Mecanica del Solido I-Estabilidad I
Clase 1

Mecánica del continuo

Es la rama de la mecánica que estudia el movimiento de sólidos, líquidos y gases bajo la hipótesis de medio continuo.

Esta idealización no tiene en cuenta la estructura atómica ó molecular.

Repaso de Cálculo Vectorial
Contenidos

Operaciones

Componentes Cartesianas dos y tres direcciones

Vector Posición

Producto Escalar

Producto Vectorial

Duración: 2 horas

Bibliografía: Estática Bedford-Fowler

Vectores - Operaciones

Suma Vectorial:

Regla del paralelogramo:

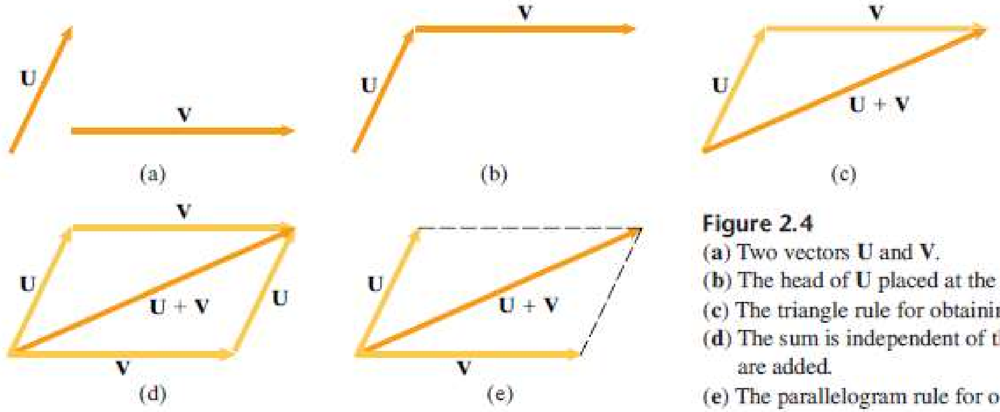
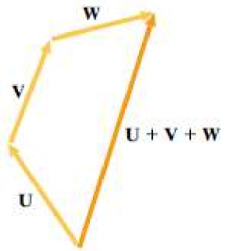


Figure 2.4
(a) Two vectors **U** and **V**.
(b) The head of **U** placed at the tail of **V**.
(c) The triangle rule for obtaining the sum of **U** and **V**.
(d) The sum is independent of the order in which the vectors are added.
(e) The parallelogram rule for obtaining the sum of **U** and **V**.

Propiedades:

Conmutativa: ***U + V = V + U***

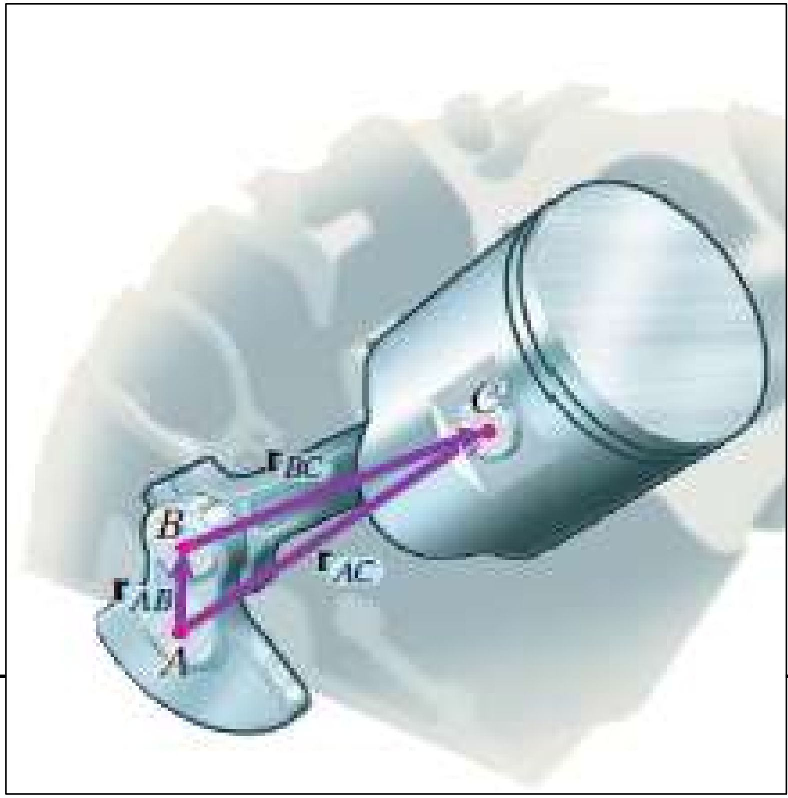
Asociativa : ***(U*** *+* ***V)*** *+* ***W*** *=* ***U*** *+****(V*** *+* ***W****)*



Vectores: Operaciones

Vector Posición de un punto en el Espacio respecto de otro.

$\boldsymbol{r}_{AB}$ = *Coordenada de B – Coordenada de A*



Vectores: Operaciones

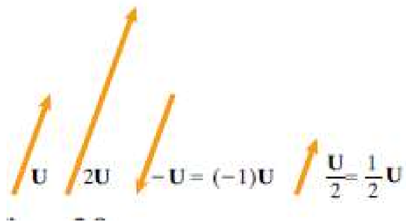
*Multiplicación de un vector por un escalar (a**U**): Es un vector. Siendo “a” escalar y **U** vector,*

*su magnitud es |a||**U**| o sea amplifica o reduce la magnitud de **U** según el valor de “a”.*

Pero la dirección o sentido depende del signo de “a”, si es positiva, mantiene dirección y sentido, si “a” es negativa tiene sentido opuesto.

*Dividir un vector **U** por un escalar “a” se define:*

$$\frac{\mathbf{U}}{a} = \left(\frac{1}{a}\right)\mathbf{U}.$$



Vectores: Operaciones

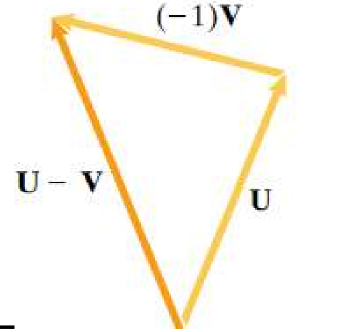
Propiedades de la Multiplicación por Escalar:

Asociativo respecto a l $a(b\mathbf{U}) = (ab)\mathbf{U},$ *ın escalar:*

Distributivo respect $(a + b)\mathbf{U} = a\mathbf{U} + b\mathbf{U},$

Asociativa con resp $a(\mathbf{U} + \mathbf{V}) = a\mathbf{U} + a\mathbf{V},$

$$\mathbf{U} - \mathbf{V} = \mathbf{U} + (-1)\mathbf{V}.$$

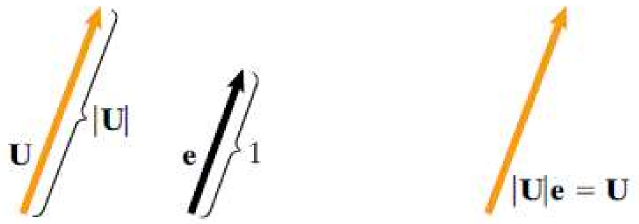


Resta Vectorial:

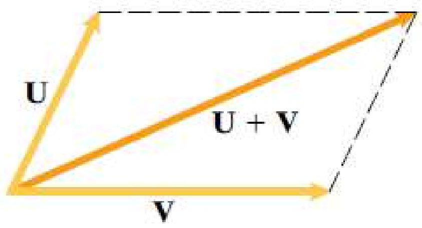
Vectores: Operaciones

Vectores Unitarios o versor: vector cuya magnitud es 1.

$$\mathbf{U} = |\mathbf{U}|\mathbf{e}. \qquad \frac{\mathbf{U}}{|\mathbf{U}|} = \mathbf{e},$$



Componentes Vectoriales: Un vector es la suma de dos vectores. Los vectores cuya suma nos da el vector son las componentes del mismo:



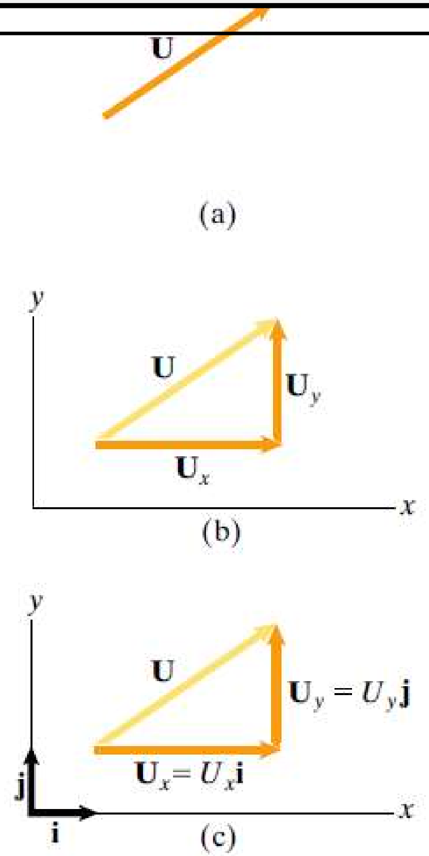
Vectores: Componentes Cartesianas

En Dos Direcciones:

$$\mathbf{U} = \mathbf{U}_x + \mathbf{U}_y$$

$$\mathbf{U} = U_x\mathbf{i} + U_y\mathbf{j}.$$

$$|\mathbf{U}| = \sqrt{U_x^2 + U_y^2}.$$

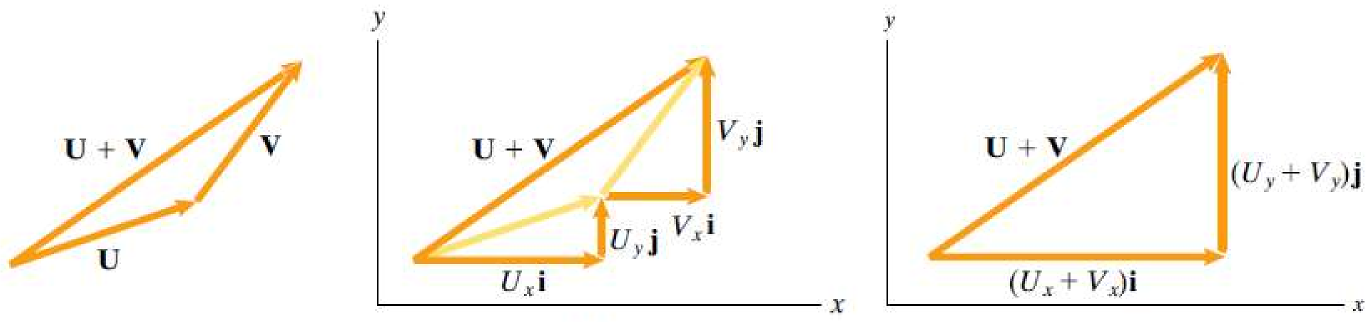


Vectores: Componentes Cartesianas

Operaciones:

Suma Ve

$$\mathbf{U} + \mathbf{V} = (U_x\mathbf{i} + U_y\mathbf{j}) + (V_x\mathbf{i} + V_y\mathbf{j})$$
$$= (U_x + V_x)\mathbf{i} + (U_y + V_y)\mathbf{j}.$$



(a) (b) (c)

