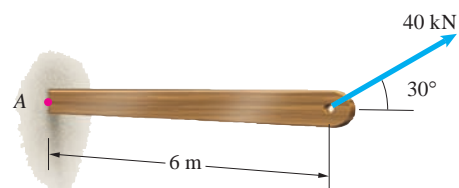


Problem 4.1 In Active Example 4.1, the 40-kN force points 30° above the horizontal. Suppose that the force points 30° below the horizontal instead. Draw a sketch of the beam with the new orientation of the force. What is the moment of the force about point A?



Solution: The perpendicular distance from A to the line of action of the force is unchanged

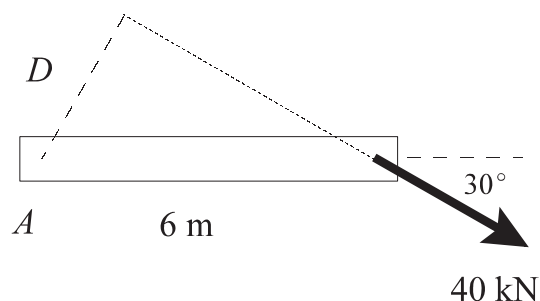
$$D = (6 \text{ m}) \sin 30^\circ = 3 \text{ m}$$

The magnitude of the moment is therefore unchanged

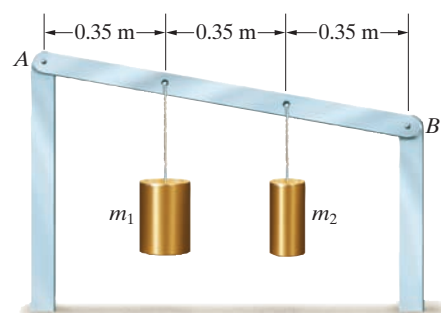
$$M = (3 \text{ m})(40 \text{ kN}) = 120 \text{ kN}\cdot\text{m}$$

However, with its new orientation, the force would tend to cause the beam to rotate about A in the clockwise direction. The moment is clockwise

$$M = 120 \text{ kN}\cdot\text{m clockwise}$$



Problem 4.2 The mass $m_1 = 20 \text{ kg}$. The magnitude of the total moment about B due to the forces exerted on bar AB by the weights of the two suspended masses is 170 N·m. What is the magnitude of the total moment due to the forces about point A?



Solution: The total moment about B is

$$M_B = m_2(9.81 \text{ m/s}^2)(0.35 \text{ m}) + (20 \text{ kg})(9.81 \text{ m/s}^2)(0.7 \text{ m})$$

$$= 170 \text{ N}\cdot\text{m}$$

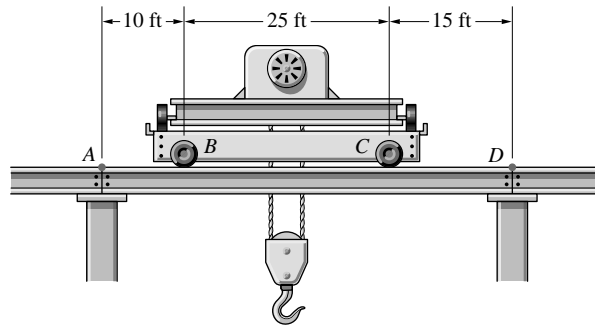
Solving, we find $m_2 = 9.51 \text{ kg}$

The moment about A is then

$$|M_A| = (20 \text{ kg})(9.81 \text{ m/s}^2)(0.35 \text{ m}) + (9.51 \text{ kg})(9.81 \text{ m/s}^2)(0.7 \text{ m})$$

$$|M_A| = 134 \text{ N}\cdot\text{m}$$

Problem 4.3 The wheels of the overhead crane exert downward forces on the horizontal I-beam at B and C . If the force at B is 40 kip and the force at C is 44 kip, determine the sum of the moments of the forces on the beam about (a) point A , (b) point D .



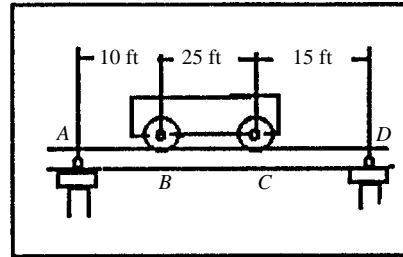
Solution: Use 2-dimensional moment strategy: determine normal distance to line of action D ; calculate magnitude DF ; determine sign. Add moments.

- (a) The normal distances from A to the lines of action are $D_{AB} = 10$ ft, and $D_{AC} = 35$ ft. The moments are clockwise (negative). Hence,

$$\sum M_A = -10(40) - 35(44) = -1940 \text{ ft-kip}$$

- (b) The normal distances from D to the lines of action are $D_{DB} = 40$ ft, and $D_{DC} = 15$ ft. The actions are positive; hence

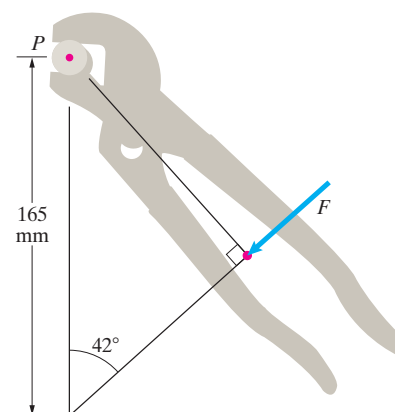
$$\sum M_D = +(40)(40) + (15)(44) = 2260 \text{ ft-kip}$$



Problem 4.4 What force F applied to the pliers is required to exert a 4 N-m moment about the center of the bolt at P ?

Solution:

$$M_P = 4 \text{ N-m} = F(0.165 \text{ m} \sin 42^\circ) \Rightarrow F = \frac{4 \text{ N-m}}{0.165 \text{ m} \sin 42^\circ} = 36.2 \text{ N}$$



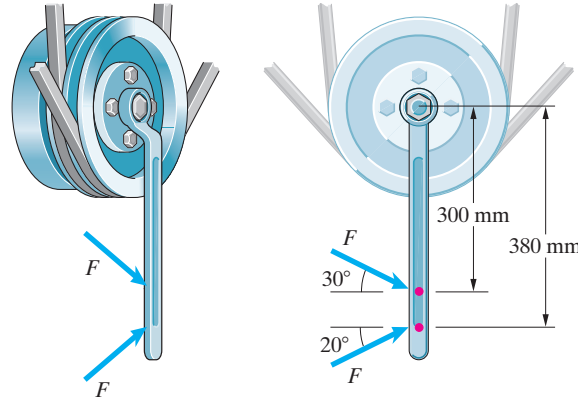
Problem 4.5 Two forces of equal magnitude F are applied to the wrench as shown. If a 50 N-m moment is required to loosen the nut, what is the necessary value of F ?

Solution:

$$\sum M_{\text{nut center}} = (F \cos 30^\circ)(0.3 \text{ m}) + (F \cos 20^\circ)(0.38 \text{ m})$$

$$= 50 \text{ N-m}$$

$$F = \frac{50 \text{ N-m}}{(0.3 \text{ m}) \cos 30^\circ + (0.38 \text{ m}) \cos 20^\circ} = 81.1 \text{ N}$$



Problem 4.6 The force $F = 8 \text{ kN}$. What is the moment of the force about P ?

Solution: The angle between the force \mathbf{F} and the x axis is

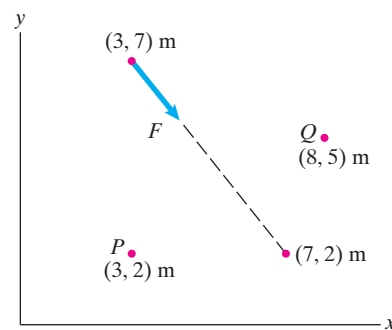
$$\alpha = \tan^{-1}(5/4) = 51.3^\circ$$

The force can then be written

$$\mathbf{F} = (8 \text{ kN})(\cos \alpha \mathbf{i} - \sin \alpha \mathbf{j}) = (5.00 \mathbf{i} - 6.25 \mathbf{j}) \text{ kN}$$

The line of action of the \mathbf{j} component passes through P , so it exerts no moment about P . The moment of the \mathbf{i} component about P is clockwise, and its magnitude is

$$M_P = (5 \text{ m})(5.00 \text{ kN}) = 25.0 \text{ kN-m} \quad \boxed{M_P = 25.0 \text{ kN-m clockwise}}$$



Problem 4.7 If the magnitude of the moment due to the force F about Q is 30 kN-m, what is F ?

Solution: The angle between the force \mathbf{F} and the x axis is

$$\alpha = \tan^{-1}(5/4) = 51.3^\circ$$

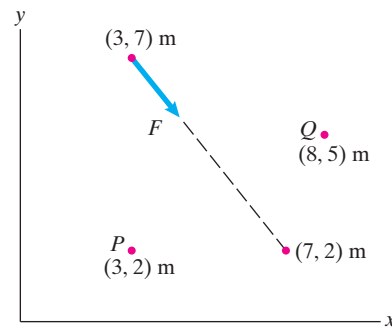
The force can then be written

$$\mathbf{F} = F(\cos \alpha \mathbf{i} - \sin \alpha \mathbf{j}) = F(0.625 \mathbf{i} - 0.781 \mathbf{j})$$

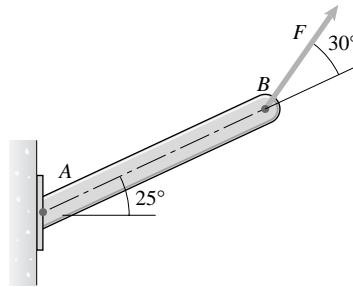
Treating counterclockwise moment as positive, the total moment about point Q is

$$M_Q = (0.781F)(5 \text{ m}) - (0.625F)(2 \text{ m}) = 30 \text{ kN-m}$$

Solving, we find $\boxed{F = 11.3 \text{ kN}}$



Problem 4.8 The support at the left end of the beam will fail if the moment about A of the 15-kN force F exceeds 18 kN·m. Based on this criterion, what is the largest allowable length of the beam?



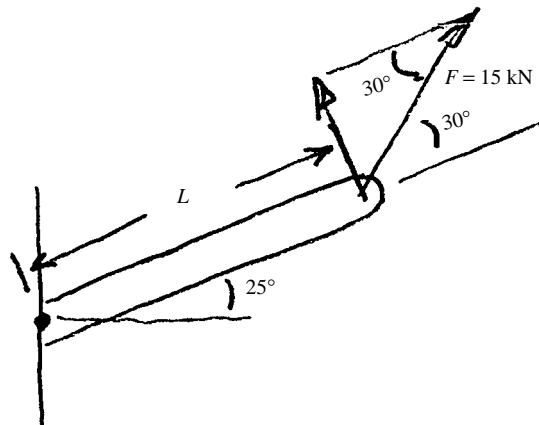
Solution:

$$M_A = L \cdot F \sin 30^\circ = L \left(\frac{15}{2} \right)$$

$$M_A = 7.5 L \text{ kN} \cdot \text{m}$$

set $M_A = M_{A_{\max}} = 18 \text{ kN} \cdot \text{m} = 7.5 L_{\max}$

$$L_{\max} = 2.4 \text{ m}$$



Problem 4.9 The length of the bar AP is 650 mm. The radius of the pulley is 120 mm. Equal forces $T = 50 \text{ N}$ are applied to the ends of the cable. What is the sum of the moments of the forces (a) about A; (b) about P.

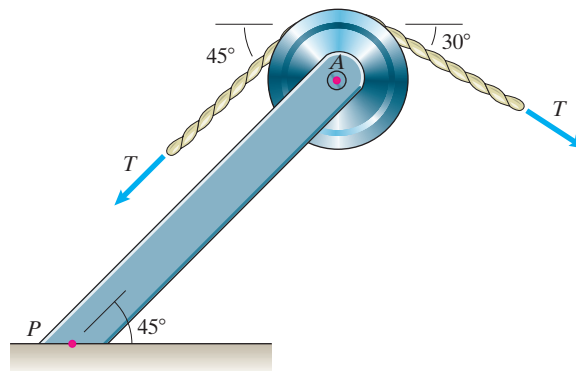
Solution:

(a) $\sum M_A = (50 \text{ N})(0.12 \text{ m}) - (50 \text{ N})(0.12 \text{ m}) = 0$

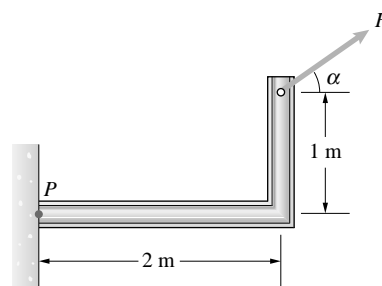
$$M_A = 0$$

(b) $\sum M_P = (50 \text{ N})(0.12 \text{ m})$
 $- (50 \text{ N} \cos 30^\circ)(0.65 \text{ m} \sin 45^\circ + 0.12 \text{ m} \cos 30^\circ)$
 $- (50 \text{ N} \sin 30^\circ)(0.65 \text{ m} \cos 45^\circ + 0.12 \text{ m} \sin 30^\circ)$

$$M_P = -31.4 \text{ N} \cdot \text{m} \quad \text{or} \quad M_P = 31.4 \text{ N} \cdot \text{m CW}$$



Problem 4.10 The force $F = 12$ kN. A structural engineer determines that the magnitude of the moment due to F about P should not exceed 5 kN-m. What is the acceptable range of the angle α ? Assume that $0 \leq \alpha \leq 90^\circ$.



Solution: We have the moment about P

$$M_P = (12 \text{ kN} \sin \alpha)(2 \text{ m}) - (12 \text{ kN} \cos \alpha)(1 \text{ m})$$

$$M_P = 12(2 \sin \alpha - \cos \alpha) \text{ kN-m}$$

The moment must not exceed 5 kN-m

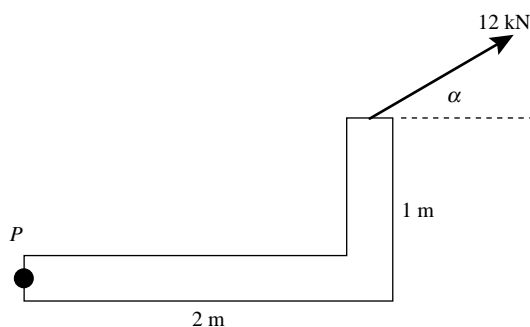
$$\text{Thus } 5 \text{ kN-m} \geq |12(2 \sin \alpha - \cos \alpha)| \text{ kN-m}$$

The limits occur when

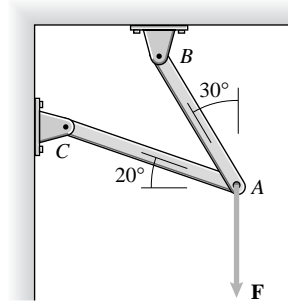
$$12(2 \sin \alpha - \cos \alpha) = 5 \Rightarrow \alpha = 37.3$$

$$12(2 \sin \alpha - \cos \alpha) = -5 \Rightarrow \alpha = 15.83^\circ$$

So we must have $15.83^\circ \leq \alpha \leq 37.3^\circ$



Problem 4.11 The length of bar AB is 350 mm. The moments exerted about points B and C by the vertical force F are $M_B = -1.75$ kN-m and $M_C = -4.20$ kN-m. Determine the force F and the length of bar AC .

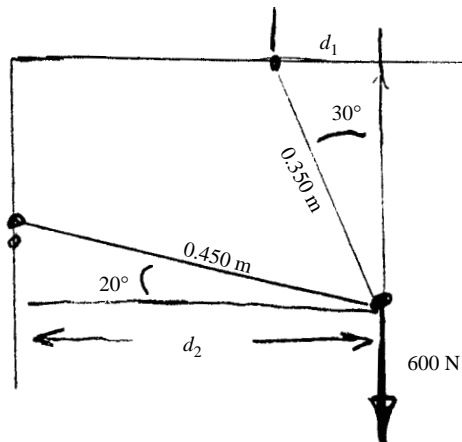
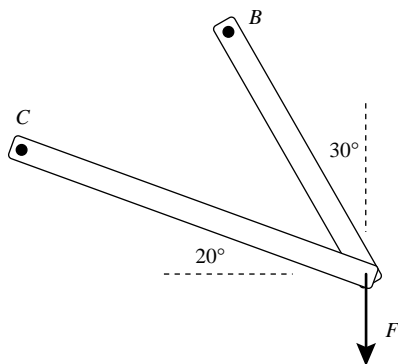


Solution: We have

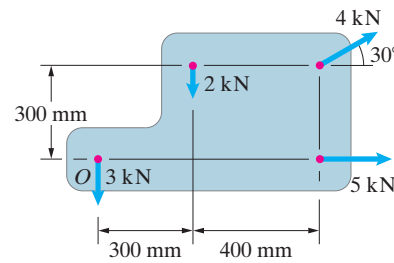
$$1.75 \text{ kN-m} = F(0.35 \text{ m}) \sin 30^\circ \Rightarrow F = 10 \text{ kN}$$

$$4.20 \text{ kN-m} = F(L_{AC}) \cos 20^\circ \Rightarrow L_{AC} = 0.447 \text{ m}$$

In summary $F = 10 \text{ kN}$, $L_{AC} = 447 \text{ mm}$



Problem 4.12 In Example 4.2, suppose that the 2-kN force points upward instead of downward. Draw a sketch of the machine part showing the orientations of the forces. What is the sum of the moments of the forces about the origin O ?



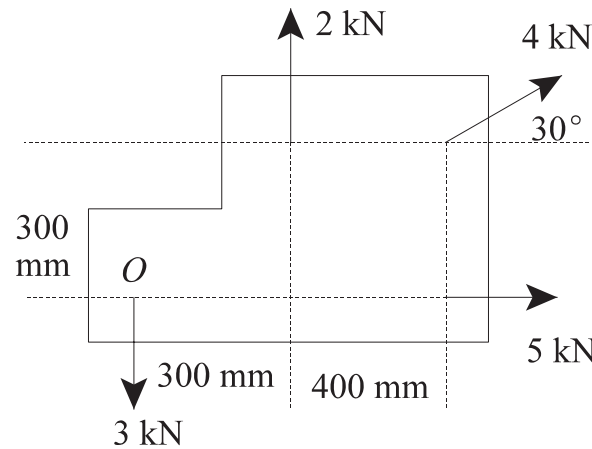
Solution: If the 2-kN force points upward, the magnitude of its moment about O does not change, but the direction of the moment changes from clockwise to counterclockwise. Treating counterclockwise moments as positive, the moment due to the 2-kN force is

$$(0.3 \text{ m})(2 \text{ kN}) = 0.6 \text{ kN}\cdot\text{m}$$

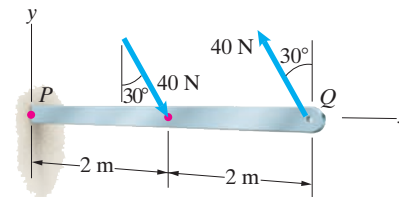
The moments due to the other forces do not change, so the sum of the moments of the four forces is

$$\sum M_O = (0.6 - 1.039 + 1.400) \text{ kN}\cdot\text{m}$$

$$\boxed{\sum M_O = 0.961 \text{ kN}\cdot\text{m}}$$



Problem 4.13 Two equal and opposite forces act on the beam. Determine the sum of the moments of the two forces (a) about point P ; (b) about point Q ; (c) about the point with coordinates $x = 7 \text{ m}$, $y = 5 \text{ m}$.



Solution:

(a)
$$M_P = -(40 \text{ N} \cos 30^\circ)(2 \text{ m}) + (40 \text{ N} \cos 30^\circ)(4 \text{ m})$$

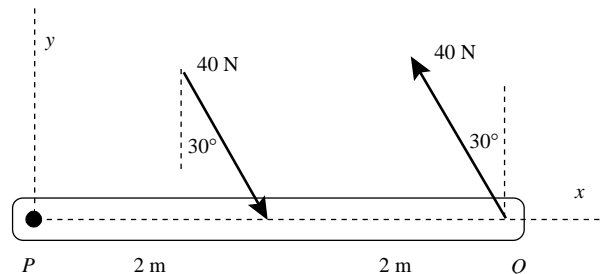
$$= 69.3 \text{ N}\cdot\text{m} \text{ (CCW)}$$

(b)
$$M_Q = (40 \text{ N} \cos 30^\circ)(2 \text{ m}) = 69.3 \text{ N}\cdot\text{m} \text{ (CCW)}$$

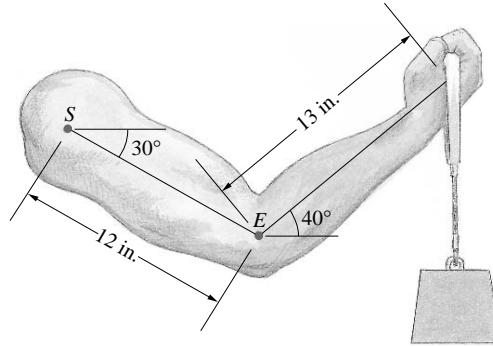
(c)
$$M = (40 \text{ N} \sin 30^\circ)(5 \text{ m}) + (40 \text{ N} \cos 30^\circ)(5 \text{ m})$$

$$- (40 \text{ N} \sin 30^\circ)(5 \text{ m}) - (40 \text{ N} \cos 30^\circ)(3 \text{ m})$$

$$= 69.3 \text{ N}\cdot\text{m} \text{ (CCW)}$$



Problem 4.14 The moment exerted about point E by the weight is 299 in-lb. What moment does the weight exert about point S ?



Solution: The key is the geometry

From trigonometry,

$$\cos 40^\circ = \frac{d_2}{13 \text{ in}}, \quad \cos 30^\circ = \frac{d_1}{12 \text{ in}}$$

Thus $d_1 = (12 \text{ in}) \cos 30^\circ$

$$d_1 = 10.39''$$

and $d_2 = (13 \text{ in}) \cos 40^\circ$

$$d_2 = 9.96''$$

We are given that

$$299 \text{ in-lb} = d_2 W = 9.96 W$$

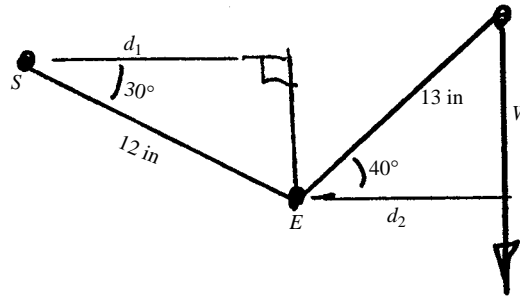
$$W = 30.0 \text{ lb}$$

Now,

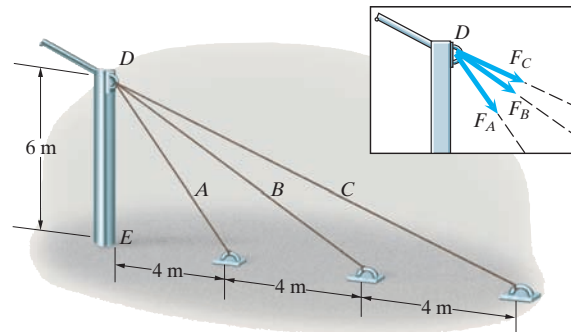
$$M_s = (d_1 + d_2)W$$

$$M_s = (20.35)(30.0)$$

$$M_s = 611 \text{ in-lb clockwise}$$



Problem 4.15 The magnitudes of the forces exerted on the pillar at D by the cables A , B , and C are equal: $F_A = F_B = F_C$. The magnitude of the total moment about E due to the forces exerted by the three cables at D is 1350 kN-m. What is F_A ?



Solution: The angles between the three cables and the pillar are

$$\alpha_A = \tan^{-1}(4/6) = 33.7^\circ$$

$$\alpha_B = \tan^{-1}(8/6) = 53.1^\circ$$

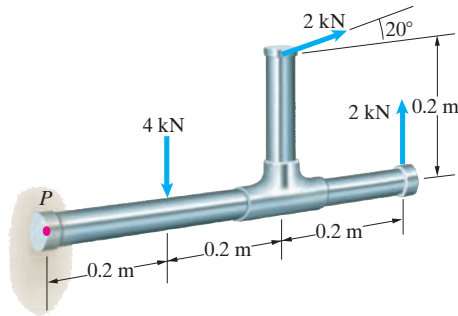
$$\alpha_C = \tan^{-1}(12/6) = 63.4^\circ$$

The vertical components of each force at point D exert no moment about E . Noting that $F_A = F_B = F_C$, the magnitude of the moment about E due to the horizontal components is

$$\sum M_E = F_A(\sin \alpha_A + \sin \alpha_B + \sin \alpha_C)(6 \text{ m}) = 1350 \text{ kN-m}$$

Solving for F_A yields $F_A = 100 \text{ kN}$

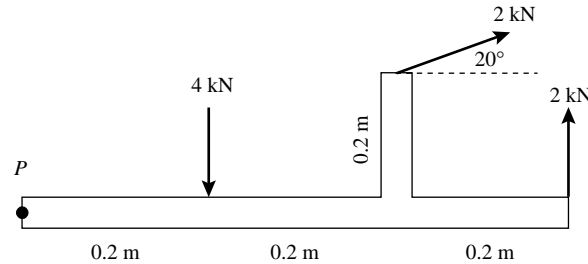
Problem 4.16 Three forces act on the piping. Determine the sum of the moments of the three forces about point P .



Solution:

$$\sum M_P = -(4 \text{ kN})(0.2 \text{ m}) + (2 \text{ kN})(0.6 \text{ m}) - (2 \text{ kN} \cos 20^\circ)(0.2 \text{ m}) + (2 \text{ kN} \sin 20^\circ)(0.4 \text{ m}) = 10.18 \text{ kN-m}$$

$$M_P = 0.298 \text{ kN-m CCW}$$

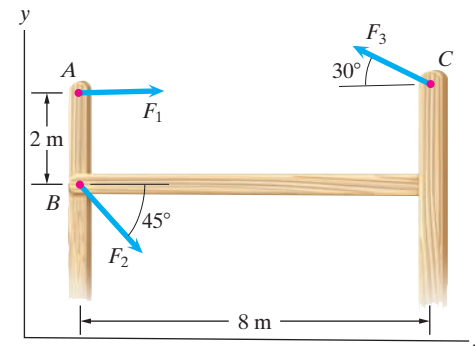


Problem 4.17 The forces $F_1 = 30 \text{ N}$, $F_2 = 80 \text{ N}$, and $F_3 = 40 \text{ N}$. What is the sum of the moments of the forces about point A ?

Solution: The moment about point A due to F_1 is zero. Treating counterclockwise moments as positive the sum of the moments is

$$\sum M_A = F_3 \sin 30^\circ (8 \text{ m}) + F_2 \cos 45^\circ (2 \text{ m})$$

$$\sum M_A = 273 \text{ N-m counterclockwise}$$



Problem 4.18 The force $F_1 = 30 \text{ N}$. The vector sum of the forces is zero. What is the sum of the moments of the forces about point A ?

Solution: The sums of the forces in the x and y directions equal zero:

$$\sum F_x : F_1 + F_2 \cos 45^\circ - F_3 \cos 30^\circ = 0$$

$$\sum F_y : -F_2 \sin 45^\circ + F_3 \sin 30^\circ = 0$$

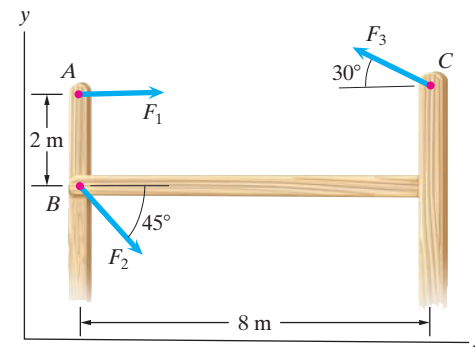
Setting $F_1 = 30 \text{ N}$ and solving yields

$$F_2 = 58.0 \text{ N}, F_3 = 82.0 \text{ N}.$$

The sum of the moments about point A is

$$\sum M_A = F_2 \sin 30^\circ (8 \text{ m}) + F_2 \cos 45^\circ (2 \text{ m}) = 410 \text{ N-m}$$

$$\sum M_A = 410 \text{ N-m counterclockwise}$$

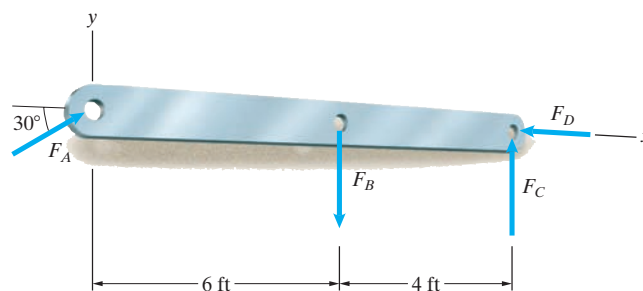


Problem 4.19 The forces $F_A = 30$ lb, $F_B = 40$ lb, $F_C = 20$ lb, and $F_D = 30$ lb. What is the sum of the moments of the forces about the origin of the coordinate system?

Solution: The moment about the origin due to F_A and F_D is zero. Treating counterclockwise moments as positive, the sum of the moments is

$$\begin{aligned}\sum M &= -F_B(6 \text{ ft}) + F_C(10 \text{ ft}) \\ &= -(40 \text{ lb})(6 \text{ ft}) + (20 \text{ lb})(10 \text{ ft}) = -40 \text{ ft}\cdot\text{lb}\end{aligned}$$

$$\boxed{\sum M = 40 \text{ ft}\cdot\text{lb clockwise}}$$



Problem 4.20 The force $F_A = 30$ lb. The vector sum of the forces on the beam is zero, and the sum of the moments of the forces about the origin of the coordinate system is zero.

- Determine the forces F_B , F_C , and F_D .
- Determine the sum of the moments of the forces about the right end of the beam.

Solution:

- The sum of the forces and the sum of the moments equals zero

$$\sum F_x : F_A \cos 30^\circ - F_D = 0$$

$$\sum F_y : F_A \sin 30^\circ - F_B + F_C = 0$$

$$\sum M_{\text{origin}} : -F_B(6 \text{ ft}) + F_C(10 \text{ ft}) = 0$$

Setting $F_A = 30$ lb and solving yields $\boxed{F_B = 37.5 \text{ lb}, F_C = 22.5 \text{ lb}, F_D = 26.0 \text{ lb}}$

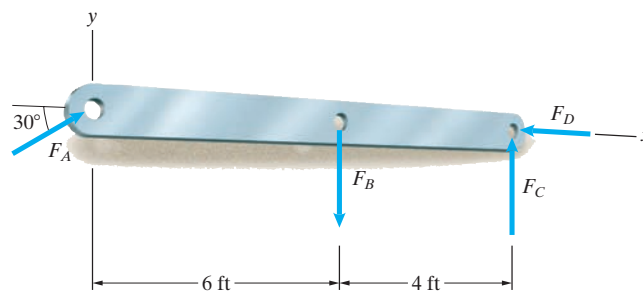
- The sum of the moments about the right end is

$$\sum M_{\text{Right End}} : F_B(4 \text{ ft}) - F_A \sin 30^\circ(10 \text{ ft})$$

$$= (37.5 \text{ lb})(4 \text{ ft}) - (30 \text{ lb})(10 \text{ ft})$$

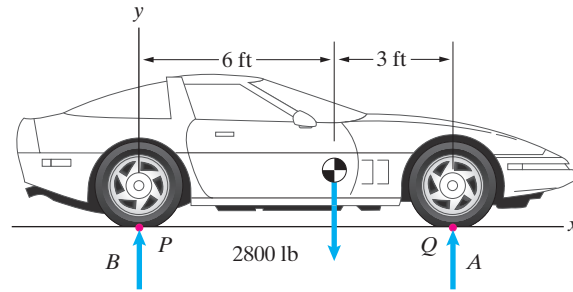
$$= 0$$

$$\boxed{\sum M_{\text{Right End}} = 0}$$



Problem 4.21 Three forces act on the car. The sum of the forces is zero and the sum of the moments of the forces about point P is zero.

- Determine the forces A and B .
- Determine the sum of the moments of the forces about point Q .



Solution:

$$\sum F_y : A + B - 2800 \text{ lb} = 0$$

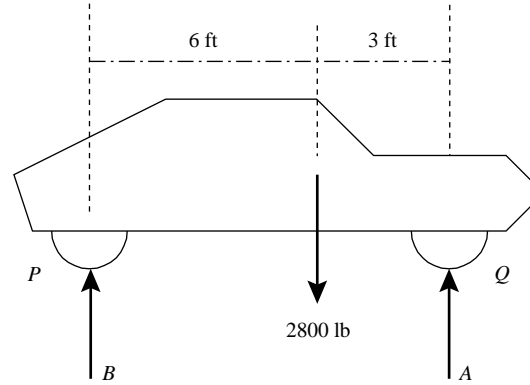
$$\sum M_P : -(2800 \text{ lb})(6 \text{ ft}) + A(9 \text{ ft}) = 0$$

- Solving we find

$$A = 1867 \text{ lb}, B = 933 \text{ lb}$$

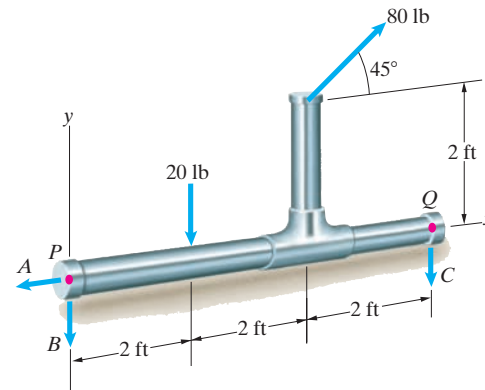
- $\sum M_Q = (2800 \text{ lb})(3 \text{ ft}) - B(9 \text{ ft}) = 0$

$$M_Q = 0$$



Problem 4.22 Five forces act on the piping. The vector sum of the forces is zero and the sum of the moments of the forces about point P is zero.

- Determine the forces A , B , and C .
- Determine the sum of the moments of the forces about point Q .



Solution:

- The conditions given in the problem are:

$$\sum F_x : -A + 80 \text{ lb} \cos 45^\circ = 0$$

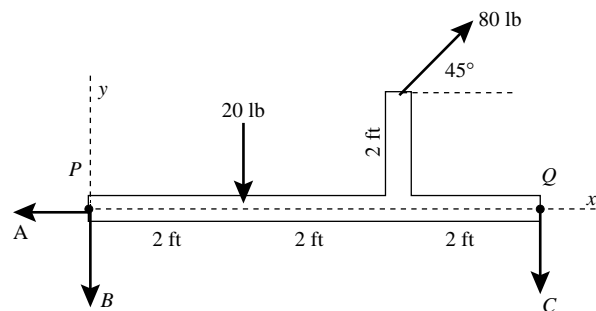
$$\sum F_y : -B - C - 20 \text{ lb} + 80 \text{ lb} \sin 45^\circ = 0$$

$$\sum M_P : -(20 \text{ lb})(2 \text{ ft}) - C(6 \text{ ft}) - (80 \text{ lb} \cos 45^\circ)(2 \text{ ft}) + (80 \text{ lb} \sin 45^\circ)(4 \text{ ft}) = 0$$

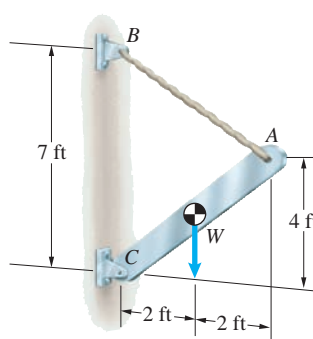
Solving we have

$$A = 56.6 \text{ lb}, B = 24.4 \text{ lb}, C = 12.19 \text{ lb}$$

- $\sum M_Q : -(80 \text{ lb} \cos 45^\circ)(2 \text{ ft}) - (80 \text{ lb} \sin 45^\circ)(2 \text{ ft}) + (20 \text{ lb})(4 \text{ ft}) + B(6 \text{ ft}) = 0$



Problem 4.23 In Example 4.3, suppose that the attachment point B is moved upward and the cable is lengthened so that the vertical distance from C to B is 9 ft. (the positions of points C and A are unchanged.) Draw a sketch of the system with the cable in its new position. What is the tension in the cable?



Solution: The angle α between the cable AB and the horizontal is.

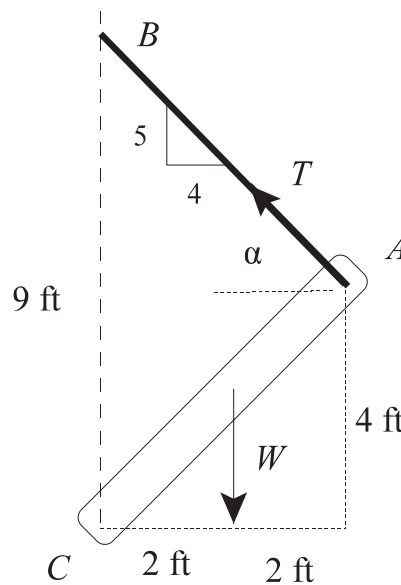
$$\alpha = \tan^{-1}(5/4) = 51.3^\circ$$

The sum of the moments about C is

$$\sum M_C : -W(2 \text{ ft}) + T \cos \alpha(4 \text{ ft}) + T \sin \alpha(4 \text{ ft}) = 0$$

Solving yields

$$T = 106.7 \text{ lb}$$

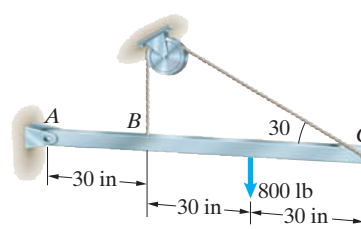


Problem 4.24 The tension in the cable is the same on both sides of the pulley. The sum of the moments about point A due to the 800-lb force and the forces exerted on the bar by the cable at B and C is zero. What is the tension in the cable?

Solution: Let T be the tension in the cable. The sum of the moments about A is

$$\sum M_A : T(30 \text{ in}) + T \sin 30^\circ(90 \text{ in}) - (800 \text{ lb})(60 \text{ in}) = 0$$

Solving yields $T = 640 \text{ lb}$



Problem 4.25 The 160-N weights of the arms AB and BC of the robotic manipulator act at their midpoints. Determine the sum of the moments of the three weights about A .

Solution: The strategy is to find the perpendicular distance from the points to the line of action of the forces, and determine the sum of the moments, using the appropriate sign of the action.

The distance from A to the action line of the weight of the arm AB is:

$$d_{AB} = (0.300) \cos 40^\circ = 0.2298 \text{ m}$$

The distance from A to the action line of the weight of the arm BC is

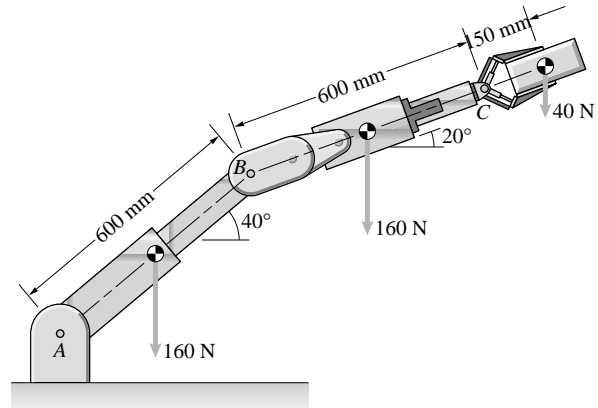
$$d_{BC} = (0.600)(\cos 40^\circ) + (0.300)(\cos 20^\circ) = 0.7415 \text{ m.}$$

The distance from A to the line of action of the force is

$$\begin{aligned} d_F &= (0.600)(\cos 40^\circ) + (0.600)(\cos 20^\circ) + (0.150)(\cos 20^\circ) \\ &= 1.1644 \text{ m.} \end{aligned}$$

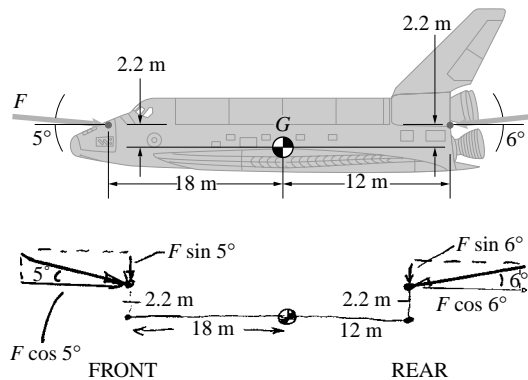
The sum of the moments about A is

$$\sum M_A = -d_{AB}(160) - d_{BC}(160) - d_F(40) = -202 \text{ N}\cdot\text{m}$$



Problem 4.26 The space shuttle's attitude thrusters exert two forces of magnitude $F = 7.70 \text{ kN}$. What moment do the thrusters exert about the center of mass G ?

Solution: The key to this problem is getting the geometry correct. The simplest way to do this is to break each force into components parallel and perpendicular to the axis of the shuttle and then to sum the moments of the components. (This will become much easier in the next section)



$$\curvearrowright +M_{\text{FRONT}_G} = (18)F \sin 5^\circ - (2.2)F \cos 5^\circ$$

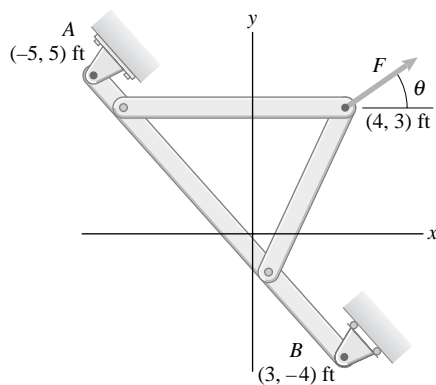
$$\curvearrowright +M_{\text{REAR}_G} = (2.2)F \cos 6^\circ - (12)F \sin 6^\circ$$

$$\curvearrowright +M_{\text{TOTAL}} = M_{\text{FRONT}} + M_{\text{REAR}}$$

$$\curvearrowright +M_{\text{TOTAL}} = -4.80 + 7.19 \text{ N}\cdot\text{m}$$

$$\curvearrowright +M_{\text{TOTAL}} = 2.39 \text{ N}\cdot\text{m}$$

Problem 4.27 The force F exerts a 200 ft-lb counter-clockwise moment about A and a 100 ft-lb clockwise moment about B . What are F and θ ?



Solution: The strategy is to resolve \mathbf{F} into x - and y -components, and compute the perpendicular distance to each component from A and B . The components of \mathbf{F} are: $\mathbf{F} = \mathbf{i}F_X + \mathbf{j}F_Y$. The vector from A to the point of application is:

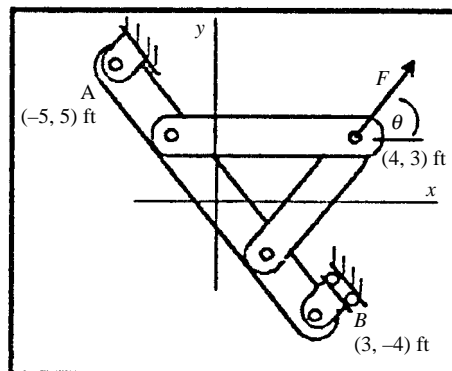
$$\mathbf{r}_{AF} = (4 - (-5))\mathbf{i} + (3 - 5)\mathbf{j} = 9\mathbf{i} - 2\mathbf{j}.$$

The perpendicular distances are $d_{AX} = 9$ ft, and $d_{AY} = 2$ ft, and the actions are positive. The moment about A is $M_A = (9)F_Y + (2)F_X = 200$ ft-lb. The vector from B to the point of application is $\mathbf{r}_{BF} = (4 - 3)\mathbf{i} + (3 - (-4))\mathbf{j} = \mathbf{i} + 7\mathbf{j}$; the distances $d_{BX} = 1$ ft and $d_{BY} = 7$ ft, the action of F_Y is positive and the action of F_X is negative. The moment about B is $M_B = (1)F_Y - (7)F_X = -100$ ft-lb. The two simultaneous equations have solution: $F_Y = 18.46$ lb and $F_X = 16.92$ lb. Take the ratio to find the angle:

$$\theta = \tan^{-1} \left(\frac{F_Y}{F_X} \right) = \tan^{-1} \left(\frac{18.46}{16.92} \right) = \tan^{-1}(1.091) = 47.5^\circ.$$

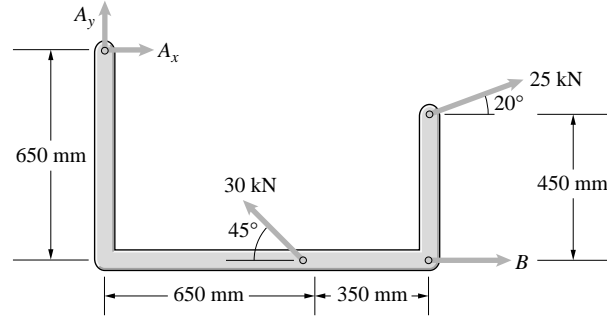
From the Pythagorean theorem

$$|\mathbf{F}| = \sqrt{F_Y^2 + F_X^2} = \sqrt{18.46^2 + 16.92^2} = 25.04 \text{ lb}$$



Problem 4.28 Five forces act on a link in the gear-shifting mechanism of a lawn mower. The vector sum of the five forces on the bar is zero. The sum of their moments about the point where the forces A_x and A_y act is zero.

- Determine the forces A_x , A_y , and B .
- Determine the sum of the moments of the forces about the point where the force B acts.



Solution: The strategy is to resolve the forces into x - and y -components, determine the perpendicular distances from B to the line of action, determine the sign of the action, and compute the moments.

The angles are measured counterclockwise from the x axis. The forces are

$$\mathbf{F}_2 = 30(\mathbf{i} \cos 135^\circ + \mathbf{j} \sin 135^\circ) = -21.21\mathbf{i} + 21.21\mathbf{j}$$

$$\mathbf{F}_1 = 25(\mathbf{i} \cos 20^\circ + \mathbf{j} \sin 20^\circ) = 23.50\mathbf{i} + 8.55\mathbf{j}.$$

- The sum of the forces is

$$\sum \mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{F}_1 + \mathbf{F}_2 = 0.$$

Substituting:

$$\sum F_x = (A_x + B_x + 23.5 - 21.2)\mathbf{i} = 0,$$

$$\text{and } \sum F_y = (A_y + 21.2 + 8.55)\mathbf{j} = 0.$$

Solve the second equation: $A_y = -29.76$ kN. The distances of the forces from A are: the triangle has equal base and altitude, hence the angle is 45° , so that the line of action of \mathbf{F}_1 passes through A . The distance to the line of action of \mathbf{B} is 0.65 m, with a positive action. The distance to the line of action of the y -component of \mathbf{F}_2 is $(0.650 + 0.350) = 1$ m, and the action is positive. The distance to the line of action of the x -component of \mathbf{F}_2 is $(0.650 - 0.450) = 0.200$ m, and the action is positive. The moment about A is

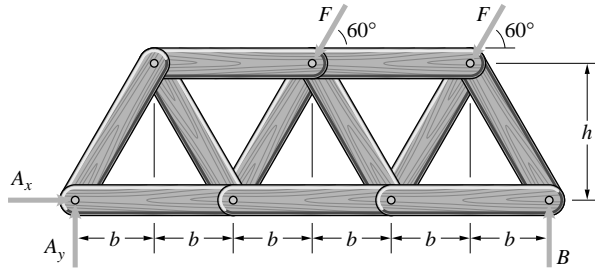
$$\sum M_A = (8.55)(1) + (23.5)(0.2) + (B_x)(0.65) = 0.$$

Solve: $B_x = -20.38$ kN. Substitute into the force equation to obtain $A_x = 18.09$ kN

- The distance from B to the line of action of the y -component of \mathbf{F}_1 is 0.350 m, and the action is negative. The distance from B to the line of action of A_x is 0.650 m and the action is negative. The distance from B to the line of action of A_y is 1 m and the action is positive. The distance from B to the line of action of the x -component of \mathbf{F}_2 is 0.450 m and the action is negative. The sum of the moments about B :

$$\begin{aligned} \sum M_B &= -(0.350)(21.21) - (0.650)(18.09) \\ &\quad + (1)(29.76) - (0.450)(23.5) = 0 \end{aligned}$$

Problem 4.29 Five forces act on a model truss built by a civil engineering student as part of a design project. The dimensions are $b = 300$ mm and $h = 400$ mm; $F = 100$ N. The sum of the moments of the forces about the point where A_x and A_y act is zero. If the weight of the truss is negligible, what is the force B ?



Solution: The x - and y -components of the force \mathbf{F} are

$$\mathbf{F} = -|\mathbf{F}|(\mathbf{i} \cos 60^\circ + \mathbf{j} \sin 60^\circ) = -|\mathbf{F}|(0.5\mathbf{i} + 0.866\mathbf{j}).$$

The distance from A to the x -component is h and the action is positive. The distances to the y -component are $3b$ and $5b$. The distance to B is $6b$. The sum of the moments about A is

$$\sum M_A = 2|\mathbf{F}|(0.5)(h) - 3b|\mathbf{F}|(0.866) - 5b|\mathbf{F}|(0.866) + 6bB = 0.$$

Substitute and solve: $B = \frac{1.6784|\mathbf{F}|}{1.8} = 93.2$ N

Problem 4.30 Consider the truss shown in Problem 4.29. The dimensions are $b = 3$ ft and $h = 4$ ft; $F = 300$ lb. The vector sum of the forces acting on the truss is zero, and the sum of the moments of the forces about the point where A_x and A_y act is zero.

