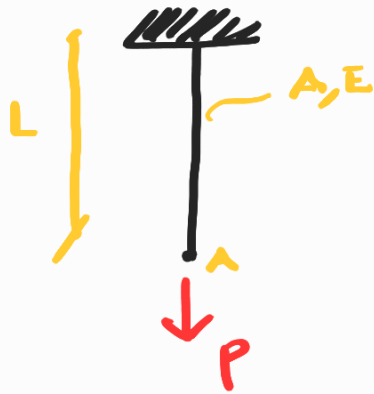


# Soluciones Axil



$$\sigma = P/A$$

$$\epsilon = \frac{P}{AE} \rightarrow \text{deformación por unidad de } L$$

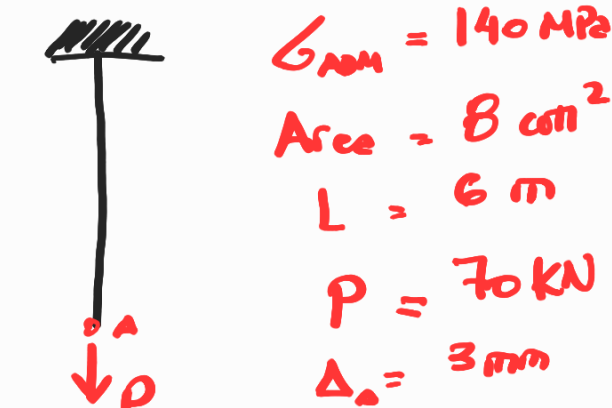
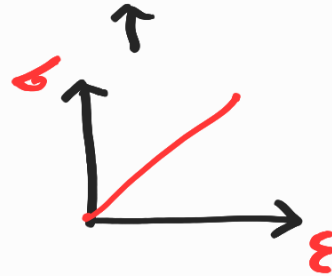
$$\Delta = \epsilon \cdot L$$

