a = 15$ ft and $b = 2$ ft, and the angle $\theta = 30^\circ$. Determine the tension in cable AB and the reactions at the pin supports C and D.



Solution: Choose a coordinate system with origin at point C, with the y axis parallel to CB. The position vectors of the labeled points are:

$$\mathbf{r}_D = 2\mathbf{i}$$

$$\mathbf{r}_E = \mathbf{r}_D + 20(\mathbf{i}\sin 30^\circ + \mathbf{j}\cos 30^\circ)$$

$$= 12\mathbf{i} + 17.3\mathbf{j},$$

$$\mathbf{r}_B = 20\mathbf{j},$$

$$\mathbf{r}_A = -15\mathbf{i}.$$

The unit vectors are:

$$\mathbf{e}_{DE} = \frac{\mathbf{r}_E - \mathbf{r}_D}{|\mathbf{r}_E - \mathbf{r}_D|} = 0.5\mathbf{i} + 0.866\mathbf{j},$$

$$\mathbf{e}_{EB} = \frac{\mathbf{r}_B - \mathbf{r}_E}{|\mathbf{r}_B - \mathbf{r}_E|} = -0.976\mathbf{i} + 0.2179\mathbf{j}.$$

$$\mathbf{e}_{CB} = \frac{\mathbf{r}_B - \mathbf{r}_C}{|\mathbf{r}_B - \mathbf{r}_C|} = 1\mathbf{j},$$

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_A - \mathbf{r}_B}{|\mathbf{r}_A - \mathbf{r}_B|} = -0.6\mathbf{i} - 0.8\mathbf{j}.$$

Isolate the juncture at E: The equilibrium conditions are

$$\sum F_x = 0.5|\mathbf{D}| - 0.976|\mathbf{T}_{EB}| = 0,$$

$$\sum F_y = 0.866|\mathbf{D}| + 0.2179|\mathbf{T}_{EB}| - 15 = 0,$$

from which

$$|\mathbf{D}| = 15.34 \text{ kip}$$

and $|\mathbf{T}_{EB}| = 7.86$ kip.

Isolate the juncture at B: The equilibrium conditions are:

$$\sum F_x = 0|\mathbf{C}| - 0.6|\mathbf{T}_{AB}| + 0.976|\mathbf{T}_{EB}|,$$

and $\sum F_y = 1|\mathbf{C}| - 0.6|\mathbf{T}_{AB}| - 0.2179|\mathbf{T}_{EB}| = 0,$

from which

$$|\mathbf{T}_{AB}| = 12.79 \text{ kip},$$

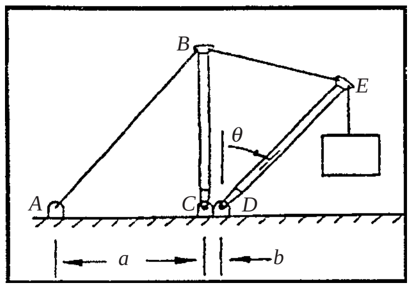
and $|\mathbf{C}| = 11.94$ kip.

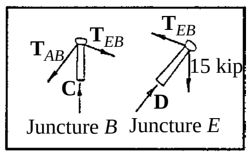
The components:

$$D_x = 0.6|\mathbf{D}| = 7.67 \text{ kip},$$

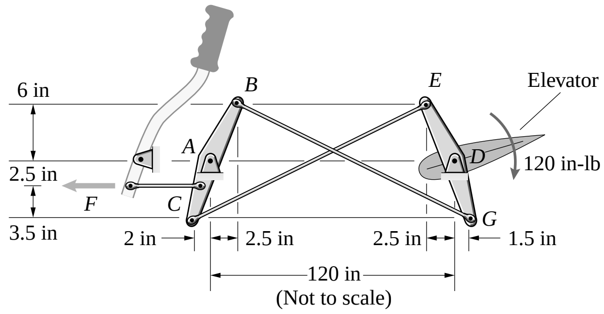
$$D_y = 0.866|\mathbf{D}| = 13.287 \text{ kip},$$

and $C_y = 1|\mathbf{C}| = 11.94$ kip

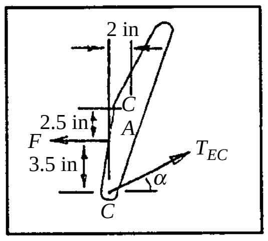


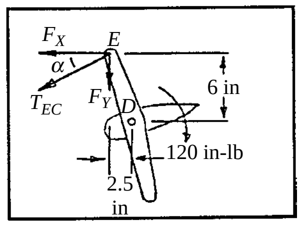


Problem 5.64 The arrangement shown controls the elevators of an airplane. (The elevators are the horizontal control surfaces in the airplane's tail.) The elevators are attached to member EDG. Aerodynamic pressures on the elevators exert a clockwise couple of 120 in-lb. Cable BG is slack, and its tension can be neglected. Determine the force F and the reactions at pin support A.



Solution: Begin at the elevator. The moment arms at E and G are 6 in. The angle of the cable EC with the horizontal is





$$\alpha = \tan^{-1}\frac{12}{119.5} = 5.734°.$$

Denote the horizontal and vertical components of the force on point E by F_X and F_Y. The sum of the moments about the pinned support on the member EG is

$$\sum M_{EG} = 2.5F_Y + 6F_X - 120 = 0.$$

This is the tension in the cable EC. Noting that

$$F_X = T_{EC}\cos\alpha,$$

and $F_Y = T_{EC}\sin\alpha$,

then $T_{EC} = \dfrac{120}{2.5\sin\alpha + 6\cos\alpha}$.

The sum of the moments about the pinned support BC is

$$\sum M_{BC} = -2T_{EC}\sin\alpha + 6T_{EC}\cos\alpha - 2.5F = 0.$$

Substituting:

$$F = \left(\frac{120}{2.5}\right)\left(\frac{6\cos\alpha - 2\sin\alpha}{6\cos\alpha + 2.5\sin\alpha}\right)$$

$$= (48)(0.9277) = 44.53 \text{ lb}.$$

The sum of the forces about the pinned joint A:

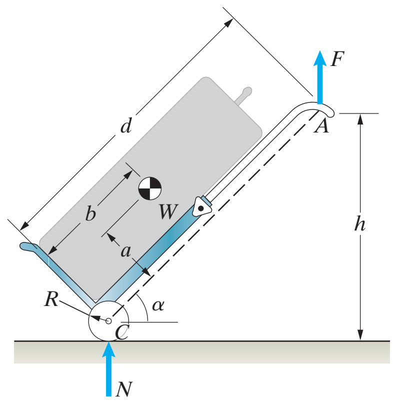
$$\sum F_x = A_x - F + T_{EC}\cos\alpha = 0$$

from which $A_x = 25.33$ lb,

$$\sum F_y = A_y + T_{EC}\sin\alpha = 0$$

from which $A_y = -1.93$ lb

Problem 5.65 In Example 5.4 suppose that $\alpha = 40°$, $d = 1$ m, $a = 200$ mm, $b = 500$ mm, $R = 75$ mm, and the mass of the luggage is 40 kg. Determine F and N.



Solution: (See Example 5.4.)

The sum of the moments about the center of the wheel:

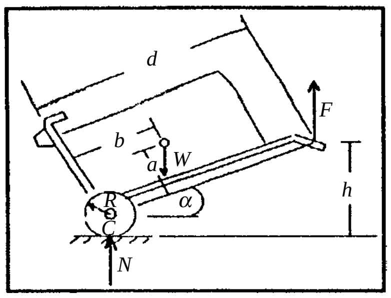
$$\sum M_C = dF\cos\alpha + aW\sin\alpha - bW\cos\alpha = 0,$$

from which $F = \dfrac{(b - a\tan\alpha)W}{d} = 130.35$ N.

The sum of the forces:

$$\sum F_Y = N - W + F = 0,$$

from which $N = 262.1$ N



Problem 5.66 In Example 5.4 suppose that $\alpha = 35°$, $d = 46$ in, $a = 10$ in, $b = 14$ in, $R = 3$ in, and you don't want the user to have to exert a force F larger than 20 lb. What is the largest luggage weight that can be placed on the carrier?

Solution: (See Example 5.4.) From the solution to Problem 5.65, the force is

$$F = \frac{(b - a\tan\alpha)W}{d}.$$

Solve for W:

$$W = \frac{Fd}{(b - a\tan\alpha)}.$$

For $F = 20$ lb,

$$W = 131.47 = 131.5 \text{ lb}$$

Problem 5.67 One of the difficulties in making design decisions is that you don't know how the user will place the luggage on the carrier in Example 5.4. Suppose you assume that the point where the weight acts may be anywhere within the "envelope" $R \le a \le 0.75c$ and $0 \le b \le 0.75d$. If $\alpha = 30°$, $c = 14$ in, $d = 48$ in, $R = 3$ in, and $W = 80$ lb, what is the largest force F the user will have to exert for any luggage placement?

Solution: (See Example 5.4.) From the solution to Problem 5.65, the force is

$$F = \frac{(b - a\tan\alpha)W}{d}.$$

The force is maximized as

$$b \to 0.75d,$$

and $a \to R$.

Thus

$$F_{\text{MAX}} = \frac{(0.75d - R\tan\alpha)W}{d} = 57.11 \text{ lb}$$

Problem 5.68 In our design of the luggage carrier in Example 5.4, we assumed a user that would hold the carrier's handle at $h = 36$ in above the floor. We assumed that $R = 3$ in, $a = 6$ in, and $b = 12$ in, and we chose the dimension $d = 4$ ft. The resulting ratio of the force the user must exert to the weight of the luggage is $F/W = 0.132$. Suppose that people with a range of heights use this carrier. Obtain a graph of F/W as a function of h for $24 \le h \le 36$ in.

Solution: (See Example 5.4.) From the solution to Problem 5.67, the force that must be exerted is

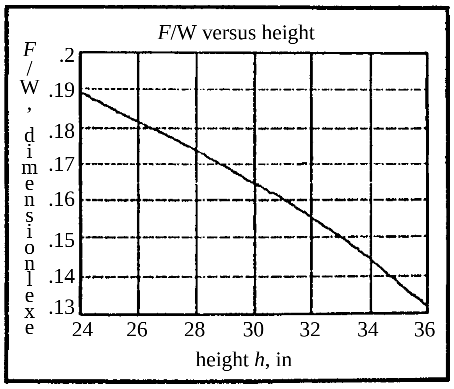
$$F = \frac{(b - a\tan\alpha)W}{d},$$

from which $\frac{F}{W} = \frac{(b - a\tan\alpha)}{d}$.

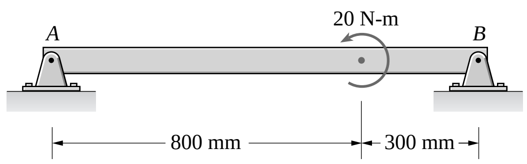
The angle a is given by

$$\alpha = \sin^{-1}\left(\frac{h - R}{d}\right).$$

The commercial package **TK Solver Plus** was used to plot a graph of $\frac{F}{W}$ as a function of h.



Problem 5.69 (a) Draw the free-body diagram of the beam and show that it is statically indeterminate. (See Active Example 5.5.)
(b) Determine as many of the reactions as possible.



Solution: (a) The free body diagram shows that there are four unknowns, whereas only three equilibrium equations can be written.
(b) The sum of moments about A is

$$\sum M_A = M + 1.1B_Y = 0,$$

from which $B_Y = -\frac{20}{1.1} = -18.18$ N.

The sum of forces in the vertical direction is

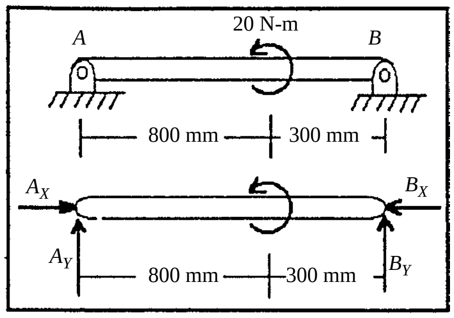
$$\sum F_Y = A_Y + B_Y = 0,$$

from which $A_Y = -B_Y = 18.18$ N.

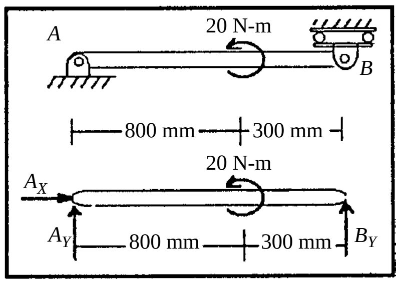
The sum of forces in the horizontal direction is

$$\sum F_X = A_X + B_X = 0,$$

from which the values of A_X and B_X are indeterminate.



Problem 5.70 Consider the beam in Problem 5.69. Choose supports at A and B so that it is not statically indeterminate. Determine the reactions at the supports.



Solution: One possibility is shown: the pinned support at B is replaced by a roller support. The equilibrium conditions are:

$$\sum F_X = A_X = 0.$$

The sum of moments about A is

$$\sum M_A = M + 1.1 B_Y = 0,$$

from which $B_Y = -\frac{20}{1.1} = -18.18$ N.

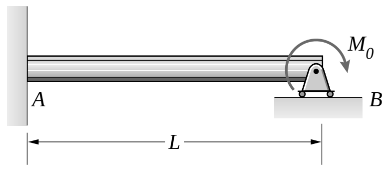
The sum of forces in the vertical direction is

$$\sum F_Y = A_Y + B_Y = 0,$$

from which $A_Y = -B_Y = 18.18$ N.

Problem 5.71 (a) Draw the free-body diagram of the beam and show that it is statically indeterminate. (The external couple M_0 is known.)

(b) By an analysis of the beam's deflection, it is determined that the vertical reaction B exerted by the roller support is related to the couple M_0 by $B = 2M_0/L$. What are the reactions at A?



Solution:

(a) $\sum F_X$: $A_X = 0$ **(1)**

$\sum F_Y$: $A_Y + B = 0$ **(2)**

$\circlearrowleft + \sum M_A$: $M_A - M_O + BL = 0$ **(3)**

Unknowns: M_A, A_X, A_Y, B.

3 Eqns in 4 unknowns

∴ *Statistically indeterminate*

(b) Given $B = 2M_O/L$ **(4)**

We now have 4 eqns in 4 unknowns and can solve.

Eqn (1) yields $A_X = 0$

Eqn (2) and Eqn (4) yield

$A_Y = -2M_O/L$

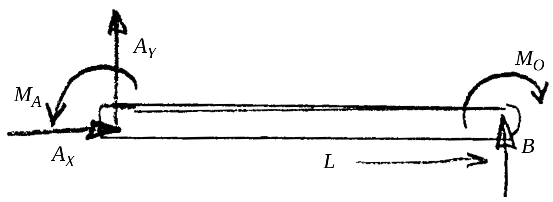
Eqn (3) and Eqn (4) yield

$M_A = M_O - 2M_O$

$M_A = -M_O$

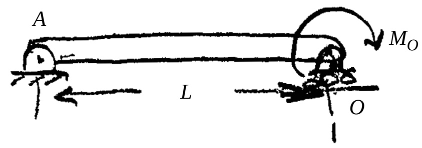
M_A was assumed counterclockwise

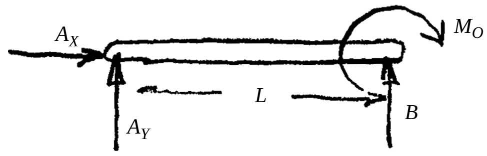
$M_A = \lvert M_O \rvert$ clockwise
$A_X = 0$
$A_Y = -2M_O/L$



Problem 5.72 Consider the beam in Problem 5.71. Choose supports at A and B so that it is not statically indeterminate. Determine the reactions at the supports.

Solution: This result is not unique. There are several possible answers





$\sum F_X$: $A_X = 0$

$\sum F_Y$: $A_Y + B_Y = 0$

$\sum M_A$: $-M_o + BL = 0$

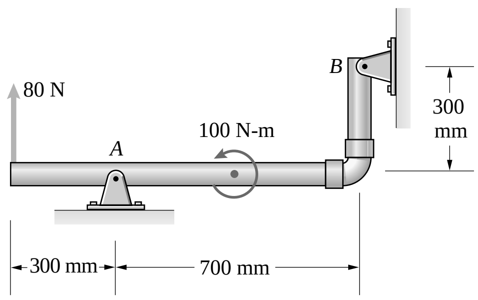
$A_X = 0$

$B = M_O/L$

$A_Y = -M_O/L$

Problem 5.73 Draw the free-body diagram of the L-shaped pipe assembly and show that it is statically indeterminate. Determine as many of the reactions as possible.

Strategy: Place the coordinate system so that the x axis passes through points A and B.



Solution: The free body diagram shows that there are four reactions, hence the system is statically indeterminate. The sum of the forces:

$$\sum F_X = (A_X + B_X) = 0,$$

and $\sum F_Y = A_Y + B_Y + F = 0.$

A strategy for solving some statically indeterminate problems is to select a coordinate system such that the indeterminate reactions vanish from the sum of the moment equations. The choice here is to locate the x axis on a line passing through both A and B, with the origin at A. Denote the reactions at A and B by A_N, A_P, B_N, and B_P, where the subscripts indicate the reactions are normal to and parallel to the new x axis. Denote

$F = 80$ N,

$M = 100$ N-m.

The length from A to B is

$$L = \sqrt{0.3^2 + 0.7^2} = 0.76157 \text{ m}.$$

The angle between the new axis and the horizontal is

$$\theta = \tan^{-1}\left(\frac{0.3}{0.7}\right) = 23.2°.$$

The moment about the point A is

$$M_A = LB_N - 0.3F + M = 0,$$

from which $B_N = \dfrac{-M + 0.3F}{L} = \dfrac{-76}{0.76157} = -99.79$ N,

from which

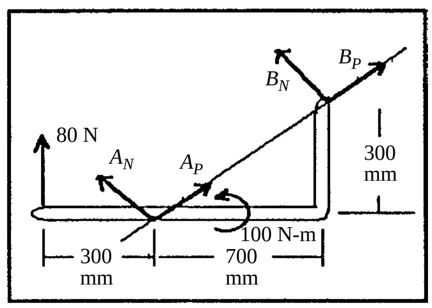
The sum of the forces normal to the new axis is

$$\sum F_N = A_N + B_N + F\cos\theta = 0,$$

from which

$$A_N = -B_N - F\cos\theta = 26.26 \text{ lb}$$

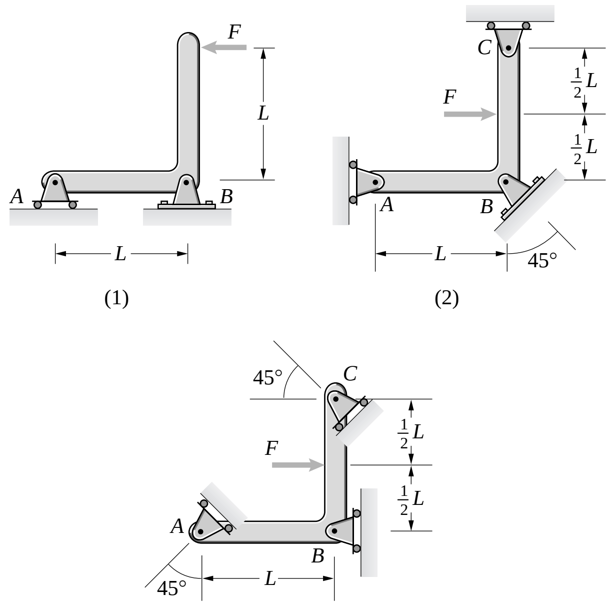
The reactions parallel to the new axis are indeterminate.



Problem 5.74 Consider the pipe assembly in Problem 5.73. Choose supports at A and B so that it is not statically indeterminate. Determine the reactions at the supports.

Solution: This problem has no unique solution.

Problem 5.75 State whether each of the L-shaped bars shown is properly or improperly supported. If a bar is properly supported, determine the reactions at its supports. (See Active Example 5.6.)



(1) (2) (3)

Solution:

(1) *is properly constrained*. The sum of the forces

$$\sum F_X = -F + B_X = 0,$$

from which $B_X = F$.

$$\sum F_Y = B_Y + A_y = 0,$$

from which $B_y = -A_y$. The sum of the moments about B:

$$\sum M_B = -LA_Y + LF = 0,$$

from which $A_Y = F$, and $B_y = -F$

(2) *is improperly constrained*. The reactions intersect at B, while the force produces a moment about B.

(3) *is properly constrained*. The forces are neither concurrent nor parallel. The sum of the forces:

$$\sum F_X = -C\cos 45^\circ - B - A\cos 45^\circ + F = 0.$$

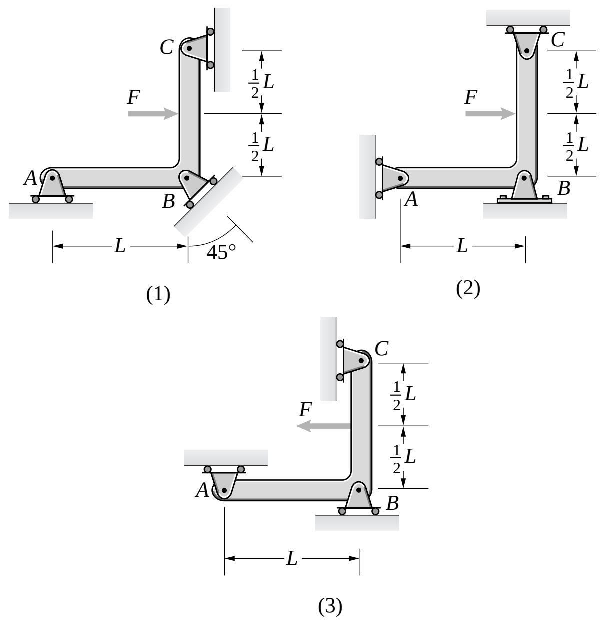
$$\sum F_Y = C\sin 45^\circ - A\sin 45^\circ = 0$$

from which $A = C$. The sum of the moments about A:

$$\sum M_A = -\tfrac{1}{2}LF + LC\cos 45^\circ + LC\sin 45^\circ = 0,$$

from which $C = \dfrac{F}{2\sqrt{2}}$. Substituting and combining: $A = \dfrac{F}{2\sqrt{2}}$, $B = \dfrac{F}{2}$

Problem 5.76 State whether each of the L-shaped bars shown is properly or improperly supported. If a bar is properly supported, determine the reactions at its supports. (See Active Example 5.6.)



Solution:

(1) *is improperly constrained.* The reactions intersect at a point P, and the force exerts a moment about that point.

(2) *is improperly constrained.* The reactions intersect at a point P and the force exerts a moment about that point.

(3) *is properly constrained*. The sum of the forces:

$$\sum F_X = C - F = 0,$$

from which $C = F$.

$$\sum F_Y = -A + B = 0,$$

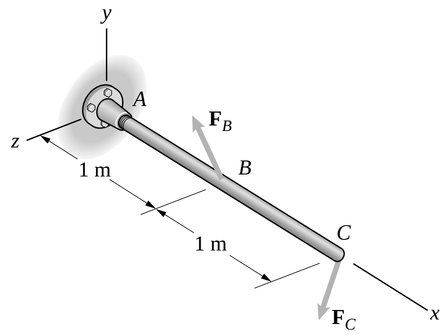
from which $A = B$. The sum of the moments about B: $LA + \frac{L}{2}F - LC = 0$, from which $A = \frac{1}{2}F$, and $B = \frac{1}{2}F$

Problem 5.77 The bar AB has a built-in support at A and is loaded by the forces

$\mathbf{F}_B = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ (kN),

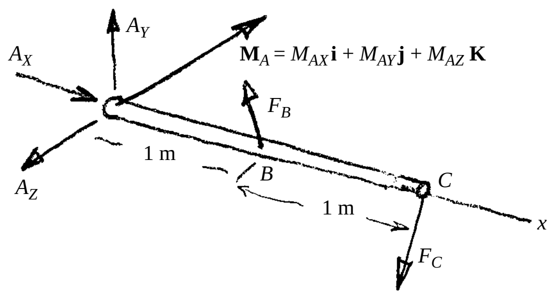
$\mathbf{F}_C = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ (kN).

(a) Draw the free-body diagram of the bar.
(b) Determine the reactions at A.



Strategy: (a) Draw a diagram of the bar isolated from its supports. Complete the free-body diagram of the bar by adding the two external forces and the reactions due to the built-in support (see Table 5.2). (b) Use the scalar equilibrium equations (5.16)–(5.21) to determine the reactions.

Solution:



$\mathbf{M}_A = M_{A_X}\mathbf{i} + M_{A_Y}\mathbf{j} + M_{A_Z}\mathbf{k}$

(b) Equilibrium Eqns (Forces)

$$\sum F_X: \quad A_X + F_{B_X} + F_{C_X} = 0$$

$$\sum F_Y: \quad A_Y + F_{B_Y} + F_{C_Y} = 0$$

$$\sum F_Z: \quad A_Z + F_{B_Z} + F_{C_Z} = 0$$

Equilibrium Equations (Moments) Sum moments about A

$\mathbf{r}_{AB} \times \mathbf{F}_B = 1\mathbf{i} \times (2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$ kN-m

$\mathbf{r}_{AB} \times \mathbf{F}_B = -3\mathbf{j} + 6\mathbf{k}$ (kN-m)

$\mathbf{r}_{AC} \times \mathbf{F}_C = 2\mathbf{i} \times (1\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ kN-m

$\mathbf{r}_{AC} \times \mathbf{F}_C = -4\mathbf{j} - 4\mathbf{k}$ (kN-m)

$x: \sum M_A: \quad M_{A_X} = 0$

$y: \sum M_A: \quad M_{A_Y} - 3 - 4 = 0$

$z: \sum M_A: \quad M_{A_Z} + 6 - 4 = 0$

Solving, we get

$A_X = -3$ kN,

$A_Y = -4$ kN,

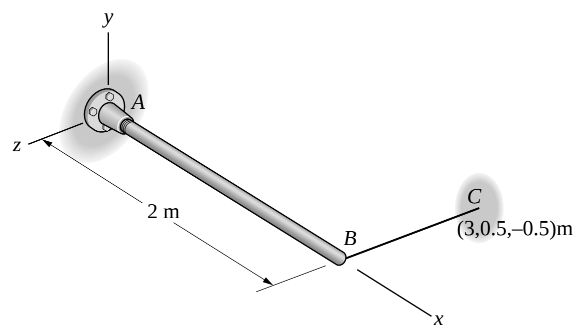
$A_Z = -5$ kN

$M_{Ax} = 0$,

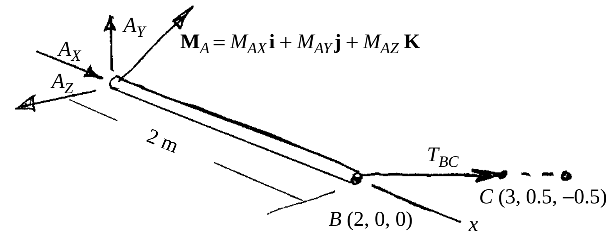
$M_{Ay} = 7$ kN-m,

$M_{Az} = -2$ kN-m

Problem 5.78 The bar AB has a built-in support at A. The tension in cable BC is 8 kN. Determine the reactions at A.



Solution:



$\mathbf{M}_A = M_{Ax}\mathbf{i} + M_{Ay}\mathbf{j} + M_{Az}\mathbf{k}$

We need the unit vector $\mathbf{e}_{BC}$

$$\mathbf{e}_{BC} = \frac{(x_C - x_B)\mathbf{i} + (y_C - y_B)\mathbf{j} + (z_C - z_B)\mathbf{k}}{\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2 + (z_C - z_B)^2}}$$

$\mathbf{e}_{BC} = 0.816\mathbf{i} + 0.408\mathbf{j} - 0.408\mathbf{k}$

$\mathbf{T}_{BC} = (8 \text{ kN})\mathbf{e}_{BC}$

$\mathbf{T}_{BC} = 6.53\mathbf{i} + 3.27\mathbf{j} - 3.27\mathbf{k}$ (kN)

The moment of $\mathbf{T}_{BC}$ about A is

$$\mathbf{M}_{BC} = \mathbf{r}_{AB} \times T_{BC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 6.53 & 3.27 & -3.27 \end{vmatrix}$$

$\mathbf{M}_{BC} = \mathbf{r}_{AB} \times \mathbf{T}_{BC} = 0\mathbf{i} + 6.53\mathbf{j} + 6.53\mathbf{k}$ (kN-m)

Equilibrium Eqns.