- Determine the forces A_x , A_y , and B .
- Determine the sum of the moments of the forces about the point where the force B acts.

Solution: The forces are resolved into x - and y -components:

$$\mathbf{F} = -300(\mathbf{i} \cos 60^\circ + \mathbf{j} \sin 60^\circ) = -150\mathbf{i} - 259.8\mathbf{j}.$$

- The sum of the forces:

$$\sum \mathbf{F} = 2\mathbf{F} + \mathbf{A} + \mathbf{B} = 0.$$

The x - and y -components:

$$\sum F_x = (A_x - 300)\mathbf{i} = 0,$$

$$\sum F_y = (-519.6 + A_y + B)\mathbf{j} = 0.$$

Solve the first: $A_x = 300$ lb. The distance from point A to the x -components of the forces is h , and the action is positive. The distances between the point A and the lines of action of the y -components of the forces are $3b$ and $5b$. The actions are negative. The distance to the line of action of the force B is $6b$. The action is positive. The sum of moments about point A is

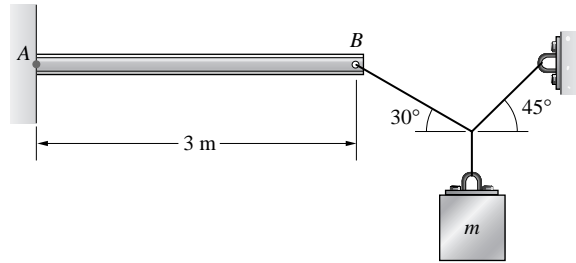
$$\sum M_A = 2(150)h - 3b(259.8) - 5b(259.8) + 6bB = 0.$$

Substitute and solve: $B = 279.7$ lb. Substitute this value into the force equation and solve: $A_y = 519.6 - 279.7 = 239.9$ lb

- The distances from B and the line of action of A_y is $6b$ and the action is negative. The distance between B and the x -component of the forces is h and the action is positive. The distance between B and the y -components of the forces is b and $3b$, and the action is positive. The sum of the moments about B :

$$\sum M_B = -6b(239.9) + 2(150)h + b(259.8) + 3b(259.8) = 0$$

Problem 4.31 The mass $m = 70$ kg. What is the moment about A due to the force exerted on the beam at B by the cable?



Solution: The strategy is to resolve the force at B into components parallel to and normal to the beam, and solve for the moment using the normal component of the force. The force at B is to be determined from the equilibrium conditions on the cable juncture O . Angles are measured from the positive x axis. The forces at the cable juncture are:

$$\mathbf{F}_{OB} = |\mathbf{F}_{OB}|(\mathbf{i} \cos 150^\circ + \mathbf{j} \sin 150^\circ) = |\mathbf{F}_{OB}|(-0.866\mathbf{i} + 0.5\mathbf{j})$$

$$\mathbf{F}_{OC} = |\mathbf{F}_{OC}|(\mathbf{i} \cos 45^\circ + \mathbf{j} \sin 45^\circ) = |\mathbf{F}_{OC}|(0.707\mathbf{i} + 0.707\mathbf{j}).$$

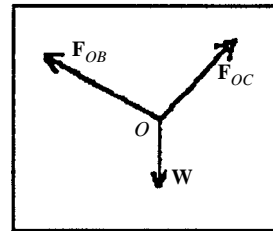
$$\mathbf{W} = (70)(9.81)(0\mathbf{i} - 1\mathbf{j}) = -686.7\mathbf{j} \text{ (N)}.$$

The equilibrium conditions are:

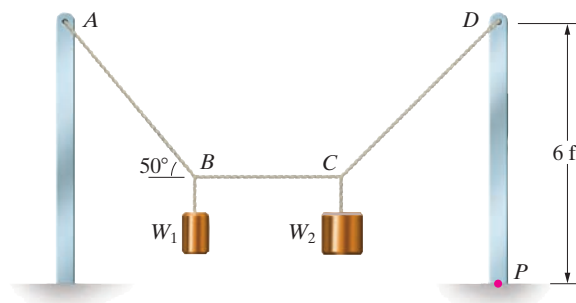
$$\sum \mathbf{F}_x = (-0.866|\mathbf{F}_{OB}| + 0.707|\mathbf{F}_{OC}|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (0.500|\mathbf{F}_{OB}| + .707|\mathbf{F}_{OC}|) - 686.7\mathbf{j} = 0.$$

Solve: $|\mathbf{F}_{OB}| = 502.70$ N. This is used to resolve the cable tension at B : $\mathbf{F}_B = 502.7(\mathbf{i} \cos 330^\circ + \mathbf{j} \sin 330^\circ) = 435.4\mathbf{i} - 251.4\mathbf{j}$. The distance from A to the action line of the y -component at B is 3 m, and the action is negative. The x -component at passes through A , so that the action line distance is zero. The moment at A is $M_A = -3(251.4) = -754.0$ N-m



Problem 4.32 The weights W_1 and W_2 are suspended by the cable system shown. The weight $W_1 = 12$ lb. The cable BC is horizontal. Determine the moment about point P due to the force exerted on the vertical post at D by the cable CD .



Solution: Isolate part of the cable system near point B . The equilibrium equations are

$$\sum F_x : T_{BC} - T_{AB} \cos 50^\circ = 0$$

$$\sum F_y : T_{AB} - 12 \text{ lb} = 0$$

Solving yields $T_{AB} = 15.7$ lb, $T_{BC} = 10.1$ lb

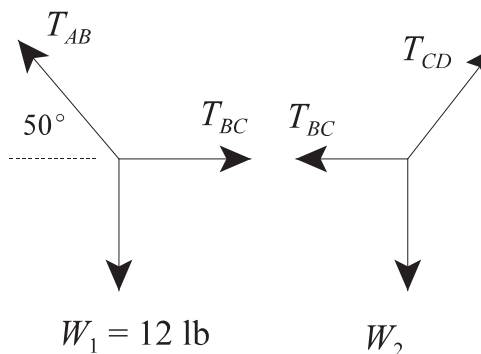
Let α be the angle between the cable CD and the horizontal. The magnitude of the moment about P due to the force exerted at D by cable CD is

$$M = T_{CD} \cos \alpha (6 \text{ ft})$$

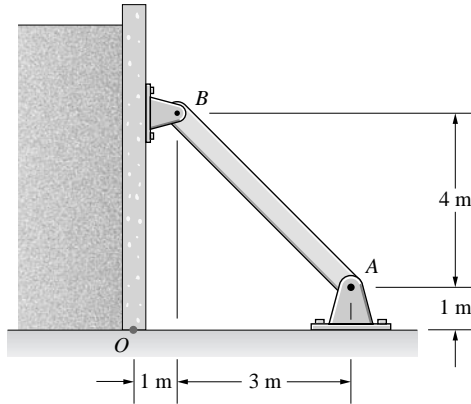
Isolate part of the cable system near point C . From the equilibrium equation

$$\sum F_x : T_{CD} \cos \alpha - T_{BC} = 0 \Rightarrow T_{CD} \cos \alpha = T_{BC} = 10.1 \text{ lb}$$

$$\text{Thus } M = (10.1 \text{ lb})(6 \text{ ft}) \quad \boxed{M = 60.4 \text{ lb}}$$

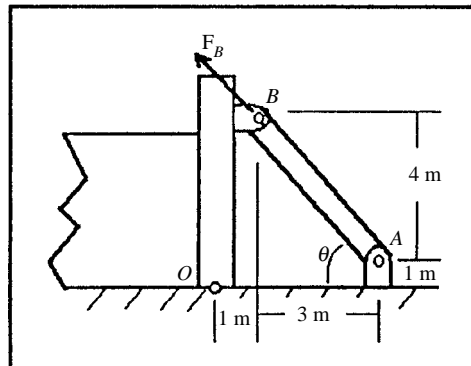


Problem 4.33 The bar AB exerts a force at B that helps support the vertical retaining wall. The force is parallel to the bar. The civil engineer wants the bar to exert a 38 kN-m moment about O . What is the magnitude of the force the bar must exert?

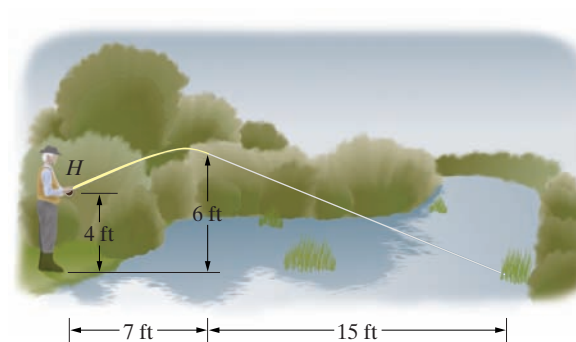


Solution: The strategy is to resolve the force at B into components parallel to and normal to the wall, determine the perpendicular distance from O to the line of action, and compute the moment about O in terms of the magnitude of the force exerted by the bar.

By inspection, the bar forms a 3, 4, 5 triangle. The angle the bar makes with the horizontal is $\cos \theta = \frac{3}{5} = 0.600$, and $\sin \theta = \frac{4}{5} = 0.800$. The force at B is $\mathbf{F}_B = |\mathbf{F}_B|(-0.600\mathbf{i} + 0.800\mathbf{j})$. The perpendicular distance from O to the line of action of the x -component is $(4 + 1) = 5$ m, and the action is positive. The distance from O to the line of action of the y -component is 1 m, and the action is positive. The moment about O is $\sum M_O = 5(0.600)|\mathbf{F}_B| + 1(0.800)|\mathbf{F}_B| = 3.8|\mathbf{F}_B| = 38$ kN, from which $|\mathbf{F}_B| = 10$ kN



Problem 4.34 A contestant in a fly-casting contest snags his line in some grass. If the tension in the line is 5 lb, what moment does the force exerted on the rod by the line exert about point H , where he holds the rod?



Solution: The strategy is to resolve the line tension into a component normal to the rod; use the length from H to tip as the perpendicular distance; determine the sign of the action, and compute the moment.

The line and rod form two right triangles, as shown in the sketch. The angles are:

$$\alpha = \tan^{-1} \left(\frac{2}{7} \right) = 15.95^\circ$$

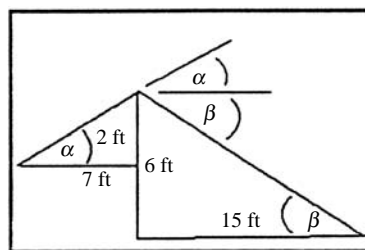
$$\beta = \tan^{-1} \left(\frac{6}{15} \right) = 21.8^\circ$$

The angle between the perpendicular distance line and the fishing line is $\theta = \alpha + \beta = 37.7^\circ$. The force normal to the distance line is $F = 5(\sin 37.7^\circ) = 3.061$ lb. The distance is $d = \sqrt{2^2 + 7^2} = 7.28$ ft, and the action is negative. The moment about H is $M_H = -7.28(3.061) = -22.3$ ft-lb. Check: The tension can be resolved into x and y components,

$$F_x = F \cos \beta = 4.642 \text{ lb}, \quad F_y = -F \sin \beta = -1.857 \text{ lb}.$$

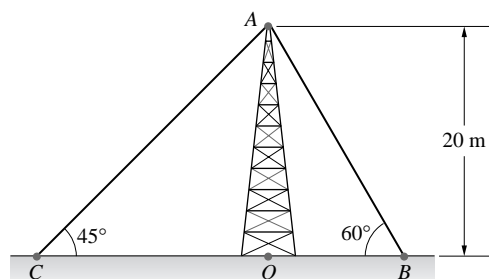
The moment is

$$M = -2F_x + 7F_y = -22.28 = -22.3 \text{ ft-lb. check.}$$



Problem 4.35 The cables AB and AC help support the tower. The tension in cable AB is 5 kN. The points A , B , C , and O are contained in the same vertical plane.

- What is the moment about O due to the force exerted on the tower by cable AB ?
- If the sum of the moments about O due to the forces exerted on the tower by the two cables is zero, what is the tension in cable AC ?



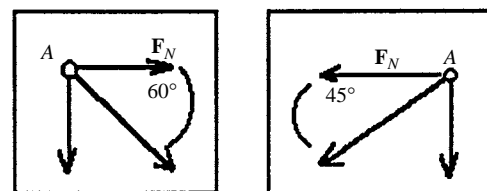
Solution: The strategy is to resolve the cable tensions into components normal to the vertical line through OA ; use the height of the tower as the perpendicular distance; determine the sign of the action, and compute the moments.

- The component normal to the line OA is $F_{BN} = 5(\cos 60^\circ) = 2.5$ kN. The action is negative. The moment about O is $M_{OA} = -2.5(20) = -50$ kN-m
- By a similar process, the normal component of the tension in the cable AC is $F_{CN} = |F_C| \cos 45^\circ = 0.707|F_C|$. The action is positive. If the sum of the moments is zero,

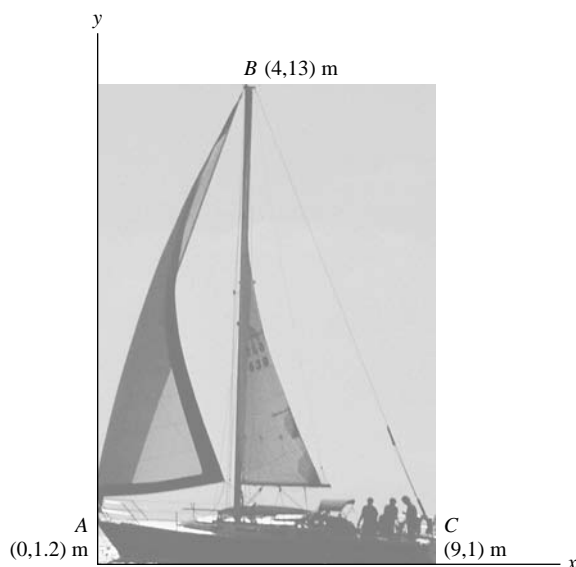
$$\sum M_O = (0.707(20)|F_C| - 50) = 0,$$

from which

$$|F_C| = \frac{50 \text{ kN m}}{(0.707)(20 \text{ m})} = 3.54 \text{ kN}$$



Problem 4.36 The cable from B to A (the sailboat's forestay) exerts a 230-N force at B . The cable from B to C (the backstay) exerts a 660-N force at B . The bottom of the sailboat's mast is located at $x = 4$ m, $y = 0$. What is the sum of the moments about the bottom of the mast due to the forces exerted at B by the forestay and backstay?



Solution: Triangle ABP

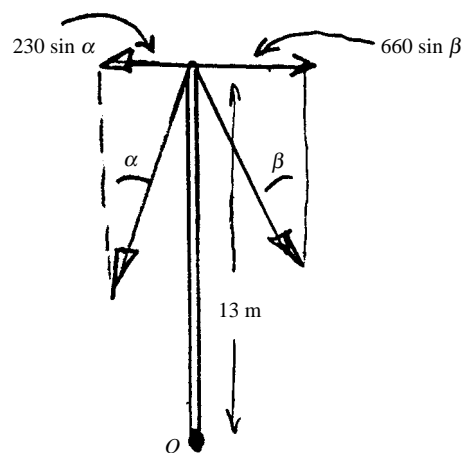
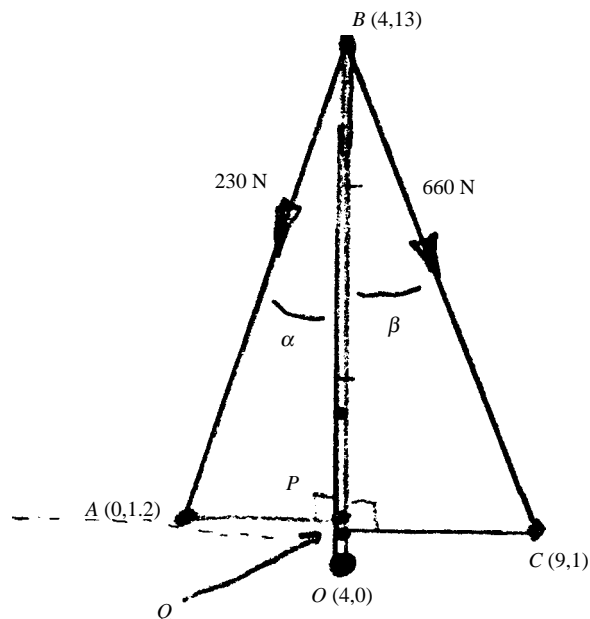
$$\tan \alpha = \frac{4}{11.8}, \quad \alpha = 18.73^\circ$$

Triangle BCQ

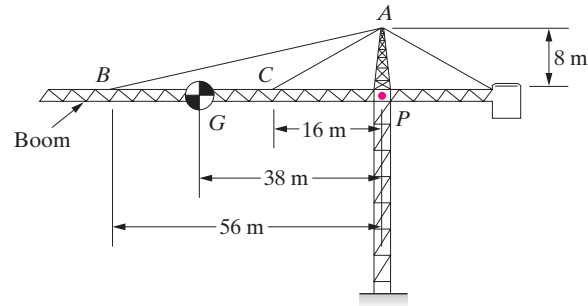
$$\tan \beta = \frac{5}{12}, \quad \beta = 22.62^\circ$$

$$\curvearrowright +M_O = (13)(230) \sin \alpha - (13)(660) \sin \beta$$

$$\curvearrowright +M_O = -2340 \text{ N}\cdot\text{m}$$



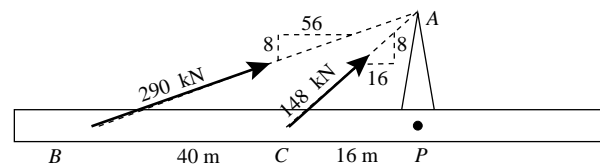
Problem 4.37 The cable AB exerts a 290-kN force on the building crane's boom at B . The cable AC exerts a 148-kN force on the boom at C . Determine the sum of the moments about P due to the forces the cables exert on the boom.



Solution:

$$\begin{aligned}\sum M_P &= -\frac{8}{\sqrt{3200}}(290 \text{ kN})(56 \text{ m}) - \frac{8}{\sqrt{320}}(148 \text{ kN})(16 \text{ m}) \\ &= -3.36 \text{ MNm}\end{aligned}$$

$$\boxed{\sum M_P = 3.36 \text{ MN-m CW}}$$



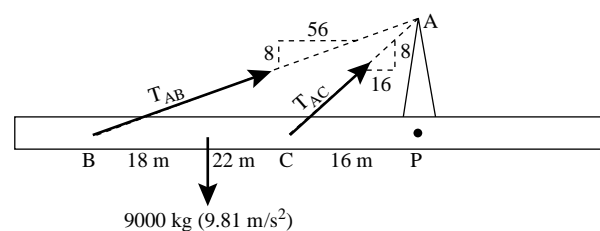
Problem 4.38 The mass of the building crane's boom in Problem 4.37 is 9000 kg. Its weight acts at G . The sum of the moments about P due to the boom's weight, the force exerted at B by the cable AB , and the force exerted at C by the cable AC is zero. Assume that the tensions in cables AB and AC are equal. Determine the tension in the cables.

Solution:

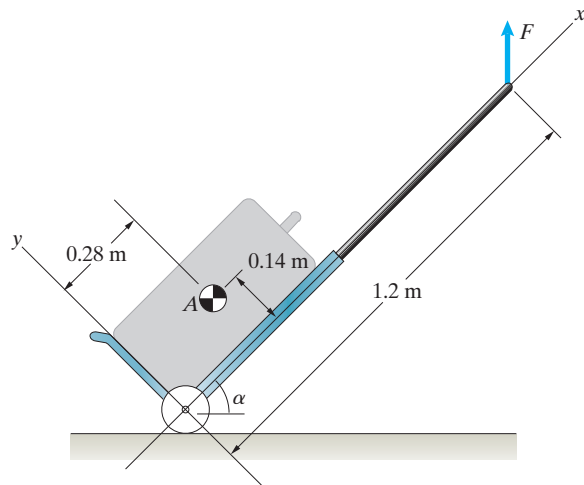
$$\begin{aligned}\sum M_P &= -\frac{8}{\sqrt{3200}}T_{AB}(56 \text{ m}) - \frac{8}{\sqrt{320}}T_{AC}(16 \text{ m}) \\ &\quad + (9000 \text{ kg})(9.81 \text{ m/s}^2)(38 \text{ m}) = 0\end{aligned}$$

using $T_{AB} = T_{AC}$ we solve and find

$$\boxed{T_{AB} = T_{AC} = 223 \text{ kN}}$$



Problem 4.39 The mass of the luggage carrier and the suitcase combined is 12 kg. Their weight acts at A. The sum of the moments about the origin of the coordinate system due to the weight acting at A and the vertical force F applied to the handle of the luggage carrier is zero. Determine the force F (a) if $\alpha = 30^\circ$; (b) if $\alpha = 50^\circ$.



Solution: O is the origin of the coordinate system

$$\sum M_O = F(1.2 \text{ m} \cos \alpha)$$

$$- (12 \text{ kg})(9.81 \text{ m/s}^2)(0.28 \cos \alpha - 0.14 \sin \alpha) = 0$$

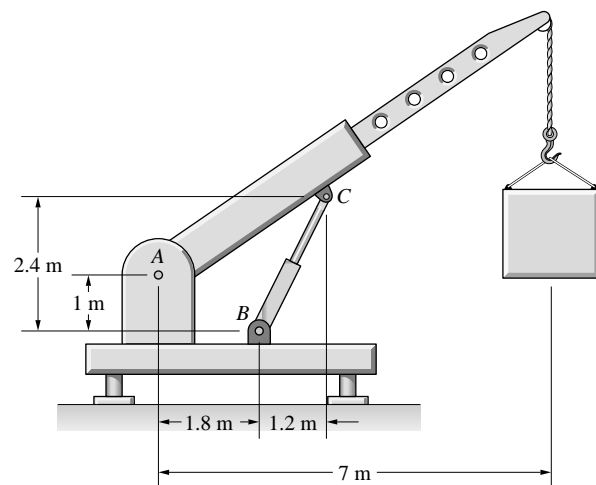
Solving we find

$$F = \frac{(12 \text{ kg})(9.81 \text{ m/s}^2)(0.28 \cos \alpha - 0.14 \sin \alpha)}{1.2 \text{ m} \cos \alpha}$$

(a) For $\alpha = 30^\circ$ We find $F = 19.54 \text{ N}$

(b) For $\alpha = 50^\circ$ We find $F = 11.10 \text{ N}$

Problem 4.40 The hydraulic cylinder BC exerts a 300-kN force on the boom of the crane at C . The force is parallel to the cylinder. What is the moment of the force about A ?



Solution: The strategy is to resolve the force exerted by the hydraulic cylinder into the normal component about the crane; determine the distance; determine the sign of the action, and compute the moment.

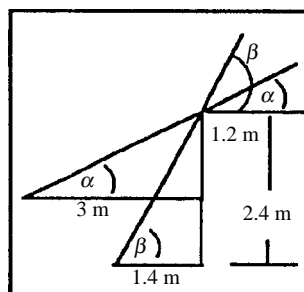
Two right triangles are constructed: The angle formed by the hydraulic cylinder with the horizontal is

$$\beta = \tan^{-1} \left(\frac{2.4}{1.2} \right) = 63.43^\circ$$

The angle formed by the crane with the horizontal is

$$\alpha = \tan^{-1} \left(\frac{1.4}{3} \right) = 25.02^\circ$$

The angle between the hydraulic cylinder and the crane is $\theta = \beta - \alpha = 38.42^\circ$. The normal component of the force is: $F_N = (300)(\sin 38.42^\circ) = 186.42 \text{ kN}$. The distance from point A is $d = \sqrt{1.4^2 + 3^2} = 3.31 \text{ m}$. The action is positive. The moment about A is $M_O = +3.31(186.42) = 617.15 \text{ kN}\cdot\text{m}$ *Check:* The force exerted by the actuator can be resolved into x - and y -components, $F_x = F \cos \beta = 134.16 \text{ kN}$, $F_y = F \sin \beta = 268.33 \text{ kN}$. The moment about the point A is $M = -1.4F_x + 3.0F_y = 617.15 \text{ kN}\cdot\text{m}$. *check.*



Problem 4.41 The hydraulic piston AB exerts a 400-lb force on the ladder at B in the direction parallel to the piston. The sum of the moments about C due to the force exerted on the ladder by the piston and the weight W of the ladder is zero. What is the weight of the ladder?

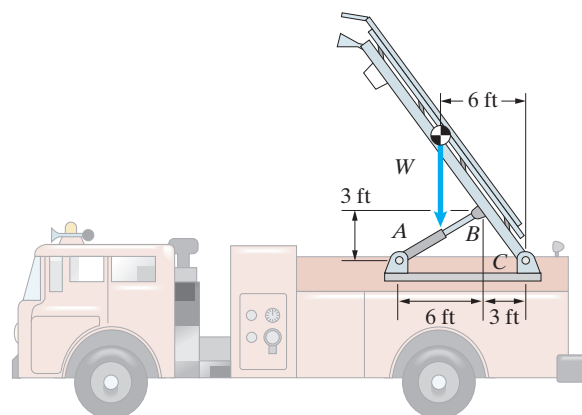
Solution: The angle between the piston AB and the horizontal is

$$\alpha = \tan^{-1}(3/6) = 26.6^\circ$$

The sum of the counterclockwise moment about C is

$$\sum M_C : W(6 \text{ ft}) - (400 \text{ lb}) \cos \alpha (3 \text{ ft}) - (400 \text{ lb}) \sin \alpha (3 \text{ ft}) = 0$$

Solving yields $W = 268 \text{ lb}$



Problem 4.42 The hydraulic cylinder exerts an 8-kN force at B that is parallel to the cylinder and points from C toward B . Determine the moments of the force about points A and D .

Solution: Use x, y coords with origin A . We need the unit vector from C to B , \mathbf{e}_{CB} . From the geometry,

$$\mathbf{e}_{CB} = 0.780\mathbf{i} - 0.625\mathbf{j}$$

The force \mathbf{F}_{CB} is given by

$$\mathbf{F}_{CB} = (0.780)8\mathbf{i} - (0.625)8\mathbf{j} \text{ kN}$$

$$\mathbf{F}_{CB} = 6.25\mathbf{i} - 5.00\mathbf{j} \text{ kN}$$

For the moments about A and D , treat the components of \mathbf{F}_{CB} as two separate forces.

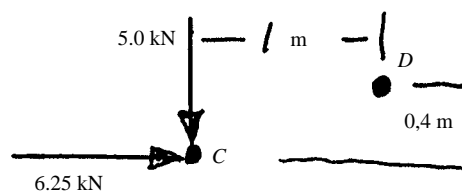
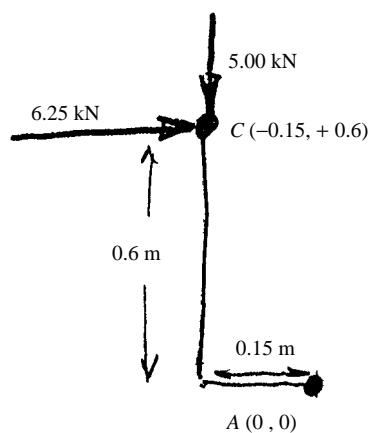
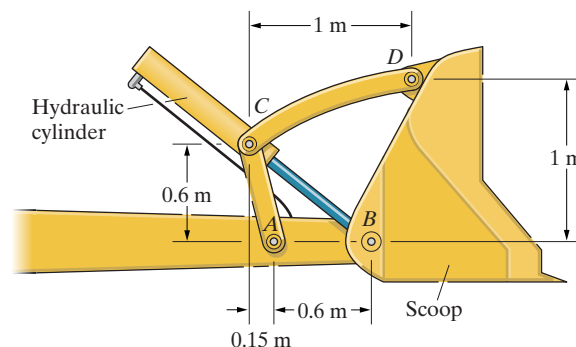
$$\curvearrowright +M_A = (5.00)(0.15) - (0.6)(6.25) \text{ kN} \cdot \text{m}$$

$$\curvearrowright +M_A = -3.00 \text{ kN} \cdot \text{m}$$

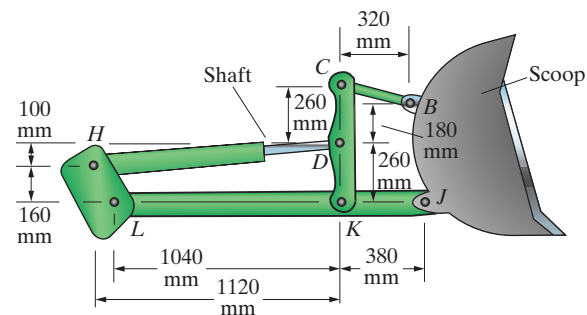
For the moment about D

$$\curvearrowright +\sum M_D = (5 \text{ kN})(1 \text{ m}) + (6.25 \text{ kN})(0.4 \text{ m})$$

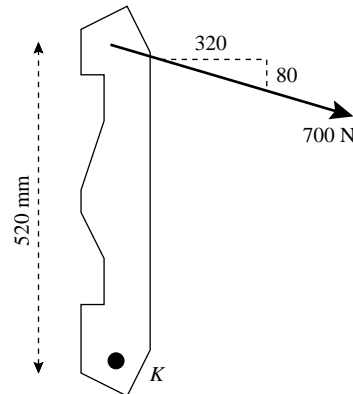
$$\curvearrowright +M_D = 7.5 \text{ kN} \cdot \text{m}$$



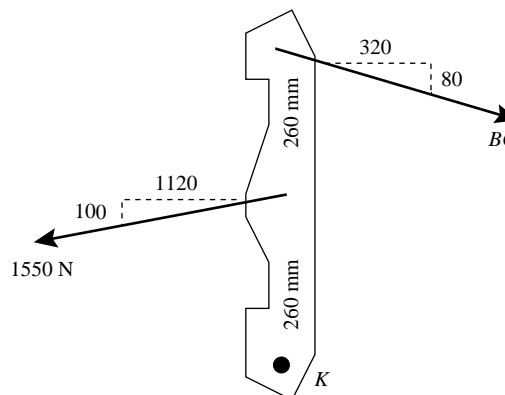
A close-up photograph of a green Newamatic 75 skid steer loader. The machine is shown from the side, highlighting its front bucket attachment and the hydraulic arm. The Newamatic logo and the number 75 are visible on the side of the machine. The loader is parked on a paved surface.


$$M_K = -\frac{320}{\sqrt{108800}}(700 \text{ N})(0.52 \text{ m}) = -353 \text{ Nm}$$

$$M_K = 353 \text{ Nm CW}$$


$$\sum M_K = \frac{1120}{\sqrt{1264400}}(1550 \text{ N})(0.26 \text{ m}) - \frac{320}{\sqrt{108800}}F(0.52) = 0$$

$F = 796 \text{ N}$



Problem 4.45 In Active Example 4.4, what is the moment of \mathbf{F} about the origin of the coordinate system?

Solution: The vector from the origin to point B is

$$\mathbf{r} = (11\mathbf{i} + 4\mathbf{k}) \text{ ft}$$

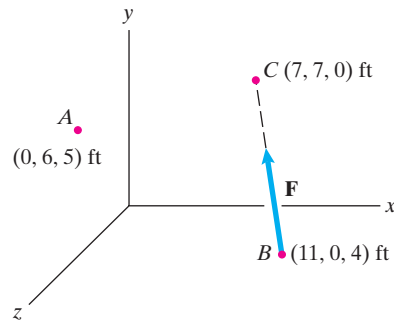
From Active Example 4.4 we know that the force \mathbf{F} is

$$\mathbf{F} = (-40\mathbf{i} + 70\mathbf{j} - 40\mathbf{k}) \text{ lb}$$

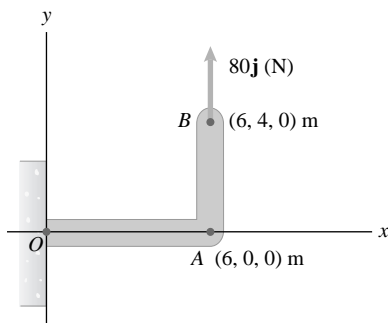
The moment of \mathbf{F} about the origin is

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 11 & 0 & 4 \\ -40 & 70 & -40 \end{vmatrix} = (-280\mathbf{i} + 280\mathbf{j} + 770\mathbf{k}) \text{ ft}\cdot\text{lb}$$

$$\boxed{M = (-280\mathbf{i} + 280\mathbf{j} + 770\mathbf{k}) \text{ ft}\cdot\text{lb}}$$



Problem 4.46 Use Eq. (4.2) to determine the moment of the 80-N force about the origin O letting \mathbf{r} be the vector (a) from O to A ; (b) from O to B .



Solution:

$$(a) \quad \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

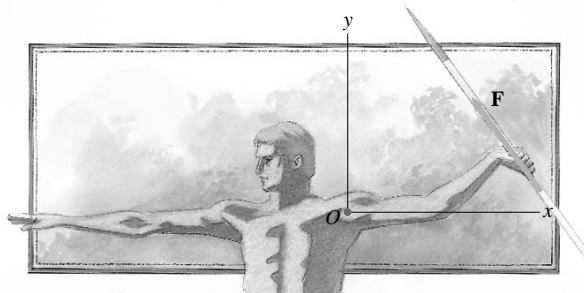
$$= 6\mathbf{i} \times 80\mathbf{j} = 480\mathbf{k} \text{ (N}\cdot\text{m)}.$$

$$(b) \quad \mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}$$

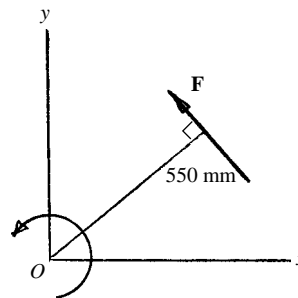
$$= (6\mathbf{i} + 4\mathbf{j}) \times 80\mathbf{j}$$

$$= 480\mathbf{k} \text{ (N}\cdot\text{m)}.$$

Problem 4.47 A bioengineer studying an injury sustained in throwing the javelin estimates that the magnitude of the maximum force exerted was $|\mathbf{F}| = 360 \text{ N}$ and the perpendicular distance from O to the line of action of \mathbf{F} was 550 mm. The vector \mathbf{F} and point O are contained in the x - y plane. Express the moment of \mathbf{F} about the shoulder joint at O as a vector.



Solution: The magnitude of the moment is $|\mathbf{F}|(0.55 \text{ m}) = (360 \text{ N})(0.55 \text{ m}) = 198 \text{ N}\cdot\text{m}$. The moment vector is perpendicular to the x - y plane, and the right-hand rule indicates it points in the positive z direction. Therefore $\mathbf{M}_O = 198\mathbf{k} \text{ (N}\cdot\text{m)}.$



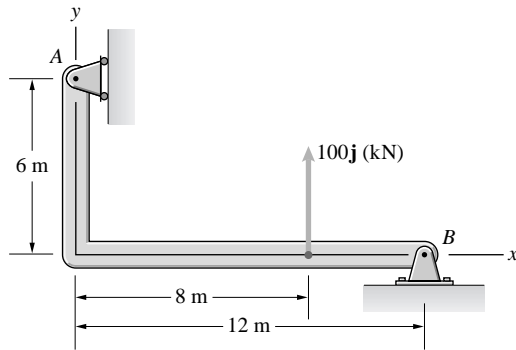
Problem 4.48 Use Eq. (4.2) to determine the moment of the 100-kN force (a) about A, (b) about B.

Solution: (a) The coordinates of A are (0,6,0). The coordinates of the point of application of the force are (8,0,0). The position vector from A to the point of application of the force is $\mathbf{r}_{AF} = (8-0)\mathbf{i} + (0-6)\mathbf{j} = 8\mathbf{i} - 6\mathbf{j}$. The force is $\mathbf{F} = 100\mathbf{j}$ (kN). The cross product is

$$\mathbf{r}_{AF} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & -6 & 0 \\ 0 & 100 & 0 \end{vmatrix} = 800\mathbf{k} \text{ (kN-m)}$$

(b) The coordinates of B are (12,0,0). The position vector from B to the point of application of the force is $\mathbf{r}_{BF} = (8-12)\mathbf{i} = -4\mathbf{i}$. The cross product is:

$$\mathbf{r}_{BF} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 0 & 0 \\ 0 & 100 & 0 \end{vmatrix} = -400\mathbf{k} \text{ (kN-m)}$$



Problem 4.49 The cable AB exerts a 200-N force on the support at A that points from A toward B. Use Eq. (4.2) to determine the moment of this force about point P in two ways: (a) letting \mathbf{r} be the vector from P to A; (b) letting \mathbf{r} be the vector from P to B.

Solution: First we express the force as a vector. The force points in the same direction as the position vector \mathbf{AB} .

$$\mathbf{AB} = (1-0.3)\mathbf{m}\mathbf{i} + (0.2-0.5)\mathbf{m}\mathbf{j} = (0.7\mathbf{i} - 0.3\mathbf{j}) \text{ m}$$

$$|\mathbf{AB}| = \sqrt{(0.7 \text{ m})^2 + (0.3 \text{ m})^2} = \sqrt{0.58} \text{ m}$$

$$\mathbf{F} = \frac{200 \text{ N}}{\sqrt{0.58}}(0.7\mathbf{i} - 0.3\mathbf{j})$$

$$(a) \quad \mathbf{M}_P = \mathbf{PA} \times \mathbf{F} = (-0.6 \text{ m}\mathbf{i} - 0.3 \text{ m}\mathbf{j}) \times \frac{200 \text{ N}}{\sqrt{0.58}}(0.7\mathbf{i} - 0.3\mathbf{j})$$

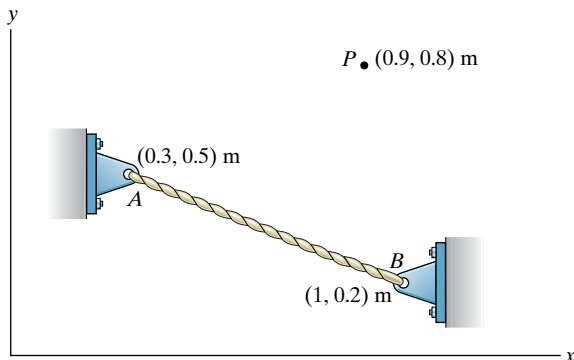
Carrying out the cross product we find

$$\boxed{\mathbf{M}_P = 102.4 \text{ N-mk}}$$

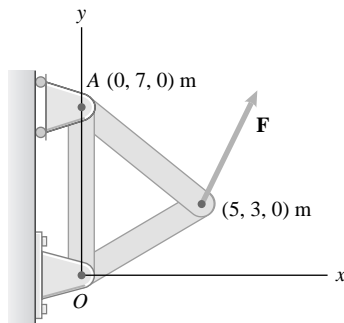
$$(b) \quad \mathbf{M}_P = \mathbf{PB} \times \mathbf{F} = (0.1 \text{ m}\mathbf{i} - 0.6 \text{ m}\mathbf{j}) \times \frac{200 \text{ N}}{\sqrt{0.58}}(0.7\mathbf{i} - 0.3\mathbf{j})$$

Carrying out the cross product we find

$$\boxed{\mathbf{M}_P = 102.4 \text{ N-mk}}$$



Problem 4.50 The line of action of \mathbf{F} is contained in the x - y plane. The moment of \mathbf{F} about O is $140\mathbf{k}$ (N-m), and the moment of \mathbf{F} about A is $280\mathbf{k}$ (N-m). What are the components of \mathbf{F} ?



Solution: The strategy is to find the moments in terms of the components of \mathbf{F} and solve the resulting simultaneous equations. The position vector from O to the point of application is $\mathbf{r}_{OF} = 5\mathbf{i} + 3\mathbf{j}$. The position vector from A to the point of application is $\mathbf{r}_{AF} = (5 - 0)\mathbf{i} + (3 - 7)\mathbf{j} = 5\mathbf{i} - 4\mathbf{j}$. The cross products:

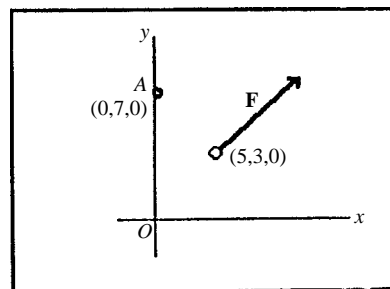
$$\mathbf{r}_{OF} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ F_X & F_Y & 0 \end{vmatrix} = (5F_Y - 3F_X)\mathbf{k} = 140\mathbf{k}, \text{ and}$$

$$\mathbf{r}_{AF} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -4 & 0 \\ F_X & F_Y & 0 \end{vmatrix} = (5F_Y + 4F_X)\mathbf{k} = 280\mathbf{k}.$$

Take the dot product of both sides with \mathbf{k} to eliminate \mathbf{k} . The simultaneous equations are:

$$5F_Y - 3F_X = 140, \quad 5F_Y + 4F_X = 280.$$

Solving: $F_Y = 40$, $F_X = 20$, from which $\mathbf{F} = 20\mathbf{i} + 40\mathbf{j}$ (N)

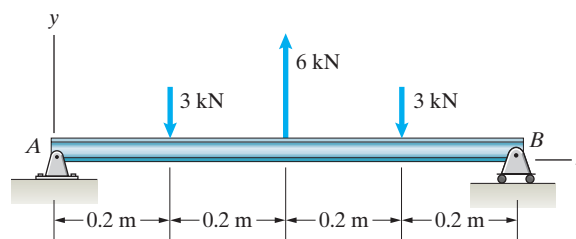


Problem 4.51 Use Eq. (4.2) to determine the sum of the moments of the three forces (a) about A , (b) about B .

Solution:

$$\begin{aligned} \text{(a)} \quad \mathbf{M}_A &= 0.2\mathbf{i} \times (-3\mathbf{j}) + 0.4\mathbf{i} \times 6\mathbf{j} + 0.6\mathbf{i} \times (-3\mathbf{j}) \\ &= \mathbf{0}. \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \mathbf{M}_B &= (-0.2\mathbf{i}) \times (-3\mathbf{j}) + (-0.4\mathbf{i}) \times 6\mathbf{j} + (-0.6\mathbf{i}) \times (-3\mathbf{j}) \\ &= \mathbf{0}. \end{aligned}$$

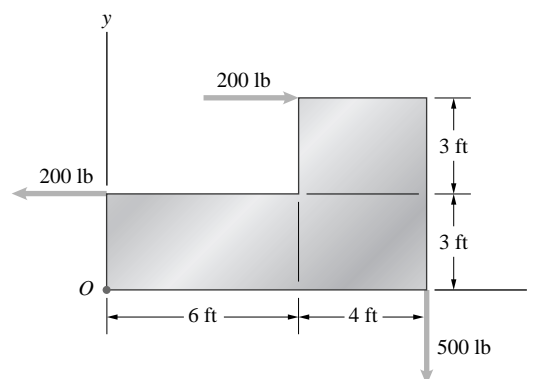


Problem 4.52 Three forces are applied to the plate. Use Eq. (4.2) to determine the sum of the moments of the three forces about the origin O .

Solution: The position vectors from O to the points of application of the forces are: $\mathbf{r}_{O1} = 3\mathbf{j}$, $\mathbf{F}_1 = -200\mathbf{i}$; $\mathbf{r}_{O2} = 10\mathbf{i}$, $\mathbf{F}_2 = -500\mathbf{j}$; $\mathbf{r}_{O3} = 6\mathbf{i} + 6\mathbf{j}$, $\mathbf{F}_3 = 200\mathbf{i}$.

The sum of the moments about O is

$$\begin{aligned} \sum \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ -200 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 0 & 0 \\ 0 & -500 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 6 & 0 \\ 200 & 0 & 0 \end{vmatrix} \text{ lb} \\ &= 600\mathbf{k} - 5000\mathbf{k} - 1200\mathbf{k} = -5600\mathbf{k} \text{ ft-lb} \end{aligned}$$



Problem 4.53 Three forces act on the plate. Use Eq. (4.2) to determine the sum of the moments of the three forces about point P .

Solution:

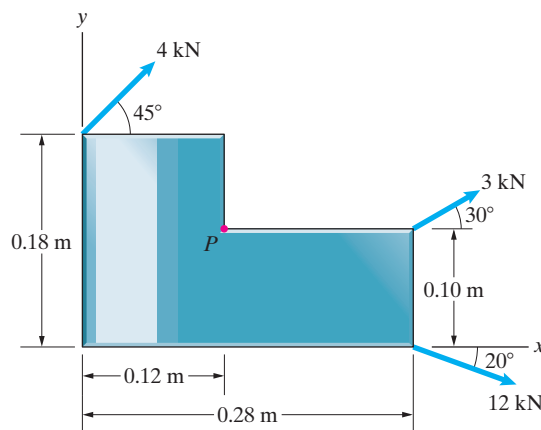
$$\mathbf{r}_1 = (-0.12\mathbf{i} + 0.08\mathbf{j}) \text{ m}, \quad \mathbf{F}_1 = (4 \cos 45^\circ \mathbf{i} + 4 \sin 45^\circ \mathbf{j}) \text{ kN}$$

$$\mathbf{r}_2 = (0.16\mathbf{i}) \text{ m}, \quad \mathbf{F}_2 = (3 \cos 30^\circ \mathbf{i} + 3 \sin 30^\circ \mathbf{j}) \text{ kN}$$

$$\mathbf{r}_3 = (0.16\mathbf{i} - 0.1\mathbf{j}) \text{ m}, \quad \mathbf{F}_3 = (12 \cos 20^\circ \mathbf{i} - 12 \sin 20^\circ \mathbf{j}) \text{ kN}$$

$$\mathbf{M}_P = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3$$

$$\mathbf{M}_P = (0.145 \text{ kN}\cdot\text{m})\mathbf{k} = (145 \text{ N}\cdot\text{m})\mathbf{k}$$



Problem 4.54 (a) Determine the magnitude of the moment of the 150-N force about A by calculating the perpendicular distance from A to the line of action of the force.

(b) Use Eq. (4.2) to determine the magnitude of the moment of the 150-N force about A .

Solution:

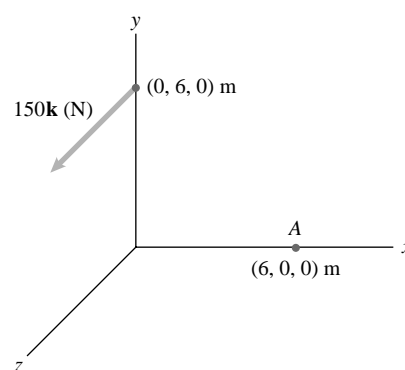
(a) The perpendicular from A to the line of action of the force lies in the x - y plane

$$d = \sqrt{6^2 + 6^2} = 8.485 \text{ m}$$

$$|\mathbf{M}| = dF = (8.485)(150) = 1270 \text{ N}\cdot\text{m}$$

(b) $\mathbf{M} = (-6\mathbf{i} + 6\mathbf{j}) \times (150\mathbf{k}) = -900\mathbf{j} + 900\mathbf{i} \text{ N}\cdot\text{m}$

$$|\mathbf{M}| = \sqrt{900^2 + 900^2} = 1270 \text{ N}\cdot\text{m}$$



Problem 4.55 (a) Determine the magnitude of the moment of the 600-N force about A by calculating the perpendicular distance from A to the line of action of the force.

(b) Use Eq. (4.2) to determine the magnitude of the moment of the 600-N force about A .

Solution:

- (a) Choose some point $P(x, 0, 0.8 \text{ m})$, on the line of action of the force. The distance from A to P is then

$$d = \sqrt{(x - 0.6 \text{ m})^2 + (0 - 0.5 \text{ m})^2 + (0.8 \text{ m} - 0.4 \text{ m})^2}$$

The perpendicular distance is the shortest distance d which occurs when $x = 0.6 \text{ m}$. We have $d = 0.6403 \text{ m}$. Thus the magnitude of the moment is

$$M = (600 \text{ N})(0.6403 \text{ m}) = 384 \text{ N}\cdot\text{m}$$

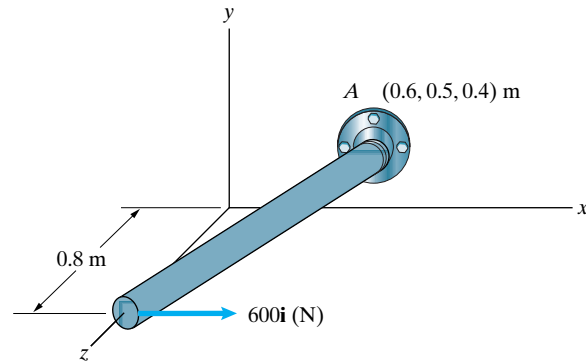
- (b) Define the point on the end of the rod to be B . Then $\mathbf{AB} = (-0.6\mathbf{i} - 0.5\mathbf{j} + 0.4\mathbf{k}) \text{ m}$ we have

$$\mathbf{M} = \mathbf{AB} \times \mathbf{F} = (-0.6\mathbf{i} - 0.5\mathbf{j} + 0.4\mathbf{k}) \text{ m} \times (600 \text{ N})\mathbf{i}$$

$$\mathbf{M} = (240\mathbf{j} + 300\mathbf{k}) \text{ N}\cdot\text{m}$$

Thus the magnitude is

$$M = \sqrt{(240 \text{ N}\cdot\text{m})^2 + (300 \text{ N}\cdot\text{m})^2} = 384 \text{ N}\cdot\text{m}$$



Problem 4.56 what is the magnitude of the moment of \mathbf{F} about point B ?