$$N = P = \int \sigma_x dA$$

$$\epsilon_x = cte, \quad \sigma_x = cte$$

$$\sigma = \epsilon E$$

LM



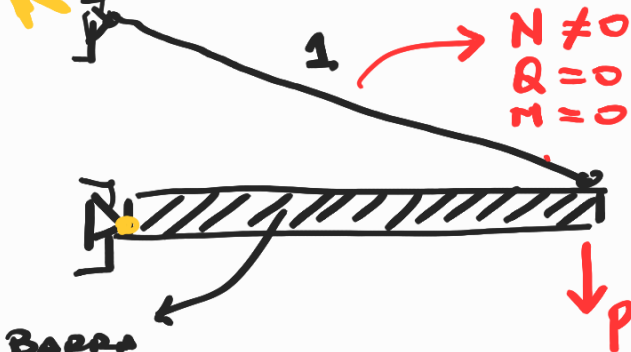
$$\sigma_{adm} = 140 \text{ MPa}$$

$$A_{ace} = 8 \text{ cm}^2$$

$$L = 6 \text{ m}$$

$$P = 70 \text{ kN} = 7 \text{ t} \rightarrow 7 \text{ poles}$$

$$\Delta_a = 3 \text{ mm}$$



$$N \neq 0$$

$$Q = 0$$

$$M = 0$$

WILLIOT

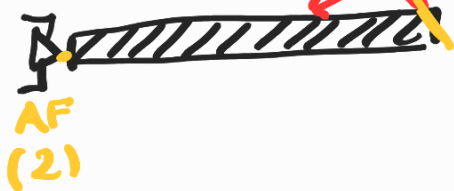
WANTO 3 EL  $\Delta 1$

BARRA  $\infty$  RIGIDO

1 SOSTÁTICO

$$GL = CV$$

DESPLAZAMIENTO



$$\sum F_v = 0$$

$$\sum F_H = 0$$

$$\sum M = 0$$

## ESTRUCTURAS HIPERESTÁTICAS

$$GL < CV$$

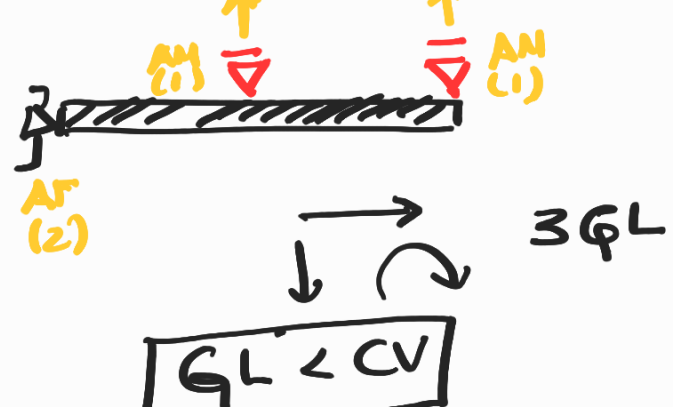
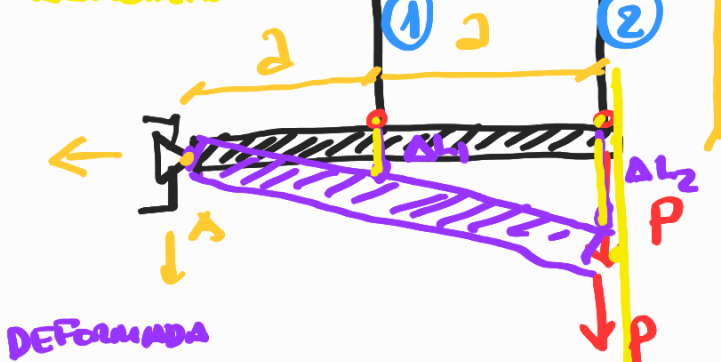
ECUACIONES DE EQUILIBRIO SON INSUFICIENTES



$N_1$

$N_2$

LCmeñito



• EQUAÇÕES DE COMPATIBILIDADE  
 $\Delta L_1, \Delta L_2 \rightarrow$



$$\boxed{\Delta L_2 = 2 \Delta L_1}$$

$$\sum M_A = 0 = N_1 \cancel{2} + N_2 \cancel{2} - P \cdot \cancel{2} = 0$$

$$\textcircled{1} \quad N_1 + 2N_2 = 2P$$

$$\Delta L_2 = 2 \Delta L_1 \quad \textcircled{2} \quad \frac{N_2 \cdot L}{EA} = \frac{2 N_1 \cdot L}{EA}$$

