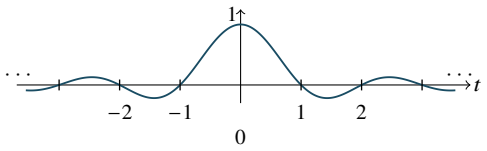
Señales útiles

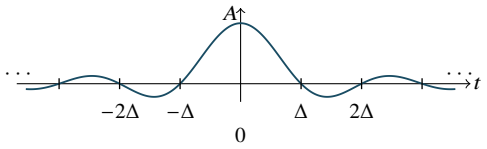
Por convención, usamos la sinc normalizada.

Sinc

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



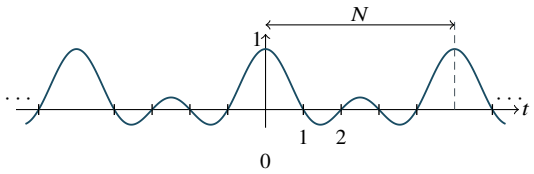
$$A \operatorname{sinc}\left(\frac{t}{\Delta}\right) = \frac{A\Delta}{\pi t} \sin\left(\frac{\pi t}{\Delta}\right)$$



Sinc periódica

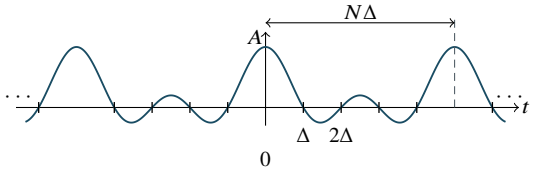
$$\operatorname{sinc}_N(t) = \frac{\operatorname{sinc}(t)}{\operatorname{sinc}(t/N)} = \frac{\sin(\pi t)}{N \sin(\pi t/N)}$$

N impar

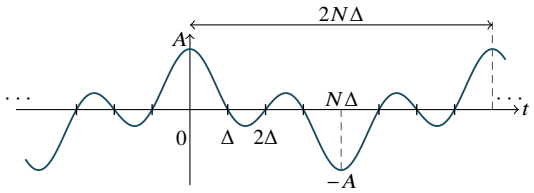


$$A \operatorname{sinc}_N\left(\frac{t}{\Delta}\right) = A \frac{\operatorname{sinc}(t/\Delta)}{\operatorname{sinc}(t/(N\Delta))} = \frac{A \sin(\pi t/\Delta)}{N \sin(\pi t/(N\Delta))}$$

N impar



N par



Fórmulas útiles

Identidades de Euler

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Suma geométrica

$$\sum_{n=N_1}^{N_2-1} \alpha^n = \frac{\alpha^{N_1} - \alpha^{N_2}}{1-\alpha}, \qquad \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}, \quad |\alpha| < 1.$$

Sistemas LTI

Convolución

	Continuo	Discreto
	$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,\mathrm{d}\tau$	$y[n] = (x * h)[n] = \sum_{k\in\mathbb{Z}} x[k]h[n-k]$

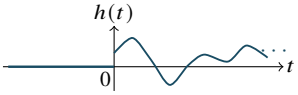
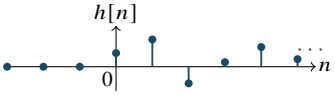
Conmutatividad $x * y = y * x$

Distributividad $(x_1 + x_2) * h = x_1 * h + x_2 * h$

Asociatividad $(x * h_1) * h_2 = x * (h_1 * h_2)$

Traslación $(\mathsf{D}_a x) * h = x * (\mathsf{D}_a h) = \mathsf{D}_a(x * h)$ (D_a: retardo de a)

Propiedades de sistemas LTI

	Continuo	Discreto
Causalidad	$h(t) = 0, \quad t < 0$ 	$h[n] = 0, \quad n < 0$ 
Estabilidad	$\int_{-\infty}^{\infty} \lvert h(t)\rvert\,\mathrm{d}t < \infty$	$\sum_{n\in\mathbb{Z}} \lvert h[n]\rvert < \infty$

Respuesta de sistemas LTI

	Continuo	Discreto
	$Y(\omega) = H(\omega)X(\omega)$	$Y(\Omega) = H(\Omega)X(\Omega)$
	$\mathrm{e}^{\mathrm{j}\omega t} \mapsto H(\omega)\mathrm{e}^{\mathrm{j}\omega t}$	$\mathrm{e}^{\mathrm{j}\Omega n} \mapsto H(\Omega)\mathrm{e}^{\mathrm{j}\Omega n}$
Autofunciones y autovalores	$\mathrm{e}^{st} \mapsto H(s)\mathrm{e}^{st}$ $s \in \mathrm{ROC}\{H(s)\}$	$z_0^n \mapsto H(z_0)z_0^n$ $z_0 \in \mathrm{ROC}\{H(z)\}$

