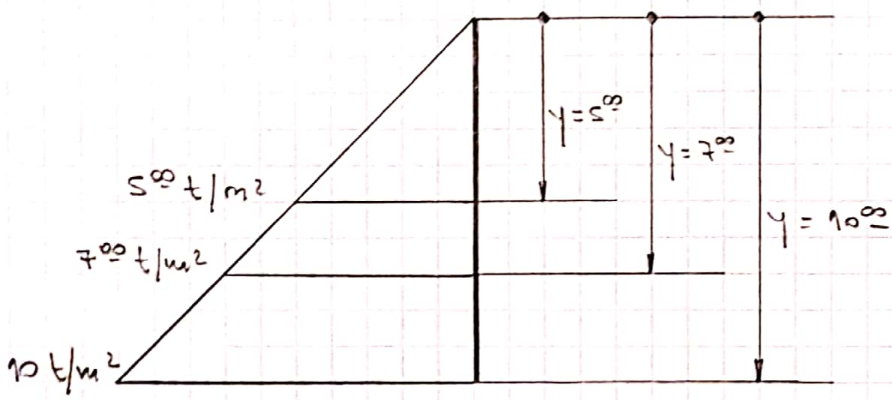
3.02.P

1)- DIAGRAMA DE PRESIONES, ACOTANDO VALORES SIGNIFICATIVOS:

$p(y) = \gamma_{liq} \cdot y$.

Esc. L = 1 : 200

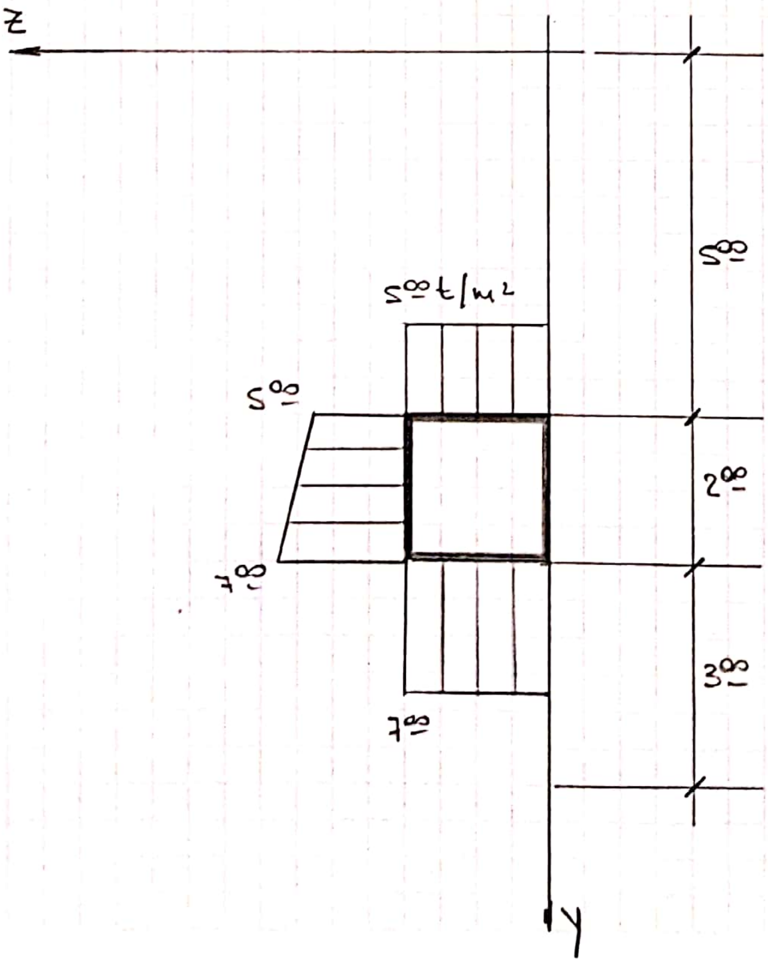
Esc. p = 2 t/m² / cm.



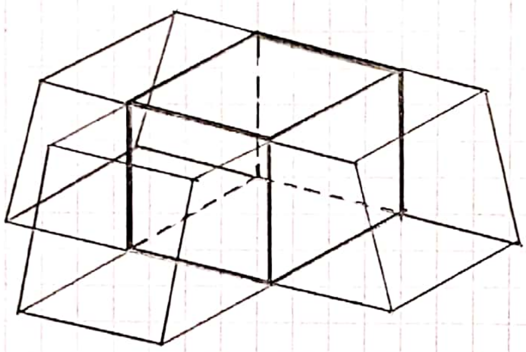
2)- DIAGRAMA DE CARGA "q_s", ACOTANDO VALORES SIGNIFICATIVOS:

Esc. L = 1:100

Esc. q_s = 4 t/m² / cm.