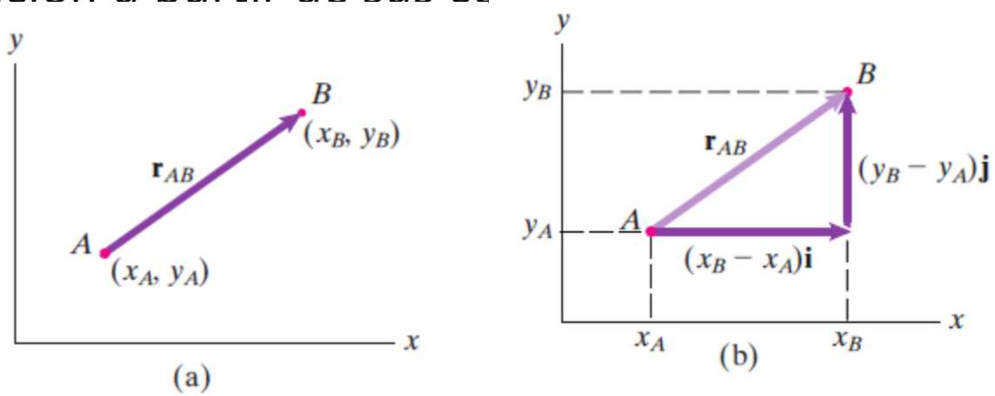
Vectores: Componentes Cartesianas

Operaciones:

Multiplicacion $a\mathbf{U} = a(U_x\mathbf{i} + U_y\mathbf{j}) = aU_x\mathbf{i} + aU_y\mathbf{j}.$

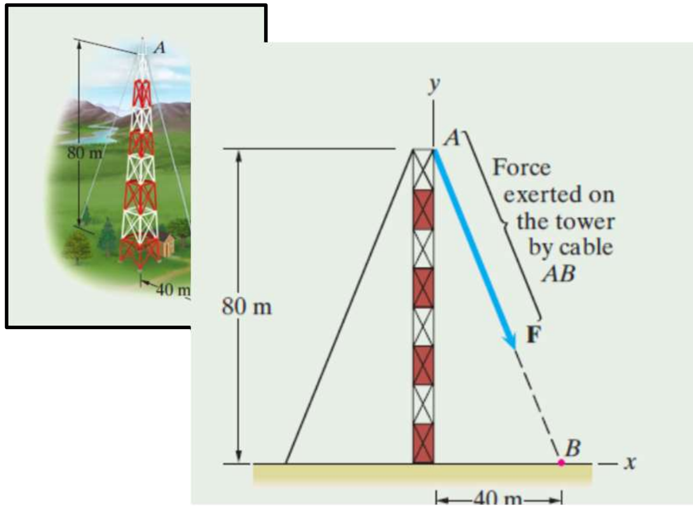
$$\mathbf{r}_{AB} = (x_B - x_A)\,\mathbf{i} + (y_B - y_A)\,\mathbf{j}.$$

Vectores posición a partir de sus componentes:



Ejemplo:

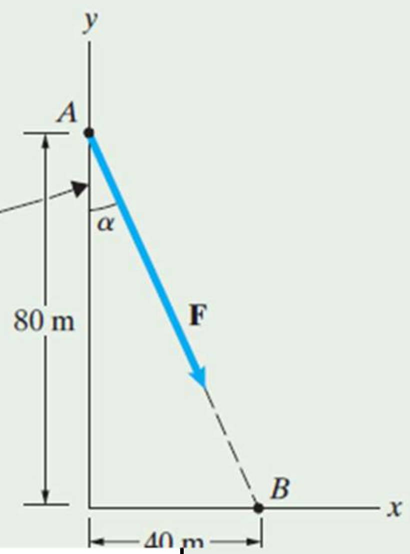
The cable from point A to point B exerts a 900-N force on the top of the television transmission tower that is represented by the vector $\mathbf{F}$. Express $\mathbf{F}$ in terms of components using the coordinate system shown.



First Method

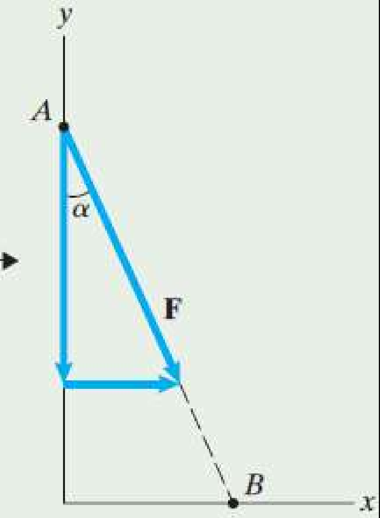
Determine the angle between **F** and the y axis:

$$\alpha = \arctan\left(\frac{40}{80}\right) = 26.6°.$$



Use trigonometry to determine **F** in terms of its components:

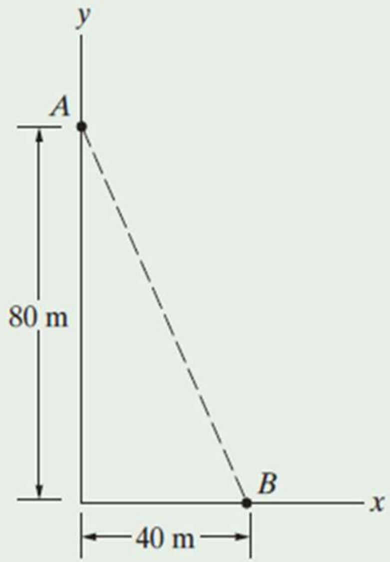
$$\begin{aligned}\mathbf{F} &= |\mathbf{F}|\sin\alpha\,\mathbf{i} - |\mathbf{F}|\cos\alpha\,\mathbf{j}\\ &= 900\sin 26.6°\,\mathbf{i} - 900\cos 26.6°\,\mathbf{j}\,(\mathrm{N})\\ &= 402\mathbf{i} - 805\mathbf{j}\,(\mathrm{N}).\end{aligned}$$



Second Method

Using the given dimensions, calculate the distance from A to B:

$$\sqrt{(40\text{ m})^2 + (80\text{ m})^2} = 89.4\text{ m}.$$

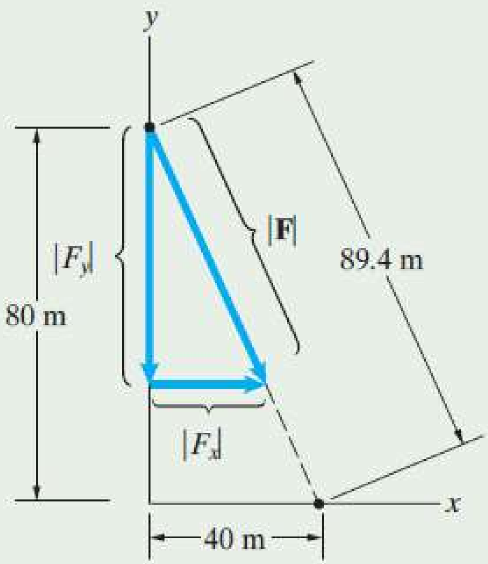


Use similar triangles to determine the components of $\mathbf{F}$:

$$\frac{|F_x|}{|\mathbf{F}|} = \frac{40\text{ m}}{89.4\text{ m}} \quad \text{and} \quad \frac{|F_y|}{|\mathbf{F}|} = \frac{80\text{ m}}{89.4\text{ m}},$$

so

$$\mathbf{F} = \frac{40}{89.4}(900\text{ N})\mathbf{i} - \frac{80}{89.4}(900\text{ N})\mathbf{j}$$
$$= 402\mathbf{i} - 805\mathbf{j}\,(\text{N}).$$



Método Recomendado

Tercer método El vector $\mathbf{r}_{AB}$ en la figura (b) es

$$\mathbf{r}_{AB} = (x_B - x_A)\,\mathbf{i} + (y_B - y_A)\,\mathbf{j} = (40 - 0)\,\mathbf{i} + (0 - 80)\,\mathbf{j}$$

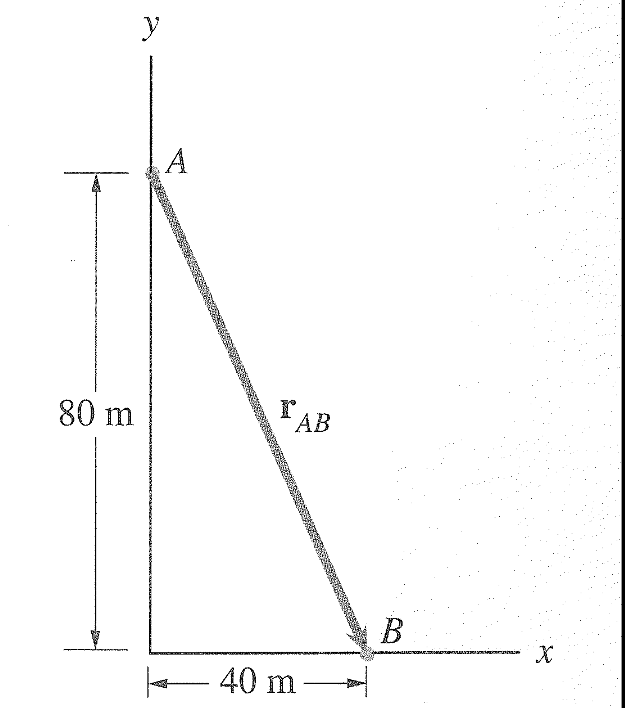
$$= 40\,\mathbf{i} - 80\,\mathbf{j} \ \ \text{(m)}.$$

Dividimos ahora este vector entre su magnitud para obtener un vector unitario $\mathbf{e}_{AB}$ que tiene la misma dirección que la fuerza $\mathbf{F}$ (Fig. c):

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{40\,\mathbf{i} - 80\,\mathbf{j}}{\sqrt{(40)^2 + (-80)^2}} = 0.447\,\mathbf{i} - 0.894\,\mathbf{j}.$$

La fuerza $\mathbf{F}$ es igual al producto de su magnitud $|\mathbf{F}|$ y $\mathbf{e}_{AB}$:

$$\mathbf{F} = |\mathbf{F}|\,\mathbf{e}_{AB} = (800)(0.447\,\mathbf{i} - 0.894\,\mathbf{j}) = 357.8\,\mathbf{i} - 715.5\,\mathbf{j} \ \ \text{(N)}.$$



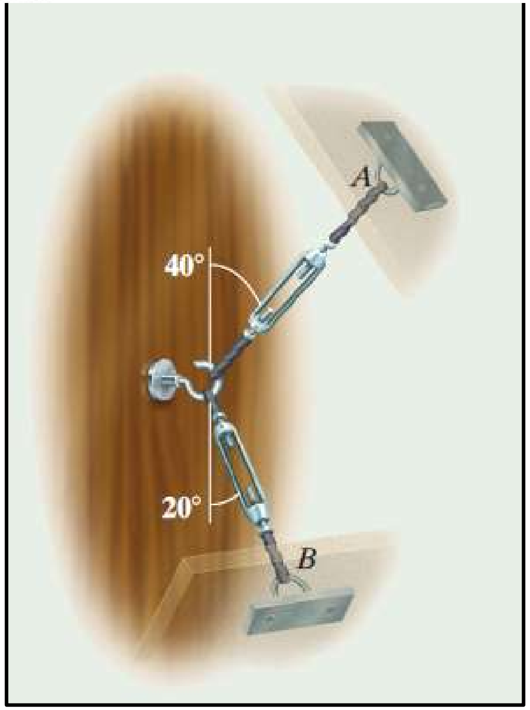
(b) Vector $\mathbf{r}_{AB}$ de A a B.