Series de Fourier

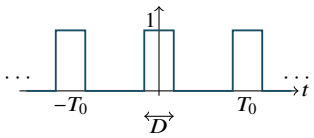
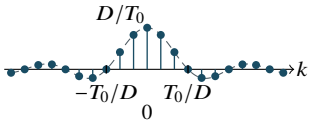
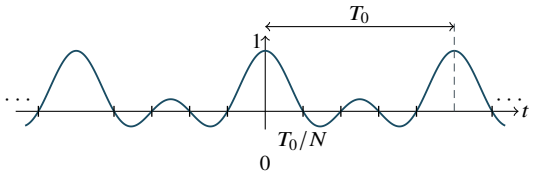
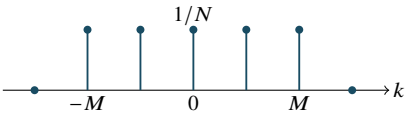
	Continuo	**Discreto**
Frecuencia fundamental	$\omega_0 = 2\pi f_0 = \dfrac{2\pi}{T_0}$	$\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{N}$
Coeficientes	$a_k = \dfrac{1}{T_0}\displaystyle\int_{T_0} x(t)\mathrm{e}^{-\mathrm{j}k\omega_0 t}\,\mathrm{d}t$	$a_k = \dfrac{1}{N}\displaystyle\sum_{n=0}^{N-1} x[n]\mathrm{e}^{-\mathrm{j}k\Omega_0 n}$
Serie	$x(t) = \displaystyle\sum_{k\in\mathbb{Z}} a_k \mathrm{e}^{\mathrm{j}k\omega_0 t}$	$x[n] = \displaystyle\sum_{k=0}^{N-1} a_k \mathrm{e}^{\mathrm{j}k\Omega_0 n}$
Periodicidad de a_k	❌	$a_{k+N} = a_k$
Convolución periódica	$(x \circledast_{T_0} y)(t) = \displaystyle\int_{T_0} x(\tau)y(t-\tau)\,\mathrm{d}\tau$	$(x \circledast_N y)[n] = \displaystyle\sum_{\ell=0}^{N-1} x[\ell]y[n-\ell]$

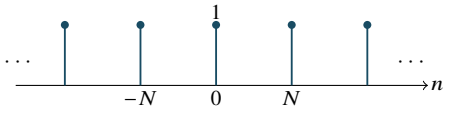
Propiedades

$x \leftrightarrow a_k, \quad y \leftrightarrow b_k$

	Continuo	**Discreto**
Retardo	$x(t-t_0) \leftrightarrow a_k \mathrm{e}^{-\mathrm{j}k\omega_0 t_0}$	$x[n-n_0] \leftrightarrow a_k \mathrm{e}^{-\mathrm{j}k\Omega_0 n_0}$
Inversión	$x(-t) \leftrightarrow a_{-k}$	$x[-n] \leftrightarrow a_{-k}$
Conjugación	$x^*(t) \leftrightarrow a^*_{-k}$	$x^*[n] \leftrightarrow a^*_{-k}$
Realidad	$x(t)$ real $\iff a_{-k} = a_k^*$	$x[n]$ real $\iff a_{-k} = a_k^*$
Modulación	$x(t)\mathrm{e}^{\mathrm{j}m\omega_0 t} \leftrightarrow a_{k-m}$	$x[n]\mathrm{e}^{\mathrm{j}m\Omega_0 n} \leftrightarrow a_{k-m}$
Producto	$x(t)y(t) \leftrightarrow \sum_{\ell\in\mathbb{Z}} a_\ell b_{k-\ell}$	$x[n]y[n] \leftrightarrow \sum_{\ell=0}^{N-1} a_\ell b_{k-\ell}$
Convolución periódica	$(x \circledast_{T_0} y)(t) \leftrightarrow T_0 a_k b_k$	$(x \circledast_N y)[n] \leftrightarrow N a_k b_k$
Derivada / diferencia	$x'(t) \leftrightarrow \mathrm{j}k\omega_0 a_k$	$x[n] - x[n-1] \leftrightarrow (1 - \mathrm{e}^{-\mathrm{j}k\Omega_0}) a_k$
Integración	$y'(t) = x(t) \quad (a_0 = 0)$ $b_k = \dfrac{a_k}{\mathrm{j}k\omega_0} \quad (k \neq 0,\ b_0 \text{ libre})$	
Parseval	$\dfrac{1}{T_0}\displaystyle\int_{T_0} \lvert x(t)\rvert^2 \,\mathrm{d}t = \sum_{k\in\mathbb{Z}} \lvert a_k\rvert^2$	$\dfrac{1}{N}\displaystyle\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \sum_{k=0}^{N-1} \lvert a_k\rvert^2$