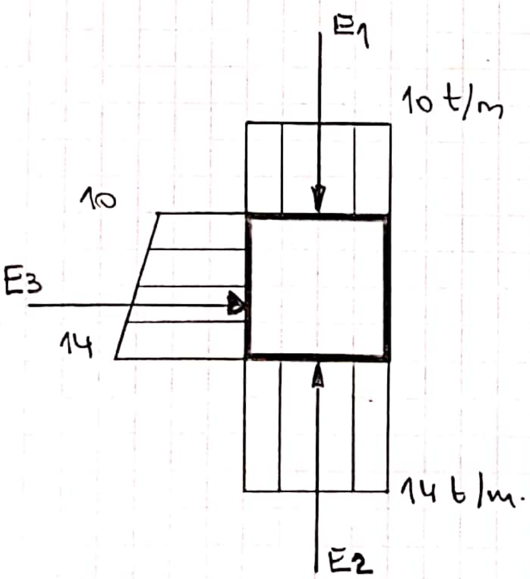
NOTA: Solamente se han representado las f_s en las caras verticales.



3)- DIAGRAMA DE LA CARGA q_L, ACOTANDO VALORES SIGNIFICATIVOS:

$$q_L = q_s \cdot b(x)$$

$$b(x) = b = cte = 2^{00}\ m.$$



NOTA: En las caras laterales, no representadas, los diagramas son iguales a frontal.

4) EMPUJE RESULTANTE SOBRE EL CUBO:

$$E = \int q_L \cdot dL.$$

Cara horizontal superior:

$$E_1 = 10\,t/m \times 2m = 20\,t\,\check{j}$$

Cara horizontal inferior:

$$E_2 = 14\,t/m \times 2m = -28\,t\,\check{j}.$$

Cara frontal:

$$E_{3.1} = 10\,t/m \times 2m = -20\,t\,\check{k}$$

$$E_{3.2} = 4\,t/m \times \frac{2m}{2} = -4\,t\,\check{k}$$

$$E_3 = E_{3.1} + E_{3.2} = -24\,t\,\check{k}.$$

$$y_3 = \left(-20 \cdot 1 - 4 \cdot \tfrac{2}{3}\, 2\right) / 24 + 5{,}00\,m.$$

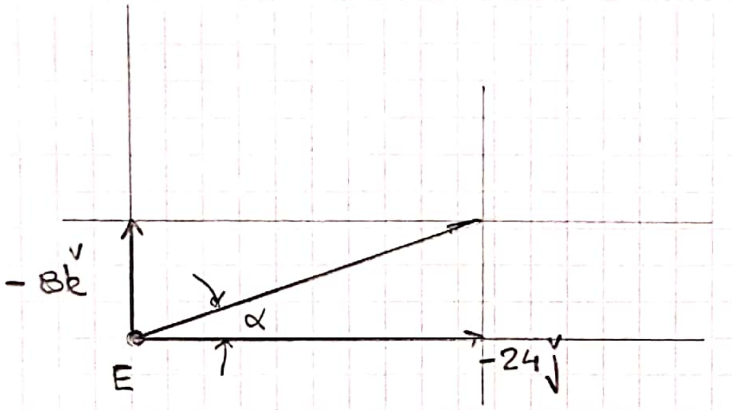
$$y_3 = 1{,}056 + 5{,}00 \rightarrow \underline{\underline{y_3 = 6{,}056}}$$

NOTA: Los empujes que se producen sobre las caras laterales se anulan mutuamente.

$$\boxed{E = -8t\,\check{j} - 24\,t\,\check{k}.}$$

PUNTOS DE LA RECTA DE ACCIÓN:

$$\underline{\underline{z_E = 1{,}00\,m}} \;;\; \underline{\underline{y_E = 6{,}056\,m}}$$



$$\operatorname{tg} \alpha = \frac{8}{24} = \frac{1}{3} \rightarrow \alpha = 18.435^\circ.$$

$$\varphi_E = 180^\circ + 18.435^\circ \rightarrow \underline{\underline{\varphi_E = 198.435^\circ.}}$$

ECUACIÓN DE LA RECTA DEL EMPUJE:

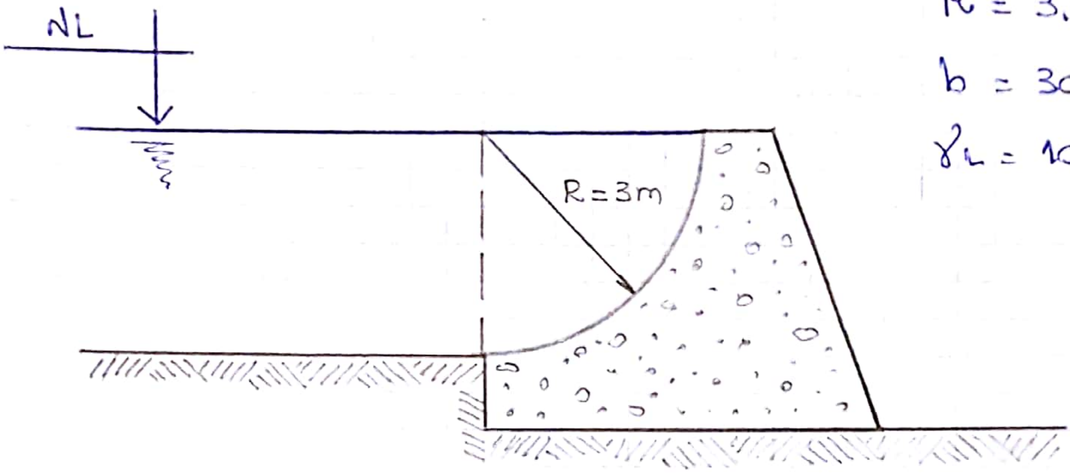
$$y = az + b.$$

$$6.056 = a \cdot 1.000 + b. \qquad a = +\frac{1}{3}.$$

$$6.056 = \frac{1}{3} \cdot 1.000 + b.$$

$$\underline{\underline{b = 5.723}}$$

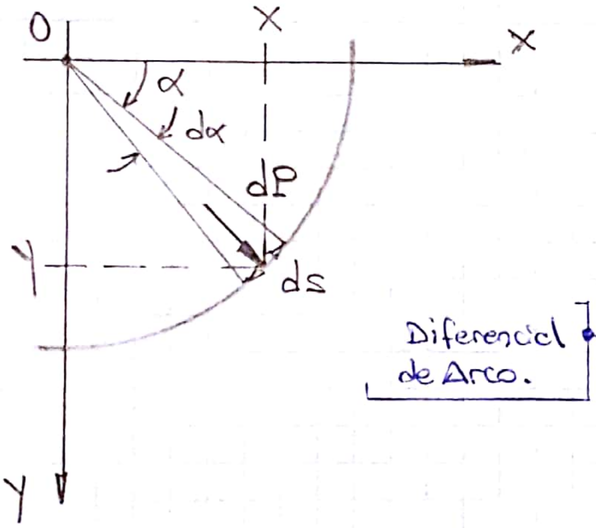
$$\boxed{y = \frac{z}{3} + 5.723.}$$



$R = 3{,}00\ m$

$b = 30{,}0\ m$

$\gamma_L = 10\ kN/m^3.$



$x = R \cdot \cos\alpha.$

$y = R \cdot \operatorname{sen}\alpha.$

$\underbrace{ds} = R \cdot d\alpha$

- **DIFERENCIAL DE FUERZA:**

$$dP = \gamma_L \cdot y \cdot b \cdot ds = \gamma_L \cdot \underbrace{R \cdot \operatorname{sen}\alpha}_{y} \cdot b \cdot \underbrace{R \cdot d\alpha}_{ds}$$

$$dP = \gamma_L \cdot b \cdot R^2 \cdot \operatorname{sen}\alpha \cdot d\alpha$$

- COMPONENTES DIFERENCIALES DE LA FUERZA:

$$\begin{cases} dP_x = dP \cdot \cos\alpha \\ dP_y = dP \cdot \operatorname{sen}\alpha \end{cases}$$

$$\begin{cases} dP_x = \gamma_L \cdot b \cdot R^2 \cdot \operatorname{sen}\alpha \cdot \cos\alpha \, d\alpha \\ dP_y = \gamma_L \cdot b \cdot R^2 \cdot \operatorname{sen}^2\alpha \cdot d\alpha \end{cases}$$