Ejercicio

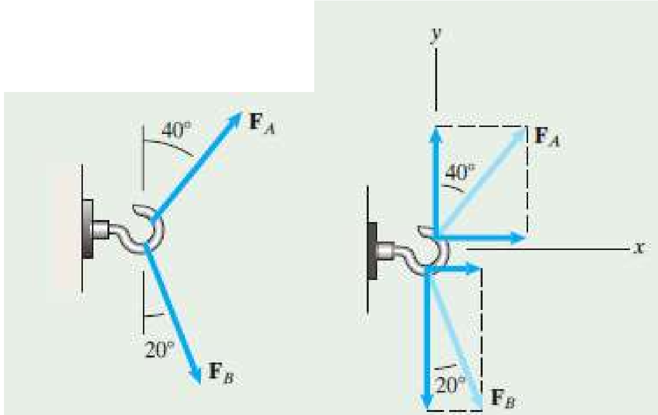
Los cables A y B de la figura 2.18 ejercen fuerzas $\mathbf{F}_A$ y $\mathbf{F}_B$ sobre el gancho. La magnitud de $\mathbf{F}_A$ es de 100 lb. La tensión en el cable B se ha ajustado para que la fuerza total $\mathbf{F}_A + \mathbf{F}_B$ sea perpendicular a la pared a la que está unido el gancho.

(a) ¿Cuál es la magnitud de $\mathbf{F}_B$?

(b) ¿Cuál es la magnitud de la fuerza total ejercida por los dos cables sobre el gancho?



$$|\mathbf{F}_A| \cos 40° = |\mathbf{F}_B| \cos 20°.$$



(a) En términos del sistema coordenado de la figura (a), las componentes de $\mathbf{F}_A$ y $\mathbf{F}_B$ son

$$\mathbf{F}_A = |\mathbf{F}_A| \text{ sen } 40° \mathbf{i} + |\mathbf{F}_A| \cos 40° \; \mathbf{j},$$

$$\mathbf{F}_B = |\mathbf{F}_B| \text{ sen } 20° \mathbf{i} + |\mathbf{F}_B| \cos 20° \; \mathbf{j}.$$

La fuerza total es

$$\mathbf{F}_A + \mathbf{F}_B = (|\mathbf{F}_A| \text{ sen } 40° - |\mathbf{F}_B| \text{ sen } 20°)\mathbf{i}$$
$$+ (|\mathbf{F}_A| \cos 40° - |\mathbf{F}_B| \cos 20°)\mathbf{j}.$$

Igualando a cero la componente de la fuerza total paralela a la pared (la componente y),

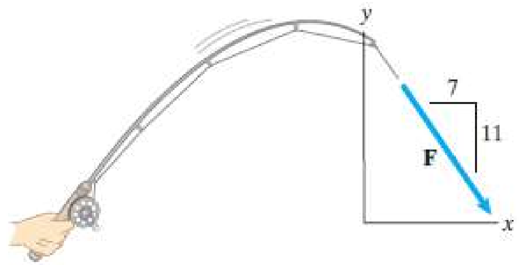
$$|\mathbf{F}_A| \cos 40° - |\mathbf{F}_B| \cos 20° = 0,$$

obtenemos una ecuación para la magnitud de $\mathbf{F}_B$:

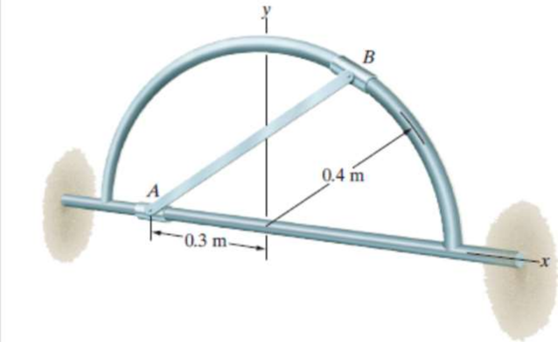
$$|\mathbf{F}_B| = \frac{|\mathbf{F}_A| \cos 40°}{\cos 20°} = \frac{(100) \cos 40°}{\cos 20°} = 81.5 \text{ lb}.$$

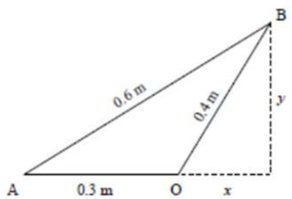
Ejercicios

2.23 A fish exerts a 10-lb force on the line that is represented by the vector **F**. Express **F** in terms of components using the coordinate system shown.

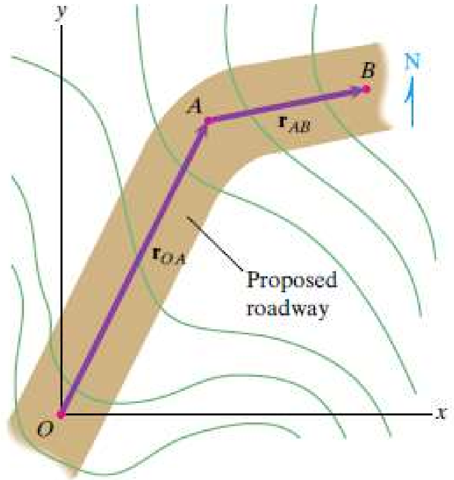


2.38 The length of the bar AB is 0.6 m. Determine the components of a unit vector $\mathbf{e}_{AB}$ that points from point A toward point B.





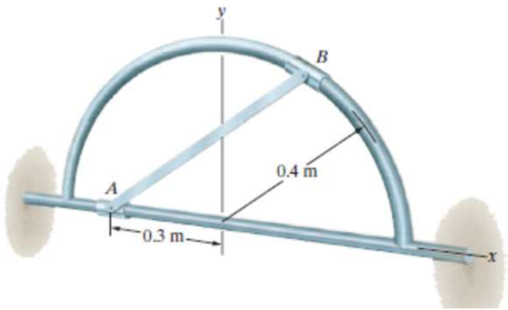
2.34 A surveyor measures the location of point A and determines that $\mathbf{r}_{OA} = 400\mathbf{i} + 800\mathbf{j}$ (m). He wants to determine the location of a point B so that $|\mathbf{r}_{AB}| = 400$ m and $|\mathbf{r}_{OA} + \mathbf{r}_{AB}| = 1200$ m. What are the cartesian coordinates of point B?



Ley del coseno =

Ejercicios:

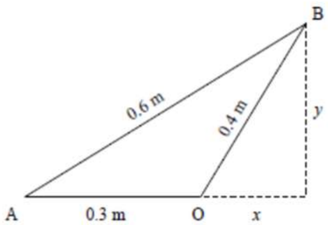
Problem 2.38 The length of the bar AB is 0.6 m. Determine the components of a unit vector $\mathbf{e}_{AB}$ that points from point A toward point B.



We have the two equations

$$(0.3\text{ m} + x)^2 + y^2 = (0.6\text{ m})^2$$

$$x^2 + y^2 = (0.4\text{ m})^2$$



Solving we find

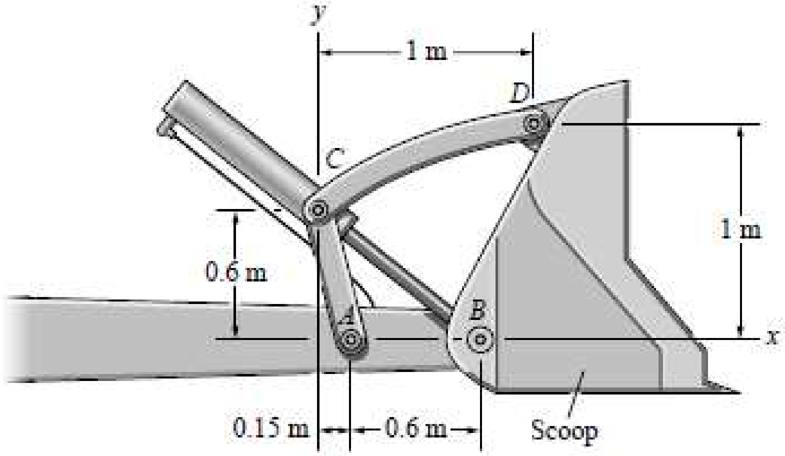
$$x = 0.183\text{ m}, \quad y = 0.356\text{ m}$$

Thus

$$\boxed{\begin{aligned}\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} &= \frac{(0.183\text{ m} - [-0.3\text{ m}])\mathbf{i} + (0.356\text{ m})\mathbf{j}}{\sqrt{(0.183\text{ m} + 0.3\text{ m})^2 + (0.356\text{ m})^2}} \\ &= (0.806\mathbf{i} + 0.593\mathbf{j})\end{aligned}}$$

Ejercicios:

Problem 2.39 Determine the components of a unit vector that is parallel to the hydraulic actuator BC and points from B toward C.



Solution: Point B is at (0.75, 0) and point C is at (0, 0.6). The vector

$$\mathbf{r}_{BC} = (x_C - x_B)\mathbf{i} + (y_C - y_B)\mathbf{j}$$

$$\mathbf{r}_{BC} = (0 - 0.75)\mathbf{i} + (0.6 - 0)\mathbf{j} \text{ (m)}$$

$$\mathbf{r}_{BC} = -0.75\mathbf{i} + 0.6\mathbf{j} \text{ (m)}$$

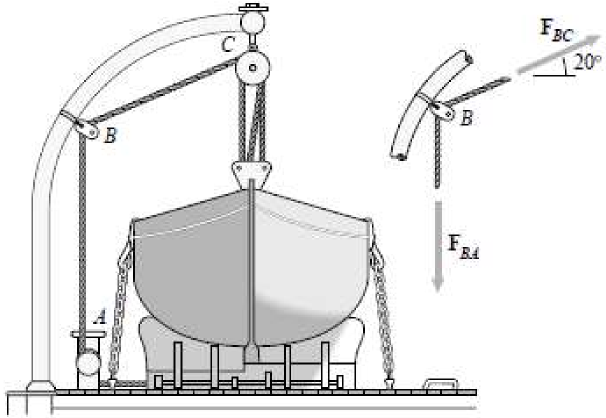
$$|\mathbf{r}_{BC}| = \sqrt{(0.75)^2 + (0.6)^2} = 0.960 \text{ (m)}$$

$$\mathbf{e}_{BC} = \frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = \frac{-0.75}{0.96}\mathbf{i} + \frac{0.6}{0.96}\mathbf{j}$$

$$\mathbf{e}_{BC} = -0.781\mathbf{i} + 0.625\mathbf{j}$$

Ejercicios:

Problem 2.44 The rope ABC exerts forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BC}$ on the block at B. Their magnitudes are equal: $|\mathbf{F}_{BA}| = |\mathbf{F}_{BC}|$. The magnitude of the total force exerted on the block at B by the rope is $|\mathbf{F}_{BA} + \mathbf{F}_{BC}| = 920$ N. Determine $|\mathbf{F}_{BA}|$ by expressing the forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BC}$ in terms of components.



Solution:

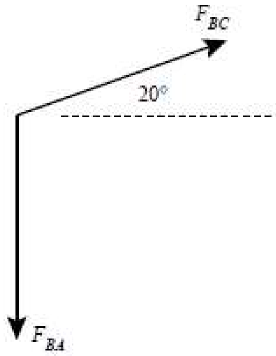
$\mathbf{F}_{BC} = F(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$

$\mathbf{F}_{BA} = F(-\mathbf{j})$

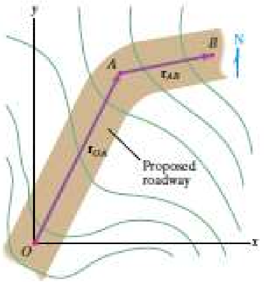
$\mathbf{F}_{BC} + \mathbf{F}_{BA} = F(\cos 20°\mathbf{i} + [\sin 20° - 1]\mathbf{j})$

Therefore

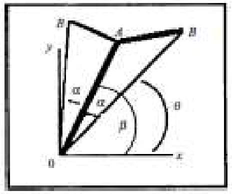
$$\boxed{(920\ \text{N})^2 = F^2(\cos^2 20° + [\sin 20° - 1]^2) \Rightarrow F = 802\ \text{N}}$$



Problem 2.34 A surveyor measures the location of point A and determines that $\mathbf{r}_{OA} = 400\mathbf{i} + 800\mathbf{j}$ (m). He wants to determine the location of a point B so that $|\mathbf{r}_{AB}| = 400$ m and $|\mathbf{r}_{OA} + \mathbf{r}_{AB}| = 1200$ m. What are the cartesian coordinates of point B?