$\sum F_X: \quad A_X + T_{BC_X} = 0$

$\sum F_Y: \quad A_Y + T_{BC_Y} = 0$

$\sum F_Z: \quad A_Z + T_{BC_Z} = 0$

$\sum M_X: \quad M_{AX} + M_{BC_X} = 0$

$\sum M_Y: \quad M_{AY} + M_{BC_Y} = 0$

$\sum M_Z: \quad M_{AZ} + M_{BC_Z} = 0$

Solving, we get

$A_X = -6.53$ (kN),

$A_Y = -3.27$ (kN),

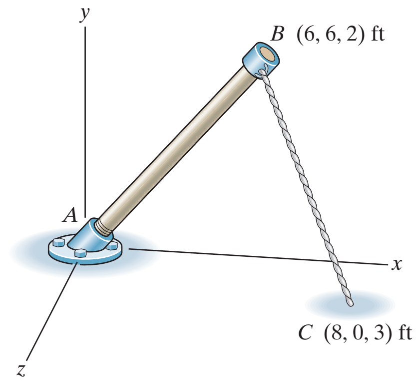
$A_Z = 3.27$ (kN)

$M_{Ax} = 0$,

$M_{Ay} = -6.53$ (kN-m),

$M_{Az} = -6.53$ (kN-m)

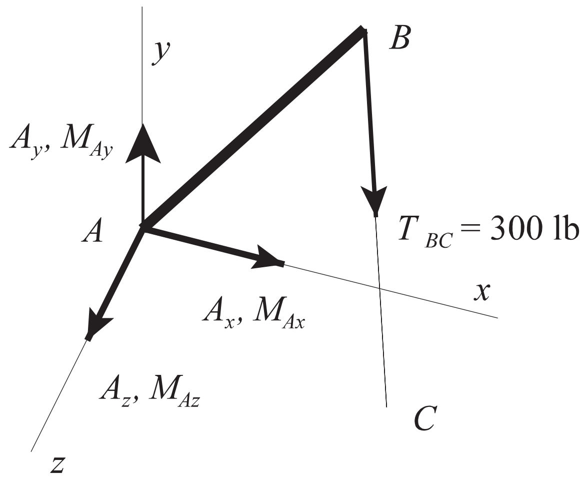
Problem 5.79 The bar AB has a fixed support at A. The collar at B is fixed to the bar. The tension in the rope BC is 300 lb. (a) Draw the free-body diagram of the bar. (b) Determine the reactions at A.



Solution:

(a) The free-body diagram is shown.

(b) We need to express the force exerted by the rope in terms of its components.



The vector from B to C is

$$\mathbf{r}_{BC} = [(8-6)\mathbf{i} + (0-6)\mathbf{j} + (3-2)\mathbf{k}] \text{ ft}$$

$$= (2\mathbf{i} - 6\mathbf{j} + \mathbf{k}) \text{ ft}$$

The force in the rope can now be written

$$\mathbf{T} = T_{BC}\frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = T_{BC}(0.312\mathbf{i} - 0.937\mathbf{j} + 0.156\mathbf{k})$$

The equilibrium equations for the bar are

$$\Sigma F_x : A_x + 0.312 T_{BC} = 0$$

$$\Sigma F_y : A_y - 0.937 T_{BC} = 0$$

$$\Sigma F_z : A_z + 0.156 T_{BC} = 0$$

$$\Sigma \mathbf{M}_A : M_{Ax}\mathbf{i} + M_{Ay}\mathbf{j} + M_{Az}\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 \text{ ft} & 6 \text{ ft} & 2 \text{ ft} \\ 0.312 T_{BC} & -0.937 T_{BC} & 0.156 T_{BC} \end{vmatrix} = 0$$

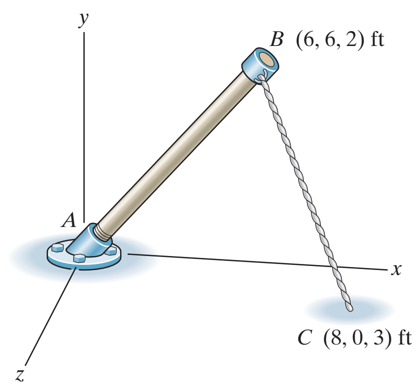
These last equations can be written as

$$M_{Ax} = -(0.218 \text{ ft})T_{BC},\ M_{Ay} = (0.312 \text{ ft})T_{BC},\ M_{Az} = (7.50 \text{ ft})T_{BC}$$

Setting $T_{BC} = 300$ lb, expanding and solving we have

$$\boxed{\begin{array}{l} A_x = -93.7 \text{ lb},\ A_y = 281 \text{ lb},\ A_z = -46.9 \text{ lb} \\ M_{Ax} = -843 \text{ ft-lb},\ M_{Ay} = 93.7 \text{ ft-lb},\ M_{Az} = 2250 \text{ ft-lb} \end{array}}$$

Problem 5.80 The bar AB has a fixed support at A. The collar at B is fixed to the bar. Suppose that you don't want the support at A to be subjected to a couple of magnitude greater than 3000 ft-lb. What is the largest allowable tension in the rope BC?



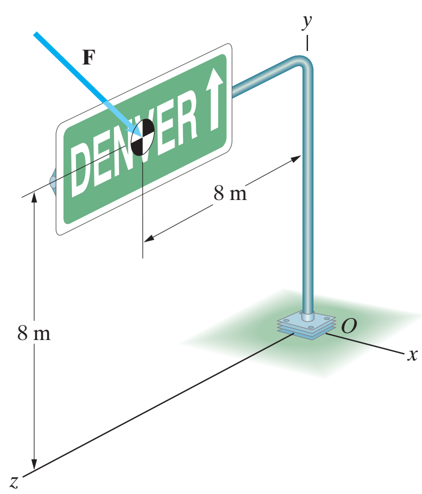
Solution: See the solution to Problem 5.79. The magnitude of the couple at A can be expressed in terms of the tension in the rope as

$$|\mathbf{M}_A| = \sqrt{M_{Ax}{}^2 + M_{Ay}{}^2 + M_{Az}{}^2}$$

$$= \sqrt{(-2.81\text{ ft})^2 + (0.312\text{ ft})^2 + (7.50\text{ ft})^2}\,T_{BC}$$

Setting $|\mathbf{M}_A| = 3000$ ft-lb and solving for T_{BC} yields $\boxed{T_{BC} = 374\text{ lb}}$

Problem 5.81 The total force exerted on the highway sign by its weight and the most severe anticipated winds is $\mathbf{F} = 2.8\mathbf{i} - 1.8\mathbf{j}$ (kN). Determine the reactions at the fixed support.



Solution: The applied load is $\mathbf{F} = (2.8\mathbf{i} - 1.8\mathbf{j})$ kN applied at $\mathbf{r} = (8\mathbf{j} + 8\mathbf{k})$ m

The force reaction at the base is

$$\mathbf{R} = O_x\mathbf{i} + O_y\mathbf{j} + O_z\mathbf{k}$$

The moment reaction at the base is

$$\mathbf{M}_O = M_{Ox}\mathbf{i} + M_{Oy}\mathbf{j} + M_{Oz}\mathbf{k}$$

For equilibrium we need

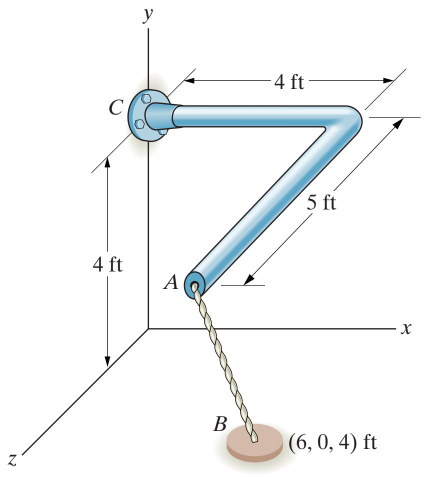
$$\sum \mathbf{F} = \mathbf{F} + \mathbf{R} = 0 \Rightarrow \begin{cases} \sum F_x : 2.8\text{ kN} + O_x = 0 \\ \sum F_y : -1.8\text{ kN} + O_y = 0 \\ \sum F_z : 0 + O_z = 0 \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} O_x = -2.8\text{ kN} \\ O_y = 1.8\text{ kN} \\ O_z = 0 \end{array}}$$

$$\sum \mathbf{M} = \mathbf{r} \times \mathbf{F} + \mathbf{M}_O = 0 \Rightarrow \begin{cases} \sum M_x : 14.4\text{ kN-m} + M_{Ox} = 0 \\ \sum M_y : 22.4\text{ kN-m} + M_{Oy} = 0 \\ \sum M_z : -22.4\text{ kN-m} + M_{Oz} = 0 \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} M_{Ox} = -14.4\text{ kN-m} \\ M_{Oy} = -22.4\text{ kN-m} \\ M_{Oz} = 22.4\text{ kN-m} \end{array}}$$

Problem 5.82 The tension in cable AB is 800 lb. Determine the reactions at the fixed support C.



Solution: The force in the cable is

$$\mathbf{F} = 800 \text{ lb} \left(\frac{2\mathbf{i} - 4\mathbf{j} - \mathbf{k}}{\sqrt{21}} \right)$$

We also have the position vector

$$\mathbf{r}_{CA} = (4\mathbf{i} + 5\mathbf{k}) \text{ ft}$$

The force reaction at the base is

$$\mathbf{R} = C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k}$$

The moment reaction at the base is

$$\mathbf{M}_C = M_{Cx}\mathbf{i} + M_{Cy}\mathbf{j} + M_{Cz}\mathbf{k}$$

For equilibrium we need

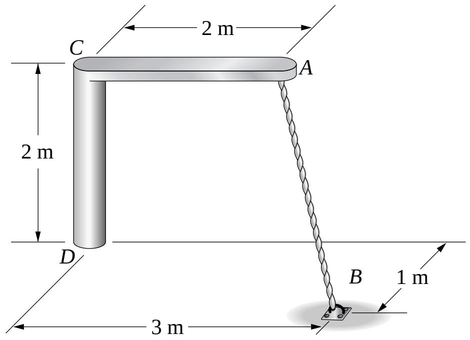
$$\sum \mathbf{F} = \mathbf{F} + \mathbf{R} = 0 \Rightarrow \begin{cases} \sum F_x : C_x + 349 \text{ lb} = 0 \\ \sum F_y : C_y - 698 \text{ lb} = 0 \\ \sum F_z : C_z - 175 \text{ lb} = 0 \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} C_x = -349 \text{ lb} \\ C_y = 698 \text{ lb} \\ C_z = 175 \text{ lb} \end{array}}$$

$$\sum \mathbf{M} = \mathbf{r} + \mathbf{F} + \mathbf{R} = 0 \Rightarrow \begin{cases} \sum M_x : M_{Cx} + 3490 \text{ ft-lb} = 0 \\ \sum M_y : M_{Cy} + 2440 \text{ ft-lb} = 0 \\ \sum M_z : M_{Cz} - 2790 \text{ ft-lb} = 0 \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} M_{Cx} = -3490 \text{ ft-lb} \\ M_{Cy} = -2440 \text{ ft-lb} \\ M_{Cz} = 2790 \text{ ft-lb} \end{array}}$$

Problem 5.83 The tension in cable AB is 24 kN. Determine the reactions in the built-in support D.



Solution: The force acting on the device is

$$\mathbf{F} = F_X\mathbf{i} + F_Y\mathbf{j} + F_Z\mathbf{k} = (24 \text{ kN})\mathbf{e}_{AB},$$

and the unit vector from A toward B is given by

$$\mathbf{e}_{AB} = \frac{1\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}}{\sqrt{6}}.$$

The force, then, is given by

$$\mathbf{F} = 9.80\mathbf{i} - 19.60\mathbf{j} + 9.80\mathbf{k} \text{ kN}.$$

The position from D to A is

$$\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + 0\mathbf{k} \text{ m}.$$

The force equations of equilibrium are

$$D_X + F_X = 0,$$

$$D_Y + F_Y = 0,$$

and $D_Z + F_Z = 0.$

The moment equation, in vector form, is

$$\sum \mathbf{M} = \mathbf{M}_D + \mathbf{r} \times \mathbf{F}.$$

Expanded, we get

$$\sum \mathbf{M} = M_{DX}\mathbf{i} + M_{DY}\mathbf{j} + M_{DZ}\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 0 \\ 9.80 & -19.60 & 9.80 \end{vmatrix} = 0.$$

The corresponding scalar equations are

$$M_{DX} + (2)(9.80) = 0,$$

$$M_{DY} - (2)(9.80) = 0,$$

and $M_{DZ} + (2)(-19.60) - (2)(9.80) = 0.$

Solving for the support reactions, we get

$$D_X = -9.80 \text{ kN},$$

$$O_Y = 19.60 \text{ kN},$$

$$O_Z = -9.80 \text{ kN}.$$

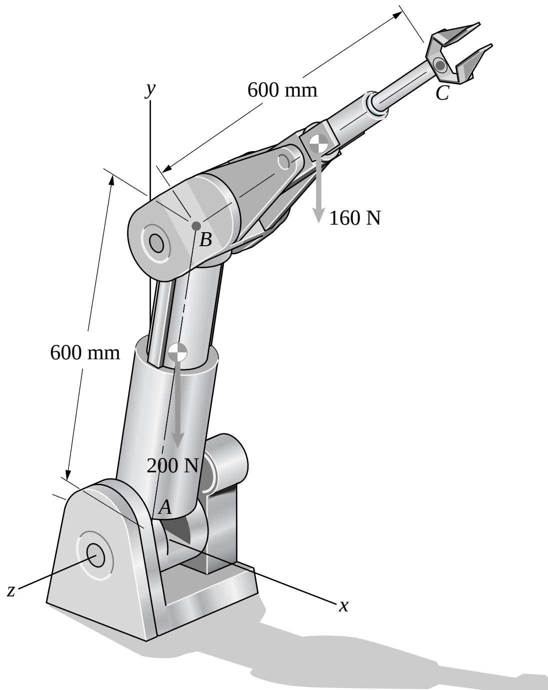
$$M_{DX} = -19.6 \text{ kN-m},$$

$$M_{DY} = 19.6 \text{ kN-m},$$

and $M_{DZ} = 58.8$ kN-m.

Problem 5.84 The robotic manipulator is stationary and the y axis is vertical. The weights of the arms AB and BC act at their midpoints. The direction cosines of the centerline of arm AB are $\cos\theta_x = 0.174$, $\cos\theta_y = 0.985$, $\cos\theta_z = 0$, and the direction cosines of the centerline of arm BC are $\cos\theta_x = 0.743$, $\cos\theta_y = 0.557$, $\cos\theta_z = -0.371$. The support at A behaves like a built-in support.

(a) What is the sum of the moments about A due to the weights of the two arms?
(b) What are the reactions at A?



Solution: Denote the center of mass of arm AB as D_1 and that of BC as D_2. We need

$\mathbf{r}_{AD}$,

$\mathbf{r}_{AB}$,

and $\mathbf{r}_{BD_2}$.

To get these, use the direction cosines to get the unit vectors $\mathbf{e}_{AB}$ and $\mathbf{e}_{BC}$. Use the relation

$$\mathbf{e} = \cos\theta_X \mathbf{i} + \cos\theta_Y \mathbf{j} + \cos\theta_Z \mathbf{k}$$

$$\mathbf{e}_{AB} = 0.174\mathbf{i} + 0.985\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{e}_{BC} = 0.743\mathbf{i} + 0.557\mathbf{j} - 0.371\mathbf{k}$$

$$\mathbf{r}_{AD_1} = 0.3\mathbf{e}_{AB} \text{ m}$$

$$\mathbf{r}_{AB} = 0.6\mathbf{e}_{AB} \text{ m}$$

$$\mathbf{r}_{BC} = 0.6\mathbf{e}_{BC} \text{ m}$$

$$\mathbf{r}_{BD_2} = 0.3\mathbf{e}_{BC} \text{ m}$$

$$\mathbf{W}_{AB} = -200\mathbf{j} \text{ N}$$

$$\mathbf{W}_{BC} = -160\mathbf{j} \text{ N}$$

Thus $\mathbf{r}_{AD_1} = 0.0522\mathbf{i} + 0.2955\mathbf{j}$ m

$$\mathbf{r}_{AB} = 0.1044\mathbf{i} + 0.5910\mathbf{j} \text{ m}$$

$$\mathbf{r}_{BD_2} = 0.2229\mathbf{i} + 0.1671\mathbf{j} - 0.1113\mathbf{k} \text{ m}$$

$$\mathbf{r}_{BC} = 0.4458\mathbf{i} + 0.3342\mathbf{j} - 0.2226\mathbf{k} \text{ m}$$

and $\mathbf{r}_{AD_2} = \mathbf{r}_{AB} + \mathbf{r}_{BD_2}$

$$\mathbf{r}_{AD_2} = 0.3273\mathbf{i} + 0.7581\mathbf{j} - 0.1113\mathbf{k} \text{ m}$$

(a) We now have the geometry determined and are ready to determine the moments of the weights about A.

$$\sum \mathbf{M}_W = \mathbf{r}_{AD1} \times \mathbf{W}_1 + \mathbf{r}_{AD2} \times \mathbf{W}_2$$

where

$$\mathbf{r}_{AD1} \times \mathbf{W}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.0522 & 0.2955 & 0 \\ 0 & -200 & 0 \end{vmatrix}$$

$$\mathbf{r}_{AD1} \times \mathbf{W}_1 = -10.44\mathbf{k} \text{ N-m}$$

and

$$\mathbf{r}_{AD2} \times \mathbf{W}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3273 & 0.7581 & -0.1113 \\ 0 & -160 & 0 \end{vmatrix}$$

$$\mathbf{r}_{AD2} \times \mathbf{W}_2 = -17.81\mathbf{i} - 52.37\mathbf{k}$$

Thus,

$$\sum \mathbf{M}_W = -17.81\mathbf{i} - 62.81\mathbf{k} \text{ (N-m)}$$

(b) Equilibrium Eqns

$$\sum F_X: \quad A_X = 0$$

$$\sum F_Y: \quad A_Y - W_1 - W_2 = 0$$

$$\sum F_Z: \quad A_Z = 0$$

Sum Moments about

$$A: \mathbf{M}_A + \sum \mathbf{M}_W = 0$$

$$\sum M_X: \quad M_{Ax} - 17.81 = 0 \text{ (N-m)}$$

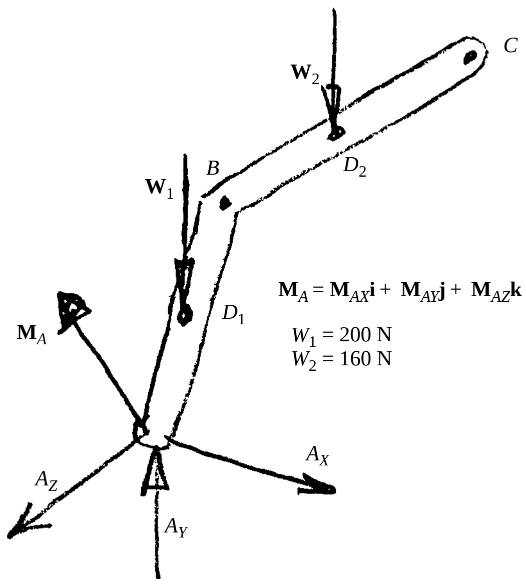
$$\sum M_Y: \quad M_{Ay} + 0 = 0$$

$$\sum M_Z: \quad M_Z - 62.81 = 0 \text{ (N-m)}$$

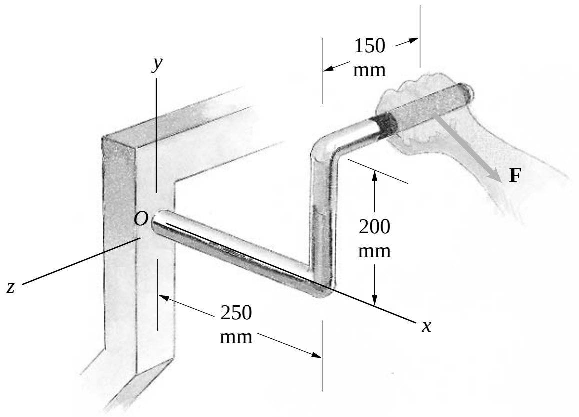
5.84 (*Continued*)

Thus: $A_X = 0,\ A_Y = 360\ \text{(N)},\ A_Z = 0,$

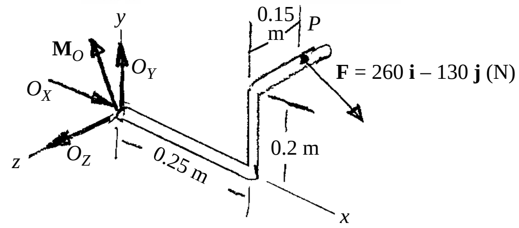
$M_{Ax} = 17.81\ \text{(N-m)},\ M_{Ay} = 0,\ M_{Az} = 62.81\ \text{(N-m)}$



Problem 5.85 The force exerted on the grip of the exercise machine is $\mathbf{F} = 260\mathbf{i} - 130\mathbf{j}$ (N). What are the reactions at the built-in support at O?



Solution:



$\mathbf{M}_O = M_{Ox}\mathbf{i} + M_{Oy}\mathbf{j} + M_{Oz}\mathbf{k}$

$\mathbf{r}_{OP} = 0.25\mathbf{i} + 0.2\mathbf{j} - 0.15\mathbf{k}$

Equilibrium (Forces)

$\sum F_X$: $O_X + F_X = O_X + 260 = 0$ (N)

$\sum F_Y$: $O_Y + F_Y = O_Y - 130 = 0$ (N)

$\sum F_Z$: $O_Z + F_Z = O_Z = 0$ (N)

Thus, $O_X = -260$ N, $O_Y = 130$ N, $O_Z = 0$

Summing Moments about O

$\sum M_X$: $M_{O_X} + M_{F_X} = 0$

$\sum M_Y$: $M_{O_Y} + M_{F_Y} = 0$

$\sum M_Z$: $M_{O_Z} + M_{F_Z} = 0$

where

$$\mathbf{M}_F = \mathbf{r}_{OP} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2 & -0.15 \\ 260 & -130 & 0 \end{vmatrix}$$

$\mathbf{M}_F = -19.5\mathbf{i} - 39\mathbf{j} - 84.5\mathbf{k}$ (N-m)

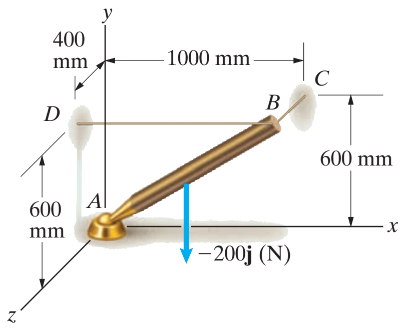
and from the moment equilibrium eqns,

$M_{O_X} = 19.5$ (N-m)

$M_{O_Y} = 39.0$ (N-m)

$M_{O_Z} = 84.5$ (N-m)

Problem 5.86 In Active Example 5.7, suppose that cable *BD* is lengthened and the attachment point *D* moved form (0, 600, 400) mm to (0, 600, 600) mm. (The end *B* of bar *AB* remains where it is.) Draw a sketch of the bar and its supports showing cable *BD* in its new position. Draw the free-body diagram of the bar and apply equilibrium to determine the tensions in the cables and the reactions at *A*.



Solution: The sketch and free-body diagram are shown.

We must express the force exerted on the bar by cable *BD* in terms of its components. The vector from *B* to *D* is

$$\mathbf{r}_{BD} = [(0-1000)\mathbf{i} + (600-600)\mathbf{j} + (600-400)\mathbf{k}]\text{ mm} = (-1000\mathbf{i} + 200\mathbf{j})\text{ mm}$$

The force exerted by cable *BD* can be expressed as

$$T_{BD}\frac{\mathbf{r}_{BD}}{|\mathbf{r}_{BD}|} = T_{BD}(-0.981\mathbf{i} + 0.196\mathbf{k})$$

The equilibrium equations are

$$\Sigma F_x : A_x - 0.981 T_{BD} = 0$$

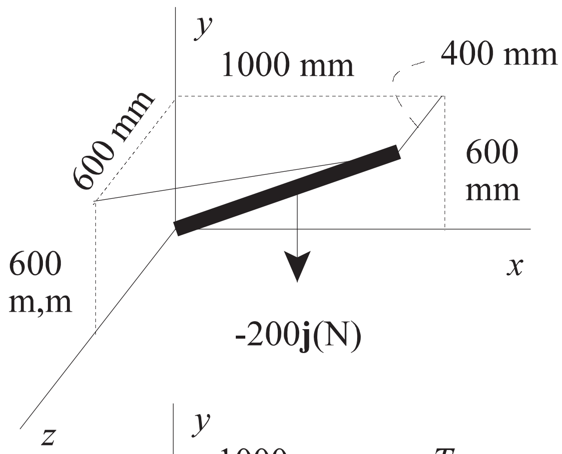
$$\Sigma F_y : A_y - 200\text{ N} = 0$$

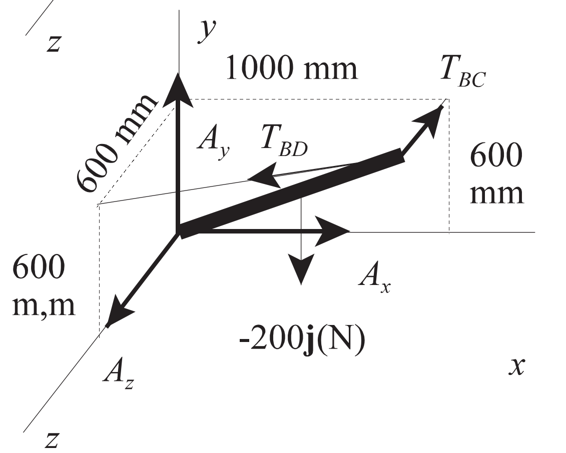
$$\Sigma F_z : A_z + 0.196 T_{BD} - T_{BC} = 0$$

$$\Sigma \mathbf{M}_A : \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0.6 & 0.4 \\ -0.981T_{BD} & 0 & 0.196T_{BD} - T_{BC} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.5 & 0.3 & 0.2 \\ 0 & -200 & 0 \end{vmatrix} = 0$$

Expanding and solving these equations, we find

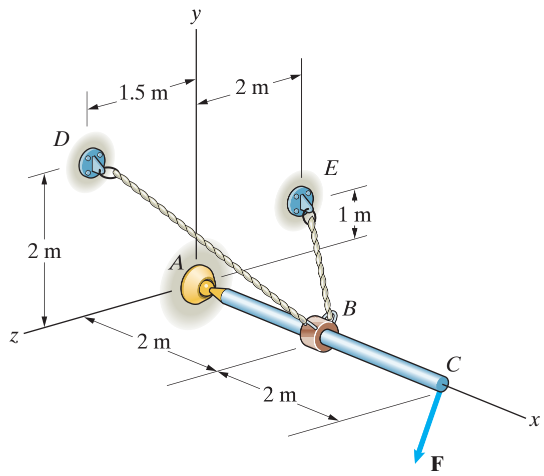
$$A_x = 166.7\text{ N}, A_y = 200\text{ N}, A_z = 66.7\text{ N}, T_{BC} = 100\text{ N},$$
$$T_{BD} = 170\text{ N}$$





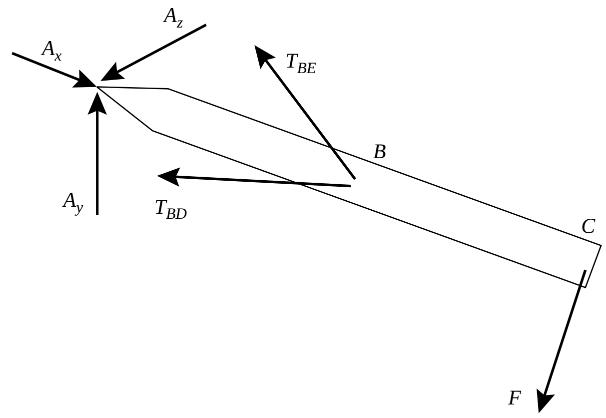
Problem 5.87 The force **F** acting on the boom *ABC* at *C* points in the direction of the unit vector $0.512\mathbf{i} - 0.384\mathbf{j} + 0.768\mathbf{k}$ and its magnitude is 8 kN. The boom is supported by a ball and socket at *A* and the cables *BD* and *BE*. The collar at *B* is fixed to the boom.

(a) Draw the free-body diagram of the boom.
(b) Determine the tensions in the cables and the reactions at *A*.