Casos particulares

Señal continua	**Coeficientes a_k**
$e^{j\omega_0 t}$	$a_1 = 1; \quad a_k = 0$ para $k \neq 1$
$\cos(\omega_0 t)$	$a_1 = a_{-1} = \frac{1}{2}; \quad a_k = 0$ para $k \neq \pm 1$
$\sin(\omega_0 t)$	$a_1 = -a_{-1} = \frac{1}{2j}; \quad a_k = 0$ para $k \neq \pm 1$
	$a_k = 1/T_0$
	$a_k = \frac{D}{T_0}\,\mathrm{sinc}\left(\frac{kD}{T_0}\right)$ 
$\mathrm{sinc}_N\left(\frac{Nt}{T_0}\right) \quad (N = 2M+1)$ 	$a_k = \begin{cases} 1/N, & \lvert k\rvert \le M, \\ 0, & \lvert k\rvert > M. \end{cases}$ 

Señal discreta	**Coeficientes a_k** $\quad m \in \mathbb{Z}$
$e^{jm\Omega_0 n}$	$a_k = \sum_{\ell\in\mathbb{Z}} \delta[k - m - \ell N]$
$\cos(m\Omega_0 n)$	$a_k = \frac{1}{2}\sum_{\ell\in\mathbb{Z}} \big(\delta[k - m - \ell N] + \delta[k + m - \ell N]\big)$
$\sin(m\Omega_0 n)$	$a_k = \frac{1}{2j}\sum_{\ell\in\mathbb{Z}} \big(\delta[k - m - \ell N] - \delta[k + m - \ell N]\big)$
	$a_k = 1/N$

Transformada de Fourier de tiempo continuo

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \mathrm{e}^{-\mathrm{j}\omega t}\, \mathrm{d}t \qquad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \mathrm{e}^{\mathrm{j}\omega t}\, \mathrm{d}\omega$$

Propiedades

	Señal	**Transformada**
Retardo	$x(t - t_0)$	$\mathrm{e}^{-\mathrm{j}\omega t_0} X(\omega)$
Modulación	$\mathrm{e}^{\mathrm{j}\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Escala temporal	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Conjugación	$x^*(t)$	$X^*(-\omega)$
Derivada	$x^{(q)}(t)$	$(\mathrm{j}\omega)^q X(\omega)$
Multiplicación por t	$tx(t)$	$\mathrm{j}X'(\omega)$
Integración	$\int_{-\infty}^{t} x(\tau)\, \mathrm{d}\tau$	$\frac{X(\omega)}{\mathrm{j}\omega} + \pi X(0)\delta(\omega)$
Convolución	$(x * y)(t)$	$X(\omega)Y(\omega)$
Producto	$x(t)y(t)$	$\frac{1}{2\pi}(X * Y)(\omega)$
Dualidad	$X(\omega)\vert_{\omega=t}$	$2\pi\, x(t)\vert_{t=-\omega}$

Simetrías

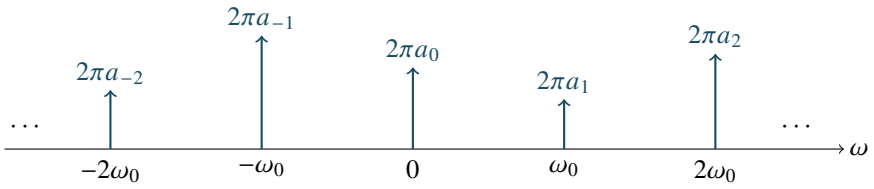
Señal	Transformada
$x(t)$ real	$X(-\omega) = X^*(\omega)$
$x(t)$ par	$X(-\omega) = X(\omega)$
$x(t)$ impar	$X(-\omega) = -X(\omega)$
$x(t)$ real y par	$X(\omega)$ real y par
$x(t)$ real e impar	$X(\omega)$ imaginaria pura e impar

Parseval

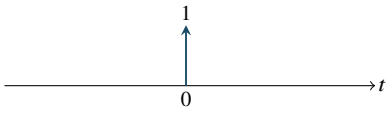
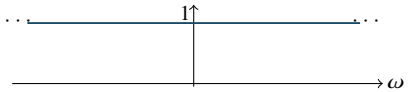
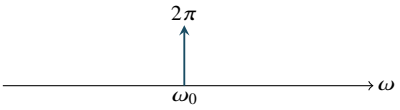
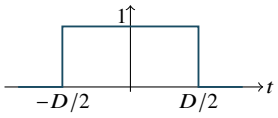
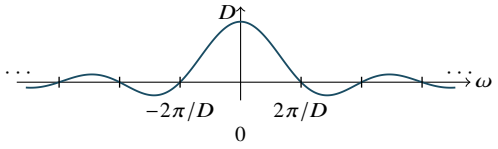
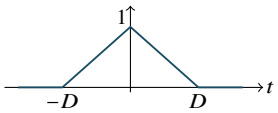
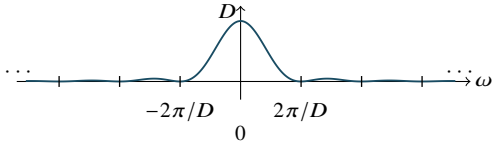
$$\int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, \mathrm{d}\omega$$