Solution: Two possibilities are: The point B lies west of point A, or point B lies east of point A, as shown. The strategy is to determine the unknown angles α, β, and θ. The magnitude of OA is



$$|\mathbf{r}_{OA}| = \sqrt{(400)^2 + (800)^2} = 894.4.$$

The angle β is determined by

$$\tan\beta = \frac{800}{400} = 2, \quad \beta = 63.4^\circ.$$

The angle α is determined from the cosine law.

$$\cos\alpha = \frac{(894.4)^2 + (1200)^2 - (400)^2}{2(894.4)(1200)} = 0.9689.$$

$\alpha = 14.3^\circ$. The angle θ is $\theta = \beta \pm \alpha = 49.12^\circ, 77.74^\circ$.

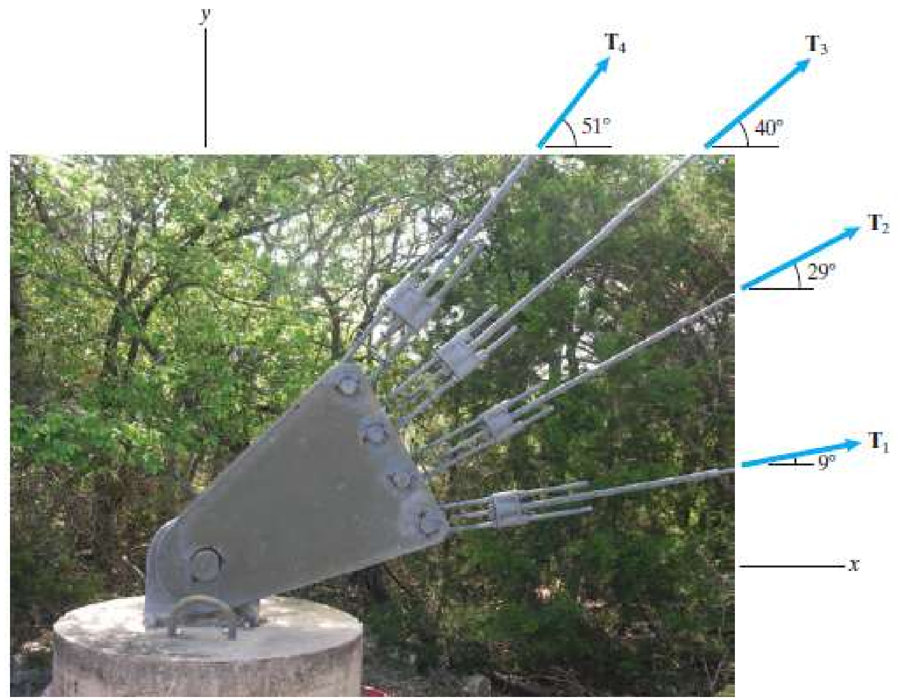
The two possible sets of coordinates of point B are

$$\begin{cases} \mathbf{r}_{OB} = 1200(\mathbf{i}\cos 77.7 + \mathbf{j}\sin 77.7) = 254.67\mathbf{i} + 1172.66\mathbf{j} \text{ (m)} \\ \mathbf{r}_{OB} = 1200(\mathbf{i}\cos 49.1 + \mathbf{j}\sin 49.1) = 785.33\mathbf{i} + 907.34\mathbf{j} \text{ (m)} \end{cases}$$

The two possibilities lead to B(254.7 m, 1172.7 m) or B(785.3 m, 907.3 m)

Ejercicios:

Problem 2.42 The magnitudes of the forces exerted by the cables are $|\mathbf{T}_1| = 2800$ lb, $|\mathbf{T}_2 = 3200$ lb, $|\mathbf{T}_3| = 4000$ lb, and $|\mathbf{T}_4| = 5000$ lb. What is the magnitude of the total force exerted by the four cables?



$$T_x = |\mathbf{T}_1| \cos 9^\circ + |\mathbf{T}_2| \cos 29^\circ |\mathbf{T}_3| \cos 40^\circ + |\mathbf{T}_4| \cos 51^\circ$$

$$T_x = (2800 \text{ lb}) \cos 9^\circ + (3200 \text{ lb}) \cos 29^\circ + (4000 \text{ lb}) \cos 40^\circ + (5000 \text{ lb}) \cos 51^\circ$$

$$T_x = 11{,}800 \text{ lb}$$

The y-component of the total force is

$$T_y = |\mathbf{T}_1| \sin 9^\circ + |\mathbf{T}_2| \sin 29^\circ + |\mathbf{T}_3| \sin 40^\circ + |\mathbf{T}_4| \sin 51^\circ$$

$$T_y = (2800 \text{ lb}) \sin 9^\circ + (3200 \text{ lb}) \sin 29^\circ + (4000 \text{ lb}) \sin 40^\circ + (5000 \text{ lb}) \sin 51^\circ$$

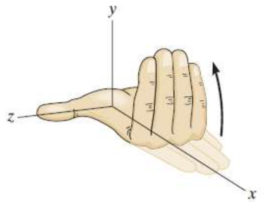
$$T_y = 8450 \text{ lb}$$

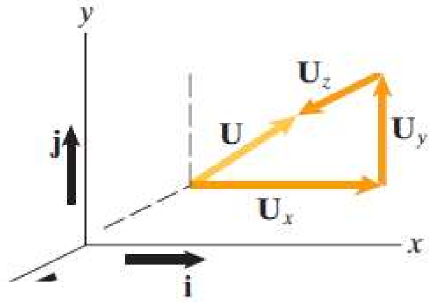
The magnitude of the total force is

$$|\mathbf{T}| = \sqrt{{T_x}^2 + {T_y}^2} = \sqrt{(11{,}800 \text{ lb})^2 + (8450 \text{ lb})^2} = 14{,}500 \text{ lb} \quad \boxed{|\mathbf{T}| = 14{,}500 \text{ lb}}$$

Vectores: Componentes en tres direcciones

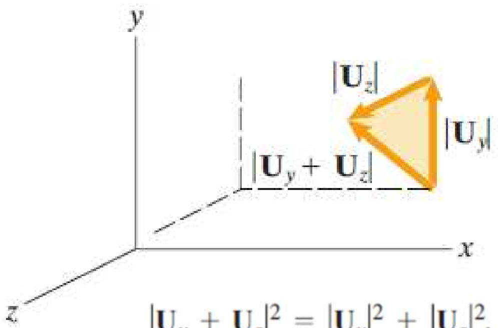
Regla de la mano derecha

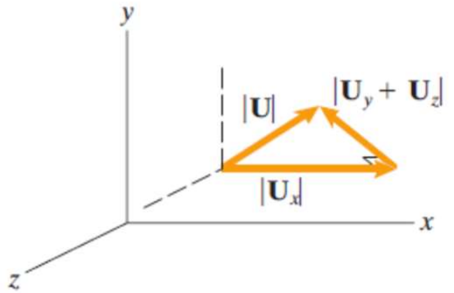




$$\mathbf{U} = \mathbf{U}_x + \mathbf{U}_y + \mathbf{U}_z.$$

$$\mathbf{U} = U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}.$$



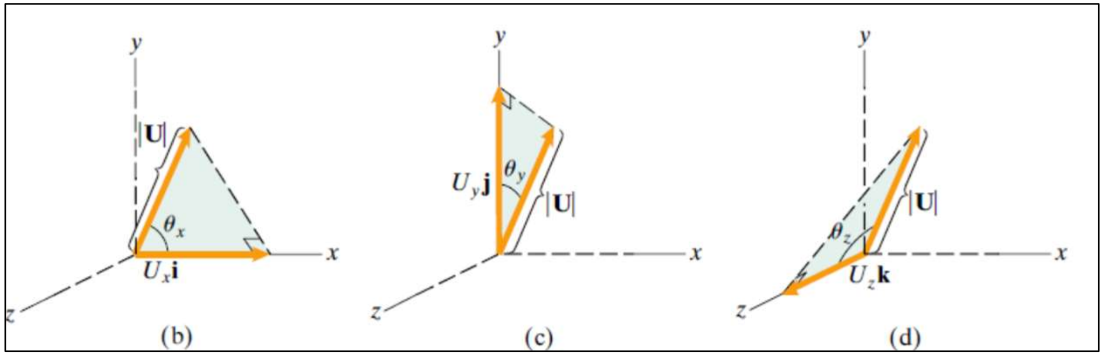


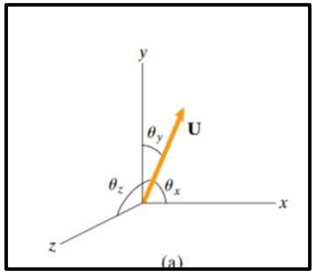
$$|\mathbf{U}_y + \mathbf{U}_z|^2 = |\mathbf{U}_y|^2 + |\mathbf{U}_z|^2.$$

$$|\mathbf{U}|^2 = |\mathbf{U}_x|^2 + |\mathbf{U}_y + \mathbf{U}_z|^2.$$

$$|\mathbf{U}|^2 = |\mathbf{U}_x|^2 + |\mathbf{U}_y|^2 + |\mathbf{U}_z|^2 = U_x^2 + U_y^2 + U_z^2.$$

Vectores: Tridimensional Cosenos Directores





$$|\mathbf{U}| = \sqrt{U_x^2 + U_y^2 + U_z^2}. \qquad U_x = |\mathbf{U}|\cos\theta_x, \quad U_y = |\mathbf{U}|\cos\theta_y, \quad U_z = |\mathbf{U}|\cos\theta_z.$$

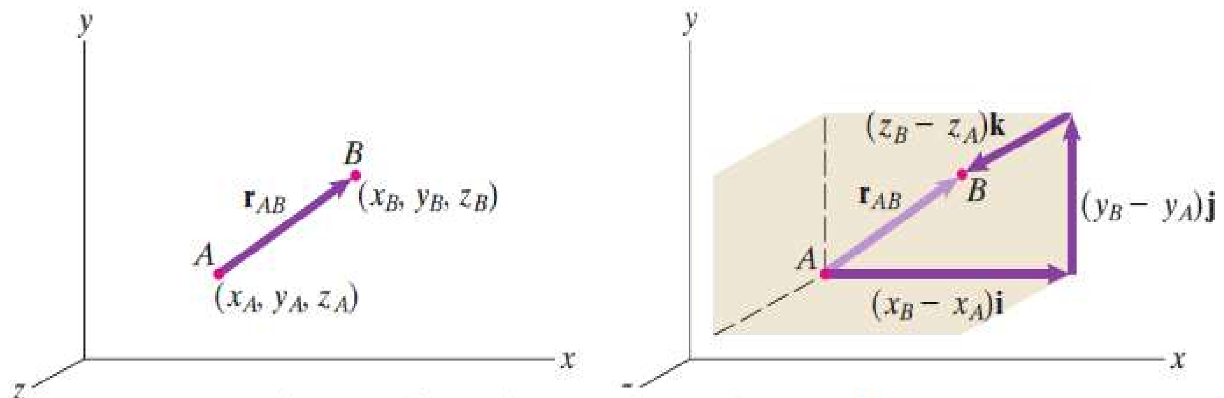
$$|\mathbf{U}|^2 = |\mathbf{U}_x|^2 + |\mathbf{U}_y|^2 + |\mathbf{U}_z|^2 = U_x^2 + U_y^2 + U_z^2. \quad ===\Rightarrow \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1.$$

$$\mathbf{U} = |\mathbf{U}|\mathbf{e}. \qquad U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k} = |\mathbf{U}|(e_x\mathbf{i} + e_y\mathbf{j} + e_z\mathbf{k} \qquad U_x = |\mathbf{U}|e_x, \quad U_y = |\mathbf{U}|e_y, \quad U_z = |\mathbf{U}|e_z.$$