Solution: The position vector from B to A is

$$\mathbf{r}_{BA} = [(4 - 8)\mathbf{i} + (4 - 1)\mathbf{j} + (2 - (-2))\mathbf{k}] \text{ ft}$$

$$\mathbf{r}_{BA} = (-4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) \text{ ft}$$

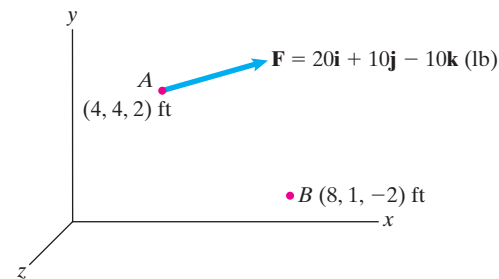
The moment of \mathbf{F} about B is

$$\mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 3 & 4 \\ 20 & 10 & -10 \end{vmatrix} = (-70\mathbf{i} + 40\mathbf{j} - 100\mathbf{k}) \text{ ft}\cdot\text{lb}$$

Its magnitude is

$$|\mathbf{M}_B| = \sqrt{(-70 \text{ ft}\cdot\text{lb})^2 + (40 \text{ ft}\cdot\text{lb})^2 + (-100 \text{ ft}\cdot\text{lb})^2} = 128 \text{ ft}\cdot\text{lb}$$

$$|\mathbf{M}_B| = 128 \text{ ft}\cdot\text{lb}$$



Problem 4.57 In Example 4.5, suppose that the attachment point C is moved to the location $(8, 2, 0)$ m and the tension in cable AC changes to 25 kN. What is the sum of the moments about O due to the forces exerted on the attachment point A by the two cables?

Solution: The position vector from A to C is

$$\mathbf{r}_{AC} = [(8 - 4)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 6)\mathbf{k}] \text{ m}$$

$$\mathbf{r}_{AC} = (4\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$$

The force exerted at A by cable AC can be written

$$\mathbf{F}_{AC} = (25 \text{ kN}) \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = (13.4\mathbf{i} + 6.68\mathbf{j} - 20.0\mathbf{k}) \text{ kN}$$

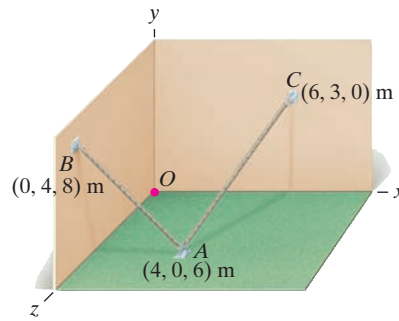
The total force exerted at A by the two cables is

$$\mathbf{F} = \mathbf{F}_{AB} + \mathbf{F}_{AC} = (6.70\mathbf{i} + 13.3\mathbf{j} - 16.7\mathbf{k}) \text{ kN}$$

The moment about O is

$$\mathbf{M}_O = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 6 \\ 6.70 & 13.3 & -16.7 \end{vmatrix} = (-80.1\mathbf{i} + 107\mathbf{j} + 53.4\mathbf{k}) \text{ kN}\cdot\text{m}$$

$$\boxed{\mathbf{M}_O = (-80.1\mathbf{i} + 107\mathbf{j} + 53.4\mathbf{k}) \text{ kN}\cdot\text{m}}$$



Problem 4.58 The rope exerts a force of magnitude $|\mathbf{F}| = 200$ lb on the top of the pole at B . Determine the magnitude of the moment of \mathbf{F} about A .

Solution: The position vector from B to C is

$$\mathbf{r}_{BC} = [(3 - 5)\mathbf{i} + (0 - 6)\mathbf{j} + (4 - 1)\mathbf{k}] \text{ ft}$$

$$\mathbf{r}_{BC} = (-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \text{ ft}$$

The force \mathbf{F} can be written

$$\mathbf{F} = (200 \text{ lb}) \frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = (-57.1\mathbf{i} - 171\mathbf{j} + 85.7\mathbf{k}) \text{ lb}$$

The moment of \mathbf{F} about A is

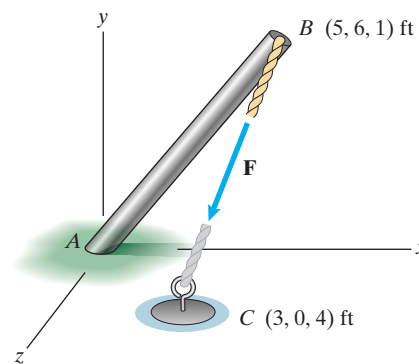
$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 6 & 1 \\ -57.1 & -171 & 85.7 \end{vmatrix}$$

$$= (686\mathbf{i} - 486\mathbf{j} - 514\mathbf{k}) \text{ ft}\cdot\text{lb}$$

Its magnitude is

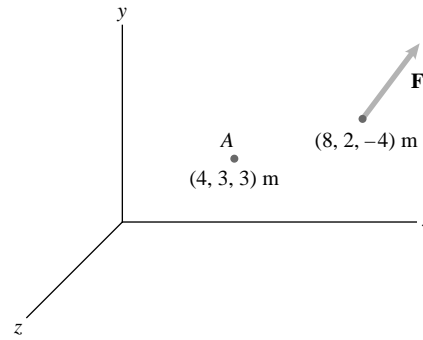
$$|\mathbf{M}_A| = \sqrt{(686 \text{ ft}\cdot\text{lb})^2 + (-486 \text{ ft}\cdot\text{lb})^2 + (-514 \text{ ft}\cdot\text{lb})^2} = 985 \text{ ft}\cdot\text{lb}$$

$$\boxed{|\mathbf{M}_A| = 985 \text{ ft}\cdot\text{lb}}$$



Problem 4.59 The force $\mathbf{F} = 30\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}$ (N).

- Determine the magnitude of the moment of \mathbf{F} about A .
- Suppose that you can change the direction of \mathbf{F} while keeping its magnitude constant, and you want to choose a direction that maximizes the moment of \mathbf{F} about A . What is the magnitude of the resulting maximum moment?



Solution: The vector from A to the point of application of \mathbf{F} is

$$\mathbf{r} = 4\mathbf{i} - 1\mathbf{j} - 7\mathbf{k} \text{ m}$$

and

$$|\mathbf{r}| = \sqrt{4^2 + 1^2 + 7^2} = 8.12 \text{ m}$$

- The moment of \mathbf{F} about A is

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & -7 \\ 30 & 20 & -10 \end{vmatrix} = 150\mathbf{i} - 170\mathbf{j} + 110\mathbf{k} \text{ N}\cdot\text{m}$$

$$|\mathbf{M}_A| = \sqrt{150^2 + 170^2 + 110^2} = 252 \text{ N}\cdot\text{m}$$

- The maximum moment occurs when $\mathbf{r} \perp \mathbf{F}$. In this case

$$|\mathbf{M}_{A_{\max}}| = |\mathbf{r}||\mathbf{F}|$$

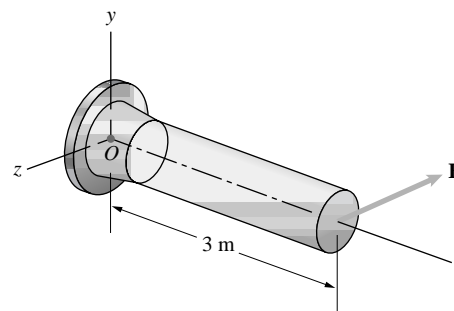
Hence, we need $|\mathbf{F}|$.

$$|\mathbf{F}| = \sqrt{30^2 + 20^2 + 10^2} = 37.4 \text{ (N)}$$

Thus,

$$|\mathbf{M}_{A_{\max}}| = (8.12)(37.4) = 304 \text{ N}\cdot\text{m}$$

Problem 4.60 The direction cosines of the force \mathbf{F} are $\cos \theta_x = 0.818$, $\cos \theta_y = 0.182$, and $\cos \theta_z = -0.545$. The support of the beam at O will fail if the magnitude of the moment of \mathbf{F} about O exceeds 100 kN-m. Determine the magnitude of the largest force \mathbf{F} that can safely be applied to the beam.



Solution: The strategy is to determine the perpendicular distance from O to the action line of \mathbf{F} , and to calculate the largest magnitude of \mathbf{F} from $M_O = D|\mathbf{F}|$. The position vector from O to the point of application of \mathbf{F} is $\mathbf{r}_{OF} = 3\mathbf{i}$ (m). Resolve the position vector into components parallel and normal to \mathbf{F} . The component parallel to \mathbf{F} is $\mathbf{r}_p = (\mathbf{r}_{OF} \cdot \mathbf{e}_F)\mathbf{e}_F$, where the unit vector \mathbf{e}_F parallel to \mathbf{F} is $\mathbf{e}_F = \mathbf{i} \cos \theta_x + \mathbf{j} \cos \theta_y + \mathbf{k} \cos \theta_z = 0.818\mathbf{i} + 0.182\mathbf{j} - 0.545\mathbf{k}$. The dot product is $\mathbf{r}_{OF} \cdot \mathbf{e}_F = 2.454$. The parallel component is $\mathbf{r}_p = 2.007\mathbf{i} + 0.4466\mathbf{j} - 1.3374\mathbf{k}$. The component normal to \mathbf{F} is $\mathbf{r}_n = \mathbf{r}_{OF} - \mathbf{r}_p = (3 - 2)\mathbf{i} - 0.4466\mathbf{j} + 1.3374\mathbf{k}$. The magnitude of the normal component is the perpendicular distance: $D = \sqrt{1^2 + 0.4466^2 + 1.337^2} = 1.7283 \text{ m}$. The maximum moment allowed is $M_O = 1.7283|\mathbf{F}| = 100 \text{ kN}\cdot\text{m}$, from which

$$|\mathbf{F}| = \frac{100 \text{ kN}\cdot\text{m}}{1.7283 \text{ m}} = 57.86 \cong 58 \text{ kN}$$

Problem 4.61 The force \mathbf{F} exerted on the grip of the exercise machine points in the direction of the unit vector $\mathbf{e} = \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$ and its magnitude is 120 N. Determine the magnitude of the moment of \mathbf{F} about the origin O .

Solution: The vector from O to the point of application of the force is

$$\mathbf{r} = 0.25\mathbf{i} + 0.2\mathbf{j} - 0.15\mathbf{k} \text{ m}$$

and the force is $\mathbf{F} = |\mathbf{F}|\mathbf{e}$

or

$$\mathbf{F} = 80\mathbf{i} - 80\mathbf{j} + 40\mathbf{k} \text{ N.}$$

The moment of \mathbf{F} about O is

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2 & -0.15 \\ 80 & -80 & 40 \end{vmatrix} \text{ N-m}$$

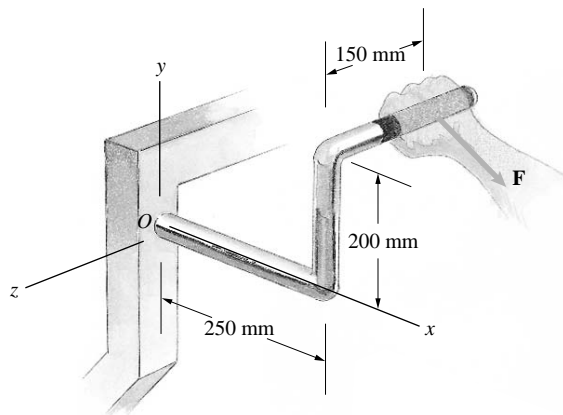
or

$$\mathbf{M}_O = -4\mathbf{i} - 22\mathbf{j} - 36\mathbf{k} \text{ N-m}$$

and

$$|\mathbf{M}_O| = \sqrt{4^2 + 22^2 + 36^2} \text{ N-m}$$

$$|\mathbf{M}_O| = 42.4 \text{ N-m}$$



Problem 4.62 The force \mathbf{F} in Problem 4.61 points in the direction of the unit vector $\mathbf{e} = \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$. The support at O will safely support a moment of 560 N-m magnitude.

- Based on this criterion, what is the largest safe magnitude of \mathbf{F} ?
- If the force \mathbf{F} may be exerted in any direction, what is its largest safe magnitude?

Solution: See the figure of Problem 4.61.

The moment in Problem 4.61 can be written as

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2 & -0.15 \\ \frac{2}{3}F & -\frac{2}{3}F & +\frac{1}{3}F \end{vmatrix} \text{ where } F = |\mathbf{F}|$$

$$\mathbf{M}_O = (-0.0333\mathbf{i} - 0.1833\mathbf{j} - 0.3\mathbf{k})F$$

And the magnitude of \mathbf{M}_O is

$$|\mathbf{M}_O| = (\sqrt{0.0333^2 + 0.1833^2 + 0.3^2})F$$

$$|\mathbf{M}_O| = 0.353 F$$

If we set $|\mathbf{M}_O| = 560 \text{ N-m}$, we can solve for $|\mathbf{F}_{\max}|$

$$560 = 0.353|\mathbf{F}_{\max}|$$

$$|\mathbf{F}_{\max}| = 1586 \text{ N}$$

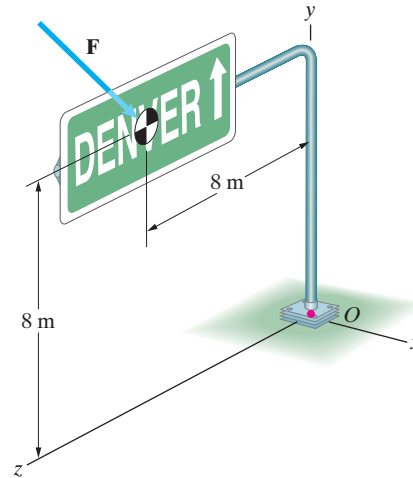
- If \mathbf{F} can be in any direction, then the worst case is when $\mathbf{r} \perp \mathbf{F}$. The moment in this case is $|\mathbf{M}_O| = |\mathbf{r}||\mathbf{F}_{\text{worst}}|$

$$|\mathbf{r}| = \sqrt{0.25^2 + 0.2^2 + 0.15^2} = 0.3536 \text{ m}$$

$$560 = (0.3536)|\mathbf{F}_{\text{worst}}|$$

$$|\mathbf{F}_{\text{worst}}| = 1584 \text{ N}$$

Problem 4.63 A civil engineer in Boulder, Colorado estimates that under the severest expected Chinook winds, the total force on the highway sign will be $\mathbf{F} = 2.8\mathbf{i} - 1.8\mathbf{j}$ (kN). Let \mathbf{M}_O be the moment due to \mathbf{F} about the base O of the cylindrical column supporting the sign. The y component of \mathbf{M}_O is called the *torsion* exerted on the cylindrical column at the base, and the component of \mathbf{M}_O parallel to the x - z plane is called the *bending moment*. Determine the magnitudes of the torsion and bending moment.



Solution: The total moment is

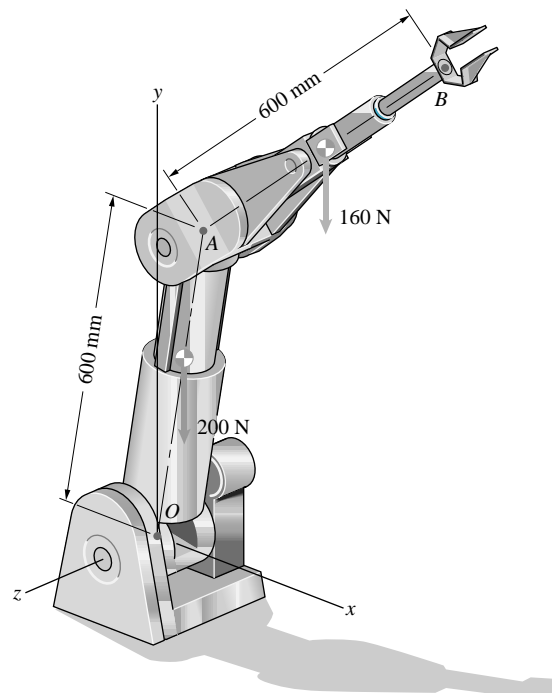
$$\mathbf{M} = (8\mathbf{j} + 8\mathbf{k}) \text{ m} \times (2.8\mathbf{i} - 1.8\mathbf{j}) \text{ kN}$$

$$= (14.4\mathbf{i} + 22.4\mathbf{j} - 22.4\mathbf{k}) \text{ kN-m}$$

We now identify

$\text{Torsion} = M_y = 22.4 \text{ kN-m}$ $\text{Bending moment} = \sqrt{M_x^2 + M_z^2}$ $= \sqrt{(14.4 \text{ kNm})^2 + (22.4 \text{ kNm})^2} = 26.6 \text{ kN-m}$
--

Problem 4.64 The weights of the arms OA and AB of the robotic manipulator act at their midpoints. The direction cosines of the centerline of arm OA are $\cos \theta_x = 0.500$, $\cos \theta_y = 0.866$, and $\cos \theta_z = 0$, and the direction cosines of the centerline of arm AB are $\cos \theta_x = 0.707$, $\cos \theta_y = 0.619$, and $\cos \theta_z = -0.342$. What is the sum of the moments about O due to the two forces?



Solution: By definition, the direction cosines are the scalar components of the unit vectors. Thus the unit vectors are $\mathbf{e}_1 = 0.5\mathbf{i} + 0.866\mathbf{j}$, and $\mathbf{e}_2 = 0.707\mathbf{i} + 0.619\mathbf{j} - 0.342\mathbf{k}$. The position vectors of the midpoints of the arms are

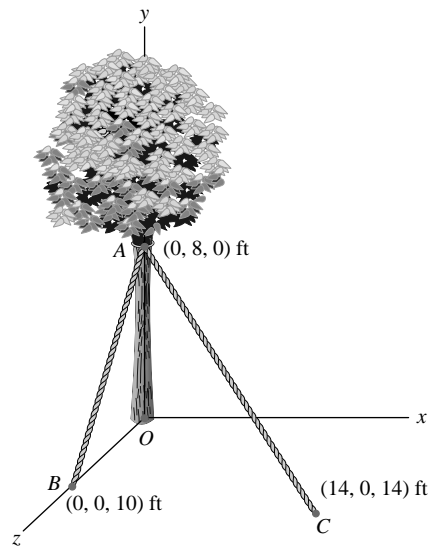
$$\mathbf{r}_1 = 0.3\mathbf{e}_1 = 0.3(0.5\mathbf{i} + 0.866\mathbf{j}) = 0.15\mathbf{i} + 0.2598\mathbf{j}$$

$$\mathbf{r}_2 = 0.6\mathbf{e}_1 + 0.3\mathbf{e}_2 = 0.512\mathbf{i} + 0.7053\mathbf{j} - 0.1026\mathbf{k}$$

The sum of moments is

$$\begin{aligned} \mathbf{M} &= \mathbf{r}_1 \times \mathbf{W}_1 + \mathbf{r}_2 \times \mathbf{W}_2 \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.2598 & 0 \\ 0 & -200 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.512 & 0.7053 & -0.1026 \\ 0 & -160 & 0 \end{vmatrix} \\ &= -16.42\mathbf{i} - 111.92\mathbf{k} \text{ (N-m)} \end{aligned}$$

Problem 4.65 The tension in cable AB is 100 lb. If you want the magnitude of the moment about the base O of the tree due to the forces exerted on the tree by the two ropes to be 1500 ft-lb, what is the necessary tension in rope AC ?



Solution: We have the forces

$$\mathbf{F}_1 = \frac{100 \text{ lb}}{\sqrt{164}}(-8\mathbf{j} + 10\mathbf{k}), \quad \mathbf{F}_2 = \frac{T_{AC}}{\sqrt{456}}(14\mathbf{i} - 8\mathbf{j} + 14\mathbf{k})$$

Thus the total moment is

$$\begin{aligned} \mathbf{M} &= (8 \text{ ft})\mathbf{j} \times (\mathbf{F}_1 + \mathbf{F}_2) = (625 \text{ ft-lb} + 5.24 \text{ ft } T_{AC})\mathbf{i} \\ &\quad - (5.24 \text{ ft})T_{AC}\mathbf{k} \end{aligned}$$

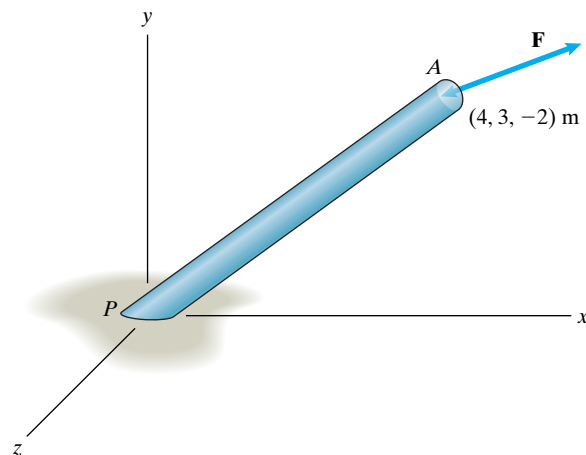
The magnitude squared is then

$$(625 \text{ ft-lb} + 5.24 \text{ ft } T_{AC})^2 + (5.24 \text{ ft } T_{AC})^2 = (1500 \text{ ft-lb})^2$$

Solving we find

$$T_{AC} = 134 \text{ lb}$$

Problem 4.66* A force \mathbf{F} acts at the top end A of the pole. Its magnitude is $|\mathbf{F}| = 6 \text{ kN}$ and its x component is $F_x = 4 \text{ kN}$. The coordinates of point A are shown. Determine the components of \mathbf{F} so that the magnitude of the moment due to \mathbf{F} about the base P of the pole is as large as possible. (There are two answers.)



Solution: The force is given by $\mathbf{F} = (4 \text{ kN}\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k})$.

Since the magnitude is constrained we must have

$$(4 \text{ kN})^2 + F_y^2 + F_z^2 = (6 \text{ kN})^2 \Rightarrow F_z = \sqrt{20 \text{ kN}^2 - F_y^2}$$

Thus we will use (suppressing the units)

$$\mathbf{F} = (4\mathbf{i} + F_y\mathbf{j} + \sqrt{20 - F_y^2}\mathbf{k})$$

The moment is now given by

$$\mathbf{M} = (4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \times \mathbf{F}$$

$$\mathbf{M} = [2F_y + 3\sqrt{20 - F_y^2}]\mathbf{i} - [8 + 4\sqrt{20 - F_y^2}]\mathbf{j} + [-12 + 4F_y]\mathbf{k}$$

The magnitude is

$$M^2 = 708 - 5F_y^2 + 64\sqrt{20 - F_y^2} + 12F_y(-8 + \sqrt{20 - F_y^2})$$

To maximize this quantity we solve $\frac{dM^2}{dF_y} = 0$ for the critical values of F_y .

There are three solutions $F_y = -4.00, -3.72, -3.38$.

The first and third solutions produce the same maximum moment. The second answer corresponds to a local minimum and is therefore discarded.

So the force that produces the largest moment is

$$\mathbf{F} = (4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \quad \text{or} \quad \mathbf{F} = (4\mathbf{i} - 3.38\mathbf{j} + 2.92\mathbf{k})$$

Problem 4.67 The force $\mathbf{F} = 5\mathbf{i}$ (kN) acts on the ring A where the cables AB , AC , and AD are joined. What is the sum of the moments about point D due to the force \mathbf{F} and the three forces exerted on the ring by the cables?

Strategy: The ring is in equilibrium. Use what you know about the four forces acting on it.

Solution: The vector from D to A is

$$\mathbf{r}_{DA} = 12\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ m.}$$

The sum of the moments about point D is given by

$$\sum \mathbf{M}_D = \mathbf{r}_{DA} \times \mathbf{F}_{AD} + \mathbf{r}_{DA} \times \mathbf{F}_{AC} + \mathbf{r}_{DA} \times \mathbf{F}_{AB} + \mathbf{r}_{DA} \times \mathbf{F}$$

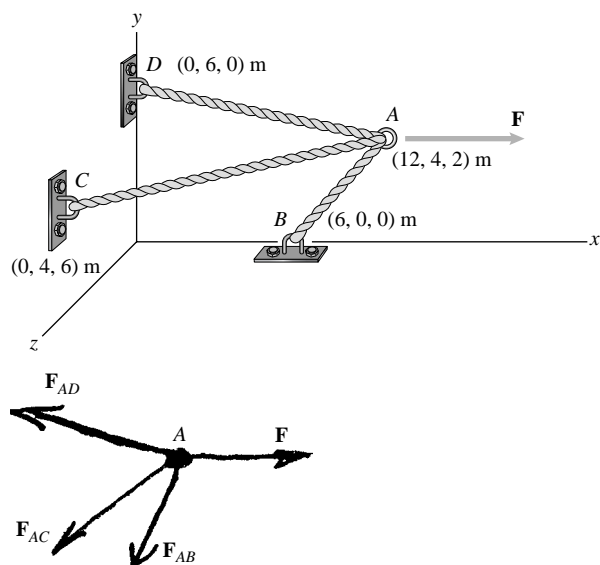
$$\sum \mathbf{M}_D = \mathbf{r}_{DA} \times (\mathbf{F}_{AD} + \mathbf{F}_{AC} + \mathbf{F}_{AB} + \mathbf{F})$$

However, we are given that ring A is in equilibrium and this implies that

$$(\mathbf{F}_{AD} + \mathbf{F}_{AC} + \mathbf{F}_{AB} + \mathbf{F}) = \mathbf{0} = 0$$

Thus,

$$\sum \mathbf{M}_D = \mathbf{r}_{DA} \times (\mathbf{0}) = 0$$



Problem 4.68 In Problem 4.67, determine the moment about point D due to the force exerted on the ring A by the cable AB .

Solution: We need to write the forces as magnitudes times the appropriate unit vectors, write the equilibrium equations for A in component form, and then solve the resulting three equations for the three unknown magnitudes. The unit vectors are of the form

$$\mathbf{e}_{AP} = \frac{(x_P - x_A)\mathbf{i} + (y_P - y_A)\mathbf{j} + (z_P - z_A)\mathbf{k}}{|\mathbf{r}_{AP}|}$$

Where P takes on values B , C , and D

Calculating the unit vectors, we get

$$\begin{cases} \mathbf{e}_{AB} = -0.802\mathbf{i} - 0.535\mathbf{j} - 0.267\mathbf{k} \\ \mathbf{e}_{AC} = -0.949\mathbf{i} + 0\mathbf{j} + 0.316\mathbf{k} \\ \mathbf{e}_{AD} = -0.973\mathbf{i} + 0.162\mathbf{j} - 0.162\mathbf{k} \end{cases}$$

From equilibrium, we have

$$F_{AB}\mathbf{e}_{AB} + F_{AC}\mathbf{e}_{AC} + F_{AD}\mathbf{e}_{AD} + 5\mathbf{i} \text{ (kN)} = 0$$

In component form, we get

$$\begin{cases} \mathbf{i}: -0.802F_{AB} - 0.949F_{AC} - 0.973F_{AD} + 5 = 0 \\ \mathbf{j}: -0.535F_{AB} + (0)F_{AC} + 0.162F_{AD} = 0 \\ \mathbf{k}: -0.267F_{AB} + 0.316F_{AC} - 0.162F_{AD} = 0 \end{cases}$$

Solving, we get

$$F_{AB} = 779.5 \text{ N}, \quad F_{AC} = 1976 \text{ N}$$

$$F_{AD} = 2569 \text{ N}$$

The vector from D to A is

$$\mathbf{r}_{DA} = 12\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ m}$$

The force \mathbf{F}_{AB} is given by

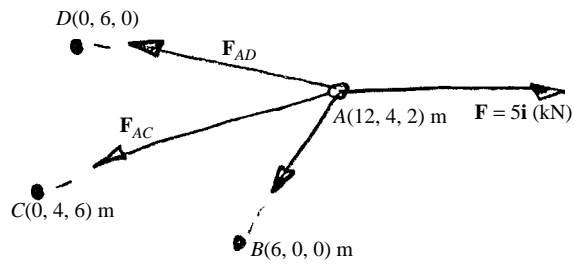
$$\mathbf{F}_{AB} = F_{AB}\mathbf{e}_{AB}$$

$$\mathbf{F}_{AB} = -0.625\mathbf{i} - 0.417\mathbf{j} - 0.208\mathbf{k} \text{ (kN)}$$

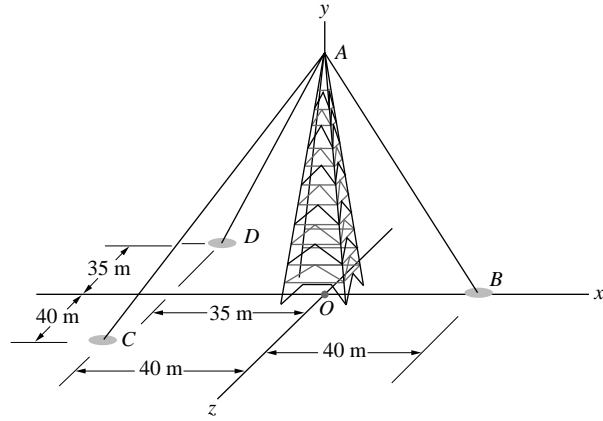
The moment about D is given by

$$\mathbf{M}_D = \mathbf{r}_{DA} \times \mathbf{F}_{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & -2 & 2 \\ -0.625 & -0.417 & -0.208 \end{vmatrix}$$

$$\mathbf{M}_D = 1.25\mathbf{i} + 1.25\mathbf{j} - 6.25\mathbf{k} \text{ (kN-m)}$$



Problem 4.69 The tower is 70 m tall. The tensions in cables AB , AC , and AD are 4 kN, 2 kN, and 2 kN, respectively. Determine the sum of the moments about the origin O due to the forces exerted by the cables at point A .



Solution: The coordinates of the points are $A(0, 70, 0)$, $B(40, 0, 0)$, $C(-40, 0, 40)$, $D(-35, 0, -35)$. The position vectors corresponding to the cables are:

$$\mathbf{r}_{AD} = (-35 - 0)\mathbf{i} + (0 - 70)\mathbf{j} + (-35 - 0)\mathbf{k}$$

$$\mathbf{r}_{AD} = -35\mathbf{i} - 70\mathbf{j} - 35\mathbf{k}$$

$$\mathbf{r}_{AC} = (-40 - 0)\mathbf{i} + (0 - 70)\mathbf{j} + (40 - 0)\mathbf{k}$$

$$\mathbf{r}_{AC} = -40\mathbf{i} - 70\mathbf{j} + 40\mathbf{k}$$

$$\mathbf{r}_{AB} = (40 - 0)\mathbf{i} + (0 - 70)\mathbf{j} + (0 - 0)\mathbf{k}$$

$$\mathbf{r}_{AB} = 40\mathbf{i} - 70\mathbf{j} + 0\mathbf{k}$$

The unit vectors corresponding to these position vectors are:

$$\begin{aligned}\mathbf{e}_{AD} &= \frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|} = \frac{-35}{85.73}\mathbf{i} - \frac{70}{85.73}\mathbf{j} - \frac{35}{85.73}\mathbf{k} \\ &= -0.4082\mathbf{i} - 0.8165\mathbf{j} - 0.4082\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{e}_{AC} &= \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = -\frac{40}{90}\mathbf{i} - \frac{70}{90}\mathbf{j} + \frac{40}{90}\mathbf{k} \\ &= -0.4444\mathbf{i} - 0.7778\mathbf{j} + 0.4444\mathbf{k}\end{aligned}$$

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{40}{80.6}\mathbf{i} - \frac{70}{80.6}\mathbf{j} + 0\mathbf{k} = 0.4962\mathbf{i} - 0.8682\mathbf{j} + 0\mathbf{k}$$

The forces at point A are

$$\mathbf{T}_{AB} = 4\mathbf{e}_{AB} = 1.9846\mathbf{i} - 3.4729\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{T}_{AC} = 2\mathbf{e}_{AC} = -0.8889\mathbf{i} - 1.5556\mathbf{j} + 0.8889\mathbf{k}$$

$$\mathbf{T}_{AD} = 2\mathbf{e}_{AD} = -0.8165\mathbf{i} - 1.6330\mathbf{j} - 0.8165\mathbf{k}$$

The sum of the forces acting at A are

$$\mathbf{T}_A = 0.2792\mathbf{i} - 6.6615\mathbf{j} + 0.07239\mathbf{k} \text{ (kN-m)}$$

The position vector of A is $\mathbf{r}_{OA} = 70\mathbf{j}$. The moment about O is $\mathbf{M} = \mathbf{r}_{OA} \times \mathbf{T}_A$

$$\begin{aligned}\mathbf{M} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 70 & 0 \\ 0.2792 & -6.6615 & 0.07239 \end{vmatrix} \\ &= (70)(0.07239)\mathbf{i} - \mathbf{j}0 - \mathbf{k}(70)(0.2792) = 5.067\mathbf{i} - 19.54\mathbf{k}\end{aligned}$$

Problem 4.70 Consider the 70-m tower in Problem 4.69. Suppose that the tension in cable AB is 4 kN, and you want to adjust the tensions in cables AC and AD so that the sum of the moments about the origin O due to the forces exerted by the cables at point A is zero. Determine the tensions.

Solution: From Varignon's theorem, the moment is zero only if the resultant of the forces normal to the vector \mathbf{r}_{OA} is zero. From Problem 4.69 the unit vectors are:

$$\mathbf{e}_{AD} = \frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|} = \frac{-35}{85.73}\mathbf{i} - \frac{70}{85.73}\mathbf{j} - \frac{35}{85.73}\mathbf{k}$$

$$= -0.4082\mathbf{i} - 0.8165\mathbf{j} - 0.4082\mathbf{k}$$

$$\mathbf{e}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = -\frac{40}{90}\mathbf{i} - \frac{70}{90}\mathbf{j} + \frac{40}{90}\mathbf{k}$$

$$= -0.4444\mathbf{i} - 0.7778\mathbf{j} + 0.4444\mathbf{k}$$

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{40}{80.6}\mathbf{i} - \frac{70}{80.6}\mathbf{j} + 0\mathbf{k} = 0.4963\mathbf{i} - 0.8685\mathbf{j} + 0\mathbf{k}$$

The tensions are $\mathbf{T}_{AB} = 4\mathbf{e}_{AB}$, $\mathbf{T}_{AC} = |\mathbf{T}_{AC}|\mathbf{e}_{AC}$, and $\mathbf{T}_{AD} = |\mathbf{T}_{AD}|\mathbf{e}_{AD}$. The components normal to \mathbf{r}_{OA} are

$$\sum \mathbf{F}_X = (-0.4082|\mathbf{T}_{AD}| - 0.4444|\mathbf{T}_{AC}| + 1.9846)\mathbf{i} = 0$$

$$\sum \mathbf{F}_Z = (-0.4082|\mathbf{T}_{AD}| + 0.4444|\mathbf{T}_{AC}|)\mathbf{k} = 0.$$

The HP-28S calculator was used to solve these equations:

$$|\mathbf{T}_{AC}| = 2.23 \text{ kN}, \quad |\mathbf{T}_{AD}| = 2.43 \text{ kN}$$

Problem 4.71 The tension in cable AB is 150 N. The tension in cable AC is 100 N. Determine the sum of the moments about D due to the forces exerted on the wall by the cables.

Solution: The coordinates of the points A, B, C are $A(8, 0, 0)$, $B(0, 4, -5)$, $C(0, 8, 5)$, $D(0, 0, 5)$. The point A is the intersection of the lines of action of the forces. The position vector \mathbf{DA} is

$$\mathbf{r}_{DA} = 8\mathbf{i} + 0\mathbf{j} - 5\mathbf{k}.$$

The position vectors \mathbf{AB} and \mathbf{AC} are

$$\mathbf{r}_{AB} = -8\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}, \quad |\mathbf{r}_{AB}| = \sqrt{8^2 + 4^2 + 5^2} = 10.247 \text{ m}.$$

$$\mathbf{r}_{AC} = -8\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}, \quad |\mathbf{r}_{AC}| = \sqrt{8^2 + 8^2 + 5^2} = 12.369 \text{ m}.$$

The unit vectors parallel to the cables are:

$$\mathbf{e}_{AB} = -0.7807\mathbf{i} + 0.3904\mathbf{j} - 0.4879\mathbf{k},$$

$$\mathbf{e}_{AC} = -0.6468\mathbf{i} + 0.6468\mathbf{j} + 0.4042\mathbf{k}.$$

The tensions are

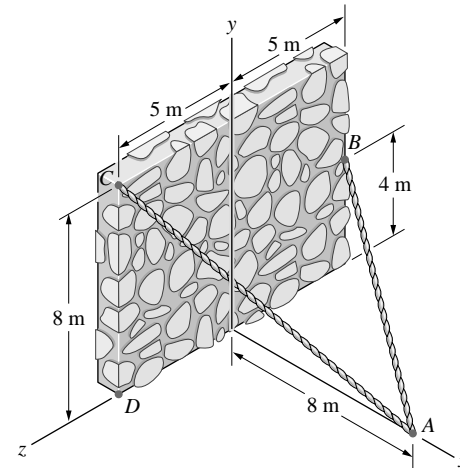
$$\mathbf{T}_{AB} = 150\mathbf{e}_{AB} = -117.11\mathbf{i} + 58.56\mathbf{j} - 73.19\mathbf{k},$$

$$\mathbf{T}_{AC} = 100\mathbf{e}_{AC} = -64.68\mathbf{i} + 64.68\mathbf{j} + 40.42\mathbf{k}.$$

The sum of the forces exerted by the wall on A is

$$\mathbf{T}_A = -181.79\mathbf{i} + 123.24\mathbf{j} - 32.77\mathbf{k}.$$

The force exerted on the wall by the cables is $-\mathbf{T}_A$. The moment about D is $\mathbf{M}_D = -\mathbf{r}_{DA} \times \mathbf{T}_A$,

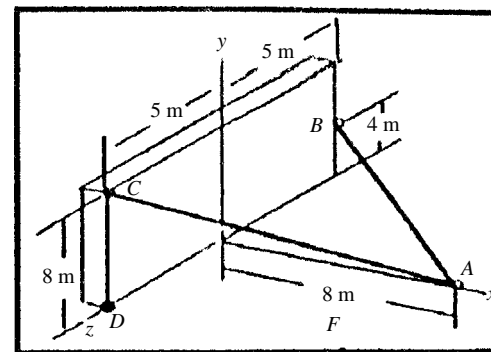


$$\mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & -5 \\ 181.79 & -123.24 & +32.77 \end{vmatrix} = (-123.24)(5)\mathbf{i}$$

$$- ((8)(+32.77) - (-5)(181.79))\mathbf{j} + (8)(-123.24)\mathbf{k}$$

$$\mathbf{M}_D = -616.2\mathbf{i} - 117.11\mathbf{j} - 985.9\mathbf{k} \text{ (N-m)}$$

(Note: An alternate method of solution is to express the moment in terms of the sum: $\mathbf{M}_D = (\mathbf{r}_{DC} \times \mathbf{T}_C) + (\mathbf{r}_{DB} \times \mathbf{T}_B)$.)



Problem 4.72 Consider the wall shown in Problem 4.71. The total force exerted by the two cables in the direction perpendicular to the wall is 2 kN. The magnitude of the sum of the moments about D due to the forces exerted on the wall by the cables is 18 kN-m. What are the tensions in the cables?

Solution: From the solution of Problem 4.71, we have $\mathbf{r}_{DA} = 8\mathbf{i} + 0\mathbf{j} - 5\mathbf{k}$. Forces in both cables pass through point A and we can use this vector to determine moments of both forces about D . The position vectors \mathbf{AB} and \mathbf{AC} are

$$\mathbf{r}_{AB} = -8\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}, \quad |\mathbf{r}_{AB}| = \sqrt{8^2 + 4^2 + 5^2} = 10.247 \text{ m.}$$

$$\mathbf{r}_{AC} = -8\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}, \quad |\mathbf{r}_{AC}| = \sqrt{8^2 + 8^2 + 5^2} = 12.369 \text{ m.}$$

The unit vectors parallel to the cables are:

$$\mathbf{e}_{AB} = -0.7807\mathbf{i} + 0.3904\mathbf{j} - 0.4879\mathbf{k},$$

$$\mathbf{e}_{AC} = -0.6468\mathbf{i} + 0.6468\mathbf{j} + 0.4042\mathbf{k}.$$

The tensions are

$$\mathbf{T}_{BA} = -T_{BA}\mathbf{e}_{AB} = -T_{BA}(-0.7807\mathbf{i} + 0.3904\mathbf{j} - 0.4879\mathbf{k}), \text{ and}$$

$$\mathbf{T}_{CA} = -T_{CA}\mathbf{e}_{AC} = -T_{CA}(-0.6468\mathbf{i} + 0.6468\mathbf{j} + 0.4042\mathbf{k}).$$

The sum of the forces exerted by the cables perpendicular to the wall is given by

$$T_{\text{Perpendicular}} = T_{AB}(0.7807) + T_{AC}(0.6468) = 2 \text{ kN.}$$

The moments of these two forces about D are given by

$$\mathbf{M}_D = (\mathbf{r}_{DA} \times \mathbf{T}_{CA}) + (\mathbf{r}_{DA} \times \mathbf{T}_{BA}) = \mathbf{r}_{DA} \times (\mathbf{T}_{CA} + \mathbf{T}_{BA}).$$

The sum of the two forces is given by

$$\mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & -5 \\ (T_{CA} + T_{CB})_X & (T_{CA} + T_{CB})_Y & (T_{CA} + T_{CB})_Z \end{vmatrix}.$$

This expression can be expanded to yield

$$\begin{aligned} \mathbf{M}_D &= 5(T_{CA} + T_{CB})_Y \mathbf{i} + [-8(T_{CA} + T_{CB})_Z - 5(T_{CA} + T_{CB})_X] \mathbf{j} \\ &\quad + 8(T_{CA} + T_{CB})_Y \mathbf{k}. \end{aligned}$$

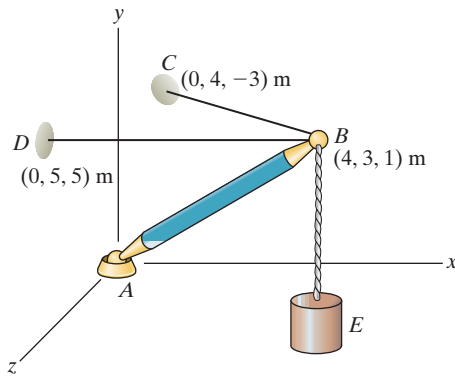
The magnitude of this vector is given as 18 kN-m. Thus, we obtain the relation

$$|\mathbf{M}_D| = \sqrt{25(T_{CA} + T_{CB})_Y^2 + [-8(T_{CA} + T_{CB})_Z - 5(T_{CA} + T_{CB})_X]^2 + 64(T_{CA} + T_{CB})_Y^2} = 18 \text{ kN-m.}$$

We now have two equations in the two tensions in the cables. Either algebraic substitution or a numerical solver can be used to give

$$\boxed{T_{BA} = 1.596 \text{ kN, and } T_{CA} = 1.166 \text{ kN.}}$$

Problem 4.73 The tension in the cable BD is 1 kN. As a result, cable BD exerts a 1-kN force on the “ball” at B that points from B toward D . Determine the moment of this force about point A .



Solution: We have the force and position vectors

$$\mathbf{F} = \frac{1 \text{ kN}}{6}(-4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}), \quad \mathbf{r} = \mathbf{AB} = (4\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \text{ m}$$

The moment is then

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = (1.667\mathbf{i} - 3.33\mathbf{j} + 3.33\mathbf{k}) \text{ kN}\cdot\text{m}$$

Problem 4.74* Suppose that the mass of the suspended object E in Problem 4.73 is 100 kg and the mass of the bar AB is 20 kg. Assume that the weight of the bar acts at its midpoint. By using the fact that the sum of the moments about point A due to the weight of the bar and the forces exerted on the “ball” at B by the three cables BC , BD , and BE is zero, determine the tensions in the cables BC and BD .

Solution: We have the following forces applied at point B .

$$\mathbf{F}_1 = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j}, \quad \mathbf{F}_2 = \frac{T_{BC}}{\sqrt{33}}(-4\mathbf{i} + \mathbf{j} - 4\mathbf{k}),$$

$$\mathbf{F}_3 = \frac{T_{BD}}{6}(-4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$$

In addition we have the weight of the bar $\mathbf{F}_4 = -(20 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j}$

The moment around point A is

$$\mathbf{M}_A = (4\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \text{ m} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3)$$

$$+ (2\mathbf{i} + 1.5\mathbf{j} + 0.5\mathbf{k}) \text{ m} \times \mathbf{F}_4 = 0$$

Carrying out the cross products and breaking into components we find

$$M_x = 1079 - 2.26T_{BC} + 1.667T_{BD} = 0$$

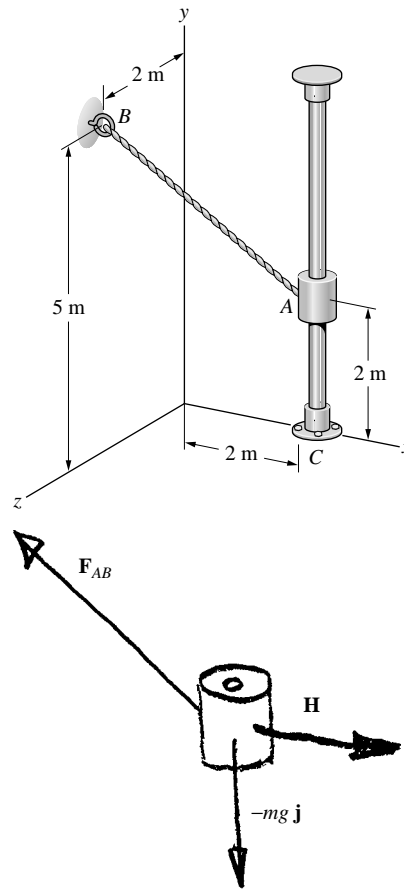
$$M_y = 2.089T_{BC} - 3.333T_{BD} = 0$$

$$M_z = -4316 + 2.785T_{BC} + 3.333T_{BD} = 0$$

Only two of these three equations are independent. Solving we find

$$T_{BC} = 886 \text{ N}, \quad T_{BD} = 555 \text{ N}$$

Problem 4.75 The 200-kg slider at A is held in place on the smooth vertical bar by the cable AB . Determine the moment about the bottom of the bar (point C with coordinates $x = 2$ m, $y = z = 0$) due to the force exerted on the slider by the cable.



Solution: The slider is in equilibrium. The smooth bar exerts no vertical forces on the slider. Hence, the vertical component of \mathbf{F}_{AB} supports the weight of the slider.

The unit vector from A to B is determined from the coordinates of points A and B $A(2, 2, 0)$, $B(0, 5, 2)$ m

Thus, $\mathbf{r}_{AB} = -2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ m

$$\mathbf{e}_{AB} = -0.485\mathbf{i} + 0.728\mathbf{j} + 0.485\mathbf{k}$$

and $\mathbf{F}_{AB} = F_{AB}\mathbf{e}_{AB}$

The horizontal force exerted by the bar on the slider is

$$\mathbf{H} = H_x\mathbf{i} + H_z\mathbf{k}$$

Equilibrium requires $\mathbf{H} + \mathbf{F}_{AB} - mg\mathbf{j} = 0$

$$\mathbf{i}: H_x - 0.485F_{AB} = 0 \quad m = 200 \text{ kg}$$

$$\mathbf{j}: 0.728F_{AB} - mg = 0 \quad g = 9.81 \text{ m/s}^2$$

$$\mathbf{k}: H_z + 0.485F_{AB} = 0$$

Solving, we get

$$F_{AB} = 2697 \text{ N} = 2.70 \text{ kN}$$

$$H_x = 1308 \text{ N} = 1.31 \text{ kN}$$

$$H_z = -1308 \text{ N} = -1.31 \text{ kN}$$

$$\mathbf{r}_{CA} = 2\mathbf{j} \text{ m}$$

$$\mathbf{F}_{AB} = F_{AB}\mathbf{e}_{AB}$$

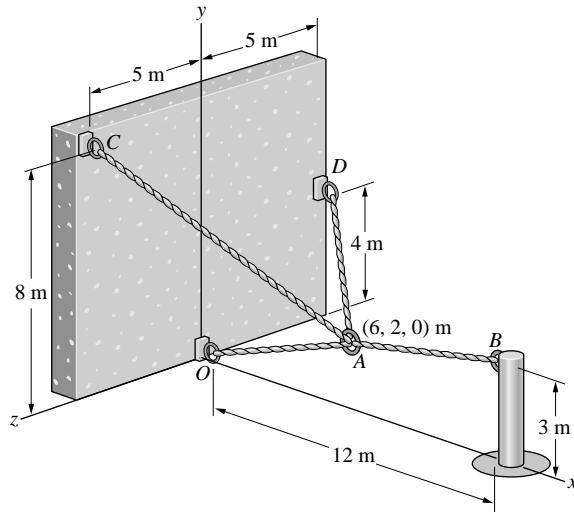
$$\mathbf{F}_{AB} = -1308\mathbf{i} + 1962\mathbf{j} + 1308\mathbf{k} \text{ N}$$

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ -1308 & 1962 & 1308 \end{vmatrix}$$

$$\mathbf{M}_C = 2616\mathbf{i} + 0\mathbf{j} + 2616\mathbf{k} \text{ N-m}$$

$$\mathbf{M}_C = 2.62\mathbf{i} + 2.62\mathbf{k} \text{ kN-m}$$

Problem 4.76 To evaluate the adequacy of the design of the vertical steel post, you must determine the moment about the bottom of the post due to the force exerted on the post at B by the cable AB . A calibrated strain gauge mounted on cable AC indicates that the tension in cable AC is 22 kN. What is the moment?