Relación con serie de Fourier

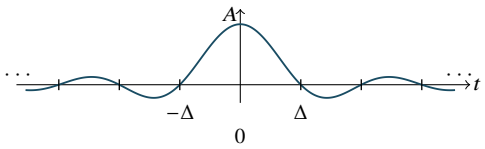
$$X(\omega) = 2\pi \sum_{k \in \mathbb{Z}} a_k \delta(\omega - k\omega_0).$$



Casos particulares

Señal $x(t)$	**Transformada** $X(\omega)$
	
$e^{j\omega_0 t}$	
$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$e^{-at}u(t)$ $(\mathrm{Re}\, a>0)$	$\frac{1}{a+j\omega}$
$\frac{t^{q-1}}{(q-1)!}e^{-at}u(t)$ $(\mathrm{Re}\, a>0)$	$\frac{1}{(a+j\omega)^q}$
$e^{-a\lvert t\rvert}$ $(a>0)$	$\frac{2a}{a^2+\omega^2}$
	
	$D\,\mathrm{sinc}^2\left(\frac{\omega D}{2\pi}\right)$ 

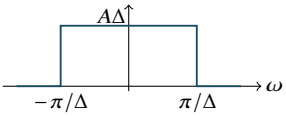
Señal $x(t)$

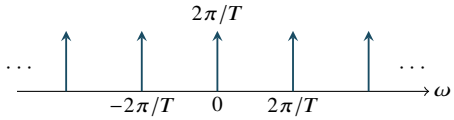




$$\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-t^2/(2\sigma^2)}$$

Transformada $X(\omega)$





$$\mathrm{e}^{-\sigma^2\omega^2/2}$$

Transformada de Fourier de tiempo discreto

$$X(\Omega) = \sum_{n\in\mathbb{Z}} x[n]\mathrm{e}^{-\mathrm{j}\Omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\mathrm{e}^{\mathrm{j}\Omega n}\,\mathrm{d}\Omega$$

Expresiones en $-\pi \le \Omega < \pi$

Extensión 2π-periódica

Propiedades

	Señal	**Transformada**
Retardo	$x[n-n_0]$	$\mathrm{e}^{-\mathrm{j}\Omega n_0}X(\Omega)$
Modulación	$\mathrm{e}^{\mathrm{j}\Omega_0 n}x[n]$	$X(\Omega-\Omega_0)$
Inversión	$x[-n]$	$X(-\Omega)$
Conjugación	$x^*[n]$	$X^*(-\Omega)$
Multiplicación por n	$nx[n]$	$\mathrm{j}X'(\Omega)$
Diferencia	$x[n]-x[n-1]$	$(1-\mathrm{e}^{-\mathrm{j}\Omega})X(\Omega)$
Acumulación	$\sum_{k=-\infty}^{n} x[k]$	$\frac{X(\Omega)}{1-\mathrm{e}^{-\mathrm{j}\Omega}} + \pi X(0)\delta(\Omega)$
Convolución	$(x*y)[n]$	$X(\Omega)Y(\Omega)$
Producto	$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\theta)Y(\Omega-\theta)\,\mathrm{d}\theta$

Simetrías

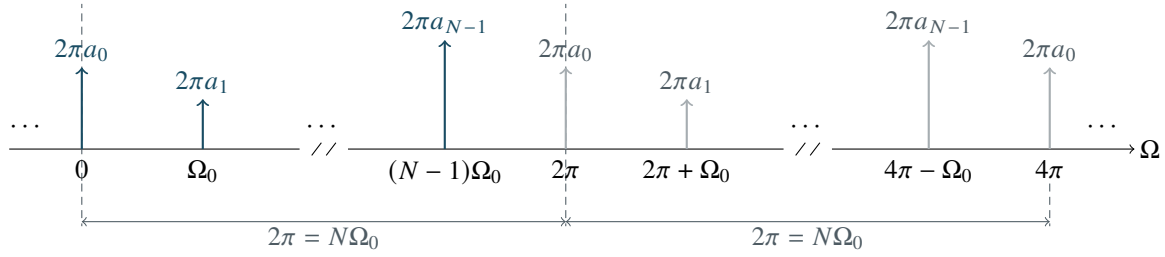
Señal	Transformada
$x[n]$ real	$X(-\Omega) = X^*(\Omega)$
$x[n]$ par	$X(-\Omega) = X(\Omega)$
$x[n]$ impar	$X(-\Omega) = -X(\Omega)$
$x[n]$ real y par	$X(\Omega)$ real y par
$x[n]$ real e impar	$X(\Omega)$ imaginaria pura e impar