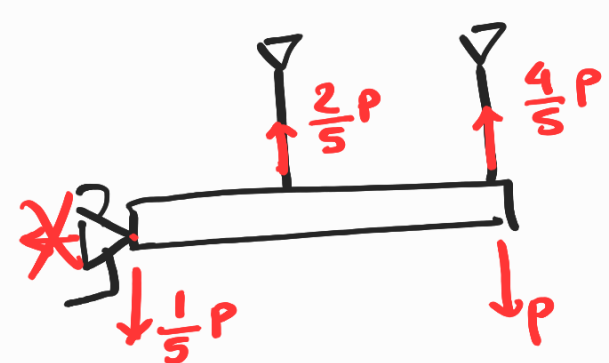
• E, A de  $1^a$  e  $2^a$  são iguais

$$N_1 + 2 \cdot 2N_1 = 2P$$

$$5N_1 = 2P \Rightarrow \boxed{N_1 = \frac{2}{5}P}$$

$$\boxed{N_2 = \frac{4}{5}P}$$

$$N_1 + N_2 = P?$$



$$N_1 \rightarrow \Delta L_1 \Rightarrow \Delta L_2 = 2 \Delta L_1$$

$$N_2 \rightarrow \Delta L_2 \Rightarrow \boxed{N_2 = 2N_1}$$

$$L_1 = L_2$$

$$E_1 = E_2$$

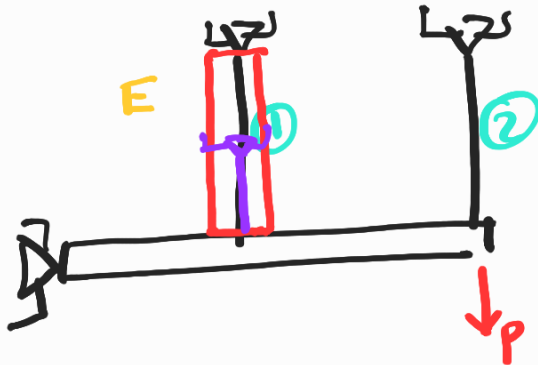
$$A_1 = A_2$$