Solution: To find the moment, we must find the force in cable AB . In order to do this, we must find the forces in cables AO and AD also. This requires that we solve the equilibrium problem at A .

Our first task is to write unit vectors \mathbf{e}_{AB} , \mathbf{e}_{AO} , \mathbf{e}_{AC} , and \mathbf{e}_{AD} . Each will be of the form

$$\mathbf{e}_{Ai} = \frac{(x_i - x_A)\mathbf{i} + (y_i - y_A)\mathbf{j} + (z_i - z_A)\mathbf{k}}{\sqrt{(x_i - x_A)^2 + (y_i - y_A)^2 + (z_i - z_A)^2}}$$

where i takes on the values B , C , D , and O . We get

$$\mathbf{e}_{AB} = 0.986\mathbf{i} + 0.164\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{e}_{AC} = -0.609\mathbf{i} + 0.609\mathbf{j} + 0.508\mathbf{k}$$

$$\mathbf{e}_{AD} = -0.744\mathbf{i} + 0.248\mathbf{j} - 0.620\mathbf{k}$$

$$\mathbf{e}_{AO} = -0.949\mathbf{i} - 0.316\mathbf{j} + 0\mathbf{k}$$

We now write the forces as

$$\mathbf{T}_{AB} = T_{AB}\mathbf{e}_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\mathbf{e}_{AC}$$

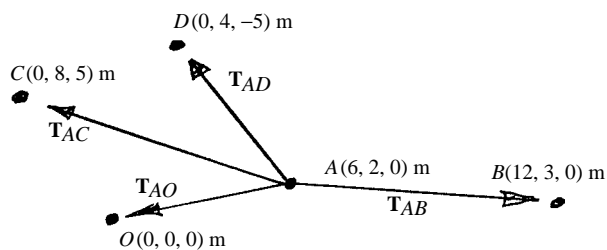
$$\mathbf{T}_{AD} = T_{AD}\mathbf{e}_{AD}$$

$$\mathbf{T}_{AO} = T_{AO}\mathbf{e}_{AO}$$

We then sum the forces and write the equilibrium equations in component form.

For equilibrium at A , $\sum \mathbf{F}_A = 0$

$$\sum \mathbf{F}_A = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{T}_{AO} = 0.$$



In component form,

$$\begin{cases} T_{AB}e_{ABx} + T_{AC}e_{ACx} + T_{AD}e_{ADx} + T_{AO}e_{AOx} = 0 \\ T_{AB}e_{ABy} + T_{AC}e_{ACy} + T_{AD}e_{ADy} + T_{AO}e_{AOy} = 0 \\ T_{AB}e_{ABz} + T_{AC}e_{ACz} + T_{AD}e_{ADz} + T_{AO}e_{AOz} = 0 \end{cases}$$

We know $T_{AC} = 22$ kN. Substituting this in, we have 3 eqns in 3 unknowns. Solving, we get

$$T_{AB} = 163.05 \text{ kN}, \quad T_{AD} = 18.01 \text{ kN}, \quad T_{AO} = 141.28 \text{ kN}$$

We now know that \mathbf{T}_{AB} is given as

$$\mathbf{T}_{AB} = T_{AB}\mathbf{e}_{AB} = 160.8\mathbf{i} + 26.8\mathbf{j} \text{ (kN)}$$

and that the force acting at B is $(-\mathbf{T}_{AB})$.

The moment about the bottom of the post is given by

$$\mathbf{M}_{\text{BOTTOM}} = \mathbf{r} \times (-\mathbf{T}_{AB}) = 3\mathbf{j} \times (-\mathbf{T}_{AB})$$

Solving, we get

$$\mathbf{M}_{\text{BOTTOM}} = 482\mathbf{k} \text{ (kN-m)}$$

Problem 4.77 The force $\mathbf{F} = 20\mathbf{i} + 40\mathbf{j} - 10\mathbf{k}$ (N). Use both of the procedures described in Example 4.7 to determine the moment due to \mathbf{F} about the z axis.

Solution: First Method: We can use Eqs. (4.5) and (4.6)

$$\mathbf{r} = (8\mathbf{i}) \text{ m}$$

$$\mathbf{F} = (20\mathbf{i} + 40\mathbf{j} - 10\mathbf{k}) \text{ N}$$

$$\mathbf{M}_{z\text{-axis}} = [\mathbf{k} \cdot (\mathbf{r} \times \mathbf{F})]\mathbf{k}$$

$$|\mathbf{M}_{z\text{-axis}}| = \mathbf{k} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 8 \text{ m} & 0 & 0 \\ 20 \text{ N} & 40 \text{ N} & -10 \text{ N} \end{vmatrix} = 320 \text{ N}\cdot\text{m}$$

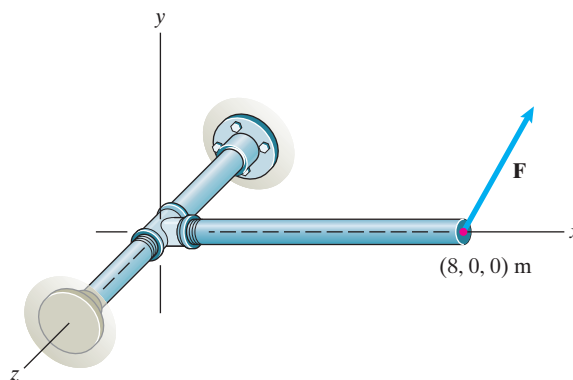
$$\mathbf{M}_{z\text{-axis}} = (320 \text{ N}\cdot\text{m})\mathbf{k}$$

Second Method: The y -component of the force is perpendicular to the plane containing the z axis and the position vector \mathbf{r} . The perpendicular distance from the z axis to the y -component of the force is 8 m. Therefore

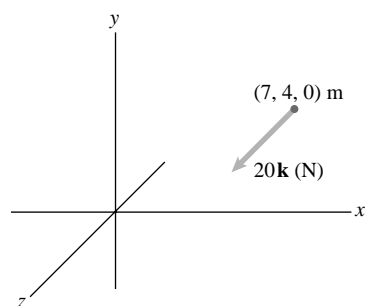
$$|\mathbf{M}_{z\text{-axis}}| = (40 \text{ N})(8 \text{ m}) = 320 \text{ N}\cdot\text{m}$$

Using the right-hand rule we see that the moment is about the $+z$ axis. Thus

$$\mathbf{M}_{z\text{-axis}} = (320 \text{ N}\cdot\text{m})\mathbf{k}$$



Problem 4.78 Use Eqs. (4.5) and (4.6) to determine the moment of the 20-N force about (a) the x axis, (b) the y axis, (c) the z axis. (First see if you can write down the results without using the equations.)



Solution: The force is parallel to the z axis. The perpendicular distance from the x axis to the line of action of the force is 4 m. The perpendicular distance from the y axis is 7 m and the perpendicular distance from the z axis is $\sqrt{4^2 + 7^2} = \sqrt{65}$ m.

By inspection, the moment about the x axis is

$$\mathbf{M}_x = (4)(20)\mathbf{i} \text{ (N}\cdot\text{m)}$$

$$\mathbf{M}_x = 80\mathbf{i} \text{ (N}\cdot\text{m)}$$

By inspection, the moment about the y axis is $\mathbf{M}_y = (7)(20)(-\mathbf{j}) \text{ N}\cdot\text{m}$

$$\mathbf{M}_y = -140\mathbf{j} \text{ (N}\cdot\text{m)}$$

By inspection, the moment about the z axis is zero since \mathbf{F} is parallel to the z axis.

$$\mathbf{M}_z = 0 \text{ (N}\cdot\text{m)}$$

Now for the calculations using (4.5) and (4.6)

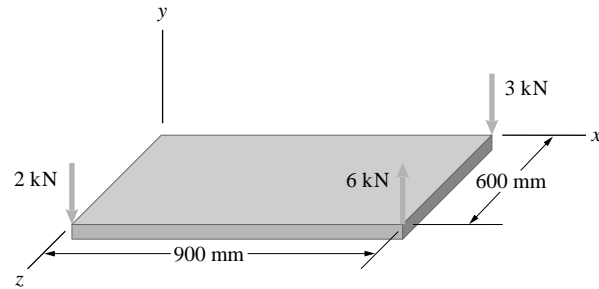
$$\mathbf{M}_L = [\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F})]\mathbf{e}$$

$$\mathbf{M}_x = \begin{vmatrix} 1 & 0 & 0 \\ 7 & 4 & 0 \\ 0 & 0 & 20 \end{vmatrix} \mathbf{i} = 80\mathbf{i} \text{ (N}\cdot\text{m)}$$

$$\mathbf{M}_y = \begin{vmatrix} 0 & 1 & 0 \\ 7 & 4 & 0 \\ 0 & 0 & 20 \end{vmatrix} \mathbf{j} = -140\mathbf{j} \text{ (N}\cdot\text{m)}$$

$$\mathbf{M}_z = \begin{vmatrix} 0 & 0 & 1 \\ 7 & 4 & 0 \\ 0 & 0 & 20 \end{vmatrix} \mathbf{k} = 0\mathbf{k} \text{ (N}\cdot\text{m)}$$

Problem 4.79 Three forces parallel to the y axis act on the rectangular plate. Use Eqs. (4.5) and (4.6) to determine the sum of the moments of the forces about the x axis. (First see if you can write down the result without using the equations.)



Solution: By inspection, the 3 kN force has no moment about the x axis since it acts through the x axis. The perpendicular distances of the other two forces from the x axis is 0.6 m. The 2 kN force has a positive moment and the 6 kN force has a negative about the x axis.

$$\sum \mathbf{M}_x = [(2)(0.6) - (6)(0.6)]\mathbf{i} \text{ kN}$$

$$\sum \mathbf{M}_x = -2.4\mathbf{i} \text{ kN}$$

Calculating the result:

$$\mathbf{M}_{3 \text{ kN}} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -3 & 0 \end{vmatrix} \mathbf{i} = 0\mathbf{i} \text{ kN}$$

$$\mathbf{M}_{2 \text{ kN}} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & .6 \\ 0 & -2 & 0 \end{vmatrix} \mathbf{i} = 1.2\mathbf{i} \text{ kN}$$

$$\mathbf{M}_{6 \text{ kN}} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & .6 \\ 0 & 6 & 0 \end{vmatrix} \mathbf{i} = -3.6\mathbf{i} \text{ kN}$$

$$\sum \mathbf{M}_x = \mathbf{M}_{3 \text{ kN}} + \mathbf{M}_{2 \text{ kN}} + \mathbf{M}_{6 \text{ kN}}$$

$$\sum \mathbf{M}_x = 0 + 1.2\mathbf{i} - 3.6\mathbf{i} \text{ (kN)}$$

$$\sum \mathbf{M}_x = -2.4\mathbf{i} \text{ (kN)}$$

Problem 4.80 Consider the rectangular plate shown in Problem 4.79. The three forces are parallel to the y axis. Determine the sum of the moments of the forces (a) about the y axis, (b) about the z axis.

Solution: (a) The magnitude of the moments about the y axis is $M = \mathbf{e}_y \cdot (\mathbf{r} \times \mathbf{F})$. The position vectors of the three forces are given in the solution to Problem 4.79. The magnitude for each force is:

$$\mathbf{e}_y \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 0.9 & 0 & 0 \\ 0 & -3 & 0 \end{vmatrix} = 0,$$

$$\mathbf{e}_y \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 0.9 & 0 & 0.6 \\ 0 & 6 & 0 \end{vmatrix} = 0,$$

$$\mathbf{e}_y \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.6 \\ 0 & -2 & 0 \end{vmatrix} = 0$$

Thus the moment about the y axis is zero, since the magnitude of each moment is zero.

(b) The magnitude of each moment about the z axis is

$$\mathbf{e}_z \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 0.9 & 0 & 0 \\ 0 & -3 & 0 \end{vmatrix} = -2.7,$$

$$\mathbf{e}_z \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 0.9 & 0 & 0.6 \\ 0 & 6 & 0 \end{vmatrix} = 5.4,$$

$$\mathbf{e}_z \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 0.6 \\ 0 & -2 & 0 \end{vmatrix} = 0.$$

Thus the moment about the z axis is

$$\sum \mathbf{M}_z = -2.7\mathbf{e}_z + 5.4\mathbf{e}_z = 2.7\mathbf{k} \text{ (kN-m)}$$

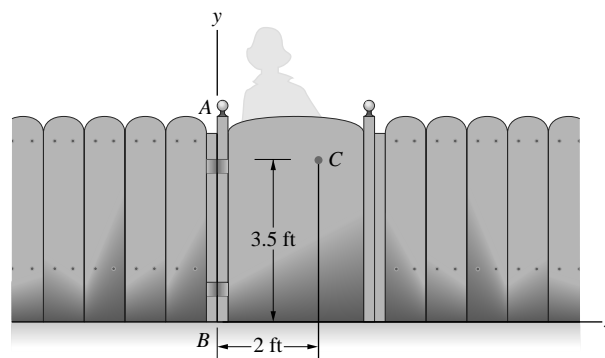
Problem 4.81 The person exerts a force $\mathbf{F} = 0.2\mathbf{i} - 0.4\mathbf{j} + 1.2\mathbf{k}$ (lb) on the gate at C . Point C lies in the x - y plane. What moment does the person exert about the gate's hinge axis, which is coincident with the y axis?

Solution:

$$\mathbf{M} = [\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F})]\mathbf{e}$$

$$\mathbf{e} = \mathbf{j}, \quad \mathbf{r} = 2\mathbf{i} \text{ ft}, \quad \mathbf{F} \text{ is given}$$

$$\mathbf{M}_y = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ .2 & -.4 & 1.2 \end{vmatrix} \mathbf{j} = -2.4\mathbf{j} \text{ (ft-lb)}$$



Problem 4.82 Four forces act on the plate. Their components are

$$\mathbf{F}_A = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \text{ (kN)},$$

$$\mathbf{F}_B = 3\mathbf{j} - 3\mathbf{k} \text{ (kN)},$$

$$\mathbf{F}_C = 2\mathbf{j} + 3\mathbf{k} \text{ (kN)},$$

$$\mathbf{F}_D = 2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k} \text{ (kN)}.$$

Determine the sum of the moments of the forces (a) about the x axis; (b) about the z axis.

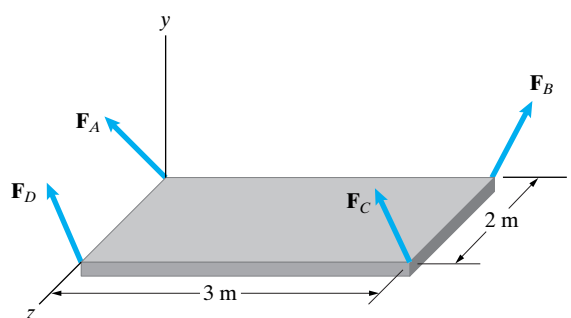
Solution: Note that \mathbf{F}_A acts at the origin so no moment is generated about the origin. For the other forces we have

$$\begin{aligned} \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 \text{ m} & 0 & 0 \\ 0 & 3 \text{ kN} & -3 \text{ kN} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 \text{ m} & 0 & 2 \text{ m} \\ 0 & 2 \text{ kN} & 3 \text{ kN} \end{vmatrix} \\ &+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \text{ m} \\ 2 \text{ kN} & 6 \text{ kN} & 4 \text{ kN} \end{vmatrix} \end{aligned}$$

$$\mathbf{M}_O = (-16\mathbf{i} + 4\mathbf{j} + 15\mathbf{k}) \text{ kN-m}$$

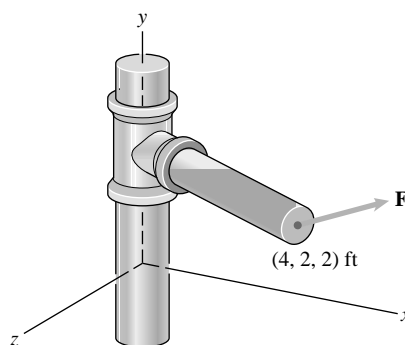
Now we find

$$M_x = \mathbf{M}_O \cdot \mathbf{i} = -16 \text{ kN-m}, \quad M_z = \mathbf{M}_O \cdot \mathbf{k} = 15 \text{ kN-m}$$



Problem 4.83 The force $\mathbf{F} = 30\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}$ (lb).

- What is the moment of \mathbf{F} about the y axis?
- Suppose that you keep the magnitude of \mathbf{F} fixed, but you change its direction so as to make the moment of \mathbf{F} about the y axis as large as possible. What is the magnitude of the resulting moment?



Solution:

$$(a) \quad M_y = \mathbf{j} \cdot [(4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \text{ ft} \times (30\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) \text{ lb}]$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ 4 \text{ ft} & 2 \text{ ft} & 2 \text{ ft} \\ 30 \text{ lb} & 20 \text{ lb} & -10 \text{ lb} \end{vmatrix} = 100 \text{ ft-lb}$$

$$\Rightarrow \mathbf{M}_y = (100 \text{ ft-lb})\mathbf{j}$$

$$(b) \quad \boxed{M_{y\max} = Fd = (\sqrt{30^2 + 20^2 + 10^2} \text{ lb})(\sqrt{4^2 + 2^2} \text{ ft}) = 167.3 \text{ ft-lb}}$$

Note that d is the distance from the y axis, not the distance from the origin.

Problem 4.84 The moment of the force \mathbf{F} shown in Problem 4.83 about the x axis is $-80\mathbf{i}$ (ft-lb), the moment about the y axis is zero, and the moment about the z axis is $160\mathbf{k}$ (ft-lb). If $F_y = 80$ lb, what are F_x and F_z ?

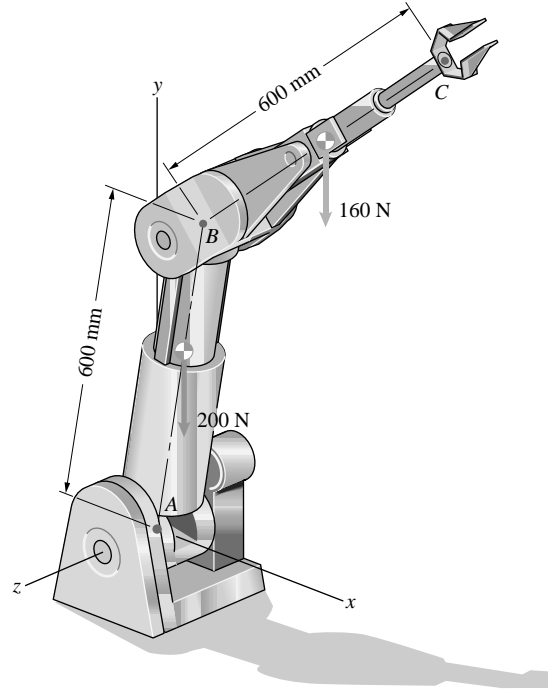
Solution: The magnitudes of the moments:

$$\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix},$$

$$\mathbf{e}_z \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 4 & 2 & -2 \\ F_x & 80 & F_z \end{vmatrix} = 320 - 2F_x = 160$$

Solve: $F_x = 80$ lb, $F_z = 40$ lb, from which the force vector is $\mathbf{F} = 80\mathbf{i} + 80\mathbf{j} + 40\mathbf{k}$

Problem 4.85 The robotic manipulator is stationary. The weights of the arms AB and BC act at their midpoints. The direction cosines of the centerline of arm AB are $\cos \theta_x = 0.500$, $\cos \theta_y = 0.866$, $\cos \theta_z = 0$, and the direction cosines of the centerline of arm BC are $\cos \theta_x = 0.707$, $\cos \theta_y = 0.619$, $\cos \theta_z = -0.342$. What total moment is exerted about the z axis by the weights of the arms?



Solution: The unit vectors along AB and AC are of the form

$$\mathbf{e} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k}.$$

The unit vectors are

$$\mathbf{e}_{AB} = 0.500\mathbf{i} + 0.866\mathbf{j} + 0\mathbf{k} \text{ and } \mathbf{e}_{BC} = 0.707\mathbf{i} + 0.619\mathbf{j} - 0.342\mathbf{k}.$$

The vector to point G at the center of arm AB is

$$\mathbf{r}_{AG} = 300(0.500\mathbf{i} + 0.866\mathbf{j} + 0\mathbf{k}) = 150\mathbf{i} + 259.8\mathbf{j} + 0\mathbf{k} \text{ mm},$$

and the vector from A to the point H at the center of arm BC is given by

$$\begin{aligned} \mathbf{r}_{AH} &= \mathbf{r}_{AB} + \mathbf{r}_{BH} = 600\mathbf{e}_{AB} + 300\mathbf{e}_{BC} \\ &= 512.1\mathbf{i} + 705.3\mathbf{j} - 102.6\mathbf{k} \text{ mm}. \end{aligned}$$

The weight vectors acting at G and H are $\mathbf{W}_G = -200\mathbf{j}$ N, and $\mathbf{W}_H = -160\mathbf{j}$ N. The moment vectors of these forces about the z axis are of the form

$$\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}.$$

Here, \mathbf{W}_G and \mathbf{W}_H take on the role of \mathbf{F} , and $\mathbf{e} = \mathbf{k}$.

Substituting into the form for the moment of the force at G , we get

$$\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 0.150 & 0.260 & 0 \\ 0 & -200 & 0 \end{vmatrix} = -30 \text{ N}\cdot\text{m}.$$

Similarly, for the moment of the force at H , we get

$$\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 0.512 & 0.705 & -0.103 \\ 0 & -160 & 0 \end{vmatrix} = -81.9 \text{ N}\cdot\text{m}.$$

The total moment about the z axis is the sum of the two moments. Hence, $M_{z\text{axis}} = -111.9 \text{ N}\cdot\text{m}$

Problem 4.86 In Problem 4.85, what total moment is exerted about the x axis by the weights of the arms?

Solution: The solution is identical to that of Problem 4.85 except that $\mathbf{e} = \mathbf{i}$. Substituting into the form for the moment of the force at G , we get

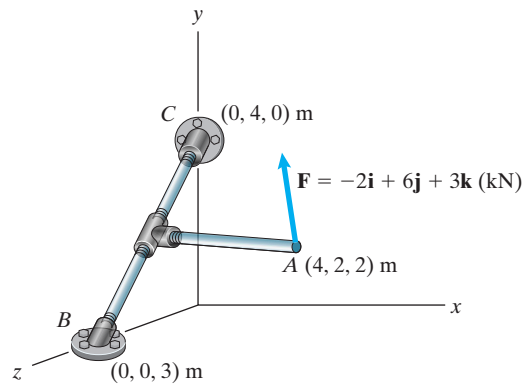
$$\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0.150 & 0.260 & 0 \\ 0 & -200 & 0 \end{vmatrix} = 0 \text{ N}\cdot\text{m}.$$

Similarly, for the moment of the force at H , we get

$$\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0.512 & 0.705 & -0.103 \\ 0 & -160 & 0 \end{vmatrix} = -16.4 \text{ N}\cdot\text{m}.$$

The total moment about the x axis is the sum of the two moments. Hence, $M_{x \text{ axis}} = -16.4 \text{ N}\cdot\text{m}$

Problem 4.87 In Active Example 4.6, suppose that the force changes to $\mathbf{F} = -2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ (kN). Determine the magnitude of the moment of the force about the axis of the bar BC .



Solution: We have the following vectors

$$\mathbf{r}_{BA} = (4\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}) \text{ m}$$

$$\mathbf{F} = (-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) \text{ kN}$$

$$\mathbf{r}_{BC} = (4\mathbf{j} - 3\mathbf{k}) \text{ m}$$

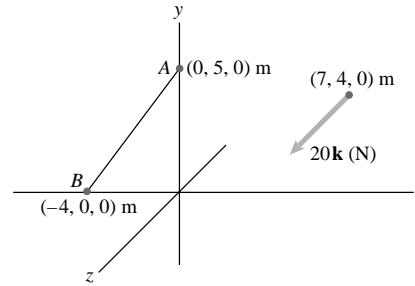
$$\mathbf{e}_{BC} = \frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = (0.8\mathbf{j} - 0.6\mathbf{k})$$

The moment of \mathbf{F} about the axis of the bar is

$$|\mathbf{M}_{BC}| = \mathbf{e}_{BC} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} 0 & 0.8 & -0.6 \\ 4 & 2 & -1 \\ -2 & 3 & 6 \end{vmatrix} = -27.2 \text{ kN}\cdot\text{m}$$

Thus $\mathbf{M}_{BC} = (-27.2 \text{ kN}\cdot\text{m})\mathbf{e}_{BC}$, $|\mathbf{M}_{BC}| = 27.2 \text{ kN}\cdot\text{m}$

Problem 4.88 Determine the moment of the 20-N force about the line AB . Use Eqs. (4.5) and (4.6), letting the unit vector \mathbf{e} point (a) from A toward B , (b) from B toward A .



Solution: First, we need the unit vector

$$\mathbf{e}_{AB} = \frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

$$\mathbf{e}_{AB} = -0.625\mathbf{i} - 0.781\mathbf{j} = -\mathbf{e}_{BA}$$

Now, the moment of the 20 \mathbf{k} (N) force about AB is given as

$$M_L = \begin{vmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \mathbf{e} \quad \text{where } \mathbf{e} \text{ is } \mathbf{e}_{AB} \text{ or } \mathbf{e}_{BA}$$

For this problem, \mathbf{r} must go from line AB to the point of application of the force. Let us use point A .

$$\mathbf{r} = (7 - 0)\mathbf{i} + (4 - 5)\mathbf{j} + (0 - 0)\mathbf{k} \text{ m}$$

$$\mathbf{r} = 7\mathbf{i} - 1\mathbf{j} + 0\mathbf{k} \text{ m}$$

Using \mathbf{e}_{AB}

$$\mathbf{M}_L = \begin{vmatrix} -0.625 & -0.781 & 0 \\ 7 & -1 & 0 \\ 0 & 0 & 20 \end{vmatrix} (-0.625\mathbf{i} - 0.781\mathbf{j})$$

$$\mathbf{M}_L = -76.1\mathbf{i} - 95.1\mathbf{j} \text{ (N}\cdot\text{m)}$$

Using \mathbf{e}_{BA}

$$\mathbf{M}_L = \begin{vmatrix} 0.625 & 0.781 & 0 \\ 7 & -1 & 0 \\ 0 & 0 & 20 \end{vmatrix} (0.625\mathbf{i} + 0.781\mathbf{j})$$

$$\mathbf{M}_L = -76.1\mathbf{i} - 95.1\mathbf{j} \text{ (N}\cdot\text{m)}$$

*Results are the same

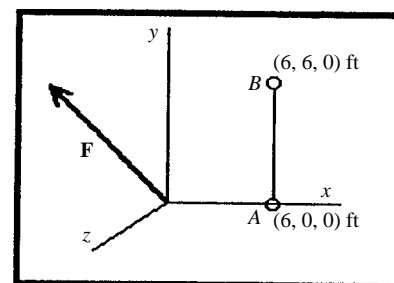
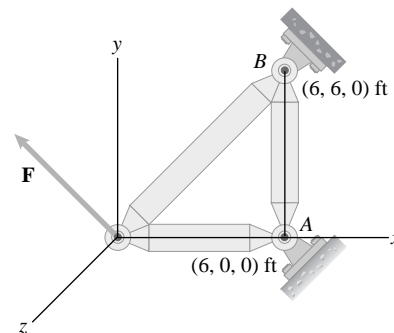
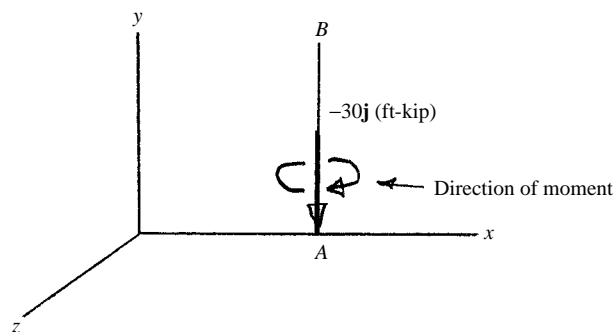
Problem 4.89 The force $\mathbf{F} = -10\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}$ (kip). Determine the moment of \mathbf{F} about the line AB . Draw a sketch to indicate the sense of the moment.

Solution: The moment of \mathbf{F} about pt. A is

$$\begin{aligned} \mathbf{M}_A &= -6\mathbf{i} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 0 & 0 \\ -10 & 5 & -5 \end{vmatrix} \\ &= -30\mathbf{j} - 30\mathbf{k} \text{ (ft}\cdot\text{kip)}. \end{aligned}$$

The unit vector \mathbf{j} is parallel to line AB , so the moment about AB is

$$\begin{aligned} \mathbf{M}_{AB} &= (\mathbf{j} \cdot \mathbf{M}_A)\mathbf{j} \\ &= -30\mathbf{j} \text{ (ft}\cdot\text{kip)}. \end{aligned}$$



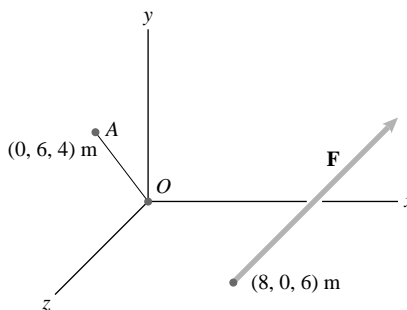
Problem 4.90 The force $\mathbf{F} = 10\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ (N). What is the moment of \mathbf{F} about the line OA ? Draw a sketch to indicate the sense of the moment.

Solution: The strategy is to determine a unit vector parallel to OA and to use this to determine the moment about OA . The vector parallel to OA is $\mathbf{r}_{OA} = 6\mathbf{j} + 4\mathbf{k}$. The magnitude: F . The unit vector parallel to OA is $\mathbf{e}_{OA} = 0.8321\mathbf{j} + 0.5547\mathbf{k}$. The vector from O to the point of application of \mathbf{F} is $\mathbf{r}_{OF} = 8\mathbf{i} + 6\mathbf{k}$. The magnitude of the moment about OA is

$$|\mathbf{M}_O| = \mathbf{e}_{OA} \cdot (\mathbf{r}_{OF} \times \mathbf{F}) = \begin{vmatrix} 0 & 0.8321 & 0.5547 \\ 8 & 0 & 6 \\ 10 & 12 & -6 \end{vmatrix}$$

$$= 89.8614 + 53.251 = 143.1 \text{ N}\cdot\text{m}.$$

The moment about OA is $\mathbf{M}_{OA} = |\mathbf{M}_{OA}|\mathbf{e}_{OA} = 119.1\mathbf{j} + 79.4\mathbf{k}$ (N-m). The sense of the moment is in the direction of the curled fingers of the right hand when the thumb is parallel to OA , pointing to A .



Problem 4.91 The tension in the cable AB is 1 kN. Determine the moment about the x axis due to the force exerted on the hatch by the cable at point B . Draw a sketch to indicate the sense of the moment.

Solution: The vector parallel to BA is

$$\mathbf{r}_{BA} = (0.4 - 1)\mathbf{i} + 0.3\mathbf{j} - 0.6\mathbf{k} = -0.6\mathbf{i} + 0.3\mathbf{j} - 0.6\mathbf{k}.$$

The unit vector parallel to BA is

$$\mathbf{e}_{BA} = -0.6667\mathbf{i} + 0.3333\mathbf{j} - 0.6667\mathbf{k}.$$

The moment about O is

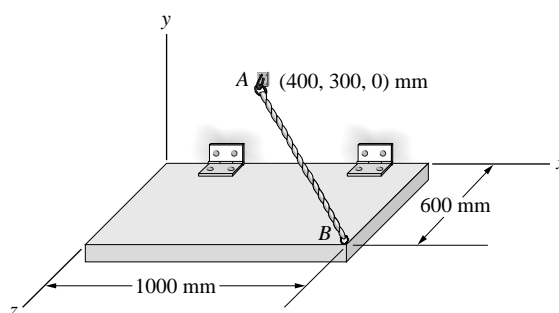
$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{T} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0.6 \\ -0.6667 & 0.3333 & -0.6667 \end{vmatrix}$$

$$\mathbf{M}_O = -0.2\mathbf{i} + 0.2667\mathbf{j} + 0.3333\mathbf{k}.$$

The magnitude is

$$|M_X| = \mathbf{e}_X \cdot \mathbf{M}_O = -0.2 \text{ kN}\cdot\text{m}.$$

The moment is $\mathbf{M}_X = -0.2\mathbf{i}$ kN-m. The sense is clockwise when viewed along the x axis toward the origin.



Problem 4.92 Determine the moment of the force applied at D about the straight line through the hinges A and B . (The line through A and B lies in the y - z plane.)

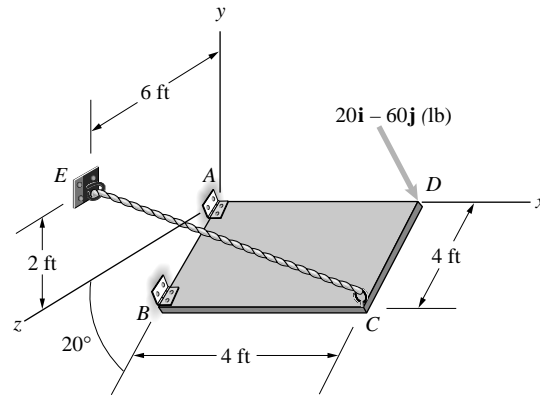
Solution: From the figure, we see that the unit vector along the line from A toward B is given by $\mathbf{e}_{AB} = -\sin 20^\circ \mathbf{j} + \cos 20^\circ \mathbf{k}$. The position vector is $\mathbf{r}_{AD} = 4\mathbf{i}$ ft, and the force vector is as shown in the figure. The moment vector of a force about an axis is of the form

$$\mathbf{e} \bullet (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}.$$

For this case,

$$\begin{aligned} \mathbf{e} \bullet (\mathbf{r} \times \mathbf{F}) &= \begin{vmatrix} 0 & -\sin 20^\circ & \cos 20^\circ \\ 4 & 0 & 0 \\ 20 & -60 & 0 \end{vmatrix} = -240 \cos 20^\circ \text{ ft-lb} \\ &= -225.5 \text{ ft-lb}. \end{aligned}$$

The negative sign is because the moment is opposite in direction to the unit vector from A to B .



Problem 4.93 In Problem 4.92, the tension in the cable CE is 160 lb. Determine the moment of the force exerted by the cable on the hatch at C about the straight line through the hinges A and B .

Solution: From the figure, we see that the unit vector along the line from A toward B is given by $\mathbf{e}_{AB} = -\sin 20^\circ \mathbf{j} + \cos 20^\circ \mathbf{k}$. The position vector is $\mathbf{r}_{BC} = 4\mathbf{i}$ ft. The coordinates of point C are $(4, -4 \sin 20^\circ, 4 \cos 20^\circ)$. The unit vector along CE is $-0.703\mathbf{i} + 0.592\mathbf{j} + 0.394\mathbf{k}$ and the force vector is as shown acting at point D .

The moment vector is a force about an axis is of the form

$$\mathbf{e} \bullet (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}.$$

For this case,

$$\mathbf{r}_{CE} = -4\mathbf{i} + 3.368\mathbf{j} + 2.242\mathbf{k}$$

$$\mathbf{T}_{CE} = 160\mathbf{e}_{CE} = -112.488\mathbf{i} + 94.715\mathbf{j} + 63.049\mathbf{k}$$

$$\begin{aligned} \mathbf{e} \bullet (\mathbf{r} \times \mathbf{F}) &= \begin{vmatrix} 0 & -\sin 20^\circ & \cos 20^\circ \\ 4 & 0 & 0 \\ -112.488 & 94.715 & 63.049 \end{vmatrix} = -240 \cos 20^\circ \text{ ft-lb} \\ &= 701 \text{ ft-lbs}. \end{aligned}$$

Problem 4.94 The coordinates of A are $(-2.4, 0, -0.6)$ m, and the coordinates of B are $(-2.2, 0.7, -1.2)$ m. The force exerted at B by the sailboat's main sheet AB is 130 N. Determine the moment of the force about the centerline of the mast (the y axis). Draw a sketch to indicate the sense of the moment.

Solution: The position vectors:

$$\mathbf{r}_{OA} = -2.4\mathbf{i} - 0.6\mathbf{k} \text{ (m)}, \quad \mathbf{r}_{OB} = -2.2\mathbf{i} + 0.7\mathbf{j} - 1.2\mathbf{k} \text{ (m)},$$

$$\begin{aligned} \mathbf{r}_{BA} &= (-2.4 + 2.2)\mathbf{i} + (0 - 0.7)\mathbf{j} + (-0.6 + 1.2)\mathbf{k} \text{ (m)} \\ &= -0.2\mathbf{i} - 0.7\mathbf{j} + 0.6\mathbf{k} \text{ (m)}. \end{aligned}$$

The magnitude is $|\mathbf{r}_{BA}| = 0.9434$ m.

The unit vector parallel to BA is

$$\mathbf{e}_{BA} = -0.2120\mathbf{i} - 0.7420\mathbf{j} + 0.6360\mathbf{k}.$$

The tension is $\mathbf{T}_{BA} = 130\mathbf{e}_{BA}$.

The moment of \mathbf{T}_{BA} about the origin is

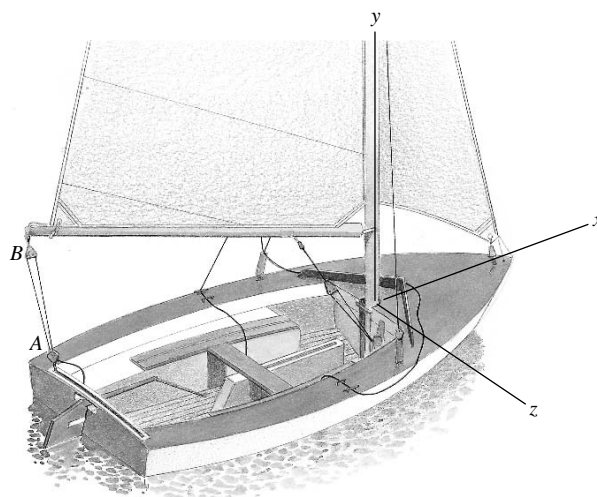
$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{T}_{BA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2.2 & 0.7 & -1.2 \\ -27.56 & -96.46 & 82.68 \end{vmatrix},$$

$$\text{or } \mathbf{M}_O = -57.88\mathbf{i} + 214.97\mathbf{j} + 231.5\mathbf{k}.$$

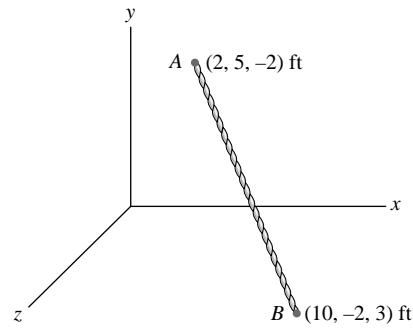
The magnitude of the moment about the y axis is

$$|\mathbf{M}_Y| = \mathbf{e}_Y \cdot \mathbf{M}_O = 214.97 \text{ N-m.}$$

The moment is $\mathbf{M}_Y = \mathbf{e}_Y(214.97) = 214.97\mathbf{j}$ N-m.



Problem 4.95 The tension in cable AB is 200 lb. Determine the moments about each of the coordinate axes due to the force exerted on point B by the cable. Draw sketches to indicate the senses of the moments.



Solution: The position vector from B to A is

$$\mathbf{r}_{BA} = (2 - 10)\mathbf{i} + [5 - (-2)]\mathbf{j} + (-2 - 3)\mathbf{k}$$

$$= -8\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \text{ (ft),}$$

So the force exerted on B is

$$\mathbf{F} = 200 \frac{\mathbf{r}_{BA}}{|\mathbf{r}_{BA}|} = -136.2\mathbf{i} + 119.2\mathbf{j} - 85.1\mathbf{k} \text{ (lb).}$$

The moment of \mathbf{F} about the origin O is

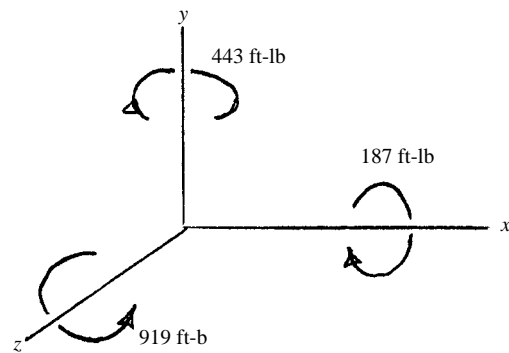
$$\begin{aligned} \mathbf{r}_{OB} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & -2 & 3 \\ -136.2 & 119.2 & -85.1 \end{vmatrix} \\ &= -187\mathbf{i} + 443\mathbf{j} + 919\mathbf{k} \text{ (ft-lb).} \end{aligned}$$

The moments about the x , y , and z axes are

$$[(\mathbf{r}_{OB} \times \mathbf{F}) \cdot \mathbf{i}]\mathbf{i} = -187\mathbf{i} \text{ (ft-lb),}$$

$$[(\mathbf{r}_{OB} \times \mathbf{F}) \cdot \mathbf{j}]\mathbf{j} = 443\mathbf{j} \text{ (ft-lb),}$$

$$[(\mathbf{r}_{OB} \times \mathbf{F}) \cdot \mathbf{k}]\mathbf{k} = 919\mathbf{k} \text{ (ft-lb).}$$



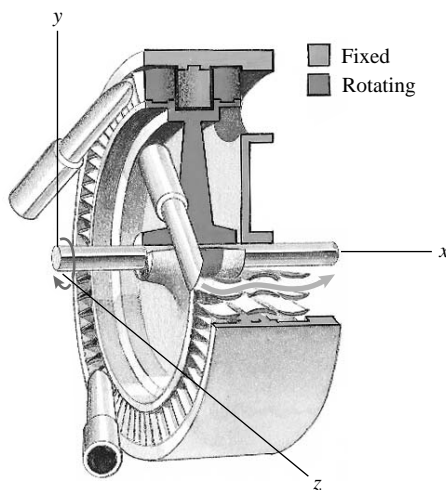
Problem 4.96 The total force exerted on the blades of the turbine by the steam nozzle is $\mathbf{F} = 20\mathbf{i} - 120\mathbf{j} + 100\mathbf{k}$ (N), and it effectively acts at the point (100, 80, 300) mm. What moment is exerted about the axis of the turbine (the x axis)?

Solution: The moment about the origin is

$$\begin{aligned} \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.08 & 0.3 \\ 20 & -120 & 100 \end{vmatrix} \\ &= 44.0\mathbf{i} - 4.0\mathbf{j} - 13.6\mathbf{k} \text{ (N-m).} \end{aligned}$$

The moment about the x axis is

$$(\mathbf{M}_O \cdot \mathbf{i})\mathbf{i} = 44.0\mathbf{i} \text{ (N-m).}$$



Problem 4.97 The pneumatic support AB holds a trunk lid in place. It exerts a 35-N force on the fixture at B that points in the direction from A toward B . Determine the magnitude of the moment of the force about the hinge axis of the lid, which is the z axis.



Solution: The vector from A to B is

$$\mathbf{r}_{AB} = [(60 - 480)\mathbf{i} + (100 - (-40))\mathbf{j} + (-30 - 40)\mathbf{k}] \text{ mm}$$

$$\mathbf{r}_{AB} = (-420\mathbf{i} + 140\mathbf{j} - 70\mathbf{k}) \text{ mm}$$

The 35-N force can be written

$$\mathbf{F} = (35 \text{ N}) \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (-32.8\mathbf{i} + 10.9\mathbf{j} - 5.47\mathbf{k}) \text{ N}$$

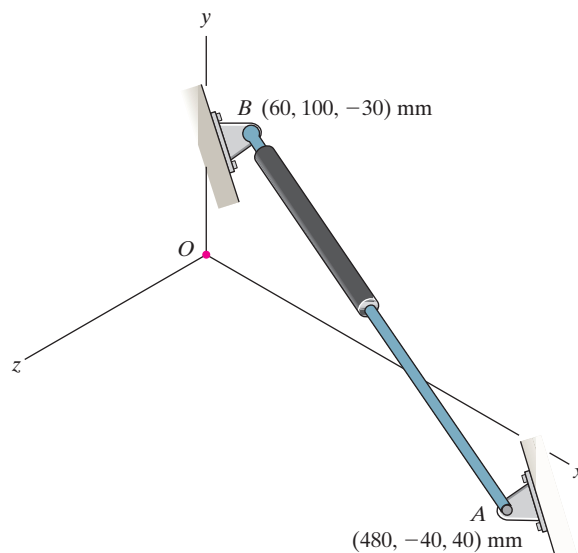
The moment about point O is

$$\begin{aligned} \mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 60 & 100 & -30 \\ -32.8 & 10.9 & -5.47 \end{vmatrix} \\ &= (-219\mathbf{i} + 1310\mathbf{j} + 3940\mathbf{k}) \text{ N-mm} \end{aligned}$$

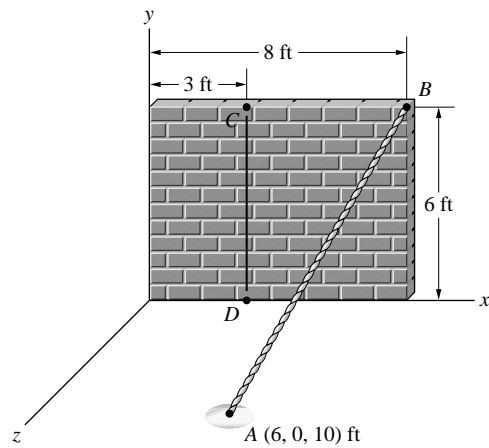
The magnitude of the moment about the z axis is

$$M_z = \mathbf{M}_O \cdot \mathbf{k} = 3940 \text{ N-mm} = 3.94 \text{ N-m}$$

$$\boxed{M_z = 3.94 \text{ N-m}}$$



Problem 4.98 The tension in cable AB is 80 lb. What is the moment about the line CD due to the force exerted by the cable on the wall at B ?



Solution: The strategy is to find the moment about the point C exerted by the force at B , and then to find the component of that moment acting along the line CD . The coordinates of the points B , C , D are $B(8, 6, 0)$, $C(3, 6, 0)$, $D(0, 0, 0)$. The position vectors are: $\mathbf{r}_{OB} = 8\mathbf{i} + 6\mathbf{j}$, $\mathbf{r}_{OC} = 3\mathbf{i} + 6\mathbf{j}$, $\mathbf{r}_{OD} = 3\mathbf{i}$. The vector parallel to CD is $\mathbf{r}_{CD} = \mathbf{r}_{OD} - \mathbf{r}_{OC} = -6\mathbf{j}$. The unit vector parallel to CD is $\mathbf{e}_{CD} = -\mathbf{j}$. The vector from point C to B is $\mathbf{r}_{CB} = \mathbf{r}_{OB} - \mathbf{r}_{OC} = 5\mathbf{i}$.