Parseval

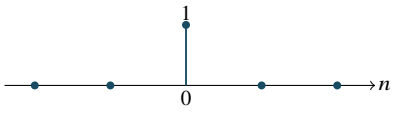
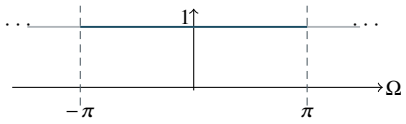
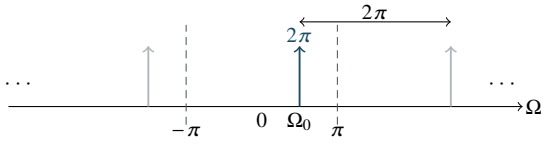
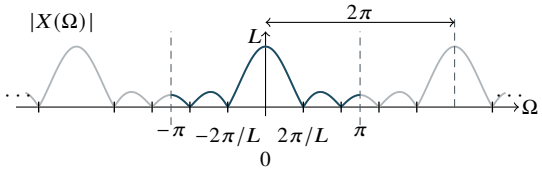
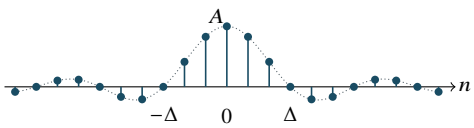
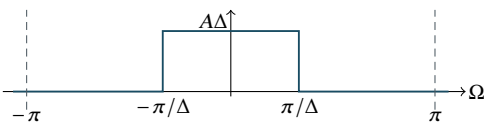
$$\sum_{n\in\mathbb{Z}} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\Omega)|^2 \,\mathrm{d}\Omega$$

Relación con serie de Fourier

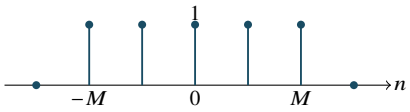
$$X(\Omega) = \sum_{\ell\in\mathbb{Z}} \left[2\pi \sum_{k=0}^{N-1} a_k \delta(\Omega - k\Omega_0 - 2\pi\ell) \right].$$



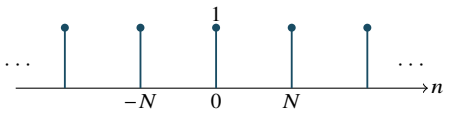
Casos particulares

Señal $x[n]$		**Transformada** $X(\Omega)$
		
$e^{j\Omega_0 n}$		
$\cos(\Omega_0 n)$		$\sum_{\ell\in\mathbb{Z}} \pi\left[\delta(\Omega-\Omega_0-2\pi\ell)+\delta(\Omega+\Omega_0-2\pi\ell)\right]$
$\sin(\Omega_0 n)$		$\sum_{\ell\in\mathbb{Z}} \frac{\pi}{j}\left[\delta(\Omega-\Omega_0-2\pi\ell)-\delta(\Omega+\Omega_0-2\pi\ell)\right]$
$u[n]$		$\frac{1}{1-e^{-j\Omega}}+\pi\delta(\Omega)$
$a^n u[n]$	$(\lvert a\rvert<1)$	$\frac{1}{1-ae^{-j\Omega}}$
$na^n u[n]$	$(\lvert a\rvert<1)$	$\frac{ae^{-j\Omega}}{(1-ae^{-j\Omega})^2}$
$\binom{n+q-1}{q-1}a^n u[n]$	$(\lvert a\rvert<1)$	$\frac{1}{(1-ae^{-j\Omega})^q}$
		$e^{-j\Omega(L-1)/2}\frac{\sin(L\Omega/2)}{\sin(\Omega/2)}$ $= Le^{-j\Omega(L-1)/2}\,\mathrm{sinc}_L\left(\frac{L\Omega}{2\pi}\right)$ 
		