$$\frac{N_2 \cdot L_2}{E_2 A_2} = \frac{2 N_1 \cdot L_1}{E_1 A_1}$$

①  $A_1 \rightarrow 2A_2$   $E_1 = E_2$   $L_2 = L_1 \rightarrow N_2 = N_1$

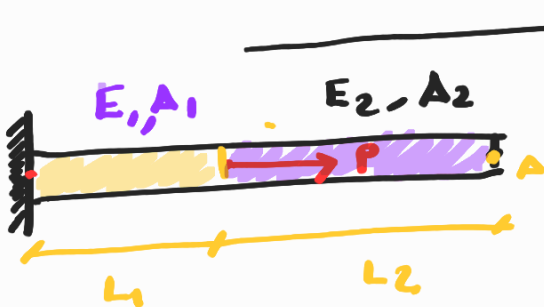
$\frac{E \cdot A}{L}$

RIGIDEZ AXIAL DE LA BARRA



$\Delta L_2 = 2 \Delta L_1$

- 1) Area  $\uparrow$
- 2) L  $\downarrow$
- 3) E  $\uparrow$

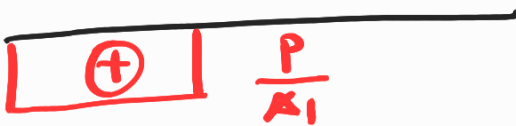


$\Delta L_A = \frac{P \cdot L_1}{E_1 \cdot A_1}$

②



③

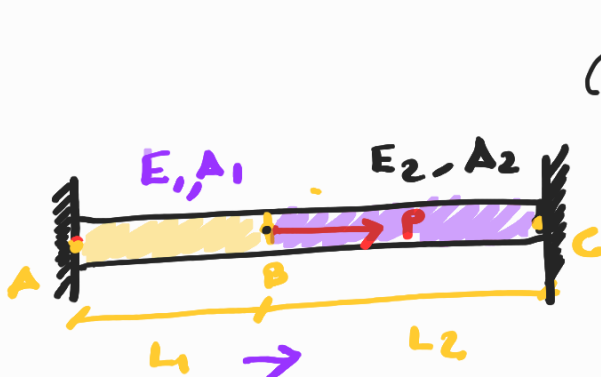