- COMPONENTES DE LA FUERZA:

$$P_x = \int dP_x = \int_0^{\pi/2} dP \cdot \cos\alpha = \int_0^{\pi/2} \gamma_L b R^2 \operatorname{sen}\alpha \cos\alpha \, d\alpha$$

$$P_x = \gamma_L b R^2 \int_0^{\pi/2} \operatorname{sen}\alpha \cos\alpha \, d\alpha = \gamma_L b R^2 \int_0^{\pi/2} \frac{2 \operatorname{sen}\alpha \cos\alpha}{2} \, d\alpha$$

$$P_x = \frac{\gamma_L b R^2}{2} \int_0^{\pi/2} \underbrace{2 \operatorname{sen}\alpha \cos\alpha}_{= \operatorname{sen} 2\alpha} \, d\alpha = \frac{\gamma_L b R^2}{2} \int_0^{\pi/2} \operatorname{sen} 2\alpha \, d\alpha$$

$$P_x = \frac{\gamma_L b R^2}{2 \cdot 2} \left(-\cos 2\alpha\right)\Big|_0^{\pi/2} = \frac{\gamma_L b R^2}{4} \left(-\cos 2\alpha\right)\Big|_0^{\pi/2}$$

$$P_x = \frac{\gamma_L b R^2}{4} \left[-\cos\left(2 \cdot \frac{\pi}{2}\right) - \left(-\cos 2 \cdot 0\right)\right]$$

$$P_x = \frac{\gamma_L b R^2}{4} \left[-\cos\pi + \cos 0\right] = \frac{\gamma_L b R^2}{4} \left[-(-1) + 1\right]$$

$$\boxed{P_x = \frac{\gamma_L b R^2}{2}}$$

¡NOTA:

$\rightarrow$ sea $\beta = 2\alpha$, luego: $\cos 2\alpha = \cos\beta$.

$$\rightarrow \frac{d\cos\beta}{d\alpha} = \frac{d\cos 2\alpha}{d\alpha} = -\operatorname{sen}\beta \cdot \frac{d\beta}{d\alpha} = -\operatorname{sen}\beta \cdot \frac{d(2\alpha)}{d\alpha} =$$

$$= -\operatorname{sen}(2\alpha) \cdot 2 = -2 \cdot \operatorname{sen}(2\alpha)$$

Por lo tanto:

$$-\frac{1}{2}\frac{d\cos\beta}{d\alpha} = \operatorname{sen}\beta \rightarrow -\frac{1}{2}\frac{d\cos(2\alpha)}{d\alpha} = \operatorname{sen}(2\alpha)$$

$$\frac{d\left[-\frac{1}{2}\cdot\cos(2\alpha)\right]}{d\alpha} = \operatorname{sen}(2\alpha) \rightarrow \underline{\int \operatorname{sen}(2\alpha) = -\frac{1}{2}\cos(2\alpha)}$$

$$P_y = \int dP_y = \int_0^{\pi/2} dP \cdot \operatorname{sen}\alpha = \int_0^{\pi/2} \gamma_L b R^2 \cdot \operatorname{sen}^2\alpha \, d\alpha$$

$$P_y = \gamma_L b R^2 \int_0^{\pi/2} \operatorname{sen}^2\alpha \, d\alpha$$

Teniendo en cuenta que:

$$\begin{cases} \operatorname{sen}^2\alpha + \cos^2\alpha = 1. \\ \operatorname{sen}^2\alpha - \cos^2\alpha = \cos 2\alpha. \end{cases}$$

sumando m.a m. $\quad 2\operatorname{sen}^2\alpha = 1 + \cos 2\alpha \rightarrow \operatorname{sen}^2\alpha = \frac{1}{2} + \frac{1}{2}\cos 2\alpha$

$$P_y = \gamma_L \cdot b \cdot R^2 \int_0^{\pi/2} \frac{1}{2}(1 + \cos 2\alpha)\, d\alpha = \gamma_L b R^2 \left[\int_0^{\pi/2} \frac{1}{2} d\alpha + \int_0^{\pi/2} \frac{\cos 2\alpha}{2} d\alpha\right]$$

$$P_y = \gamma_L b R^2 \left[\frac{1}{2}\alpha\Big|_0^{\pi/2} + \frac{\operatorname{sen}(2\alpha)}{4}\Big|_0^{\pi/2}\right]$$

$$P_y = \gamma_L b R^2 \left[\left(\frac{1}{2}\frac{\pi}{2} - \frac{1}{2}\cdot 0 \right) + \left(\frac{\text{sen}(2\cdot\pi/2)}{4} - \frac{\text{sen}(2\cdot 0)}{4} \right) \right]$$

$$P_y = \gamma_L b . R^2 \left[\left(\frac{\pi}{4} - 0 \right) + (0 - 0) \right]$$

$$\boxed{P_y = \frac{\pi \gamma_L b R^2}{4}}$$

~~Fuerza Resultante~~

NOTA:

→ SEA: $\beta = 2\alpha$; LUEGO $\text{sen}\,\beta = \text{sen}(2\alpha)$

→ $$\frac{d\,\text{sen}\,\beta}{d\alpha} = \frac{d\,\text{sen}(2\alpha)}{d\alpha} = \cos\beta\,\frac{d\beta}{d\alpha} = \cos(2\alpha)\,\frac{d(2\alpha)}{d\alpha} =$$

$$= \cos(2\alpha) \cdot 2 = 2 . \cos 2\alpha .$$

→ POR LO TANTO:

$$\frac{d\,\text{sen}(2\alpha)}{d\alpha} = 2 . \cos(2\alpha)$$

$$\text{sen}(2\alpha) = \int 2\cos(2\alpha) = 2 . \int \cos 2\alpha$$

$$\frac{1}{2}\,\text{sen}(2\alpha) = \int \cos(2\alpha)$$

→ CON LO CUAL:

$$\int \frac{1}{2}\cos(2\alpha) = \frac{1}{2}\int \cos(2\alpha) = \frac{1}{2}\cdot\frac{1}{2}\,\text{sen}(2\alpha) = \frac{1}{4}\,\text{sen}\,2\alpha$$

- **FUERZA RESULTANTE $\bar{P} = \bar{R}$:**

$$P^2 = P_x^2 + P_y^2 \longrightarrow P = \sqrt{P_x^2 + P_y^2}$$

$$P = \sqrt{\frac{\gamma_L^2 b^2 R^4}{4} + \frac{\pi^2 \gamma_L^2 b^2 R^4}{16}}$$

$$P = \sqrt{\frac{\gamma_L^2 b^2 R^4}{4}\left(1 + \frac{\pi^2}{4}\right)} = \frac{\gamma_L b R^2}{2}\sqrt{1 + \frac{\pi^2}{4}}$$

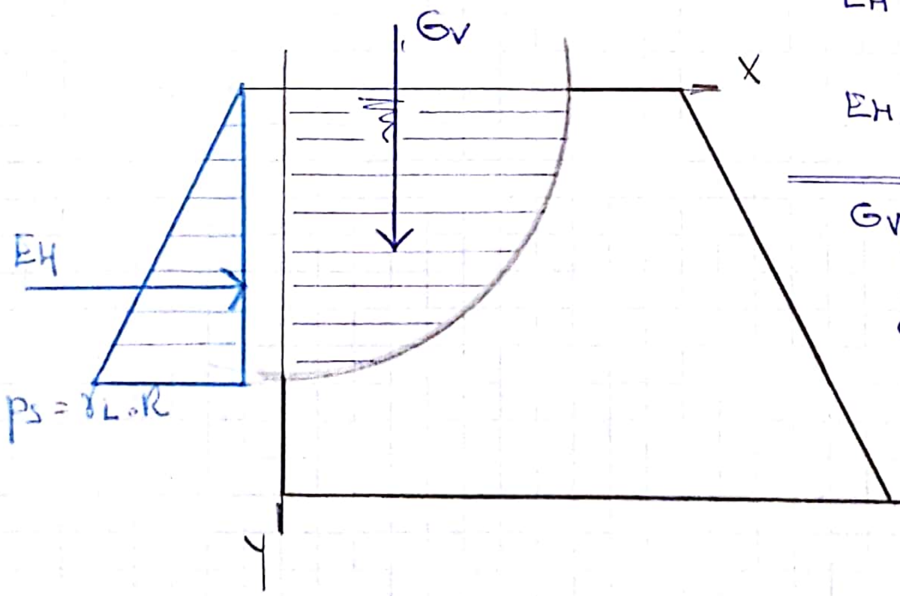
REEMPLAZANDO VALORES:

$$P = R = \frac{10\ kN/m^3 \cdot 30m \cdot (3m)^2}{2}\sqrt{1 + \frac{\pi^2}{4}}$$

$$P = R = 1350\ kN \cdot (1,8621)$$

$$\boxed{P = R = 2513,83\ kN.}$$

- **OTRA FORMA DE CALCULAR**



$$E_H = \frac{\gamma_L . R . R . b}{2}$$

$$E_H = \frac{\gamma_L b R^2}{2}$$

$$G_V = \frac{\pi R^2}{4} . \gamma_L . b$$

$$G_V = \frac{\pi \gamma_L b R^2}{4}$$

PESO DEL CUARTO DE CÍRCULO POR LA LONGITUD 'b'!