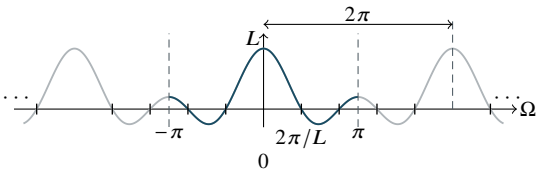
Señal $x[n]$

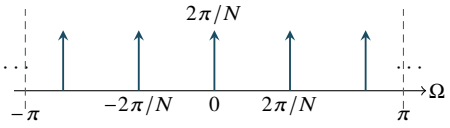


$$L = 2M + 1$$

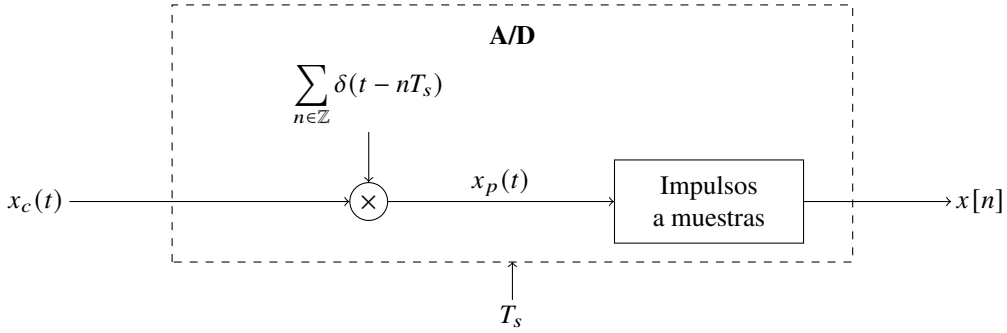


Transformada $X(\Omega)$





Muestreo

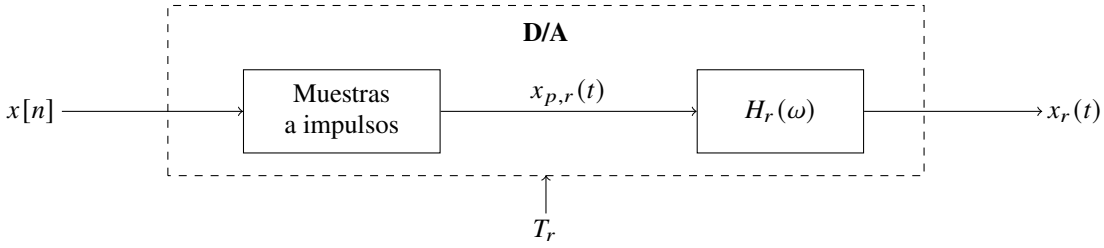


$$x[n] = x_c(nT_s), \qquad \omega_s = 2\pi f_s = \frac{2\pi}{T_s}, \qquad \Omega = \omega T_s.$$

$$x_p(t) = x_c(t)\sum_{n\in\mathbb{Z}} \delta(t-nT_s) = \sum_{n\in\mathbb{Z}} x[n]\delta(t-nT_s).$$

$$X_p(\omega) = \frac{1}{T_s}\sum_{k\in\mathbb{Z}} X_c(\omega - k\omega_s), \qquad X(\Omega) = X_p\left(\frac{\Omega}{T_s}\right) = \frac{1}{T_s}\sum_{k\in\mathbb{Z}} X_c\left(\frac{\Omega - 2\pi k}{T_s}\right).$$

Interpolación ideal



$$H_r(\omega) = \quad T_r \uparrow \quad -\pi/T_r \quad \pi/T_r \quad \omega$$

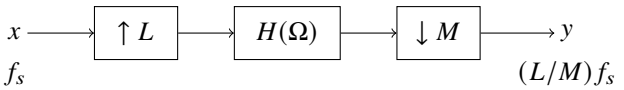
$$x_{p,r}(t) = \sum_{n\in\mathbb{Z}} x[n]\delta(t-nT_r),$$

$$x_r(t) = \sum_{n\in\mathbb{Z}} x[n]\,\mathrm{sinc}\left(\frac{t-nT_r}{T_r}\right).$$

$X_c(\omega) = 0$ para $|\omega| > \omega_M$, $\quad \omega_s > 2\omega_M$: $\quad T_r = T_s \implies x_r(t) = x_c(t)$.

Cambio de tasa

		Señal	**Transformada**
Sobremuestreo	$\uparrow L$	$x_{\uparrow L}[n] = \begin{cases} x[n/L], & n \in L\mathbb{Z}, \\ 0, & n \notin L\mathbb{Z} \end{cases}$	$X_{\uparrow L}(\Omega) = X(L\Omega)$
Submuestreo	$\downarrow M$	$x_{\downarrow M}[n] = x[Mn]$	$X_{\downarrow M}(\Omega) = \frac{1}{M}\sum_{r=0}^{M-1} X\left(\frac{\Omega + 2\pi r}{M}\right)$



$$H(\Omega) = \begin{cases} L, & |\Omega| < \min(\pi/L, \pi/M), \\ 0, & \text{resto de } [-\pi, \pi]. \end{cases}$$

DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] \mathrm{e}^{-\mathrm{j}2\pi kn/N} \qquad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \mathrm{e}^{\mathrm{j}2\pi kn/N}$$

$$0 \leq n, k < N$$

Convolución circular

$$[r]_N = r \bmod N, \qquad (x \circledast_N y)[n] = \sum_{r=0}^{N-1} x[r] y[[n-r]_N]$$

Propiedades

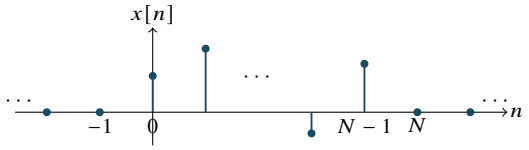
Señal	DFT
$x[[n-m]_N]$	$\mathrm{e}^{-\mathrm{j}2\pi km/N} X[k]$
$\mathrm{e}^{\mathrm{j}2\pi mn/N} x[n]$	$X[[k-m]_N]$
$x[[-n]_N]$	$X[[-k]_N]$
$x^*[n]$	$X^*[[-k]_N]$
$(x \circledast_N y)[n]$	$X[k]Y[k]$
$x[n]y[n]$	$\frac{1}{N}(X \circledast_N Y)[k]$
$X[k]\|_{k=n}$	$N\, x[n]\|_{n=[-k]_N}$