④

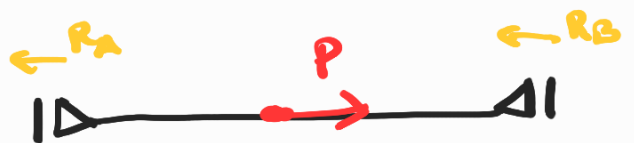


$\epsilon = \frac{dU}{dx}$

⑤



HIPERESTATICO



①  $\sum F_H = 0$  ✓

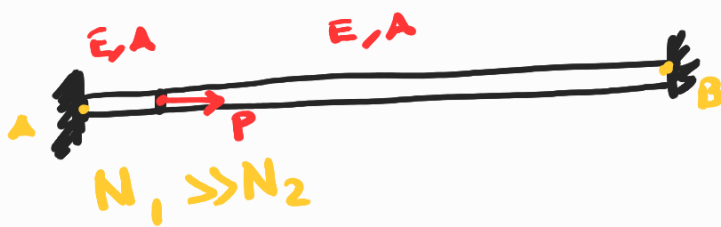
②  $|\Delta L_1| = |\Delta L_2|$  ✓

$\delta_A = 0$   
 $\delta_C = 0$

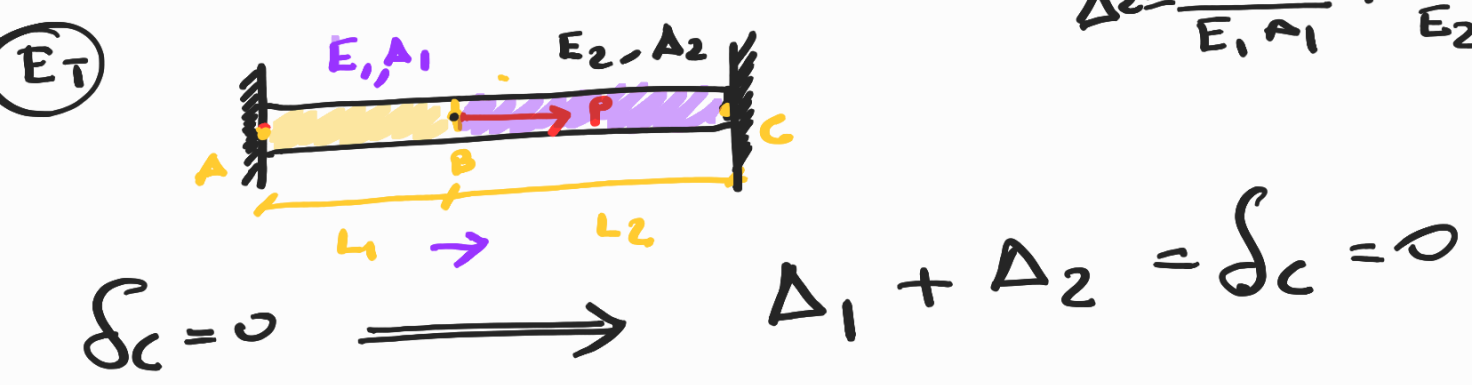
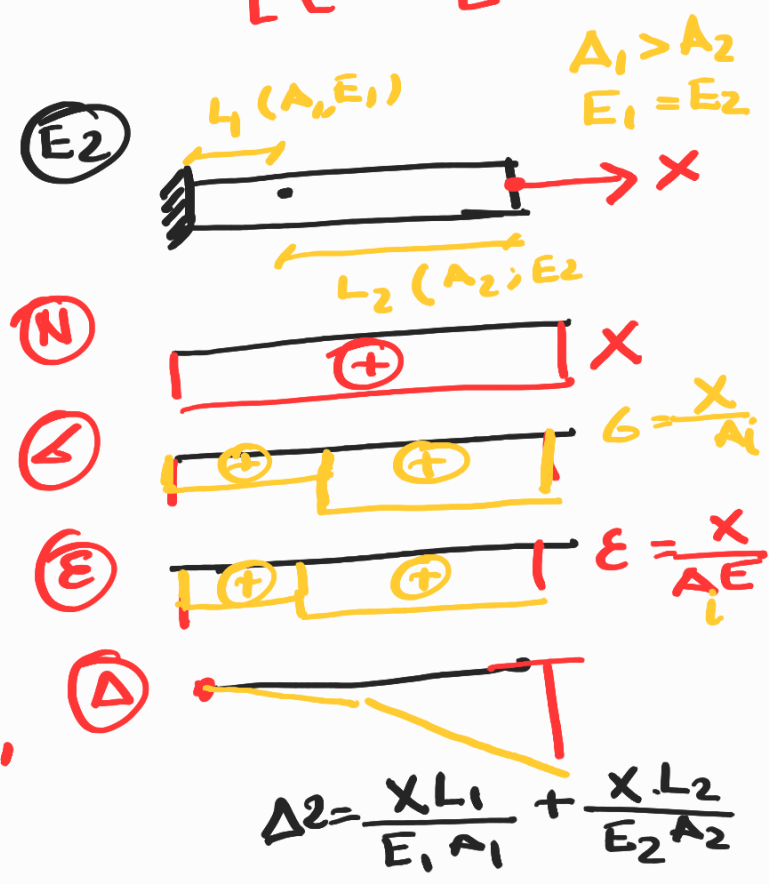
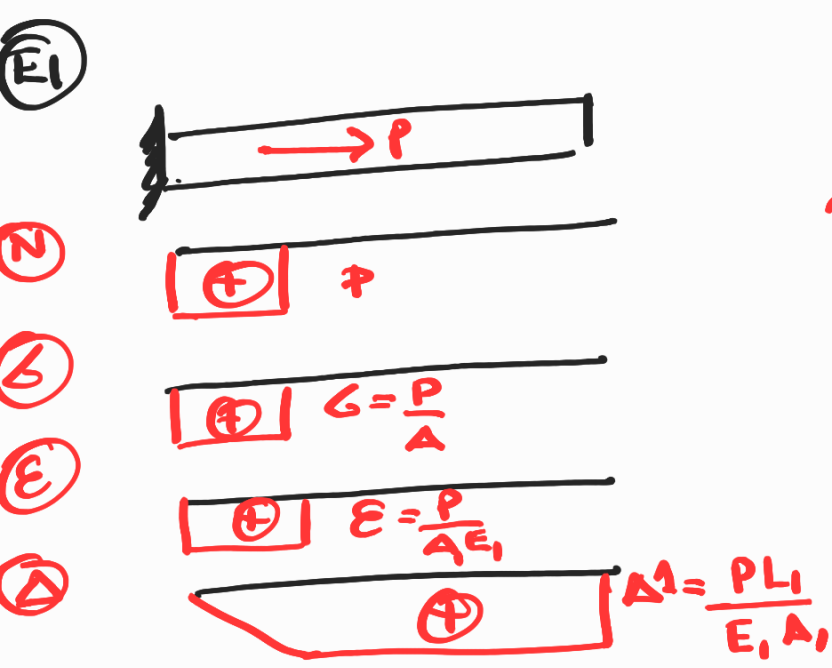
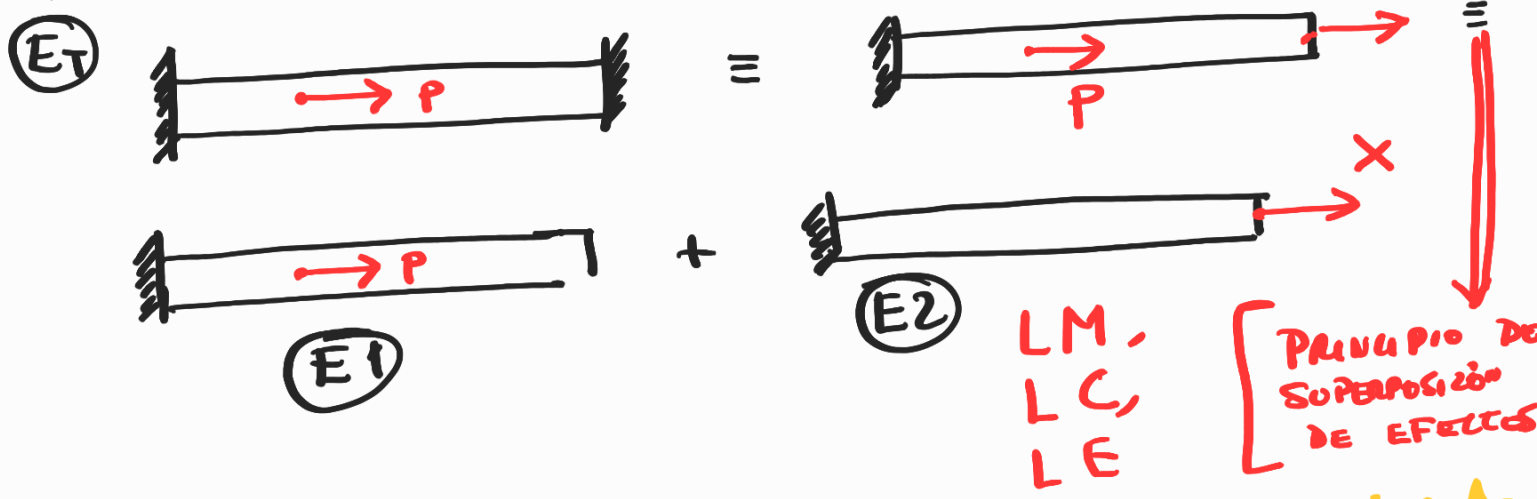
LA RIGIDEZ AXIAL DE LA BARRA