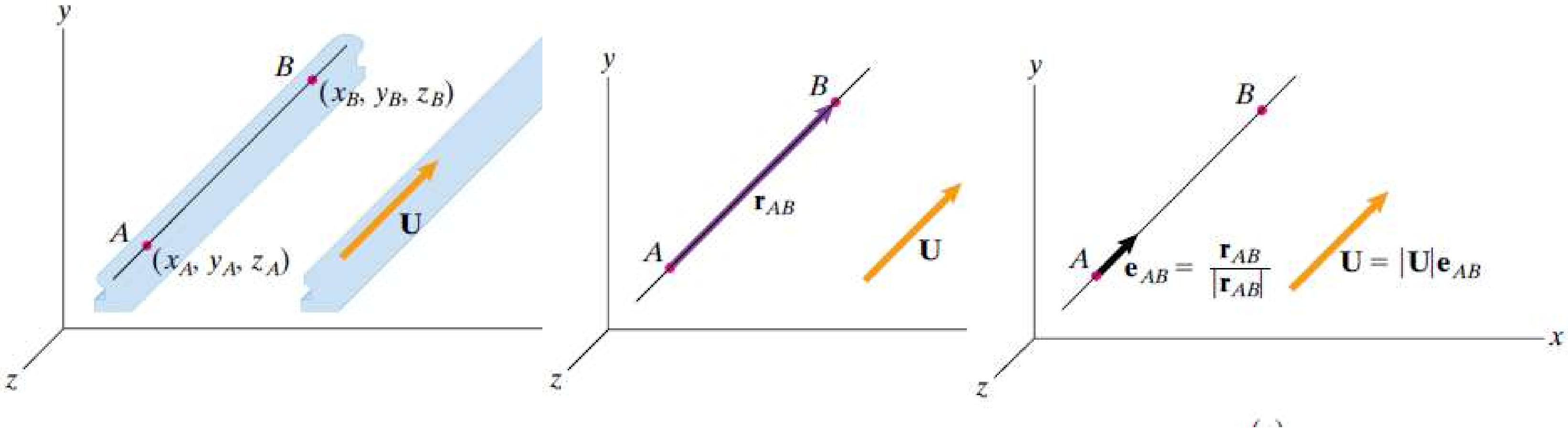
$$\boxed{\cos\theta_x = e_x, \quad \cos\theta_y = e_y, \quad \cos\theta_z = e_z.}$$

Componentes del versor unitario asociado a **U**

Vectores: Vector Posición

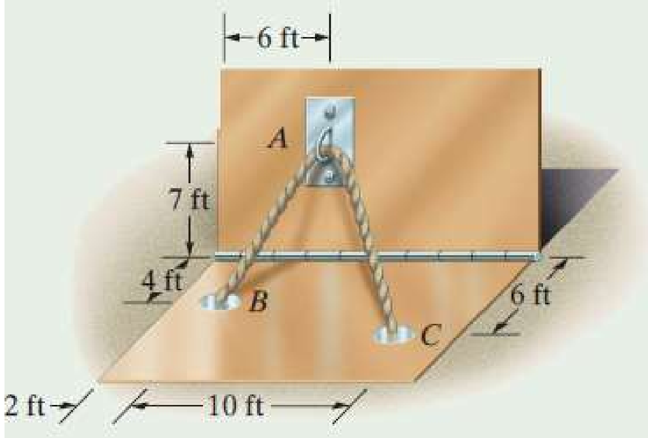


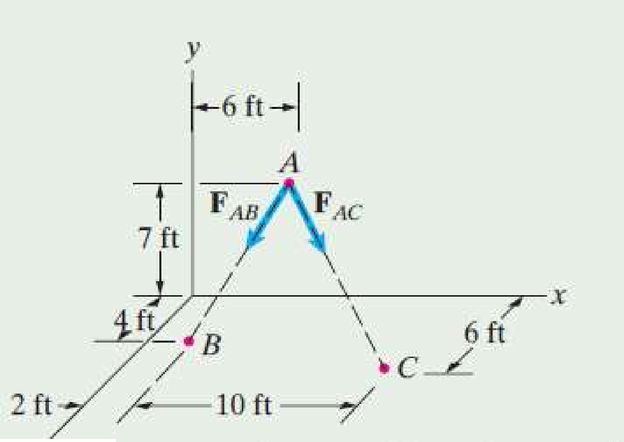
$$\mathbf{r}_{AB} = (x_B - x_A)\,\mathbf{i} + (y_B - y_A)\,\mathbf{j} + (z_B - z_A)\,\mathbf{k}.$$ (b)



Ejercicio

The rope extends from point B through a metal loop attached to the wall at A to point C. The rope exerts forces $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ on the loop at A with magnitudes $|\mathbf{F}_{AB}| = |\mathbf{F}_{AC}| = 200$ lb. What is the magnitude of the total force $\mathbf{F} = \mathbf{F}_{AB} + \mathbf{F}_{AC}$ exerted on the loop by the rope?





$$\mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$
$$= (2 - 6)\mathbf{i} + (0 - 7)\mathbf{j} + (4 - 0)\mathbf{k}$$
$$= -4\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}\ (\text{ft})$$

$$\mathbf{r}_{AC} = (x_C - x_A)\mathbf{i} + (y_C - y_A)\mathbf{j} + (z_C - z_A)\mathbf{k}$$
$$= (12 - 6)\mathbf{i} + (0 - 7)\mathbf{j} + (6 - 0)\mathbf{k}$$
$$= 6\mathbf{i} - 7\mathbf{j} + 6\mathbf{k}\ (\text{ft}).$$

Ejercicio

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -0.444\mathbf{i} - 0.778\mathbf{j} + 0.444\mathbf{k},$$

$$\mathbf{e}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = 0.545\mathbf{i} - 0.636\mathbf{j} + 0.545\mathbf{k}.$$

$$\mathbf{F}_{AB} = (200 \text{ lb})\mathbf{e}_{AB} = -88.9\mathbf{i} - 155.6\mathbf{j} + 88.9\mathbf{k} \text{ (lb)},$$

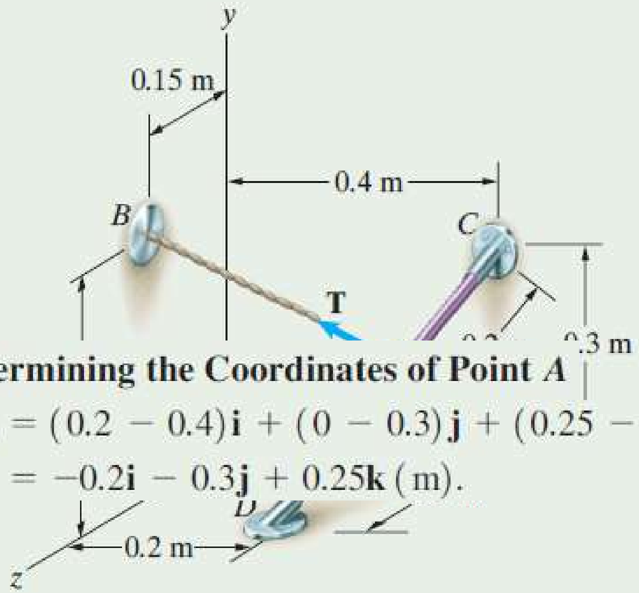
$$\mathbf{F}_{AC} = (200 \text{ lb})\mathbf{e}_{AC} = 109.1\mathbf{i} - 127.3\mathbf{j} + 109.1\mathbf{k} \text{ (lb)}.$$

$$\mathbf{F} = \mathbf{F}_{AB} + \mathbf{F}_{AC} = 20.2\mathbf{i} - 282.8\mathbf{j} + 198.0\mathbf{k} \text{ (lb)},$$

$$|\mathbf{F}| = \sqrt{(20.2)^2 + (-282.8)^2 + (198.0)^2} = 346 \text{ lb}.$$

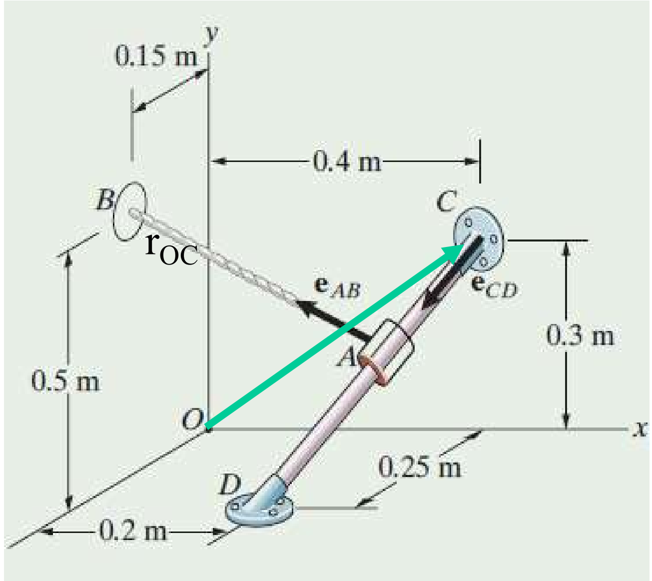
Ejercicio

The cable *AB* exerts a 50-N force **T** on the collar at *A*. Express **T** in terms of components.



Determining the Coordinates of Point *A*

$$\mathbf{r}_{CD} = (0.2 - 0.4)\,\mathbf{i} + (0 - 0.3)\,\mathbf{j} + (0.25 - 0)\,\mathbf{k}$$
$$= -0.2\mathbf{i} - 0.3\mathbf{j} + 0.25\mathbf{k}\ (\mathrm{m}).$$

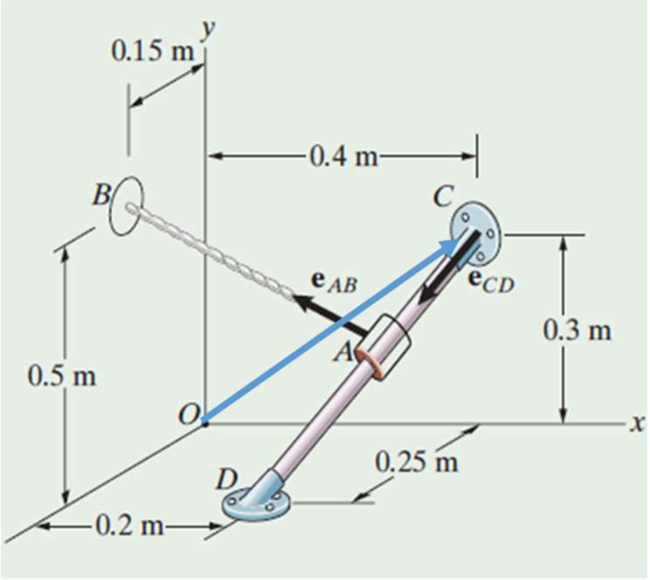


$$\mathbf{r}_{OC} = 0.4\mathbf{i} + 0.3\mathbf{j}\ (\mathrm{m})$$

$$\mathbf{r}_{CA} = (0.2\ \mathrm{m})\mathbf{e}_{CD} = -0.091\mathbf{i} - 0.137\mathbf{j} + 0.114\mathbf{k}\ (\mathrm{m}).$$

$$\mathbf{e}_{CD} = \frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = \frac{-0.2\mathbf{i} - 0.3\mathbf{j} + 0.25\mathbf{k}}{\sqrt{(-0.2)^2 + (-0.3)^2 + (0.25)^2}}$$
$$= -0.456\mathbf{i} - 0.684\mathbf{j} + 0.570\mathbf{k}.$$

$$\mathbf{r}_{OA} = \mathbf{r}_{OC} + \mathbf{r}_{CA} = (0.4\mathbf{i} + 0.3\mathbf{j}) + (-0.091\mathbf{i} - 0.137\mathbf{j} + 0.114\mathbf{k})$$
$$= 0.309\mathbf{i} + 0.163\mathbf{j} + 0.114\mathbf{k}\ (\mathrm{m}).$$



Determining the Components of T Using the coordinates of point A, we find that the position vector from A to B is

$$\mathbf{r}_{AB} = (0 - 0.309)\mathbf{i} + (0.5 - 0.163)\mathbf{j} + (0.15 - 0.114)\mathbf{k}$$
$$= -0.309\mathbf{i} + 0.337\mathbf{j} + 0.036\mathbf{k}\ (\mathrm{m}).$$

Dividing this vector by its magnitude, we obtain the unit vector $\mathbf{e}_{AB}$ (Fig. a).