Parseval

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2.$$

Relación con transformada y serie de Fourier

Soporte acotado: $0 \le n \le N-1$



$$\Longrightarrow \qquad X[k] = X(\Omega)|_{\Omega=2\pi k/N}\,.$$

Para cualquier $x[n]$ periódica de período N, con coeficientes de serie a_k:

$$X[k] = Na_k.$$

Periodización

$$x_N[n] = \sum_{\ell\in\mathbb{Z}} x[n-\ell N].$$

$$x_0[n] = \begin{cases} x_N[n], & 0 \le n \le N-1, \\ 0, & \text{resto}, \end{cases} \qquad \Longrightarrow \qquad X_N[k] = X_0(\Omega)|_{\Omega=2\pi k/N}\,.$$

Frecuencias

$$\Omega_k = \frac{2\pi k}{N}, \qquad \Delta f = \frac{f_s}{N}, \qquad f_k = \begin{cases} k f_s/N, & 0 \le k < N/2, \\ (k-N) f_s/N, & N/2 \le k < N. \end{cases}$$

Ventanas y espectrograma

$$S[m,k] = \sum_{r=0}^{L-1} x[mR+r]\,w[r]\,\mathrm{e}^{-\mathrm{j}2\pi kr/N} \qquad \text{espectrograma: } |S[m,k]|^2.$$

L: longitud de ventana; R: salto entre bloques; $N \ge L$.

Transformada de Laplace

$$X(s) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-st}\,\mathrm{d}t \qquad x(t) = \frac{1}{2\pi\mathrm{j}}\int_{\sigma-\mathrm{j}\infty}^{\sigma+\mathrm{j}\infty} X(s)\mathrm{e}^{st}\,\mathrm{d}s$$

$$\mathrm{ROC}\{X(s)\} = \{s : \sigma_1 < \mathrm{Re}\, s < \sigma_2\}, \qquad \sigma_1 < \sigma < \sigma_2.$$

Propiedades

Operación	Señal	Transformada	ROC
Linealidad	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	$\supseteq \mathrm{ROC}\{X_1(s)\} \cap \mathrm{ROC}\{X_2(s)\}$
Desplazamiento	$x(t - t_0)$	$\mathrm{e}^{-st_0}X(s)$	$\mathrm{ROC}\{X(s)\}$
Exponencial	$\mathrm{e}^{s_0 t}x(t)$	$X(s - s_0)$	$\mathrm{ROC}\{X(s)\} + s_0$
Escala / inversión	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\{s : s/a \in \mathrm{ROC}\{X(s)\}\}, a \in \mathbb{R}$
Conjugación	$x^*(t)$	$X^*(s^*)$	$\mathrm{ROC}\{X(s)\}$
Convolución	$(x_1 * x_2)(t)$	$X_1(s)X_2(s)$	$\supseteq \mathrm{ROC}\{X_1(s)\} \cap \mathrm{ROC}\{X_2(s)\}$
Derivada	$x'(t)$	$sX(s)$	$\supseteq \mathrm{ROC}\{X(s)\}$
Integral	$\int_{-\infty}^{t} x(\tau)\,\mathrm{d}\tau$	$\frac{X(s)}{s}$	$\supseteq \mathrm{ROC}\{X(s)\} \cap \{\mathrm{Re}\, s > 0\}$
Multiplicación por t	$t\,x(t)$	$-X'(s)$	$\supseteq \mathrm{ROC}\{X(s)\}$

Casos particulares

$q = 1, 2, \ldots$; $p \in \mathbb{C}$; $\omega_0 \in \mathbb{R}$.

Señal	Transformada	ROC
$\delta(t - t_0)$	e^{-st_0}	$\mathbb{C}$
$e^{pt}u(t)$	$\dfrac{1}{s-p}$	$\operatorname{Re} s > \operatorname{Re} p$
$-e^{pt}u(-t)$	$\dfrac{1}{s-p}$	$\operatorname{Re} s < \operatorname{Re} p$
$\dfrac{t^{q-1}}{(q-1)!}e^{pt}u(t)$	$\dfrac{1}{(s-p)^q}$	$\operatorname{Re} s > \operatorname{Re} p$
$-\dfrac{t^{q-1}}{(q-1)!}e^{pt}u(-t)$	$\dfrac{1}{(s-p)^q}$	$\operatorname{Re} s < \operatorname{Re} p$
$e^{pt}\cos(\omega_0 t)u(t)$	$\dfrac{s-p}{(s-p)^2+\omega_0^2}$	$\operatorname{Re} s > \operatorname{Re} p$
$e^{pt}\sin(\omega_0 t)u(t)$	$\dfrac{\omega_0}{(s-p)^2+\omega_0^2}$	$\operatorname{Re} s > \operatorname{Re} p$

Valores inicial y final

$x(t) = 0$ para $t < 0$, sin impulsos en $t = 0$.

$$x(0^+) = \lim_{s\to+\infty} sX(s), \qquad \lim_{t\to\infty} x(t) = \lim_{s\to 0^+} sX(s).$$

Valor final, X racional: $\operatorname{Re} p < 0$ para todo polo p de $sX(s)$.