$$\frac{EA}{L}$$



# MÉTODO DE LAS INCÓGNITAS ESTÁTICAS (MIE)



$$\frac{PL_1}{E_1 A_1} + X \left( \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right) = 0$$

$$X = - \frac{\frac{PL_1}{E_1 A_1}}{\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2}}$$

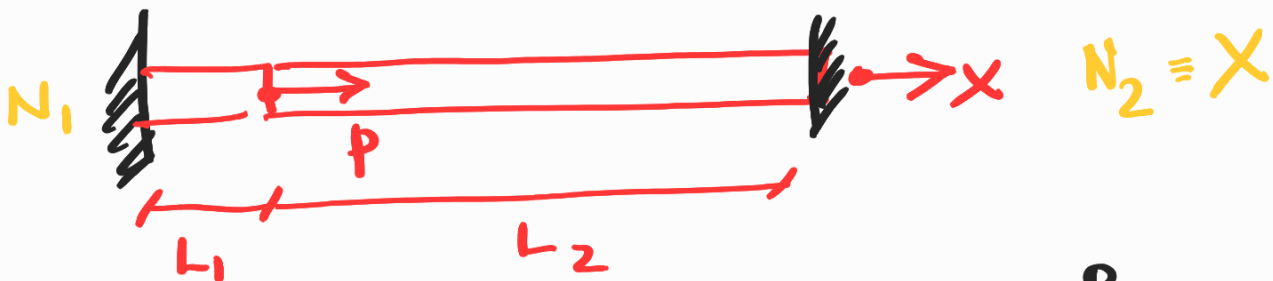
$$R_A + R_C = P$$

$$\begin{aligned} E_1 &= E_2 \\ A_1 &= A_2 \end{aligned}$$

 $\Rightarrow$ 

$$\frac{PL_1}{E_1 A_1} + \frac{X}{E_1 A_1} (L_1 + L_2) = 0$$

$$X = \frac{-P L_1 L_2}{(L_1 + L_2)}$$



$$N_1 + N_2 = P$$

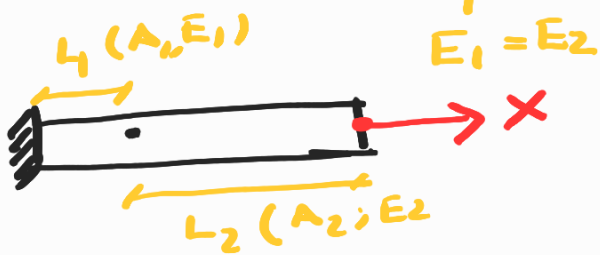
$$N_1 = \frac{-P L_2}{(L_1 + L_2)}$$

$$\frac{P L_2}{(L_1 + L_2)} + \frac{P L_1}{(L_1 + L_2)} = P$$

$$\frac{P L_2 + P L_1}{(L_1 + L_2)} = P$$

$$P \frac{(L_2 + L_1)}{(L_1 + L_2)} = P$$

(E2)



(N)



(L)



(E)

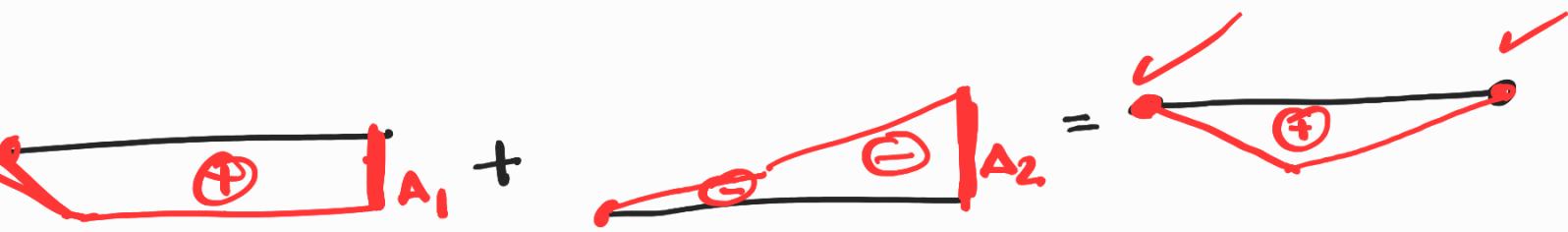


(Δ)



$$\Delta_2 = \frac{X L_1}{E_1 A_1} + \frac{X L_2}{E_2 A_2}$$

$X \rightarrow$  negative



$$\Delta_1 + \Delta_2 = 0$$