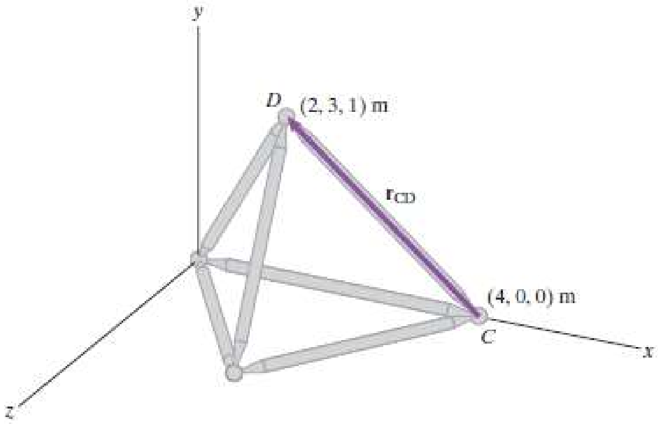
$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{-0.309\mathbf{i} + 0.337\mathbf{j} + 0.036\mathbf{k}\ (\mathrm{m})}{\sqrt{(-0.309\ \mathrm{m})^2 + (0.337\ \mathrm{m})^2 + (0.036\ \mathrm{m})^2}}$$
$$= -0.674\mathbf{i} + 0.735\mathbf{j} + 0.079\mathbf{k}.$$

The force $\mathbf{T}$ is

$$\mathbf{T} = |\mathbf{T}|\mathbf{e}_{AB} = (50\ \mathrm{N})(-0.674\mathbf{i} + 0.735\mathbf{j} + 0.079\mathbf{k})$$
$$= -33.7\mathbf{i} + 36.7\mathbf{j} + 3.9\mathbf{k}\ (\mathrm{N}).$$

Ejercicio

Problem 2.67 In Active Example 2.6, suppose that you want to redesign the truss, changing the position of point D so that the magnitude of the vector $\mathbf{r}_{CD}$ from point C to point D is 3 m. To accomplish this, let the coordinates of point D be $(2, y_D, 1)$ m, and determine the value of y_D so that $|\mathbf{r}_{CD}| = 3$ m. Draw a sketch of the truss with point D in its new position. What are the new directions cosines of $\mathbf{r}_{CD}$?



(a)

Solution: The vector $\mathbf{r}_{CD}$ and the magnitude $|\mathbf{r}_{CD}|$ are

$$\mathbf{r}_{CD} = ([2\text{ m} - 4\text{ m}]\mathbf{i} + [y_D - 0]\mathbf{j} + [1\text{ m} - 0]\mathbf{k}) = (-2\text{ m})\mathbf{i} + (y_D)\mathbf{j} + (1\text{ m})\mathbf{k}$$

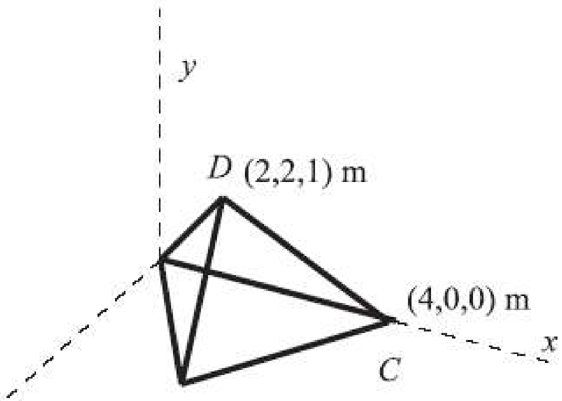
$$|\mathbf{r}_{CD}| = \sqrt{(-2\text{ m})^2 + (y_{CD})^2 + (1\text{ m})^2} = 3\text{ m}$$

Solving we find $y_{CD} = \sqrt{(3\text{ m})^2 - (-2\text{ m})^2 - (1\text{ m})^2} = 2$ m

$\boxed{y_{CD} = 2\text{ m}}$

The new direction cosines of $\mathbf{r}_{CD}$.

$$\boxed{\begin{aligned} \cos\theta_x &= -2/3 = -0.667 \\ \cos\theta_y &= 2/3 = 0.667 \\ \cos\theta_z &= 1/3 = 0.333 \end{aligned}}$$



Ejercicio

Problem 2.68 A force vector is given in terms of its components by $\mathbf{F} = 10\mathbf{i} - 20\mathbf{j} - 20\mathbf{k}$ (N).

(a) What are the direction cosines of **F**?
(b) Determine the components of a unit vector **e** that has the same direction as **F**.

Solution:

$$\mathbf{F} = (10\mathbf{i} - 20\mathbf{j} - 20\mathbf{k})\ \text{N}$$

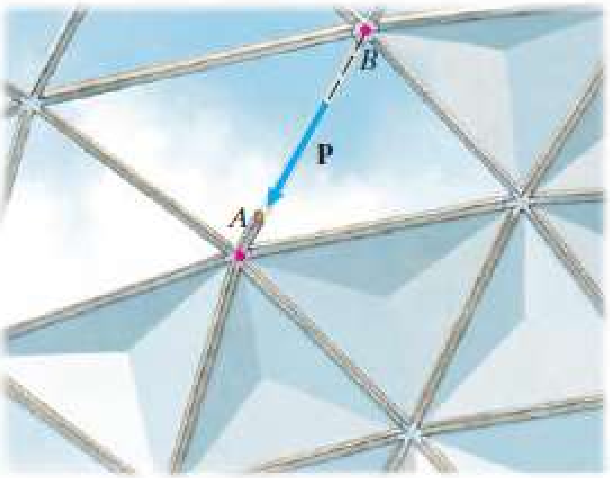
$$F = \sqrt{(10\ \text{N})^2 + (-20\ \text{N})^2 + (-20\ \text{N})^2} = 30\ \text{N}$$

(a) $$\boxed{\cos\theta_x = \frac{10\ \text{N}}{30\ \text{N}} = 0.333, \quad \cos\theta_y = \frac{-20\ \text{N}}{30\ \text{N}} = -0.667, \quad \cos\theta_z = \frac{-20\ \text{N}}{30\ \text{N}} = -0.667}$$

(b) $$\boxed{\mathbf{e} = (0.333\mathbf{i} - 0.667\mathbf{j} - 0.667\mathbf{k})}$$

Ejercicio

Problem 2.87 An engineer calculates that the magnitude of the axial force in one of the beams of a geodesic dome is $|\mathbf{P}| = 7.65$ kN. The cartesian coordinates of the endpoints A and B of the straight beam are $(-12.4, 22.0, -18.4)$ m and $(-9.2, 24.4, -15.6)$ m, respectively. Express the force $\mathbf{P}$ in terms of scalar components.



Solution: The components of the position vector from B to A are

$$\mathbf{r}_{BA} = (x_A - x_B)\mathbf{i} + (y_A - y_B)\mathbf{j} + (z_A - z_B)\mathbf{k}$$

$$= (-12.4 + 9.2)\mathbf{i} + (22.0 - 24.4)\mathbf{j}$$

$$+ (-18.4 + 15.6)\mathbf{k}$$

$$= -3.2\mathbf{i} - 2.4\mathbf{j} - 2.8\mathbf{k} \text{ (m)}.$$

Dividing this vector by its magnitude, we obtain a unit vector that points from B toward A:

$$\mathbf{e}_{BA} = -0.655\mathbf{i} - 0.492\mathbf{j} - 0.573\mathbf{k}.$$

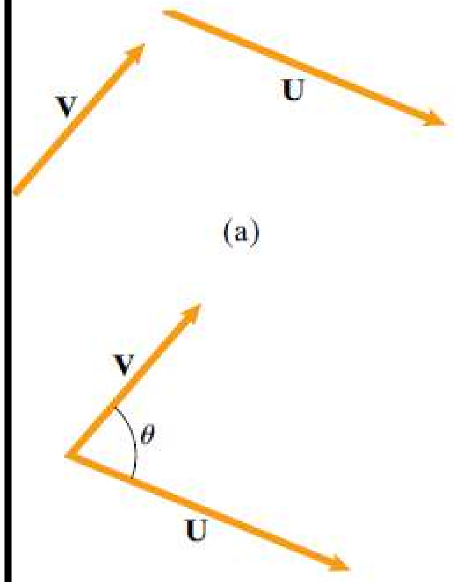
Therefore

$$\mathbf{P} = |\mathbf{P}|\mathbf{e}_{BA}$$

$$= 7.65\,\mathbf{e}_{BA}$$

$$= -5.01\mathbf{i} - 3.76\mathbf{j} - 4.39\mathbf{k} \text{ (kN)}.$$

Vectores: Producto Escalar o Punto



(a) (b)

$$\mathbf{U} \cdot \mathbf{V} = |\mathbf{U}||\mathbf{V}| \cos \theta.$$

Propiedades:

Conmutativa

$$\mathbf{U} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{U},$$

Asociativa Respecto a multiplicar por escalar

$$a(\mathbf{U} \cdot \mathbf{V}) = (a\mathbf{U}) \cdot \mathbf{V} = \mathbf{U} \cdot (a\mathbf{V}),$$

Distributiva respecto a la suma vectorial

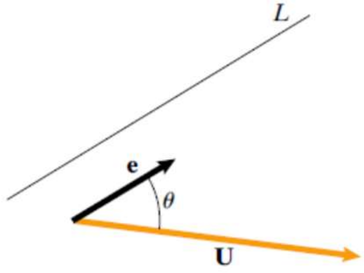
$$\mathbf{U} \cdot (\mathbf{V} + \mathbf{W}) = \mathbf{U} \cdot \mathbf{V} + \mathbf{U} \cdot \mathbf{W},$$

$$\begin{array}{lll} \mathbf{i} \cdot \mathbf{i} = 1, & \mathbf{i} \cdot \mathbf{j} = 0, & \mathbf{i} \cdot \mathbf{k} = 0, \\ \mathbf{j} \cdot \mathbf{i} = 0, & \mathbf{j} \cdot \mathbf{j} = 1, & \mathbf{j} \cdot \mathbf{k} = 0, \\ \mathbf{k} \cdot \mathbf{i} = 0, & \mathbf{k} \cdot \mathbf{j} = 0, & \mathbf{k} \cdot \mathbf{k} = 1. \end{array}$$

$$\begin{aligned} \mathbf{U} \cdot \mathbf{V} &= (U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}) \cdot (V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}) \\ &= U_xV_x(\mathbf{i} \cdot \mathbf{i}) + U_xV_y(\mathbf{i} \cdot \mathbf{j}) + U_xV_z(\mathbf{i} \cdot \mathbf{k}) \\ &\quad + U_yV_x(\mathbf{j} \cdot \mathbf{i}) + U_yV_y(\mathbf{j} \cdot \mathbf{j}) + U_yV_z(\mathbf{j} \cdot \mathbf{k}) \\ &\quad + U_zV_x(\mathbf{k} \cdot \mathbf{i}) + U_zV_y(\mathbf{k} \cdot \mathbf{j}) + U_zV_z(\mathbf{k} \cdot \mathbf{k}). \end{aligned}$$