The position vector of A is $\mathbf{r}_{OA} = 6\mathbf{i} + 10\mathbf{k}$. The vector parallel to BA is $\mathbf{r}_{BA} = \mathbf{r}_{OA} - \mathbf{r}_{OB} = -2\mathbf{i} - 6\mathbf{j} + 10\mathbf{k}$. The magnitude is $|\mathbf{r}_{BA}| = 11.832$ ft. The unit vector parallel to BA is

$$\mathbf{e}_{BA} = -0.1690\mathbf{i} - 0.5071\mathbf{j} + 0.8452\mathbf{k}.$$

The tension acting at B is

$$\mathbf{T}_{BA} = 80\mathbf{e}_{BA} = -13.52\mathbf{i} - 40.57\mathbf{j} + 67.62\mathbf{k}.$$

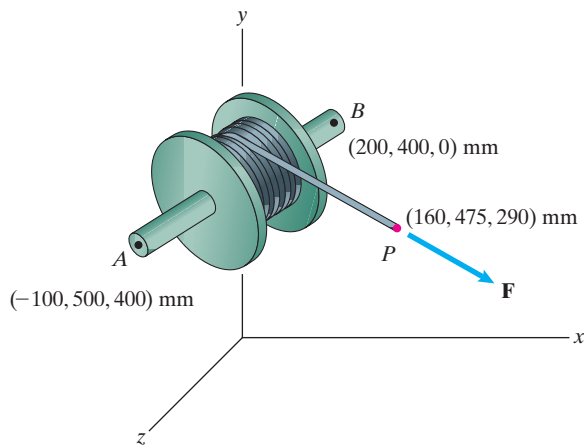
The magnitude of the moment about CD due to the tension acting at B is

$$|\mathbf{M}_{CD}| = \mathbf{e}_{CD} \cdot (\mathbf{r}_{CB} \times \mathbf{T}_{BA}) = \begin{vmatrix} 0 & -1 & 0 \\ 5 & 0 & 0 \\ -13.52 & -40.57 & 67.62 \end{vmatrix}$$

$$= 338.1 \text{ (ft lb)}.$$

The moment about CD is $\mathbf{M}_{CD} = 338.1\mathbf{e}_{CD} = -338.1\mathbf{j}$ (ft lb). The sense of the moment is along the curled fingers of the right hand when the thumb is parallel to CD , pointing toward D .

Problem 4.99 The magnitude of the force \mathbf{F} is 0.2 N and its direction cosines are $\cos \theta_x = 0.727$, $\cos \theta_y = -0.364$, and $\cos \theta_z = 0.582$. Determine the magnitude of the moment of \mathbf{F} about the axis AB of the spool.



Solution: We have

$$\mathbf{r}_{AB} = (0.3\mathbf{i} - 0.1\mathbf{j} - 0.4\mathbf{k}) \text{ m},$$

$$r_{AB} = \sqrt{(0.3)^2 + (0.1)^2 + (0.4)^2} \text{ m} = \sqrt{0.26} \text{ m}$$

$$\mathbf{e}_{AB} = \frac{1}{\sqrt{0.26}}(0.3\mathbf{i} - 0.1\mathbf{j} - 0.4\mathbf{k})$$

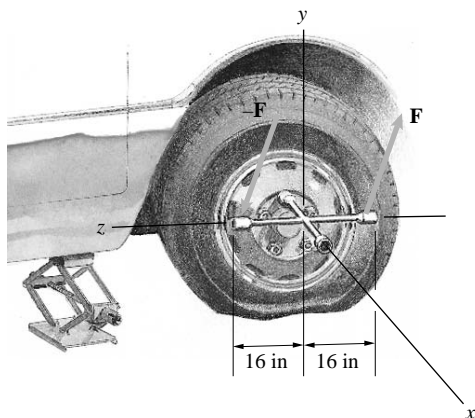
$$\mathbf{F} = 0.2 \text{ N}(0.727\mathbf{i} - 0.364\mathbf{j} + 0.582\mathbf{k})$$

$$\mathbf{r}_{AP} = (0.26\mathbf{i} - 0.025\mathbf{j} - 0.11\mathbf{k}) \text{ m}$$

Now the magnitude of the moment about the spool axis AB is

$M_{AB} = \frac{0.2 \text{ N}}{\sqrt{0.26}}$	0.3	-0.1	-0.4	$= 0.0146 \text{ N-m}$
	0.26 m	-0.025 m	-0.11 m	
	0.727	-0.364	0.582	

Problem 4.100 A motorist applies the two forces shown to loosen a lug nut. The direction cosines of \mathbf{F} are $\cos \theta_x = \frac{4}{13}$, $\cos \theta_y = \frac{12}{13}$, and $\cos \theta_z = \frac{3}{13}$. If the magnitude of the moment about the x axis must be 32 ft-lb to loosen the nut, what is the magnitude of the forces the motorist must apply?



Solution: The unit vectors for the forces are the direction cosines. The position vector of the force \mathbf{F} is $\mathbf{r}_{OF} = -1.333\mathbf{k}$ ft. The magnitude of the moment due to \mathbf{F} is

$$|\mathbf{M}_{OF}| = \mathbf{e}_x \cdot (\mathbf{r}_{OF} \times \mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & -1.333 \\ 0.3077F & 0.9231F & 0.2308F \end{vmatrix}$$

$$|\mathbf{M}_{OF}| = 1.230F \text{ ft-lb.}$$

The magnitude of the moment due to $-\mathbf{F}$ is

$$|\mathbf{M}_{-OF}| = \mathbf{e}_x \cdot (\mathbf{r}_{-OF} \times -\mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1.333 \\ -0.3077F & -0.9231F & -0.2308F \end{vmatrix} = 1.230F \text{ ft-lb.}$$

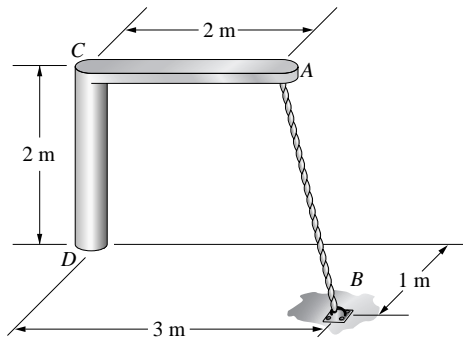
The total moment about the x axis is

$$\sum \mathbf{M}_x = 1.230F\mathbf{i} + 1.230F\mathbf{i} = 2.46F\mathbf{i},$$

from which, for a total magnitude of 32 ft-lb, the force to be applied is

$$F = \frac{32}{2.46} = 13 \text{ lb}$$

Problem 4.101 The tension in cable AB is 2 kN. What is the magnitude of the moment about the shaft CD due to the force exerted by the cable at A ? Draw a sketch to indicate the sense of the moment about the shaft.



Solution: The strategy is to determine the moment about C due to A , and determine the component parallel to CD . The moment is determined from the distance CA and the components of the tension, which is to be found from the magnitude of the tension and the unit vector parallel to AB . The coordinates of the points A , B , C , and D are: $A (2, 2, 0)$, $B (3, 0, 1)$, $C (0, 2, 0)$, and $D (0,0,0)$. The unit vector parallel to CD is by inspection $\mathbf{e}_{CD} = -\mathbf{j}$. The position vectors parallel to DC , DA , and DB :

$$\mathbf{r}_{DC} = 2\mathbf{j}, \quad \mathbf{r}_{DA} = 2\mathbf{i} + 2\mathbf{j}, \quad \mathbf{r}_{DB} = 3\mathbf{i} + 1\mathbf{k}.$$

The vector parallel to CA is $\mathbf{r}_{CA} = 2\mathbf{i}$. The vector parallel to AB is

$$\mathbf{r}_{AB} = \mathbf{r}_{DB} - \mathbf{r}_{DA} = 1\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}.$$

The magnitude: $|\mathbf{r}_{AB}| = 2.4495$ m. The unit vector parallel to AB is

$$\mathbf{e}_{AB} = 0.4082\mathbf{i} - 0.8165\mathbf{j} + 0.4082\mathbf{k}.$$

The tension is

$$\mathbf{T}_{AB} = 2\mathbf{e}_{AB} = 0.8165\mathbf{i} - 1.633\mathbf{j} + 0.8165\mathbf{k}.$$

The magnitude of the moment about CD is

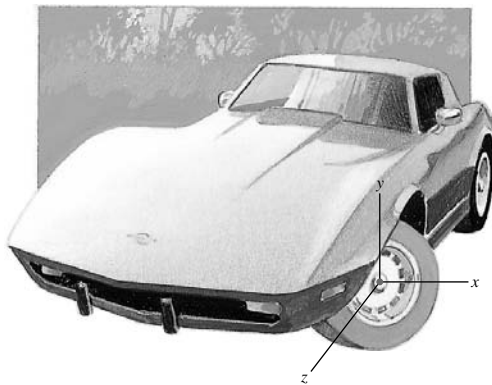
$$|\mathbf{M}_{CD}| = \mathbf{e}_{CD} \cdot (\mathbf{r}_{CA} \times \mathbf{T}_{AB}) = \begin{vmatrix} 0 & -1 & 0 \\ 2 & 0 & 0 \\ 0.8164 & -1.633 & 0.8165 \end{vmatrix} = 1.633 \text{ kN-m.}$$

The moment about CD is

$$\mathbf{M}_{CD} = \mathbf{e}_{CD}|\mathbf{M}_{CD}| = -1.633\mathbf{j} \text{ (kN-m).}$$

The sense is in the direction of the curled fingers of the right hand when the thumb is parallel to DC , pointed toward D .

Problem 4.102 The axis of the car's wheel passes through the origin of the coordinate system and its direction cosines are $\cos \theta_x = 0.940$, $\cos \theta_y = 0$, $\cos \theta_z = 0.342$. The force exerted on the tire by the road effectively acts at the point $x = 0$, $y = -0.36$ m, $z = 0$ and has components $\mathbf{F} = -720\mathbf{i} + 3660\mathbf{j} + 1240\mathbf{k}$ (N). What is the moment of \mathbf{F} about the wheel's axis?



Solution: We have to determine the moment about the axle where a unit vector along the axle is

$$\mathbf{e} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k}$$

$$\mathbf{e} = 0.940\mathbf{i} + 0\mathbf{j} + 0.342\mathbf{k}$$

The vector from the origin to the point of contact with the road is

$$\mathbf{r} = 0\mathbf{i} - 0.36\mathbf{j} + 0\mathbf{k} \text{ m}$$

The force exerted at the point of contact is

$$\mathbf{F} = -720\mathbf{i} + 3660\mathbf{j} + 1240\mathbf{k} \text{ N}$$

The moment of the force \mathbf{F} about the axle is

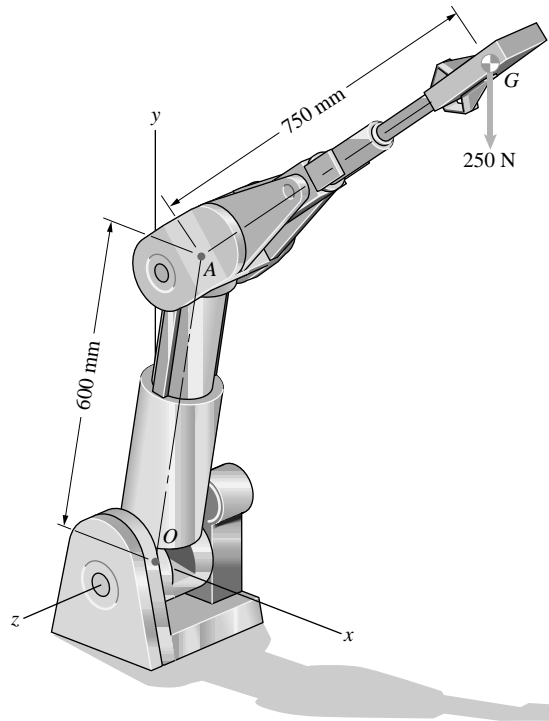
$$\mathbf{M}_{AXLE} = [\mathbf{e} \cdot (\mathbf{r} \times \mathbf{F})]\mathbf{e}$$

$$\mathbf{M}_{AXLE} = \begin{vmatrix} 0.940 & 0 & 0.342 \\ 0 & -0.36 & 0 \\ -720 & +3660 & +1240 \end{vmatrix} (0.940\mathbf{i} + 0.342\mathbf{k}) \text{ (N-m)}$$

$$\mathbf{M}_{AXLE} = (-508.26)(0.940\mathbf{i} + 0.342\mathbf{k}) \text{ (N-m)}$$

$$\mathbf{M}_{AXLE} = -478\mathbf{i} - 174\mathbf{k} \text{ (N-m)}$$

Problem 4.103 The direction cosines of the centerline OA are $\cos \theta_x = 0.500$, $\cos \theta_y = 0.866$, and $\cos \theta_z = 0$, and the direction cosines of the line AG are $\cos \theta_x = 0.707$, $\cos \theta_y = 0.619$, and $\cos \theta_z = -0.342$. What is the moment about OA due to the 250-N weight? Draw a sketch to indicate the sense of the moment about the shaft.



Solution: By definition, the direction cosines are the scalar components of the unit vectors. Thus the unit vectors are

$$\mathbf{e}_1 = 0.5\mathbf{i} + 0.866\mathbf{j}, \text{ and } \mathbf{e}_2 = 0.707\mathbf{i} + 0.619\mathbf{j} - 0.341\mathbf{k}.$$

The force is $\mathbf{W} = 250\mathbf{j}$ (N). The position vector of the 250 N weight is

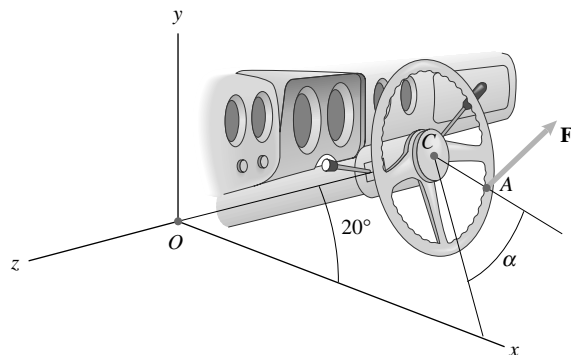
$$\mathbf{r}_W = 0.600\mathbf{e}_1 + 0.750\mathbf{e}_2 = 0.8303\mathbf{i} + 0.9839\mathbf{j} - 0.2565\mathbf{k}$$

The moment about OA is

$$\begin{aligned} \mathbf{M}_{OA} &= \mathbf{e}_{OA}(\mathbf{e}_{OA} \cdot (\mathbf{r}_W \times \mathbf{W})) \\ &= \begin{vmatrix} 0.5 & 0.866 & 0 \\ 0.8303 & 0.9839 & -0.2565 \\ 0 & -250 & 0 \end{vmatrix} \mathbf{e}_1 = -32.06\mathbf{e}_1 \\ &= -16\mathbf{i} - 27.77\mathbf{j} \text{ (N-m)} \end{aligned}$$

The moment is anti parallel to the unit vector parallel to OA , with the sense of the moment in the direction of the curled fingers when the thumb of the right hand is directed oppositely to the direction of the unit vector.

Problem 4.104 The radius of the steering wheel is 200 mm. The distance from O to C is 1 m. The center C of the steering wheel lies in the $x-y$ plane. The driver exerts a force $\mathbf{F} = 10\mathbf{i} + 10\mathbf{j} - 5\mathbf{k}$ (N) on the wheel at A . If the angle $\alpha = 0$, what is the magnitude of the moment about the shaft OC ? Draw a sketch to indicate the sense of the moment about the shaft.



Solution: The strategy is to determine the moment about C , and then determine its component about OC . The radius vectors parallel to OC and CA are:

$$\mathbf{r}_{OC} = 1(\mathbf{i} \cos 20^\circ + \mathbf{j} \sin 20^\circ + 0\mathbf{k}) = 0.9397\mathbf{i} + 0.3420\mathbf{j}.$$

The line from C to the x axis is perpendicular to OC since it lies in the plane of the steering wheel. The unit vector from C to the x axis is

$$\mathbf{e}_{CX} = \mathbf{i} \cos(20 - 90) + \mathbf{j} \sin(20 - 90) = 0.3420\mathbf{i} - 0.9397\mathbf{j},$$

where the angle is measured positive counterclockwise from the x axis. The vector parallel to CA is

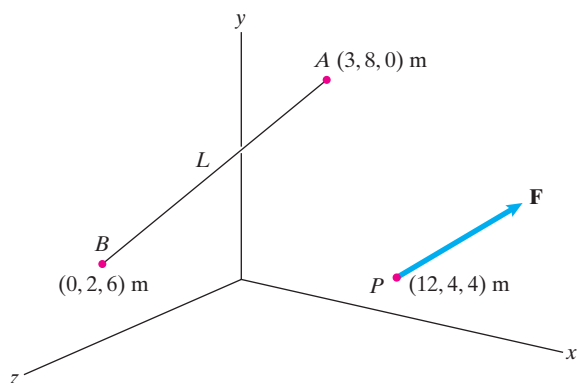
$$\mathbf{r}_{CA} = 0.2\mathbf{e}_{CX} = +0.0684\mathbf{i} - 0.1879\mathbf{j} \text{ (m)}.$$

The magnitude of the moment about OC

$$\begin{aligned} |\mathbf{M}_{OC}| &= \mathbf{e}_{OC} \cdot (\mathbf{r}_{CA} \times \mathbf{F}) = \begin{vmatrix} 0.9397 & 0.3420 & 0 \\ 0.0684 & -0.1879 & 0 \\ 10 & 10 & -5 \end{vmatrix} \\ &= 0.9998 = 1 \text{ N-m}. \end{aligned}$$

The sense of the moment is in the direction of the curled fingers of the right hand if the thumb is parallel to OC , pointing from O to C .

Problem 4.105* The magnitude of the force \mathbf{F} is 10 N. Suppose that you want to choose the direction of the force \mathbf{F} so that the magnitude of its moment about the line L is a maximum. Determine the components of \mathbf{F} and the magnitude of its moment about L . (There are two solutions for \mathbf{F} .)



Solution: The moment of the general force $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$ about the line is developed by

$$\mathbf{e}_{BA} = \frac{3\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}}{9} = \frac{1}{3}(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}),$$

$$\mathbf{r}_{BP} = (12\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \text{ m},$$

$$M_{BA} = \mathbf{e}_{BA} \cdot (\mathbf{r}_{BP} \times \mathbf{F})$$

$$\text{This expression simplifies to } M_{BA} = -\frac{22}{3} \text{ m} (F_y + F_z)$$

$$\text{We also have the constraint that } (10 \text{ N})^2 = F_x^2 + F_y^2 + F_z^2$$

Since F_x does not contribute to the moment we set it equal to zero. Solving the constraint equation for F_z and substituting this into the expression for the moment we find

$$M_{BA} = -\frac{22}{3} (F_y \pm \sqrt{100 - F_y^2}). \Rightarrow \frac{dM_{BA}}{dF_y} = 0$$

$$\Rightarrow F_y = \pm 5\sqrt{2} \text{ N} \Rightarrow F_z = \pm 5\sqrt{2}$$

We thus have two answers:

$$\boxed{\mathbf{F} = (7.07\mathbf{j} + 7.07\mathbf{k}) \text{ N} \quad \text{or} \quad \mathbf{F} = -(7.07\mathbf{j} + 7.07\mathbf{k})}$$

Problem 4.106 The weight W causes a tension of 100 lb in cable CD . If $d = 2$ ft, what is the moment about the z axis due to the force exerted by the cable CD at point C ?

Solution: The strategy is to use the unit vector parallel to the bar to locate point C relative to the origin, and then use this location to find the unit vector parallel to the cable CD . With the tension resolved into components about the origin, the moment about the origin can be resolved into components along the z axis. Denote the top of the bar by T and the bottom of the bar by B . The position vectors of the ends of the bar are:

$$\mathbf{r}_{OB} = 3\mathbf{i} + 0\mathbf{j} + 10\mathbf{k}, \quad \mathbf{r}_{OT} = 12\mathbf{i} + 10\mathbf{j} + 0\mathbf{k}.$$

The vector from the bottom to the top of the bar is

$$\mathbf{r}_{BT} = \mathbf{r}_{OT} - \mathbf{r}_{OB} = 9\mathbf{i} + 10\mathbf{j} - 10\mathbf{k}.$$

The magnitude:

$$|\mathbf{r}_{BT}| = \sqrt{9^2 + 10^2 + 10^2} = 16.763 \text{ ft}.$$

The unit vector parallel to the bar, pointing toward the top, is

$$\mathbf{e}_{BT} = 0.5369\mathbf{i} + 0.5965\mathbf{j} - 0.5965\mathbf{k}.$$

The position vector of the point C relative to the bottom of the bar is

$$\mathbf{r}_{BC} = 2\mathbf{e}_{BT} = 1.074\mathbf{i} + 1.193\mathbf{j} - 1.193\mathbf{k}.$$

The position vector of point C relative to the origin is

$$\mathbf{r}_{OC} = \mathbf{r}_{OB} + \mathbf{r}_{BC} = 4.074\mathbf{i} + 1.193\mathbf{j} + 8.807\mathbf{k}.$$

The position vector of point D is

$$\mathbf{r}_{OD} = 0\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}.$$

The vector parallel to CD is

$$\mathbf{r}_{CD} = \mathbf{r}_{OD} - \mathbf{r}_{OC} = -4.074\mathbf{i} + 1.807\mathbf{j} - 8.807\mathbf{k}.$$

The magnitude is

$$|\mathbf{r}_{CD}| = \sqrt{4.074^2 + 1.807^2 + 8.807^2} = 9.87 \text{ ft}.$$

The unit vector parallel to CD is

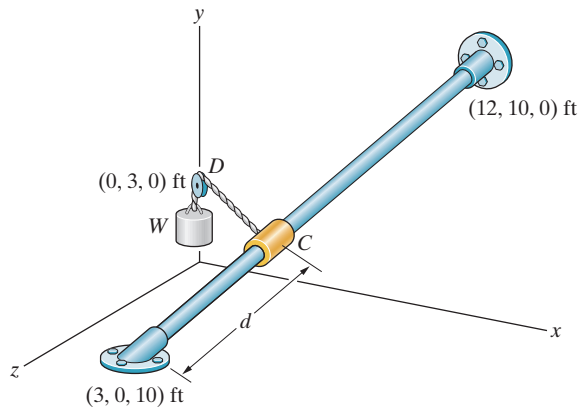
$$\mathbf{e}_{CD} = -0.4127\mathbf{i} + 0.1831\mathbf{j} - 0.8923\mathbf{k}.$$

The tension is

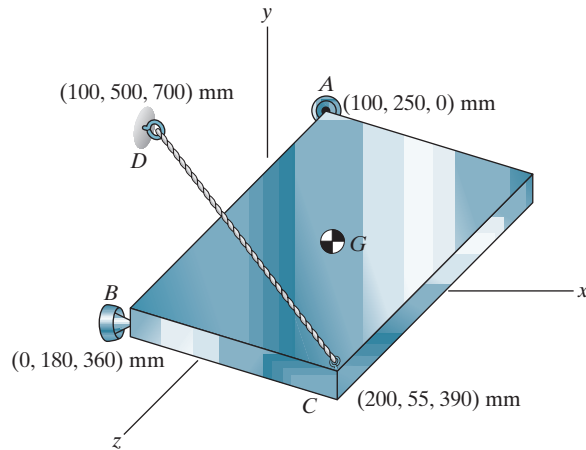
$$\mathbf{T}_{CD} = 100\mathbf{e}_{CD} = -41.27\mathbf{i} + 18.31\mathbf{j} - 89.23\mathbf{k} \text{ lb}.$$

The magnitude of the moment about the z axis is

$$\begin{aligned} |\mathbf{M}_O| &= \mathbf{e}_z \cdot (\mathbf{r}_{OC} \times \mathbf{T}_{CD}) = \begin{vmatrix} 0 & 0 & 1 \\ 4.074 & 1.193 & 8.807 \\ -41.27 & 18.31 & -89.23 \end{vmatrix} \\ &= 123.83 \text{ ft lb} \end{aligned}$$



Problem 4.107* The y axis points upward. The weight of the 4-kg rectangular plate acts at the midpoint G of the plate. The sum of the moments about the straight line through the supports A and B due to the weight of the plate and the force exerted on the plate by the cable CD is zero. What is the tension in the cable?



Solution: Note that the coordinates of point G are $(150, 152.5, 195)$.

We calculate the moment about the line BA due to the two forces as follows.

$$\mathbf{e}_{BA} = \frac{0.1\mathbf{i} + 0.07\mathbf{j} - 0.36\mathbf{k}}{\sqrt{0.1445}}$$

$$\mathbf{r}_1 = (0.2\mathbf{i} - 0.125\mathbf{j} + 0.03\mathbf{k}) \text{ m},$$

$$\mathbf{F}_1 = T_{CD} \frac{(-0.1\mathbf{i} + 0.445\mathbf{j} + 0.31\mathbf{k})}{\sqrt{0.304125}}$$

$$\mathbf{r}_2 = (0.15\mathbf{i} - 0.0275\mathbf{j} - 0.165\mathbf{k}) \text{ m},$$

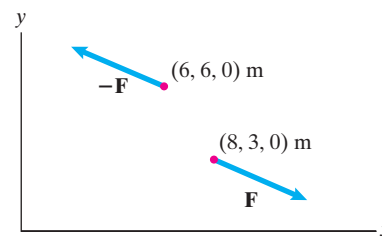
$$\mathbf{F}_2 = -(4 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j}$$

$$M_{BA} = \mathbf{e}_{BA} \cdot (\mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2)$$

The moment reduces to

$$M_{BA} = 3.871 \text{ N}\cdot\text{m} - (0.17793 \text{ m})T_{CD} = 0 \Rightarrow T_{CD} = 21.8 \text{ N}$$

Problem 4.108 In Active Example 4.9, suppose that the point of application of the force \mathbf{F} is moved from $(8, 3, 0)$ m to $(8, 8, 0)$ m. Draw a sketch showing the new position of the force. From your sketch, will the moment due to the couple be clockwise or counterclockwise? Calculate the moment due to the couple. Represent the moment by its magnitude and a circular arrow indicating its direction.



Solution: From Active Example 4.9 we know that

$$\mathbf{F} = (10\mathbf{i} - 4\mathbf{j}) \text{ N}$$

From the sketch, it is evident that the moment will be clockwise.

The moment due to the couple is the sum of the moments of the two forces about *any point*. If we determine the sum of the moments about the point of application of one of the forces, the moment due to that force is zero and we only need to determine the moment due to the other force.

Let us determine the moment about the point of application of the force \mathbf{F} . The vector from the point of application of \mathbf{F} to the point of application of the force $-\mathbf{F}$ is

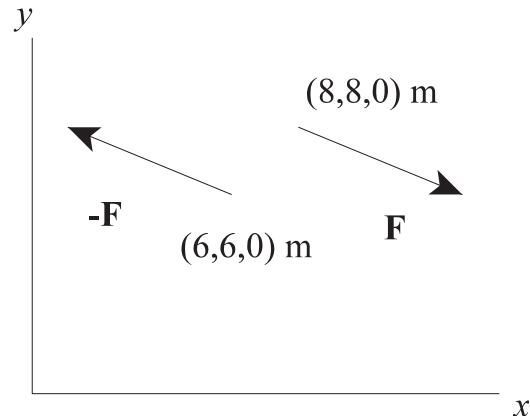
$$\mathbf{r} = [(6 - 8)\mathbf{i} + (6 - 3)\mathbf{j}] \text{ m} = (-2\mathbf{i} + 3\mathbf{j}) \text{ m}$$

The sum of the moments of the two forces is

$$\mathbf{M} = \mathbf{r} \times (-\mathbf{F}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 0 \\ -10 & 4 & 0 \end{vmatrix} = -28\mathbf{k} \text{ N}\cdot\text{m}$$

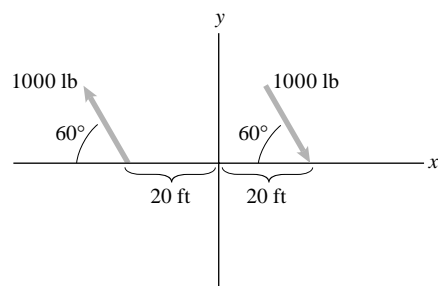
The magnitude of the moment is 28 N·m. Pointing the thumb of the right hand *into* the page, the right-hand rule indicates that the moment is clockwise.

$$M = 28 \text{ N}\cdot\text{m} \text{ clockwise}$$



Problem 4.109 The forces are contained in the x – y plane.

- Determine the moment of the couple and represent it as shown in Fig. 4.28c.
- What is the sum of the moments of the two forces about the point $(10, -40, 20)$ ft?



Solution: The right hand force is

$$\mathbf{F} = [1000 \text{ (lb)}](\cos 60^\circ \mathbf{i} - \sin 60^\circ \mathbf{j})$$

$$\mathbf{F} = +500\mathbf{i} - 867\mathbf{j} \text{ lb.}$$

The vector from the x intercept of the left force to that of the right force is $\mathbf{r} = 40\mathbf{i}$ ft.

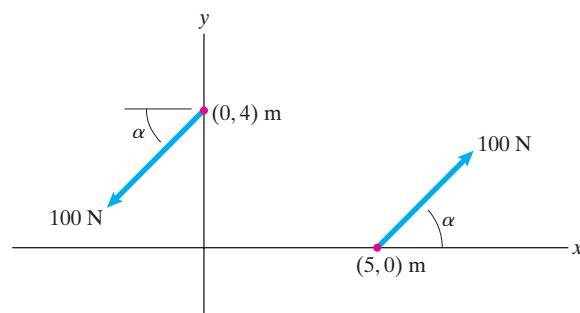
The moment is $\mathbf{M}_C = \mathbf{r} \times \mathbf{F}$

$$\mathbf{M}_C = 40\mathbf{i} \times (500\mathbf{i} - 867\mathbf{j}) \text{ (ft-lb)}$$

$$\mathbf{M}_C = -34700 \text{ (ft-lb)} \mathbf{k}$$

or $\mathbf{M}_C = -34700 \text{ (ft-lb)}$ clockwise

Problem 4.110 The moment of the couple is 600 k (N-m). What is the angle α ?



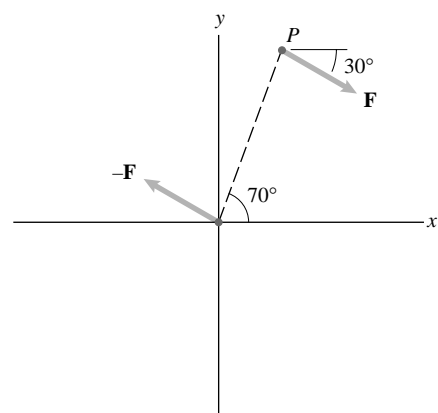
Solution:

$$M = (100 \text{ N} \cos \alpha)(4 \text{ m}) + (100 \text{ N} \sin \alpha)(5 \text{ m}) = 600 \text{ N-m}$$

Solving yields two answers:

$$\alpha = 30.9^\circ \text{ or } \alpha = 71.8^\circ$$

Problem 4.111 Point P is contained in the x – y plane, $|\mathbf{F}| = 100 \text{ N}$, and the moment of the couple is $-500\mathbf{k}$ (N-m). What are the coordinates of P ?



Solution: The force is

$$\mathbf{F} = 100(\mathbf{i} \cos(-30^\circ) + \mathbf{j} \sin(-30^\circ)) = 86.6\mathbf{i} - 50\mathbf{j}.$$

Let r be the distance OP . The vector parallel to OP is

$$\mathbf{r} = r(\mathbf{i} \cos 70^\circ + \mathbf{j} \sin 70^\circ) = r(0.3420\mathbf{i} + 0.9397\mathbf{j}).$$

The moment is

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3420r & 0.9397r & 0 \\ 86.6 & -50.0 & 0 \end{vmatrix} = -98.48r\mathbf{k}.$$

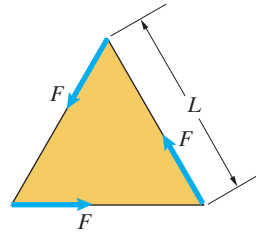
From which, $r = \frac{500}{98.48} = 5.077 \text{ m}$. From above,

$$\mathbf{r} = 5.077(0.3420\mathbf{i} + 0.9397\mathbf{j}).$$

The coordinates of P are

$$x = 5.077(0.3420) = 1.74 \text{ m}, \quad y = 5.077(0.9397) = 4.77 \text{ m}$$

Problem 4.112 Three forces of equal magnitude are applied parallel to the sides of an equilateral triangle. (a) Show that the sum of the moments of the forces is the same about any point. (b) Determine the magnitude of the sum of the moments.

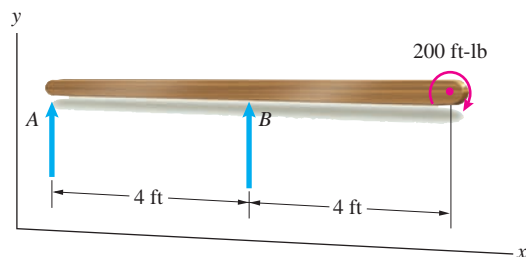


Solution:

- (a) Resolving one of the forces into vector components parallel to the other two forces results in two equal and opposite forces with the same line of action and one couple. Therefore the moment due to the forces is the same about any point.
- (b) Determine the moment about one of the vertices of the triangle. A vertex lies on the line of action of two of the forces, so the moment due to them is zero. The perpendicular distance to the line of action of the third force is $L \cos 30^\circ$, so the magnitude of the moment due to the three force is

$$M = FL \cos 30^\circ$$

Problem 4.113 In Example 4.10, suppose that the 200 ft-lb couple is counterclockwise instead of clockwise. Draw a sketch of the beam showing the forces and couple acting on it. What are the forces A and B ?

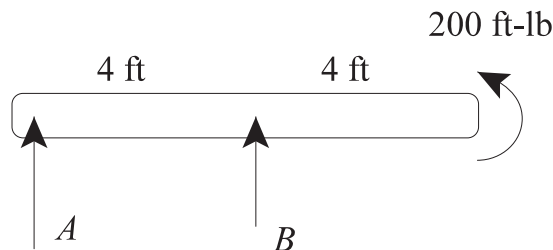


Solution: In Example 4.10 we are given that the sum of the forces is zero and the sum of the moments is zero. Thus

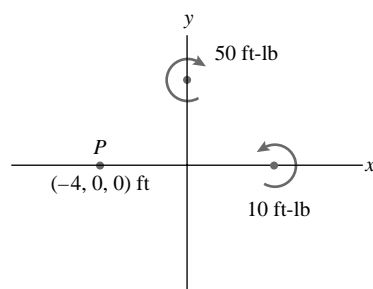
$$\Sigma F_y = A + B = 0$$

$$\Sigma M_A = B(4 \text{ ft}) + 200 \text{ ft-lb} = 0$$

Solving we find $A = 50 \text{ lb}, B = -50 \text{ lb}$



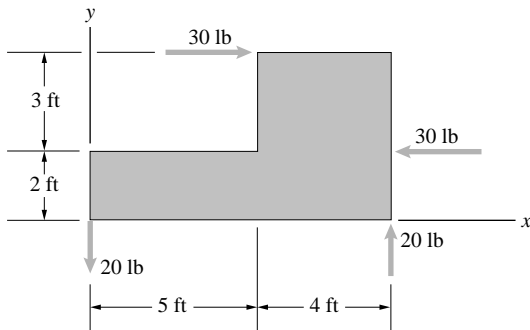
Problem 4.114 The moments of two couples are shown. What is the sum of the moments about point P ?



Solution: The moment of a couple is the same anywhere in the plane. Hence the sum about the point P is

$$\Sigma \mathbf{M} = -50\mathbf{k} + 10\mathbf{k} = -40\mathbf{k} \text{ ft lb}$$

Problem 4.115 Determine the sum of the moments exerted on the plate by the two couples.



Solution: The moment due to the 30 lb couple, which acts in a clockwise direction is

$$\mathbf{M}_{30} = -3(30)\mathbf{k} = -90\mathbf{k} \text{ ft lb.}$$

The moment due to the 20 lb couple, which acts in a counterclockwise direction, is

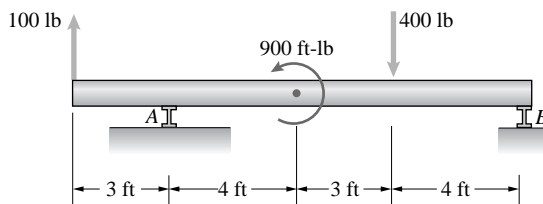
$$\mathbf{M}_{20} = 9(20)\mathbf{k} = 180\mathbf{k} \text{ ft lb.}$$

The sum of the moments is

$$\sum \mathbf{M} = -90\mathbf{k} + 180\mathbf{k} = +90\mathbf{k} \text{ ft lb.}$$

The sum of the moments is the same anywhere on the plate.

Problem 4.116 Determine the sum of the moments exerted about A by the couple and the two forces.



Solution: Let the x axis point to the right and the y axis point upward in the plane of the page. The moments of the forces are

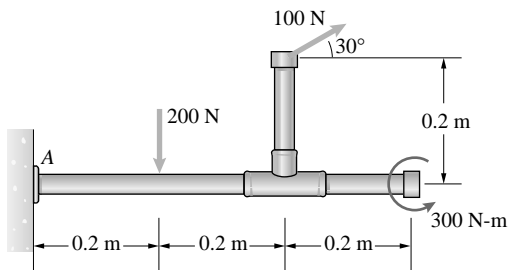
$$\mathbf{M}_{100} = (-3\mathbf{i}) \times (100\mathbf{j}) = -300\mathbf{k} \text{ (ft-lb),}$$

$$\text{and } \mathbf{M}_{400} = (7\mathbf{i}) \times (-400\mathbf{j}) = -2800\mathbf{k} \text{ (ft-lb).}$$

The moment of the couple is $\mathbf{M}_C = 900\mathbf{k}$ (ft-lb). Summing the moments, we get

$$\boxed{\mathbf{M}_{\text{Total}} = -2200\mathbf{k} \text{ (ft-lb)}}$$

Problem 4.117 Determine the sum of the moments exerted about A by the couple and the two forces.



Solution:

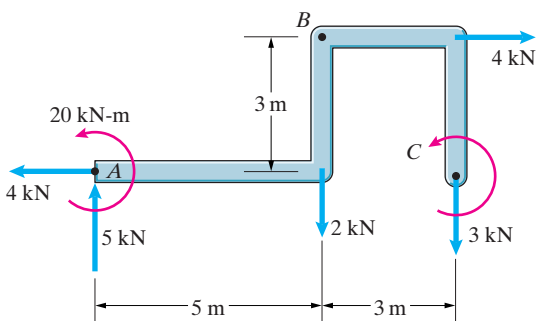
$$\begin{aligned} \sum \mathbf{M}_A &= (0.2\mathbf{i}) \times (-200\mathbf{j}) + (0.4\mathbf{i} + 0.2\mathbf{j}) \\ &\quad \times (86.7\mathbf{i} + 50\mathbf{j}) + 300\mathbf{k} \text{ (N-m)} \end{aligned}$$

$$\sum \mathbf{M}_A = -40\mathbf{k} + 2.66\mathbf{k} + 300\mathbf{k} \text{ (N-m)}$$

$$\sum \mathbf{M}_A = 262.7\mathbf{k} \text{ (N-m)} \simeq 263\mathbf{k} \text{ (N-m)}$$

Problem 4.118 The sum of the moments about point A due to the forces and couples acting on the bar is zero.

- What is the magnitude of the couple C ?
- Determine the sum of the moments about point B due to the forces and couples acting on the bar.



Solution:

$$\begin{aligned} \text{(a) } \sum M_A &= 20 \text{ kN-m} - (2 \text{ kN})(5 \text{ m}) - (4 \text{ kN})(3 \text{ m}) \\ &\quad - (3 \text{ kN})(8) + C = 0 \end{aligned}$$

$$\boxed{C = 26 \text{ kN-m}}$$

$$\begin{aligned} \text{(b) } \sum M_B &= -(3 \text{ kN})(3 \text{ m}) - (4 \text{ kN})(3 \text{ m}) - (5 \text{ kN})(5 \text{ m}) \\ &\quad + 20 \text{ kN-m} + 26 \text{ kN-m} = 0 \end{aligned}$$

Problem 4.119 In Example 4.11, suppose that instead of acting in the positive z direction, the upper 20-N force acts in the positive x axis direction. Instead of acting in the negative z axis direction, let the lower 20-N force act in the negative x axis direction. Draw a sketch of the pipe showing the forces acting on it. Determine the sum of the moments exerted on the pipe by the two couples.

Solution: The magnitude of the moment of the 20-N couple is unchanged,

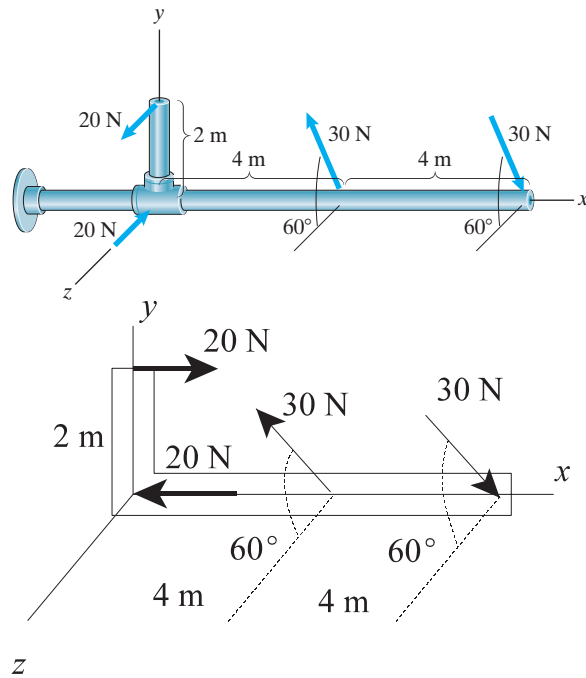
$$(2 \text{ m})(20 \text{ N}) = 40 \text{ N}\cdot\text{m}.$$

The direction of the moment vector is perpendicular to the x - y plane, and the right-hand rule indicates that it points in the negative z axis direction. The moment of the 20-N couple is $(-40 \text{ N}\cdot\text{m}) \mathbf{k}$.

The sum of the moments exerted on the pipe by the two couples is

$$\Sigma \mathbf{M} = (-40 \text{ N}\cdot\text{m}) \mathbf{k} + (30 \text{ N}) \cos 60^\circ (4 \text{ m}) \mathbf{j} - (30 \text{ N}) \sin 60^\circ (4 \text{ m}) \mathbf{k}$$

$$\Sigma \mathbf{M} = (60\mathbf{j} - 144\mathbf{k}) \text{ N}\cdot\text{m}$$



Problem 4.120 (a) What is the moment of the couple?
(b) Determine the perpendicular distance between the lines of action of the two forces.

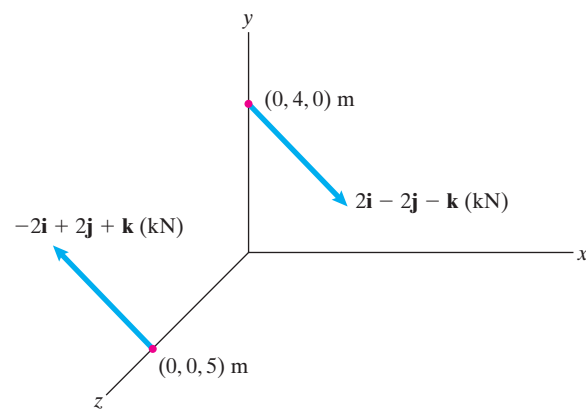
Solution:

$$\begin{aligned} \mathbf{M} &= (4\mathbf{j} - 5\mathbf{k}) \text{ m} \times (2\mathbf{i} - 2\mathbf{j} - \mathbf{k}) \text{ kN} \\ &= (-14\mathbf{i} - 10\mathbf{j} - 8\mathbf{k}) \text{ kN}\cdot\text{m} \end{aligned}$$

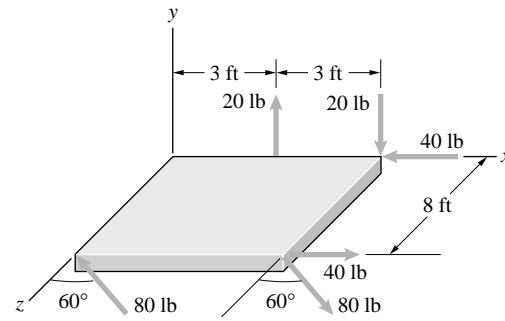
$$(b) \quad M = \sqrt{(-14)^2 + (-10)^2 + (-8)^2} \text{ kN}\cdot\text{m} = 18.97 \text{ kN}\cdot\text{m}$$

$$F = \sqrt{(2)^2 + (-2)^2 + (-1)^2} \text{ kN} = 3 \text{ kN}$$

$$M = Fd \Rightarrow d = \frac{M}{F} = \frac{18.97 \text{ kN}\cdot\text{m}}{3 \text{ kN}} = 6.32 \text{ m}$$



Problem 4.121 Determine the sum of the moments exerted on the plate by the three couples. (The 80-lb forces are contained in the x - z plane.)



Solution: The moments of two of the couples can be determined from inspection:

$$\mathbf{M}_1 = -(3)(20)\mathbf{k} = -60\mathbf{k} \text{ ft lb.}$$

$$\mathbf{M}_2 = (8)(40)\mathbf{j} = 320\mathbf{j} \text{ ft lb}$$

The forces in the 3rd couple are resolved:

$$\mathbf{F} = (80)(\mathbf{i} \sin 60^\circ + \mathbf{k} \cos 60^\circ) = 69.282\mathbf{i} + 40\mathbf{k}$$

The two forces in the third couple are separated by the vector

$$\mathbf{r}_3 = (6\mathbf{i} + 8\mathbf{k}) - (8\mathbf{k}) = 6\mathbf{i}$$

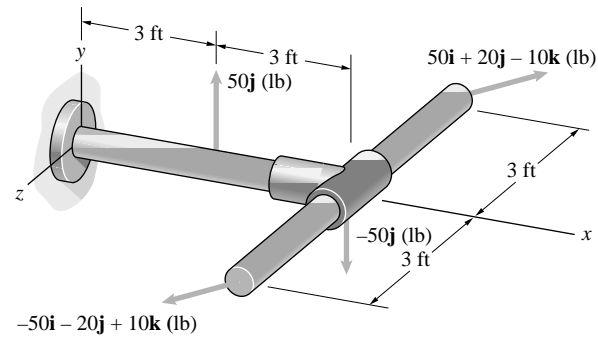
The moment is

$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ 69.282 & 0 & 40 \end{vmatrix} = -240\mathbf{j}.$$

The sum of the moments due to the couples:

$$\sum \mathbf{M} = -60\mathbf{k} + 320\mathbf{j} - 240\mathbf{j} = 80\mathbf{j} - 60\mathbf{k} \text{ ft lb}$$

Problem 4.122 What is the magnitude of the sum of the moments exerted on the T -shaped structure by the two couples?



Solution: The moment of the 50 lb couple can be determined by inspection:

$$\mathbf{M}_1 = -(50)(3)\mathbf{k} = -150\mathbf{k} \text{ ft lb.}$$

The vector separating the other two force is $\mathbf{r} = 6\mathbf{k}$. The moment is

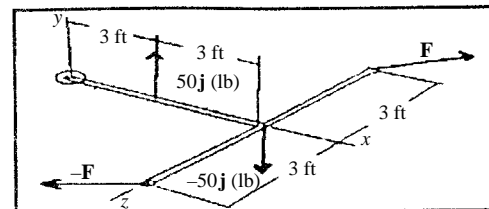
$$\mathbf{M}_2 = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 50 & 20 & -10 \end{vmatrix} = -120\mathbf{i} + 300\mathbf{j}.$$

The sum of the moments is

$$\sum \mathbf{M} = -120\mathbf{i} + 300\mathbf{j} - 150\mathbf{k}.$$

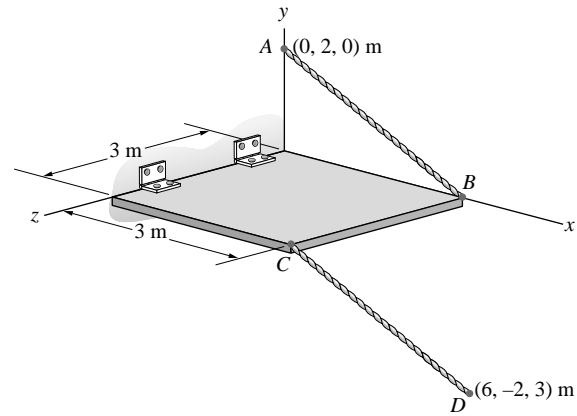
The magnitude is

$$|\mathbf{M}| = \sqrt{120^2 + 300^2 + 150^2} = 356.23 \text{ ft lb}$$



Problem 4.123 The tension in cables AB and CD is 500 N.

- Show that the two forces exerted by the cables on the rectangular hatch at B and C form a couple.
- What is the moment exerted on the plate by the cables?