Solution:

(a) The free-body diagram



(b) We identify the following forces, position vectors, and reactions

$$\mathbf{r}_{AC} = 4 \text{ mi}, \ \mathbf{F} = 8 \text{ kN}(0.512\mathbf{i} - 0.384\mathbf{j} + 0.768\mathbf{k})$$

$$\mathbf{r}_{AB} = 2 \text{ mi}, \begin{cases} \mathbf{T}_{BD} = T_{BD}\left(\dfrac{-2\mathbf{i} + 2\mathbf{j} + 1.5\mathbf{k}}{\sqrt{10.25}}\right) \\ \mathbf{T}_{BE} = T_{BE}\left(\dfrac{-2\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{3}\right) \end{cases}$$

$$\mathbf{R} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Force equilibrium requires:

$$\sum \mathbf{F} = \mathbf{R} + \mathbf{T}_{BD} + \mathbf{T}_{BE} + \mathbf{F} = 0.$$

In component form we have

$$\sum F_x : A_x + 8 \text{ kN}(0.512) - \frac{2}{\sqrt{10.25}}T_{BD} - \frac{2}{3}T_{BE} = 0$$

$$\sum F_y : A_y - 8 \text{ kN}(0.384) + \frac{2}{\sqrt{10.25}}T_{BD} + \frac{1}{3}T_{BE} = 0$$

$$\sum F_z : A_z + 8 \text{ kN}(0.768) + \frac{1.5}{\sqrt{10.25}}T_{BD} - \frac{2}{3}T_{BE} = 0$$

Moment equilibrium requires:

$$\sum \mathbf{M}_A = \mathbf{r}_{AB} \times (\mathbf{T}_{BD} + \mathbf{T}_{BE}) + \mathbf{r}_{AC} \times \mathbf{F} = 0.$$

In components:

$$\sum M_x : 0 = 0$$

$$\sum M_y : -8 \text{ kN}(0.768)(4 \text{ m}) - \frac{1.5}{\sqrt{10.25}}T_{BD}(2 \text{ m}) + \frac{2}{3}T_{BE}(2 \text{ m}) = 0$$

$$\sum M_z : -8 \text{ kN}(0.384)(4 \text{ m}) + \frac{2}{\sqrt{10.25}}T_{BD}(2 \text{ m}) + \frac{1}{3}T_{BE}(2 \text{ m}) = 0$$

Solving five equations for the five unknowns we find

$$\boxed{A_x = 8.19 \text{ kN}, \ A_y = -3.07 \text{ kN}, \ A_z = 6.14 \text{ kN}, \ T_{BD} = 0, \ T_{BE} = 18.43 \text{ kN}}$$

Problem 5.88 The cables *BD* and *BE* in Problem 5.87 will each safely support a tension of 25 kN. Based on this criterion, what is the largest acceptable magnitude of the force **F**?

Solution: We have the force and distances:

$$\mathbf{r}_{AC} = 4 \text{ m}\mathbf{i}, \ \mathbf{F} = F(0.512\mathbf{i} - 0.384\mathbf{j} + 0.768\mathbf{k})$$

$$\mathbf{r}_{AB} = 2 \text{ m}\mathbf{i}, \ \begin{cases} \mathbf{T}_{BD} = T_{BD}\left(\dfrac{-2\mathbf{i} + 2\mathbf{j} + 1.5\mathbf{k}}{\sqrt{10.25}}\right) \\ \mathbf{T}_{BE} = T_{BE}\left(\dfrac{-2\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{3}\right) \end{cases}$$

The moment equations are

$$\sum M_y : -F(0.768)(4 \text{ m}) - \frac{1.5}{\sqrt{10.25}}T_{BD}(2 \text{ m}) + \frac{2}{3}T_{BE}(2 \text{ m}) = 0$$

$$\sum M_z : -F(0.384)(4 \text{ m}) + \frac{2}{\sqrt{10.25}}T_{BD}(2 \text{ m}) + \frac{1}{3}T_{BE}(2 \text{ m}) = 0$$

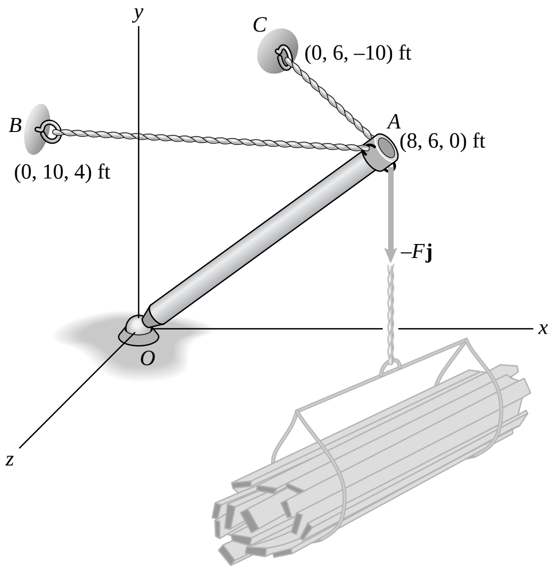
Solving we find

$$T_{BE} = 2.304F, \ T_{BD} = 0$$

Thus:

$$\boxed{25 \text{ kN} = 2.304F \Rightarrow F = 10.85 \text{ kN}}$$

Problem 5.89 The suspended load exerts a force $F = 600$ lb at A, and the weight of the bar OA is negligible. Determine the tensions in the cables and the reactions at the ball and socket support O.



Solution: From the diagram, the important points in this problem are A (8, 6, 0), B (0, 10, 4), C (0, 6, −10), and the origin O (0, 0, 0) with all dimensions in ft. We need unit vectors in the directions A to B and A to C. Both vectors are of the form

$$\mathbf{e}_{AP} = (x_P - x_A)\mathbf{i} + (y_P - y_A)\mathbf{j} + (z_P - z_A)\mathbf{k},$$

where P can be either A or B. The forces in cables AB and AC are

$$\mathbf{T}_{AB} = T_{AB}\,\mathbf{e}_{AB} = T_{ABX}\mathbf{i} + T_{ABY}\mathbf{j} + T_{ABZ}\mathbf{k},$$

and $\mathbf{T}_{AC} = T_{AC}\,\mathbf{e}_{AB} = T_{ACX}\mathbf{i} + T_{ACY}\mathbf{j} + T_{ACZ}\mathbf{k}$.

The weight force is

$$\mathbf{F} = 0\mathbf{i} - 600\mathbf{j} + 0\mathbf{k},$$

and the support force at the ball joint is

$$\mathbf{S} = S_X\mathbf{i} + S_Y\mathbf{j} + S_Z\mathbf{k}.$$

The vector form of the force equilibrium equation (which gives three scalar equations) for the bar is

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{F} + \mathbf{S} = 0.$$

Let us take moments about the origin. The moment equation, in vector form, is given by

$$\sum \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T}_{AB} + \mathbf{r}_{OA} \times \mathbf{T}_{AC} + \mathbf{r}_{OA} \times \mathbf{F} = \mathbf{0},$$

where $\mathbf{r}_{OA} = 8\mathbf{i} + 6\mathbf{j} + 0\mathbf{k}$.

The cross products are evaluated using the form

$$\mathbf{M} = \mathbf{r} \times \mathbf{H} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 6 & 0 \\ H_X & H_Y & H_Z \end{vmatrix},$$

where $\mathbf{H}$ can be any of the three forces acting at point A. The vector moment equation provides another three equations of equilibrium. Once we have evaluated and applied the unit vectors, we have six vector equations of equilibrium in the five unknowns T_{AB}, T_{AC}, S_X, S_Y, and S_Z (there is one redundant equation since all forces pass through the line OA). Solving these equations yields the required values for the support reactions at the origin.

If we carry through these operations in the sequence described, we get the following vectors:

$$\mathbf{e}_{AB} = -0.816\mathbf{i} + 0.408\mathbf{j} + 0.408\mathbf{k},$$

$$\mathbf{e}_{AC} = -0.625\mathbf{i} + 0\mathbf{j} - 0.781\mathbf{k},$$

$$\mathbf{T}_{AB} = -387.1\mathbf{i} + 193.5\mathbf{j} + 193.5\mathbf{k} \text{ lb},$$

$$|\mathbf{T}_{AB}| = 474.1 \text{ lb},$$

$$\mathbf{T}_{AC} = -154.8\mathbf{i} + 0\mathbf{j} - 193.5\mathbf{k} \text{ lb},$$

$$|\mathbf{T}_{AC}| = 247.9 \text{ lb},$$

$$\mathbf{M}_{AB} = \mathbf{r}_{OA} \times \mathbf{T}_{AB} = 1161\mathbf{i} - 1548\mathbf{j} + 3871\mathbf{k} \text{ ft-lb},$$

$$\mathbf{M}_{AC} = \mathbf{r}_{OA} \times \mathbf{T}_{AC} = -1161\mathbf{i} + 1548\mathbf{j} + 929\mathbf{k} \text{ ft-lb},$$

and $\mathbf{S} = 541.9\mathbf{i} + 406.5\mathbf{j} + 0\mathbf{k}$ lb

Problem 5.90 In Problem 5.89, suppose that the suspended load exerts a force $F = 600$ lb at A and bar OA weighs 200 lb. Assume that the bar's weight acts at its midpoint. Determine the tensions in the cables and the reactions at the ball and socket support O.

Solution: Point G is located at (4, 3, 0) and the position vector of G with respect to the origin is

$$\mathbf{r}_{OG} = 4\mathbf{i} + 3\mathbf{j} + 0\mathbf{k} \text{ ft.}$$

The weight of the bar is

$$\mathbf{W}_B = 0\mathbf{i} - 200\mathbf{j} + 0\mathbf{k} \text{ lb,}$$

and its moment around the origin is

$$\mathbf{M}_{WB} = 0\mathbf{i} + 0\mathbf{j} - 800\mathbf{k} \text{ ft-lb.}$$

The mathematical representation for all other forces and moments from Problem 5.89 remain the same (the numbers change!). Each equation of equilibrium has a new term reflecting the addition of the weight of the bar. The new force equilibrium equation is

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{F} + \mathbf{S} + \mathbf{W}_B = 0.$$

The new moment equilibrium equation is

$$\sum \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T}_{AB} + \mathbf{r}_{OA} \times \mathbf{T}_{AC}$$

$$+ \mathbf{r}_{OA} \times \mathbf{F} + \mathbf{r}_{OG} \times \mathbf{W}_B = 0.$$

As in Problem 5.89, the vector equilibrium conditions can be reduced to six scalar equations of equilibrium. Once we have evaluated and applied the unit vectors, we have six vector equations of equilibrium in the five unknowns T_{AB}, T_{AC}, S_X, S_Y, and S_Z (As before, there is one redundant equation since all forces pass through the line OA). Solving these equations yields the required values for the support reactions at the origin.

If we carry through these operations in the sequence described, we get the following vectors:

$$\mathbf{e}_{AB} = -0.816\mathbf{i} + 0.408\mathbf{j} + 0.408\mathbf{k},$$

$$\mathbf{e}_{AC} = -0.625\mathbf{i} + 0\mathbf{j} - 0.781\mathbf{k},$$

$$\mathbf{T}_{AB} = -451.6\mathbf{i} + 225.8\mathbf{j} + 225.8\mathbf{k} \text{ lb,}$$

$$|\mathbf{T}_{AB}| = 553.1 \text{ lb,}$$

$$\mathbf{T}_{AC} = -180.6\mathbf{i} + 0\mathbf{j} - 225.8\mathbf{k} \text{ lb,}$$

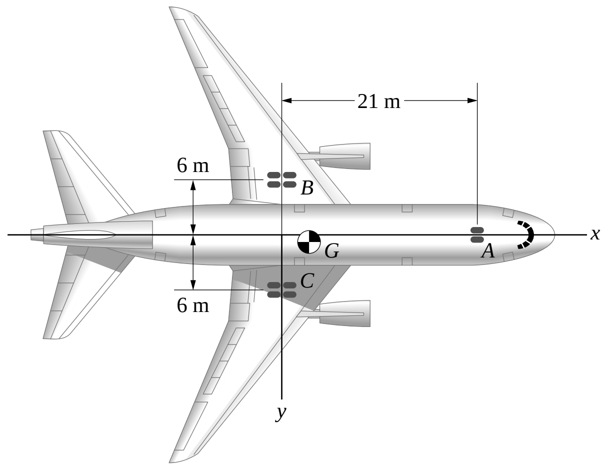
$$|\mathbf{T}_{AC}| = 289.2 \text{ lb,}$$

$$\mathbf{M}_{AB} = \mathbf{r}_{OA} \times \mathbf{T}_{AB} = 1355\mathbf{i} - 1806\mathbf{j} + 4516\mathbf{k} \text{ ft-lb,}$$

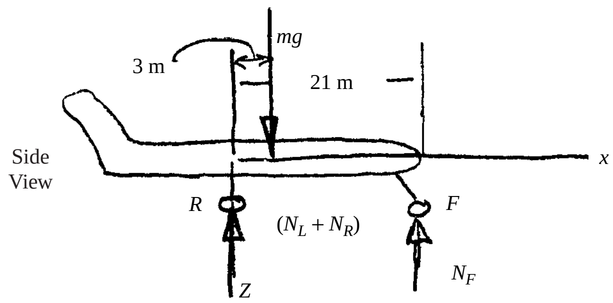
$$\mathbf{M}_{AC} = \mathbf{r}_{OA} \times \mathbf{T}_{AC} = -1354\mathbf{i} + 1806\mathbf{j} + 1084\mathbf{k} \text{ ft-lb,}$$

and $\mathbf{S} = 632.3\mathbf{i} + 574.2\mathbf{j} + 0\mathbf{k}$ lb

Problem 5.91 The 158,000-kg airplane is at rest on the ground ($z = 0$ is ground level). The landing gear carriages are at A, B, and C. The coordinates of the point G at which the weight of the plane acts are (3, 0.5, 5) m. What are the magnitudes of the normal reactions exerted on the landing gear by the ground?



Solution:



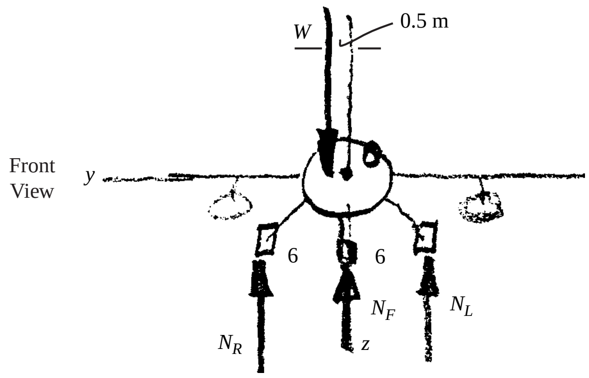
$$\sum F_Y = (N_L + N_R) + N_F - W = 0$$

$$\sum M_R = -3\ \text{mg} + 21 N_F = 0$$

Solving,

$$N_F = 221.4\ \text{kN} \quad \textbf{(1)}$$

$$(N_L + N_R) = 1328.6\ \text{kN} \quad \textbf{(2)}$$



$$\sum F_Y = N_R + N_L + N_F - W = 0$$

(same equation as before)

$$\circlearrowleft + \sum M_O = 0.5\ \text{W} - 6(N_R) + 6(N_L) = 0 \quad \textbf{(3)}$$

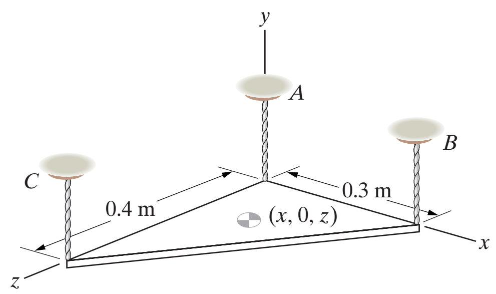
Solving (1), (2), and (3), we get

$N_F = 221.4$ kN

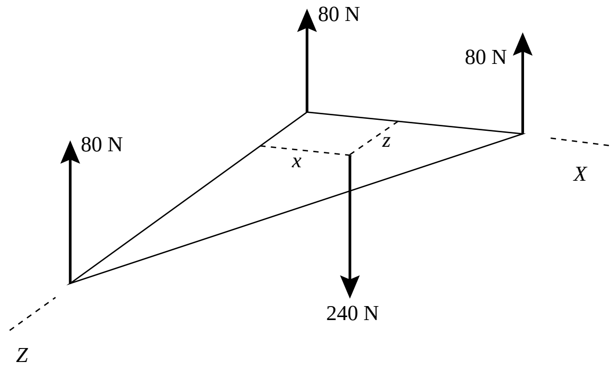
$N_R = 728.9$ kN

$N_L = 599.7$ kN

Problem 5.92 The horizontal triangular plate is suspended by the three vertical cables A, B, and C. The tension in each cable is 80 N. Determine the x and z coordinates of the point where the plate's weight effectively acts.



Solution:

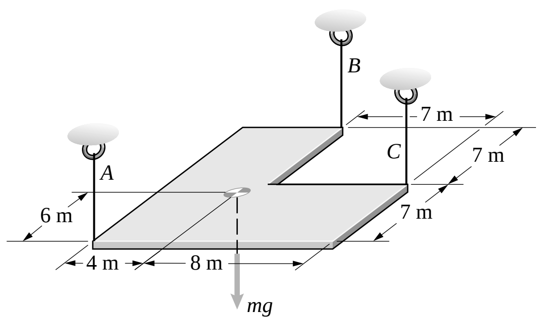


$\sum M_x : (240\text{ N})z - (80\text{ N})(0.4\text{ m}) = 0$

$\sum M_z : (80\text{ N})(0.3\text{ m}) - (240\text{ N})x = 0$

Solving $\boxed{x = 0.1\text{ m},\ z = 0.1333\text{ m}}$

Problem 5.93 The 800-kg horizontal wall section is supported by the three vertical cables, A, B, and C. What are the tensions in the cables?



Solution: All dimensions are in m and all forces are in N. Forces A, B, C, and W act on the wall at (0, 0, 0), (5, 14, 0), (12, 7, 0), and (4, 6, 0), respectively. All forces are in the z direction. The force equilibrium equation in the z direction is $A + B + C - W = 0$. The moments are calculated from

$$\mathbf{M}_B = \mathbf{r}_{OB} \times B\mathbf{k},$$

$$\mathbf{M}_C = \mathbf{r}_{OC} \times C\mathbf{k},$$

and $\mathbf{M}_G = \mathbf{r}_{OG} \times (-W)\mathbf{k}$.

The moment equilibrium equation is

$$\sum \mathbf{M}_O = \mathbf{M}_B + \mathbf{M}_C + \mathbf{M}_G = 0.$$

Carrying out these operations, we get

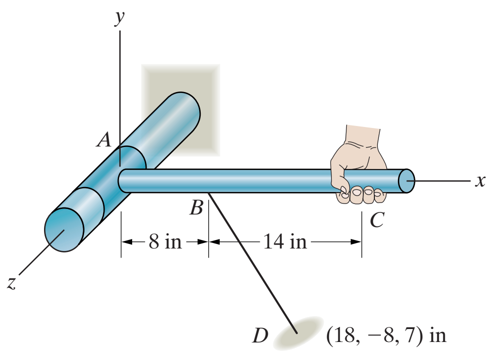
$$A = 3717\text{ N},$$

$$B = 2596\text{ N},$$

$$C = 1534\text{ N},$$

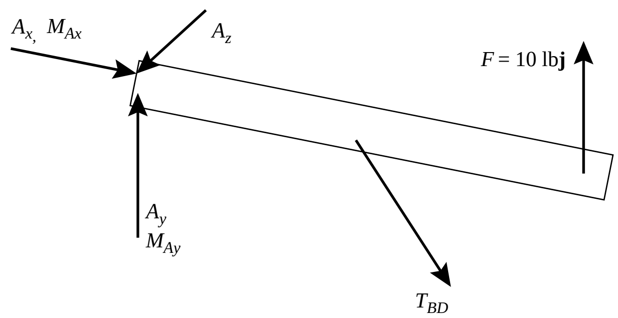
and $W = 7848$ N.

Problem 5.94 The bar AC is supported by the cable BD and a bearing at A that can rotate about the z axis. The person exerts a force $\mathbf{F} = 10\mathbf{j}$ (lb) at C. Determine the tension in the cable and the reactions at A.



Solution: The force in the cable is

$$\mathbf{T}_{BD} = T_{BD}\left(\frac{10\mathbf{i} - 8\mathbf{j} + 7\mathbf{k}}{\sqrt{213}}\right)$$



We have the following six equilibrium equations

$$\sum F_x : A_x + \frac{10}{\sqrt{213}} T_{BD} = 0$$

$$\sum F_y : A_y - \frac{8}{\sqrt{213}} T_{BD} + 10 \text{ lb} = 0$$

$$\sum F_z : A_z + \frac{7}{\sqrt{213}} T_{BD} = 0$$

$$\sum M_x : M_{Ax} = 0$$

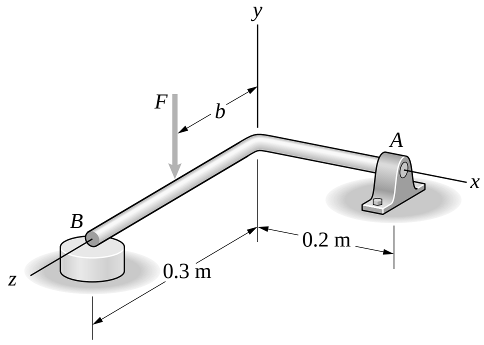
$$\sum M_y : M_{Ay} - \frac{7}{\sqrt{213}} T_{BD}(8 \text{ in}) = 0$$

$$\sum M_z : \frac{8}{\sqrt{213}} T_{BD}(8 \text{ in}) + (10 \text{ lb})(22 \text{ in}) = 0$$

Solving we find

$$\boxed{\begin{array}{l} A_x = -34.4 \text{ lb},\ A_y = 17.5 \text{ lb},\ A_z = -24.1 \text{ lb} \\ M_{Ax} = 0,\ M_{Ay} = 192.5 \text{ lb in},\ T_{BD} = 50.2 \text{ lb} \end{array}}$$

Problem 5.95 The L-shaped bar is supported by a bearing at A and rests on a smooth horizontal surface at B. The vertical force $F = 4$ kN and the distance $b = 0.15$ m. Determine the reactions at A and B.



Solution: *Equilibrium Eqns*:

$\sum F_X$: $O = 0$

$\sum F_Y$: $A_Y + B - F = 0$

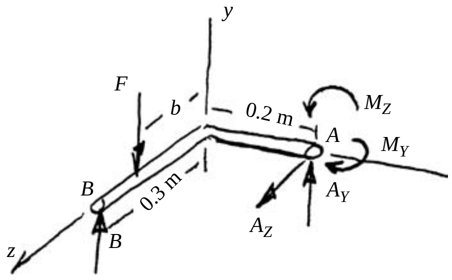
$\sum F_Z$: $A_Z = 0$

Sum moments around A

x: $Fb - 0.3B = (4)(0.15) - 0.3B = 0$

y: $M_{A_Y} = 0$

z: $M_{A_Z} + 0.2F - 0.2B = 0$



Solving,

$A_X = 0$,

$A_Y = 2$ (kN),

$A_Z = 0$

$M_{A_X} = 0$,

$M_{A_Y} = 0$,

$M_{A_Z} = -0.4$ (kN-m)

Problem 5.96 In Problem 5.95, the vertical force $F = 4$ kN and the distance $b = 0.15$ m. If you represent the reactions at A and B by an equivalent system consisting of a single force, what is the force and where does its line of action intersect the $x-z$ plane?

Solution: We want to represent the forces at A & B by a single force. From Prob. 5.95

$\mathbf{A} = +2\mathbf{j}$ (kN),

$\mathbf{B} = +2\mathbf{j}$ (kN)

$\mathbf{M}_A = -0.4\mathbf{k}$ (kN-m)

We want a single equivalent force, $\mathbf{R}$ that has the same resultant force and moment about A as does the set $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{M}_A$.

$\mathbf{R} = \mathbf{A} + \mathbf{B} = 4\mathbf{j}$ (kN)

Let $\mathbf{R}$ pierce the $x-z$ plane at (x_R, z_R)

$\sum M_X$: $-z_R R = -0.3B$

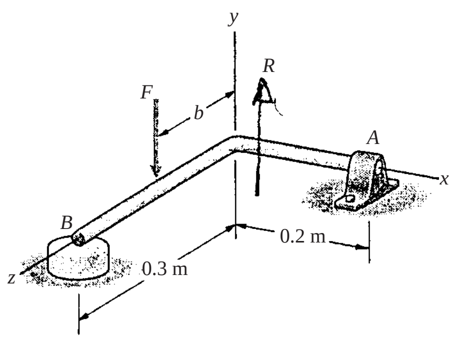
$\sum M_Z$: $-x_R R = 0.2A_Y$

$z_R(4) = (+0.3)(2)$

$z_R = +0.15$ m

$x_R(4) = 0.2(2)$

$x_R = 0.1$ m



Problem 5.97 In Problem 5.95, the vertical force $F =$ 4 kN. The bearing at A will safely support a force of 2.5-kN magnitude and a couple of 0.5 kN-m magnitude. Based on these criteria, what is the allowable range of the distance b?

Solution: The solution to Prob. 5.95 produced the relations

$$A_Y + B - F = 0 \quad (F = 4\ kN)$$

$$Fb - 0.3B = 0$$

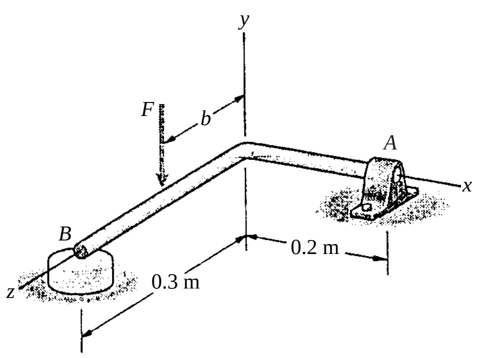
$$M_{A_Z} + 0.2F - 0.2B = 0$$

$$A_X = A_Z = M_{A_X} = M_{A_Y} = 0$$

Set the force at A to its limit of 2.5 kN and solve for b. In this case, $M_{A_Z} = -0.5$ (kN-m) which is at the moment limit. The value for b is $b = 0.1125$ m

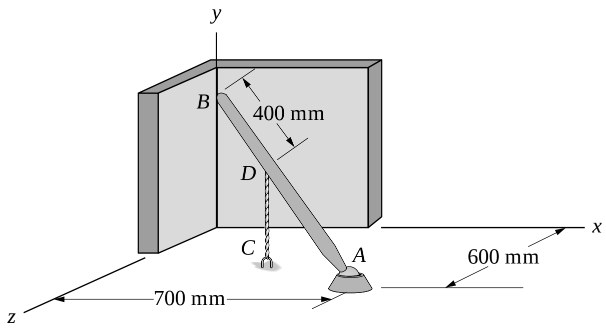
We make A_Y unknown, b unknown, and B unknown ($F = 4$ kN, $M_{A_Y} = +0.5$ (kN-m), and solve we get $A_Y = -2.5$ at $b = 0.4875$ m However, 0.3 is the physical limit of the device.

Thus, $0.1125 \text{ m} \le b \le 0.3 \text{ m}$



Problem 5.98 The 1.1-m bar is supported by a ball and socket support at A and the two smooth walls. The tension in the vertical cable CD is 1 kN.

(a) Draw the free-body diagram of the bar.
(b) Determine the reactions at A and B.



Solution:

(a) The ball and socket cannot support a couple reaction, but can support a three force reaction. The smooth surface supports one-force normal to the surface. The cable supports one force parallel to the cable.

(b) The strategy is to determine the moments about A, which will contain only the unknown reaction at B. This will require the position vectors of B and D relative to A, which in turn will require the unit vector parallel to the rod. The angle formed by the bar with the horizontal is required to determine the coordinates of B:

$$\alpha = \cos^{-1}\left(\frac{\sqrt{0.6^2 + 0.7^2}}{1.1}\right) = 33.1^\circ.$$

The coordinates of the points are: A (0.7, 0, 0.6), B (0, 1.1 ($\sin 33.1^\circ$), 0) = (0, 0.6, 0), from which the vector parallel to the bar is

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = -0.7\mathbf{i} + 0.6\mathbf{j} - 0.6\mathbf{k}\ \text{(m)}.$$

The unit vector parallel to the bar is

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{1.1} = -0.6364\mathbf{i} + 0.5455\mathbf{j} - 0.5455\mathbf{k}.$$

The vector location of the point D relative to A is

$$\mathbf{r}_{AD} = (1.1 - 0.4)\mathbf{e}_{AB} = 0.7\mathbf{e}_{AB}$$

$$= -0.4455\mathbf{i} + 0.3819\mathbf{j} - 0.3819\mathbf{k}.$$

The reaction at B is horizontal, with unknown x-component and z-components. The sum of the moments about A is

$$\sum \mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{B} + \mathbf{r}_{AD} \times \mathbf{D} = 0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.7 & 0.6 & -0.6 \\ B_X & 0 & B_Z \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.4455 & 0.3819 & -0.3819 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

Expand and collect like terms:

$$\sum \mathbf{M}_A = (0.6B_Z - 0.3819)\mathbf{i} - (0.6B_X - 0.7B_Z)\mathbf{j}$$

$$+(-0.6B_X + 0.4455)\mathbf{k} = 0.$$

From which,

$$B_Z = \frac{0.3819}{0.6} = 0.6365\ \text{kN},$$

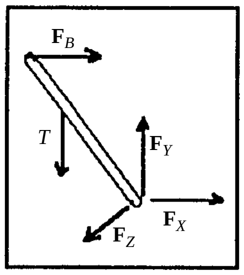
$$B_X = \frac{0.4455}{0.6} = 0.7425\ \text{kN}.$$

The reactions at A are determined from the sums of the forces:

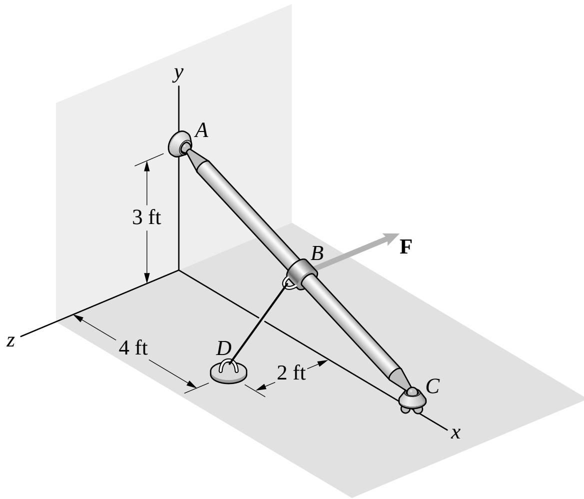
$$\sum \mathbf{F}_X = (B_X + A_X)\mathbf{i} = 0,\ \text{from which } A_X = -0.7425\ \text{kN}.$$

$$\sum \mathbf{F}_Y = (A_Y - 1)\mathbf{j} = 0,\ \text{from which } A_Y = 1\ \text{kN}.$$

$$\sum \mathbf{F}_Z = (B_Z + A_Z)\mathbf{k} = 0,\ \text{from which } A_Z = -0.6365\ \text{kN}$$



Problem 5.99 The 8-ft bar is supported by a ball and socket at A, the cable BD, and a roller support at C. The collar at B is fixed to the bar at its midpoint. The force $\mathbf{F} = -50\mathbf{k}$ (lb). Determine the tension in the cable BD and the reactions at A and C.