Vectores: Producto Escalar o Punto

$$\cos\theta = \frac{\mathbf{U}\cdot\mathbf{V}}{|\mathbf{U}||\mathbf{V}|} = \frac{U_xV_x + U_yV_y + U_zV_z}{|\mathbf{U}||\mathbf{V}|}.$$



The Parallel Component In terms of the angle θ between $\mathbf{U}$ and the vector component $\mathbf{U}_p$, the magnitude of $\mathbf{U}_p$ is

$$|\mathbf{U}_p| = |\mathbf{U}|\cos\theta. \qquad (2.25)$$

Let $\mathbf{e}$ be a unit vector parallel to L (Fig. 2.23). The dot product of $\mathbf{e}$ and $\mathbf{U}$ is

$$\mathbf{e}\cdot\mathbf{U} = |\mathbf{e}||\mathbf{U}|\cos\theta = |\mathbf{U}|\cos\theta.$$

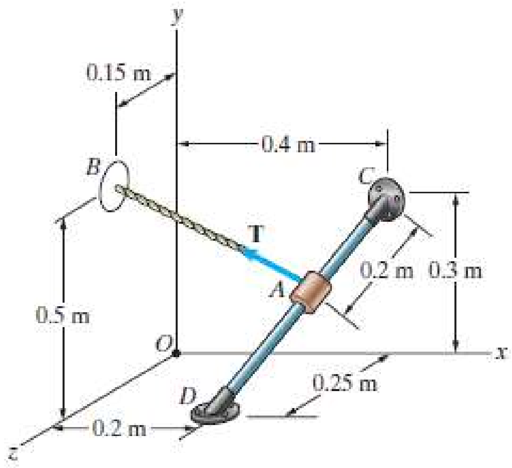
Comparing this result with Eq. (2.25), we see that the magnitude of $\mathbf{U}_p$ is

$$|\mathbf{U}_p| = \mathbf{e}\cdot\mathbf{U}.$$

Therefore the parallel vector component, or projection of $\mathbf{U}$ onto L, is

$$\mathbf{U}_p = (\mathbf{e}\cdot\mathbf{U})\mathbf{e}. \qquad (2.26)$$

Problem 2.117 The rope AB exerts a 50-N force $\mathbf{T}$ on collar A. Determine the vector component of $\mathbf{T}$ parallel to the bar CD.



Solution: We have the following vectors

$$\mathbf{r}_{CD} = (-0.2\mathbf{i} - 0.3\mathbf{j} + 0.25\mathbf{k}) \text{ m}$$

$$\mathbf{e}_{CD} = \frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = (-0.456\mathbf{i} - 0.684\mathbf{j} + 0.570\mathbf{k})$$

$$\mathbf{r}_{OB} = (0.5\mathbf{j} + 0.15\mathbf{k}) \text{ m}$$

$$\mathbf{r}_{OC} = (0.4\mathbf{i} + 0.3\mathbf{j}) \text{ m}$$

$$\mathbf{r}_{OA} = \mathbf{r}_{OC} + (0.2 \text{ m})\mathbf{e}_{CD} = (0.309\mathbf{i} + 0.163\mathbf{j} + 0.114\mathbf{k}) \text{ m}$$

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA} = (-0.309\mathbf{i} + 0.337\mathbf{j} + 0.036\mathbf{k}) \text{ m}$$

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (0.674\mathbf{i} + 0.735\mathbf{j} + 0.079\mathbf{k})$$

We can now write the force $\mathbf{T}$ and determine the vector component parallel to CD.

$$\mathbf{T} = (50 \text{ N})\mathbf{e}_{AB} = (-33.7\mathbf{i} + 36.7\mathbf{j} + 3.93\mathbf{k}) \text{ N}$$

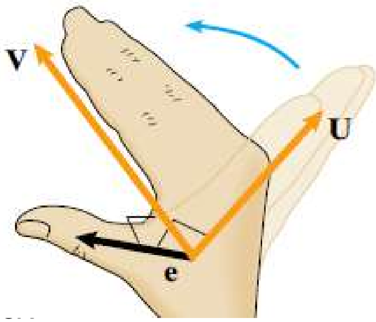
$$\mathbf{T}_p = (\mathbf{e}_{CD} \cdot \mathbf{T})\mathbf{e}_{CD} = (3.43\mathbf{i} + 5.14\mathbf{j} - 4.29\mathbf{k}) \text{ N}$$

$$\boxed{\mathbf{T}_p = (3.43\mathbf{i} + 5.14\mathbf{j} - 4.29\mathbf{k}) \text{ N}}$$

Vectores: Producto Vectorial o Cruz

Definicio $\mathbf{U} \times \mathbf{V} = |\mathbf{U}||\mathbf{V}| \sin\theta\, \mathbf{e}.$

Propiedades



No es conmutativa

$$\mathbf{U} \times \mathbf{V} = -\mathbf{V} \times \mathbf{U}.$$

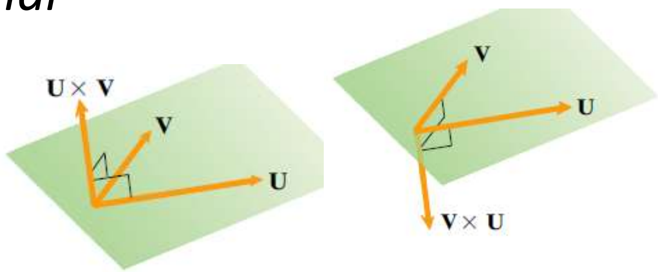
Es asociativa respecto a la multiplicación por es

$$a(\mathbf{U} \times \mathbf{V}) = (a\mathbf{U}) \times \mathbf{V} = \mathbf{U} \times (a\mathbf{V})$$

Es distributiva respecto a la suma vectorial

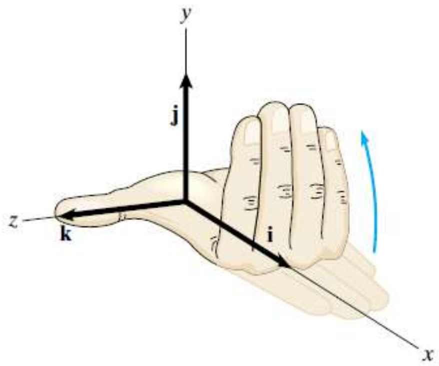
$$\mathbf{U} \times (\mathbf{V} + \mathbf{W}) = (\mathbf{U} \times \mathbf{V}) + (\mathbf{U} \times \mathbf{W})$$

$$\begin{array}{lll} \mathbf{i} \times \mathbf{i} = \mathbf{0}, & \mathbf{i} \times \mathbf{j} = \mathbf{k}, & \mathbf{i} \times \mathbf{k} = -\mathbf{j}, \\ \mathbf{j} \times \mathbf{i} = -\mathbf{k}, & \mathbf{j} \times \mathbf{j} = \mathbf{0}, & \mathbf{j} \times \mathbf{k} = \mathbf{i}, \\ \mathbf{k} \times \mathbf{i} = \mathbf{j}, & \mathbf{k} \times \mathbf{j} = -\mathbf{i}, & \mathbf{k} \times \mathbf{k} = \mathbf{0}. \end{array}$$

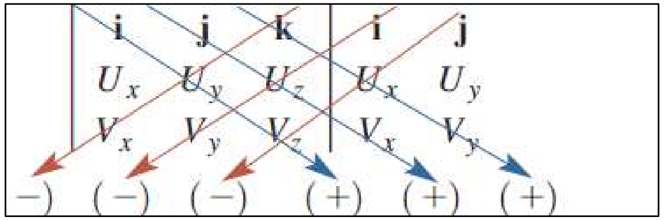


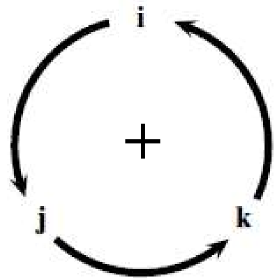
Vectores: Producto Vectorial o Cruz

$$
\begin{aligned}
\mathbf{U} \times \mathbf{V} &= (U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}) \times (V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}) \\
&= U_xV_x(\mathbf{i} \times \mathbf{i}) + U_xV_y(\mathbf{i} \times \mathbf{j}) + U_xV_z(\mathbf{i} \times \mathbf{k}) \\
&\quad + U_yV_x(\mathbf{j} \times \mathbf{i}) + U_yV_y(\mathbf{j} \times \mathbf{j}) + U_yV_z(\mathbf{j} \times \mathbf{k}) \\
&\quad + U_zV_x(\mathbf{k} \times \mathbf{i}) + U_zV_y(\mathbf{k} \times \mathbf{j}) + U_zV_z(\mathbf{k} \times \mathbf{k}).
\end{aligned}
$$



$$
\boxed{\begin{aligned}\mathbf{U} \times \mathbf{V} = (U_yV_z - U_zV_y)\mathbf{i} - (U_xV_z - U_zV_x)\mathbf{j} \\ + (U_xV_y - U_yV_x)\mathbf{k}.\end{aligned}}
$$



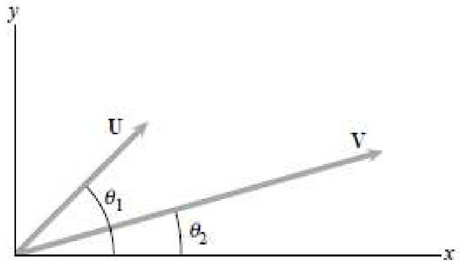


(a)

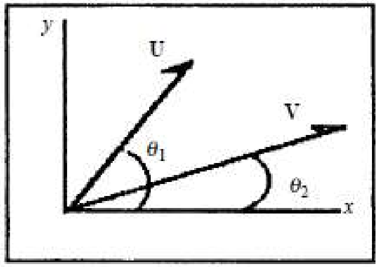
$$
\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix} = \begin{aligned} &U_yV_z\mathbf{i} + U_zV_x\mathbf{j} + U_xV_y\mathbf{k} \\ &-U_yV_x\mathbf{k} - U_zV_y\mathbf{i} - U_xV_z\mathbf{j}. \end{aligned}
$$

Ejercicios

Problem 2.132 By evaluating the cross product $\mathbf{U} \times \mathbf{V}$, prove the identity $\sin(\theta_1 - \theta_2) = \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$.



Solution: Assume that both **U** and **V** lie in the x-y plane. The strategy is to use the definition of the cross product (Eq. 2.28) and the Eq. (2.34), and equate the two. From Eq. (2.28) $\mathbf{U} \times \mathbf{V} = |\mathbf{U}||\mathbf{V}| \sin(\theta_1 - \theta_2)\mathbf{e}$. Since the positive z-axis is out of the paper, and $\mathbf{e}$ points into the paper, then $\mathbf{e} = -\mathbf{k}$. Take the dot product of both sides with $\mathbf{e}$, and note that $\mathbf{k} \cdot \mathbf{k} = 1$. Thus



$$\sin(\theta_1 - \theta_2) = -\left(\frac{(\mathbf{U} \times \mathbf{V}) \cdot \mathbf{k}}{|\mathbf{U}||\mathbf{V}|}\right)$$

The vectors are:

$$\mathbf{U} = |\mathbf{U}|(\mathbf{i}\cos\theta_1 + \mathbf{j}\sin\theta_2), \text{ and } \mathbf{V} = |\mathbf{V}|(\mathbf{i}\cos\theta_2 + \mathbf{j}\sin\theta_2).$$

The cross product is

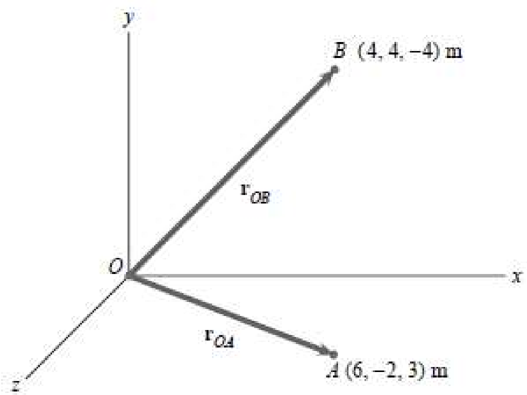
$$\mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ |\mathbf{U}|\cos\theta_1 & |\mathbf{U}|\sin\theta_1 & 0 \\ |\mathbf{V}|\cos\theta_2 & |\mathbf{V}|\sin\theta_2 & 0 \end{vmatrix}$$

$$= \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(|\mathbf{U}||\mathbf{V}|)(\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1)$$

Substitute into the definition to obtain: $\sin(\theta_1 - \theta_2) = \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$. Q.E.D.