Sistemas racionales

$$H(s) = \frac{B(s)}{A(s)} = K\frac{\prod_m (s - z_m)}{\prod_\ell (s - p_\ell)}, \qquad H(\omega) = H(s)|_{s=j\omega}, \quad j\mathbb{R} \subseteq \mathrm{ROC}\{H(s)\}.$$

$$\text{Causal:} \quad \mathrm{ROC}\{H(s)\} = \{s : \operatorname{Re} s > \max_\ell \operatorname{Re} p_\ell\}.$$

$$\text{Estable:} \quad \deg B \le \deg A, \quad j\mathbb{R} \subseteq \mathrm{ROC}\{H(s)\}.$$

$$\text{Causal y estable:} \quad \deg B \le \deg A, \quad \operatorname{Re} p_\ell < 0, \quad \mathrm{ROC}\{H(s)\} \text{ causal.}$$

B/A reducida: sin factores comunes.

Transformada Z

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad x[n] = \frac{1}{2\pi \mathrm{j}} \oint_{|z|=r} X(z) z^{n-1} \, \mathrm{d}z$$

$$\mathrm{ROC}\{X(z)\} = \{z : r_1 < |z| < r_2\}, \qquad r_1 < r < r_2, \quad \text{sentido antihorario.}$$

Propiedades

Operación	Señal	Transformada	ROC
Linealidad	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\supseteq \mathrm{ROC}\{X_1(z)\} \cap \mathrm{ROC}\{X_2(z)\}$
Desplazamiento	$x[n - n_0]$	$z^{-n_0} X(z)$	$\mathrm{ROC}\{X(z)\}$, salvo 0 e ∞
Exponencial	$a^n x[n]$	$X(z/a)$	$\lvert a\rvert r_1 < \lvert z\rvert < \lvert a\rvert r_2$
Inversión temporal	$x[-n]$	$X(1/z)$	$1/r_2 < \lvert z\rvert < 1/r_1$
Conjugación	$x^*[n]$	$X^*(z^*)$	$\mathrm{ROC}\{X(z)\}$
Convolución	$(x_1 * x_2)[n]$	$X_1(z)X_2(z)$	$\supseteq \mathrm{ROC}\{X_1(z)\} \cap \mathrm{ROC}\{X_2(z)\}$
Multiplicación por n	$n\,x[n]$	$-zX'(z)$	$\supseteq \mathrm{ROC}\{X(z)\}$
Diferencia	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	$\supseteq \mathrm{ROC}\{X(z)\}$, salvo 0 e ∞
Acumulación	$\sum_{m=-\infty}^{n} x[m]$	$\frac{X(z)}{1 - z^{-1}}$	$\supseteq \mathrm{ROC}\{X(z)\} \cap \{\lvert z\rvert > 1\}$

Casos particulares

$q = 1, 2, \ldots$; $a \in \mathbb{C}$; $r > 0$; $\Omega_0 \in \mathbb{R}$.

Señal	Transformada	ROC
$\delta[n]$	1	$\mathbb{C}$
$a^n u[n]$	$\dfrac{1}{1 - az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\dfrac{1}{1 - az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\binom{n+q-1}{q-1} a^n u[n]$	$\dfrac{1}{(1 - az^{-1})^q}$	$\lvert z\rvert > \lvert a\rvert$
$r^n \cos(\Omega_0 n) u[n]$	$\dfrac{1 - r\cos(\Omega_0) z^{-1}}{1 - 2r\cos(\Omega_0) z^{-1} + r^2 z^{-2}}$	$\lvert z\rvert > r$
$r^n \sin(\Omega_0 n) u[n]$	$\dfrac{r\sin(\Omega_0) z^{-1}}{1 - 2r\cos(\Omega_0) z^{-1} + r^2 z^{-2}}$	$\lvert z\rvert > r$

Valores inicial y final

$x[n] = 0$ para $n < 0$.

$$x[0] = \lim_{z\to\infty} X(z), \qquad \lim_{n\to\infty} x[n] = \lim_{z\to 1} (1 - z^{-1}) X(z).$$

Valor final, X racional: $\lvert p\rvert < 1$ para todo polo p de $(1 - z^{-1})X(z)$.

Sistemas racionales

$$H(z) = \frac{B(z)}{A(z)} = K\frac{\prod_m (z - z_m)}{\prod_\ell (z - p_\ell)}, \qquad H(\Omega) = H(z)\big|_{z=e^{j\Omega}}, \quad \{\lvert z\rvert = 1\} \subseteq \mathrm{ROC}\{H(z)\}.$$

Causal: $\mathrm{ROC}\{H(z)\} = \{z : \lvert z\rvert > \max_\ell \lvert p_\ell\rvert\}, \quad \infty \in \mathrm{ROC}\{H(z)\}.$

Estable: $\{\lvert z\rvert = 1\} \subseteq \mathrm{ROC}\{H(z)\}.$

Causal y estable: $\lvert p_\ell\rvert < 1, \quad \mathrm{ROC}\{H(z)\}$ causal. (B/A reducida: sin factores comunes)