Solution: The strategy is to determine the sum of the moments about A, which will involve the unknown reactions at B and C. This will require the unit vectors parallel to the rod and parallel to the cable.

The angle formed by the rod is

$$\alpha = \sin^{-1}\left(\frac{3}{8}\right) = 22^\circ.$$

The vector positions are:

$$\mathbf{r}_A = 3\mathbf{j},$$

$$\mathbf{r}_D = 4\mathbf{i} + 2\mathbf{k}$$

and $\mathbf{r}_C = (8\cos 22^\circ)\mathbf{i} = 7.4162\mathbf{i}$.

The vector parallel to the rod is

$$\mathbf{r}_{AC} = \mathbf{r}_C - \mathbf{r}_A = 7.4162\mathbf{i} - 3\mathbf{j}.$$

The unit vector parallel to the rod is

$$\mathbf{e}_{AC} = 0.9270\mathbf{i} - 0.375\mathbf{j}.$$

The location of B is

$$\mathbf{r}_{AB} = 4\mathbf{e}_{AC} = 3.7081\mathbf{i} - 1.5\mathbf{j}.$$

The vector parallel to the cable is

$$\mathbf{r}_{BD} = \mathbf{r}_D - (\mathbf{r}_A + \mathbf{r}_{AB}) = 0.2919\mathbf{i} - 1.5\mathbf{j} + 2\mathbf{k}.$$

The unit vector parallel to the cable is

$$\mathbf{e}_{BD} = 0.1160\mathbf{i} - 0.5960\mathbf{j} + 0.7946\mathbf{k}.$$

The tension in the cable is $\mathbf{T} = |\mathbf{T}|\mathbf{e}_{BD}$. The reaction at the roller support C is normal to the x–z plane. The sum of the moments about A

$$\sum \mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F} + \mathbf{r}_{AB} \times \mathbf{T} + \mathbf{r}_{AC} \times \mathbf{C} = 0$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.7081 & -1.5 & 0 \\ 0 & 0 & -50 \end{vmatrix}$$

$$+ |\mathbf{T}| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.7081 & -1.5 & 0 \\ 0.1160 & -0.5960 & 0.7946 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7.4162 & -3 & 0 \\ 0 & C_Y & 0 \end{vmatrix} = 0$$

$$= 75\mathbf{i} + 185.4\mathbf{j} + |\mathbf{T}|(-1.192\mathbf{i} - 2.9466\mathbf{j} - 2.036\mathbf{k})$$

$$+ 7.4162 C_Y \mathbf{k} = 0,$$

from which $|\mathbf{T}| = \dfrac{75}{1.192} = 62.92$ lb

$$C_Y = \frac{2.036|\mathbf{T}|}{7.4162} = 17.27 \text{ lb}.$$

The reaction at A is determined from the sums of forces:

$$\sum \mathbf{F}_X = (A_X + 0.1160|\mathbf{T}|)\mathbf{i} = 0,$$

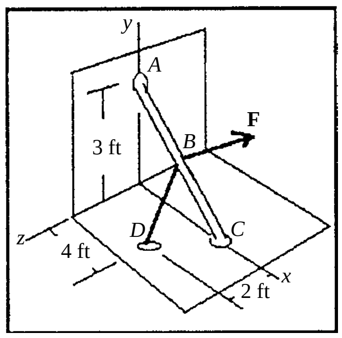
from which $A_X = -7.29$ lb,

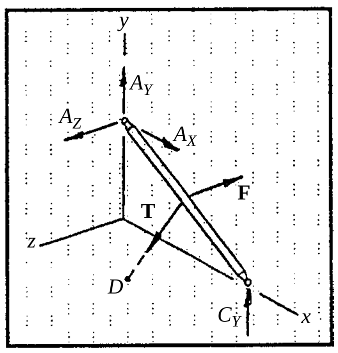
$$\sum \mathbf{F}_Y = (A_Y - 0.5960|\mathbf{T}| + C_Y)\mathbf{j} = 0,$$

from which $A_Y = 20.23$ lb

$$\sum \mathbf{F}_Z = (A_Z + 0.7946|\mathbf{T}| - 50)\mathbf{k} = 0,$$

from which $A_Z = 0$ lb





Problem 5.100 Consider the 8-ft bar in Problem 5.99. The force $\mathbf{F} = F_y\mathbf{j} - 50\mathbf{k}$ (lb). What is the largest value of F_y for which the roller support at C will remain on the floor?

Solution: From the solution to Problem 5.99, the sum of the moments about A is

$$\sum \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.7081 & -1.5 & 0 \\ 0 & F_Y & -50 \end{vmatrix}$$

$$+ |\mathbf{T}| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.7081 & -1.5 & 0 \\ 0.1160 & -0.5960 & 0.7946 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7.4162 & -3 & 0 \\ 0 & C_Y & 0 \end{vmatrix} = 0$$

$$= 75\mathbf{i} + 185.4\mathbf{j} + 3.7081 F_Y \mathbf{k}$$

$$+ |\mathbf{T}|(-1.192\mathbf{i} - 2.9466\mathbf{j} - 2.036\mathbf{k})$$

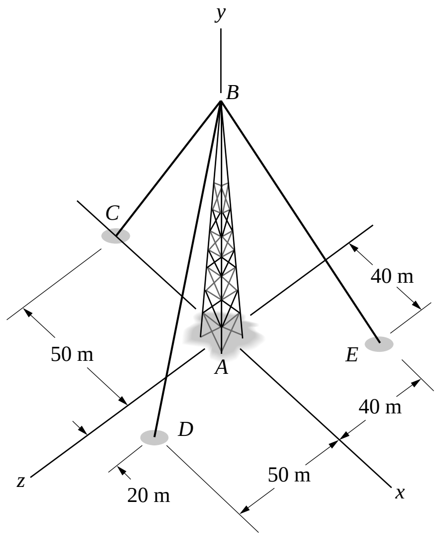
$$+ 7.4162 C_Y \mathbf{k} = 0,$$

from which, $|\mathbf{T}| = \dfrac{75}{1.192} = 62.92$ lb.

Collecting terms in $\mathbf{k}$, $3.7081 F_Y + 2.384|\mathbf{T}| - 7.4162 C_Y = 0$.

For $C_Y = 0$, $F_Y = \dfrac{128.11}{3.708} = 34.54$ lb

Problem 5.101 The tower is 70 m tall. The tension in each cable is 2 kN. Treat the base of the tower A as a built-in support. What are the reactions at A?



Solution: The strategy is to determine moments about A due to the cables. This requires the unit vectors parallel to the cables.

The coordinates of the points are:

$A(0, 0, 0)$, $B(0, 70, 0)$, $C(-50, 0, 0)$,

$D(20, 0, 50)$, $E(40, 0, -40)$.

The unit vectors parallel to the cables, directed from B to the points E, D, and C

$\mathbf{r}_{BE} = 40\mathbf{i} - 70\mathbf{j} - 40\mathbf{k}$,

$\mathbf{r}_{BD} = 20\mathbf{i} - 70\mathbf{j} + 50\mathbf{k}$,

$\mathbf{r}_{BC} = -50\mathbf{i} - 70\mathbf{j}$.

The unit vectors parallel to the cables, pointing from B, are:

$\mathbf{e}_{BE} = 0.4444\mathbf{i} - 0.7778\mathbf{j} - 0.4444\mathbf{k}$,

$\mathbf{e}_{BD} = 0.2265\mathbf{i} - 0.7926\mathbf{j} + 0.5661\mathbf{k}$,

$\mathbf{e}_{BC} = -0.5812\mathbf{i} - 0.8137\mathbf{j} + 0\mathbf{k}$.

The tensions in the cables are:

$\mathbf{T}_{BD} = 2\mathbf{e}_{BD} = 0.4529\mathbf{i} - 1.5852\mathbf{j} + 1.1323\mathbf{k}$ (kN),

$\mathbf{T}_{BE} = 2\mathbf{e}_{BE} = 0.8889\mathbf{i} - 1.5556\mathbf{j} - 0.8889\mathbf{k}$ (kN),

$\mathbf{T}_{BC} = 2\mathbf{e}_{BC} = -1.1625\mathbf{i} - 1.6275\mathbf{j} - 0\mathbf{k}$.

The sum of the moments about A is

$$\sum \mathbf{M}_A = \mathbf{M}^A + \mathbf{r}_{AB} \times \mathbf{T}_{BE}$$

$$+ \quad \mathbf{r}_{AB} \times \mathbf{T}_{BD} + \mathbf{r}_{AB} \times \mathbf{T}_{BC} = 0$$

$$= \mathbf{M}^A + \mathbf{r}_{AB} \times (\mathbf{T}_{BE} + \mathbf{T}_{BC} + \mathbf{T}_{BD})$$

$$\sum \mathbf{M}_A = \mathbf{M}^A + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 70 & 0 \\ 0.1793 & -4.7682 & 0.2434 \end{vmatrix} = 0$$

$$= (M_X^A + 17.038)\mathbf{i} + (M_Y^A + 0)\mathbf{j}$$

$$+ (M_Z^A - 12.551)\mathbf{k} = 0$$

from which

$M_X^A = -17.038$ kN-m,

$M_Y^A = 0$,

$M_Z^A = 12.551$ kN-m.

The force reactions at A are determined from the sums of forces. (Note that the sums of the cable forces have already been calculated and used above.)

$$\sum \mathbf{F}_X = (A_X + 0.17932)\mathbf{i} = 0,$$

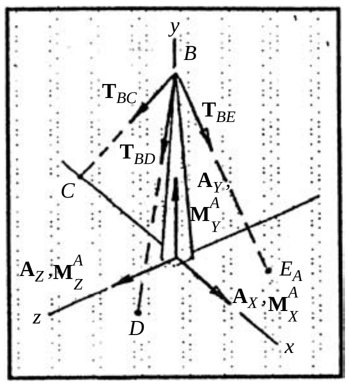
from which $A_X = -0.179$ kN,

$$\sum \mathbf{F}_Y = (A_Y - 4.7682)\mathbf{j} = 0,$$

from which $A_Y = 4.768$ kN,

$$\sum \mathbf{F}_Z = (A_Z + 0.2434)\mathbf{k} = 0,$$

from which $A_Z = -0.2434$ kN



Problem 5.102 Consider the tower in Problem 5.101. If the tension in cable BC is 2 kN, what must the tensions in cables BD and BE be if you want the couple exerted on the tower by the built-in support at A to be zero? What are the resulting reactions at A?

Solution: From the solution to Problem 5.101, the sum of the moments about A is given by

$$\sum \mathbf{M}_A = \mathbf{M}^A + \mathbf{r}_{AB} \times (\mathbf{T}_{BE} + \mathbf{T}_{BC} + \mathbf{T}_{BD}) = 0.$$

If the couple $\mathbf{M}^A = 0$, then the cross product is zero, which is possible only if the vector sum of the cable tensions is zero in the x and z directions. Thus, from Problem 5.101,

$$\mathbf{e}_x \cdot (\mathbf{T}_{BC} + |\mathbf{T}_{BE}|\mathbf{e}_{BE} + |\mathbf{T}_{BD}|\mathbf{e}_{BD}) = 0,$$

and $\mathbf{e}_z \cdot (\mathbf{T}_{BC} + |\mathbf{T}_{BE}|\mathbf{e}_{BE} + |\mathbf{T}_{BD}|\mathbf{e}_{BD}) = 0.$

Two simultaneous equations in two unknowns result;

$$0.4444|\mathbf{T}_{BE}| + 0.2265|\mathbf{T}_{BD}| = 1.1625$$

$$-0.4444|\mathbf{T}_{BE}| + 0.5661|\mathbf{T}_{BD}| = 0.$$

Solve:

$$|\mathbf{T}_{BE}| = 1.868 \text{ kN},$$

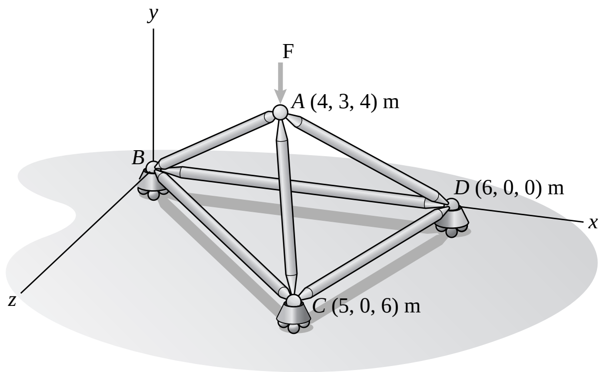
$$|\mathbf{T}_{BD}| = 1.467 \text{ kN}.$$

The reactions at A oppose the sum of the cable tensions in the x-, y-, and z-directions.

$$A_X = 0, \quad A_Y = 4.243 \text{ kN}, \quad A_Z = 0.$$

(These results are to be expected if there is no moment about A.)

Problem 5.103 The space truss has roller supports at B, C, and D and is subjected to a vertical force $F =$ 20 kN at A. What are the reactions at the roller supports?



Solution:

The key to this solution is expressing the forces in terms of unit vectors and magnitudes-then using the method of joints in three dimensions. The points A, B, C, and D are located at

$A(4, 3, 4)$ m, $B(0, 0, 0)$ m,

$C(5, 0, 6)$ m, $D(6, 0, 0)$ m

we need $\mathbf{e}_{AB}$, $\mathbf{e}_{AC}$, $\mathbf{e}_{AD}$, $\mathbf{e}_{BC}$, $\mathbf{e}_{BD}$, and $\mathbf{e}_{CD}$. Use the form

$$\mathbf{e}_{PQ} = \frac{(x_Q - x_P)\mathbf{i} + (y_Q - y_P)\mathbf{j} + (z_Q - z_P)\mathbf{k}}{[(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2]^{1/2}}$$

$$\mathbf{e}_{AB} = -0.625\mathbf{i} - 0.469\mathbf{j} - 0.625\mathbf{k}$$

$$\mathbf{e}_{AC} = 0.267\mathbf{i} - 0.802\mathbf{j} + 0.535\mathbf{k}$$

$$\mathbf{e}_{AD} = 0.371\mathbf{i} - 0.557\mathbf{j} - 0.743\mathbf{k}$$

$$\mathbf{e}_{BC} = 0.640\mathbf{i} + 0\mathbf{j} + 0.768\mathbf{k}$$

$$\mathbf{e}_{BD} = 1\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{e}_{CD} = 0.164\mathbf{i} + 0\mathbf{j} - 0.986\mathbf{k}$$

We will write each force as a magnitude times the appropriate unit vector.

$$\mathbf{T}_{AB} = T_{AB}\mathbf{e}_{AB},\ \mathbf{T}_{AC} = T_{AC}\mathbf{e}_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\mathbf{e}_{AD},\ \mathbf{T}_{BC} = T_{BC}\mathbf{e}_{BC}$$

$$\mathbf{T}_{BD} = T_{BD}\mathbf{e}_{BD},\ \mathbf{T}_{CD} = T_{CD}\mathbf{e}_{CD}$$

Each force will be written in component form, i.e.

$$\left.\begin{aligned} T_{AB_X} &= T_{AB}e_{AB_X} \\ T_{AB_Y} &= T_{AB}e_{AB_Y} \\ T_{AB_Z} &= T_{AB}e_{AB_Z} \end{aligned}\right\} \text{etc.}$$

Joint A: $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{F} = 0$

$$T_{AB_X} + T_{AC_X} + T_{AD_X} = 0$$

$$T_{AB_Y} + T_{AC_Y} + T_{AD_Y} - 20 = 0$$

$$T_{AB_Z} + T_{AC_Z} + T_{AD_Z} = 0$$

Joint B: $-\mathbf{T}_{AB} + \mathbf{T}_{BC} + \mathbf{T}_{BD} + N_B\mathbf{j} = 0$

Joint C: $-\mathbf{T}_{AC} - \mathbf{T}_{BC} + T_{CD} + N_C\mathbf{j} = 0$

Joint D: $-\mathbf{T}_{AD} - \mathbf{T}_{BD} - T_{CD} + N_D\mathbf{j} = 0$

Solving for all the unknowns, we get

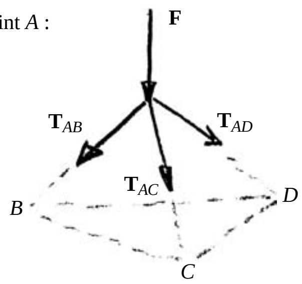
$N_B = 4.44$ kN
$N_C = 2.22$ kN
$N_D = 13.33$ kN

Also, $T_{AB} = -9.49$ kN, $T_{AC} = -16.63$ kN

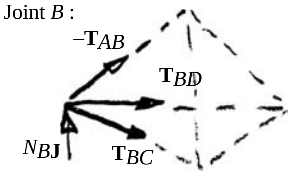
$T_{AD} = -3.99$ kN, $T_{BC} = 7.71$ kN

$T_{BD} = 0.99$ kN, $T_{CD} = 3.00$ kN

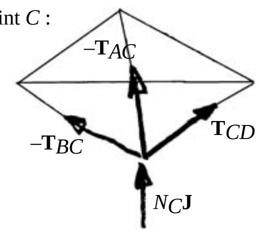
Joint A:



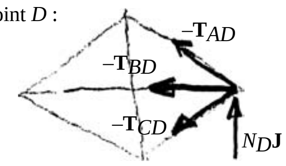
Joint B:



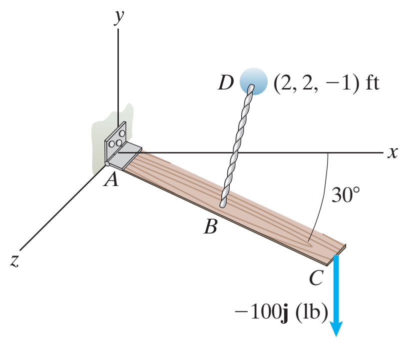
Joint C:



Joint D:



Problem 5.104 In Example 5.8, suppose that the cable *BD* is lengthened and the attachment point *B* is moved to the end of the bar at *C*. The positions of the attachment point *D* and the bar are unchanged. Draw a sketch of the bar showing cable *BD* in its new position. Draw the free-body diagram of the bar and apply equilibrium to determine the tension in the cable and the reactions at *A*.



Solution: The sketch and free-body diagram are shown. We must express the force exerted on the bar by cable *BD* in terms of its components. The bar *AC* is 4 ft long. The vector from *C* to *D* is

$$\mathbf{r}_{CD} = [(2 - 4\cos 30^\circ)\mathbf{i} + (2 - \{-4\sin 30^\circ\})\mathbf{j}$$

$$+ (-1 - 0)\mathbf{j}]\ \text{ft}$$

$$\mathbf{r}_{CD} = (-1.46\mathbf{i} + 4\mathbf{j} - \mathbf{k})\ \text{ft}$$

The force exerted by the cable *CD* can be expressed

$$T\frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = T(-0.335\mathbf{i} + 0.914\mathbf{j} - 0.229\mathbf{k})$$

The equilibrium equations are

$$\Sigma F_x : A_x - 0.335T = 0$$

$$\Sigma F_y : A_y + 0.914T - 100\ \text{lb} = 0$$

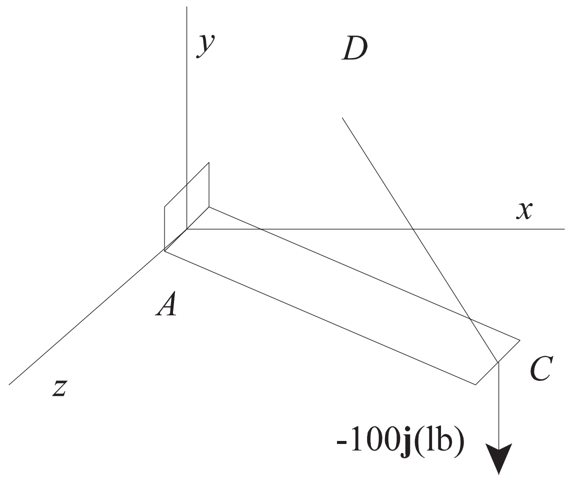
$$\Sigma F_z : A_z - 0.229T = 0$$

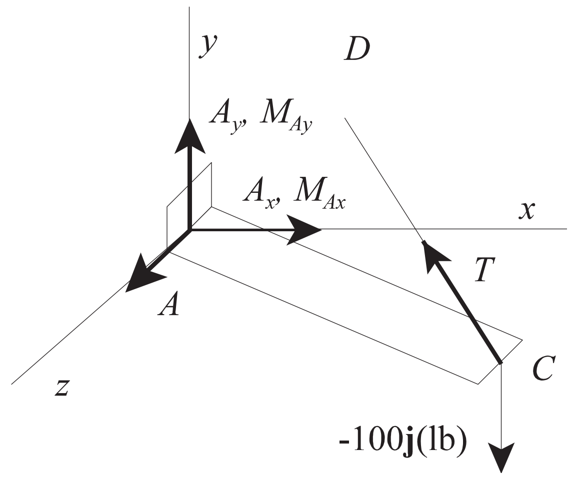
$$\Sigma \mathbf{M}_A : M_{Ax}\mathbf{i} + M_{Ay}\mathbf{j}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.464 & -2 & 0 \\ -0.335T & 0.914T - 100 & -0.229T \end{vmatrix} = 0$$

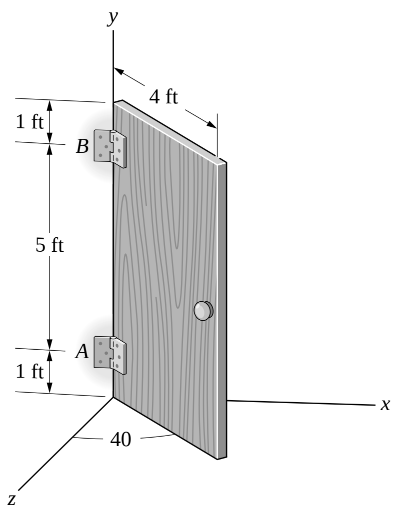
Expanding the determinant and solving the six equations, we obtain

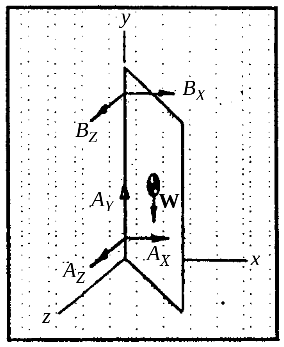
$$\boxed{\begin{array}{l} T = 139\ \text{lb},\ A_x = 46.4\ \text{lb},\ A_y = -26.8\ \text{lb},\ A_z = 31.7\ \text{lb} \\ M_{Ax} = -63.4\ \text{ft-lb},\ M_{Ay} = -110\ \text{ft-lb} \end{array}}$$





Problem 5.105 The 40-lb door is supported by hinges at A and B. The y axis is vertical. The hinges do not exert couples on the door, and the hinge at B does not exert a force parallel to the hinge axis. The weight of the door acts at its midpoint. What are the reactions at A and B?





Solution: The position vector of the midpoint of the door:

$$\mathbf{r}_{CM} = (2\cos 50^\circ)\mathbf{i} + 3.5\mathbf{j} + (2\cos 40^\circ)\mathbf{k}$$

$$= 1.2856\mathbf{i} + 3.5\mathbf{j} + 1.532\mathbf{k}.$$

The position vectors of the hinges:

$$\mathbf{r}_A = \mathbf{j}, \quad \mathbf{r}_B = 6\mathbf{j}.$$

The forces are: $\mathbf{W} = -40\mathbf{j}$,

$$\mathbf{A} = A_X\mathbf{i} + A_Y\mathbf{j} + A_Z\mathbf{k},$$

$$\mathbf{B} = B_X\mathbf{i} + B_Z\mathbf{k}.$$

The position vectors relative to A are

$$\mathbf{r}_{ACM} = \mathbf{r}_{CM} - \mathbf{r}_A = 1.2856\mathbf{i} + 2.5\mathbf{j} + 1.532\mathbf{k},$$

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = 5\mathbf{j}.$$

The sum of the moments about A

$$\sum \mathbf{M}_A = \mathbf{r}_{ACM} \times \mathbf{W} + \mathbf{r}_{AB} \times \mathbf{B}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2856 & 2.5 & 1.532 \\ 0 & -40 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ B_X & 0 & B_Z \end{vmatrix} = 0$$

$$\sum \mathbf{M}_A = (5B_Z + 40(1.532))\mathbf{i} + (-5B_X - 40(1.285))\mathbf{k} = 0,$$

from which $B_Z = \dfrac{-40(1.532)}{5} = -12.256$ lb

and $\quad B_X = \dfrac{-40(1.285)}{5} = -10.28$ lb.

The reactions at A are determined from the sums of forces:

$$\sum \mathbf{F}_X = (A_X + B_X)\mathbf{i} = 0,$$

from which $A_X = 10.28$ lb,

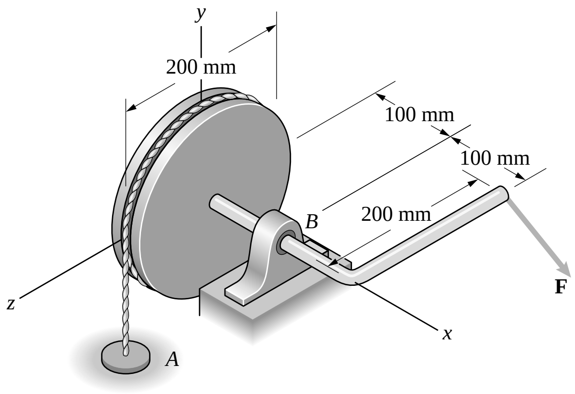
$$\sum \mathbf{F}_Y = (A_Y - 40)\mathbf{j} = 0,$$

from which $A_Y = 40$ lb,

$$\sum \mathbf{F}_Z = (A_Z + B_Z)\mathbf{k} = 0,$$

from which $A_Z = 12.256$ lb

Problem 5.106 The vertical cable is attached at A. Determine the tension in the cable and the reactions at the bearing B due to the force $\mathbf{F} = 10\mathbf{i} - 30\mathbf{j} - 10\mathbf{k}$ (N).



Solution: The position vector of the point of application of the force is

$\mathbf{r}_F = 0.2\mathbf{i} - 0.2\mathbf{k}$.

The position vector of the bearing is

$\mathbf{r}_B = 0.1\mathbf{i}$.

The position vector of the cable attachment to the wheel is

$\mathbf{r}_C = 0.1\mathbf{k}$.

he position vectors relative to B are:

$\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = -0.1\mathbf{i} + 0.1\mathbf{k}$,

$\mathbf{r}_{BF} = \mathbf{r}_F - \mathbf{r}_B = 0.1\mathbf{i} - 0.2\mathbf{k}$.

The sum of the moments about the bearing B is

$$\sum \mathbf{M}_B = M_B + \mathbf{r}_{BF} \times \mathbf{F} + \mathbf{r}_{BC} \times \mathbf{C} = 0,$$

$$\text{or } \sum \mathbf{M}_B = M_B + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0 & -0.2 \\ 10 & -30 & -10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.1 & 0 & 0.1 \\ 0 & -T & 0 \end{vmatrix}$$

$$= (-6 + 0.1T)\mathbf{i} + (M_{BY} - 1)\mathbf{j}$$

$$+ (M_{BZ} - 3 + 0.1T)\mathbf{k} = 0,$$

from which $T = \dfrac{6}{0.1} = 60$ N,

$$M_{BY} = +1 \text{ N-m},$$

$$M_{BZ} = -0.1T + 3 = -3 \text{ N-m}.$$

The force reactions at the bearing are determined from the sums of forces:

$$\sum \mathbf{F}_X = (B_X + 10)\mathbf{i} = 0,$$

from which $B_X = -10$ N.

$$\sum \mathbf{F}_Y = (B_Y - 30 - 60)\mathbf{j} = 0,$$

from which $B_Y = 90$ N.

$$\sum \mathbf{F}_Z = (B_Z - 10)\mathbf{j} = 0,$$

from which $B_Z = 10$ N.