Ejercicios

Problem 2.134 (a) What is the cross product $\mathbf{r}_{OA} \times \mathbf{r}_{OB}$? (b) Determine a unit vector $\mathbf{e}$ that is perpendicular to $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$.



Solution: The two radius vectors are

$$\mathbf{r}_{OB} = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}, \quad \mathbf{r}_{OA} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

(a) The cross product is

$$\mathbf{r}_{OA} \times \mathbf{r}_{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -2 & 3 \\ 4 & 4 & -4 \end{vmatrix} = \mathbf{i}(8 - 12) - \mathbf{j}(-24 - 12)$$

$$+ \mathbf{k}(24 + 8)$$

$$= -4\mathbf{i} + 36\mathbf{j} + 32\mathbf{k}\ (\mathrm{m}^2)$$

The magnitude is

$$|\mathbf{r}_{OA} \times \mathbf{r}_{OB}| = \sqrt{4^2 + 36^2 + 32^2} = 48.33\ \mathrm{m}^2$$

(b) The unit vector is

$$\mathbf{e} = \pm\left(\frac{\mathbf{r}_{OA} \times \mathbf{r}_{OB}}{|\mathbf{r}_{OA} \times \mathbf{r}_{OB}|}\right) = \pm(-0.0828\mathbf{i} + 0.7448\mathbf{j} + 0.6621\mathbf{k})$$

(Two vectors.)

Ejercicios

Problem 2.135 For the points O, A, and B in Problem 2.134, use the cross product to determine the length of the shortest straight line from point B to the straight line that passes through points O and A.

Solution:

$$\mathbf{r}_{OA} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \text{ (m)}$$

$$\mathbf{r}_{OB} = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k} \text{ (m)}$$

$$\mathbf{r}_{OA} \times \mathbf{r}_{OB} = \mathbf{C}$$

($\mathbf{C}$ is $\perp$ to both $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$)

$$\mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -2 & 3 \\ 4 & 4 & -4 \end{vmatrix} = \begin{matrix} (+8-12)\mathbf{i} \\ +(12+24)\mathbf{j} \\ +(24+8)\mathbf{k} \end{matrix}$$

$$\mathbf{C} = -4\mathbf{i} + 36\mathbf{j} + 32\mathbf{k}$$

$\mathbf{C}$ is $\perp$ to both $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$. Any line $\perp$ to the plane formed by $\mathbf{C}$ and $\mathbf{r}_{OA}$ will be parallel to the line BP on the diagram. $\mathbf{C} \times \mathbf{r}_{OA}$ is such a line. We then need to find the component of $\mathbf{r}_{OB}$ in this direction and compute its magnitude.

$$\mathbf{C} \times \mathbf{r}_{OA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & +36 & 32 \\ 6 & -2 & 3 \end{vmatrix}$$

$$\mathbf{C} = 172\mathbf{i} + 204\mathbf{j} - 208\mathbf{k}$$

The unit vector in the direction of D C is

eD = D/|D| $\quad 8\mathbf{i} + 0.603\mathbf{j} - 0.614\mathbf{k}$

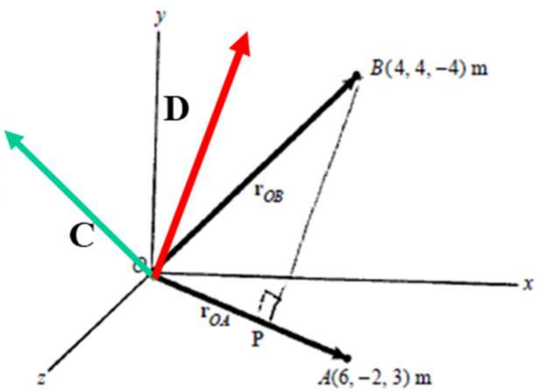
(The magnitude of $\mathbf{C}$ is 338.3)

We now want to find the length of the projection, P, of line OB in direction $\mathbf{e}_c$.

$$P = \mathbf{r}_{OB} \cdot \mathbf{e}_C$$

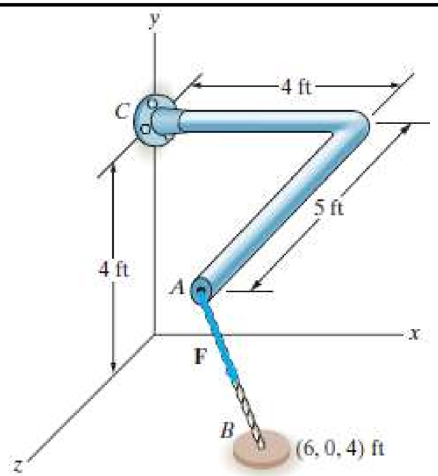
$$= (4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}) \cdot \mathbf{e}_C$$

$$P = 6.90 \text{ m}$$



Ejercicios

Problem 2.126 The two segments of the L-shaped bar are parallel to the x and z axes. The rope AB exerts a force of magnitude $|\mathbf{F}| = 500$ lb on the bar at A. Determine the cross product $\mathbf{r}_{CA} \times \mathbf{F}$, where $\mathbf{r}_{CA}$ is the position vector form point C to point A.



Solution: We need to determine the force $\mathbf{F}$ in terms of its components. The vector from A to B is used to define $\mathbf{F}$.

$\mathbf{r}_{AB} = (2\mathbf{i} - 4\mathbf{j} - \mathbf{k})$ ft

$$\mathbf{F} = (500 \text{ lb})\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (500 \text{ lb})\frac{(2\mathbf{i} - 4\mathbf{j} - \mathbf{k})}{\sqrt{(2)^2 + (-4)^2 + (-1)^2}}$$

$$\mathbf{F} = (218\mathbf{i} - 436\mathbf{j} - 109\mathbf{k}) \text{ lb}$$

Also we have $\mathbf{r}_{CA} = (4\mathbf{i} + 5\mathbf{k})$ ft

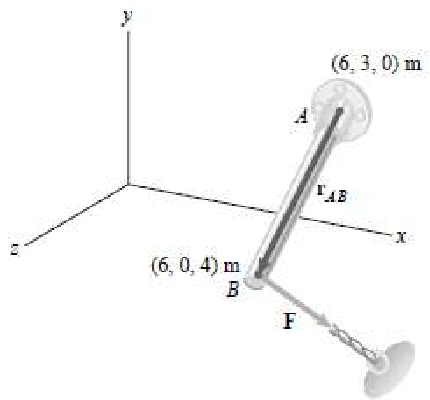
Therefore

$$\mathbf{r}_{CA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 5 \\ 218 & -436 & -109 \end{vmatrix} = (2180\mathbf{i} + 1530\mathbf{j} - 1750\mathbf{k}) \text{ ft-lb}$$

$$\boxed{\mathbf{r}_{CA} \times \mathbf{F} = (2180\mathbf{i} + 1530\mathbf{j} - 1750\mathbf{k}) \text{ ft-lb}}$$

Ejercicios

Problem 2.131 The force $\mathbf{F} = 10\mathbf{i} - 4\mathbf{j}$ (N). Determine the cross product $\mathbf{r}_{AB} \times \mathbf{F}$.



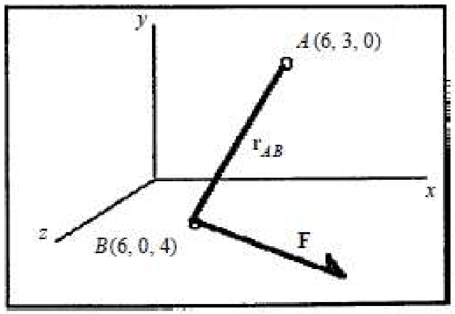
Solution: The position vector is

$$\mathbf{r}_{AB} = (6-6)\mathbf{i} + (0-3)\mathbf{j} + (4-0)\mathbf{k} = 0\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

The cross product:

$$\mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & 4 \\ 10 & -4 & 0 \end{vmatrix} = \mathbf{i}(16) - \mathbf{j}(-40) + \mathbf{k}(30)$$

$$= 16\mathbf{i} + 40\mathbf{j} + 30\mathbf{k} \text{ (N-m)}$$



Triple Producto Mixto

Definicion:

$$\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}).$$

$$\begin{aligned}
\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) &= (U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ V_x & V_y & V_z \\ W_x & W_y & W_z \end{vmatrix} \\
&= (U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}) \cdot [(V_yW_z - V_zW_y)\mathbf{i} \\
&\quad - (V_xW_z - V_zW_x)\mathbf{j} + (V_xW_y - V_yW_x)\mathbf{k}] \\
&= U_x(V_yW_z - V_zW_y) - U_y(V_xW_z - V_zW_x) \\
&\quad + U_z(V_xW_y - V_yW_x).
\end{aligned}$$

$$\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) = \begin{vmatrix} U_x & U_y & U_z \\ V_x & V_y & V_z \\ W_x & W_y & W_z \end{vmatrix}.$$

$$\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) = -\mathbf{W} \cdot (\mathbf{V} \times \mathbf{U}).$$

Ejercicios

Problem 2.146 The vectors $\mathbf{U} = \mathbf{i} + U_Y\mathbf{j} + 4\mathbf{k}$, $\mathbf{V} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, and $\mathbf{W} = -3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ are coplanar (they lie in the same plane). What is the component U_y?

Solution: Since the non-zero vectors are coplanar, the cross product of any two will produce a vector perpendicular to the plane, and the dot product with the third will vanish, by definition of the dot product. Thus $\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) = 0$, for example.

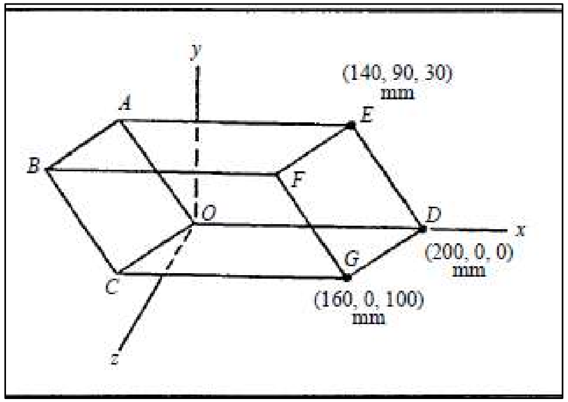
$$\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) = \begin{vmatrix} 1 & U_Y & 4 \\ 2 & 1 & -2 \\ -3 & 1 & -2 \end{vmatrix}$$

$$= 1(-2+2) - (U_Y)(-4-6) + (4)(2+3)$$

$$= +10U_Y + 20 = 0$$

Thus $U_Y = -2$

Problem 2.144 Use the mixed triple product to calculate the volume of the parallelepiped.



$$\mathbf{r}_{OA} \cdot (\mathbf{r}_{OC} \times \mathbf{r}_{OD}) = \begin{vmatrix} -60 & 90 & 30 \\ -40 & 0 & 100 \\ 200 & 0 & 0 \end{vmatrix}$$

$$= -60(0) + 90(200)(100) + (30)(0) \text{ mm}^3$$

$$= 1{,}800{,}000 \text{ mm}^3$$

Fin