Solution: One condition for a couple is that the sum of a pair of forces vanish; another is for a non-zero moment to be the same anywhere. The first condition is demonstrated by determining the unit vectors parallel to the action lines of the forces. The vector position of point B is $\mathbf{r}_B = 3\mathbf{i}$ m. The vector position of point A is $\mathbf{r}_A = 2\mathbf{j}$. The vector parallel to cable AB is

$$\mathbf{r}_{BA} = \mathbf{r}_A - \mathbf{r}_B = -3\mathbf{i} + 2\mathbf{j}.$$

The magnitude is:

$$|\mathbf{r}_{AB}| = \sqrt{3^2 + 2^2} = 3.606 \text{ m}.$$

The unit vector:

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -0.8321\mathbf{i} + 0.5547\mathbf{j}.$$

The tension is

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|\mathbf{e}_{AB} = -416.05\mathbf{i} + 277.35\mathbf{j}.$$

The vector position of points C and D are:

$$\mathbf{r}_C = 3\mathbf{i} + 3\mathbf{k}, \quad \mathbf{r}_D = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}.$$

The vector parallel to the cable CD is $\mathbf{r}_{CD} = \mathbf{r}_D - \mathbf{r}_C = 3\mathbf{i} - 2\mathbf{j}$. The magnitude is $|\mathbf{r}_{CD}| = 3.606$ m, and the unit vector parallel to the cable CD is $\mathbf{e}_{CD} = +0.8321\mathbf{i} - 0.5547\mathbf{j}$. The magnitude of the tension in the two cables is the same, and $\mathbf{e}_{BA} = -\mathbf{e}_{CD}$, hence the sum of the tensions vanish on the plate. The second condition is demonstrated by determining the moment at any point on the plate. By inspection, the distance between the action lines of the forces is

$$\mathbf{r}_{CB} = \mathbf{r}_B - \mathbf{r}_C = 3\mathbf{i} - 3\mathbf{i} - 3\mathbf{k} = -3\mathbf{k}.$$

The moment is

$$\begin{aligned} \mathbf{M} = \mathbf{r}_{CB} \times \mathbf{T}_{AB} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -3 \\ -416.05 & 277.35 & 0 \end{vmatrix} \\ &= 832.05\mathbf{i} - 1248.15\mathbf{j} \text{ (N-m)}. \end{aligned}$$

The moment about the origin is

$$\mathbf{M}_O = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{T}_{AB} = \mathbf{r}_{CB} \times \mathbf{T}_{AB},$$

which is identical with the above expression for the moment. Let \mathbf{r}_{PC} and \mathbf{r}_{PB} be the distances to points C and B from an arbitrary point P on the plate. Then $\mathbf{M}_P = (\mathbf{r}_{PB} - \mathbf{r}_{PC}) \times \mathbf{T}_{AB} = \mathbf{r}_{CB} \times \mathbf{T}_{AB}$ which is identical to the above expression. Thus the moment is the same everywhere on the plate, and the forces form a couple.

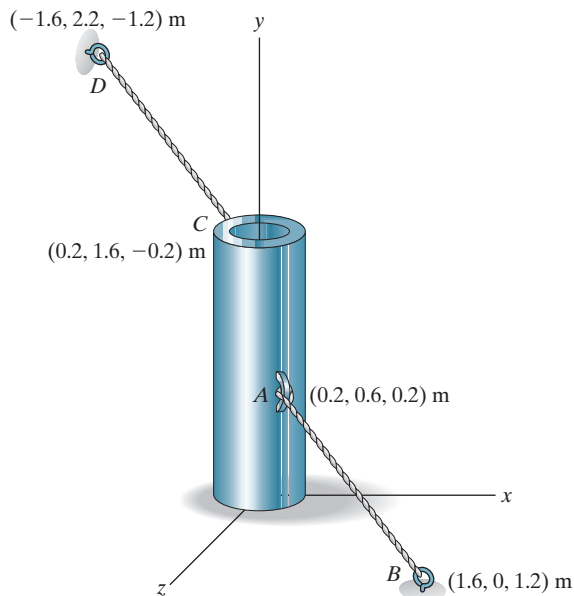
Problem 4.124 The cables AB and CD exert a couple on the vertical pipe. The tension in each cable is 8 kN. Determine the magnitude of the moment the cables exert on the pipe.

Solution:

$$\mathbf{F}_{AB} = 8 \text{ kN} \frac{(1.4\mathbf{i} - 0.6\mathbf{j} + 1.0\mathbf{k})}{\sqrt{3.32}}, \quad \mathbf{r}_{DB} = (3.2\mathbf{i} - 2.2\mathbf{j} + 2.4\mathbf{k}) \text{ m}$$

$$\mathbf{M} = \mathbf{r}_{BD} \times \mathbf{F}_{AB} = (-3.34\mathbf{i} + 0.702\mathbf{j} + 5.09\mathbf{k}) \text{ kN}\cdot\text{m}$$

$$\Rightarrow M = 6.13 \text{ kN}\cdot\text{m}$$



Problem 4.125 The bar is loaded by the forces

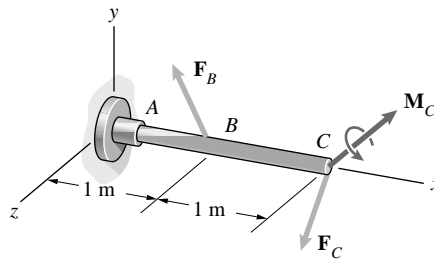
$$\mathbf{F}_B = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k} \text{ (kN)},$$

$$\mathbf{F}_C = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ (kN)},$$

and the couple

$$\mathbf{M}_C = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \text{ (kN}\cdot\text{m)}.$$

Determine the sum of the moments of the two forces and the couple about A.



Solution: The moments of the two forces about A are given by

$$\mathbf{M}_{FB} = (1\mathbf{i}) \times (2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \text{ (kN}\cdot\text{m)} = 0\mathbf{i} - 3\mathbf{j} + 6\mathbf{k} \text{ (kN}\cdot\text{m)} \text{ and}$$

$$\mathbf{M}_{FC} = (2\mathbf{i}) \times (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \text{ (kN}\cdot\text{m)} = 0\mathbf{i} - 4\mathbf{j} - 4\mathbf{k} \text{ (kN}\cdot\text{m)}.$$

Adding these two moments and

$$\mathbf{M}_C = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \text{ (kN}\cdot\text{m)},$$

$$\text{we get } \mathbf{M}_{\text{TOTAL}} = 2\mathbf{i} - 6\mathbf{j} + 0\mathbf{k} \text{ (kN}\cdot\text{m)}$$

Problem 4.126 In Problem 4.125, the forces

$$\mathbf{F}_B = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k} \text{ (kN)},$$

$$\mathbf{F}_C = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ (kN)},$$

and the couple

$$\mathbf{M}_C = M_{Cy}\mathbf{j} + M_{Cz}\mathbf{k} \text{ (kN}\cdot\text{m)}.$$

Determine the values for M_{Cy} and M_{Cz} , so that the sum of the moments of the two forces and the couple about A is zero.

Solution: From the solution to Problem 4.125, the sum of the moments of the two forces about A is

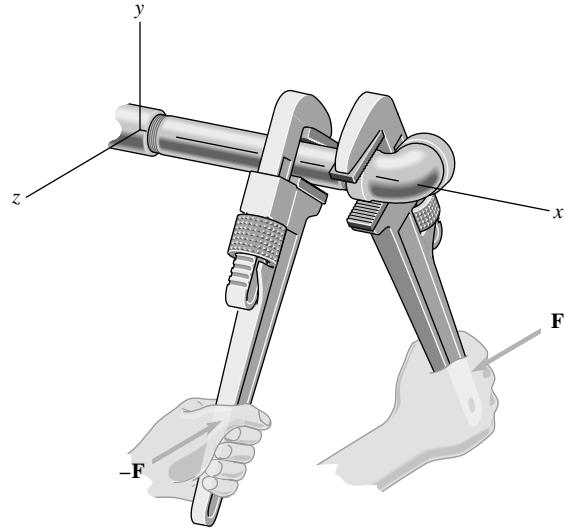
$$\mathbf{M}_{\text{Forces}} = 0\mathbf{i} - 7\mathbf{j} + 2\mathbf{k} \text{ (kN}\cdot\text{m)}.$$

The required moment, \mathbf{M}_C , must be the negative of this sum.

$$\text{Thus } M_{Cy} = 7 \text{ (kN}\cdot\text{m)}, \text{ and } M_{Cz} = -2 \text{ (kN}\cdot\text{m)}.$$

Problem 4.127 Two wrenches are used to tighten an elbow fitting. The force $\mathbf{F} = 10\mathbf{k}$ (lb) on the right wrench is applied at $(6, -5, -3)$ in, and the force $-\mathbf{F}$ on the left wrench is applied at $(4, -5, 3)$ in.

- Determine the moment about the x axis due to the force exerted on the right wrench.
- Determine the moment of the couple formed by the forces exerted on the two wrenches.
- Based on the results of (a) and (b), explain why two wrenches are used.



Solution: The position vector of the force on the right wrench is $\mathbf{r}_R = 6\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}$. The magnitude of the moment about the x axis is

$$|\mathbf{M}_R| = \mathbf{e}_x \cdot (\mathbf{r}_R \times \mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 6 & -5 & -3 \\ 0 & 0 & 10 \end{vmatrix} = -50 \text{ in lb}$$

- The moment about the x axis is

$$\mathbf{M}_R = |\mathbf{M}_R|\mathbf{e}_x = -50\mathbf{i} \text{ (in lb)}.$$

- The moment of the couple is

$$\mathbf{M}_C = (\mathbf{r}_R - \mathbf{r}_L) \times \mathbf{F}_R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -6 \\ 0 & 0 & 10 \end{vmatrix} = -20\mathbf{j} \text{ in lb}$$

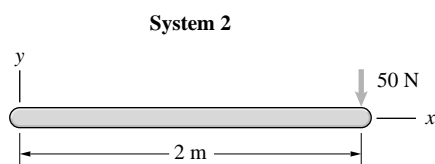
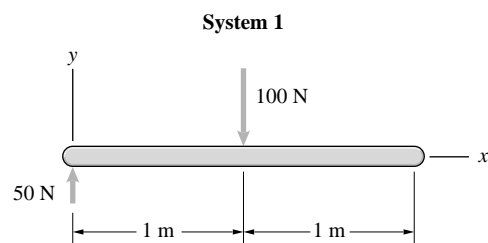
- The objective is to apply a moment to the elbow relative to connecting pipe, and zero resultant moment to the pipe itself. A resultant moment about the x axis will affect the joint at the origin. However the use of two wrenches results in a net zero moment about the x axis the moment is absorbed at the juncture of the elbow and the pipe. This is demonstrated by calculating the moment about the x axis due to the left wrench:

$$|\mathbf{M}_L| = \mathbf{e}_x \cdot (\mathbf{r}_L \times \mathbf{F}_L) = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -5 & 3 \\ 0 & 0 & -10 \end{vmatrix} = 50 \text{ in lb}$$

from which $\mathbf{M}_{XL} = 50\mathbf{i}$ in lb, which is opposite in direction and equal in magnitude to the moment exerted on the x axis by the right wrench. The left wrench force is applied 2 in nearer the origin than the right wrench force, hence the moment must be absorbed by the space between, where it is wanted.

Problem 4.128 Two systems of forces act on the beam. Are they equivalent?

Strategy: Check the two conditions for equivalence. The sums of the forces must be equal, and the sums of the moments about an arbitrary point must be equal.



Solution: The strategy is to check the two conditions for equivalence: (a) the sums of the forces must be equal and (b) the sums of the moments about an arbitrary point must be equal. The sums of the forces of the two systems: $\sum \mathbf{F}_x = 0$, (both systems) and

$$\sum \mathbf{F}_{y1} = -100\mathbf{j} + 50\mathbf{j} = -50\mathbf{j} \text{ (N)}$$

$$\sum \mathbf{F}_{y2} = -50\mathbf{j} \text{ (N)}.$$

The sums of the forces are equal. The sums of the moments about the left end are:

$$\sum \mathbf{M}_1 = -(1)(100)\mathbf{k} = -100\mathbf{k} \text{ (N-m)}$$

$$\sum \mathbf{M}_2 = -(2)(50)\mathbf{k} = -100\mathbf{k} \text{ (N-m)}.$$

The sums of the moments about the left end are equal. Choose any point P at the same distance $\mathbf{r} = x\mathbf{i}$ from the left end on each beam. The sums of the moments about the point P are

$$\sum \mathbf{M}_1 = (-50x + 100(x - 1))\mathbf{k} = (50x - 100)\mathbf{k} \text{ (N-m)}$$

$$\sum \mathbf{M}_2 = (-50(2 - x))\mathbf{k} = (50x - 100)\mathbf{k} \text{ (N-m)}.$$

Thus the sums of the moments about any point on the beam are equal for the two sets of forces; the systems are equivalent. Yes

Problem 4.129 Two systems of forces and moments act on the beam. Are they equivalent?

Solution: The sums of the forces are:

$$\sum F_x = 0 \text{ (both systems)}$$

$$\sum F_{y1} = 10\mathbf{j} - 20\mathbf{j} = -10\mathbf{j} \text{ (lb)}$$

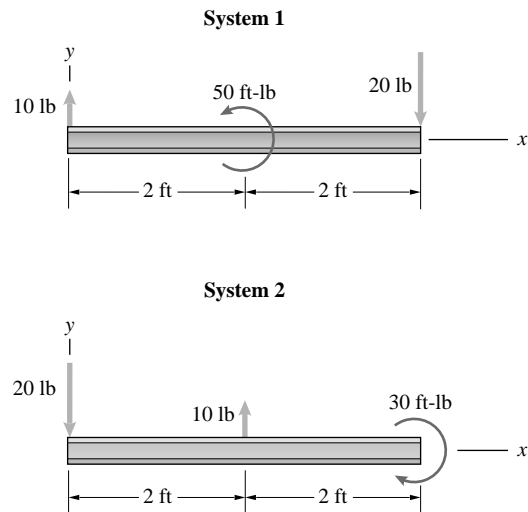
$$\sum F_{y2} = -20\mathbf{j} + 10\mathbf{j} = -10\mathbf{j} \text{ (lb)}$$

Thus the sums of the forces are equal. The sums of the moments about the left end are:

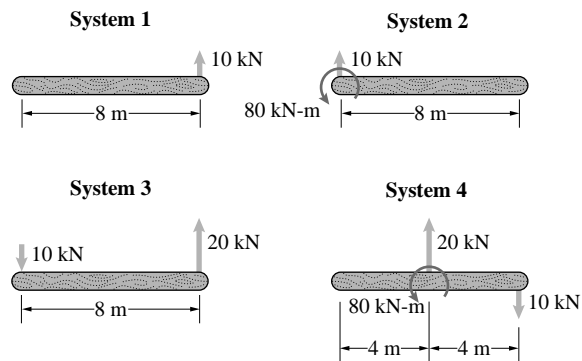
$$\sum \mathbf{M}_1 = (-20)(4)\mathbf{k} + 50\mathbf{k} = -30\mathbf{k} \text{ (ft lb)}$$

$$\sum \mathbf{M}_2 = (+10(2))\mathbf{k} - 30\mathbf{k} = -10\mathbf{k} \text{ (ft lb)}$$

The sums of the moments are not equal, hence the systems are not equivalent. No



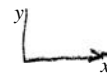
Problem 4.130 Four systems of forces and moments act on an 8-m beam. Which systems are equivalent?



Solution: For equivalence, the sum of the forces and the sum of the moments about some point (the left end will be used) must be the same.

	System 1	System 2	System 3	System 4
$\sum \mathbf{F} \text{ (kN)}$	$10\mathbf{j}$	$10\mathbf{j}$	$10\mathbf{j}$	$10\mathbf{j}$
$\sum \mathbf{M}_L \text{ (kN-m)}$	$80\mathbf{k}$	$80\mathbf{k}$	$160\mathbf{k}$	$80\mathbf{k}$

Systems 1, 2, and 4 are equivalent.



Problem 4.131 The four systems shown in Problem 4.130 can be made equivalent by adding a couple to one of the systems. Which system is it, and what couple must be added?

Solution: From the solution to 4.130, all systems have

$$\sum \mathbf{F} = 10\mathbf{j} \text{ kN}$$

and systems 1, 2, and 4 have

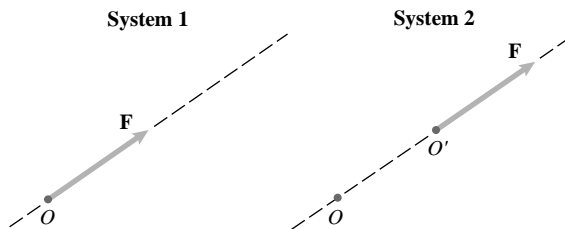
$$\sum \mathbf{M}_L = 80\mathbf{k} \text{ (kN-m)}$$

system 3 has

$$\sum \mathbf{M}_L = 160\mathbf{k} \text{ (kN-m)}.$$

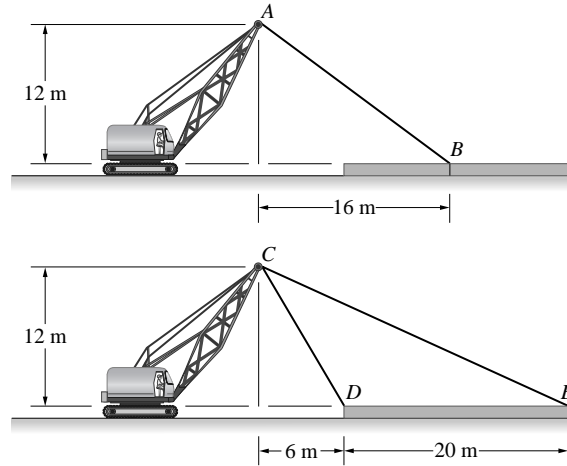
Thus, we need to add a couple $\mathbf{M} = -80\mathbf{k}$ (kN-m) to system 3 (clockwise moment).

Problem 4.132 System 1 is a force \mathbf{F} acting at a point O . System 2 is the force \mathbf{F} acting at a different point O' along the same line of action. Explain why these systems are equivalent. (This simple result is called the *principle of transmissibility*.)



Solution: The sum of forces is obviously equal for both systems. Let P be any point on the dashed line. The moment about P is the cross product of the distance from P to the line of action of a force times the force, that is, $\mathbf{M} = \mathbf{r}_{PL} \times \mathbf{F}$, where \mathbf{r}_{PL} is the distance from P to the line of action of \mathbf{F} . Since both systems have the same line of action, and the forces are equal, the systems are equivalent.

Problem 4.133 The vector sum of the forces exerted on the log by the cables is the same in the two cases. Show that the systems of forces exerted on the log are equivalent.



Solution: The angle formed by the single cable with the positive x axis is

$$\theta = 180^\circ - \tan^{-1} \left(\frac{12}{16} \right) = 143.13^\circ.$$

The single cable tension is

$$\begin{aligned} \mathbf{T}_1 &= |\mathbf{T}|(\mathbf{i} \cos 143.13^\circ + \mathbf{j} \sin 143.13^\circ) \\ &= |\mathbf{T}|(-0.8\mathbf{i} + 0.6\mathbf{j}). \end{aligned}$$

The position vector to the center of the log from the left end is $\mathbf{r}_c = 10\mathbf{i}$. The moment about the end of the log is

$$\mathbf{M} = \mathbf{r} \times \mathbf{T}_1 = |\mathbf{T}| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 0 & 0 \\ -0.8 & 0.6 & 0 \end{vmatrix} = |\mathbf{T}|(6)\mathbf{k} \text{ (N-m)}.$$

For the two cables, the angles relative to the positive x axis are

$$\theta_1 = 180^\circ - \tan^{-1} \left(\frac{12}{6} \right) = 116.56^\circ, \text{ and}$$

$$\theta_2 = 180^\circ - \tan^{-1} \left(\frac{12}{26} \right) = 155.22^\circ.$$

The two cable vectors are

$$\begin{aligned} \mathbf{T}_L &= |\mathbf{T}_L|(\mathbf{i} \cos 116.56^\circ + \mathbf{j} \sin 116.56^\circ) \\ &= |\mathbf{T}_L|(-0.4472\mathbf{i} + 0.8945\mathbf{j}), \end{aligned}$$

$$\begin{aligned} \mathbf{T}_R &= |\mathbf{T}_R|(\mathbf{i} \cos 155.22^\circ + \mathbf{j} \sin 155.22^\circ) \\ &= |\mathbf{T}_R|(-0.9079\mathbf{i} + 0.4191\mathbf{j}). \end{aligned}$$

Since the vector sum of the forces in the two systems is equal, two simultaneous equations are obtained:

$$0.4472|\mathbf{T}_L| + 0.9079|\mathbf{T}_R| = 0.8|\mathbf{T}_1|, \text{ and}$$

$$0.8945|\mathbf{T}_L| + 0.4191|\mathbf{T}_R| = 0.6|\mathbf{T}_1|$$

Solve:

$$|\mathbf{T}_L| = 0.3353|\mathbf{T}_1|, \text{ and}$$

$$|\mathbf{T}_R| = 0.7160|\mathbf{T}_1|.$$

The tension in the right hand cable is $\mathbf{T}_R = |\mathbf{T}_1|(0.7160)(-0.9079\mathbf{i} + 0.4191\mathbf{j}) = |\mathbf{T}_1|(-0.6500\mathbf{i} + 0.3000\mathbf{j})$. The position vector of the right end of the log is $\mathbf{r}_R = 20\mathbf{i}$ m relative to the left end. The moments about the left end of the log for the second system are

$$\mathbf{M}_2 = \mathbf{r}_R \times \mathbf{T}_R = |\mathbf{T}_1| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 20 & 0 & 0 \\ -0.6500 & 0.3000 & 0 \end{vmatrix} = |\mathbf{T}_1|(6)\mathbf{k} \text{ (N-m)}.$$

This is equal to the moment about the left end of the log for System 1, hence the systems are equivalent.

Problem 4.134 Systems 1 and 2 each consist of a couple. If they are equivalent, what is F ?

Solution: For couples, the sum of the forces vanish for both systems. For System 1, the two forces are located at $\mathbf{r}_{11} = 4\mathbf{i}$, and $\mathbf{r}_{12} = +5\mathbf{j}$. The forces are $\mathbf{F}_1 = 200(\mathbf{i} \cos 30^\circ + \mathbf{j} \sin 30^\circ) = 173.21\mathbf{i} + 100\mathbf{j}$. The moment due to the couple in System 1 is

$$\mathbf{M}_1 = (\mathbf{r}_{11} - \mathbf{r}_{12}) \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -5 & 0 \\ 173.21 & 100 & 0 \end{vmatrix} = 1266.05\mathbf{k} \text{ (N-m)}.$$

For System 2, the positions of the forces are $\mathbf{r}_{21} = 2\mathbf{i}$, and $\mathbf{r}_{22} = 5\mathbf{i} + 4\mathbf{j}$. The forces are

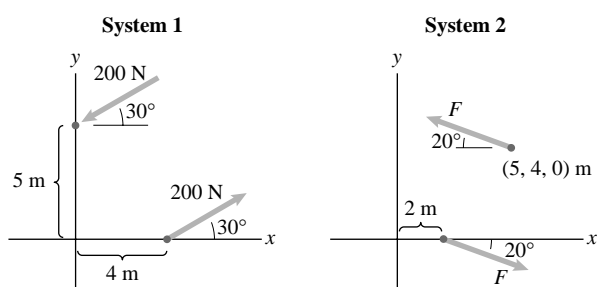
$$\mathbf{F}_2 = F(\mathbf{i} \cos(-20^\circ) + \mathbf{j} \sin(-20^\circ)) = F(0.9397\mathbf{i} - 0.3420\mathbf{j}).$$

The moment of the couple in System 2 is

$$\mathbf{M}_2 = (\mathbf{r}_{21} - \mathbf{r}_{22}) \times \mathbf{F}_2 = F \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -4 & 0 \\ 0.9397 & -0.3420 & 0 \end{vmatrix} = 4.7848F\mathbf{k},$$

from which, if the systems are to be equivalent,

$$F = \frac{1266}{4.7848} = 264.6 \text{ N}$$



Problem 4.135 Two equivalent systems of forces and moments act on the L-shaped bar. Determine the forces F_A and F_B and the couple M .

Solution: The sums of the forces for System 1 are

$$\sum F_X = 50, \text{ and}$$

$$\sum F_Y = -F_A + 60.$$

The sums of the forces for System 2 are

$$\sum F_X = F_B, \text{ and}$$

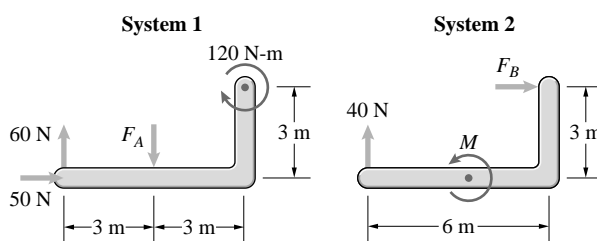
$$\sum F_Y = 40.$$

For equivalent systems: $F_B = 50 \text{ N}$, and $F_A = 60 - 40 = 20 \text{ N}$.

The sum of the moments about the left end for

System 1 is

$$\sum M_1 = -(3)F_A - 120 = -180 \text{ N-m}.$$



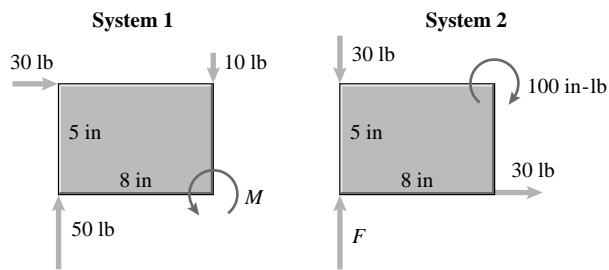
The sum of the moments about the left end for

System 2 is

$$\sum M_2 = -(3)F_B + M = -150 + M \text{ N-m}.$$

Equating the sums of the moments, $M = 150 - 180 = -30 \text{ N-m}$

Problem 4.136 Two equivalent systems of forces and moments act on the plate. Determine the force F and the couple M .



Solution: The sums of the forces for System 1 are

$$\sum F_X = 30 \text{ lb,}$$

$$\sum F_Y = 50 - 10 = 40 \text{ lb.}$$

The sums of the forces for System 2 are

$$\sum F_X = 30 \text{ lb,}$$

$$\sum F_Y = F - 30 \text{ lb.}$$

For equivalent forces, $F = 30 + 40 = 70 \text{ lb}$. The sum of the moments about the lower left corner for System 1 is

$$\sum M_1 = -(5)(30) - (8)(10) + M = -230 + M \text{ in lb.}$$

The sum of the moments about the lower left corner for System 2 is

$$\sum M_2 = -100 \text{ in lb.}$$

Equating the sum of moments, $M = 230 - 100 = 130 \text{ in lb}$

Problem 4.137 In Example 4.13, suppose that the 30-kN vertical force in system 1 is replaced by a 230-kN vertical force. Draw a sketch of the new system 1. If you represent system 1 by a single force \mathbf{F} as in system 3, at what position D on the x axis must the force be placed?

Solution: The first step is to represent system 1 by a single force \mathbf{F} acting at the origin and a couple M (system 2). The force \mathbf{F} must equal the sum of the forces in system 1:

$$(\Sigma \mathbf{F})_2 = (\Sigma \mathbf{F})_1$$

$$\mathbf{F} = (230 \text{ kN}) \mathbf{j} + (20\mathbf{i} + 20\mathbf{j}) \text{ kN}$$

$$\mathbf{F} = (20\mathbf{i} + 250\mathbf{j}) \text{ kN}$$

The moment about the origin in system 2 is M . Therefore M must equal the sum of the moments about the origin due to the forces and moments in system 1:

$$(\Sigma M)_2 = (\Sigma M)_1$$

$$M = (230 \text{ kN})(3 \text{ m}) + (20 \text{ kN})(5 \text{ m}) + (210 \text{ kN-m}) = 1000 \text{ kN-m}$$

The next step is to represent system 2 by system 3.

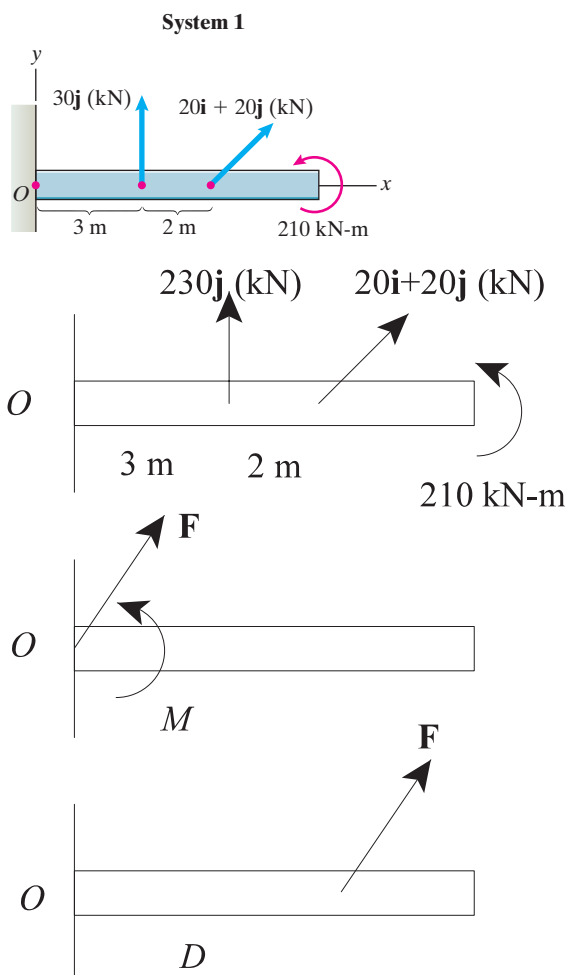
The sums of the forces in the two systems are equal. The sums of the moments about the origin must be equal. The \mathbf{j} component of \mathbf{F} is 250 kN, so

$$(\Sigma M)_3 = (\Sigma M)_2$$

$$(1000 \text{ kN-m}) = (250 \text{ kN})D$$

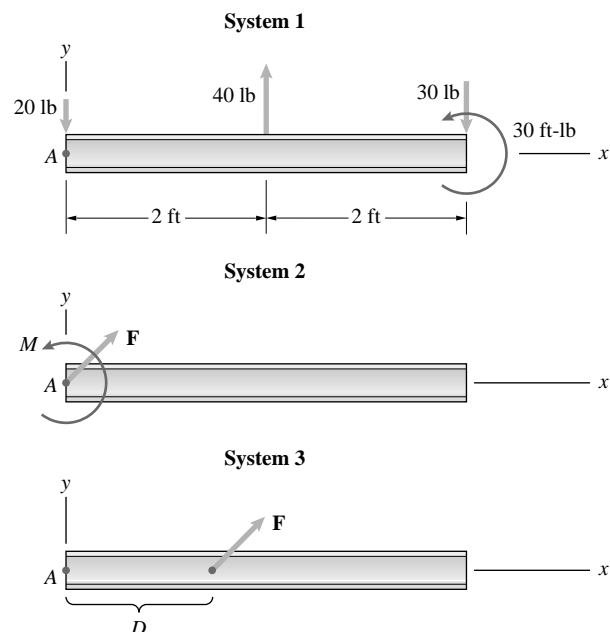
$$D = \frac{1000 \text{ kN-m}}{250 \text{ kN}} = 4 \text{ m}$$

$$\boxed{D = 4 \text{ m}}$$



Problem 4.138 Three forces and a couple are applied to a beam (system 1).

- If you represent system 1 by a force applied at A and a couple (system 2), what are \mathbf{F} and M ?
- If you represent system 1 by the force \mathbf{F} (system 3), what is the distance D ?



Solution: The sum of the forces in System 1 is

$$\sum \mathbf{F}_X = 0\mathbf{i},$$

$$\sum \mathbf{F}_Y = (-20 + 40 - 30)\mathbf{j} = -10\mathbf{j} \text{ lb.}$$

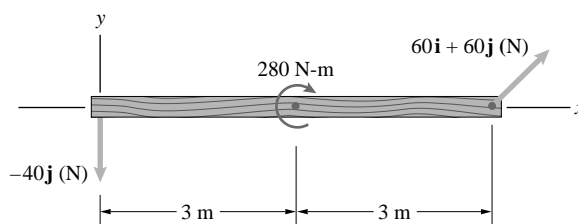
The sum of the moments about the left end for System 1 is

$$\sum \mathbf{M}_1 = (2(40) - 4(30) + 30)\mathbf{k} = -10\mathbf{k} \text{ ft lb.}$$

- For System 2, the force at A is $\mathbf{F} = -10\mathbf{j}$ lb. The moment at A is $\mathbf{M}_2 = -10\mathbf{k}$ ft lb.
- For System 3 the force at D is $\mathbf{F} = -10\mathbf{j}$ lb. The distance D is the ratio of the magnitude of the moment to the magnitude of the force, where the magnitudes are those in System 1:

$$D = \frac{10}{10} = 1 \text{ ft}$$

Problem 4.139 Represent the two forces and couple acting on the beam by a force \mathbf{F} . Determine \mathbf{F} and determine where its line of action intersects the x axis.



Solution: We first represent the system by an equivalent system consisting of a force \mathbf{F} at the origin and a couple M :

This system is equivalent if

$$\mathbf{F} = -40\mathbf{j} + 60\mathbf{i} + 60\mathbf{j}$$

$$= 60\mathbf{i} + 20\mathbf{j} \text{ (N),}$$

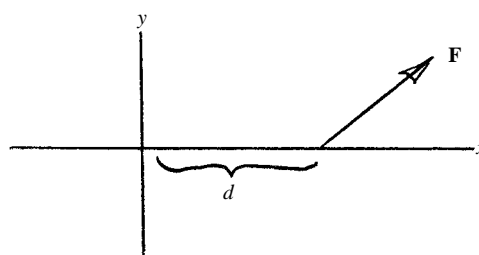
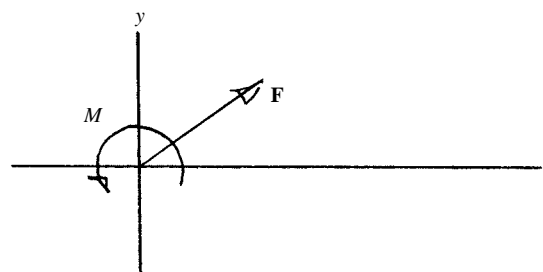
$$M = -280 + (6)(60)$$

$$= 80 \text{ N-m.}$$

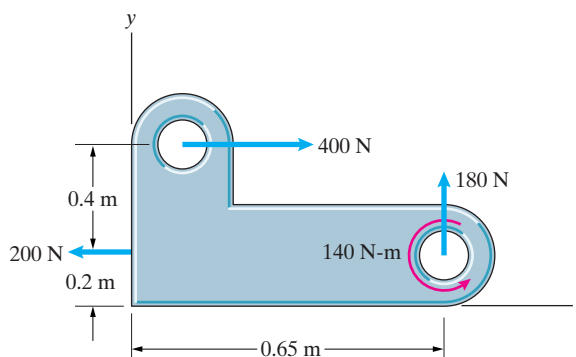
We then represent this system by an equivalent system consisting of \mathbf{F} alone:

For equivalence, $M = d(F_y)$, so

$$d = \frac{M}{F_y} = \frac{80}{20} = 4 \text{ m.}$$



Problem 4.140 The bracket is subjected to three forces and a couple. If you represent this system by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x axis?



Solution: We locate a single equivalent force along the x axis a distance d to the right of the origin. We must satisfy the following three equations:

$$\sum F_x = 400 \text{ N} - 200 \text{ N} = R_x$$

$$\sum F_y = 180 \text{ N} = R_y$$

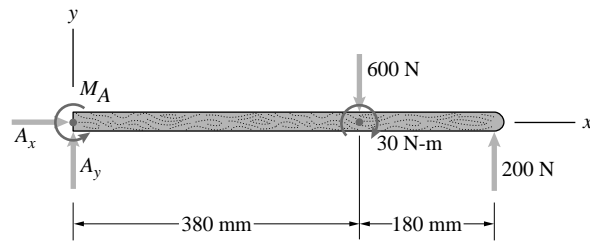
$$\begin{aligned} \sum M_O &= -(400 \text{ N})(0.6 \text{ m}) + (200 \text{ N})(0.2 \text{ m}) + (180 \text{ N})(0.65 \text{ m}) \\ &\quad + 140 \text{ Nm} = R_y d \end{aligned}$$

Solving we find

$$\boxed{R_x = 200 \text{ N}, R_y = 180 \text{ N}, d = 0.317 \text{ m}}$$

Problem 4.141 The vector sum of the forces acting on the beam is zero, and the sum of the moments about the left end of the beam is zero.

- Determine the forces A_x and A_y , and the couple M_A .
- Determine the sum of the moments about the right end of the beam.
- If you represent the 600-N force, the 200-N force, and the 30 N-m couple by a force \mathbf{F} acting at the left end of the beam and a couple M , what are \mathbf{F} and M ?



Solution: (a) The sum of the forces is

$$\sum \mathbf{F}_X = A_x \mathbf{i} = 0 \text{ and}$$

$$\sum \mathbf{F}_Y = (A_y - 600 + 200) \mathbf{j} = 0,$$

from which $A_y = 400$ N. The sum of the moments is

$$\sum \mathbf{M}_L = (M_A - 0.38(600) - 30 + 0.560(200)) \mathbf{k} = 0,$$

from which $M_A = 146$ N-m. (b) The sum of the moments about the right end of the beam is

$$\sum \mathbf{M}_R = 0.18(600) - 30 + 146 - 0.56(400) = 0.$$

(c) The sum of the forces for the new system is

$$\sum \mathbf{F}_Y = (A_y + F) \mathbf{j} = 0,$$

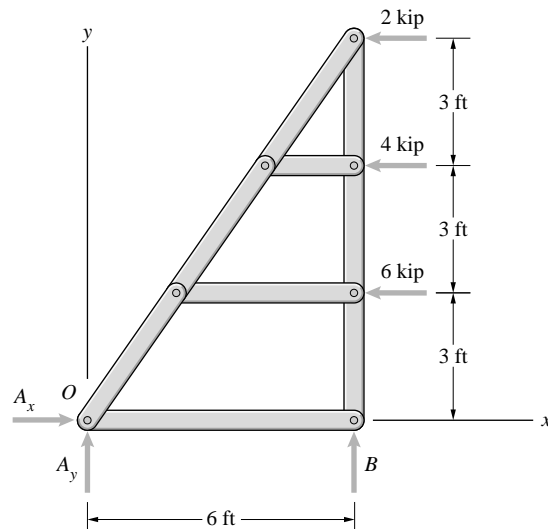
from $F = -A_y = -400$ N, or $\mathbf{F} = -400 \mathbf{j}$ N. The sum of the moments for the new system is

$$\sum \mathbf{M} = (M_A + M) = 0,$$

from which $M = -M_A = -146$ N-m

Problem 4.142 The vector sum of the forces acting on the truss is zero, and the sum of the moments about the origin O is zero.

- Determine the forces A_x , A_y , and B .
- If you represent the 2-kip, 4-kip, and 6-kip forces by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the y axis?
- If you replace the 2-kip, 4-kip, and 6-kip forces by the force you determined in (b), what are the vector sum of the forces acting on the truss and the sum of the moments about O ?



Solution: (a) The sum of the forces is

$$\sum \mathbf{F}_X = (A_x - 2 - 4 - 6) \mathbf{i} = 0,$$

from which $A_x = 12$ kip

$$\sum \mathbf{F}_Y = (A_y + B) \mathbf{j} = 0.$$

The sum of the moments about the origin is

$$\sum \mathbf{M}_O = (3)(6) + (6)(4) + (9)(2) + 6(B) = 0,$$

from which $B = -10 \mathbf{j}$ kip. (b) Substitute into the force balance equation to obtain $A_y = -B = 10$ kip. (b) The force in the new system will replace the 2, 4, and 6 kip forces, $\mathbf{F} = (-2 - 4 - 6) \mathbf{i} = -12 \mathbf{i}$ kip. The force must match the moment due to these forces: $FD = 3(6) + (6)(4) + (9)(2) = 60$ kip ft, from which $D = \frac{60}{12} = 5$ ft, or the action line intersects the y axis 5 ft above the origin. (c) The new system is equivalent to the old one, hence the sum of the forces vanish and the sum of the moments about O are zero.

Problem 4.143 The distributed force exerted on part of a building foundation by the soil is represented by five forces. If you represent them by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x axis?

Solution: The equivalent force must equal the sum of the forces exerted by the soil:

$$\mathbf{F} = (80 + 35 + 30 + 40 + 85)\mathbf{j} = 270\mathbf{j} \text{ kN}$$

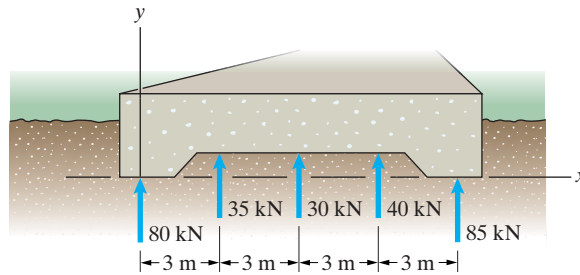
The sum of the moments about any point must be equal for the two systems. The sum of the moments are

$$\sum M = 3(35) + 6(30) + 9(40) + 12(85) = 1665 \text{ kN}\cdot\text{m}.$$

Equating the moments for the two systems $FD = 1665 \text{ kN}\cdot\text{m}$ from which

$$D = \frac{1665 \text{ kN}\cdot\text{m}}{270 \text{ kN}} = 6.167 \text{ m}.$$

Thus the action line intersects the x axis at a distance $D = 6.167 \text{ m}$ to the right of the origin.



Problem 4.144 At a particular instant, aerodynamic forces distributed over the airplane's surface exert the 88-kN and 16-kN vertical forces and the 22 kN-m counterclockwise couple shown. If you represent these forces and couple by a system consisting of a force \mathbf{F} acting at the center of mass G and a couple M , what are \mathbf{F} and M ?

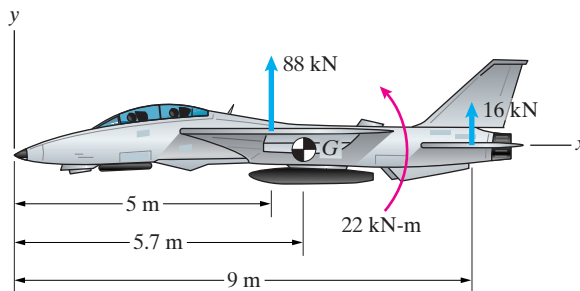
Solution:

$$\sum F_y = 88 \text{ kN} + 16 \text{ kN} = R_y$$

$$\sum M_G = -(88 \text{ kN})(0.7 \text{ m}) + (16 \text{ kN})(3.3 \text{ m}) + 22 \text{ kN}\cdot\text{m} = M$$

Solving we find

$$R_y = 104 \text{ kN}, M = 13.2 \text{ kN}\cdot\text{m}$$



Problem 4.145 If you represent the two forces and couple acting on the airplane in Problem 4.144 by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x axis?

Solution:

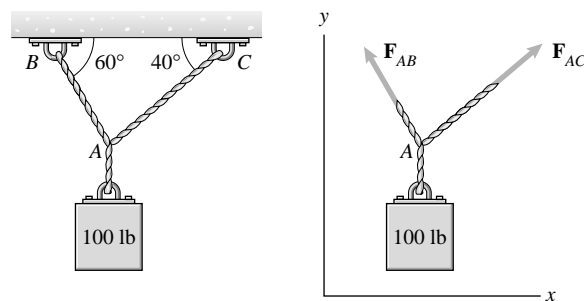
$$\sum F_y = 88 \text{ kN} + 16 \text{ kN} = R_y$$

$$\sum M_{\text{Origin}} = (88 \text{ kN})(5 \text{ m}) + (16 \text{ kN})(9 \text{ m}) + 22 \text{ kN}\cdot\text{m} = R_y x$$

Solving we find

$$\mathbf{F} = R_y \mathbf{j} = 104 \text{ kN} \mathbf{j}, x = 5.83 \text{ m}$$

Problem 4.146 The system is in equilibrium. If you represent the forces \mathbf{F}_{AB} and \mathbf{F}_{AC} by a force \mathbf{F} acting at A and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?

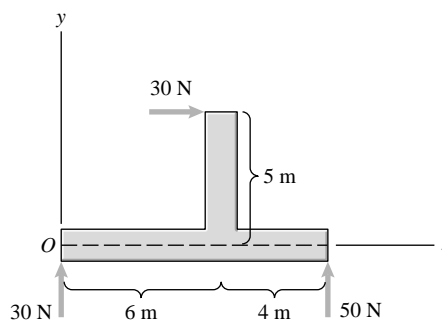


Solution: The sum of the forces acting at A is in opposition to the weight, or $\mathbf{F} = |\mathbf{W}|\mathbf{j} = 100\mathbf{j}$ lb.

The moment about point A is zero.

Problem 4.147 Three forces act on a beam.

- Represent the system by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} .
- Represent the system by a single force. Where does the line of action of the force intersect the x axis?



Solution: (a) The sum of the forces is

$$\sum \mathbf{F}_x = 30\mathbf{i} \text{ N, and}$$

$$\sum \mathbf{F}_y = (30 + 50)\mathbf{j} = 80\mathbf{j} \text{ N.}$$

The equivalent at O is $\mathbf{F} = 30\mathbf{i} + 80\mathbf{j}$ (N). The sum of the moments about O :

$$\sum M = (-5(30) + 10(50)) = 350 \text{ N}\cdot\text{m}$$

- The solution of Part (a) is the single force. The intersection is the moment divided by the y -component of force: $D = \frac{350}{80} = 4.375 \text{ m}$

Problem 4.148 The tension in cable AB is 400 N, and the tension in cable CD is 600 N.

- If you represent the forces exerted on the left post by the cables by a force \mathbf{F} acting at the origin O and a couple M , what are \mathbf{F} and M ?
- If you represent the forces exerted on the left post by the force \mathbf{F} alone, where does its line of action intersect the y axis?

Solution: From the right triangle, the angle between the positive x axis and the cable AB is

$$\theta = -\tan^{-1}\left(\frac{400}{800}\right) = -26.6^\circ.$$

The tension in AB is

$$\mathbf{T}_{AB} = 400(\mathbf{i}\cos(-26.6^\circ) + \mathbf{j}\sin(-26.6^\circ)) = 357.77\mathbf{i} - 178.89\mathbf{j} \text{ (N)}.$$

The angle between the positive x axis and the cable CD is

$$\alpha = -\tan^{-1}\left(\frac{300}{800}\right) = -20.6^\circ.$$

The tension in CD is

$$\mathbf{T}_{CD} = 600(\mathbf{i}\cos(-20.6^\circ) + \mathbf{j}\sin(-20.6^\circ)) = 561.8\mathbf{i} - 210.67\mathbf{j}.$$

The equivalent force acting at the origin O is the sum of the forces acting on the left post:

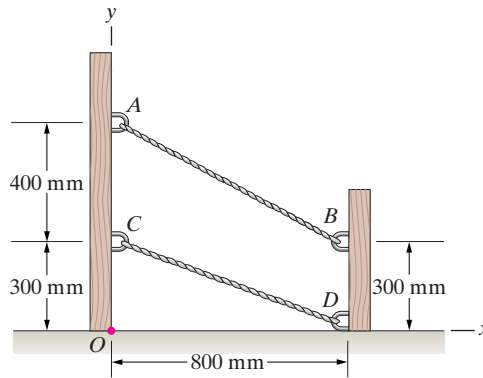
$$\begin{aligned}\mathbf{F} &= (357.77 + 561.8)\mathbf{i} + (-178.89 - 210.67)\mathbf{j} \\ &= 919.6\mathbf{i} - 389.6\mathbf{j} \text{ (N)}.\end{aligned}$$

The sum of the moments acting on the left post is the product of the moment arm and the x -component of the tensions:

$$\sum \mathbf{M} = -0.7(357.77)\mathbf{k} - 0.3(561.8)\mathbf{k} = -419\mathbf{k} \text{ N}\cdot\text{m}$$

Check: The position vectors at the point of application are $\mathbf{r}_{AB} = 0.7\mathbf{j}$, and $\mathbf{r}_{CD} = 0.3\mathbf{j}$. The sum of the moments is

$$\begin{aligned}\sum \mathbf{M} &= (\mathbf{r}_{AB} \times \mathbf{T}_{AB}) + (\mathbf{r}_{CD} \times \mathbf{T}_{CD}) \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.7 & 0 \\ 357.77 & -178.89 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.3 & 0 \\ 561.8 & -210.67 & 0 \end{vmatrix} \\ &= -0.7(357.77)\mathbf{k} - 0.3(561.8)\mathbf{k} = -419\mathbf{k}\end{aligned}$$



Check. (b) The equivalent single force retains the same scalar components, but must act at a point that duplicates the sum of the moments. The distance on the y axis is the ratio of the sum of the moments to the x -component of the equivalent force. Thus

$$D = \frac{419}{919.6} = 0.456 \text{ m}$$

Check: The moment is

$$\mathbf{M} = \mathbf{r}_F \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & D & 0 \\ 919.6 & -389.6 & 0 \end{vmatrix} = -919.6D\mathbf{k} = -419\mathbf{k},$$

$$\text{from which } D = \frac{419}{919.6} = 0.456 \text{ m, Check.}$$

Problem 4.149 Consider the system shown in Problem 4.148. The tension in each of the cables AB and CD is 400 N. If you represent the forces exerted on the right post by the cables by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the y axis?