Problem 5.107 In Problem 5.106, suppose that the z component of the force **F** is zero, but otherwise **F** is unknown. If the couple exerted on the shaft by the bearing at B is $\mathbf{M}_B = 6\mathbf{j} - 6\mathbf{k}$ N-m, what are the force **F** and the tension in the cable?

Solution: From the diagram of Problem 5.106, the force equilibrium equation components are

$$\sum F_x = B_X + F_X = 0,$$

$$\sum F_y = B_Y + F_Y = 0,$$

and $\sum F_z = B_Z + F_Z = 0,$

where $F_Z = 0$ is given in the problem statement. The moment equations can be developed by inspection of the figure also. They are

$$\sum M_x = M_{BX} + M_{AX} + M_{FX} = 0,$$

$$\sum M_Y = M_{BY} + M_{AY} + M_{FY} = 0,$$

and $\sum M_Z = M_{BZ} + M_{AZ} + M_{FZ} = 0,$

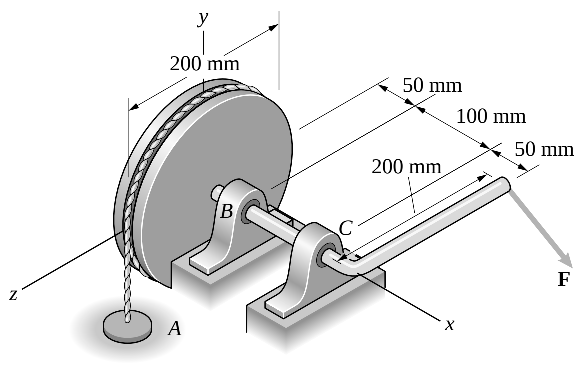
where $M_B = 6\mathbf{j} - 6\mathbf{k}$ N-m. Note that $M_{BX} = 0$ can be inferred. The moments which need to be substituted into the moment equations are

$$M_A = (0.1)A\mathbf{i} + 0\mathbf{j} + (0.1)A\mathbf{k} \text{ N-m},$$

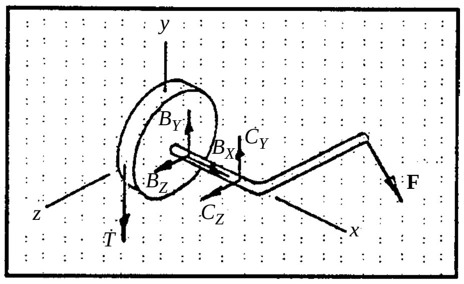
and $M_F = (0.2)F_Y\mathbf{i} - (0.2)F_X\mathbf{j} + (0.1)F_Y\mathbf{k}$ N-m.

Substituting these values into the equilibrium equations, we get $\mathbf{F} = 30\mathbf{i} - 60\mathbf{j} + 0\mathbf{k}$ N, and $A = 120$ N.

Problem 5.108 The device in Problem 5.106 is badly designed because of the couples that must be supported by the bearing at B, which would cause the bearing to “bind”. (Imagine trying to open a door supported by only one hinge.) In this improved design, the bearings at B and C support no couples, and the bearing at C does not exert a force in the x direction. If the force $\mathbf{F} = 10\mathbf{i} - 30\mathbf{j} - 10\mathbf{k}$ (N), what are the tension in the vertical cable and the reactions at the bearings B and C?



Solution: The position vectors relative to the bearing B are: the position vector of the cable attachment to the wheel is



$$\mathbf{r}_{BT} = -0.05\mathbf{i} + 0.1\mathbf{k}.$$

The position vector of the bearing C is:

$$\mathbf{r}_{BC} = 0.1\mathbf{i}.$$

The position vector of the point of application of the force is:

$$\mathbf{r}_{BF} = 0.15\mathbf{i} - 0.2\mathbf{k}.$$

The sum of the moments about B is

$$\sum \mathbf{M}_B = \mathbf{r}_{BT} \times \mathbf{T} + \mathbf{r}_{BC} \times \mathbf{C} + \mathbf{r}_{BF} \times \mathbf{F} = 0$$

$$\sum \mathbf{M}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.05 & 0 & 0.1 \\ 0 & -T & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0 & 0 \\ 0 & C_Y & C_Z \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0 & -0.2 \\ 10 & -30 & -10 \end{vmatrix} = 0$$

$$\sum \mathbf{M}_B = (0.1T - 6)\mathbf{i} + (-0.1C_Z + 1.5 - 2)\mathbf{j}$$

$$+ (0.05T + 0.1C_Y - 4.5)\mathbf{k} = 0.$$

From which: $T = 60$ N,

$$C_Z = \frac{-0.5}{0.1} = -5 \text{ N},$$

$$C_Y = \frac{4.5 - 0.05T}{0.1} = 15 \text{ N}.$$

The reactions at B are found from the sums of forces:

$$\sum \mathbf{F}_X = (B_X + 10)\mathbf{i} = 0,$$

from which $B_X = -10$ N.

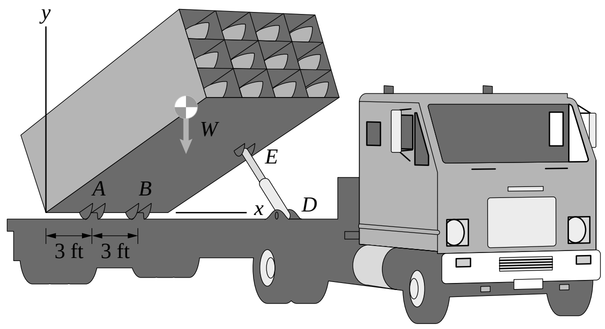
$$\sum \mathbf{F}_Y = (B_Y + C_Y - T - 30)\mathbf{j} = 0,$$

from which $B_Y = 75$ N.

$$\sum \mathbf{F}_Z = (B_Z + C_Z - 10)\mathbf{k} = 0,$$

from which $B_Z = 15$ N

Problem 5.109 The rocket launcher is supported by the hydraulic jack DE and the bearings A and B. The bearings lie on the x axis and support shafts parallel to the x axis. The hydraulic cylinder DE exerts a force on the launcher that points along the line from D to E. The coordinates of D are (7, 0, 7) ft, and the coordinates of E are (9, 6, 4) ft. The weight $W = 30$ kip acts at (4.5, 5, 2) ft. What is the magnitude of the reaction on the launcher at E?



Solution: The position vectors of the points D, E and W are

$$\mathbf{r}_D = 7\mathbf{i} + 7\mathbf{k},$$

$$\mathbf{r}_E = 9\mathbf{i} + 6\mathbf{j} + 4\mathbf{k} \text{ (ft)},$$

$$\mathbf{r}_W = 4.5\mathbf{i} + 5\mathbf{j} + 2\mathbf{k} \text{ (ft)}.$$

The vector parallel to DE is

$$\mathbf{r}_{DE} = \mathbf{r}_E - \mathbf{r}_D = 2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}.$$

The unit vector parallel to DE is

$$\mathbf{e}_{DE} = 0.2857\mathbf{i} + 0.8571\mathbf{j} - 0.4286\mathbf{k}.$$

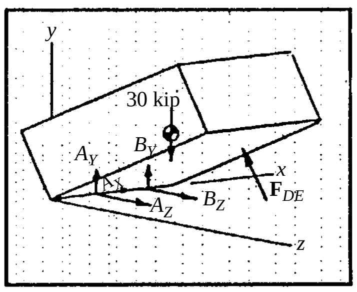
Since the bearings cannot exert a moment about the x axis, the sum of the moments due to the weight and the jack force must be zero about the x axis. The sum of the moments about the x axis is:

$$\sum M_X = \begin{vmatrix} 1 & 0 & 0 \\ 4.5 & 5 & 2 \\ 0 & -30 & 0 \end{vmatrix} + |\mathbf{F}_{DE}| \begin{vmatrix} 1 & 0 & 0 \\ 9 & 6 & 4 \\ 0.2857 & 0.8571 & -0.4286 \end{vmatrix} = 0$$

$$= 60 - 6|\mathbf{F}_{DE}| = 0.$$

From which

$$|\mathbf{F}_{DE}| = \frac{60}{6} = 10 \text{ kip}$$



Problem 5.110 Consider the rocket launcher described in Problem 5.109. The bearings at A and B do not exert couples, and the bearing B does not exert a force in the x direction. Determine the reactions at A and B.

Solution: See the solution of Problem 5.109. The force $\mathbf{F}_{DE}$ can be written

$$\mathbf{F}_{DE} = F_{DE}(0.2857\mathbf{i} + 0.8571\mathbf{j} - 0.4286\mathbf{k}).$$

The equilibrium equations are

$$\sum F_X = A_X + 0.2857F_{DE} = 0,$$

$$\sum F_Y = A_Y + B_Y + 0.8571F_{DE} - 30 = 0,$$

$$\sum F_Z = A_Z + B_Z - 0.4286F_{DE} = 0,$$

$$\sum \mathbf{M}_{\text{(origin)}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ A_X & A_Y & A_Z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ 0 & B_Y & B_Z \end{vmatrix}$$

$$+ F_{DE} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 0 & 7 \\ 0.2857 & 0.8571 & -0.4286 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4.5 & 5 & 2 \\ 0 & -30 & 0 \end{vmatrix} = 0$$

The components of the moment eq. are

$$-5.9997F_{DE} + 60 = 0,$$

$$-3A_Z - 6B_Z + 5.0001F_{DE} = 0,$$

$$3A_Y + 6B_Y + 5.9997F_{DE} - 135 = 0.$$

Solving, we obtain

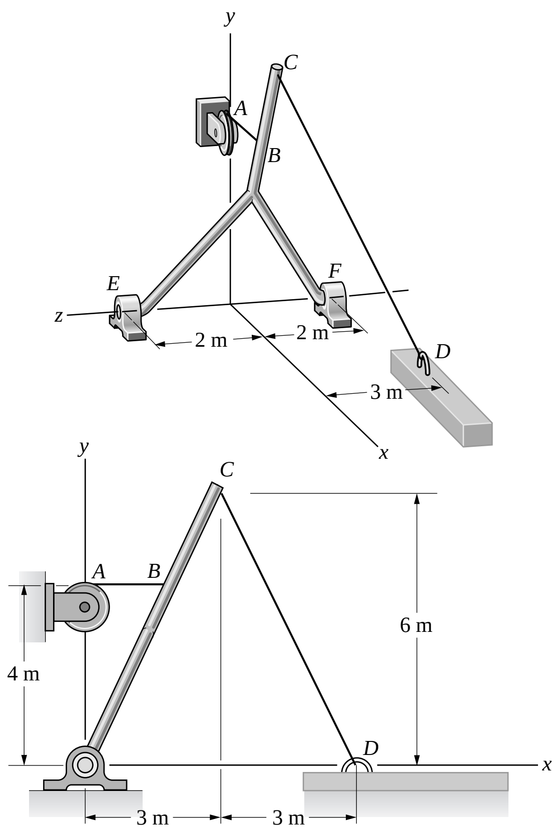
$$F_{DE} = 10.00 \text{ kip}, \ A_X = -2.86 \text{ kip},$$

$$A_Y = 17.86 \text{ kip}, \ A_Z = -8.09 \text{ kip},$$

$$B_Y = 3.57 \text{ kip}, \ B_Z = 12.38 \text{ kip}.$$

Problem 5.111 The crane's cable CD is attached to a stationary object at D. The crane is supported by the bearings E and F and the horizontal cable AB. The tension in cable AB is 8 kN. Determine the tension in the cable CD.

Strategy: Since the reactions exerted on the crane by the bearings do not exert moments about the z axis, the sum of the moments about the z axis due to the forces exerted on the crane by the cables AB and CD equals zero. (See the discussion at the end of Example 5.9.)



Solution: The position vector from C to D is

$$\mathbf{r}_{CD} = 3\mathbf{i} - 6\mathbf{j} - 3\mathbf{k} \text{ (m)},$$

so we can write the force exerted at C by cable CD as

$$\mathbf{T}_{CD} = T_{CD}\frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = T_{CD}(0.408\mathbf{i} - 0.816\mathbf{j} - 0.408\mathbf{k}).$$

The coordinates of pt. B are $x = \frac{4}{6}(3) = 2$ m, $y = 4$ m.

The moment about the origin due to the forces exerted by the two cables is

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 0 \\ -8 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 6 & 0 \\ 0.408T_{CD} & -0.816T_{CD} & -0.408T_{CD} \end{vmatrix}$$

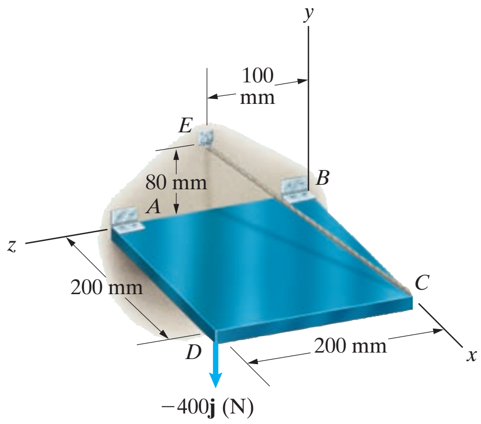
$$= 32\mathbf{k} - 2.448T_{CD}\mathbf{i} + 1.224T_{CD}\mathbf{j} - 4.896T_{CD}\mathbf{k}.$$

The moment about the z axis is

$$\mathbf{k}\cdot\mathbf{M}_O = 32 - 4.896T_{CD} = 0,$$

so $T_{CD} = 6.54$ kN.

Problem 5.112 In Example 5.9, suppose that the cable CE is shortened and its attachment point E is moved to the point (0, 80, 0) mm. The plate remains in the same position. Draw a sketch of the plate and its supports showing the new position of cable CE. Draw the free-body diagram of the plate and apply equilibrium to determine the reactions at the hinges and the tension in the cable.



Solution: The sketch and free-body diagram are shown. The vector from C to E is

$$\mathbf{r}_{CE} = [(0-200)\mathbf{i} + (80-0)\mathbf{j} + (0-0)\mathbf{k}]\ \text{mm}$$

$$= (-200\mathbf{i} + 80\mathbf{j})\ \text{mm}$$

The force exerted by cable CE can be expresses as

$$T\frac{\mathbf{r}_{CE}}{|\mathbf{r}_{CE}|} = T(-0.928\mathbf{i} + 0.371\mathbf{j})$$

The equilibrium equations for the plate are

$$\Sigma F_x : A_x + B_x - 0.928T = 0$$

$$\Sigma F_y : A_y + B_y + 0.371T - 400\ \text{N} = 0$$

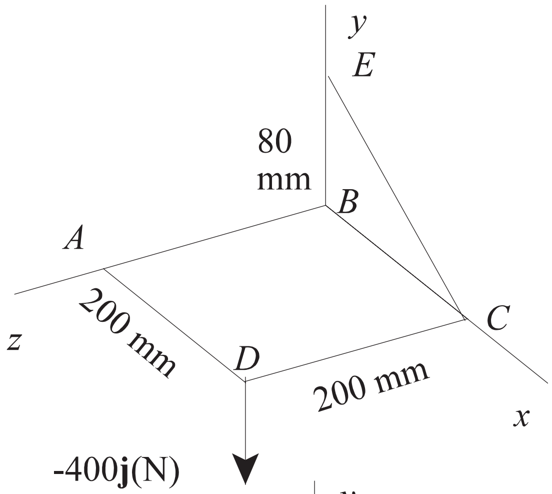
$$\Sigma F_z : A_z + B_z = 0$$

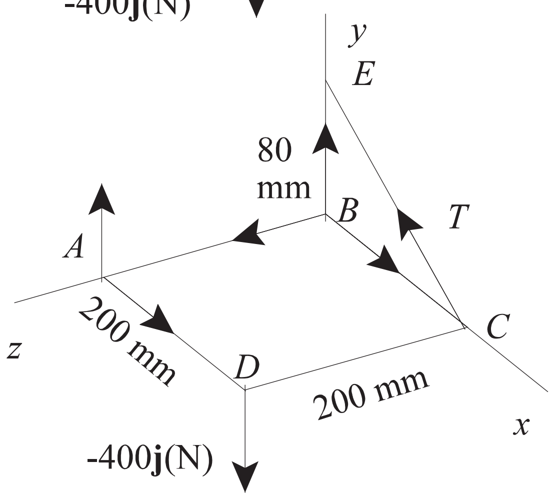
$$\Sigma \mathbf{M}_B : \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0 \\ -0.928T & 0.371T & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.2 \\ A_x & A_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0.2 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

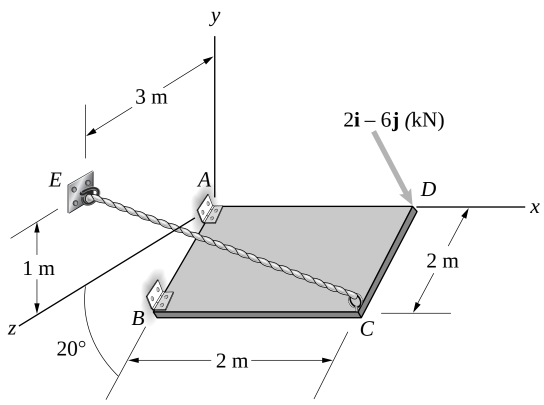
Expanding and solving we find

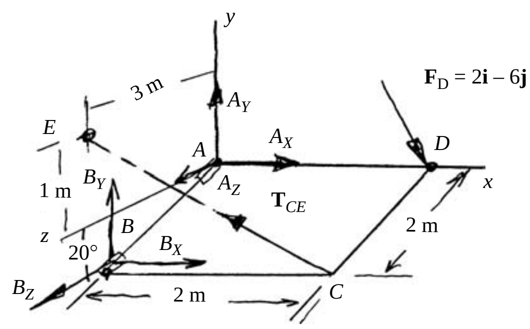
$A_x = 0, A_y = 400\ \text{N}, T = 1080\ \text{N}$
$B_x = 1000\ \text{N}, B_y = -400\ \text{N}, B_z = 0,$





Problem 5.113 The plate is supported by hinges at A and B and the cable CE, and it is loaded by the force at D. The edge of the plate to which the hinges are attached lies in the y–z plane, and the axes of the hinges are parallel to the line through points A and B. The hinges do not exert couples on the plate. What is the tension in cable CE?





Solution:

$$\sum \mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{F}_D + \mathbf{T}_{CE} = 0$$

However, we just want tension in CE. This quantity is the only unknown in the moment equation about the line AB. To get this, we need the unit vector along CE.

Point C is at $(2, -2\sin 20^\circ, 2\cos 20^\circ)$ Point E is at (0, 1, 3)

$$\mathbf{e}_{CE} = \frac{\mathbf{r}_{CE}}{|\mathbf{r}_{CE}|}$$

$$\mathbf{e}_{CE} = -0.703\mathbf{i} + 0.592\mathbf{j} + 0.394\mathbf{k}$$

We also need the unit vector $\mathbf{e}_{AB}$. A(0, 0, 0), $B(0, -2\sin 20^\circ, 2\cos 20^\circ)$

$$\mathbf{e}_{AB} = 0\mathbf{i} - 0.342\mathbf{j} + 0.940\mathbf{k}$$

The moment of $\mathbf{F}_D$ about A (a point on AB) is

$$\mathbf{M}_{F_D} = \mathbf{r}_{AD} \times \mathbf{F}_{D_1} = (2\mathbf{i}) \times (2\mathbf{i} - 6\mathbf{j})$$

$$\mathbf{M}_{F_D} = -12\mathbf{k}$$

The moment of $\mathbf{T}_{CE}$ about B (another point on line CE) is

$$\mathbf{M}_{T_{CE}} = \mathbf{r}_{BC} \times T_{CE}\mathbf{e}_{CE} = 2\mathbf{i} \times T_{CE}\mathbf{e}_{CE},$$

where $\mathbf{e}_{CE}$ is given above.

The moment of $\mathbf{F}_D$ about line AB is

$$M_{FD_{AB}} = \mathbf{M}_{FD} \cdot \mathbf{e}_{AB}$$

$$M_{FD_{AB}} = -11.27 \text{ N-m}$$

The moment of T_{CE} about line AB is

$$M_{CE_{AB}} = T_{CE}(2\mathbf{i} \times \mathbf{e}_{CE}) \cdot \mathbf{e}_{AB}$$

$$M_{CE_{AB}} = T_{CE}(-0.788\mathbf{j} + 1.184\mathbf{k}) \cdot \mathbf{e}_{AB}$$

$$M_{CE_{AB}} = 1.382 T_{CE}$$

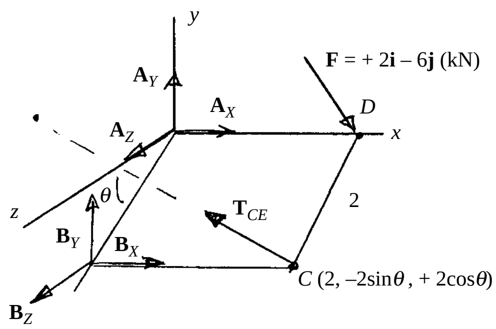
The sum of the moments about line AB is zero. Hence

$$M_{FD_{AB}} + M_{CE_{AB}} = 0$$

$$-11.27 + 1.382 T_{CE} = 0$$

$$T_{CE} = 8.15 \text{ kN}$$

Problem 5.114 In Problem 5.113, the hinge at B does not exert a force on the plate in the direction of the hinge axis. What are the magnitudes of the forces exerted on the plate by the hinges at A and B?



Solution: From the solution to Problem 5.113, $T_{CE} = 8.15$ kN Also, from that solution,

$$\mathbf{e}_{AB} = 0\mathbf{i} - 0.342\mathbf{j} + 0.940\mathbf{k}$$

We are given that the force at force at hinge B does not exert a force parallel to AB at B. This implies

$$\mathbf{B} \cdot \mathbf{e}_{AB} = 0.$$

$$\mathbf{B} \cdot \mathbf{e}_{AB} = -0.342B_Y + 0.940B_Z = 0 \quad \textbf{(1)}$$

We also had, in the solution to Problem 5.113

$$\mathbf{e}_{CE} = -0.703\mathbf{i} + 0.592\mathbf{j} + 0.394\mathbf{k}$$

and $\mathbf{T}_{CE} = T_{CE}\mathbf{e}_{CE}$ (kN)

For Equilibrium,

$$\sum \mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{T}_{CE} + \mathbf{F} = 0$$

$$\sum F_X: \quad A_X + B_X + T_{CE}e_{CE_X} + 2 = 0 \text{ (kN)} \quad \textbf{(2)}$$

$$\sum F_Y: \quad A_Y + B_Y + T_{CE}e_{CE_Y} - 6 = 0 \text{ (kN)} \quad \textbf{(3)}$$

$$\sum F_Z: \quad A_Z + B_Z + T_{CE}e_{CE_Z} = 0 \text{ (kN)} \quad \textbf{(4)}$$

Summing Moments about A, we have

$$\mathbf{r}_{AD} \times \mathbf{F} + \mathbf{r}_{AC} \times \mathbf{T}_{CE} + \mathbf{r}_{AB} \times \mathbf{B} = 0$$

$$\mathbf{r}_{AD} \times \mathbf{F} = 2\mathbf{i} \times (2\mathbf{i} - 6\mathbf{j}) = -12\mathbf{k} \text{ (kN)}$$

$$\mathbf{r}_{AC} \times \mathbf{T}_{CE} = (-2\sin\theta T_Z - 2\cos\theta T_Y)\mathbf{i}$$

$$+ (2\cos\theta T_X - 2T_Z)\mathbf{j}$$

$$+ (2T_Y + 2T_X\sin\theta)\mathbf{k}$$

$$\mathbf{r}_{CE} \times \mathbf{B} = (-2B_Z\sin\theta - 2B_Y\cos\theta)\mathbf{i}$$

$$+ (2B_X\cos\theta)\mathbf{j} + (+2B_X\sin\theta)\mathbf{k}$$

$$\sum \mathbf{M}_A = 0,$$

Hence

$$x: \quad -2\sin\theta T_Z - 2\cos\theta T_Y - 2B_Z\sin\theta$$

$$-2B_Y\cos\theta = 0 \quad \textbf{(5)}$$

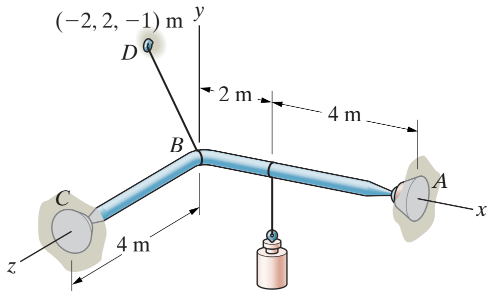
$$y: \quad 2\cos\theta T_X - 2T_Z + 2B_X\cos\theta = 0 \quad \textbf{(6)}$$

$$z: \quad -12 + 2T_Y + 2T_X\sin\theta + 2B_X\sin\theta = 0 \quad \textbf{(7)}$$

Solving Eqns (1)–(7), we get

$|\mathbf{A}| = 8.53$ (kN), $|\mathbf{B}| = 10.75$ (kN)

Problem 5.115 The bar *ABC* is supported by ball and socket supports at *A* and *C* and the cable *BD*. The suspended mass is 1800 kg. Determine the tension in the cable.



Solution: We take moments about the line *AC* to eliminate the reactions at *A* and *C*.

We have

$$\mathbf{r}_{AB} = -(4\text{ m})\mathbf{k},\ \ \mathbf{T}_{BD} = T_{BD}\left(\frac{-2\mathbf{i}+2\mathbf{j}-\mathbf{k}}{3}\right)$$

$$\mathbf{r}_{AW} = (2\mathbf{i}-4\mathbf{k})\text{ m},\ \ \mathbf{W} = -(1800\text{ kg})(9.81\text{ m/s}^2)\mathbf{j}$$

$$\mathbf{e}_{CA} = \frac{6\mathbf{i}-4\mathbf{k}}{\sqrt{52}} = \frac{1}{\sqrt{13}}(3\mathbf{i}-2\mathbf{k})$$

The one equilibrium equation we need is

$$\sum M_{AC} = \mathbf{e}_{CA}\cdot(\mathbf{r}_{AB}\times\mathbf{T}_{AB}+\mathbf{r}_{AW}\times\mathbf{W}) = 0$$

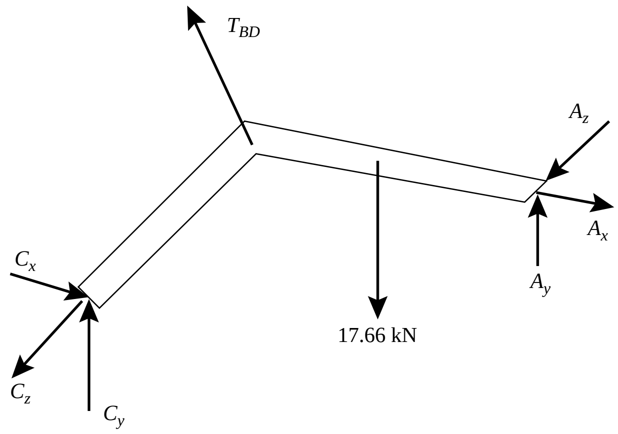
This equation reduces to the scalar equation

$$\frac{1}{\sqrt{13}}(3\mathbf{i}-2\mathbf{k})\cdot\left(\left[\frac{8}{3}\text{ m}\right]T_{BD}\mathbf{i}+\left[\frac{8}{3}\text{ m}\right]T_{BD}\mathbf{j}-[4\text{ m}][17.66\text{ kN}]\mathbf{i}\right.$$
$$\left.-[2\text{ m}][17.66\text{ kN}]\mathbf{k}\right) = 0$$

$$\frac{1}{\sqrt{2}}\left(3\left\{\left[\frac{8}{3}\text{ m}\right]T_{BD}-[4\text{ m}][17.66\text{ kN}]\right\}+2\{[2\text{ m}][17.66\text{ kN}]\}\right) = 0$$

Solving we find

$$\boxed{T_{BD} = 17.66\text{ kN}}$$



Problem 5.116* In Problem 5.115, assume that the ball and socket support at A is designed so that it exerts no force parallel to the straight line from A to C. Determine the reactions at A and C.

Solution: We have

$$\mathbf{T}_{BD} = T_{BD}\left(\frac{-2\mathbf{i}+2\mathbf{j}-\mathbf{k}}{3}\right)$$

There are 7 unknowns. We have the following 6 equilibrium equations

$$\sum F_x : A_x + C_x - \frac{2}{3}T_{BD} = 0$$

$$\sum F_y : A_y + C_y + \frac{2}{3}T_{BD} - 17.66 \text{ kN} = 0$$

$$\sum F_z : A_z + C_z - \frac{1}{3}T_{BD} = 0$$

$$\sum M_x : -C_y(4 \text{ m}) = 0$$

$$\sum M_y : C_x(4 \text{ m}) - A_z(6 \text{ m}) = 0$$

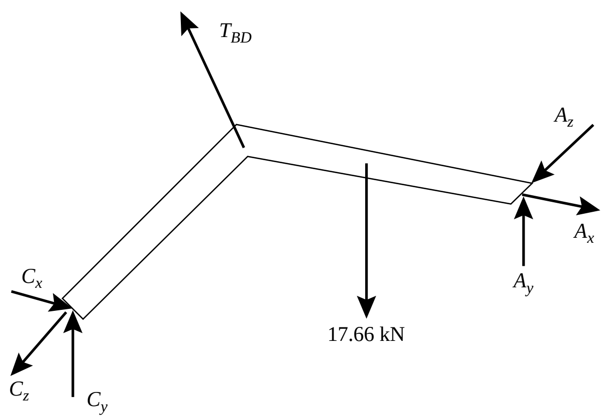
$$\sum M_z : A_y(6 \text{ m}) - (17.66 \text{ kN})(2 \text{ m}) = 0$$

The last equation comes from the fact that the ball and socket at A exerts no force in the direction of the line from A to C

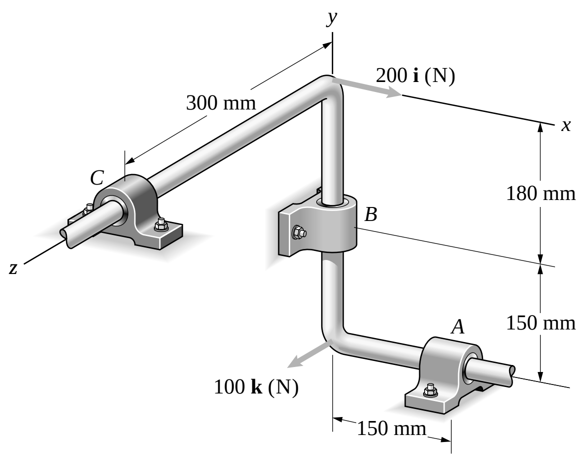
$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot \left(\frac{-6\mathbf{i}+4\mathbf{k}}{\sqrt{52}}\right) = \frac{1}{\sqrt{52}}(-6A_x + 4A_z) = 0$$

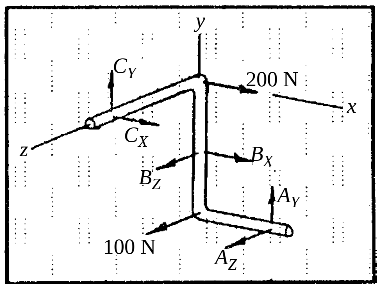
Solving these 7 equations we find

$$\boxed{\begin{array}{l} A_x = 3.62 \text{ kN},\ A_y = 5.89 \text{ kN},\ A_z = 5.43 \text{ kN} \\ C_x = 8.15 \text{ kN},\ C_y = 0,\ C_z = 0.453 \text{ kN} \\ T_{BD} = 17.66 \text{ kN} \end{array}}$$



Problem 5.117 The bearings at A, B, and C do not exert couples on the bar and do not exert forces in the direction of the axis of the bar. Determine the reactions at the bearings due to the two forces on the bar.





Solution: The strategy is to take the moments about A and solve the resulting simultaneous equations. The position vectors of the bearings relative to A are:

$$\mathbf{r}_{AB} = -0.15\mathbf{i} + 0.15\mathbf{j},$$

$$\mathbf{r}_{AC} = -0.15\mathbf{i} + 0.33\mathbf{j} + 0.3\mathbf{k}.$$

Denote the lower force by subscript 1, and the upper by subscript 2:

$$\mathbf{r}_{A1} = -0.15\mathbf{i},$$

$$\mathbf{r}_{A2} = -0.15\mathbf{i} + 0.33\mathbf{j}.$$

The sum of the moments about A is:

$$\sum \mathbf{M}_A = \mathbf{r}_{A1} \times \mathbf{F}_1 + \mathbf{r}_{AB} \times \mathbf{B} + \mathbf{r}_{A2} \times \mathbf{F}_2 + \mathbf{r}_{AC} \times \mathbf{C} = 0$$

$$\sum \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0 & 0 \\ 0 & 0 & 100 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.15 & 0 \\ B_X & 0 & B_Z \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.33 & 0 \\ 200 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.33 & 0.3 \\ C_X & C_Y & 0 \end{vmatrix} = 0$$

$$\sum \mathbf{M}_A = (0.15B_Z - 0.3C_Y)\mathbf{i} + (15 + 0.15B_Z + 0.3C_X)\mathbf{j}$$

$$+ (-0.15B_X - 66 - 0.15C_Y - 0.33C_X)\mathbf{k} = 0.$$

This results in three equations in four unknowns; an additional equation is provided by the sum of the forces in the x-direction (which cannot have a reaction term due to A)

$$\sum \mathbf{F}_X = (B_X + C_X + 200)\mathbf{i} = 0.$$

The four equations in four unknowns:

$$0B_X + 0.15B_Z + 0C_X - 0.3C_Z = 0$$

$$0B_X + 0.15B_Z + 0.3C_X + 0C_Y = -15$$

$$-0.15B_X + 0B_Z - 0.33C_X - 0.15C_Y = 66$$

$$B_X + 0B_Z + C_X + 0C_Z = -200.$$

(The HP-28S hand held calculator was used to solve these equations.) The solution:

$$B_X = 750 \text{ N},$$

$$B_Z = 1800 \text{ N},$$

$$C_X = -950 \text{ N},$$

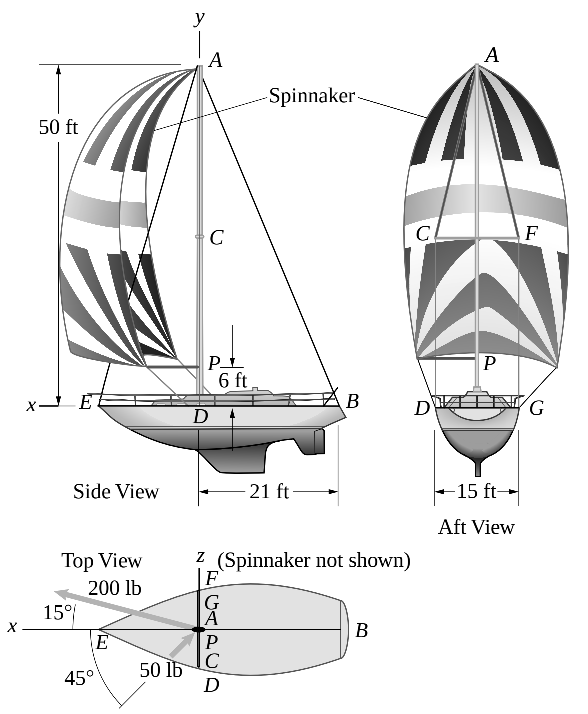
$$C_Y = 900 \text{ N}.$$

The reactions at A are determined by the sums of forces:

$$\sum \mathbf{F}_Y = (A_Y + C_Y)\mathbf{j} = 0, \text{ from which } A_Y = -C_Y = -900 \text{ N}$$

$$\sum \mathbf{F}_Z = (A_Z + B_Z + 100)\mathbf{k} = 0, \text{ from which } A_Z = -1900 \text{ N}$$

Problem 5.118 The support that attaches the sailboat's mast to the deck behaves like a ball and socket support. The line that attaches the spinnaker (the sail) to the top of the mast exerts a 200-lb force on the mast. The force is in the horizontal plane at 15° from the centerline of the boat. (See the top view.) The spinnaker pole exerts a 50-lb force on the mast at P. The force is in the horizontal plane at 45° from the centerline. (See the top view.) The mast is supported by two cables, the back stay AB and the port shroud ACD. (The forestay AE and the starboard shroud AFG are slack, and their tensions can be neglected.) Determine the tensions in the cables AB and CD and the reactions at the bottom of the mast.



Solution: Although the dimensions are not given in the sketch, assume that the point C is at the midpoint of the mast (25 ft above the deck). The position vectors for the points A, B, C, D, and P are:

$\mathbf{r}_A = 50\mathbf{j}$,

$\mathbf{r}_B = -21\mathbf{i}$,

$\mathbf{r}_P = 6\mathbf{j}$,

$\mathbf{r}_C = 25\mathbf{j} - 7.5\mathbf{k}$.

The vector parallel to the backstay AB is

$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = -21\mathbf{i} - 50\mathbf{j}$.

The unit vector parallel to backstay AB is

$\mathbf{e}_{AB} = -0.3872\mathbf{i} - 0.9220\mathbf{j}$.

The vector parallel to AC is

$\mathbf{r}_{AC} = \mathbf{r}_C - \mathbf{r}_A = -25\mathbf{j} - 7.5\mathbf{k}$.

The forces acting on the mast are: (1) The force due to the spinnaker at the top of the mast:

$\mathbf{F}_A = 200(\mathbf{i}\cos 15^\circ + \mathbf{k}\cos 75^\circ) = 193.19\mathbf{i} + 51.76\mathbf{k}$.

(2) The reaction due to the backstay:

$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|\mathbf{e}_{AB}$

(3) The reaction due to the shroud:

$\mathbf{T}_{AC} = |\mathbf{T}_{AC}|\mathbf{e}_{AC}$

(4) The force acting on the cross spar CE:

$\mathbf{T}_{CE} = -(\mathbf{k} \cdot \mathbf{T}_{AC})\mathbf{k} = 0.2873|\mathbf{T}_{AC}|\mathbf{k}$.

(5) The force due to the spinnaker pole:

$\mathbf{F}_P = 50(-0.707\mathbf{i} + 0.707\mathbf{k}) = -35.35\mathbf{i} + 35.35\mathbf{k}$.

The sum of the moments about the base of the mast is

$$\mathbf{M}_Q = \mathbf{r}_A \times \mathbf{F}_A + \mathbf{r}_A \times \mathbf{T}_{AB} + \mathbf{r}_A \times \mathbf{T}_{AC} + \mathbf{r}_C \times \mathbf{T}_{CE} + \mathbf{r}_P \times \mathbf{F}_P = 0$$

$$\sum \mathbf{M}_Q = \mathbf{r}_A \times (\mathbf{F}_A + \mathbf{T}_{AB} + \mathbf{T}_{AC}) + \mathbf{r}_C \times \mathbf{T}_{CE} + \mathbf{r}_P \times \mathbf{F}_P = 0.$$

From above,

$$\mathbf{F}_A + \mathbf{T}_{AB} + \mathbf{T}_{AC} = F_{TX}\mathbf{i} + F_{TY}\mathbf{j} + F_{TZ}\mathbf{k}$$

$$= (193.2 - (0.3872)|\mathbf{T}_{AB}|)\mathbf{i} + (-0.922|\mathbf{T}_{AB}| - 0.9578|\mathbf{T}_{AC}|)\mathbf{j} + (51.76 - 0.2873|\mathbf{T}_{AC}|)\mathbf{k}$$

$$\sum \mathbf{M}_Q = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 50 & 50 \\ F_{TX} & F_{TY} & F_{TZ} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 25 & -7.5 \\ 0 & 0 & 0.2873|\mathbf{T}_{AC}| \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 6 \\ -35.35 & 0 & 35.35 \end{vmatrix} = 0$$

$$= (50F_{TZ} + (25)(0.2873)|\mathbf{T}_{AC}| + 212.1)\mathbf{i} + (-50F_{TX} + 212.1)\mathbf{k} = 0.$$

Substituting and collecting terms:

$(2800 - 7.1829|\mathbf{T}_{AC}|)\mathbf{i} + (-9447.9 + 19.36|\mathbf{T}_{AB}|)\mathbf{k} = 0$,

from which

$$|\mathbf{T}_{AC}| = \frac{2800}{7.1829} = 389.81 \text{ lb},$$

$|\mathbf{T}_{AB}| = 488.0$ lb.

5.118 (*Continued*)

The tension in cable CD is the vertical component of the tension in AC,

$|\mathbf{T}_{CD}| = |\mathbf{T}_{AC}|(\mathbf{j} \cdot \mathbf{e}_{AC}) = |\mathbf{T}_{AC}|(0.9578) = 373.37$ lb.

The reaction at the base is found from the sums of the forces:

$$\sum \mathbf{F}_X = (Q_X + 193.19 - 35.35 - |\mathbf{T}_{AB}|(0.3872)) = 0,$$

from which $Q_X = 31.11$ lb

$$\sum \mathbf{F}_Y = (Q_Y - 0.922|\mathbf{T}_{AB}| - 0.9578|\mathbf{T}_{AC}|)\mathbf{j} = 0,$$

from which $Q_Y = 823.24$ lb

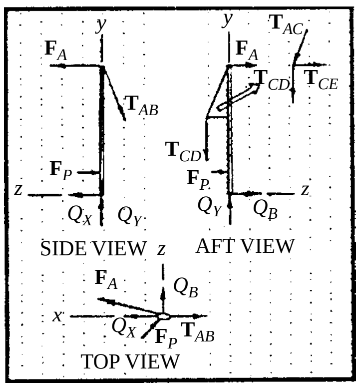
$$\sum \mathbf{F}_Z = (Q_Z + 51.76 + (0.2873|\mathbf{T}_{AC}|$$

$$- 0.2873|\mathbf{T}_{AC}| + 35.35))\mathbf{k} = 0,$$

from which $Q_Z = -87.11$ lb

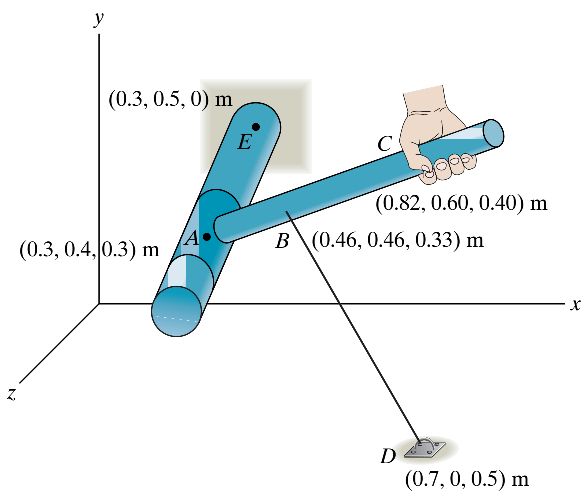
Collecting the terms, the reaction is

$\mathbf{Q} = 31.14\mathbf{i} + 823.26\mathbf{j} - 87.12\mathbf{k}$ (lb)



Problem 5.119* The bar *AC* is supported by the cable *BD* and a bearing at *A* that can rotate about the axis *AE*. The person exerts a force $\mathbf{F} = 50\mathbf{j}$ (N) at *C*. Determine the tension in the cable.

Strategy: Use the fact that the sum of the moments about the axis *AE* due to the forces acting on the free-body diagram of the bar must equal zero.



Solution: We will take moments about the line *AE* in order to eliminate all of the reactions at the bearing *A*. We have:

$$\mathbf{e}_{AE} = \frac{0.1\mathbf{j} - 0.3\mathbf{k}}{\sqrt{0.1}} = 0.316\mathbf{j} - 0.949\mathbf{k}$$

$$\mathbf{r}_{AB} = (0.16\mathbf{i} + 0.06\mathbf{j} + 0.03\mathbf{k})m,$$

$$\mathbf{T}_{BD} = T_{BD}\left(\frac{0.24\mathbf{i} - 0.46\mathbf{j} + 0.17\mathbf{k}}{\sqrt{0.2981}}\right)$$

$$\mathbf{r}_{AC} = (0.52\mathbf{i} + 0.2\mathbf{j} + 0.1\mathbf{k})m, \qquad \mathbf{F} = (50\mathbf{j})N$$

Then the equilibrium equation is

$$\sum M_{AE} = \mathbf{e}_{AE} \cdot (\mathbf{r}_{AB} \times \mathbf{T}_{BD} + \mathbf{r}_{AC} \times \mathbf{F}) = 0$$

This reduces to the single scalar equation

$$\boxed{T_{BD} = 174.5 \text{ N}}$$

Problem 5.120* In Problem 5.119, determine the reactions at the bearing *A*.

Strategy: Write the couple exerted on the free-body diagram of the bar by the bearing as $\mathbf{M}_A = M_{Ax}\mathbf{i} + M_{Ay}\mathbf{j} + M_{Az}\mathbf{k}$. Then, in addition to the equilibrium equations, obtain an equation by requiring the component of $\mathbf{M}_A$ parallel to the axis *AE* to equal zero.

Solution: See the previous problem for setup. We add the reactions (force, moment)

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}, \ \ \mathbf{M}_A = M_{Ax}\mathbf{i} + M_{Ay}\mathbf{j} + M_{Az}\mathbf{k}$$

This gives us too many reaction moments. We will add the constrain that

$$\mathbf{M}_A \cdot \mathbf{e}_{AE} = 0$$

We have the following 6 equilibrium equations:

$$\sum \mathbf{F} = A + \mathbf{T}_{BD} + \mathbf{F} = 0 \Rightarrow \begin{cases} \sum F_x : A_x + 0.440T_{BD} = 0 \\ \sum F_y : A_y - 0.843T_{BD} + 50 \text{ N} = 0 \\ \sum F_x : A_z + 0.311T_{BD} = 0 \end{cases}$$

$$\sum \mathbf{M}_A = \mathbf{M}_A + \mathbf{r}_{AB} \times \mathbf{T}_{BD} + \mathbf{r}_{AC} \times \mathbf{F} = 0$$

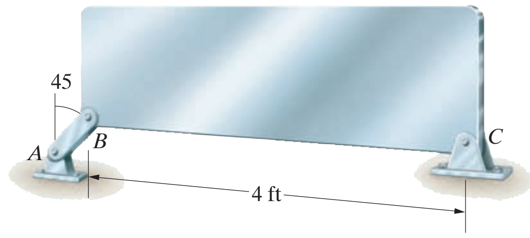
$$\Rightarrow \begin{cases} \sum M_{Ax} : M_{Ax} - 5 \text{ N-m} + (0.0440 \text{ m})T_{BD} = 0 \\ \sum M_{Ay} : M_{Ay} - (0.0366 \text{ m})T_{BD} = 0 \\ \sum M_{Az} : M_{Az} + 26 \text{ N-m} - (0.161 \text{ m})T_{BD} = 0 \end{cases}$$

$$\mathbf{e}_{AE} \cdot \mathbf{M}_A = 0 \quad \Rightarrow \ 0.316M_{Ay} - 0.949M_{Az} = 0$$

Solving these 7 equations we find

$$\boxed{\begin{array}{l} A_x = -76.7 \text{ N}, \ A_y = 97.0 \text{ N}, \ A_z = -54.3 \text{ N} \\ M_{Ax} = -2.67 \text{ N-m}, \ M_{Ay} = 6.39 \text{ N-m}, \ M_{Az} = 2.13 \text{ N-m} \end{array}}$$

Problem 5.121 In Active Example 5.10, suppose that the support at A is moved so that the angle between the bar AB and the vertical decreases from 45° to 30°. The position of the rectangular plate does not change. Draw the free-body diagram of the plate showing the point P where the lines of action of the three forces acting on the plate intersect. Determine the magnitudes of the reactions on the plate at B and C.



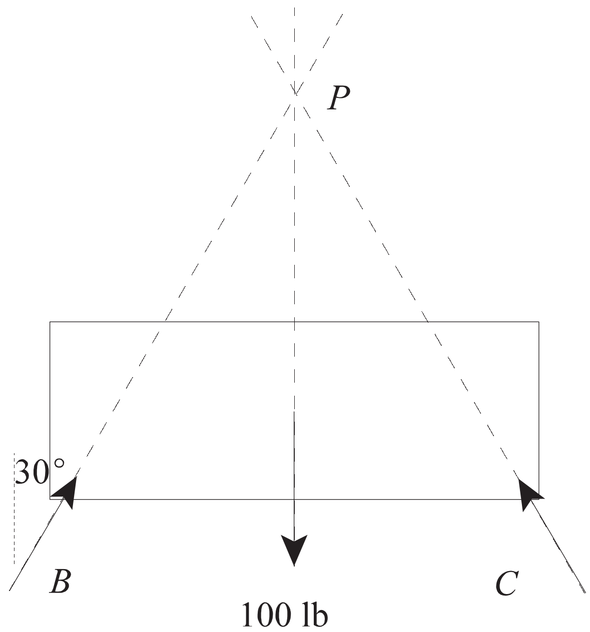
Solution: The equilibrium equations are

$\Sigma F_x : B \sin 30^\circ - C \sin 30^\circ = 0$

$\Sigma F_y : B \cos 30^\circ + C \cos 30^\circ - 100 \text{ lb} = 0$

Solving yields

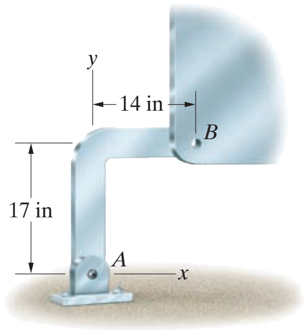
$\boxed{B = C = 57.7 \text{ lb}}$



Problem 5.122 The magnitude of the reaction exerted on the L-shaped bar at B is 60 lb. (See Example 5.11.)

(a) What is the magnitude of the reaction exerted on the bar by the support at A?

(b) What are the x and y components of the reaction exerted on the bar by the support at A?



Solution: The angle between the line AB and the x axis is

$$\theta = \tan^{-1}(17/14) = 50.5^\circ$$

(a) The bar is a two-force member, so the magnitude of the reaction at A is

$$\boxed{A = 60 \text{ lb}}$$

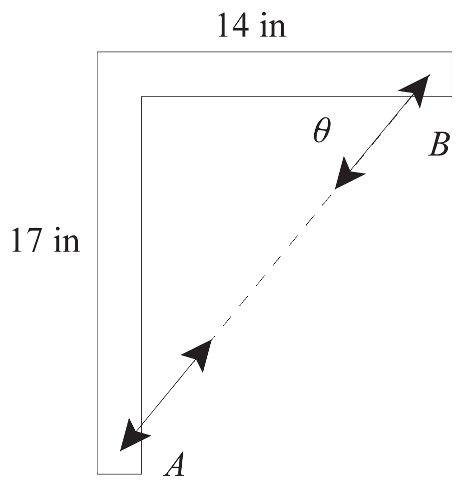
(b) The reaction at A must be parallel to the line from A to B, but it may point either from A toward B or from B toward A.
In the first case, the components are

$$A_x = (60 \text{ lb}) \cos\theta, A_y = (60 \text{ lb}) \sin\theta$$

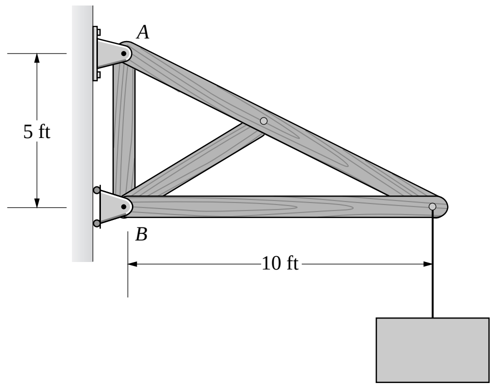
In the second case the components are

$$A_x = -(60 \text{ lb}) \cos\theta, A_y = -(60 \text{ lb}) \sin\theta$$

Thus $\boxed{A_x = 38.1 \text{ lb}, A_y = 46.3 \text{ lb or } A_x = -38.1 \text{ lb}, A_y = -46.3 \text{ lb}}$



Problem 5.123 The suspended load weighs 1000 lb. The structure is a three-force member if its weight is neglected. Use this fact to determine the magnitudes of the reactions at A and B.

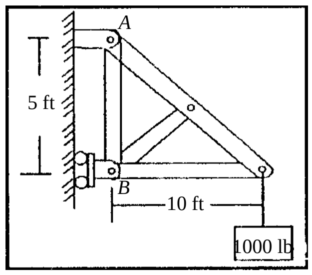


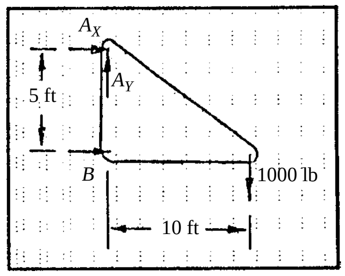
Solution: The pin support at A is a two-force reaction, and the roller support at B is a one force reaction. The moment about A is $M_A = 5B - 10(1000) = 0$, from which the magnitude at B is $B = 2000$ lb. The sums of the forces:

$$\sum F_X = A_X + B = A_X + 2000 = 0, \text{ from which } A_X = -2000 \text{ lb.}$$

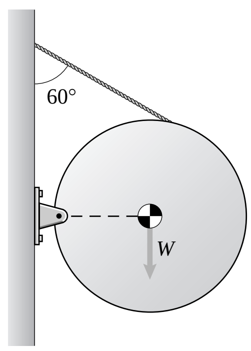
$$\sum F_Y = A_Y - 1000 = 0, \text{ from which } A_Y = 1000 \text{ lb.}$$

The magnitude at A is $A = \sqrt{2000^2 + 1000^2} = 2236$ lb





Problem 5.124 The weight $W = 50$ lb acts at the center of the disk. Use the fact that the disk is a three-force member to determine the tension in the cable and the magnitude of the reaction at the pin support.



Solution: Denote the magnitude of the reaction at the pinned joint by B. The sums of the forces are:

$$\sum F_X = B_X - T \sin 60^\circ = 0,$$

and $\sum F_Y = B_Y + T \cos 60^\circ - W = 0.$

The perpendicular distance to the action line of the tension from the center of the disk is the radius R. The sum of the moments about the center of the disk is $M_C = -RB_Y + RT = 0$, from which $B_Y = T$. Substitute into the sum of the forces to obtain: $T + T(0.5) - W = 0$, from which

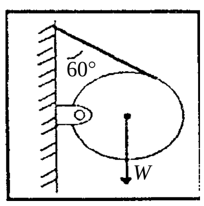
$$T = \frac{2}{3}W = 33.33 \text{ lb.}$$

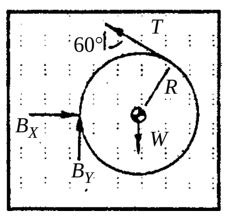
Substitute into the sum of forces to obtain

$$B_X = T \sin 60^\circ = 28.86 \text{ lb.}$$

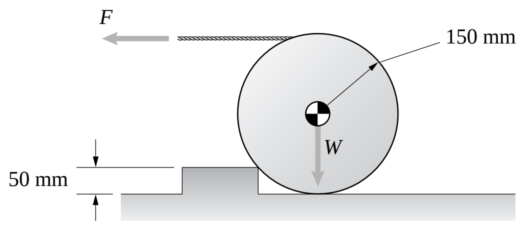
The magnitude of the reaction at the pinned joint is

$$B = \sqrt{33.33^2 + 28.86^2} = 44.1 \text{ lb}$$

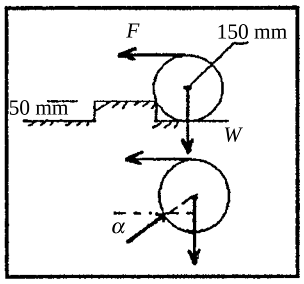




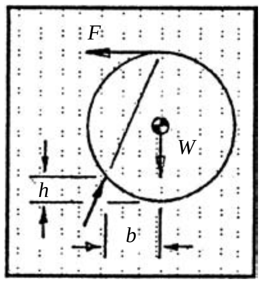
Problem 5.125 The weight $W = 40$ N acts at the center of the disk. The surfaces are rough. What force F is necessary to lift the disk off the floor?



Solution: The reaction at the obstacle acts through the center of the disk (see sketch) Denote the contact point by B. When the moment is zero about the point B, the disk is at the verge of leaving the floor, hence the force at this condition is the force required to lift the disk. The perpendicular distance from B to the action line of the weight is $d = R\cos\alpha$, where α is given by (see sketch)



$$\alpha = \sin^{-1}\left(\frac{R-h}{R}\right) = \sin^{-1}\left(\frac{150-50}{150}\right) = 41.81^\circ.$$



The perpendicular distance to the action line of the force is

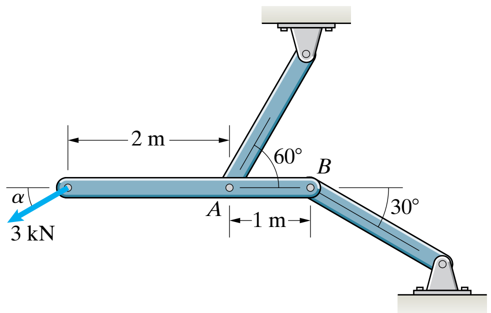
$$D = 2R - h = 300 - 50 = 250 \text{ mm}.$$

The sum of the moments about the contact point is

$$M_B = -(R\cos\alpha)W + (2R-h)F = 0,$$

from which $\boxed{F = \dfrac{(150\cos 41.81^\circ)W}{250} = 0.4472W = 17.88 \text{ N}}$

Problem 5.126 Use the fact that the horizontal bar is a three-force member to determine the angle α and the magnitudes of the reactions at A and B. Assume that $0 \le \alpha \le 90^\circ$.