Solution: From the solution of Problem 4.148, the tensions are

$$\mathbf{T}_{AB} = -400(\mathbf{i} \cos(-26.6^\circ) + \mathbf{j} \sin(-26.6^\circ)) = -357.77\mathbf{i} + 178.89\mathbf{j},$$

and

$$\mathbf{T}_{CD} = -400(\mathbf{i} \cos(-20.6^\circ) + \mathbf{j} \sin(-20.6^\circ)) = -374.42\mathbf{i} + 140.74\mathbf{j}.$$

The equivalent force is equal to the sum of these forces:

$$\begin{aligned}\mathbf{F} &= (-357.77 - 374.42)\mathbf{i} + (178.77 + 140.74)\mathbf{j} \\ &= -732.19\mathbf{i} + 319.5\mathbf{j} \text{ (N)}.\end{aligned}$$

The sum of the moments about O is

$$\sum \mathbf{M} = 0.3(357.77) + 0.8(140.74 + 178.89)\mathbf{k} = 363\mathbf{k} \text{ (N-m)}.$$

The intersection is $D = \frac{363}{732.19} = 0.496 \text{ m}$ on the positive y axis.

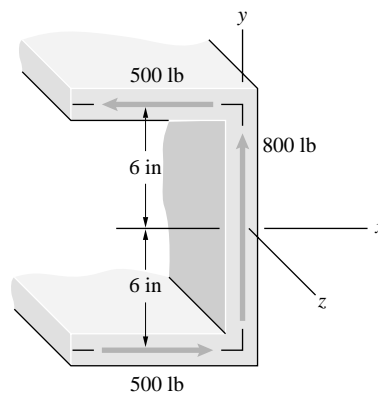
Problem 4.150 If you represent the three forces acting on the beam cross section by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x axis?

Solution: The sum of the forces is

$$\sum \mathbf{F}_x = (500 - 500)\mathbf{i} = 0.$$

$$\sum \mathbf{F}_y = 800\mathbf{j}.$$

Thus a force and a couple with moment $\mathbf{M} = 500\mathbf{k} \text{ ft lb}$ act on the cross section. The equivalent force is $\mathbf{F} = 800\mathbf{j}$ which acts at a positive x axis location of $D = \frac{500}{800} = 0.625 \text{ ft} = 7.5 \text{ in}$ to the right of the origin.



Problem 4.151 In Active Example 4.12, suppose that the force \mathbf{F}_B is changed to $\mathbf{F}_B = 20\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}$ (kN), and you want to represent system 1 by an equivalent system consisting of a force \mathbf{F} acting at the point P with coordinates $(4, 3, -2)$ m and a couple \mathbf{M} (system 2). Determine \mathbf{F} and \mathbf{M} .

Solution: From Active Example 4.12 we know that

$$\mathbf{F}_A = (-10\mathbf{i} + 10\mathbf{j} - 15\mathbf{k}) \text{ kN}$$

$$\mathbf{M}_C = (-90\mathbf{i} + 150\mathbf{j} + 60\mathbf{k}) \text{ kN}\cdot\text{m}$$

The force \mathbf{F} must equal the sum of the forces in system 1:

$$(\Sigma \mathbf{F})_2 = (\Sigma \mathbf{F})_1 :$$

$$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B = (10\mathbf{i} - 5\mathbf{j} + 15\mathbf{k}) \text{ kN}$$

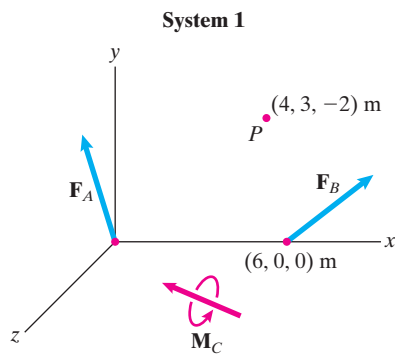
In system 2, the sum of the moments about P is \mathbf{M} . Therefore equivalence requires that \mathbf{M} be equal to the sum of the moments about point P due to the forces and moments in system 1:

$$(\Sigma \mathbf{M}_P)_2 = (\Sigma \mathbf{M}_P)_1 :$$

$$\mathbf{M} = \left[\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -3 & 2 \\ -10 & 10 & -15 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 2 \\ 20 & -15 & 30 \end{vmatrix} + (-90\mathbf{i} + 150\mathbf{j} + 60\mathbf{k}) \right] \text{ kN}\cdot\text{m}$$

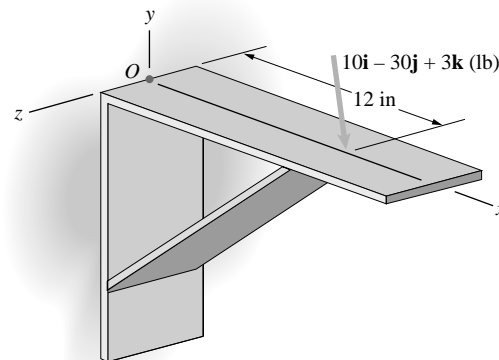
$$\mathbf{M} = (-125\mathbf{i} + 50\mathbf{j} + 20\mathbf{k}) \text{ kN}\cdot\text{m}$$

$$\text{Thus } \boxed{\mathbf{F} = (10\mathbf{i} - 5\mathbf{j} + 15\mathbf{k}) \text{ kN}, \mathbf{M} = (-125\mathbf{i} + 50\mathbf{j} + 20\mathbf{k}) \text{ kN}\cdot\text{m}}$$



Problem 4.152 The wall bracket is subjected to the force shown.

- Determine the moment exerted by the force about the z axis.
- Determine the moment exerted by the force about the y axis.
- If you represent the force by a force \mathbf{F} acting at O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?



Solution:

- The moment about the z axis is negative,

$$M_Z = -1(30) = -30 \text{ ft}\cdot\text{lb},$$

- The moment about the y axis is negative,

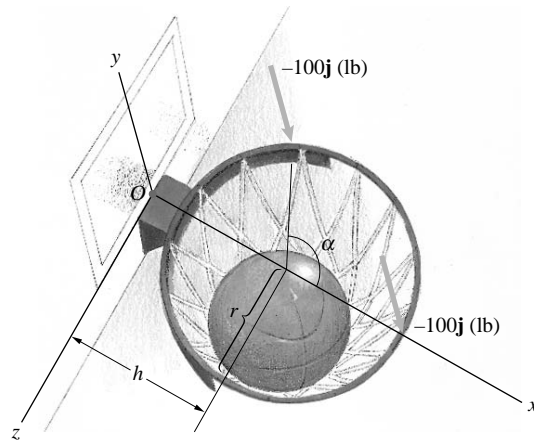
$$M_Y = -1(3) = -3 \text{ ft}\cdot\text{lb}$$

- The equivalent force at O must be equal to the force at $x = 12$ in, thus $\mathbf{F}_{EQ} = 10\mathbf{i} - 30\mathbf{j} + 3\mathbf{k}$ (lb)

The couple moment must equal the moment exerted by the force at $x = 12$ in. This moment is the product of the moment arm and the y - and z -components of the force: $\mathbf{M} = -1(30)\mathbf{k} - 1(3)\mathbf{j} = -3\mathbf{j} - 30\mathbf{k}$ (ft lb).

Problem 4.153 A basketball player executes a “slam dunk” shot, then hangs momentarily on the rim, exerting the two 100-lb forces shown. The dimensions are $h = 14\frac{1}{2}$ in, and $r = 9\frac{1}{2}$ in, and the angle $\alpha = 120^\circ$.

- If you represent the forces he exerts by a force \mathbf{F} acting at O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?
- The glass backboard will shatter if $|\mathbf{M}| > 4000$ in-lb. Does it break?



Solution: The equivalent force at the origin must equal the sum of the forces applied: $\mathbf{F}_{EQ} = -200\mathbf{j}$. The position vectors of the points of application of the forces are $\mathbf{r}_1 = (h + r)\mathbf{i}$, and $\mathbf{r}_2 = \mathbf{i}(h + r \cos \alpha) - \mathbf{k}r \sin \alpha$. The moments about the origin are

$$\mathbf{M} = (\mathbf{r}_1 \times \mathbf{F}_1) + (\mathbf{r}_2 \times \mathbf{F}_2) = (\mathbf{r}_1 + \mathbf{r}_2) \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2h + r(1 + \cos \alpha) & 0 & -r \sin \alpha \\ 0 & -100 & 0 \end{vmatrix}$$

$$= -100(r \sin \alpha)\mathbf{i} - 100(2h + r(1 + \cos \alpha))\mathbf{k}.$$

For the values of h , r , and α given, the moment is $\mathbf{M} = -822.72\mathbf{i} - 3375\mathbf{k}$ in-lb. This is the couple moment required. (b) The magnitude of the moment is $|\mathbf{M}| = \sqrt{822.72^2 + 3375^2} = 3473.8$ in-lb. The backboard does not break.

Problem 4.154 In Example 4.14, suppose that the 30-lb upward force in system 1 is changed to a 25-lb upward force. If you want to represent system 1 by a single force \mathbf{F} (system 2), where does the line of action of \mathbf{F} intersect the x - z plane?

Solution: The sum of the forces in system 2 must equal the sum of the forces in system 1:

$$(\Sigma \mathbf{F})_2 = (\Sigma \mathbf{F})_1$$

$$\mathbf{F} = (20 + 25 - 10)\mathbf{j} \text{ lb}$$

$$\mathbf{F} = 35\mathbf{j} \text{ lb}$$

The sum of the moments about a point in system 2 must equal the sum of the moments about the same point in system 1. We sum moments about the origin.

$$(\Sigma \mathbf{M})_2 = (\Sigma \mathbf{M})_1$$

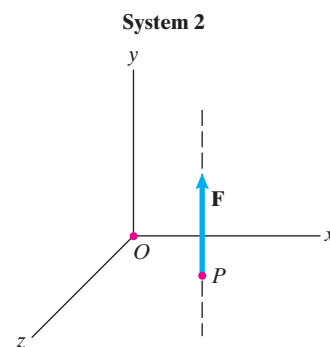
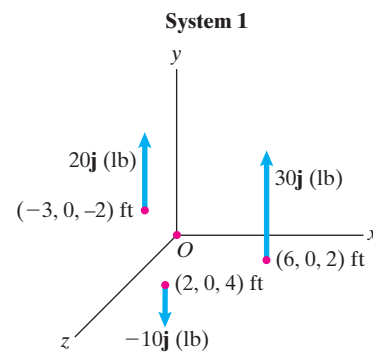
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ 0 & 35 & 0 \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 2 \\ 0 & 25 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 4 \\ 0 & -10 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 0 & -2 \\ 0 & 20 & 0 \end{vmatrix}$$

Expanding the determinants results in the equations

$$-35z = -50 + 40 + 40$$

$$35x = 150 - 20 - 60$$

$$\text{Solving yields } \boxed{x = 2.00 \text{ ft}, z = -0.857 \text{ ft}}$$



Problem 4.155 The normal forces exerted on the car's tires by the road are

$$\mathbf{N}_A = 5104\mathbf{j} \text{ (N)},$$

$$\mathbf{N}_B = 5027\mathbf{j} \text{ (N)},$$

$$\mathbf{N}_C = 3613\mathbf{j} \text{ (N)},$$

$$\mathbf{N}_D = 3559\mathbf{j} \text{ (N)}.$$

If you represent these forces by a single equivalent force \mathbf{N} , what is \mathbf{N} , and where does its line of action intersect the x - z plane?

Solution: We must satisfy the following three equations

$$\sum F_y : 5104 \text{ N} + 5027 \text{ N} + 3613 \text{ N} + 3559 \text{ N} = R_y$$

$$\sum M_x : (5104 \text{ N} + 3613 \text{ N})(0.8 \text{ m})$$

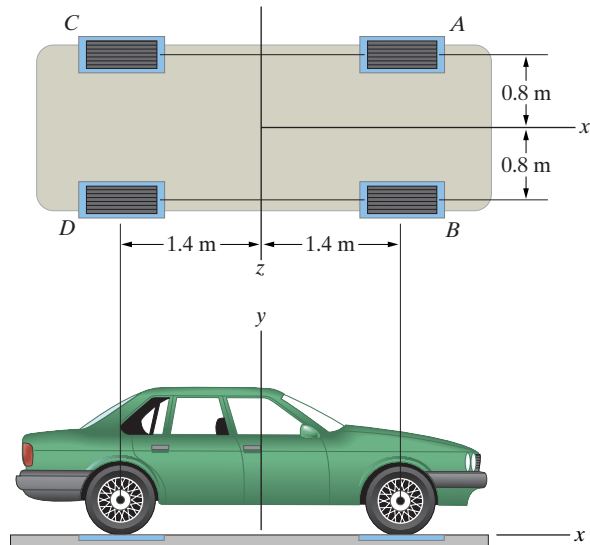
$$- (5027 \text{ N} + 3559 \text{ N})(0.8 \text{ m}) = -R_y z$$

$$\sum M_z : (5104 \text{ N} + 5027 \text{ N})(1.4 \text{ m})$$

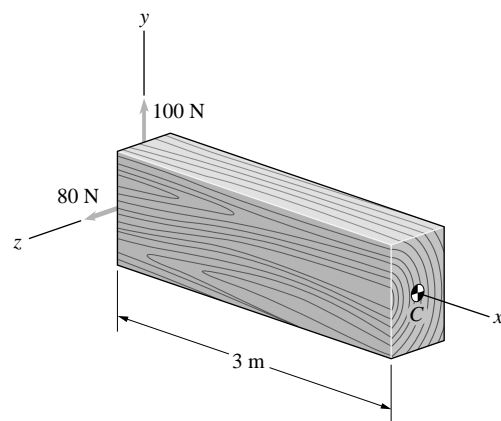
$$- (3613 \text{ N} + 3559 \text{ N})(1.4 \text{ m}) = R_y x$$

Solving we find

$$R_y = 17303 \text{ N}, \quad x = 0.239 \text{ m}, \quad z = -0.00606 \text{ m}$$



Problem 4.156 Two forces act on the beam. If you represent them by a force \mathbf{F} acting at C and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?



Solution: The equivalent force must equal the sum of forces: $\mathbf{F} = 100\mathbf{j} + 80\mathbf{k}$. The equivalent couple is equal to the moment about C :

$$\sum \mathbf{M} = (3)(80)\mathbf{j} - (3)(100)\mathbf{k} = 240\mathbf{j} - 300\mathbf{k}$$

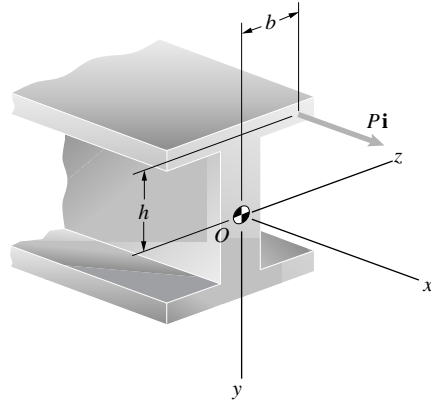
Problem 4.157 An axial force of magnitude P acts on the beam. If you represent it by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?

Solution: The equivalent force at the origin is equal to the applied force $\mathbf{F} = P\mathbf{i}$. The position vector of the applied force is $\mathbf{r} = -h\mathbf{j} + b\mathbf{k}$. The moment is

$$\mathbf{M} = (\mathbf{r} \times \mathbf{P}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -h & +b \\ P & 0 & 0 \end{vmatrix} = bP\mathbf{j} + hP\mathbf{k}.$$

This is the couple at the origin.

(Note that in the sketch the axis system has been rotated 180 about the x axis; so that up is negative and right is positive for y and z .)



Problem 4.158 The brace is being used to remove a screw.

- If you represent the forces acting on the brace by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?
- If you represent the forces acting on the brace by a force \mathbf{F}' acting at a point P with coordinates (x_P, y_P, z_P) and a couple \mathbf{M}' , what are \mathbf{F}' and \mathbf{M}' ?

Solution: (a) Equivalent force at the origin O has the same value as the sum of forces,

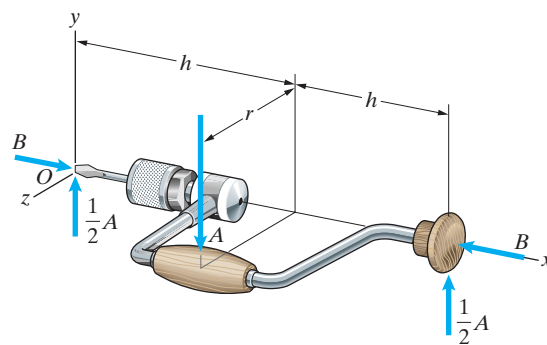
$$\sum \mathbf{F}_X = (B - B)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_Y = (-A + \frac{1}{2}A + \frac{1}{2}A)\mathbf{j} = 0,$$

thus $\mathbf{F} = 0$. The equivalent couple moment has the same value as the moment exerted on the brace by the forces,

$$\sum \mathbf{M}_O = (rA)\mathbf{i}.$$

Thus the couple at O has the moment $\mathbf{M} = rA\mathbf{i}$. (b) The equivalent force at (x_P, y_P, z_P) has the same value as the sum of forces on the brace, and the equivalent couple at (x_P, y_P, z_P) has the same moment as the moment exerted on the brace by the forces: $\mathbf{F} = 0$, $\mathbf{M} = rA\mathbf{i}$.



Problem 4.159 Two forces and a couple act on the cube. If you represent them by a force \mathbf{F} acting at point P and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?

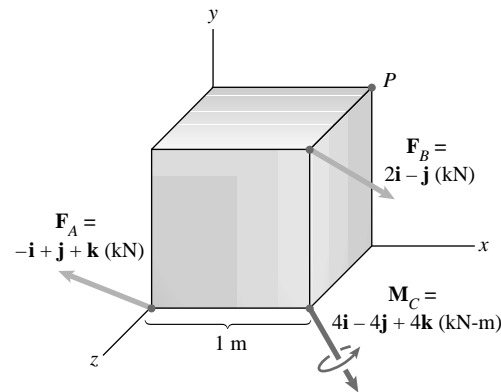
Solution: The equivalent force at P has the value of the sum of forces,

$$\sum \mathbf{F} = (2 - 1)\mathbf{i} + (1 - 1)\mathbf{j} + \mathbf{k}, \mathbf{F}_P = \mathbf{i} + \mathbf{k} \text{ (kN)}.$$

The equivalent couple at P has the moment exerted by the forces and moment about P . The position vectors of the forces relative to P are:

$\mathbf{r}_A = -\mathbf{i} - \mathbf{j} + \mathbf{k}$, and $\mathbf{r}_B = +\mathbf{k}$. The moment of the couple:

$$\begin{aligned} \sum \mathbf{M} &= (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_B \times \mathbf{F}_B) + \mathbf{M}_C \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 2 & -1 & 0 \end{vmatrix} + \mathbf{M}_C \\ &= 3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ (kN-m)}. \end{aligned}$$



Problem 4.160 The two shafts are subjected to the torques (couples) shown.

- If you represent the two couples by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?
- What is the magnitude of the total moment exerted by the two couples?

Solution: The equivalent force at the origin is zero, $\mathbf{F} = 0$ since there is no resultant force on the system. Represent the couples of 4 kN-m and 6 kN-m magnitudes by the vectors \mathbf{M}_1 and \mathbf{M}_2 . The couple at the origin must equal the sum:

$$\sum \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2.$$

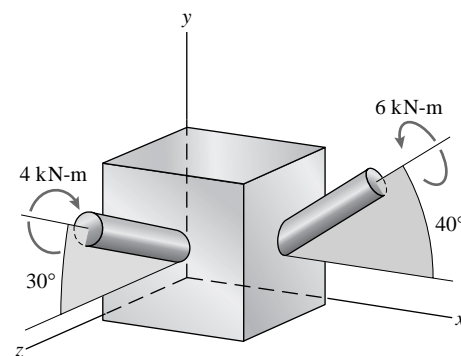
The sense of \mathbf{M}_1 is (see sketch) negative with respect to both y and z , and the sense of \mathbf{M}_2 is positive with respect to both x and y .

$$\mathbf{M}_1 = 4(-\mathbf{j} \sin 30^\circ - \mathbf{k} \cos 30^\circ) = -2\mathbf{j} - 3.464\mathbf{k},$$

$$\mathbf{M}_2 = 6(\mathbf{i} \cos 40^\circ + \mathbf{j} \sin 40^\circ) = 4.5963\mathbf{i} + 3.8567\mathbf{j}.$$

Thus the couple at the origin is $\mathbf{M}_O = 4.6\mathbf{i} + 1.86\mathbf{j} - 3.46\mathbf{k}$ (kN-m)

(b) The magnitude of the total moment exerted by the two couples is $|\mathbf{M}_O| = \sqrt{4.6^2 + 1.86^2 + 3.46^2} = 6.05$ (kN-m)



Problem 4.161 The two systems of forces and moments acting on the bar are equivalent. If

$$\mathbf{F}_A = 30\mathbf{i} + 30\mathbf{j} - 20\mathbf{k} \text{ (kN)},$$

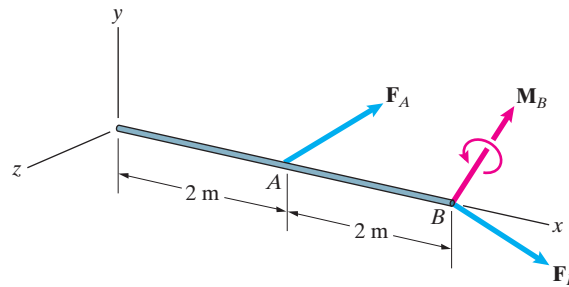
$$\mathbf{F}_B = 40\mathbf{i} - 20\mathbf{j} + 25\mathbf{k} \text{ (kN)},$$

$$\mathbf{M}_B = 10\mathbf{i} + 40\mathbf{j} - 10\mathbf{k} \text{ (kN-m)},$$

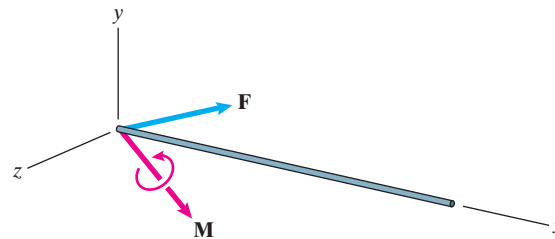
what are \mathbf{F} and \mathbf{M} ?

Solution:

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_A + \mathbf{F}_B = (70\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}) \text{ kN} \\ \mathbf{M} &= (2 \text{ m}) \times \mathbf{F}_A + (4 \text{ m}) \times \mathbf{F}_B + \mathbf{M}_B \\ &= (10\mathbf{i} - 20\mathbf{j} - 30\mathbf{k}) \text{ kNm}\end{aligned}$$



System 1



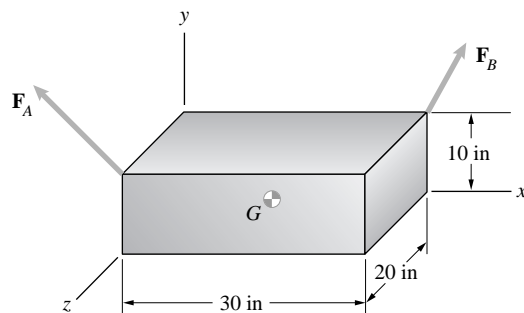
System 2

Problem 4.162 Point G is at the center of the block. The forces are

$$\mathbf{F}_A = -20\mathbf{i} + 10\mathbf{j} + 20\mathbf{k} \text{ (lb)},$$

$$\mathbf{F}_B = 10\mathbf{j} - 10\mathbf{k} \text{ (lb)}.$$

If you represent the two forces by a force \mathbf{F} acting at G and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?



Solution: The equivalent force is the sum of the forces:

$$\begin{aligned}\sum \mathbf{F} &= (-20\mathbf{i} + (10 + 10)\mathbf{j} + (20 - 10)\mathbf{k}) \\ &= -20\mathbf{i} + 20\mathbf{j} + 10\mathbf{k} \text{ (lb)}.\end{aligned}$$

The equivalent couple is the sum of the moments about G . The position vectors are:

$$\mathbf{r}_A = -15\mathbf{i} + 5\mathbf{j} + 10\mathbf{k} \text{ (in)},$$

$$\mathbf{r}_B = 15\mathbf{i} + 5\mathbf{j} - 10\mathbf{k}.$$

The sum of the moments:

$$\begin{aligned}\sum \mathbf{M}_G &= (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_B \times \mathbf{F}_B) \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -15 & 5 & 10 \\ -20 & 10 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & 5 & -10 \\ 0 & 10 & -10 \end{vmatrix} \\ &= 50\mathbf{i} + 250\mathbf{j} + 100\mathbf{k} \text{ (in lb)}\end{aligned}$$

Problem 4.163 The engine above the airplane's fuselage exerts a thrust $T_0 = 16$ kip, and each of the engines under the wings exerts a thrust $T_U = 12$ kip. The dimensions are $h = 8$ ft, $c = 12$ ft, and $b = 16$ ft. If you represent the three thrust forces by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?

Solution: The equivalent thrust at the point G is equal to the sum of the thrusts:

$$\sum T = 16 + 12 + 12 = 40 \text{ kip}$$

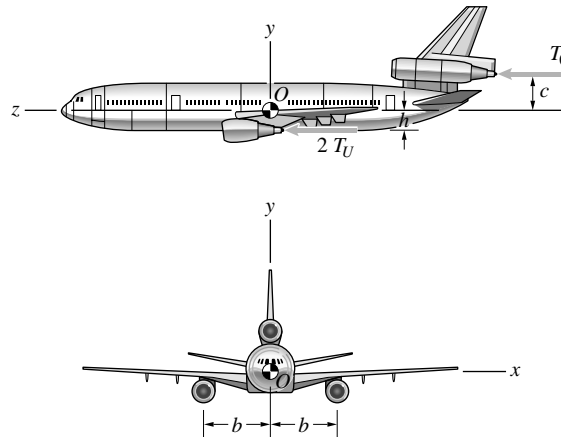
The sum of the moments about the point G is

$$\begin{aligned} \sum \mathbf{M} &= (\mathbf{r}_{1U} \times \mathbf{T}_U) + (\mathbf{r}_{2U} \times \mathbf{T}_U) + (\mathbf{r}_O \times \mathbf{T}_O) \\ &= (\mathbf{r}_{1U} + \mathbf{r}_{2U}) \times \mathbf{T}_U + (\mathbf{r}_O \times \mathbf{T}_O). \end{aligned}$$

The position vectors are $\mathbf{r}_{1U} = +b\mathbf{i} - h\mathbf{j}$, $\mathbf{r}_{2U} = -b\mathbf{i} - h\mathbf{j}$, and $\mathbf{r}_O = +c\mathbf{j}$. For $h = 8$ ft, $c = 12$ ft, and $b = 16$ ft, the sum of the moments is

$$\sum \mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -16 & 0 \\ 0 & 0 & 12 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 12 & 0 \\ 0 & 0 & 16 \end{vmatrix} = (-192 + 192)\mathbf{i} = 0.$$

Thus the equivalent couple is $\mathbf{M} = 0$



Problem 4.164 Consider the airplane described in Problem 4.163 and suppose that the engine under the wing to the pilot's right loses thrust.

- If you represent the two remaining thrust forces by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?
- If you represent the two remaining thrust forces by the force \mathbf{F} alone, where does its line of action intersect the x - y plane?

Solution: The sum of the forces is now

$$\sum \mathbf{F} = 12 + 16 = 28\mathbf{k} \text{ (kip)}.$$

The sum of the moments is now:

$$\sum \mathbf{M} = (\mathbf{r}_{2U} \times \mathbf{T}_U) + (\mathbf{r}_O \times \mathbf{T}_O).$$

For $h = 8$ ft, $c = 12$ ft, and $b = 16$ ft, using the position vectors for the engines given in Problem 4.163, the equivalent couple is

$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 16 & -8 & 0 \\ 0 & 0 & 12 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 12 & 0 \\ 0 & 0 & 16 \end{vmatrix} = 96\mathbf{i} - 192\mathbf{j} \text{ (ft kip)}$$

(b) The moment of the single force is

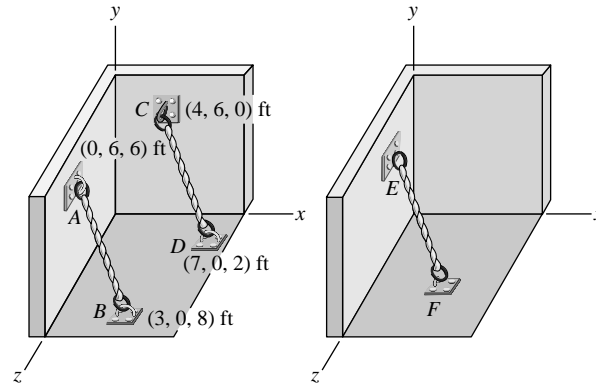
$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ 0 & 0 & 28 \end{vmatrix} = 28y\mathbf{i} - 28x\mathbf{j} = 96\mathbf{i} - 192\mathbf{j}.$$

From which

$$x = \frac{192}{28} = 6.86 \text{ ft, and } y = \frac{96}{28} = 3.43 \text{ ft.}$$

As to be expected, z can have any value, corresponding to any point on the line of action. Arbitrarily choose $z = 0$, so that the coordinates of the point of action are (6.86, 3.43, 0).

Problem 4.165 The tension in cable AB is 100 lb, and the tension in cable CD is 60 lb. Suppose that you want to replace these two cables by a single cable EF so that the force exerted on the wall at E is equivalent to the two forces exerted by cables AB and CD on the walls at A and C . What is the tension in cable EF , and what are the coordinates of points E and F ?



Solution: The position vectors of the points A , B , C , and D are

$$\mathbf{r}_A = 0\mathbf{i} + 6\mathbf{j} + 6\mathbf{k},$$

$$\mathbf{r}_B = 3\mathbf{i} + 0\mathbf{j} + 8\mathbf{k},$$

$$\mathbf{r}_C = 4\mathbf{i} + 6\mathbf{j} + 0\mathbf{k}, \text{ and}$$

$$\mathbf{r}_D = 7\mathbf{i} + 0\mathbf{j} + 2\mathbf{k}.$$

The unit vectors parallel to the cables are obtained as follows:

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k},$$

$$|\mathbf{r}_{AB}| = \sqrt{3^2 + 6^2 + 2^2} = 7,$$

from which

$$\mathbf{e}_{AB} = 0.4286\mathbf{i} - 0.8571\mathbf{j} + 0.2857\mathbf{k}.$$

$$\mathbf{r}_{CD} = \mathbf{r}_D - \mathbf{r}_C = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k},$$

$$|\mathbf{r}_{CD}| = \sqrt{3^2 + 6^2 + 2^2} = 7,$$

from which

$$\mathbf{e}_{CD} = 0.4286\mathbf{i} - 0.8571\mathbf{j} + 0.2857\mathbf{k}.$$

Since $\mathbf{e}_{AB} = \mathbf{e}_{CD}$, the cables are parallel. To duplicate the force, the single cable EF must have the same unit vector.

The force on the wall at point A is

$$\mathbf{F}_A = 100\mathbf{e}_{AB} = 42.86\mathbf{i} - 85.71\mathbf{j} + 28.57\mathbf{k} \text{ (lb)}.$$

The force on the wall at point C is

$$\mathbf{F}_C = 60\mathbf{e}_{CD} = 25.72\mathbf{i} - 51.43\mathbf{j} + 17.14\mathbf{k} \text{ (lb)}.$$

The total force is

$$\mathbf{F}_{EF} = 68.58\mathbf{i} - 137.14\mathbf{j} + 45.71\mathbf{k} \text{ (lb)},$$

$$|\mathbf{F}_{EF}| = 160 \text{ lb}.$$

For the systems to be equivalent, the moments about the origin must be the same. The moments about the origin are

$$\sum \mathbf{M}_O = (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_C \times \mathbf{F}_C)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 6 \\ 42.86 & -85.71 & 28.57 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & 0 \\ 25.72 & -51.43 & 17.14 \end{vmatrix}$$

$$= 788.57\mathbf{i} + 188.57\mathbf{j} - 617.14\mathbf{k}.$$

This result is used to establish the coordinates of the point E . For the one cable system, the end E is located at $x = 0$. The moment is

$$\mathbf{M}_1 = \mathbf{r} \times \mathbf{F}_{EF} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 68.58 & -137.14 & 45.71 \end{vmatrix}$$

$$= (45.71y + 137.14z)\mathbf{i} + (68.58z)\mathbf{j} - (68.58y)\mathbf{k}$$

$$= 788.57\mathbf{i} + 188.57\mathbf{j} - 617.14\mathbf{k},$$

from above. From which

$$y = \frac{617.14}{68.58} = 8.999 \dots = 9 \text{ ft}$$

$$z = \frac{188.57}{68.58} = 2.75 \text{ ft}.$$

Thus the coordinates of point E are $E(0, 9, 2.75)$ ft. The coordinates of the point F are found as follows: Let L be the length of cable EF . Thus, from the definition of the unit vector, $y_F - y_E = Le_y$ with the condition that $y_F = 0$, $L = \frac{9}{0.8571} = 10.5$ ft. The other coordinates are $x_F - x_E = Le_x$, from which $x_F = 0 + 10.5(0.4286) = 4.5$ ft $z_F - z_E = Le_z$, from which $z_F = 2.75 + 10.5(0.2857) = 5.75$ ft. The coordinates of F are $F(4.5, 0, 5.75)$ ft.

Problem 4.166 The distance $s = 4$ m. If you represent the force and the 200-N-m couple by a force \mathbf{F} acting at origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?

Solution: The equivalent force at the origin is

$$\mathbf{F} = 100\mathbf{i} + 20\mathbf{j} - 20\mathbf{k}.$$

The strategy is to establish the position vector of the action point of the force relative to the origin O for the purpose of determining the moment exerted by the force about the origin. The position of the top of the bar is

$\mathbf{r}_T = 2\mathbf{i} + 6\mathbf{j} + 0\mathbf{k}$. The vector parallel to the bar, pointing toward the base, is $\mathbf{r}_{TB} = 2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$, with a magnitude of $|\mathbf{r}_{TB}| = 7$. The unit vector parallel to the bar is

$$\mathbf{e}_{TB} = 0.2857\mathbf{i} - 0.8571\mathbf{j} + 0.4286\mathbf{k}.$$

The vector from the top of the bar to the action point of the force is

$$\mathbf{r}_{TF} = s\mathbf{e}_{TB} = 4\mathbf{e}_{TB} = 1.1429\mathbf{i} - 3.4286\mathbf{j} + 1.7143\mathbf{k}.$$

The position vector of the action point from the origin is

$$\mathbf{r}_F = \mathbf{r}_T + \mathbf{r}_{TF} = 3.1429\mathbf{i} + 2.5714\mathbf{j} + 1.7143\mathbf{k}.$$

The moment of the force about the origin is

$$\begin{aligned} \mathbf{M}_F = \mathbf{r} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.1429 & 2.5714 & 1.7143 \\ 100 & 20 & -20 \end{vmatrix} \\ &= -85.71\mathbf{i} + 234.20\mathbf{j} - 194.3\mathbf{k}. \end{aligned}$$

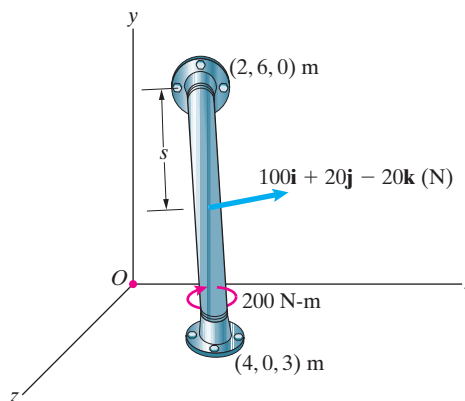
The couple is obtained from the unit vector and the magnitude. The sense of the moment is directed positively toward the top of the bar.

$$\mathbf{M}_C = -200\mathbf{e}_{TB} = -57.14\mathbf{i} + 171.42\mathbf{j} - 85.72\mathbf{k}.$$

The sum of the moments is

$$\mathbf{M} = \mathbf{M}_F + \mathbf{M}_C = -142.86\mathbf{i} + 405.72\mathbf{j} - 280\mathbf{k}.$$

This is the moment of the equivalent couple at the origin.



Problem 4.167 The force \mathbf{F} and couple \mathbf{M} in system 1 are

$$\mathbf{F} = 12\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \text{ (lb)},$$

$$\mathbf{M} = 4\mathbf{i} + 7\mathbf{j} + 4\mathbf{k} \text{ (ft-lb)}.$$

Suppose you want to represent system 1 by a wrench (system 2). Determine the couple \mathbf{M}_p and the coordinates x and z where the line of action of the force intersects the x - z plane.

Solution: The component of \mathbf{M} that is parallel to \mathbf{F} is found as follows: The unit vector parallel to \mathbf{F} is

$$\mathbf{e}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = 0.9231\mathbf{i} + 0.3077\mathbf{j} - 0.2308\mathbf{k}.$$

The component of \mathbf{M} parallel to \mathbf{F} is

$$\mathbf{M}_p = (\mathbf{e}_F \cdot \mathbf{M})\mathbf{e}_F = 4.5444\mathbf{i} + 1.5148\mathbf{j} - 1.1361\mathbf{k} \text{ (ft-lb)}.$$

The component of \mathbf{M} normal to \mathbf{F} is

$$\mathbf{M}_N = \mathbf{M} - \mathbf{M}_p = -0.5444\mathbf{i} + 5.4858\mathbf{j} + 5.1361\mathbf{k} \text{ (ft-lb)}.$$

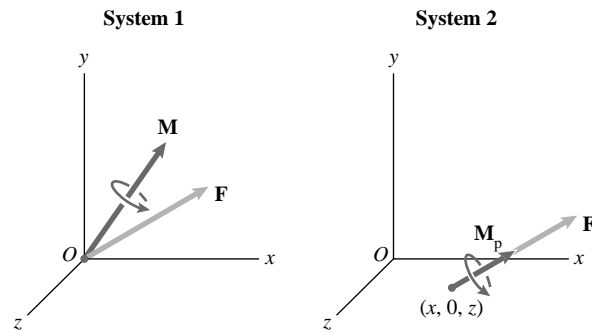
The moment of \mathbf{F} must produce a moment equal to the normal component of \mathbf{M} . The moment is

$$\mathbf{M}_F = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 12 & 4 & -3 \end{vmatrix} = -(4z)\mathbf{i} + (3x + 12z)\mathbf{j} + (4x)\mathbf{k},$$

from which

$$z = \frac{-0.5444}{-4} = 0.1361 \text{ ft}$$

$$x = \frac{5.1362}{4} = 1.2840 \text{ ft}$$



Problem 4.168 A system consists of a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , where

$$\mathbf{F} = 10\mathbf{i} \text{ (lb)}, \quad \mathbf{M} = 20\mathbf{j} \text{ (ft-lb)}.$$

If you represent the system by a wrench consisting of the force \mathbf{F} and a parallel couple \mathbf{M}_p , what is \mathbf{M}_p , and where does the line of action of \mathbf{F} intersect the y - z plane?

Solution: The component of \mathbf{M} parallel to \mathbf{F} is zero, since $\mathbf{M}_p = (\mathbf{e}_F \cdot \mathbf{M})\mathbf{e}_F = 0$. The normal component is equal to \mathbf{M} . The equivalent force must produce the same moment as the normal component

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 10 & 0 & 0 \end{vmatrix} = (10z)\mathbf{j} - (10y)\mathbf{k} = 20\mathbf{j},$$

$$\text{from which } z = \frac{20}{10} = 2 \text{ ft and } y = 0$$

Problem 4.169 A system consists of a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , where

$$\mathbf{F} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} \text{ (N)}, \quad \mathbf{M} = 10\mathbf{i} + 8\mathbf{j} - 4\mathbf{k} \text{ (N-m)}.$$

If you represent it by a wrench consisting of the force \mathbf{F} and a parallel couple \mathbf{M}_p , (a) determine \mathbf{M}_p , and determine where the line of action of \mathbf{F} intersects (b) the x - z plane, (c) the y - z plane.

Solution: The unit vector parallel to \mathbf{F} is

$$\mathbf{e}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = 0.1826\mathbf{i} + 0.3651\mathbf{j} + 0.9129\mathbf{k}.$$

(a) The component of \mathbf{M} parallel to \mathbf{F} is

$$\mathbf{M}_p = (\mathbf{e}_F \cdot \mathbf{M})\mathbf{e}_F = 0.2\mathbf{i} + 0.4\mathbf{j} + 1.0\mathbf{k} \text{ (N-m)}.$$

The normal component is

$$\mathbf{M}_N = \mathbf{M} - \mathbf{M}_p = 9.8\mathbf{i} + 7.6\mathbf{j} - 5\mathbf{k}.$$

The moment of the force about the origin must be equal to the normal component of the moment. (b) The intersection with the x - z plane:

$$\begin{aligned} \mathbf{M}_N = \mathbf{r} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 1 & 2 & 5 \end{vmatrix} = -(2z)\mathbf{i} - (5x - z)\mathbf{j} + (2x)\mathbf{k} \\ &= 9.8\mathbf{i} + 7.6\mathbf{j} - 5\mathbf{k}, \end{aligned}$$

from which

$$z = \frac{9.8}{-2} = -4.9 \text{ m, and } x = \frac{5}{-2} = -2.5 \text{ m}$$

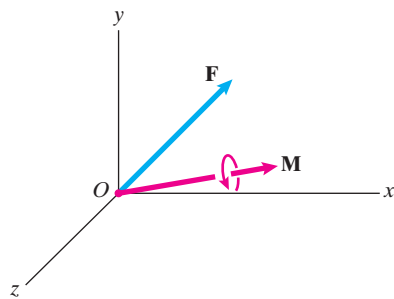
(c) The intersection with the y - z plane is

$$\begin{aligned} \mathbf{M}_N = \mathbf{r} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 1 & 2 & 5 \end{vmatrix} = (5y - 2z)\mathbf{i} + (z)\mathbf{j} - (y)\mathbf{k} \\ &= 9.8\mathbf{i} + 7.6\mathbf{j} - 5\mathbf{k}, \end{aligned}$$

from which

$$y = 5 \text{ m and } z = 7.6 \text{ m}$$

Problem 4.170 Consider the force \mathbf{F} acting at the origin O and the couple \mathbf{M} given in Example 4.15. If you represent this system by a wrench, where does the line of action of the force intersect the x - y plane?



Solution: From Example 4.15 the force and moment are $\mathbf{F} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ (N), and $\mathbf{M} = 12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ (N-m).

The normal component of the moment is

$$\mathbf{M}_N = 7.592\mathbf{i} - 4.816\mathbf{j} + 3.061\mathbf{k} \text{ (N-m)}.$$

The moment produced by the force must equal the normal component:

$$\begin{aligned} \mathbf{M}_N = \mathbf{r} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ 3 & 6 & 2 \end{vmatrix} \\ &= (2y)\mathbf{i} - (2x)\mathbf{j} + (6x - 3y)\mathbf{k} = 7.592\mathbf{i} - 4.816\mathbf{j} + 3.061\mathbf{k}, \end{aligned}$$

from which

$$x = \frac{4.816}{2} = 2.408 \text{ m and } y = \frac{7.592}{2} = 3.796 \text{ m}$$

Problem 4.171 Consider the force \mathbf{F} acting at the origin O and the couple \mathbf{M} given in Example 4.15. If you represent this system by a wrench, where does the line of action of the force intersect the plane $y = 3$ m?

Solution: From Example 4.15 (see also Problem 4.170) the force is $\mathbf{F} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, and the normal component of the moment is

$$\mathbf{M}_N = 7.592\mathbf{i} - 4.816\mathbf{j} + 3.061\mathbf{k}.$$

The moment produced by the force must be equal to the normal component:

$$\begin{aligned}\mathbf{M}_N = \mathbf{r} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 3 & z \\ 3 & 6 & 2 \end{vmatrix} = (6 - 6z)\mathbf{i} - (2x - 3z)\mathbf{j} + (6x - 9)\mathbf{k} \\ &= 7.592\mathbf{i} - 4.816\mathbf{j} + 3.061\mathbf{k},\end{aligned}$$

from which

$$x = \frac{9 + 3.061}{6} = 2.01 \text{ m and } z = \frac{6 - 7.592}{6} = -0.2653 \text{ m}$$

Problem 4.172 A wrench consists of a force of magnitude 100 N acting at the origin O and a couple of magnitude 60 N-m. The force and couple point in the direction from O to the point (1, 1, 2) m. If you represent the wrench by a force \mathbf{F} acting at point (5, 3, 1) m and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?

Solution: The vector parallel to the force is $\mathbf{r}_F = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, from which the unit vector parallel to the force is $\mathbf{e}_F = 0.4082\mathbf{i} + 0.4082\mathbf{j} + 0.8165\mathbf{k}$. The force and moment at the origin are

$$\mathbf{F} = |\mathbf{F}|\mathbf{e}_F = 40.82\mathbf{i} + 40.82\mathbf{j} + 81.65\mathbf{k} \text{ (N), and}$$

$$\mathbf{M} = 24.492\mathbf{i} + 24.492\mathbf{j} + 48.99\mathbf{k} \text{ (N-m)}.$$

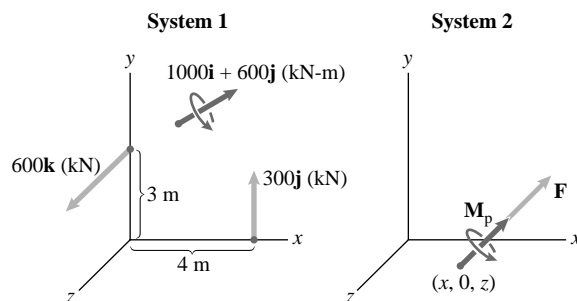
The force and moment are parallel. At the point (5, 3, 1) m the equivalent force is equal to the force at the origin, given above. The moment of this force about the origin is

$$\begin{aligned}\mathbf{M}_F = \mathbf{r} \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 1 \\ 40.82 & 40.82 & 81.65 \end{vmatrix} \\ &= 204.13\mathbf{i} - 367.43\mathbf{j} + 81.64\mathbf{k}.\end{aligned}$$

For the moments to be equal in the two systems, the added equivalent couple must be

$$\mathbf{M}_C = \mathbf{M} - \mathbf{M}_F = -176.94\mathbf{i} + 391.92\mathbf{j} - 32.65\mathbf{k} \text{ (N-m)}$$

Problem 4.173 System 1 consists of two forces and a couple. Suppose that you want to represent it by a wrench (system 2). Determine the force \mathbf{F} , the couple \mathbf{M}_p , and the coordinates x and z where the line of action of \mathbf{F} intersects the x - z plane.



Solution: The sum of the forces in System 1 is $\mathbf{F} = 300\mathbf{j} + 600\mathbf{k}$ (N). The equivalent force in System 2 must have this value. The unit vector parallel to the force is $\mathbf{e}_F = 0.4472\mathbf{j} + 0.8944\mathbf{k}$. The sum of the moments in System 1 is

$$\begin{aligned}\mathbf{M} &= 600(3)\mathbf{i} + 300(4)\mathbf{k} + 1000\mathbf{i} + 600\mathbf{j} \\ &= 2800\mathbf{i} + 600\mathbf{j} + 1200\mathbf{k} \text{ (kN m)}.\end{aligned}$$

The component parallel to the force is

$$\mathbf{M}_p = 599.963\mathbf{j} + 1199.93\mathbf{k} \text{ (kN-m)} = 600\mathbf{j} + 1200\mathbf{k} \text{ (kN-m)}.$$

The normal component is $\mathbf{M}_N = \mathbf{M} - \mathbf{M}_p = 2800\mathbf{i}$. The moment of the force

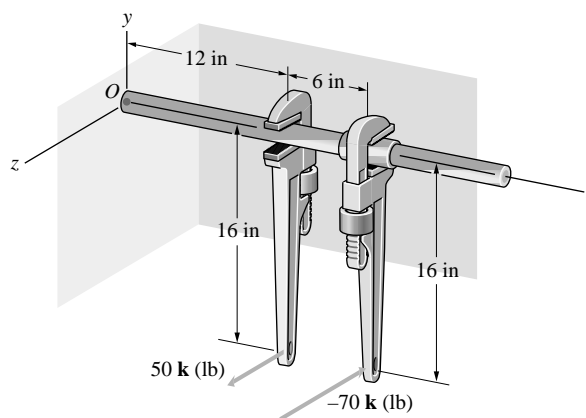
$$\mathbf{M}_N = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 0 & 300 & 600 \end{vmatrix} = -300z\mathbf{i} - 600x\mathbf{j} + 300x\mathbf{k} = 2800\mathbf{i},$$

from which

$$x = 0, \quad z = \frac{2800}{-300} = -9.333 \text{ m}$$

Problem 4.174 A plumber exerts the two forces shown to loosen a pipe.

- What total moment does he exert about the axis of the pipe?
- If you represent the two forces by a force \mathbf{F} acting at O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?
- If you represent the two forces by a wrench consisting of the force \mathbf{F} and a parallel couple \mathbf{M}_p , what is \mathbf{M}_p , and where does the line of action of \mathbf{F} intersect the x - y plane?



Solution: The sum of the forces is

$$\sum \mathbf{F} = 50\mathbf{k} - 70\mathbf{k} = -20\mathbf{k} \text{ (lb)}.$$

- The total moment exerted on the pipe is

$$\mathbf{M} = 16(20)\mathbf{i} = 320\mathbf{i} \text{ (ft lb)}.$$

- The equivalent force at O is $\mathbf{F} = -20\mathbf{k}$. The sum of the moments about O is

$$\sum \mathbf{M}_O = (\mathbf{r}_1 \times \mathbf{F}_1) + (\mathbf{r}_2 \times \mathbf{F}_2)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & -16 & 0 \\ 0 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 18 & -16 & 0 \\ 0 & 0 & -70 \end{vmatrix}$$

$$= 320\mathbf{i} + 660\mathbf{j}.$$

- The unit vector parallel to the force is $\mathbf{e}_F = \mathbf{k}$, hence the moment parallel to the force is $\mathbf{M}_p = (\mathbf{e}_F \cdot \mathbf{M})\mathbf{e}_F = 0$, and the moment normal to the force is $\mathbf{M}_N = \mathbf{M} - \mathbf{M}_p = 320\mathbf{i} + 660\mathbf{j}$. The force at the location of the wrench must produce this moment for the wrench to be equivalent.

$$\mathbf{M}_N = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ 0 & 0 & -20 \end{vmatrix} = -20y\mathbf{i} + 20x\mathbf{j} = 320\mathbf{i} + 660\mathbf{j},$$

$$\text{from which } x = \frac{660}{-20} = -33 \text{ in, } y = \frac{320}{-20} = -16 \text{ in}$$

Problem 4.175 The Leaning Tower of Pisa is approximately 55 m tall and 7 m in diameter. The horizontal displacement of the top of the tower from the vertical is approximately 5 m. Its mass is approximately 3.2×10^6 kg. If you model the tower as a cylinder and assume that its weight acts at the center, what is the magnitude of the moment exerted by the weight about the point at the center of the tower's base?



Solution: The unstretched length of spring is 1 m and the spring constant is $k = 20 \frac{\text{N}}{\text{m}}$. Assume that the bar is a quarter circle, with a radius of 4 m. The stretched length of the spring is found from the Pythagorean Theorem: The height of the attachment point is $h = 4 \sin \alpha$ m, and the distance from the center is $4 \cos \alpha$. The stretched length of the spring is

$$L = \sqrt{(3 - h)^2 + (4 \cos \alpha)^2} \text{ m.}$$

The spring force is $F = (20)(L - 1)$ N. The angle that the spring makes with a vertical line parallel to A is

$$\beta = \tan^{-1} \left(\frac{3 - h}{4 \cos \alpha} \right).$$

The horizontal component of the spring force is $F_X = F \cos \beta$ N. The vertical component of the force is $F_Y = F \sin \beta$ N. The displacement of the attachment point to the left of point A is $d = 4(1 - \cos \alpha)$ m, hence the action of the vertical component is negative, and the action of the horizontal component is positive. The moment about A is

$$\sum M_A = -dF_Y + hF_X.$$

Collecting terms and equations,

$$h = 4 \sin \alpha \text{ m,}$$

$$F_Y = F \sin \beta \text{ N,}$$

$$F_X = F \cos \beta \text{ N,}$$

$$F = (20)(L - 1) \text{ N,}$$

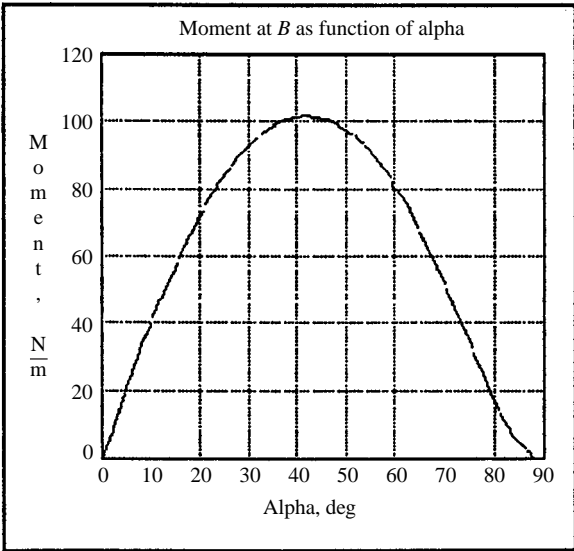
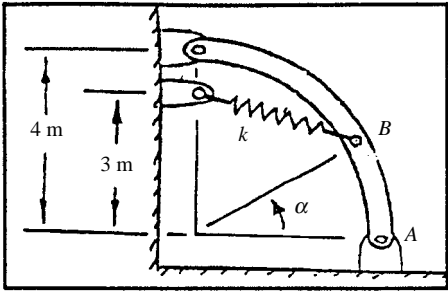
$$L = \sqrt{(3 - h)^2 + (4 \cos \alpha)^2} \text{ m,}$$

$$\beta = \tan^{-1} \left(\frac{3 - h}{4 \cos \alpha} \right).$$

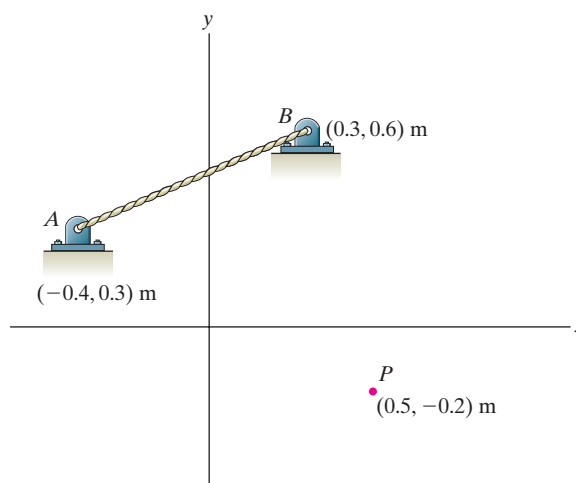
A programmable calculator or a commercial package such as **TK Solver®** or **Mathcad®** is almost essential to the solution of this and the following problems. The commercial package **TK Solver PLUS** was used here to plot the graph of M against α . Using the graph as a guide, the following tabular values were taken about the maximum:

α , deg	Moment, N-m
41.5	101.463
42.0	101.483
42.5	101.472

The maximum value of the moment is estimated at $M_B = 101.49$ N-m, which occurs at approximately $\alpha = 42.2^\circ$



Problem 4.176 The cable AB exerts a 300-N force on the support A that points from A toward B . Determine the magnitude of the moment the force exerts about point P .



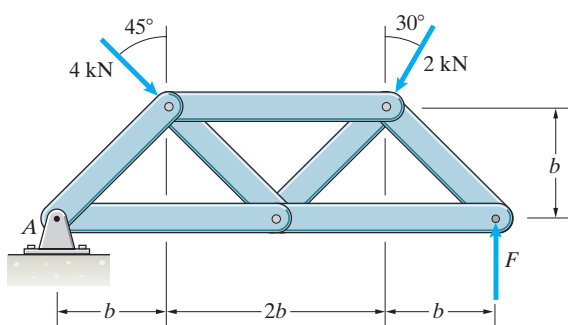
Solution:

$$\mathbf{F} = (300 \text{ N}) \frac{0.7\mathbf{i} + 0.3\mathbf{j}}{\sqrt{0.58}},$$

$$\mathbf{r}_{PA} = (-0.9\mathbf{i} + 0.5\mathbf{j}) \text{ m}$$

$$\mathbf{M}_P = \mathbf{r}_{PA} \times \mathbf{F} = -(244 \text{ Nm})\mathbf{k} \Rightarrow \boxed{M_P = 244 \text{ Nm}}$$

Problem 4.177 Three forces act on the structure. The sum of the moments due to the forces about A is zero. Determine the magnitude of the force F .



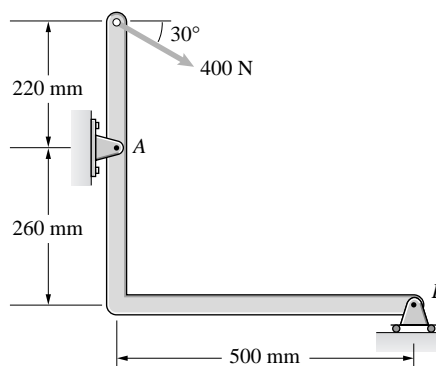
Solution:

$$\begin{aligned} \sum M_A &= -(4 \text{ kN})(\sqrt{2}b) - (2 \text{ kN} \cos 30^\circ)3b \\ &\quad + (2 \text{ kN} \sin 30^\circ)b + F(4b) = 0 \end{aligned}$$

Solving we find

$$\boxed{F = 2.463 \text{ kN}}$$

Problem 4.178 Determine the moment of the 400-N force (a) about A , (b) about B .



Solution: Use the two dimensional description of the moment. The vertical and horizontal components of the 200 N force are

$$F_y = -400 \sin 30^\circ = -200 \text{ N},$$

$$F_x = +400 \cos 30^\circ = 346.41 \text{ N}.$$

- (a) The moment arm from A to the line of action of the horizontal component is 0.22 m. The moment arm from A to the vertical component is zero. The moment about A is negative,

$$M_A = -0.22(346.41) = -76.21 \text{ N-m}$$

- (b) The perpendicular distances to the lines of action of the vertical and horizontal components of the force from B are $d_1 = 0.5 \text{ m}$, and $d_2 = 0.48 \text{ m}$. The action of the vertical component is positive, and the action of the horizontal component is negative. The sum of the moments: $M_B = +0.5(200) - 0.48(346.41) = -66.28 \text{ N-m}$

Problem 4.179 Determine the sum of the moments exerted about A by the three forces and the couple.

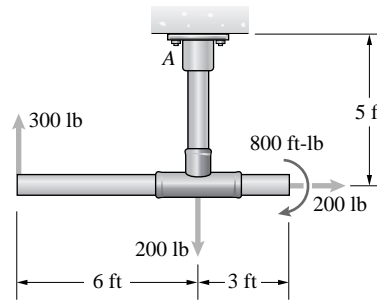
Solution: Establish coordinates with origin at A, x horizontal, and y vertical with respect to the page. The moment exerted by the couple is the same about any point. The moment of the 300 lb force about A is $\mathbf{M}_{300} = (-6\mathbf{i} - 5\mathbf{j}) \times (300\mathbf{j}) = -1800\mathbf{k}$ ft-lb.

The moment of the downward 200 lb force about A is zero since the line of action of the force passes through A. The moment of the 200 lb force which pulls to the right is

$$\mathbf{M}_{200} = (3\mathbf{i} - 5\mathbf{j}) \times (200\mathbf{i}) = 1000\mathbf{k} \text{ (ft-lb)}.$$

The moment of the couple is $\mathbf{M}_C = -800\mathbf{k}$ (ft-lb). Summing the four moments, we get

$$\mathbf{M}_A = (-1800 + 0 + 1000 - 800)\mathbf{k} = -1600\mathbf{k} \text{ (ft-lb)}$$



Problem 4.180 In Problem 4.179, if you represent the three forces and the couple by an equivalent system consisting of a force \mathbf{F} acting at A and a couple \mathbf{M} , what are the magnitudes of \mathbf{F} and \mathbf{M} ?

Solution: The equivalent force will be equal to the sum of the forces and the equivalent couple will be equal to the sum of the moments about A. From the solution to Problem 4.189, the equivalent couple will be $\mathbf{C} = \mathbf{M}_A = -1600\mathbf{k}$ (ft-lb). The equivalent force will be $\mathbf{F}_{\text{EQUIV}} = 200\mathbf{i} - 200\mathbf{j} + 300\mathbf{j} = 200\mathbf{i} + 100\mathbf{j}$ (lb)

Problem 4.181 The vector sum of the forces acting on the beam is zero, and the sum of the moments about A is zero.

- What are the forces A_x , A_y , and B ?
- What is the sum of the moments about B ?

Solution: The vertical and horizontal components of the 400 N force are:

$$F_X = 400 \cos 30^\circ = 346.41 \text{ N},$$

$$F_Y = 400 \sin 30^\circ = 200 \text{ N}.$$

The sum of the forces is

$$\sum F_X = A_X + 346.41 = 0,$$

$$\text{from which } A_X = -346.41 \text{ N}$$

$$\sum F_Y = A_Y + B - 200 = 0.$$

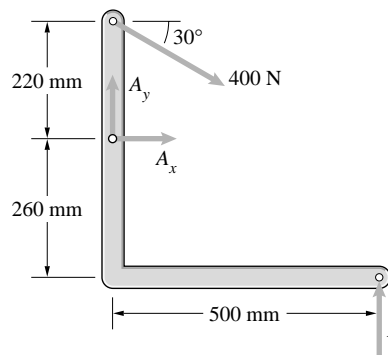
The sum of the moments about A is

$$\sum M_A = 0.5B - 0.22(346.41) = 0,$$

from which $B = 152.42 \text{ N}$. Substitute into the force equation to get $A_Y = 200 - B = 47.58 \text{ N}$

- The moments about B are

$$M_B = -0.5A_Y - 0.48(346.41) - 0.26A_X + 0.5(200) = 0$$



Problem 4.182 The hydraulic piston BC exerts a 970-lb force on the boom at C in the direction parallel to the piston. The angle $\alpha = 40^\circ$. The sum of the moment about A due to the force exerted on the boom by the piston and the weight of the suspended load is zero. What is the weight of the suspended load?

Solution: The horizontal (x) and vertical (y) coordinates of point C relative to point B are

$$x = (9 \text{ ft}) \cos \alpha - (6 \text{ ft}) = 0.894 \text{ ft}$$

$$y = (9 \text{ ft}) \sin \alpha = 5.79 \text{ ft}$$

The angle between the piston BC and the horizontal is

$$\beta = \tan^{-1}(y/x) = 81.2^\circ$$

The horizontal and vertical components of the force exerted by the piston at C are

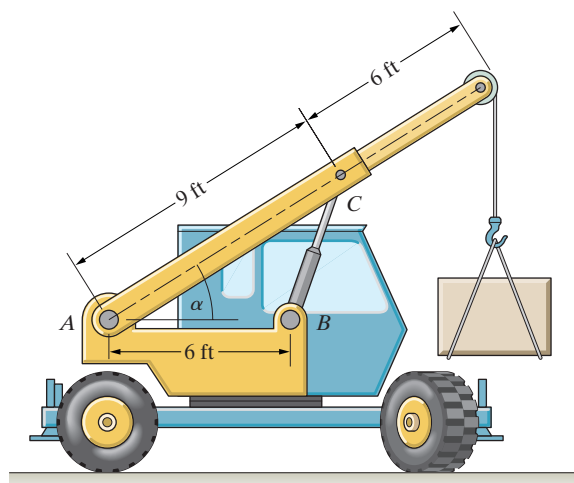
$$C_x = (970 \text{ lb}) \cos \beta = 148 \text{ lb}$$

$$C_y = (970 \text{ lb}) \sin \beta = 959 \text{ lb}$$

The sum of the moments about A due to the piston force and the suspended weight W is

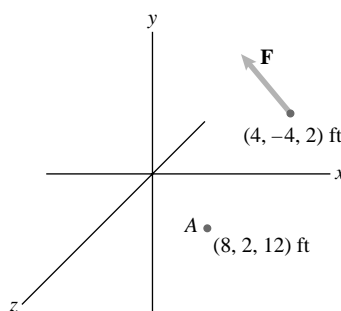
$$\Sigma M_A = -W(15 \text{ ft}) \cos \alpha + C_y (9 \text{ ft}) \cos \alpha - C_x (9 \text{ ft}) \sin \alpha = 0$$

Solving, yields $W = 501 \text{ lb}$



Problem 4.183 The force $\mathbf{F} = -60\mathbf{i} + 60\mathbf{j}$ (lb).

- Determine the moment of \mathbf{F} about point A .
- What is the perpendicular distance from point A to the line of action of \mathbf{F} ?



Solution: The position vector of A and the point of action are

$$\mathbf{r}_A = 8\mathbf{i} + 2\mathbf{j} + 12\mathbf{k} \text{ (ft)}, \text{ and } \mathbf{r}_F = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

The vector from A to F is

$$\begin{aligned} \mathbf{r}_{AF} &= \mathbf{r}_F - \mathbf{r}_A = (4 - 8)\mathbf{i} + (-4 - 2)\mathbf{j} + (2 - 12)\mathbf{k} \\ &= -4\mathbf{i} - 6\mathbf{j} - 10\mathbf{k}. \end{aligned}$$

- The moment about A is

$$\begin{aligned} \mathbf{M}_A &= \mathbf{r}_{AF} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -6 & -10 \\ -60 & 60 & 0 \end{vmatrix} \\ &= 600\mathbf{i} + 600\mathbf{j} - 600\mathbf{k} \text{ (ft lb)} \end{aligned}$$

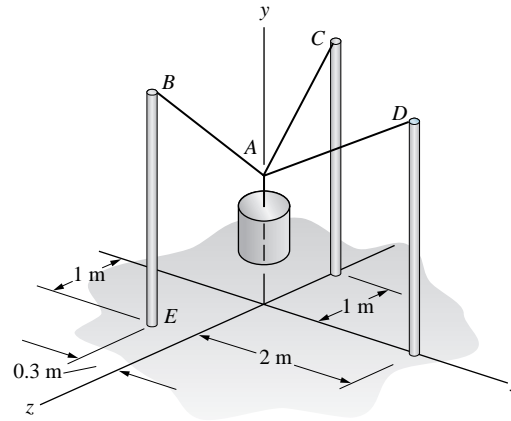
- The magnitude of the moment is

$$|\mathbf{M}_A| = \sqrt{600^2 + 600^2 + 600^2} = 1039.3 \text{ ft lb}.$$

The magnitude of the force is $|\mathbf{F}| = \sqrt{60^2 + 60^2} = 84.8528 \text{ lb}$.
The perpendicular distance from A to the line of action is

$$D = \frac{1039.3}{84.8528} = 12.25 \text{ ft}$$

Problem 4.184 The 20-kg mass is suspended by cables attached to three vertical 2-m posts. Point A is at (0, 1.2, 0) m. Determine the moment about the base E due to the force exerted on the post BE by the cable AB.



Solution: The strategy is to develop the simultaneous equations in the unknown tensions in the cables, and use the tension in AB to find the moment about E. This strategy requires the unit vectors parallel to the cables. The position vectors of the points are:

$$\mathbf{r}_{OA} = 1.2\mathbf{j},$$

$$\mathbf{r}_{OB} = -0.3\mathbf{i} + 2\mathbf{j} + 1\mathbf{k},$$

$$\mathbf{r}_{OC} = 2\mathbf{j} - 1\mathbf{k},$$

$$\mathbf{r}_{OD} = 2\mathbf{i} + 2\mathbf{j},$$

$$\mathbf{r}_{OE} = -0.3\mathbf{i} + 1\mathbf{k}.$$

The vectors parallel to the cables are:

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA} = -0.3\mathbf{i} + 0.8\mathbf{j} + 1\mathbf{k},$$

$$\mathbf{r}_{AC} = \mathbf{r}_{OC} - \mathbf{r}_{OA} = +0.8\mathbf{j} - 1\mathbf{k},$$

$$\mathbf{r}_{AD} = \mathbf{r}_{OD} - \mathbf{r}_{OA} = +2\mathbf{i} + 0.8\mathbf{j}.$$

The unit vectors parallel to the cables are:

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -0.2281\mathbf{i} + 0.6082\mathbf{j} + 0.7603\mathbf{k} :$$

$$\mathbf{e}_{AC} = 0\mathbf{i} + 0.6247\mathbf{j} - 0.7809\mathbf{k},$$

$$\mathbf{e}_{AD} = +0.9284\mathbf{i} + 0.3714\mathbf{j} + 0\mathbf{k}.$$

The tensions in the cables are

$$\mathbf{T}_{AB} = T_{AB}\mathbf{e}_{AB},$$

$$\mathbf{T}_{AC} = T_{AC}\mathbf{e}_{AC}, \text{ and}$$

$$\mathbf{T}_{AD} = T_{AD}\mathbf{e}_{AD}.$$

The equilibrium conditions are $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} = \mathbf{W}$. Collect like terms in $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\sum \mathbf{F}_X = (-0.2281T_{AB} + 0T_{AC} + 0.9284T_{AD})\mathbf{i} = 0$$

$$\sum \mathbf{F}_Y = (+0.6082 \cdot T_{AB} + 0.6247 \cdot T_{AC} + 0.3714 \cdot T_{AD} - 196.2)\mathbf{j} = 0$$

$$\sum \mathbf{F}_Z = (+0.7603 \cdot T_{AB} - 0.7809 \cdot T_{AC} + 0 \cdot T_{AD})\mathbf{k} = 0$$

Solve:

$$T_{AB} = 150.04 \text{ N},$$

$$T_{AC} = 146.08 \text{ N},$$

$$T_{AD} = 36.86 \text{ N}.$$

The moment about E is

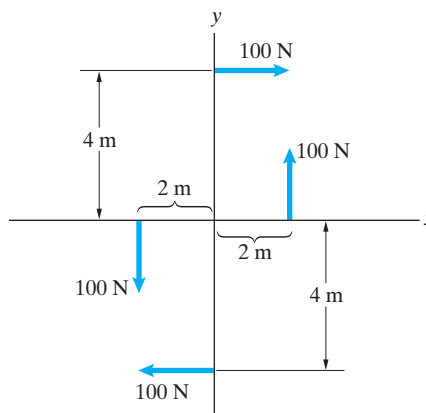
$$\mathbf{M}_E = \mathbf{r}_{EB} \times (-T_{AB}\mathbf{e}_{AB}) = -T_{AB}(\mathbf{r}_{EB} \times \mathbf{e}_{AB})$$

$$= -150 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ -0.2281 & +0.6082 & +0.7603 \end{vmatrix}$$

$$= -228\mathbf{i} - 68.43\mathbf{k} \text{ (N-m)}$$

Problem 4.185 What is the total moment due to the two couples?

- Express the answer by giving the magnitude and stating whether the moment is clockwise or counterclockwise.
- Express the answer as a vector.



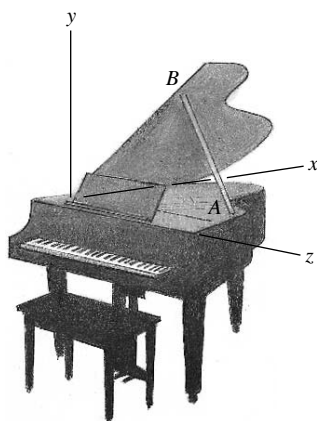
Solution:

- The couple in which the forces are 4 m apart exerts a counterclockwise moment of magnitude $(100 \text{ N})(4 \text{ m}) = 400 \text{ N}\cdot\text{m}$. The couple in which the forces are 8 m apart exerts a clockwise moment of magnitude $(100 \text{ N})(8 \text{ m}) = 800 \text{ N}\cdot\text{m}$. The sum of their moments is a clockwise moment of 400 N·m.
- The vector representation of the clockwise moment of 400 N·m magnitude is $-400\mathbf{k}$ (N·m). This expression can also be obtained by calculating the sum of the moments of the four forces about any point. The sum of the moments about the origin is

$$\begin{aligned} \mathbf{M} &= (2 \text{ m})\mathbf{i} \times (100 \text{ N})\mathbf{j} + (-2 \text{ m})\mathbf{i} \times (-100 \text{ N})\mathbf{j} \\ &\quad + (4 \text{ m})\mathbf{j} \times (100 \text{ N})\mathbf{i} + (-4 \text{ m})\mathbf{j} \times (-100 \text{ N})\mathbf{i} \\ &= (-400 \text{ N}\cdot\text{m})\mathbf{k} \end{aligned}$$

(a) 400 N·m clockwise (b) $-400\mathbf{k}$ N·m

Problem 4.186 The bar AB supporting the lid of the grand piano exerts a force $\mathbf{F} = -6\mathbf{i} + 35\mathbf{j} - 12\mathbf{k}$ (lb) at B . The coordinates of B are (3, 4, 3) ft. What is the moment of the force about the hinge line of the lid (the x axis)?



Solution: The position vector of point B is $\mathbf{r}_{OB} = 3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$. The moment about the x axis due to the force is

$$M_X = \mathbf{e}_X \cdot (\mathbf{r}_{OB} \times \mathbf{F}) = \mathbf{i} \cdot (\mathbf{r}_{OB} \times \mathbf{F})$$

$$M_X = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 4 & 3 \\ -6 & 35 & -12 \end{vmatrix} = -153 \text{ ft}\cdot\text{lb}$$

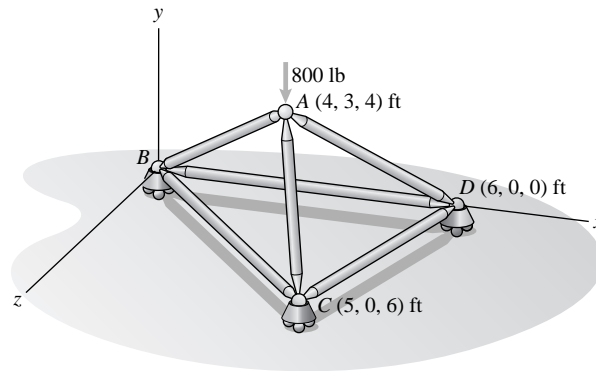
Problem 4.187 Determine the moment of the vertical 800-lb force about point C .

Solution: The force vector acting at A is $\mathbf{F} = -800\mathbf{j}$ (lb) and the position vector from C to A is

$$\begin{aligned}\mathbf{r}_{CA} &= (x_A - x_C)\mathbf{i} + (y_A - y_C)\mathbf{j} + (z_A - z_C)\mathbf{k} \\ &= (4 - 5)\mathbf{i} + (3 - 0)\mathbf{j} + (4 - 6)\mathbf{k} = -1\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \text{ (ft)}.\end{aligned}$$

The moment about C is

$$M_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & -2 \\ 0 & -800 & 0 \end{vmatrix} = -1600\mathbf{i} + 0\mathbf{j} + 800\mathbf{k} \text{ (ft-lb)}$$



Problem 4.188 In Problem 4.187, determine the moment of the vertical 800-lb force about the straight line through points C and D .

Solution: In Problem 4.197, we found the moment of the 800 lb force about point C to be given by

$$\mathbf{M}_C = -1600\mathbf{i} + 0\mathbf{j} + 800\mathbf{k} \text{ (ft-lb)}.$$

The vector from C to D is given by

$$\begin{aligned}\mathbf{r}_{CD} &= (x_D - x_C)\mathbf{i} + (y_D - y_C)\mathbf{j} + (z_D - z_C)\mathbf{k} \\ &= (6 - 5)\mathbf{i} + (0 - 0)\mathbf{j} + (0 - 6)\mathbf{k} \\ &= 1\mathbf{i} + 0\mathbf{j} - 6\mathbf{k} \text{ (ft)},\end{aligned}$$

and its magnitude is

$$|\mathbf{r}_{CD}| = \sqrt{1^2 + 6^2} = \sqrt{37} \text{ (ft)}.$$

The unit vector from C to D is given by

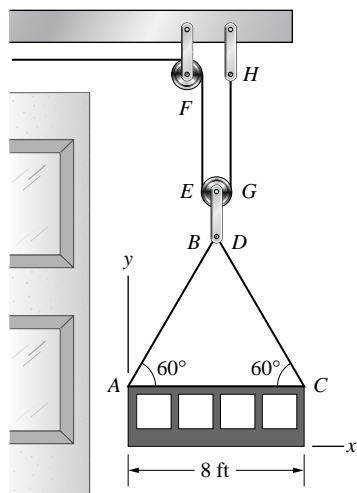
$$\mathbf{e}_{CD} = \frac{1}{\sqrt{37}}\mathbf{i} - \frac{6}{\sqrt{37}}\mathbf{k}.$$

The moment of the 800 lb vertical force about line CD is given by

$$\begin{aligned}M_{CD} &= \left(\frac{1}{\sqrt{37}}\mathbf{i} - \frac{6}{\sqrt{37}}\mathbf{k} \right) \cdot (-1600\mathbf{i} + 0\mathbf{j} + 800\mathbf{k}) \text{ (ft-lb)} \\ &= \left(\frac{-1600 - 4800}{\sqrt{37}} \right) \text{ (ft-lb)}.\end{aligned}$$

Carrying out the calculations, we get $M_{CD} = -1052 \text{ (ft-lb)}$

Problem 4.189 The system of cables and pulleys supports the 300-lb weight of the work platform. If you represent the upward force exerted at E by cable EF and the upward force exerted at G by cable GH by a single equivalent force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x axis?



Solution: The cable-pulley combination does not produce a moment. Hence the equivalent force does not. The equivalent force is equal to the total supported weight, or $\mathbf{F} = +\frac{600}{2}\mathbf{j} = 300\mathbf{j}$ (lb). The force occurs at midpoint of the platform width, $x = \frac{8}{2} = 4$ ft

Problem 4.190 Consider the system in Problem 4.189.

- What are the tensions in cables AB and CD ?
- If you represent the forces exerted by the cables at A and C by a single equivalent force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x axis?

Solution: The vertical component of the tension in each cable must equal half the weight supported.

$T_{AB} \sin 60^\circ = 150$ lb, from which $T_{AB} = \frac{150}{\sin 60^\circ} = 173.2$ lb. By symmetry, the tension $T_{CD} = 173.2$ lb.

The single force must equal the sum of the vertical components; since there is no resultant moment produced by the cables, the force is $\mathbf{F} = 300\mathbf{j}$ lb and it acts at the platform width midpoint $x = 4$ ft.

Problem 4.191 The two systems are equivalent. Determine the forces A_x and A_y , and the couple M_A .

Solution: The sum of the forces for System 1 is

$$\sum \mathbf{F}_X = (A_X + 20)\mathbf{i},$$

$$\sum \mathbf{F}_Y = (A_Y + 30)\mathbf{j}.$$

The sum of forces for System 2 is

$$\sum \mathbf{F}_X = (-20)\mathbf{i} \text{ and}$$

$$\sum \mathbf{F}_Y = (80 - 10)\mathbf{j}.$$

Equating the two systems:

$$A_X + 20 = -20 \text{ from which } A_X = -40 \text{ N}$$

$$A_Y + 30 = 80 - 10 \text{ from which } A_Y = 40 \text{ N}$$

The sum of the moments about the left end for System 1 is

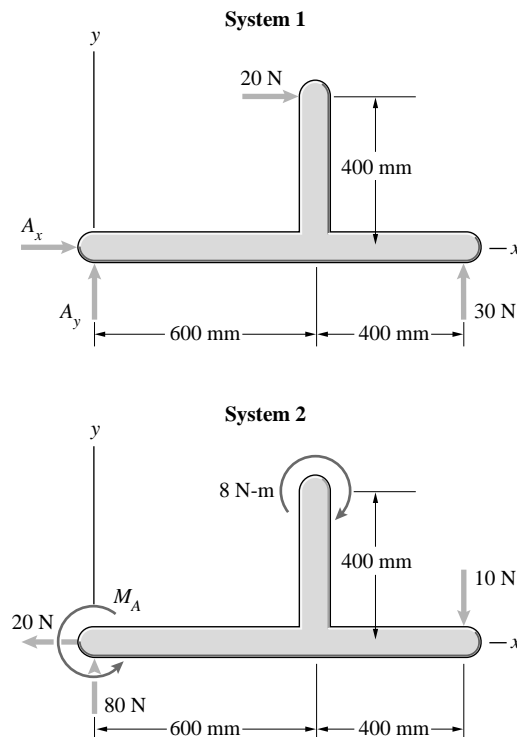
$$\sum M_1 = -(0.4)(20) + 30(1) = 22 \text{ N}\cdot\text{m}.$$

The sum of moments about the left end for System 2 is

$$\sum M_2 = M_A - 10(1) - 8 = M_A - 18.$$

Equating the moments for the two systems:

$$M_A = 18 + 22 = 40 \text{ N}\cdot\text{m}$$



Problem 4.192 If you represent the equivalent systems in Problem 4.191 by a force \mathbf{F} acting at the origin and a couple M , what are \mathbf{F} and M ?

Solution: Summing the forces in System 1, $\mathbf{F} = (A_X + 20)\mathbf{i} + (A_Y + 30)\mathbf{j}$. Substituting from the solution in Problem 4.201, $\mathbf{F} = -20\mathbf{i} + 70\mathbf{j}$. The moment is $\mathbf{M} = -20(0.4)\mathbf{k} + 30\mathbf{k} = 22\mathbf{k} \text{ (N}\cdot\text{m)}$

Problem 4.193 If you represent the equivalent systems in Problem 4.191 by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x axis?

Solution: The force is $\mathbf{F} = -20\mathbf{i} + 70\mathbf{j}$. The moment to be represented is

$$\mathbf{M} = (\mathbf{r} \times \mathbf{F}) = 22\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & 0 \\ -20 & 70 & 0 \end{vmatrix} = 70x\mathbf{k},$$

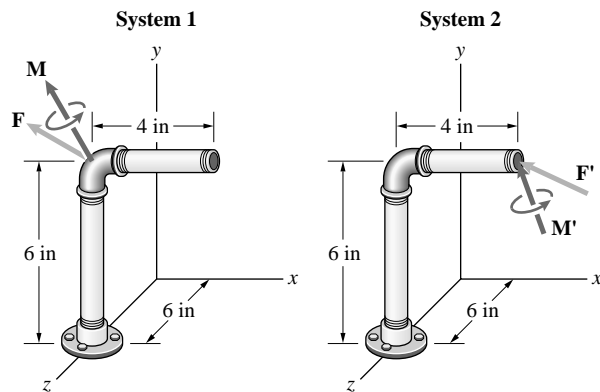
$$\text{from which } x = \frac{22}{70} = 0.3143 \text{ m}$$

Problem 4.194 The two systems are equivalent. If

$$\mathbf{F} = -100\mathbf{i} + 40\mathbf{j} + 30\mathbf{k} \text{ (lb)},$$

$$\mathbf{M}' = -80\mathbf{i} + 120\mathbf{j} + 40\mathbf{k} \text{ (in-lb)},$$

determine \mathbf{F}' and \mathbf{M} .



Solution: The sum of forces in the two systems must be equal, thus $\mathbf{F}' = \mathbf{F} = -100\mathbf{i} + 40\mathbf{j} + 30\mathbf{k}$ (lb).

The moment for the unprimed system is $\mathbf{M}_T = \mathbf{r} \times \mathbf{F} + \mathbf{M}$.

The moment for the primed system is $\mathbf{M}'_T = \mathbf{r}' \times \mathbf{F} + \mathbf{M}'$.

The position vectors are $\mathbf{r} = 0\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$, and $\mathbf{r}' = 4\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$. Equating the moments and solving for the unknown moment

$$\begin{aligned} \mathbf{M} &= \mathbf{M}' + (\mathbf{r}' - \mathbf{r}) \times \mathbf{F} = -80\mathbf{i} + 120\mathbf{j} + 40\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ -100 & 40 & 30 \end{vmatrix} \\ &= -80\mathbf{i} + 120\mathbf{j} + 40\mathbf{k} - 120\mathbf{j} + 160\mathbf{k} \\ &= -80\mathbf{i} + 200\mathbf{k} \text{ (in-lb)} \end{aligned}$$

Problem 4.195 The tugboats *A* and *B* exert forces $F_A = 1 \text{ kN}$ and $F_B = 1.2 \text{ kN}$ on the ship. The angle $\theta = 30^\circ$. If you represent the two forces by a force \mathbf{F} acting at the origin *O* and a couple *M*, what are \mathbf{F} and *M*?

Solution: The sums of the forces are:

$$\sum F_x = (1 + 1.2 \cos 30^\circ)\mathbf{i} = 2.0392\mathbf{i} \text{ (kN)}$$

$$\sum F_y = (1.2 \sin 30^\circ)\mathbf{j} = 0.6\mathbf{j} \text{ (kN)}.$$

The equivalent force at the origin is

$$\mathbf{F}_{EQ} = 2.04\mathbf{i} + 0.6\mathbf{j}$$

The moment about *O* is $\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F}_A + \mathbf{r}_B \times \mathbf{F}_B$. The vector positions are

$$\mathbf{r}_A = -25\mathbf{i} + 60\mathbf{j} \text{ (m)}, \text{ and}$$

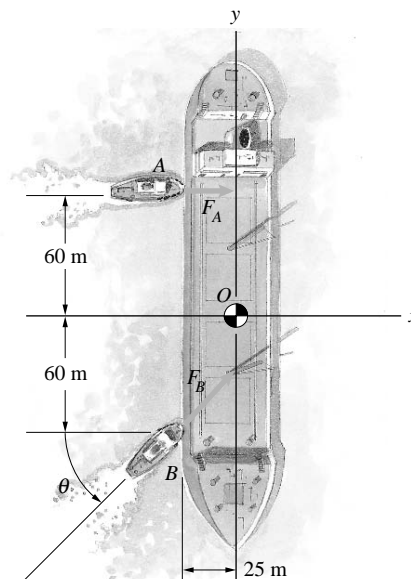
$$\mathbf{r}_B = -25\mathbf{i} - 60\mathbf{j} \text{ (m)}.$$

The moment:

$$\begin{aligned} \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -25 & 60 & 0 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -25 & -60 & 0 \\ 1.0392 & 0.6 & 0 \end{vmatrix} \\ &= -12.648\mathbf{k} = -12.6\mathbf{k} \text{ (kN-m)} \end{aligned}$$

Check: Use a two dimensional description: The moment is

$$\begin{aligned} M_O &= -(25)F_B \sin 30^\circ + (60)(F_B \cos 30^\circ) - (60)(F_A) \\ &= 39.46F_B - 60F_A = -12.6 \text{ kN-m} \end{aligned}$$



Problem 4.196 The tugboats A and B in Problem 4.195 exert forces $F_A = 600$ N and $F_B = 800$ N on the ship. The angle $\theta = 45^\circ$. If you represent the two forces by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the y axis?

Solution: The equivalent force is

$$\mathbf{F} = (0.6 + 0.8 \cos 45^\circ)\mathbf{i} + 0.8 \sin 45^\circ\mathbf{j} = 1.1656\mathbf{i} + 0.5656\mathbf{j} \text{ (kN)}.$$

The moment produced by the two forces is

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F}_A + \mathbf{r}_B \times \mathbf{F}_B.$$

The vector positions are

$$\mathbf{r}_A = -25\mathbf{i} + 60\mathbf{j} \text{ (m)}, \text{ and } \mathbf{r}_B = -25\mathbf{i} - 60\mathbf{j} \text{ (m)}.$$

The moment:

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -25 & 60 & 0 \\ 0.6 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -25 & -60 & 0 \\ 0.5656 & 0.5656 & 0 \end{vmatrix} = -16.20\mathbf{k} \text{ (kN-m)}$$

Check: Use a two dimensional description:

$$\begin{aligned} M_O &= -(25)F_B \sin 45^\circ + (60)F_B \cos 45^\circ - 60F_A \\ &= 24.75F_B - 60F_A = -16.20 \text{ kN-m.} \end{aligned}$$

The single force must produce this moment.

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & 0 \\ 1.1656 & 0.5656 & 0 \end{vmatrix} = -1.1656y\mathbf{k} = -16.20\mathbf{k},$$

from which

$$y = \frac{16.20}{1.1656} = 13.90 \text{ m}$$

Problem 4.197 The tugboats A and B in Problem 4.195 want to exert two forces on the ship that are equivalent to a force \mathbf{F} acting at the origin O of 2-kN magnitude. If $F_A = 800$ N, determine the necessary values of F_B and angle θ .

Solution: The equivalent force at the origin is $(F_A + F_B \cos \theta)^2 + (F_B \sin \theta)^2 = 2000^2$. The moment about the origin due to F_A and F_B must be zero:

$$\sum M_O = -60F_A + 60F_B \cos \theta - 25F_B \sin \theta = 0.$$

These are two equations in two unknowns $F_B \sin \theta$ and $F_B \cos \theta$. For brevity write $x = F_B \cos \theta$, $y = F_B \sin \theta$, so that the two equations become $x^2 + 2F_A x + F_A^2 + y^2 = 2000^2$ and $60x - 25y - 60F_A = 0$. Eliminate y by solving each equation for y^2 and equating the results:

$$y^2 = 2000^2 - x^2 - 2F_A x - F_A^2 = \left(-\frac{60}{25}F_A + \frac{60}{25}x\right)^2.$$

Reduce to obtain the quadratic in x :

$$\begin{aligned} &\left[1 + \left(\frac{60}{25}\right)^2\right]x^2 + 2F_A \left[1 - \left(\frac{60}{25}\right)^2\right]x \\ &+ \left[1 + \left(\frac{60}{25}\right)^2\right]F_A^2 - 2000^2 = 0. \end{aligned}$$

Substitute $F_A = 800$ N to obtain $6.76x^2 - 7616x + 326400 = 0$. In canonical form: $x^2 + 2bx + c = 0$, where $b = -563.31$, and $c = 48284.0$, with the solutions $x = -b \pm \sqrt{b^2 - c} = 1082.0, = 44.62$. From the second equation, $y = -1812.9, = 676.81$. The force F_B has two solutions: Solve for F_B and θ : (1)

$$F_B = \sqrt{44.6^2 + 1812.9^2} = 1813.4 \text{ N}$$

at the angle

$$\theta = \tan^{-1} \left(\frac{-1812.9}{44.6} \right) = -88.6^\circ, \text{ and (2)}$$

$$F_B = \sqrt{676.8^2 + 1082.0^2} = 1276.2 \text{ N},$$

at the angle

$$\theta = \tan^{-1} \left(\frac{676.8}{1082.0} \right) = 32.0^\circ$$

Problem 4.198 If you represent the forces exerted by the floor on the table legs by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?

Solution: The sum of the forces is the equivalent force at the origin. $\mathbf{F} = (50 + 48 + 50 + 42)\mathbf{j} = 190\mathbf{j}$ (N). The position vectors of the legs are, numbering the legs counterclockwise from the lower left in the sketch:

$$\mathbf{r}_1 = +1\mathbf{k},$$

$$\mathbf{r}_2 = 2\mathbf{i} + 1\mathbf{k},$$

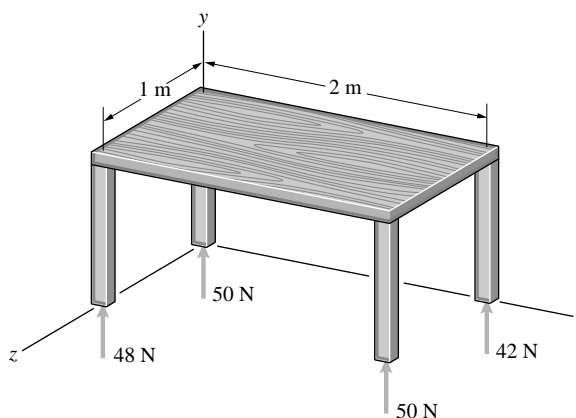
$$\mathbf{r}_3 = 2\mathbf{i},$$

$$\mathbf{r}_4 = 0.$$

The sum of the moments about the origin is

$$\begin{aligned} \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 48 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 1 \\ 0 & 50 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & 42 & 0 \end{vmatrix} \\ &= -98\mathbf{i} + 184\mathbf{k} \text{ (N-m)}. \end{aligned}$$

This is the couple that acts at the origin.



Problem 4.199 If you represent the forces exerted by the floor on the table legs in Problem 4.198 by a force \mathbf{F} , what is \mathbf{F} , and where does its line of action intersect the x - z plane?

Solution: From the solution to Problem 4.198 the equivalent force is $\mathbf{F} = 190\mathbf{j}$. This force must produce the moment $\mathbf{M} = -98\mathbf{i} + 184\mathbf{k}$ obtained in Problem 4.198.

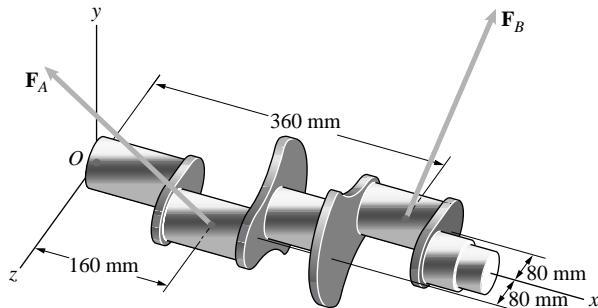
$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 0 & 190 & 0 \end{vmatrix} = -190z\mathbf{i} + 190x\mathbf{k} = -98\mathbf{i} + 184\mathbf{k},$$

from which

$$x = \frac{184}{190} = 0.9684 \text{ m and}$$

$$z = \frac{98}{190} = 0.5158 \text{ m}$$

Problem 4.200 Two forces are exerted on the crankshaft by the connecting rods. The direction cosines of \mathbf{F}_A are $\cos \theta_x = -0.182$, $\cos \theta_y = 0.818$, and $\cos \theta_z = 0.545$, and its magnitude is 4 kN. The direction cosines of \mathbf{F}_B are $\cos \theta_x = 0.182$, $\cos \theta_y = 0.818$, and $\cos \theta_z = -0.545$, and its magnitude is 2 kN. If you represent the two forces by a force \mathbf{F} acting at the origin O and a couple \mathbf{M} , what are \mathbf{F} and \mathbf{M} ?



Solution: The equivalent force is the sum of the forces:

$$\mathbf{F}_A = 4(-0.182\mathbf{i} + 0.818\mathbf{j} + 0.545\mathbf{k})$$

$$= -0.728\mathbf{i} + 3.272\mathbf{j} + 2.18\mathbf{k} \text{ (kN)}$$

$$\mathbf{F}_B = 2(0.182\mathbf{i} + 0.818\mathbf{j} - 0.545\mathbf{k}) = 0.364\mathbf{i} + 1.636\mathbf{j} - 1.09\mathbf{k} \text{ (kN)}$$

$$\text{The sum: } \mathbf{F}_A + \mathbf{F}_B = -0.364\mathbf{i} + 4.908\mathbf{j} + 1.09\mathbf{k} \text{ (kN)}$$

The equivalent couple is the sum of the moments. $\mathbf{M} = \mathbf{r}_A \times \mathbf{F}_A + \mathbf{r}_B \times \mathbf{F}_B$. The position vectors are:

$$\mathbf{r}_A = 0.16\mathbf{i} + 0.08\mathbf{k},$$

$$\mathbf{r}_B = 0.36\mathbf{i} - 0.08\mathbf{k}.$$

The sum of the moments:

$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.16 & 0 & 0.08 \\ -0.728 & 3.272 & 2.180 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.36 & 0 & -0.08 \\ 0.364 & 1.636 & -1.090 \end{vmatrix}$$

$$\mathbf{M} = -0.1309\mathbf{i} - 0.0438\mathbf{j} + 1.1125\mathbf{k} \text{ (kN-m)}$$

Problem 4.201 If you represent the two forces exerted on the crankshaft in Problem 4.200 by a wrench consisting of a force \mathbf{F} and a parallel couple \mathbf{M}_p , what are \mathbf{F} and \mathbf{M}_p , and where does the line of action of \mathbf{F} intersect the x - z plane?

Solution: From the solution to Problem 4.200,

$$\mathbf{F} = -0.364\mathbf{i} + 4.908\mathbf{j} + 1.09\mathbf{k} \text{ (kN) and}$$

$$\mathbf{M} = -0.1309\mathbf{i} - 0.0438\mathbf{j} + 1.1125\mathbf{k} \text{ (kN-m)}.$$

The unit vector parallel to \mathbf{F} is

$$\mathbf{e}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = -0.0722\mathbf{i} + 0.9737\mathbf{j} + 0.2162\mathbf{k}.$$

The moment parallel to the force is

$$\mathbf{M}_p = (\mathbf{e}_F \cdot \mathbf{M})\mathbf{e}_F.$$

Carrying out the operations:

$$\mathbf{M}_p = 0.2073\mathbf{e}_F = -0.01497\mathbf{i} + 0.2019\mathbf{j} + 0.0448\mathbf{k} \text{ (kN-m)}.$$

This is the equivalent couple parallel to \mathbf{F} .

The component of the moment perpendicular to \mathbf{F} is

$$\mathbf{M}_N = \mathbf{M} - \mathbf{M}_p = -0.1159\mathbf{i} - 0.2457\mathbf{j} + 1.0688\mathbf{k}.$$

The force exerts this moment about the origin.

$$\begin{aligned} \mathbf{M}_N &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -0.364 & 4.908 & 1.09 \end{vmatrix} \\ &= (-4.908z)\mathbf{i} - (1.09x + 0.364z)\mathbf{j} + (4.908x)\mathbf{k} \\ &= -0.1159\mathbf{i} - 0.2457\mathbf{j} + 1.0688\mathbf{k}. \end{aligned}$$

From which

$$x = \frac{1.0688}{4.908} = 0.2178 \text{ m,}$$

$$z = \frac{0.1159}{4.908} = 0.0236 \text{ m}$$