Solution: The forces at A and B are parallel to the respective bars since these bars are 2-force members. Since the horizontal bar is a 3-force member, all of the forces must intersect at a point. Thus we have the following picture:

From geometry we see that

$$d = 1 \text{ m} \ \cos 30^\circ$$

$$d \sin 30^\circ = e \sin\alpha$$

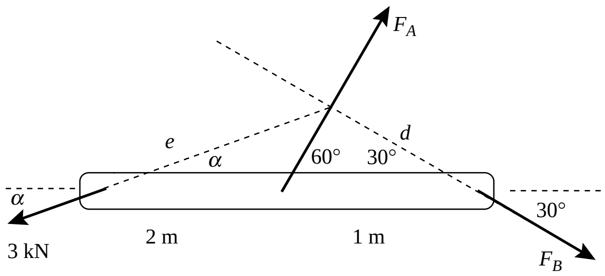
$$d \cos 30^\circ + e \cos\alpha = 3 \text{ m}$$

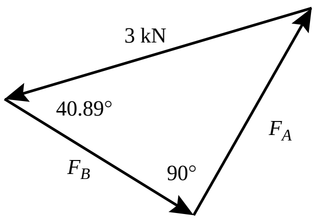
Solving we find

$$\boxed{\alpha = 10.89^\circ}$$

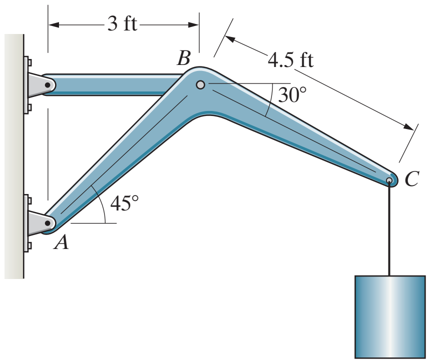
To find the other forces we look at the force triangle

$$\boxed{\begin{aligned} F_B &= 3 \text{ kN} \ \cos 40.89^\circ = 2.27 \text{ kN} \\ F_A &= 3 \text{ kN} \ \sin 40.89^\circ = 1.964 \text{ kN} \end{aligned}}$$





Problem 5.127 The suspended load weighs 600 lb. Use the fact that *ABC* is a three-force member to determine the magnitudes of the reactions at *A* and *B*.



Solution: All of the forces must intersect at a point.

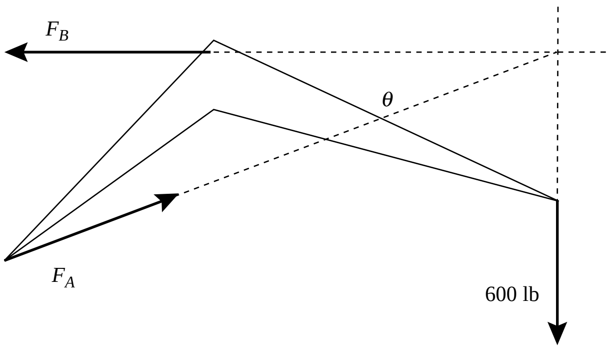
From geometry

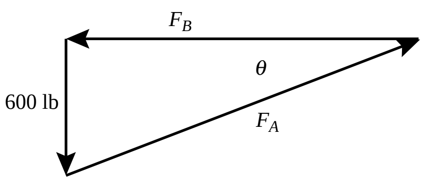
$$\tan\theta = \frac{3\text{ ft}}{(3 + 4.5\cos 30°)\,ft} = 0.435$$

$$\Rightarrow \qquad \theta = 23.5°$$

Now using the force triangle we find

$$\boxed{\begin{aligned} F_B &= 600\text{ lb}\cot\theta = 1379\text{ lb} \\ F_A &= 600\text{ lb}\csc\theta = 1504\text{ lb} \end{aligned}}$$

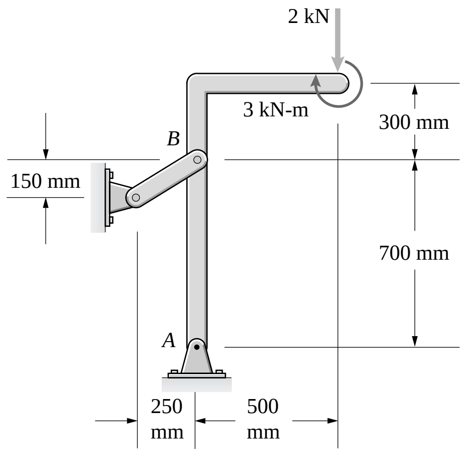




Problem 5.128 (a) Is the L-shaped bar a three-force member?

(b) Determine the magnitudes of the reactions at A and B.

(c) Are the three forces acting on the L-shaped bar concurrent?



Solution: (a) No. The reaction at B is one-force, and the reaction at A is two-force. The couple keeps the L-shaped bar from being a three force member.(b) The angle of the member at B with the horizontal is

$$\alpha = \tan^{-1}\left(\frac{150}{250}\right) = 30.96^\circ.$$

The sum of the moments about A is

$$\sum M_A = -3 - 0.5(2) + 0.7B\cos\alpha = 0,$$

from which $B = 6.6637$ kN. The sum of forces:

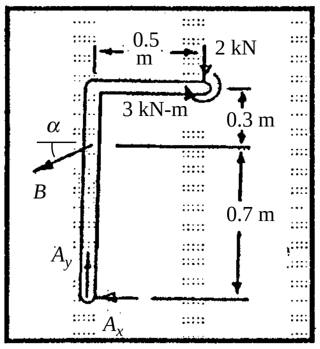
$$\sum F_X = A_X + B\cos\alpha = 0,$$

from which $A_X = -5.7143$ kN.

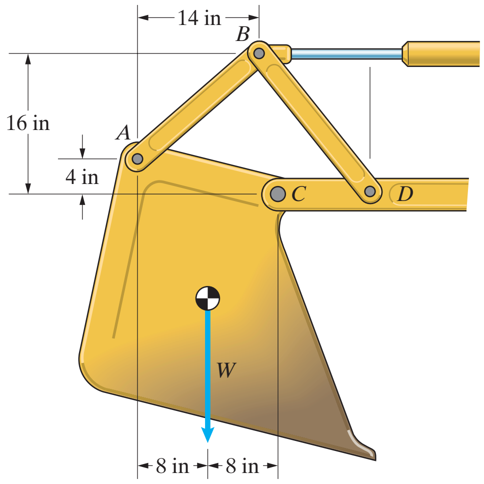
$$\sum F_Y = A_Y - B\sin\alpha - 2 = 0,$$

from which $A_Y = 5.4281$ kN. The magnitude at A:

$A = \sqrt{5.71^2 + 5.43^2} = 7.88$ kN (c) No, by inspection.



Problem 5.129 The hydraulic piston exerts a horizontal force at B to support the weight $W = 1500$ lb of the bucket of the excavator. Determine the magnitude of the force the hydraulic piston must exert. (The vector sum of the forces exerted at B by the hydraulic piston, the two-forces member AB, and the two-force member BD must equal zero.)



Solution: See the solution to Problem 5.23.

The angle between the two-force member AB and the horizontal is

$$\alpha = \tan^{-1}(12/14) = 40.6^\circ$$

and the magnitude of the force exerted at B by member AB is

$$T_{AB} = 892 \text{ lb}$$

in the direction from B toward A. Let F be the force exerted by the piston, and let T_{BD} be the force exerted at B by member BD in the direction from B toward D. The angle between member BD and the horizontal is

$$\beta = \tan^{-1}(16/12) = 53.1^\circ$$

The sum of the forces at B is

$$\Sigma F_x : F + T_{BD}\cos\beta - T_{AB}\cos\alpha = 0$$

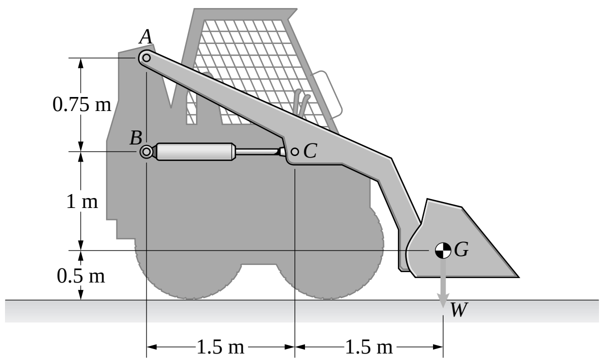
$$\Sigma F_y : -T_{BD}\sin\beta - T_{AB}\sin\alpha = 0$$

Solving yields $T_{BD} = -726$ lb, $F = 1110$ lb

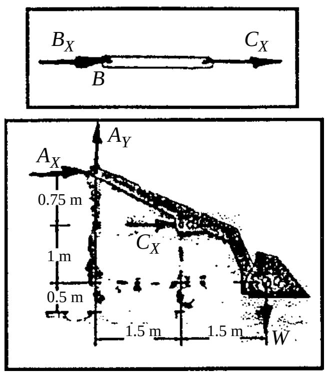
Thus $\boxed{F = 1110 \text{ lb}}$

Problem 5.130 The member ACG of the front-end loader is subjected to a load $W = 2$ kN and is supported by a pin support at A and the hydraulic cylinder BC. Treat the hydraulic cylinder as a two-force member.

(a) Draw a free-body diagrams of the hydraulic cylinder and the member ACG.
(b) Determine the reactions on the member ACG.



Solution: This is a very simple Problem. The free body diagrams are shown at the right. From the free body diagram of the hydraulic cylinder, we get the equation $B_X + C_X = 0$. This will enable us to find B_X once the loads on member ACG are known. From the diagram of ACG, the equilibrium equations are



$$\sum F_x = A_X + C_X = 0,$$

$$\sum F_y = A_Y - W = 0,$$

and $\sum M_A = (0.75)C_X - (3)W = 0.$

Using the given value for W and solving these equations, we get

$A_X = -8$ kN,

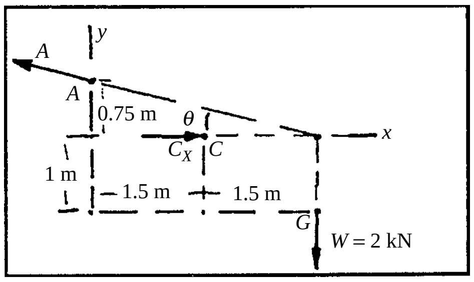
$A_Y = 2$ kN,

$C_X = 8$ kN,

and $B_X = -8$ kN.

Problem 5.131 In Problem 5.130, determine the reactions of the member ACG by using the fact that it is a three-force member.

Solution: The easiest way to do this is take advantage of the fact that for a three force member, the three forces must be concurrent. The fact that the force at C is horizontal and the weight is vertical make it very easy to find the point of concurrency. We then use this point to determine the direction of the force through A. We can even know which direction this force must take along its line — it must have an upward component to support the weight — which is down. From the geometry, we can determine the angle between the force A and the horizontal.



$\tan\theta = 0.75/3,$

or $\theta = 14.04^{\circ}$.

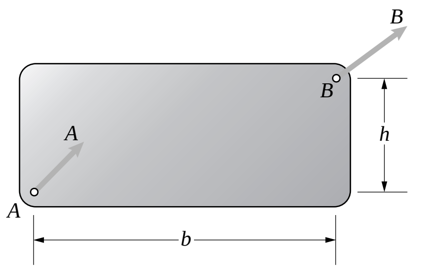
Using this, we can write force equilibrium equations in the form

$\sum F_x = -A\cos\theta + C_X = 0,$ and

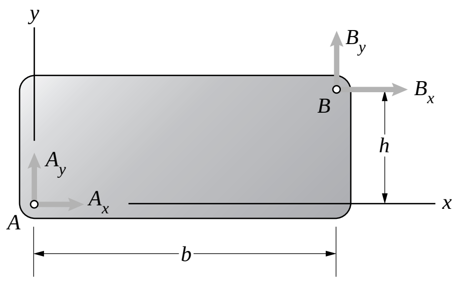
$\sum F_y = A\sin\theta - W = 0.$

Solving these equations, we get $A = 8.246$ kN, and $C_X = 8$ kN. The components of A are as calculated in Problem 5.130.

Problem 5.132 A rectangular plate is subjected to two forces A and B (Fig. a). In Fig. b, the two forces are resolved into components. By writing equilibrium equations in terms of the components A_x, A_y, B_x, and B_y, show that the two forces A and B are equal in magnitude, opposite in direction, and directed along the line between their points of application.



(a)



(b)

Solution: The sum of forces:

$$\sum F_X = A_X + B_X = 0,$$

from which $A_X = -B_X$

$$\sum F_Y = A_Y + B_Y = 0,$$

from which $A_Y = -B_y$. These last two equations show that A and B are equal and opposite in direction, (if the components are equal and opposite, the vectors are equal and opposite). To show that the two vectors act along the line connecting the two points, determine the angle of the vectors relative to the positive x axis. The sum of the moments about A is

$$M_A = B_x(h) - bB_y = 0,$$

from which the angle of direction of B is

$$\tan^{-1}\left(\frac{B_Y}{B_X}\right) = \tan^{-1}\left(\frac{h}{b}\right) = \alpha_B.$$

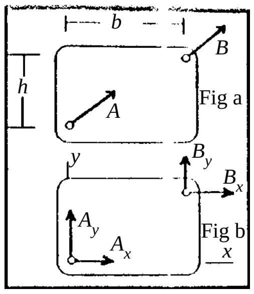
or $(180 + \alpha_B)$. Similarly, by substituting A:

$$\tan^{-1}\left(\frac{A_Y}{A_X}\right) = \tan^{-1}\left(\frac{h}{b}\right) = \alpha_A,$$

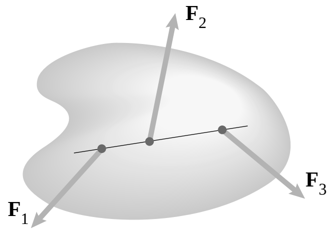
or $(180 + \alpha_A)$. But

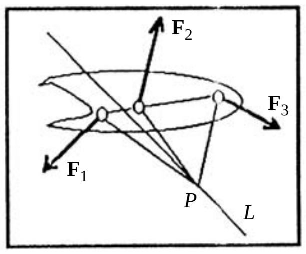
$$\alpha = \tan^{-1}\left(\frac{h}{b}\right)$$

describes direction of the line from A to B. The two vectors are opposite in direction, therefore the angles of direction of the vectors is one of two possibilities: B is directed along the line from A to B, and A is directed along the same line, oppositely to B.



Problem 5.133 An object in equilibrium is subjected to three forces whose points of application lie on a straight line. Prove that the forces are coplanar.





Solution: The strategy is to show that for a system in equilibrium under the action of forces alone, any two of the forces must lie in the same plane, hence all three must be in the same plane, since the choice of the two was arbitrary. Let P be a point in a plane containing the straight line and one of the forces, say $\mathbf{F}_2$. Let L also be a line, not parallel to the straight line, lying in the same plane as $\mathbf{F}_2$, passing through P. Let $\mathbf{e}$ be a vector parallel to this line L. First we show that the sum of the moments about any point in the plane is equal to the sum of the moments about one of the points of application of the forces. The sum of the moments about the point P:

$$\mathbf{M} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3 = 0,$$

where the vectors are the position vectors of the points of the application of the forces relative to the point P. (The position vectors lie in the plane.) Define

$$\mathbf{d}_{12} = \mathbf{r}_2 - \mathbf{r}_1,$$

and $\mathbf{d}_{13} = \mathbf{r}_3 - \mathbf{r}_1$.

Then the sum of the moments can be rewritten,

$$\mathbf{M} = \mathbf{r}_1 \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3)$$

$$+ \mathbf{d}_{12} \times \mathbf{F}_2 + \mathbf{d}_{13} \times \mathbf{F}_3 = 0.$$

Since the system is in equilibrium,

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0,$$

and the sum of moments reduces to

$$\mathbf{M} = \mathbf{d}_{12} \times \mathbf{F}_2 + \mathbf{d}_{13} \times \mathbf{F}_3 = 0,$$

which is the moment about the point of application of $\mathbf{F}_1$. (The vectors $\mathbf{d}_{12}$, $\mathbf{d}_{13}$ are parallel to the line L.) The component of the moment parallel to the line L is

$$\mathbf{e} \cdot (\mathbf{d}_{12} \times \mathbf{F}_2)\mathbf{e} + \mathbf{e} \cdot (\mathbf{d}_{13} \times \mathbf{F}_3)\mathbf{e} = 0,$$

or $\mathbf{F}_2 \cdot (\mathbf{d}_{12} \times \mathbf{e})\mathbf{e} + \mathbf{F}_3 \cdot (\mathbf{d}_{13} \times \mathbf{e})\mathbf{e} = 0$.

But by definition, $\mathbf{F}_2$ lies in the same plane as the line L, hence it is normal to the cross product $\mathbf{d}_{12} \times \mathbf{e} \neq 0$, and the term

$$\mathbf{F}_2 \cdot (\mathbf{d}_{12} \times \mathbf{e}) = 0.$$

But this means that

$$\mathbf{F}_3 \cdot (\mathbf{d}_{13} \times \mathbf{e})\mathbf{e} = 0,$$

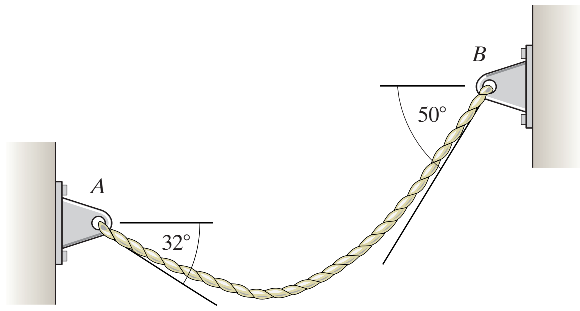
which implies that $\mathbf{F}_3$ also lies in the same plane as $\mathbf{F}_2$, since

$$\mathbf{d}_{13} \times \mathbf{e} \neq 0.$$

Thus the two forces lie in the same plane. Since the choice of the point about which to sum the moments was arbitrary, this process can be repeated to show that $\mathbf{F}_1$ lies in the same plane as $\mathbf{F}_2$. *Thus all forces lie in the same plane.*

Problem 5.134 The suspended cable weighs 12 lb.

(a) Draw the free-body diagram of the cable. (The tensions in the cable at A and B are *not* equal.)
(b) Determine the tensions in the cable at A and B.
(c) What is the tension in the cable at its lowest point?



Solution:

(a) The FBD
(b) The equilibrium equations

$$\sum F_x : -T_A \cos 32^\circ + T_B \cos 50^\circ = 0$$

$$\sum F_y : T_A \sin 32^\circ + T_B \sin 50 - 12 \text{ lb} = 0$$

Solving we find

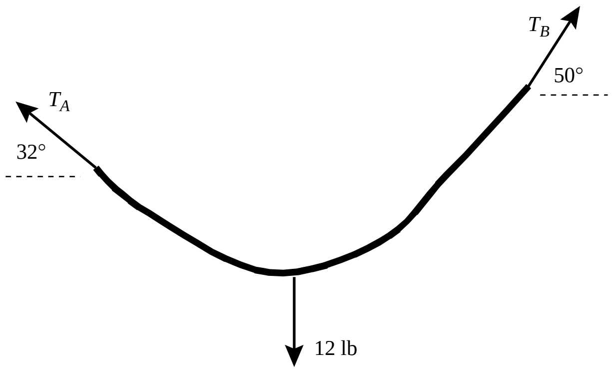
$$\boxed{T_A = 7.79 \text{ lb}, \;\; T_B = 10.28 \text{ lb}}$$

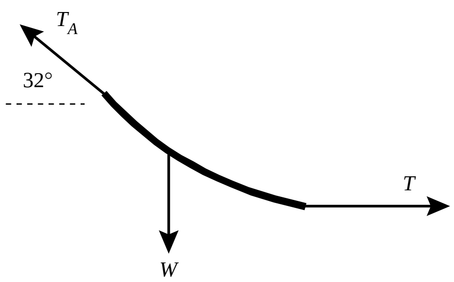
(c) Consider the FBD where W represents only a portion of the total weight. We have

$$\sum F_x : -T_A \cos 32^\circ + T = 0$$

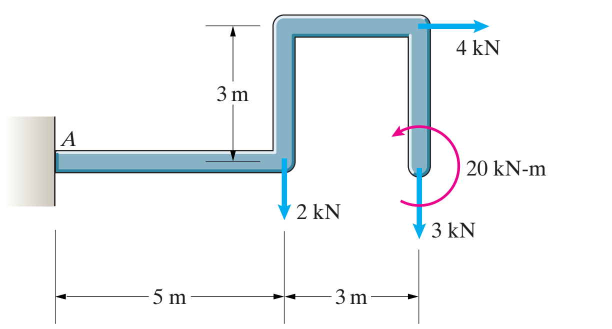
Solving

$$\boxed{T = 6.61 \text{ lb}}$$





Problem 5.135 Determine the reactions at the fixed support.



Solution: The equilibrium equations

$$\sum F_x : A_x + 4 \text{ kN} = 0$$

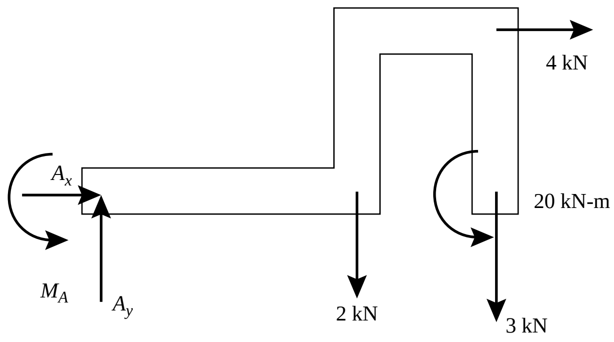
$$\sum F_y : A_y - 2 \text{ kN} - 3 \text{ kN} = 0$$

$$\sum M_A : M_A - (2 \text{ kN})(5 \text{ m}) - (4 \text{ kN})(3 \text{ m})$$

$$- (3 \text{ kN})(8 \text{ m}) + 20 \text{ kN-m} = 0$$

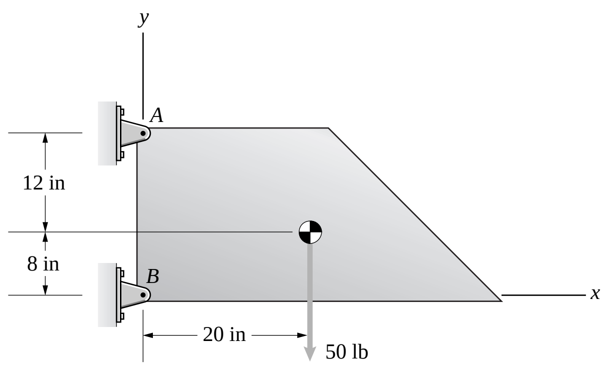
Solving

$$\boxed{A_x = -4 \text{ kN}, \; A_y = 5 \text{ kN}, \; M_A = 26 \text{ kN-m}}$$



Problem 5.136 (a) Draw the free-body diagram of the 50-lb plate, and explain why it is statically indeterminate.

(b) Determine as many of the reactions at A and B as possible.



Solution:

(a) The pin supports at A and B are two-force supports, thus there are four unknown reactions A_X, A_Y, B_X, and B_Y, but only three equilibrium equations can be written, two for the forces, and one for the moment. Thus there are four unknowns and only three equations, so the system is indeterminate.

(b) Sums the forces:

$$\sum F_X = A_X + B_X = 0,$$

or $A_X = -B_X$, and

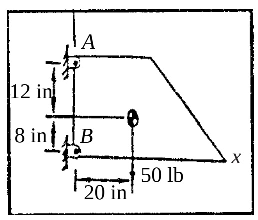
$$\sum F_Y = A_Y + B_Y - 50 = 0.$$

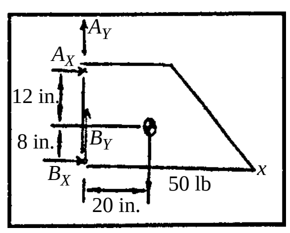
The sum of the moments about B

$$M_B = -20A_X - 50(20) = 0,$$

from which $A_X = -50$ lb,

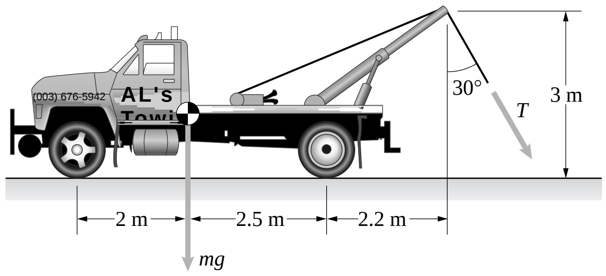
and from the sum of forces $B_X = 50$ lb.





Problem 5.137 The mass of the truck is 4 Mg. Its wheels are locked, and the tension in its cable is $T = 10$ kN.

(a) Draw the free-body diagram of the truck.
(b) Determine the normal forces exerted on the truck's wheels by the road.



Solution: The weight is $4000(9.81) = 39.24$ kN. The sum of the moments about B

$$\sum M_B = -3T\sin 30^\circ - 2.2T\cos 30^\circ + 2.5W - 4.5A_N = 0$$

from which

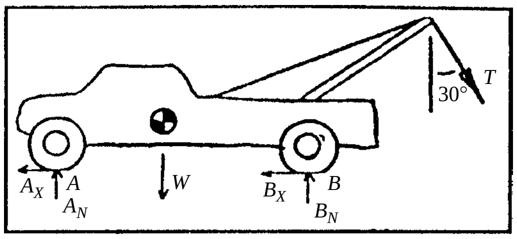
$$A_N = \frac{2.5W - T(3\sin 30^\circ + 2.2\cos 30^\circ)}{4.5}$$

$$= \frac{64.047}{4.5} = 14.23 \text{ N}$$

The sum of the forces:

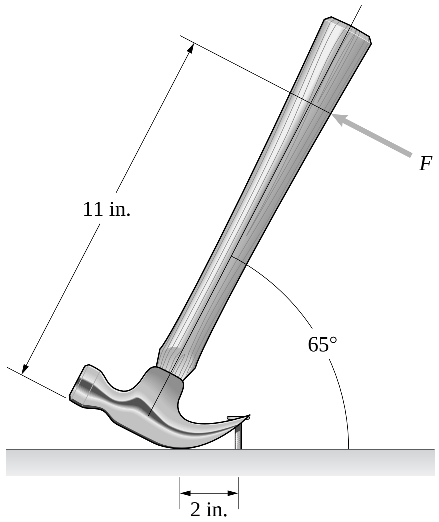
$$\sum F_Y = A_N - W + B_N - T\cos 30^\circ = 0,$$

from which $B_N = T\cos 30^\circ - A_N + W = 33.67$ N



Problem 5.138 Assume that the force exerted on the head of the nail by the hammer is vertical, and neglect the hammer's weight.

(a) Draw the free-body diagram of the hammer.
(b) If $F = 10$ lb, what are the magnitudes of the forces exerted on the nail by the hammer and the normal and friction forces exerted on the floor by the hammer?



Solution: Denote the point of contact with the floor by B. The perpendicular distance from B to the line of action of the force is 11 in. The sum of the moments about B is $M_B = 11F - 2F_N = 0$, from which the force exerted by the nail head is $F_N = \frac{11F}{2} = 5.5F$. The sum of the forces:

$$\sum F_X = -F\cos 25 + H_x = 0,$$

from which the friction force exerted on the hammer is $H_X = 0.9063F$.

$$\sum F_Y = N_H - F_N + F\sin 25^\circ = 0,$$

from which the normal force exerted by the floor on the hammer is $N_H = 5.077F$

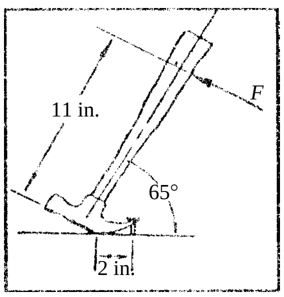
If the force on the handle is

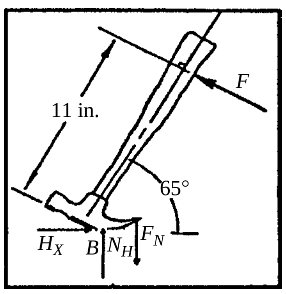
$F = 10$ lb,

then $F_N = 55$ lb,

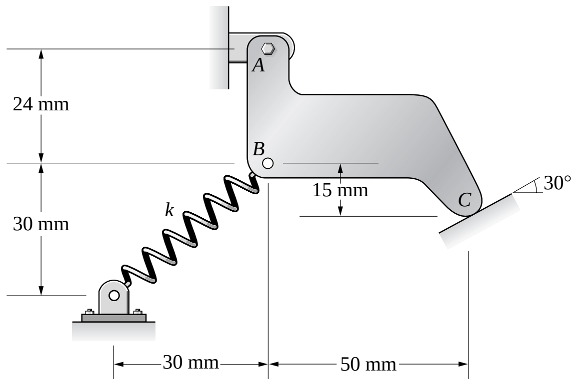
$H_X = 9.063$ lb,

and $N_H = 50.77$ lb





Problem 5.139 The spring constant is $k = 9600$ N/m and the unstretched length of the spring is 30 mm. Treat the bolt at A as a pin support and assume that the surface at C is smooth. Determine the reactions at A and the normal force at C.



Solution: The length of the spring is

$$l = \sqrt{30^2 + 30^2} \text{ mm} = \sqrt{1800} \text{ mm}$$

$$l = 42.4 \text{ mm} = 0.0424 \text{ m}$$

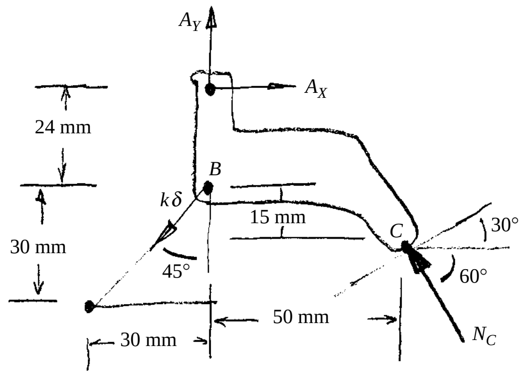
The spring force is $k\delta$ where $\delta = l - l_0$. l_0 is give as 30 mm. (We must be careful because the units for k are given as N/m) We need to use length units as all mm or all meters). k is given as 9600 N/m. Let us use $l_0 = 0.0300$ m and $l = 0.0424$ m

Equilibrium equations:

$$\sum F_X = 0: \quad A_X - k(l - l_0)\sin 45^\circ - N_C \cos 60^\circ = 0$$

$$\sum F_Y = 0: \quad A_Y - k(l - l_0)\cos 45^\circ + N_C \sin 60^\circ = 0$$

$$\sum M_B = 0: \quad (-0.024)A_X + (0.050)(N_C \sin 60^\circ) - (0.015)(N_C \cos 60^\circ) = 0$$



Solving, we get

$A_X = 126.7$ N

$A_Y = 10.5$ N

$N_C = 85.1$ N

Problem 5.140 The engineer designing the release mechanism shown in Problem 5.139 wants the normal force exerted at C to be 120 N. If the unstretched length of the spring is 30 mm, what is the necessary value of the spring constant k?

Solution: Refer to the solution of Problem 5.139. The equilibrium equations derived were

$$\sum F_X = 0: \quad A_X - k(l - l_0)\sin 45 - N_C \cos 60^\circ = 0$$

$$\sum F_Y = 0: \quad A_Y - k(l - l_0)\cos 45 + N_C \sin 60^\circ = 0$$

$$\sum M_B = 0: \quad -0.024A_X + 0.050N_C \sin 60^\circ$$

$$-0.015N_C \cos 60^\circ = 0$$

where $l = 0.0424$ m, $l_0 = 0.030$ m, $N_C = 120$ N, and A_X, A_Y, and k are unknowns.

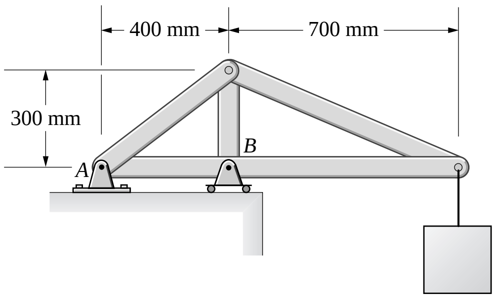
Solving, we get

$A_X = 179.0$ N,

$A_Y = 15.1$ N,

$k = 13500$ N/m

Problem 5.141 The truss supports a 90-kg suspended object. What are the reactions at the supports A and B?



Solution: Treat the truss as a single element. The pin support at A is a two force reaction support; the roller support at B is a single force reaction. The sum of the moments about A is

$$M_A = B(400) - W(1100) = 0,$$

from which $B = \dfrac{1100W}{400} = 2.75W$

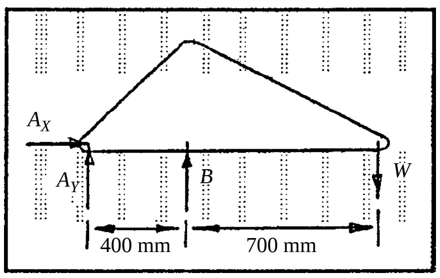
$$B = 2.75(90)(9.81) = 2427.975 = 2.43 \text{ kN}.$$

The sum of the forces:

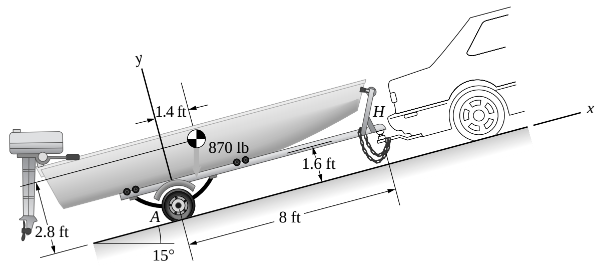
$$\sum F_X = A_X = 0$$

$$\sum F_Y = A_Y + B - W = 0,$$

from which $A_Y = W - B = 882.9 - 2427.975 = -1.545$ kN



Problem 5.142 The trailer is parked on a 15° slope. Its wheels are free to turn. The hitch H behaves like a pin support. Determine the reactions at A and H.



Solution: The coordinate system has the x axis parallel to the road. The wheels are a one force reaction normal to the road, the pin H is a two force reaction. The position vectors of the points of the center of mass and H are:

$\mathbf{r}_W = 1.4\mathbf{i} + 2.8\mathbf{j}$ ft and

$\mathbf{r}_H = 8\mathbf{i} + 1.6\mathbf{j}$.

The angle of the weight vector realtive to the positive x axis is

$\alpha = 270^\circ - 15^\circ = 255^\circ$.

The weight has the components

$$\mathbf{W} = W(\mathbf{i}\cos 255^\circ + \mathbf{j}\sin 255^\circ) = 870(-0.2588\mathbf{i} - 0.9659\mathbf{j})$$

$$= -225.173\mathbf{i} - 840.355\mathbf{j} \text{ (lb)}.$$

The sum of the moments about H is

$$\mathbf{M}_H = (\mathbf{r}_W - \mathbf{r}_H) \times \mathbf{W} + (\mathbf{r}_A - \mathbf{r}_H) \times \mathbf{A},$$

$$\mathbf{M}_H = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6.6 & 1.2 & 0 \\ -225.355 & -840.355 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & -1.6 & 0 \\ 0 & A_Y & 0 \end{vmatrix} = 0$$

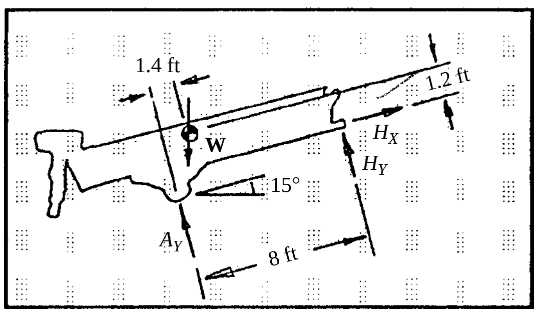
$$= 5816.55 - 8A_Y = 0,$$

from which $A_Y = \dfrac{5816.55}{8} = 727.1$ lb.

The sum of the forces is

$$\sum \mathbf{F}_X = (H_X - 225.173)\mathbf{i} = 0, \text{ from which } H_X = 225.2 \text{ lb},$$

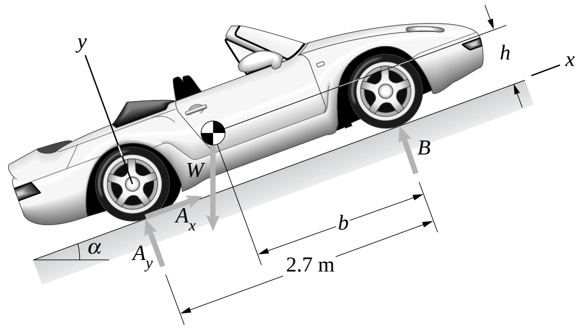
$$\sum \mathbf{F}_Y = (A_Y + H_Y - 840.355)\mathbf{j} = 0, \text{ from which } H_Y = 113.3 \text{ lb}$$



Problem 5.143 To determine the location of the point where the weight of a car acts (the *center of mass*), an engineer places the car on scales and measures the normal reactions at the wheels for two values of α, obtaining the following results:

α	A_y (kN)	B (kN)
10°	10.134	4.357
20°	10.150	3.677

What are the distances b and h?



Solution: The position vectors of the cm and the point B are

$$\mathbf{r}_{CM} = (2.7 - b)\mathbf{i} + h\mathbf{j},$$

$$\mathbf{r}_B = 2.7\mathbf{i}.$$

The angle between the weight and the positive x axis is $\beta = 270 - \alpha$. The weight vector at each of the two angles is

$$\mathbf{W}_{10} = W(\mathbf{i}\cos 260^\circ + \mathbf{j}\sin 260^\circ)$$

$$\mathbf{W}_{10} = W(-0.1736\mathbf{i} - 0.9848\mathbf{j})$$

$$\mathbf{W}_{20} = W(\mathbf{i}\cos 250^\circ + \mathbf{j}\sin 250^\circ) \text{ or}$$

$$\mathbf{W}_{20} = W(-0.3420\mathbf{i} - 0.9397\mathbf{j})$$

The weight W is found from the sum of forces:

$$\sum F_Y = A_Y + B_Y + W\sin\beta = 0,$$

from which $W_\beta = \dfrac{A_Y + B_Y}{\sin\beta}$.

Taking the values from the table of measurements:

$$W_{10} = -\frac{10.134 + 4.357}{\sin 260^\circ} = 14.714 \text{ kN},$$

$$\left[check: W_{20} = -\frac{10.150 + 3.677}{\sin 250^\circ} = 14.714 \text{ kN } check\right]$$

The moments about A are

$$\mathbf{M}_A = \mathbf{r}_{CM} \times \mathbf{W} + \mathbf{r}_B \times \mathbf{B} = 0.$$

Taking the values at the two angles:

$$\mathbf{M}_A^{10} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 - b & h & 0 \\ -2.5551 & -14.4910 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 0 & 0 \\ 0 & 4.357 & 0 \end{vmatrix} = 0$$

$$= 14.4903b + 2.5551h - 27.3618 = 0$$

$$\mathbf{M}_A^{20} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 - b & h & 0 \\ -5.0327 & -13.8272 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 0 & 0 \\ 0 & 3.677 & 0 \end{vmatrix}$$

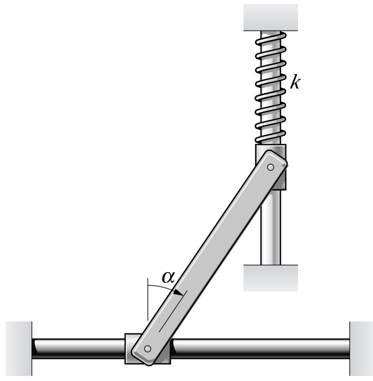
$$= 013.8272b + 5.0327h - 27.4054 = 0$$

These two simultaneous equations in two unknowns were solved using the HP-28S hand held calculator.

$b = 1.80$ m,

$h = 0.50$ m

Problem 5.144 The bar is attached by pin supports to collars that slide on the two fixed bars. Its mass is 10 kg, it is 1 m in length, and its weight acts at its midpoint. Neglect friction and the masses of the collars. The spring is unstretched when the bar is vertical ($\alpha = 0$), and the spring constant is $k = 100$ N/m. Determine the values of α in the range $0 \le \alpha \le 60^\circ$ at which the bar is in equilibrium.



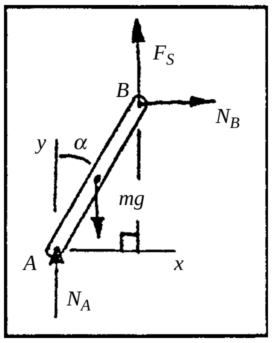
Solution: The force exerted by the spring is given by $F_S = k(L - L\cos\alpha)$. The equations of equilibrium, from the free body diagram, are

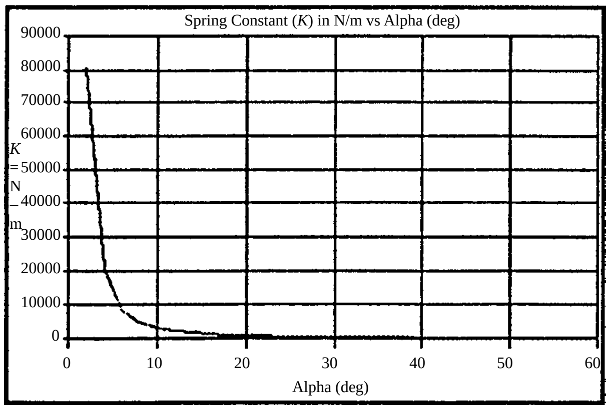
$$\sum F_x = N_B = 0,$$

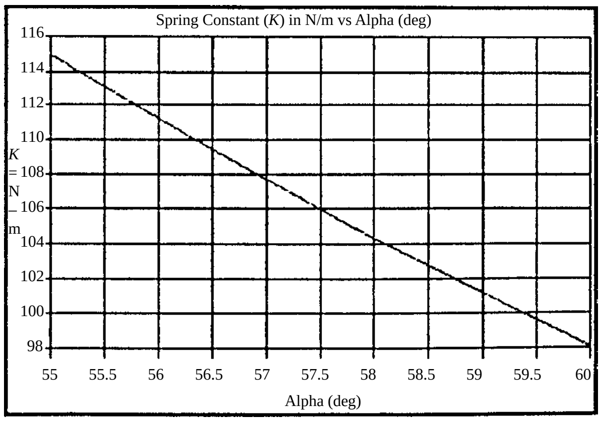
$$\sum F_y = F_S + N_A - mg = 0,$$

and $\sum M_B = (-L\sin\alpha)N_A + \left(\frac{L}{2}\sin\alpha\right) mg = 0.$

These equations can be solved directly with most numerical solvers and the required plot can be developed. The plot over the given α range is shown at the left and a zoom-in is given at the right. The solution and the plot were developed with the ***TK Solver Plus*** commercial software package. From the plot, the required equilibrium value is $\alpha \cong 59.4^\circ$.

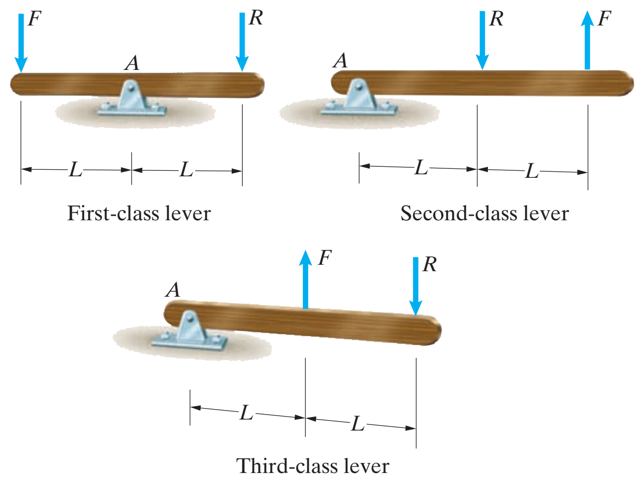






Problem 5.145 With each of the devices shown you can support a load R by applying a force F. They are called levers of the first, second, and third class.

(a) The ration R/F is called the *mechanical advantage*. Determine the mechanical advantage of each lever.
(b) Determine the magnitude of the reaction at A for each lever. (Express you answers in terms of F)



Solution:

(a) First Class $\Sigma M_A : LF - LR = 0 \Rightarrow F = R$ $\boxed{R/F = 1}$

Second Class $\Sigma M_A : 2LF - LR = 0 \Rightarrow F = R/2$ $\boxed{R/F = 2}$

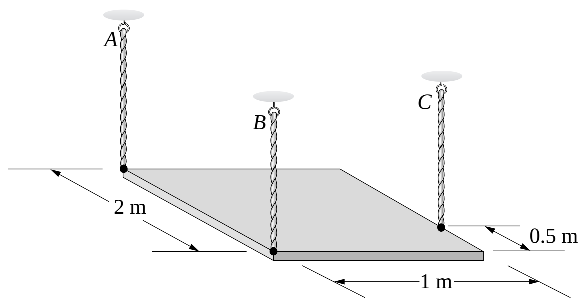
Third Class $\Sigma M_A : LF - 2LR = 0 \Rightarrow F = 2R$ $\boxed{R/F = 1/2}$

(b) First Class $\Sigma F_y : -F + A - R = 0 \Rightarrow$ $\boxed{A = 2F}$

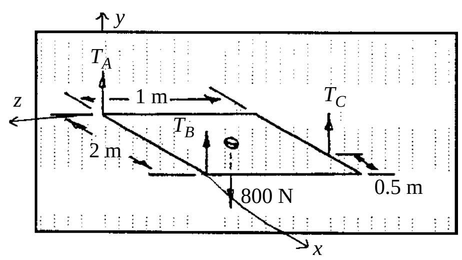
Second Class $\Sigma F_y : A - R + F = 0 \Rightarrow$ $\boxed{A = F}$

Third Class $\Sigma F_y : A + F - R = 0 \Rightarrow$ $\boxed{A = -F/2}$

Problem 5.146 The force exerted by the weight of the horizontal rectangular plate is 800 N. The weight of the plate acts at its midpoint. If you represent the reactions exerted on the plate by the three cables by a single equivalent force, what is the force, and where does its line of action intersect the plate?



Solution: The equivalent force must equal the sum of the reactions: $F_{EQ} = T_A + T_B + T_C$. $F_{EQ} = 300 + 100 + 400 = 800$ N. The moment due to the action of the equivalent force must equal the moment due to the reactions: The moment about A is

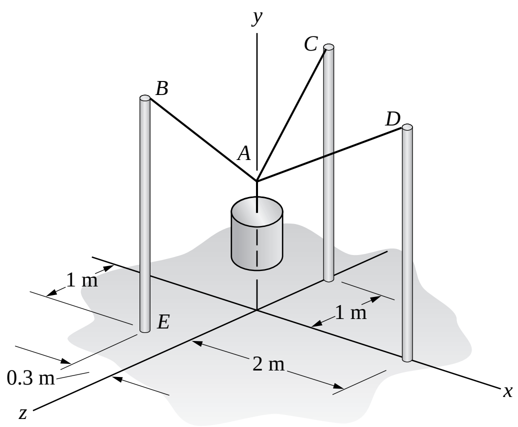


$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & 100 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 0 & -1 \\ 0 & 400 & 0 \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 0 & 800 & 0 \end{vmatrix}$$

$$\mathbf{M}_A = 400\mathbf{i} + 800\mathbf{k} = -(800z)\mathbf{i} + (800x)\mathbf{k},$$

from which $z = -0.5$ m, and $x = 1$ m, which corresponds to the midpoint of the plate. *Thus the equivalent force acts upward at the midpoint of the plate.*

Problem 5.147 The 20-kg mass is suspended by cables attached to three vertical 2-m posts. Point A is at (1, 1.2, 0) m. Determine the reactions at the built-in support at E.



Solution: All distances will be in meters, all forces in Newtons, and all moments in Newton-meters. To solve the three dimensional point equilibrium problem at A, we will need unit vectors $\mathbf{e}_{AB}$, $\mathbf{e}_{AC}$, and $\mathbf{e}_{AD}$. To determine these, we need the coordinates of points A, B, C, and D. The rest of the problem will require knowing where points E, G (under C), and H(under D) are located. From the diagram, the required point locations are A (0, 1.2, 0), B (−0.3, 2, 1), C (0, 2, −1), D (2, 2, 0), E (−0.3, 0, 1), G(0, 0, −1), and H(2, 0, 0). The required unit vectors are calculated from the coordinates of the points of the ends of the lines defining the vector. These are

$$\mathbf{e}_{AB} = -0.228\mathbf{i} + 0.608\mathbf{j} + 0.760\mathbf{k},$$

$$\mathbf{e}_{AC} = 0\mathbf{i} + 0.625\mathbf{j} - 0.781\mathbf{k},$$

and $\mathbf{e}_{AD} = 0.928\mathbf{i} + 0.371\mathbf{j} + 0\mathbf{k}$.

The force $\mathbf{T}_{AB}$ in cable AB can be written as

$$\mathbf{T}_{AB} = T_{ABX}\mathbf{i} + T_{ABY}\mathbf{j} + T_{ABZ}\mathbf{k},$$

where $T_{ABX} = |\mathbf{T}_{AB}|e_{ABX}$, etc. Similar equations can be written for the forces in AC and AD. The free body diagram of point A yields the following three equations of equilibrium.

$$\sum F_x = T_{ABX} + T_{ACX} + T_{ADX} = 0,$$

$$\sum F_y = T_{ABY} + T_{ACY} + T_{ADY} - W = 0,$$

and $\sum F_z = T_{ABZ} + T_{ACZ} + T_{ADZ} = 0,$

where $W = mg = (20)(9.81) = 196.2$ N. Solving the equations above after making the substitutions related to the force components yields the tensions in the cables. They are

$$|\mathbf{T}_{AB}| = 150 \text{ N},$$

$|\mathbf{T}_{AC}| = 146$ N, and

$$|\mathbf{T}_{AD}| = 36.9 \text{ N}.$$

Now that we know the tensions in the cables, we are ready to tackle the reactions at E (also G and H). The first step is to draw the free body diagram of the post EB and to write the equations of equilibrium for the post. A key point is to note that the force on the post from cable AB is opposite in direction to the force found in the first part of the problem. The equations of equilibrium for post EB are

$$\sum F_x = E_X - T_{ABX} = 0,$$

$$\sum F_y = E_Y - T_{ABY} = 0,$$

$$\sum F_z = E_Z - T_{ABZ} = 0,$$

and, summing moments around the base point E,

$$\sum \mathbf{M} = \mathbf{M}_E + (2\mathbf{j}) \times (-\mathbf{T}_{AB}) = 0.$$

The couple $\mathbf{M}_E$ is the couple exerted on the post by the built in support. Solving these equations, we get

$$\mathbf{E} = -34.2\mathbf{i} + 91.3\mathbf{j} + 114.1\mathbf{k} \text{ N}$$

and $\mathbf{M}_E = 228.1\mathbf{i} + 0\mathbf{j} + 68.44\mathbf{k}$ N-m.

Also, $|\mathbf{M}_E| = 238.2$ N-m.

Using a procedure identical to that followed for post EB above, we can find the built-in support forces and moments for posts CG and DH. The results for CG are:

$$G = 0\mathbf{i} + 91.3\mathbf{j} - 114.1\mathbf{k} \text{ N}$$

and $\mathbf{M}_G = -228.1\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$ N-m.

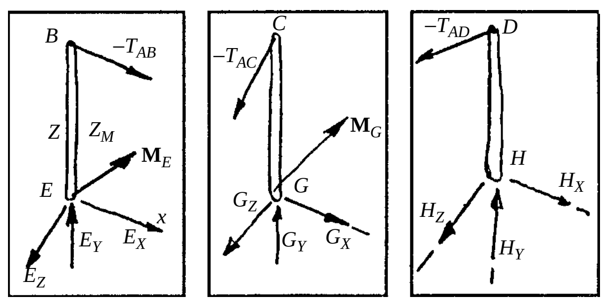
Also, $|\mathbf{M}_G| = 228.1$ N-m.

The results for DH are:

$$\mathbf{H} = 34.2\mathbf{i} + 13.7\mathbf{j} + 0 \text{ kN}$$

and $\mathbf{M}_H = 0\mathbf{i} + 0\mathbf{j} + 68.4\mathbf{k}$ N-m.

Also, $|\mathbf{M}_G| = 68.4$ N-m

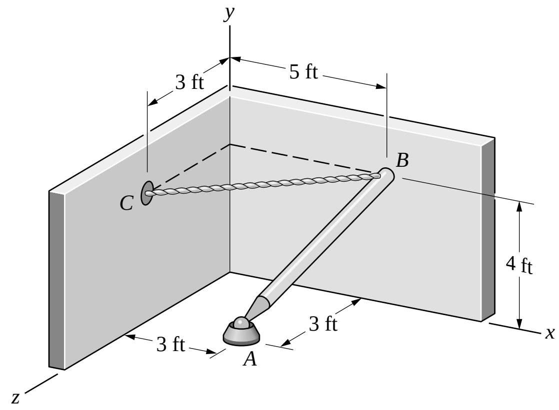


Problem 5.148 In Problem 5.147, the built-in support of each vertical post will safely support a couple of 800 N-m magnitude. Based on this criterion, what is the maximum safe value of the suspended mass?

Solution: We have all of the information necessary to solve this problem in the solution to Problem 5.147 above. All of the force and moment equations are linear and we know from the solution that a 20 kg mass produces a couple of magnitude 238.2 N-m at support E and that the magnitudes of the couples at the other two supports are smaller than this. All we need to do is scale the Problem. The scale factor is $f = 800/238.2 = 3.358$ and the maximum value for the suspended mass is $m_{\max} = 20f = 67.16$ kg

Problem 5.149 The 80-lb bar is supported by a ball and socket support at A, the smooth wall it leans against, and the cable BC. The weight of the bar acts at its midpoint.

(a) Draw the free-body diagram of the bar.
(b) Determine the tension in cable BC and the reactions at A.



Solution: (a) The ball and socket is a three reaction force support; the cable and the smooth wall are each one force reaction supports. (b) The coordinates of the points A, B and C are A (3, 0, 3), B (5, 4, 0), and C(0, 4, 3).

The vector parallel to the bar is

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}.$$

The length of the bar is

$$|\mathbf{r}_{AB}| = \sqrt{2^2 + 4^2 + 3^2} = 5.3852.$$

The unit vector parallel to the bar is

$$\mathbf{e}_{AB} = 0.3714\mathbf{i} + 0.7428\mathbf{j} - 0.5571\mathbf{k}.$$

The vector parallel to the cable is

$$\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = -5\mathbf{i} + 3\mathbf{k}.$$

The unit vector parallel to the cable is

$$\mathbf{e}_{BC} = -0.8575\mathbf{i} + 0.5145\mathbf{k}.$$

The cable tension is $\mathbf{T} = |\mathbf{T}|\mathbf{e}_{BC}$. The point of application of the weight relative to A is

$$\mathbf{r}_{AW} = 2.6936\mathbf{e}_{AB}$$

$$\mathbf{r}_{AW} = 1.000\mathbf{i} + 2.000\mathbf{j} - 1.500\mathbf{k}.$$

The reaction at B is $\mathbf{B} = |\mathbf{B}|\mathbf{k}$, since it is normal to a wall in the $y-z$ plane. The sum of the moments about A is

$$\mathbf{M}_A = \mathbf{r}_{AW} \times \mathbf{W} + \mathbf{r}_{AB} \times \mathbf{B} + \mathbf{r}_{AB} \times \mathbf{T} = 0$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1.5 \\ 0 & -80 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -3 \\ 0 & 0 & |\mathbf{B}| \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -3 \\ -0.8575 & 0 & 0.5145 \end{vmatrix} |\mathbf{T}| = 0$$

$$\mathbf{M}_A = (-120 + 4|\mathbf{B}| + 2.058|\mathbf{T}|)\mathbf{i} - (2|\mathbf{B}| - 1.544|\mathbf{T}|)\mathbf{j}$$

$$+ (3.43|\mathbf{T}| - 80)\mathbf{k} = 0.$$

Solve:

$$|\mathbf{T}| = \frac{80}{3.43} = 23.32 \text{ lb}$$

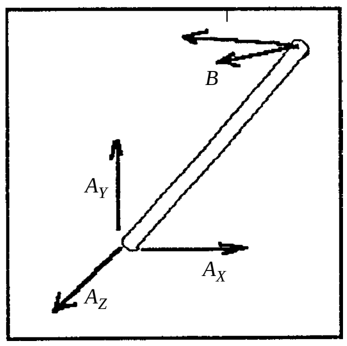
$$|\mathbf{B}| = \frac{120 - 2.058|\mathbf{T}|}{4} = 18.00 \text{ lb}.$$

The reactions at A are found from the sums of forces:

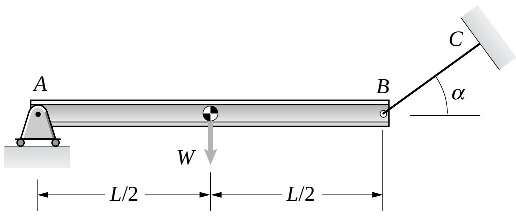
$$\sum F_X = A_X - |\mathbf{T}|(0.8575) = 0 \text{ from which } A_X = 20 \text{ lb}$$

$$\sum F_Y = A_Y - 80 = 0, \text{ from which } A_Y = 80 \text{ lb}$$

$$\sum F_Z = A_Z + |\mathbf{T}|(0.5145) + |\mathbf{B}| = 0, \text{ from which } A_Z = -30 \text{ lb}$$



Problem 5.150 The horizontal bar of weight W is supported by a roller support at A and the cable BC. Use the fact that the bar is a three-force member to determine the angle α, the tension in the cable, and the magnitude of the reaction at A.



Solution: The sum of the moments about B is

$$M_B = -LA_Y + \left(\frac{L}{2}\right) W = 0,$$

from which $A_Y = \frac{W}{2}$. The sum of the forces:

$$\sum F_X = T\cos\alpha = 0,$$

from which $\mathbf{T} = 0$ or $\cos\alpha = 0$. The choice is made from the sum of forces in the y-direction:

$$\sum F_Y = A_Y - W + T\sin\alpha = 0,$$

from which $T\sin\alpha = W - A_Y = \frac{W}{2}$. This equation cannot be satisfied if $T = 0$, hence $\cos\alpha = 0$, or $\alpha = 90^\circ$, and $T = \frac{W}{2}$